

Hand-on-12

a) Aggregate method

⇒ In the aggregate method, we analyze the total cost for a seq of operations and then divide it by the no of operations to get the average cost (amortized cost)

1. Inserting an element without resizing for each insertion where there is enough space in the table the cost is constant $O(1)$

2. Resizing: every time the table doubles in size the cost is proportional to the no of elements being copied to the new table if the table size is k before doubling, copying all k elements takes $O(k)$ time

Total cost of n insertions
⇒ The cost of inserting n elements $O(n)$ for insertion

⇒ The cost of resizing the first doubling involves copying 1 element the next involves copying 2, then 4, and so on

⇒ The sum of these doubling operations
 $O(1 + 2 + 4 + 8 + \dots + 2^{n-1}) \sim O(n)$

→ Amortized cost per Insertion = $O(n) + O(n)$
So, using aggregate method the amortized
time complexity for inserting n elements is
 $O(1)$

b) Accounting method

⇒ The accounting method assigns "credit"
to each operation to account for the cost
of future expensive operations

↳ Assigning credit

each insertion will be charged 3 credit
⇒ 2 credit for the insertion itself, which
pays constant time $O(1)$ operations
⇒ 1 credit to help for the cost of future
resizing operations

2. cost of insertion

⇒ When no resizing happens, the cost
is exactly 1 credit for the insertion

→ When a resizing happens if cost $O(n)$
for copying K elements but since we
have credit saved for each previous
insertion we have enough credit to
cover the resizing

Resizing & cost

- When the table doubled the cost of copying element doubled as well
- The total no of credits that we collect is 3 credits per insertion full n insertions resulting in $3n$ credits
- Each resizing is covered by the second credit. The total no of resizing operations is proportional to the no of doublings (about $\log n$ times)

Final Amortized cost

- Inserting n element cost 1 credit each
- Total credits collected = $3n$
- Cost of each resizing operation is already covered by saved credits

Amortized cost per insertion

$$= \frac{3n}{n}$$

$$= O(1)$$

∴ $O(1)$ is the amortized time complexity for inserting n element using accounting method for a dynamic table that doubles