3) mathematically derive the average runtime complexity of the non-random pivot version of Quicksort

A:- Recurrance relation is

$$T(n) - T(k) + T(n-k-1) + O(n)$$

here

$T(n)$ = Time complexity of Quicksort for an array of size n

K = number of element in the left subarray

$n-K-1$ = no of element in right array

$O(n)$ = represents time array

considering Average case

$$T(n) = 2T(n/2) + O(n)$$

here $2T(n/2)$ represents avg time for recursively

so $T(n) = O(n) + 2 \cdot T(n/2)$

$T(n) = O(n) + 2\left(O\left(\frac{n}{2}\right) + 2 \cdot T(n/4)\right)$

$T(n) = O(n) + 2\left(O\left(\frac{n}{2}\right)\right) + 4 \cdot T(n/4)$

$T(n) =$

$\therefore T(n) = k \cdot O\left(\frac{n}{2^k}\right) + 2^k \cdot T\left(\frac{n}{2^k}\right)$

Therefore, Runtime complexity of Quick sort for Average case is $O(n\log n)$

$$T(n) = O(n \log n)$$