Linear Regression

Bike Sharing Assignment

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Assignment-Based Subjective Questions

Q1: From your analysis of the categorical variables from the dataset, what could you infer about their effect on the dependent variable?

Observation for categorical variables:

- The year box plots indicates that more bikes are rent during 2019.
- The season box plots indicates that more bikes are rent during fall season.
- The working day and holiday box plots indicate that more bikes are rent during normal working days than on weekends or holidays.
- The month box plots indicates that more bikes are rent during september month.
- The weekday box plots indicates that more bikes are rent during saturday.
- The weathersit box plots indicates that more bikes are rent during Clear, Few clouds, Partly cloudy weather.

Q2: Why is it important to use drop_first=True during dummy variable creation?

drop_first=True helps in reducing the extra column created during dummy variable creation. Hence, it reduces the correlations created among dummy variables.

If there is a column which has 10 unique categorical values or labels, using pd.getdummies() we convert them into a binary vector which makes 10 columns, one column for each unique value of our original column and wherever this value is true for a row it is indicated as 1 else 0.

if drop_first is true it removes the first column which is created for the first unique value of a column.

Q3: Looking at the pair-plot among the numerical variables, which one has the highest correlation with the target variable?

By looking at the pair plot **temp variable has the highest (0.63) correlation** with target variable **'cnt'**.

Q4: How did you validate the assumptions of Linear Regression after building the model on the training set?

We use following methods to validate the Linear regression assumptions:

Assumption 1: Linear relationship between target and feature variables.

We create scatter plots to analyse the linear relationship .

Assumption 2: Autocorelation in residuals

We use Durbin Watson test, in which if:
0 < DW < 2 : positive autocorrelation
2 < DW < 4 : negative autocorrelation

Assumption 3: No Heteroskedasticity

Again, we use scatter plot to analyze if the pattern is a funnel-like patter or not. If most of the points are centered around Zero, so we do not have any heteroskedasticity.

Assumption 4: No Multicollinearity

Here, we check VIF value. If VIF value is low, there is no multicollinearity.

Assumption 5: Residuals must be normally distributed

We create a normal distribution plot for residuals and make sure the residuals are normally distributed.

Q5: Based on the final model, which are the top 3 features contributing significantly towards explaining the demand of the shared bikes?

The Top 3 features contributing significantly towards the demands of share bikes are:

1. weathersit_Light_Snow(negative correlation).
2. yr_2019(Positive correlation).
3. temp(Positive correlation).

General Subjective Questions

Q1 : Explain the linear regression algorithm in detail.

Linear Regression is a machine learning algorithm which is based on **supervised learning** category. It finds a best linear-fit relationship on any given data, between independent (Target) and dependent (Predictor) variables. In other words, it creates the best straight-line fitting to the provided data to find the best linear relationship between the independent and dependent variables. Mostly it uses Sum of Squared Residuals Method.

Linear regression is of the 2 types:

1. Simple Linear Regression
2. Multiple Linear Regression

Simple Linear Regression:

It explains the relationship between a dependent variable and only one independent variable using a straight line. The straight line is plotted on the scatter plot of these two points.

Formula for the Simple Linear Regression:

$$Y=\beta0+\beta1X1 +\epsilon$$

Multiple Linear Regression:

It shows the relationship between one dependent variable and several independent variables. The objective of multiple regression is to find a linear equation that can best determine the value of dependent variable Y for different values independent variables in X. It fits a ‘hyperplane’ instead of a straight line.

Formula for the Multiple Linear Regression:

$$Y=\beta0+\beta1X1+\beta2X2+\ldots+\beta pXp+\epsilon$$

The equation of the best fit regression line $Y = \beta_0 + \beta_1 X$ can be found by the following two methods:

· Differentiation

· Gradient descent

We can use **statsmodels** or **SKLearn** libraries in python for the linear regression.

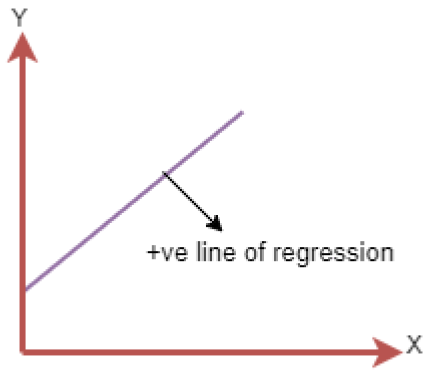
Linear Regression Line

A linear line showing the relationship between the dependent and independent variables is called a regression line. A regression line can show two types of relationship:

1. Positive Linear Relationship:
2. Negative Linear Relationship:

Positive Linear Relationship:

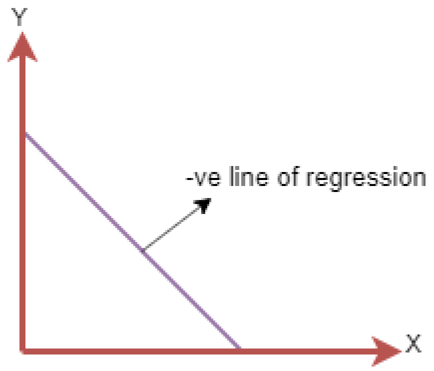
If the dependent variable increases on the Y-axis and independent variable increases on X-axis, then such a relationship is termed as a Positive linear relationship.



The line equation will be: **$Y= a_0+a_1x$**

Negative Linear Relationship:

If the dependent variable decreases on the Y-axis and independent variable increases on the X-axis, then such a relationship is called a negative linear relationship.



The line of equation will be: **$Y = -a_0 + a_1 x$**

Finding the best fit line:

When working with linear regression, our main goal is to find the best fit line that means the error between predicted values and actual values should be minimized. The best fit line will have the least error.

The different values for weights or the coefficient of lines (a_0, a_1) gives a different line of regression, so we need to calculate the best values for a_0 and a_1 to find the best fit line, so to calculate this we use cost function.

Cost function

- The different values for weights or coefficient of lines (a_0, a_1) gives the different line of regression, and the cost function is used to estimate the values of the coefficient for the best fit line.
- Cost function optimizes the regression coefficients or weights. It measures how a linear regression model is performing.
- We can use the cost function to find the accuracy of the mapping function, which maps the input variable to the output variable. This mapping function is also known as Hypothesis function.

For Linear Regression, we use the Mean Squared Error (MSE) cost function, which is the average of squared error occurred between the predicted values and actual values. It can be written as:
For the above linear equation, MSE can be calculated as:

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$$

Where,

N=Total number of observation

Yi = Actual value

$(a1x_i+a_0)$= Predicted value.

Residuals:

The distance between the actual value and predicted values is called residual. If the observed points are far from the regression line, then the residual will be high, and so cost function will high. If the scatter points are close to the regression line, then the residual will be small and hence the cost function.

Gradient Descent:

- Gradient descent is used to minimize the MSE by calculating the gradient of the cost function.
- A regression model uses gradient descent to update the coefficients of the line by reducing the cost function.
- It is done by a random selection of values of coefficient and then iteratively update the values to reach the minimum cost function.

Model Performance:

The Goodness of fit determines how the line of regression fits the set of observations. The process of finding the best model out of various models is called **optimization**. It can be achieved by below method:

R-squared method:

- R-squared is a statistical method that determines the goodness of fit.
- It measures the strength of the relationship between the dependent and independent variables on a scale of 0-100%.

- The high value of R-square determines the less difference between the predicted values and actual values and hence represents a good model.
- It is also called a coefficient of determination, or coefficient of multiple determination for multiple regression.
- It can be calculated from the below formula:

$$\text{R-squared} = \frac{\text{Explained variation}}{\text{Total Variation}}$$

Assumptions of Linear Regression

Below are some important assumptions of Linear Regression. These are some formal checks while building a Linear Regression model, which ensures to get the best possible result from the given dataset.

- **Linear relationship between the features and target:**
  Linear regression assumes the linear relationship between the dependent and independent variables.

- **Small or no multicollinearity between the features:**
  Multicollinearity means high-correlation between the independent variables. Due to multicollinearity, it may difficult to find the true relationship between the predictors and target variables. Or we can say, it is difficult to determine which predictor variable is affecting the target variable and which is not. So, the model assumes either little or no multicollinearity between the features or independent variables.

- **Homoscedasticity Assumption:**
  Homoscedasticity is a situation when the error term is the same for all the values of independent variables. With homoscedasticity, there should be no clear pattern distribution of data in the scatter plot.

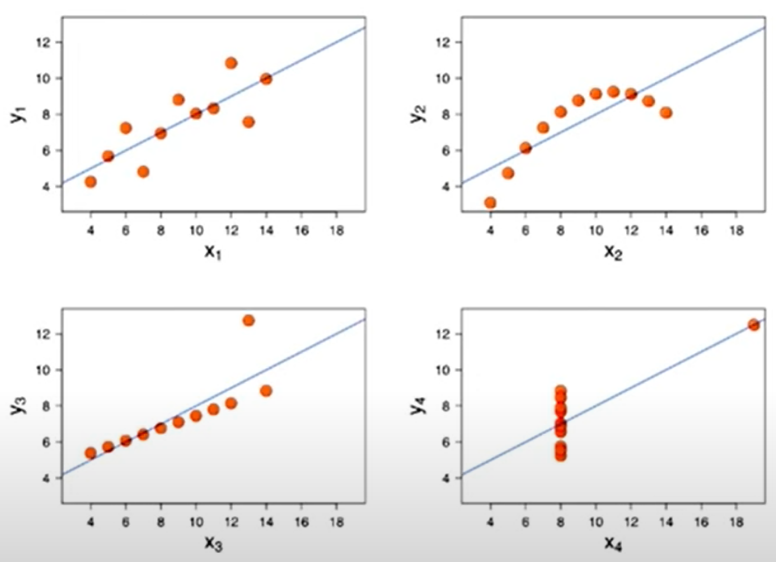
- **Normal distribution of error terms:**
  Linear regression assumes that the error term should follow the normal distribution pattern. If error terms are not normally distributed, then confidence intervals will become either too wide or too narrow, which may cause difficulties in finding coefficients.
  It can be checked using the q-q plot. If the plot shows a straight line without any deviation, which means the error is normally distributed.

- **No autocorrelations:**
  The linear regression model assumes no autocorrelation in error terms. If there will be any correlation in the error term, then it will drastically reduce the accuracy of the model. Autocorrelation usually occurs if there is a dependency between residual errors.

Q2: Explain the Anscombe's quartet in detail.

Anscombe's quartet comprises four data sets that have nearly identical simple descriptive statistics, yet have very different distributions and appear very different when graphed.

I		II		III		IV	
x	y	x	y	x	y	x	y
10.0	8.04	10.0	9.14	10.0	7.46	8.0	6.58
8.0	6.95	8.0	8.14	8.0	6.77	8.0	5.76
13.0	7.58	13.0	8.74	13.0	12.74	8.0	7.71
9.0	8.81	9.0	8.77	9.0	7.11	8.0	8.84
11.0	8.33	11.0	9.26	11.0	7.81	8.0	8.47
14.0	9.96	14.0	8.10	14.0	8.84	8.0	7.04
6.0	7.24	6.0	6.13	6.0	6.08	8.0	5.25
4.0	4.26	4.0	3.10	4.0	5.39	19.0	12.50
12.0	10.84	12.0	9.13	12.0	8.15	8.0	5.56
7.0	4.82	7.0	7.26	7.0	6.42	8.0	7.91
5.0	5.68	5.0	4.74	5.0	5.73	8.0	6.89



Anscombe's Quartet can be defined as **a group of four data sets which are nearly identical in simple descriptive statistics, but there are some peculiarities in the dataset that fools the regression model if built.** They have very different distributions and appear differently when plotted on scatter plots.

It was constructed in 1973 by statistician Francis Anscombe to illustrate the **importance of plotting the graphs before analyzing and model building**, and the effect of other observations on statistical properties.There are these four data set plots which have nearly same statistical observations, which provides same statistical information that involves variance, and mean of all x,y points in all four datasets.

This tells us about the importance of visualising the data before applying various algorithms out there to build models out of them which suggests that the data features must be plotted in order to see the distribution of the samples that can help you identify the various anomalies present in the data like outliers, diversity of the data, linear separability of the data, etc. Also, the Linear

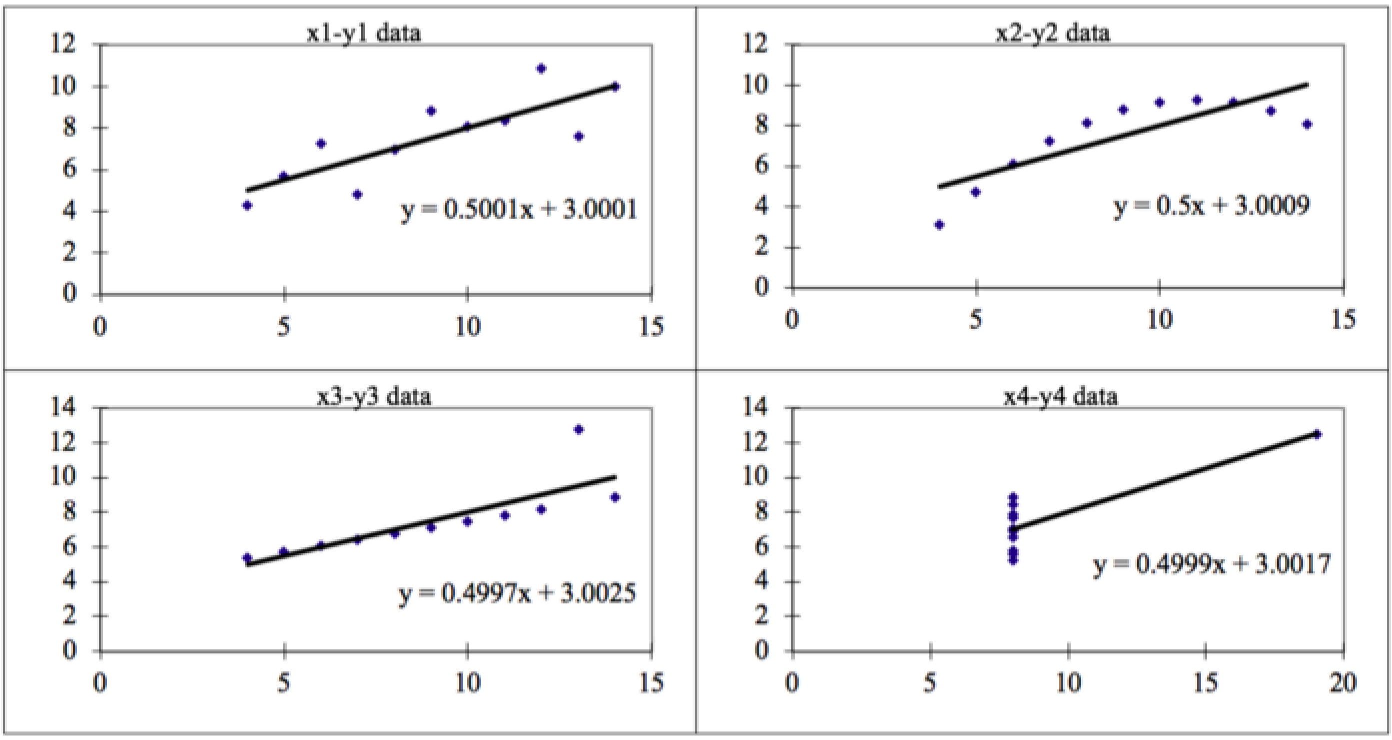
Regression can be only be considered a fit for the data with linear relationships and is incapable of handling any other kind of datasets. These four plots can be defined as follows:

Anscombe's Data											
Observation	x1	y1		x2	y2		x3	y3		x4	y4
1	10	8.04		10	9.14		10	7.46		8	6.58
2	8	6.95		8	8.14		8	6.77		8	5.76
3	13	7.58		13	8.74		13	12.74		8	7.71
4	9	8.81		9	8.77		9	7.11		8	8.84
5	11	8.33		11	9.26		11	7.81		8	8.47
6	14	9.96		14	8.1		14	8.84		8	7.04
7	6	7.24		6	6.13		6	6.08		8	5.25
8	4	4.26		4	3.1		4	5.39		19	12.5
9	12	10.84		12	9.13		12	8.15		8	5.56
10	7	4.82		7	7.26		7	6.42		8	7.91
11	5	5.68		5	4.74		5	5.73		8	6.89

The statistical information for all these four datasets are approximately similar and can be computed as follows:

Anscombe's Data											
Observation	x1	y1		x2	y2		x3	y3		x4	y4
1	10	8.04		10	9.14		10	7.46		8	6.58
2	8	6.95		8	8.14		8	6.77		8	5.76
3	13	7.58		13	8.74		13	12.74		8	7.71
4	9	8.81		9	8.77		9	7.11		8	8.84
5	11	8.33		11	9.26		11	7.81		8	8.47
6	14	9.96		14	8.1		14	8.84		8	7.04
7	6	7.24		6	6.13		6	6.08		8	5.25
8	4	4.26		4	3.1		4	5.39		19	12.5
9	12	10.84		12	9.13		12	8.15		8	5.56
10	7	4.82		7	7.26		7	6.42		8	7.91
11	5	5.68		5	4.74		5	5.73		8	6.89
					Summary Statistics						
N	11	11		11	11		11	11		11	11
mean	9.00	7.50		9.00	7.500909		9.00	7.50		9.00	7.50
SD	3.16	1.94		3.16	1.94		3.16	1.94		3.16	1.94
r	0.82			0.82			0.82			0.82	

When these models are plotted on a scatter plot, all datasets generates a different kind of plot that is not interpretable by any regression algorithm which is fooled by these peculiarities and can be seen as follows:



The four datasets can be described as:

1. **Dataset 1:** this fits the linear regression model pretty well.
2. **Dataset 2:** this could not fit linear regression model on the data quite well as the data is non-linear.
3. **Dataset 3:** shows the outliers involved in the dataset which cannot be handled by linear regression model.
4. **Dataset 4:** shows the outliers involved in the dataset which cannot be handled by linear regression model.

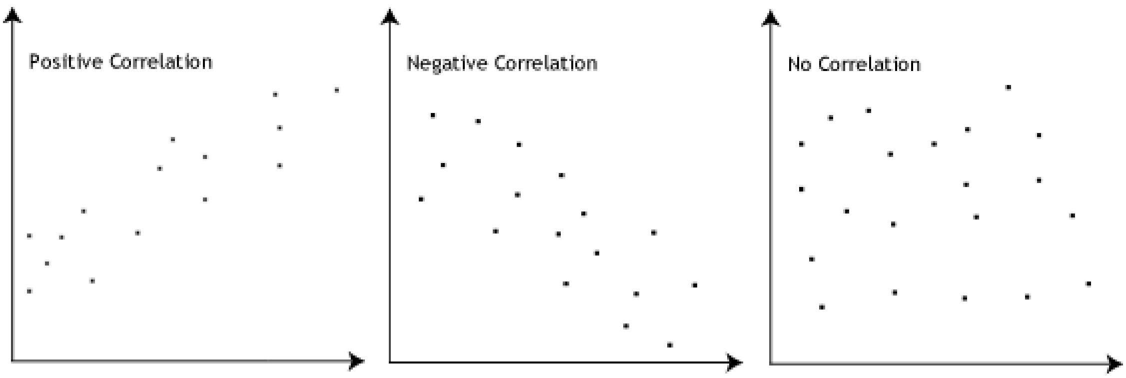
Hence, all the important features in the dataset must be visualised before implementing any machine learning algorithm on them which will help to make a good fit model.

Q3: What is Pearson's R?

In statistics, the Pearson correlation coefficient (PCC), also referred to as Pearson's r, the Pearson product-moment correlation coefficient (PPMCC), or the bivariate correlation, is a measure of linear correlation between two sets of data. It is the covariance of two variables, divided by the product of their standard deviations; thus it is essentially a normalised measurement of the covariance, such that the result always has a value between −1 and 1.

The Pearson's correlation coefficient varies between -1 and +1 where:

- r = 1 means the data is perfectly linear with a positive slope (i.e., both variables tend to change in the same direction)
- r = -1 means the data is perfectly linear with a negative slope (i.e., both variables tend to change in different directions)
- r = 0 means there is no linear association
- r > 0 < 5 means there is a weak association
- r > 5 < 8 means there is a moderate association
- r > 8 means there is a strong association



PEARSON'S FORMULA

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}}$$

Here,

- r = correlation coefficient
- x_i = values of the x-variable in a sample
- $\bar{x}$ = mean of the values of the x-variable

- y_i = values of the y-variable in a sample
- $\bar{y}$ = mean of the values of the y-variable

Q4: What is scaling? Why is scaling performed? What is the difference between normalized scaling and standardized scaling?

Scaling is a step of data Pre-Processing which is applied to independent variables to normalize the data within a particular range. It also helps in speeding up the calculations in an algorithm.

Why is scaling performed?
Most of the times, collected data set contains features highly varying in magnitudes, units and range. If scaling is not done then algorithm only takes magnitude in account and not units hence incorrect modelling. To solve this issue, we have to do scaling to bring all the variables to the same level of magnitude.

It is important to note that scaling just affects the coefficients and none of the other parameters like t-statistic, F-statistic, p-values, R-squared, etc.

Normalized Scaling:
It brings all of the data in the range of 0 and 1. **sklearn.preprocessing.MinMaxScaler** helps to implement normalization in python.

$$\text{MinMax Scaling: } x = \frac{x - min(x)}{max(x) - min(x)}$$

Standardized Scaling:
Standardization replaces the values by their Z scores. It brings all of the data into a standard normal distribution which has mean (μ) zero and standard deviation one (σ).

$$\text{Standardisation: } x = \frac{x - mean(x)}{sd(x)}$$

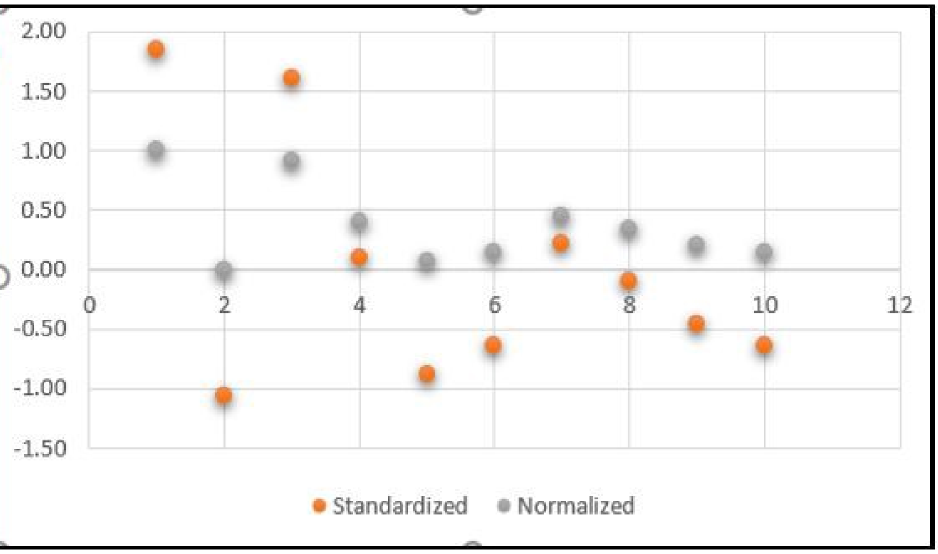
sklearn.preprocessing.scale helps to implement standardization in python.

One disadvantage of normalization over standardization is that it loses some information in the data, especially about outliers.

Example:

Below shows example of Standardized and Normalized scaling on original values.

Original	Standardized	Normalized
59	1.85	1.000
11	-1.06	0.000
55	1.61	0.917
30	0.09	0.396
14	-0.88	0.063
18	-0.64	0.146
32	0.21	0.438
27	-0.09	0.333
21	-0.46	0.208
18	-0.64	0.146



Difference b/w Normalization and Standardization scaling

#	Normalization	Standardization
1	Minimum and maximum value of features are used for scaling	Mean and standard deviation is used for scaling.
2	It is used when features are of different scales.	It is used when we want to ensure zero mean and unit standard deviation.
3	Scales values between [0, 1] or [-1, 1].	It is not bounded to a certain range.
4	It is really affected by outliers.	It is much less affected by outliers.
5	Scikit-Learn provides a transformer called MinMaxScaler for Normalization.	Scikit-Learn provides a transformer called StandardScaler for standardization.
6	This transformation squishes the n-dimensional data into an n-dimensional unit hypercube.	It translates the data to the mean vector of original data to the origin and squishes or expands.

7	It is useful when we don’t know about the distribution	It is useful when the feature distribution is Normal or Gaussian.
8	It is a often called as Scaling Normalization	It is a often called as Z-Score Normalization.

Q5: You might have observed that sometimes the value of VIF is infinite. Why does this happen?

The value of VIF is calculated by the below formula:

$$VIF_i = \frac{1}{1-{R_i}^2}$$

Where, ‘i’ refers to the ith variable.
If R-squared value is equal to 1 then the denominator of the above formula become 0 and the overall value become infinite. It denotes perfect correlation in variables.

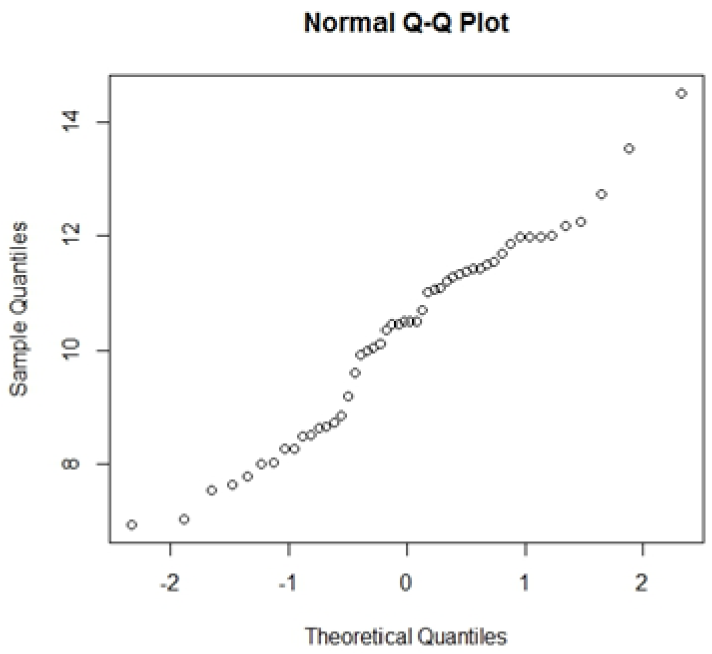
Hence, If there is perfect correlation, then VIF = infinity. This shows a perfect correlation between two independent variables. In the case of perfect correlation, we get R2 =1, which lead to 1/(1-R2) infinity. To solve this problem we need to drop one of the variables from the dataset which is causing this perfect multicollinearity.

An infinite VIF value indicates that the corresponding variable may be expressed exactly by a linear combination of other variables (which show an infinite VIF as well).

Q6: What is a Q-Q plot? Explain the use and importance of a Q-Q plot in linear regression

The Q-Q plot or quantile-quantile plot is a graphical technique for determining if two data sets come from populations with a common distribution.
A Q-Q plot is a scatterplot created by plotting two sets of quantiles against one another. If both sets of quantiles came from the same distribution, we should see the points forming a line that’s

roughly straight. Here’s an example of a Normal Q-Q plot when both sets of quantiles truly come from Normal distributions.



Use of Q-Q plot in Linear Regression:

The Q-Q plot is used to see if the points lie approximately on the line. If they don’t, it means, our residuals aren’t Gaussian (Normal) and thus, our errors are also not Gaussian.

Importance of Q-Q plot: Below are the points:

1. The sample sizes do not need to be equal.

2. Many distributional aspects can be simultaneously tested. For example, shifts in location, shifts in scale, changes in symmetry, and the presence of outliers.

3. The q-q plot can provide more insight into the nature of the difference than analytical methods.