Time Series Lab and assignment

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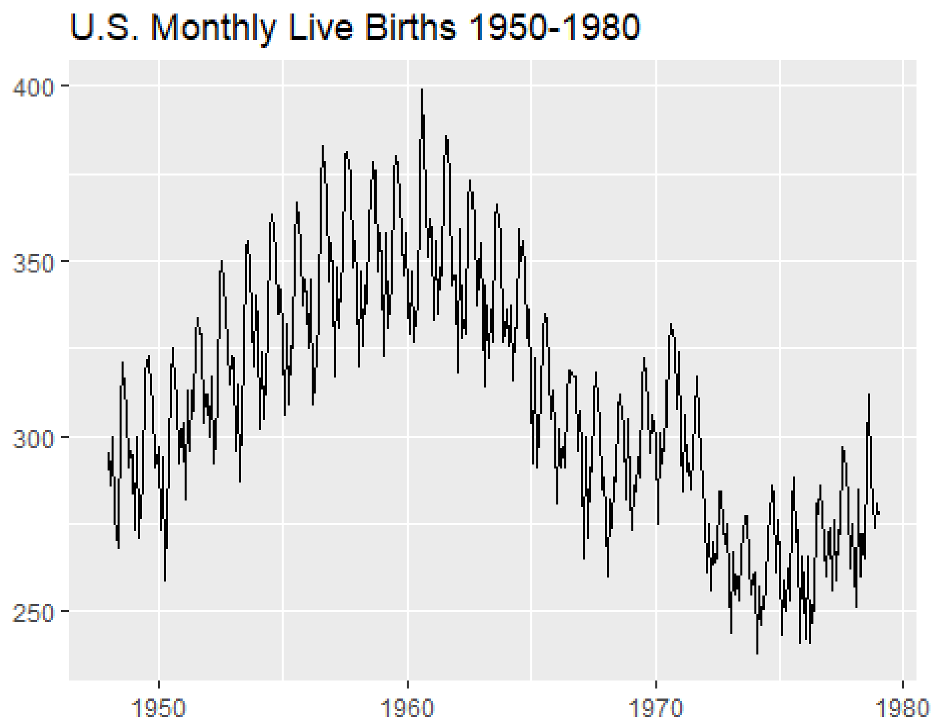
TimeSeries Lab & Assignment

Dataset: "birth" from libraty astsa, U.S. Monthly Live Births 1950-1980

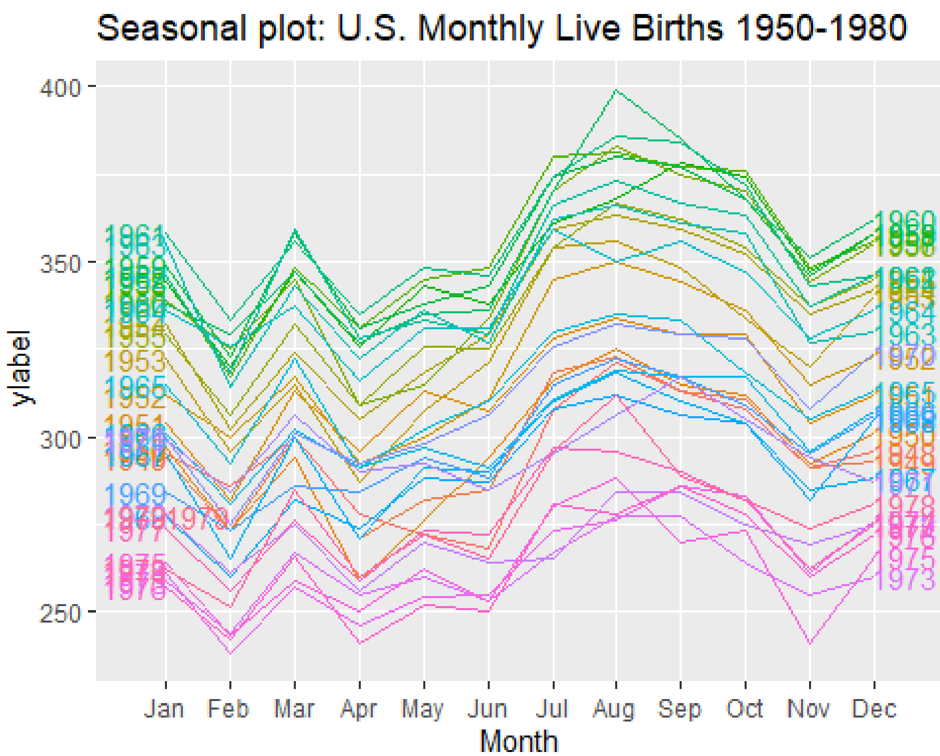
```
library(astsa)
data(birth)

#plot(birth)
library(ggplot2)
library(ggfortify)
library(dplyr)

##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union

birth %>%
  autoplot() + ggtitle("U.S. Monthly Live Births 1950-1980")
```



A seasonal plot is similar to a time plot except that the data are plotted against the individual “seasons” in which the data were observed.

```
library(forecast)

## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo

## Registered S3 methods overwritten by 'forecast':
##   method                 from
##   autoplot.Arima         ggfortify
##   autoplot.acf           ggfortify
##   autoplot.ar            ggfortify
##   autoplot.bats          ggfortify
##   autoplot.decomposed.ts ggfortify
##   autoplot.ets           ggfortify
##   autoplot.forecast      ggfortify
##   autoplot.stl           ggfortify
##   autoplot.ts            ggfortify
##   fitted.ar              ggfortify
##   fortify.ts             ggfortify
##   residuals.ar           ggfortify

##
## Attaching package: 'forecast'
```

```
## The following object is masked from 'package:astsa':
##
##     gas

ggseasonplot(birth, year.labels=TRUE, year.labels.left=TRUE) +
  ylab(" ylabel") +
  ggtitle("Seasonal plot: U.S. Monthly Live Births 1950-1980")
```



We are going to try a few things to get a feeling about the cyclical nature of the dataset.

There seems to be a yearly cycle. We can try adding monthly variables or use a sin and/or cosing with the right frequency for a year repetition.

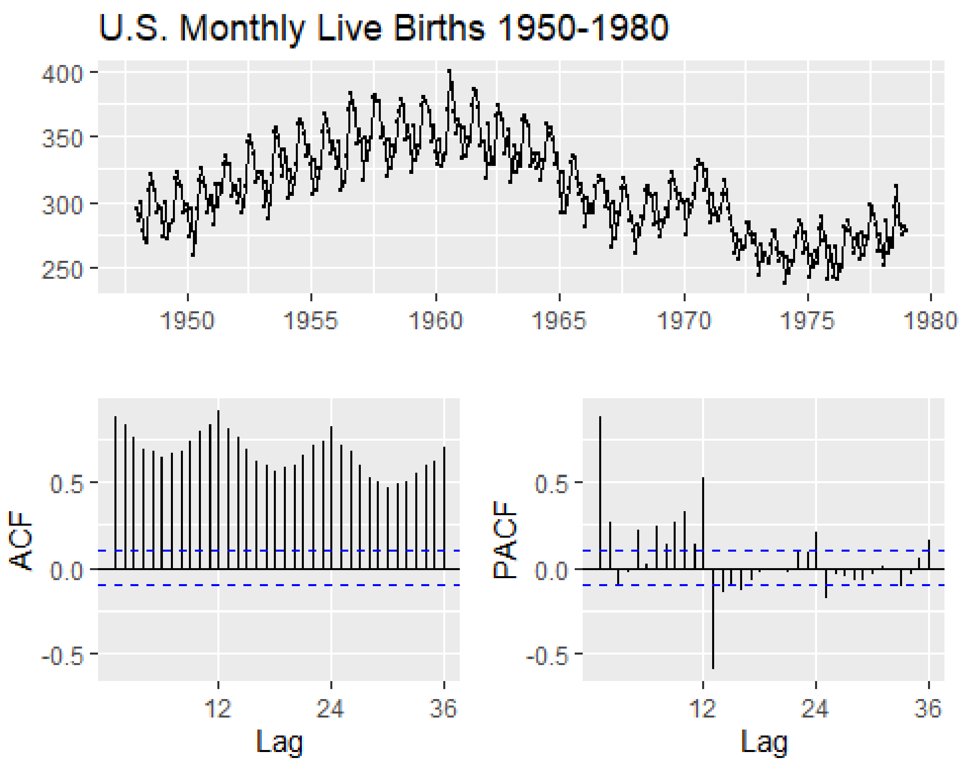
Note: I added numbers to the names of the month because otherwise r will order them alphabetically.

```
n<-length(birth)
#n=373, n/12 = 31.08
month<-
rep(c("01Jan","02Feb","03Mar","04Apr","05May","06Jun","07Jul","08Aug","09Sep"
,"10Oct","11Nov","12Dec"),32)[1:n]
times<-1:n

# we won't use all the monthly dummy variables because Jan = when all other
are 0
#X<-as.data.frame(cbind(times,Feb,Mar,Apr,May,Jun,Jul,Aug,Sep,Oct,Nov,Dec))
X<-data.frame(times=times,month=month)
```

```r
# alternatively, seasons can be created with sin, cos
sint=sin(2*pi*times/12)
cost=cos(2*pi*times/12)
X_jan=data.frame(times=times,sint=sint,cost=cost,Jan=rep(c(1,0,0,0,0,0,0,0,0,
0,0,0),32)[1:n])
```

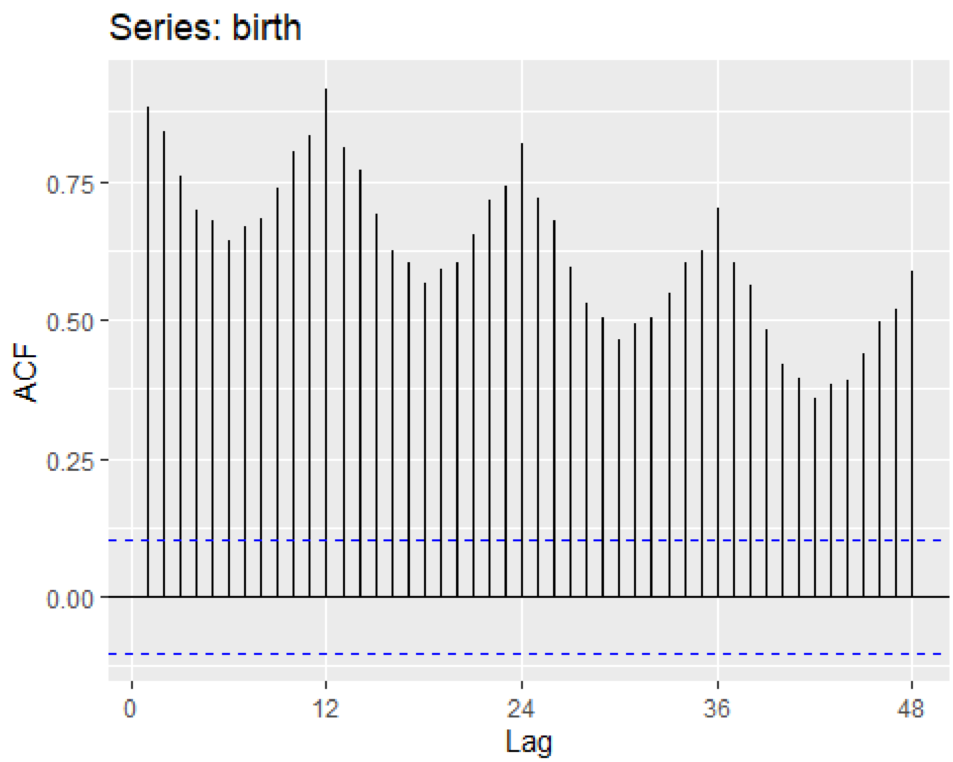
Let's look at the auto correlation function and partial auto correlation functions

```r
#acf(birth)
#pacf(birth)
birth %>% ggtsdisplay(main="U.S. Monthly Live Births 1950-1980")
```

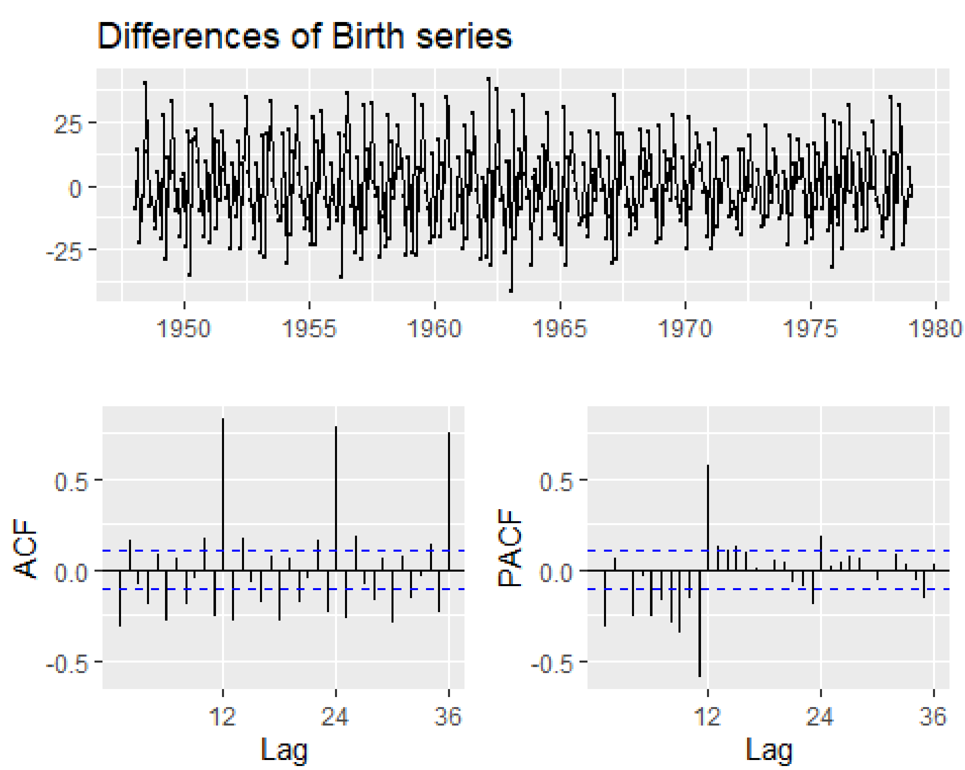


Ussing ggplot2

```r
ggAcf(birth,lag=48) # default is lag=24
```

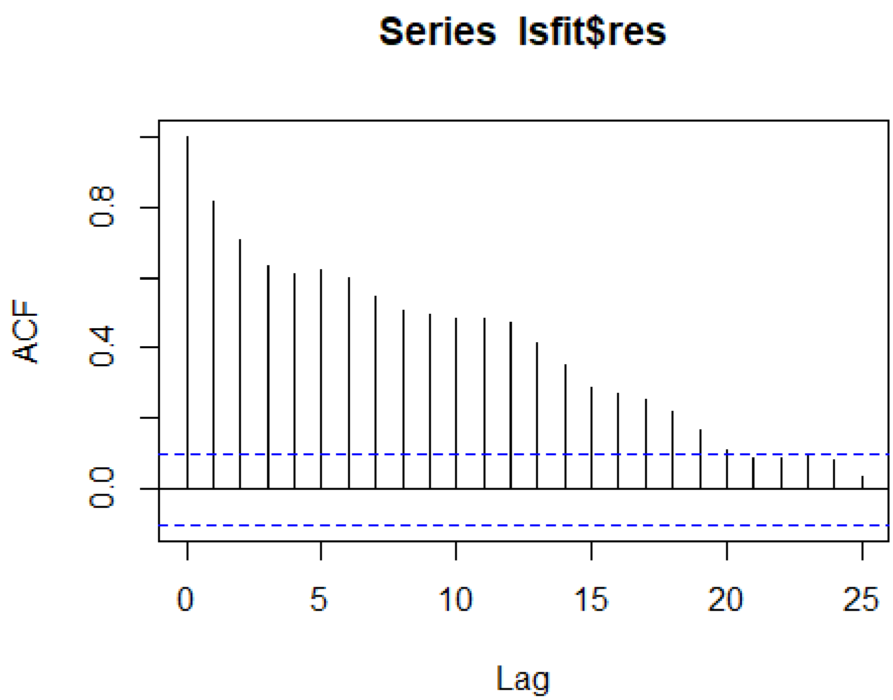



```r
#acf(diff(birth,1))
birth %>% diff() %>% ggtsdisplay(main="Differences of Birth series")
```



Let's fit a model with monthly dummy variables. There is a curve trend that is beyond quadratic.

```
lsfit=lm(birth~poly(times,3)+month,
 #          Feb+Mar+Apr+May+Jun+Jul+Aug+Sep+Oct+Nov+Dec,
         data=X)
summary(lsfit)

##
## Call:
## lm(formula = birth ~ poly(times, 3) + month, data = X)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -30.806  -8.521  -1.008   9.051  41.496
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)        307.478      2.164 142.119  < 2e-16 ***
## poly(times, 3)1 -356.462     12.241 -29.120  < 2e-16 ***
## poly(times, 3)2 -369.891     12.239 -30.222  < 2e-16 ***
## poly(times, 3)3  245.762     12.247  20.066  < 2e-16 ***
## month02Feb        -20.826      3.084  -6.752 5.90e-11 ***
## month03Mar          2.731      3.084   0.885   0.3766
## month04Apr        -17.837      3.084  -5.783 1.60e-08 ***
## month05May         -6.853      3.084  -2.222   0.0269 *
## month06Jun         -6.284      3.084  -2.038   0.0423 *
## month07Jul         19.869      3.084   6.442 3.79e-10 ***
## month08Aug         27.219      3.084   8.826  < 2e-16 ***
## month09Sep         23.154      3.084   7.507 4.84e-13 ***
## month10Oct         16.705      3.084   5.416 1.12e-07 ***
## month11Nov         -2.998      3.084  -0.972   0.3316
## month12Dec          6.398      3.084   2.074   0.0388 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 12.24 on 358 degrees of freedom
## Multiple R-squared:  0.884,  Adjusted R-squared:  0.8795
## F-statistic: 194.9 on 14 and 358 DF,  p-value: < 2.2e-16

acf(lsfit$res)
```



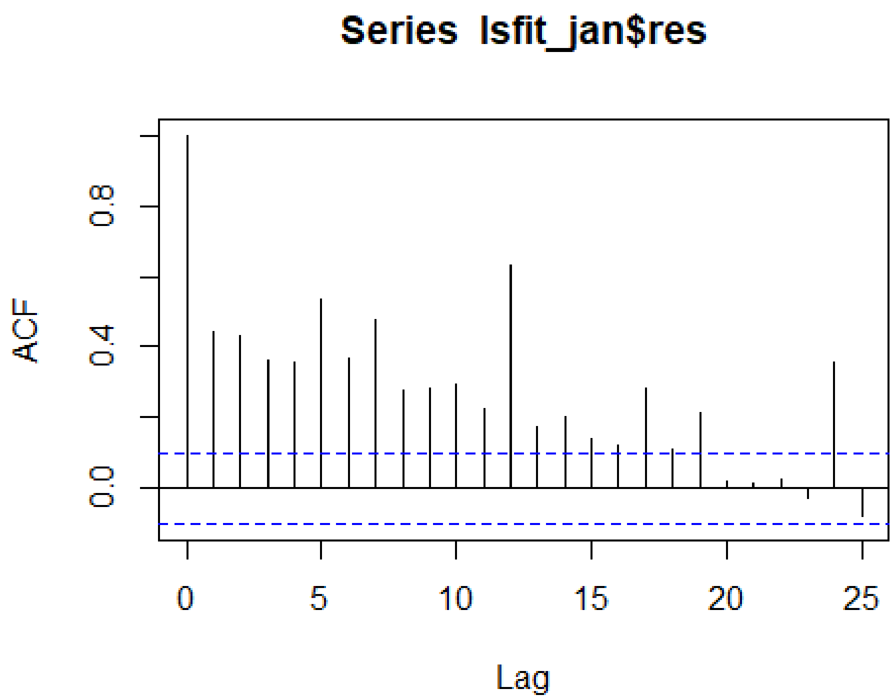
Although this looks like a good fit, we see that the residuals have autocorrelation.

Let’s also fit a model with sin and cos to model cyclical nature.

```
lsfit_jan=lm(birth~poly(times,3)+sint+cost+Jan,data=X_jan) #you remove
sin/cos and do all months
summary(lsfit_jan)

##
## Call:
## lm(formula = birth ~ poly(times, 3) + sint + cost + Jan, data = X_jan)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -33.58 -11.16  -1.32  10.30  48.34
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)        310.2118     0.8069 384.462  < 2e-16 ***
## poly(times, 3)1 -356.6168    14.7680 -24.148  < 2e-16 ***
## poly(times, 3)2 -369.8896    14.7665 -25.049  < 2e-16 ***
## poly(times, 3)3  245.5296    14.7736  16.620  < 2e-16 ***
## sint              -18.0085     1.1130 -16.181  < 2e-16 ***
## cost               -2.5458     1.1695  -2.177  0.03013 *
## Jan                 8.4756     3.0262   2.801  0.00537 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 14.76 on 366 degrees of freedom
## Multiple R-squared:  0.8274, Adjusted R-squared:  0.8246
## F-statistic: 292.5 on 6 and 366 DF,  p-value: < 2.2e-16
```

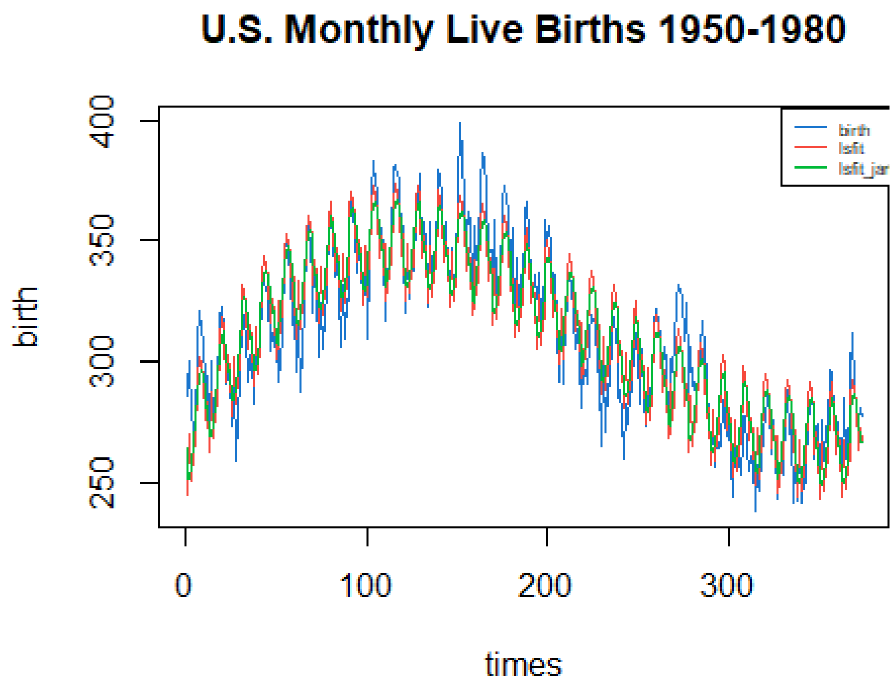
```
acf(lsfit_jan$res)
```



Same problem with this model, we also see that the residuals still have autocorrelation.

Let’s plot both models:

```
plot(times,birth,type="l",main="U.S. Monthly Live Births 1950-1980",col=4)

lines(times,lsfit$fitted.values,col=2)
lines(times,lsfit_jan$fitted,col=3)
legend(329,405,c("birth","lsfit","lsfit_jan"),col=c(4,2,3),lty=1,cex=.5)
```




```
#df<-data.frame(fit=lsfit$fitted.values, times=times)
#df2<-data.frame(fit=lsfit_jan$fitted.values, times=times)
#birth %>%
#  autoplot(,col="darkgrey") +
#  ggtitle("U.S. Monthly Live Births 1950-1980") +
#  geom_line(data=df,aes(x=time(birth),y=fit),col=2)+
#  geom_line(data=df2,aes(x=time(birth),y=fit),col=3)
```

Which model performs better?

```
aic<-round(c(AIC(lsfit), AIC(lsfit_jan)),2)
bic<-round(c(BIC(lsfit), BIC(lsfit_jan)),2)
adjr2<-round(c(summary(lsfit)$ad,summary(lsfit_jan)$ad),2)
rbind(c("lsfit", "lsfit_jan"), aic,bic,adjr2)

##       [,1]      [,2]
##       "lsfit"   "lsfit_jan"
## aic   "2943.52" "3075.81"
## bic   "3006.26" "3107.19"
## adjr2 "0.88"    "0.82"
```

Now let's try the time series model with auto-regressive, integrated, moving averages and cyclic components:

```
library(forecast)
birthmod<-auto.arima(birth)
birthmod
```

```
## Series: birth
## ARIMA(0,1,2)(1,1,1)[12]
##
## Coefficients:
##           ma1      ma2    sar1     sma1
##       -0.3984  -0.1632  0.1018  -0.8434
## s.e.   0.0512   0.0486  0.0713   0.0476
##
## sigma^2 = 46.1:  log likelihood = -1204.93
## AIC=2419.86   AICc=2420.03   BIC=2439.29
```

The result is ARIMA(0,1,2)(1,1,1)[12] We also see the aic and the bic metrics and this model performed better that the ones we did earlier.

Equation corresponding to the time series model:

$$(I - sar1B^{12})(I - B^{12})(I - B)y_t = (I + sma1B^{12})(I + ma1B + ma2B^2)w_t$$

where $\{w_t\}$ are the random errors.

Plugging in the numbers:

$$(I - 0.1018B^{12})(I - B^{12})(I - B)y_t = (I - 0.8434B^{12})(I - 0.3984B - 0.1632B^2)w_t$$

Or

$$(I - B^{12})(I - B)y_t - 0.1018B^{12}(I - B^{12})(I - B)y_t$$
$$= (I - 0.3984B - 0.1632B^2)w_t - 0.8434B^{12}(I - 0.3984B - 0.1632B^2)w_t$$

$$(I - B^{12})(y_t - y_{t-1}) - 0.1018B^{12}(I - B^{12})(y_t - y_{t-1})$$
$$= (w_t - 0.3984w_{t-1} - 0.1632w_{t-2}) - 0.8434(w_{t-12} - 0.3984w_{t-13} - 0.1632w_{t-14})$$

$$(y_t - y_{t-1}) - (y_{t-12} - y_{t-13}) - 0.1018B^{12}\big((y_t - y_{t-1}) - (y_{t-12} - y_{t-13})\big)$$
$$= (w_t - 0.3984w_{t-1} - 0.1632w_{t-2}) - 0.8434(w_{t-12} - 0.3984w_{t-13} - 0.1632w_{t-14})$$

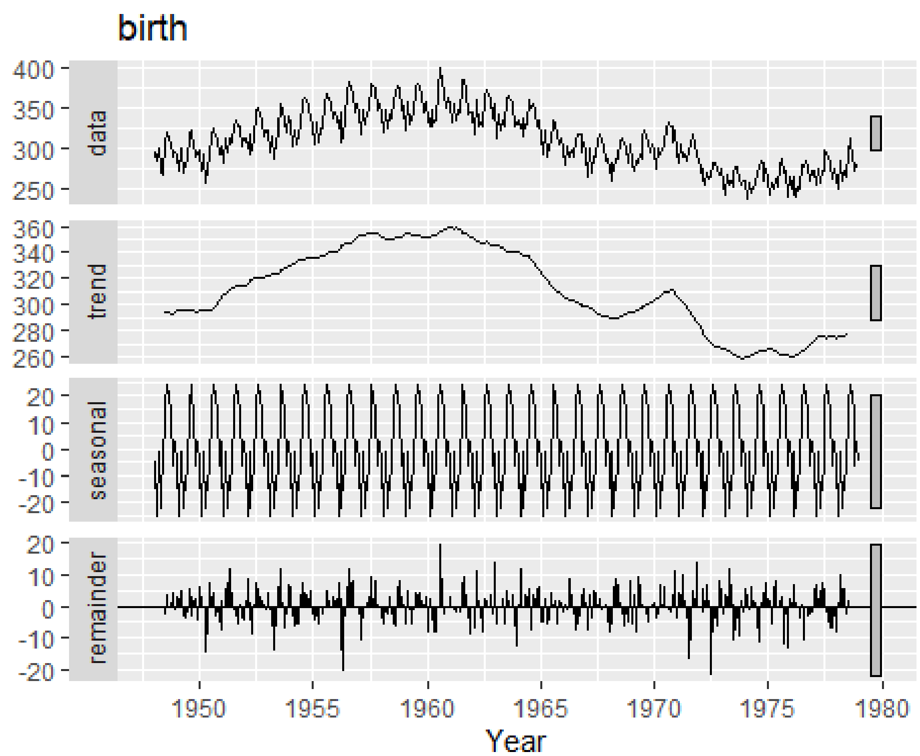
$$(y_t - y_{t-1}) - (y_{t-12} - y_{t-13}) - 0.1018\big((y_{t-12} - y_{t-13}) - (y_{t-24} - y_{t-25})\big)$$
$$= w_t - 0.3984w_{t-1} - 0.1632w_{t-2} - 0.8434w_{t-12} + 0.8434 * 0.3984w_{t-13} + 0.8434 * 0.1632w_{t-14}$$

$$(y_t$$
$$= y_{t-1} + (y_{t-12} - y_{t-13}) + 0.1018\big((y_{t-12} - y_{t-13}) - (y_{t-24} - y_{t-25})\big) + w_t - 0.3984w_{t-1} - 0.1632w_{t-2} - 0.8434w_{t-12} + 0.8434 * 0.3984w_{t-13} + 0.8434 * 0.1632w_{t-14}$$

We see that this is quite a complicated structure that captures a yearly cycle plus a 2 year cycle. That seems to account for the curved patterns we observed in the plot of the values.

Let’s see the decomposition of the cycles:

```r
birth %>% decompose() %>%
  autoplot() + xlab("Year") +
  ggtitle("birth")
```




```r
#dbirth<-decompose(birth)
#plot(dbirth)
```

We see the trend (2nd plot), the seasonal component (3rd plot) and the random part (4th plot). The 1st plot is the original series.

- Trend: the trend-cycle component T_t is a m-moving average, where m is the cycle. In our case of montly data, $m = 12$. (moving average = average of previous m-observations)

- Detrended series: Calculate the detrended series as $y_t - T_t$

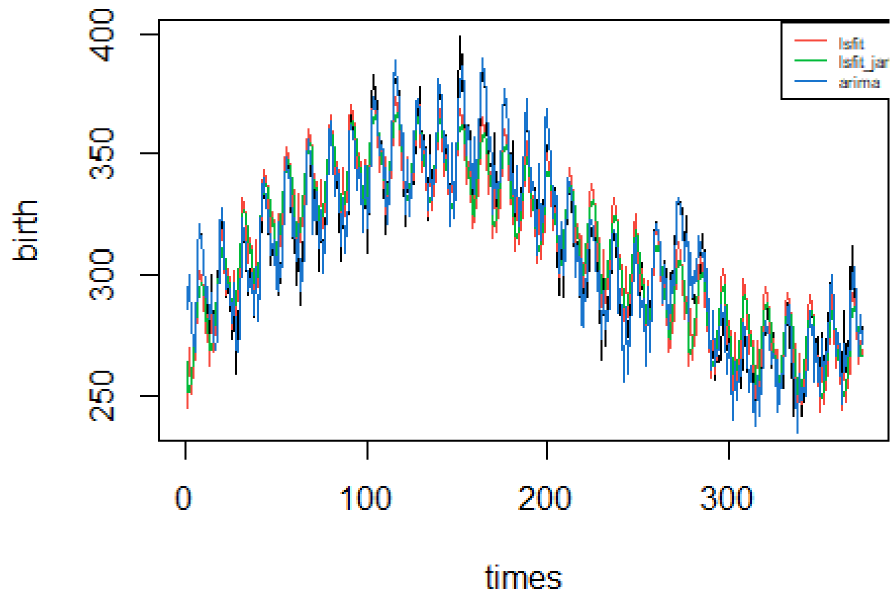
- Seasonal component: the seasonal component for each season is the average of the detrended values for that season. This gives a series called S_t.

- Error: The remainder component is calculated by subtracting the estimated seasonal and trend-cycle components: $R_t = Y_t - T_t - S_t$.

Let's plot the fitted values of the 3 models:

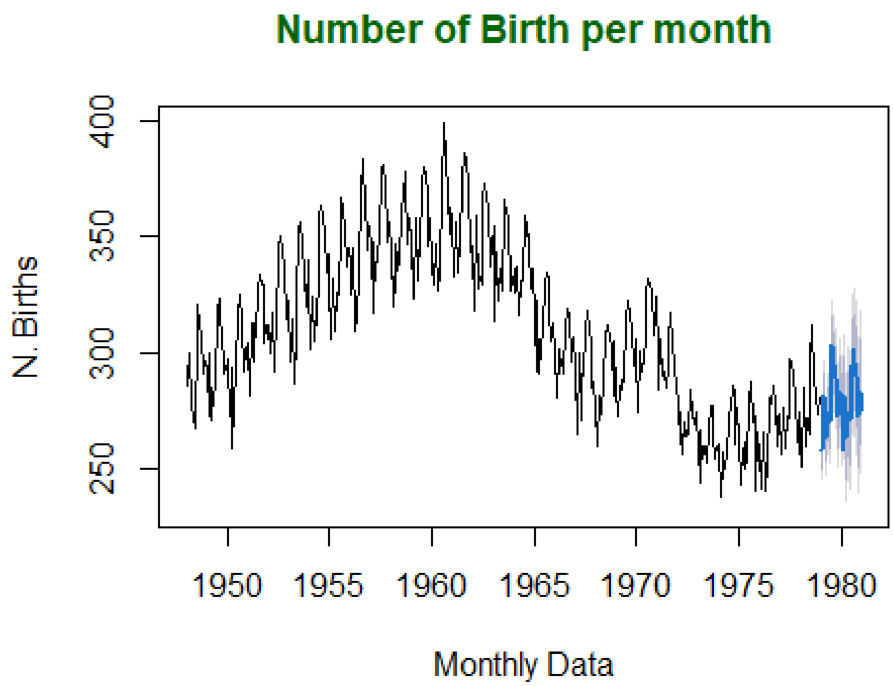
```r
plot(times,birth,type="l") #plot on original scale
#lines(times,birth) #add lines to existing plot
lines(times,lsfit$fitted.values,col=2) #undo log for fitted model
```

```
lines(times,lsfit_jan$fitted,col=3) #undo log for fitted model
lines(times,birthmod$fitted,col=4)
legend(329,405,c("lsfit","lsfit_jan","arima"),col=c(2,3,4),lty=1,cex=.5)
```



Now let's use our arima model to do forecasts:

```
plot(forecast(birthmod, 24), xlab ="Monthly Data",
     ylab ="N. Births",
     main ="Number of Birth per month", col.main ="darkgreen")
```

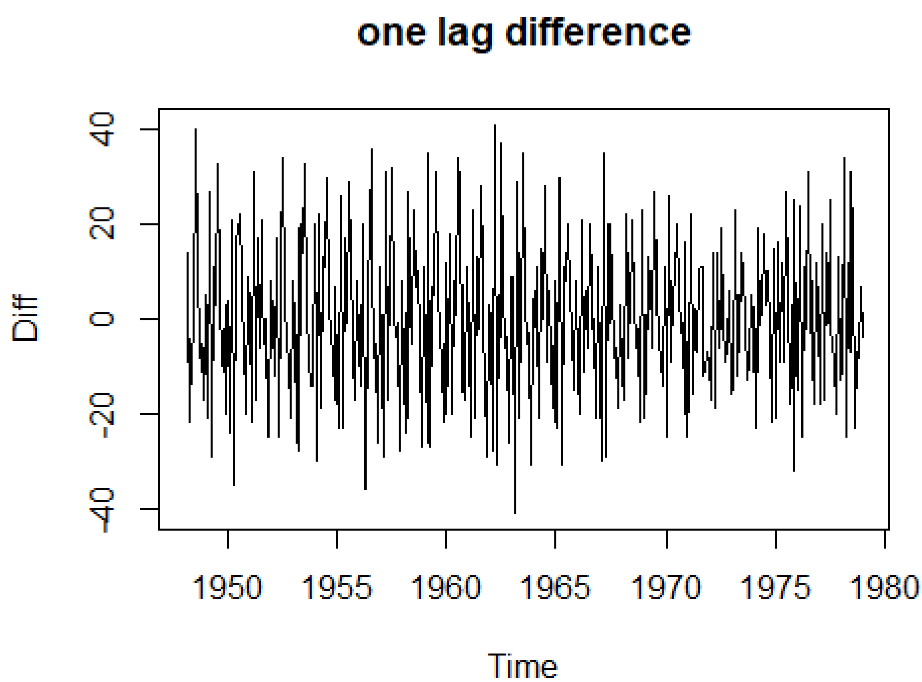


Let's check that the errors do not have any auto-correlation:

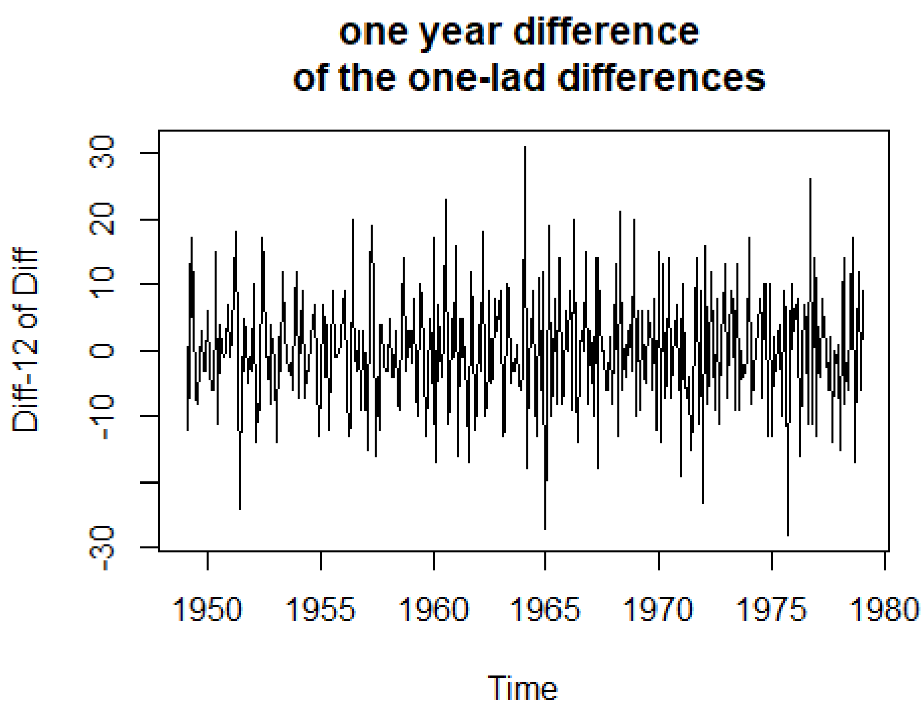
```
acf(birthmod$residuals)
```

Just for the heck of it, let's look at the differences involved in the arima model:

```
plot(diff(birth,1),main="one lag difference",ylab="Diff")
```




```
plot(diff( diff(birth,1) ,12),main="one year difference \n of the one-lad
differences",ylab="Diff-12 of Diff")
```



Time Series Assignment

We will fit a model to the log of the Australian wine sales.

- Plot wine and log(wine).
- Plot the auto correlation and partial auto correlation functions for log(wine).
- Just as we did for "birth", fit a model allowing for a term for each month and time.
- Just as we did for "birth", fit a model using sin and cos to model seasonality and time.
- compute the aic, bic and adjusted r^2 corresponding to both models.
- Use auto.arima() to obtain the arima model.
- compare the aic and bic of the arima model to the previous 2 models.
- Write down the equation corresponding to the arima model.
- Plot the decomposition of the series.
- Plot the fitted values of all 3 models over the values of wine. Remember that your models were for log(wine) but you are plotting wine, so you need to adjust your fitted values.

- plot the predicted values for the next 12 months.
- auto.arima does not work with covariates. But we can use the structure it developed to add one or several covarites. Consider the models:
  - Arima(y, order = c(1,1,1), xreg = X) and
  - Arima(y, order = c(1,0,1), xreg = X) where X is the data frame with times and the monthly dummy variables

```
wine=c(
.46400E+03,
.67500E+03,
.70300E+03,
.88700E+03,
.11390E+04,
.10770E+04,
.13180E+04,
.12600E+04,
.11200E+04,
.96300E+03,
.99600E+03,
.96000E+03,
.53000E+03,
.88300E+03,
.89400E+03,
.10450E+04,
.11990E+04,
.12870E+04,
.15650E+04,
.15770E+04,
.10760E+04,
.91800E+03,
.10080E+04,
.10630E+04,
.54400E+03,
.63500E+03,
.80400E+03,
.98000E+03,
.10180E+04,
.10640E+04,
.14040E+04,
.12860E+04,
.11040E+04,
.99900E+03,
.99600E+03,
.10150E+04,
.61500E+03,
.72200E+03,
.83200E+03,
.97700E+03,
```

```
.12700E+04,
.14370E+04,
.15200E+04,
.17080E+04,
.11510E+04,
.93400E+03,
.11590E+04,
.12090E+04,
.69900E+03,
.83000E+03,
.99600E+03,
.11240E+04,
.14580E+04,
.12700E+04,
.17530E+04,
.22580E+04,
.12080E+04,
.12410E+04,
.12650E+04,
.18280E+04,
.80900E+03,
.99700E+03,
.11640E+04,
.12050E+04,
.15380E+04,
.15130E+04,
.13780E+04,
.20830E+04,
.13570E+04,
.15360E+04,
.15260E+04,
.13760E+04,
.77900E+03,
.10050E+04,
.11930E+04,
.15220E+04,
.15390E+04,
.15460E+04,
.21160E+04,
.23260E+04,
.15960E+04,
.13560E+04,
.15530E+04,
.16130E+04,
.81400E+03,
.11500E+04,
.12250E+04,
.16910E+04,
.17590E+04,
.17540E+04,
```

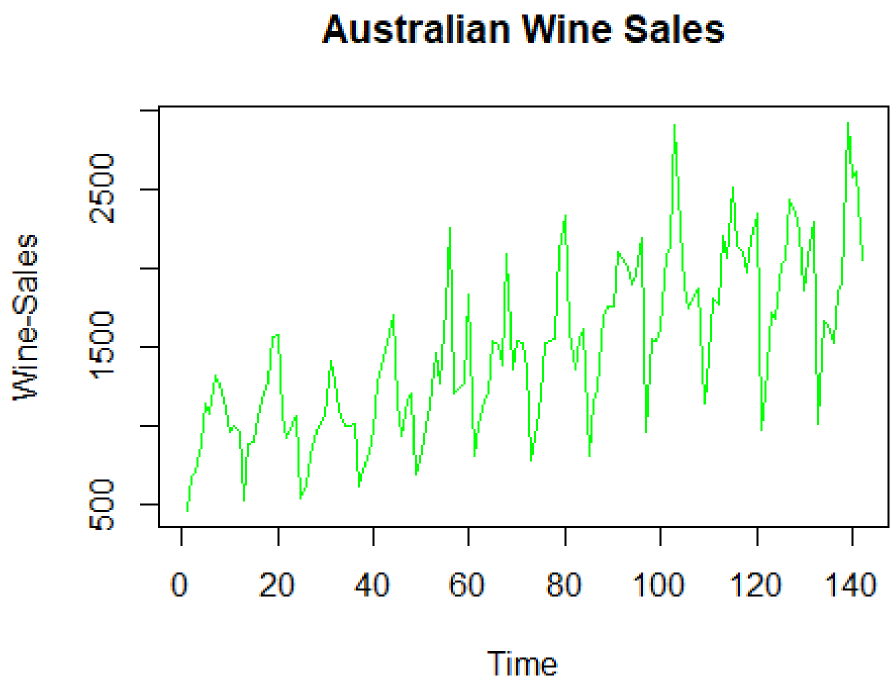
```
.21000E+04,
.20620E+04,
.20120E+04,
.18970E+04,
.19640E+04,
.21860E+04,
.96600E+03,
.15490E+04,
.15380E+04,
.16120E+04,
.20780E+04,
.21370E+04,
.29070E+04,
.22490E+04,
.18830E+04,
.17390E+04,
.18280E+04,
.18680E+04,
.11380E+04,
.14300E+04,
.18090E+04,
.17630E+04,
.22000E+04,
.20670E+04,
.25030E+04,
.21410E+04,
.21030E+04,
.19720E+04,
.21810E+04,
.23440E+04,
.97000E+03,
.11990E+04,
.17180E+04,
.16830E+04,
.20250E+04,
.20510E+04,
.24390E+04,
.23530E+04,
.22300E+04,
.18520E+04,
.21470E+04,
.22860E+04,
.10070E+04,
.16650E+04,
.16420E+04,
.15250E+04,
.18380E+04,
.18920E+04,
.29200E+04,
.25720E+04,
```

```
.26170E+04,
.20470E+04)

y=log(wine)
times=1:142

Jan=rep(c(1,0,0,0,0,0,0,0,0,0,0,0),12)[1:142]
Feb=rep(c(0,1,0,0,0,0,0,0,0,0,0,0),12)[1:142]
Mar=rep(c(0,0,1,0,0,0,0,0,0,0,0,0),12)[1:142]
Apr=rep(c(0,0,0,1,0,0,0,0,0,0,0,0),12)[1:142]
May=rep(c(0,0,0,0,1,0,0,0,0,0,0,0),12)[1:142]
Jun=rep(c(0,0,0,0,0,1,0,0,0,0,0,0),12)[1:142]
Jul=rep(c(0,0,0,0,0,0,1,0,0,0,0,0),12)[1:142]
Aug=rep(c(0,0,0,0,0,0,0,1,0,0,0,0),12)[1:142]
Sep=rep(c(0,0,0,0,0,0,0,0,1,0,0,0),12)[1:142]
Oct=rep(c(0,0,0,0,0,0,0,0,0,1,0,0),12)[1:142]
Nov=rep(c(0,0,0,0,0,0,0,0,0,0,1,0),12)[1:142]
Dec=rep(c(0,0,0,0,0,0,0,0,0,0,0,1),12)[1:142]
sint=sin(2*pi*times/12)
cost=cos(2*pi*times/12)
X=cbind(times,Feb,Mar,Apr,May,Jun,Jul,Aug,Sep,Oct,Nov,Dec)  #sin and cos and
constant for Jan;
X_jan=cbind(times,sint,cost,Jan)  #sin and cos and constant for Jan;
```

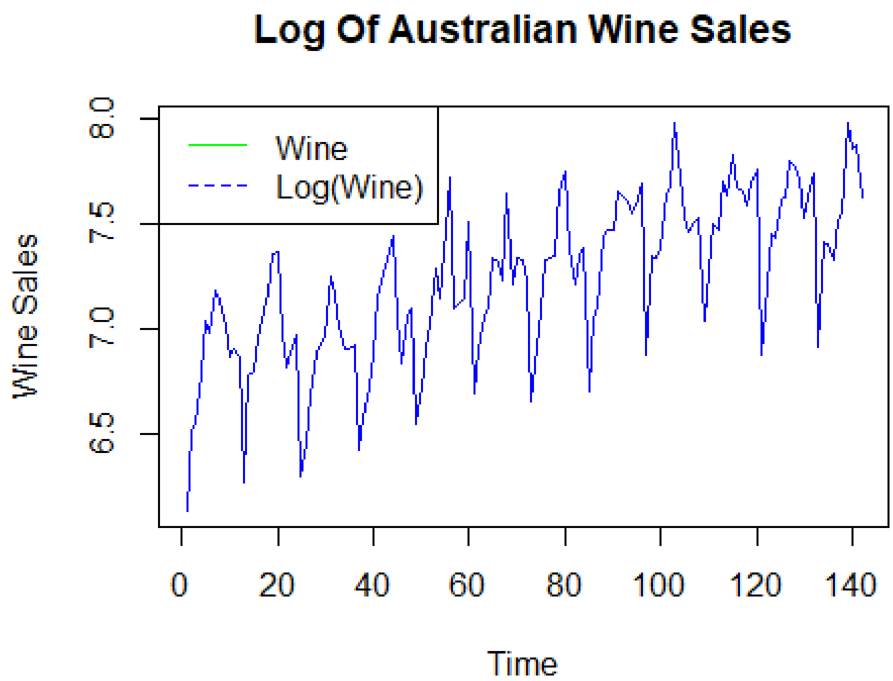
We will fit a model to the log of the Australian wine sales.

- Plot wine and log(wine).

```
library(ggplot2)

if (!requireNamespace("astsa", quietly = TRUE)) {
  install.packages("astsa")
}

# Loading required libraries
library(astsa)

# Plotting wine and log(wine)
plot(wine, type = "l", col = "green", ylab = "Wine-Sales", xlab = "Time",
main = "Australian Wine Sales")
```

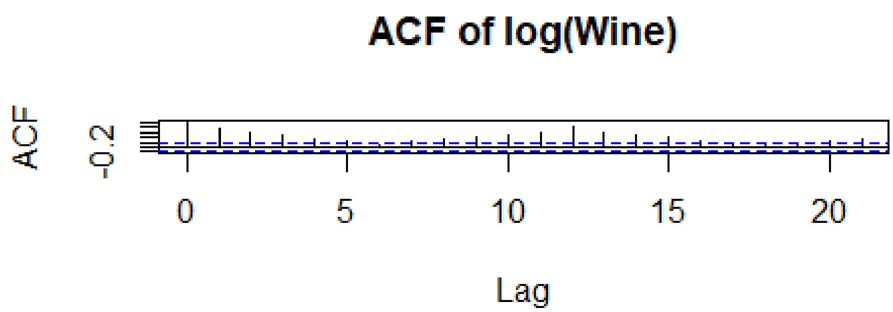


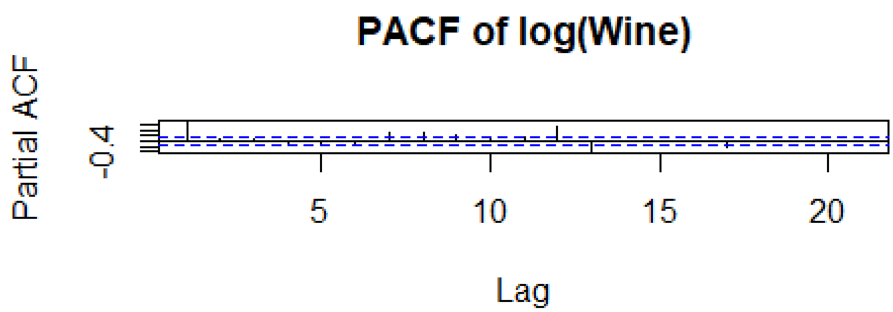

```
plot(log(wine), type = "l", col = "blue", ylab = "Wine Sales", xlab = "Time",
main = "Log Of Australian Wine Sales")
legend("topleft", legend = c("Wine", "Log(Wine)"), col = c("green", "blue"),
lty = c(1, 2))
```



* Plot the auto correlation and partial auto correlation functions for log(wine).

```
# Calculating and plotting ACF and PACF for log(wine)
par(mfrow = c(2, 1))
acf(log(wine), main = "ACF of log(Wine)")
pacf(log(wine), main = "PACF of log(Wine)")
```





- Just as we did for "birth", fit a model allowing for a term for each month and time.

```
# Fitting a model with a term for each month and time
lm_model <- lm(log(wine) ~ poly(times,3)  + Jan + Feb + Mar + Apr + May + Jun
+ Jul + Aug + Sep + Oct + Nov + Dec, data = data.frame(times,Jan, Feb, Mar,
Apr, May, Jun, Jul, Aug, Sep, Oct, Nov, Dec, log(wine)))

# Displaying the summary of the model
summary(lm_model)

##
## Call:
## lm(formula = log(wine) ~ poly(times, 3) + Jan + Feb + Mar + Apr +
##     May + Jun + Jul + Aug + Sep + Oct + Nov + Dec, data =
data.frame(times,
##     Jan, Feb, Mar, Apr, May, Jun, Jul, Aug, Sep, Oct, Nov, Dec,
##     log(wine)))
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.29932 -0.06594 -0.01251  0.06661  0.28057
##
## Coefficients: (1 not defined because of singularities)
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)      7.331166   0.031477 232.905  < 2e-16 ***
## poly(times, 3)1  3.087022   0.104514  29.537  < 2e-16 ***
## poly(times, 3)2 -0.165071   0.104539  -1.579 0.116813
```

```
## poly(times, 3)3 -0.502162   0.104869  -4.788 4.60e-06 ***
## Jan             -0.689645   0.043614 -15.812  < 2e-16 ***
## Feb             -0.394616   0.043595  -9.052 2.08e-15 ***
## Mar             -0.267949   0.043581  -6.148 9.39e-09 ***
## Apr             -0.156363   0.043571  -3.589 0.000473 ***
## May              0.013090   0.043566   0.300 0.764313
## Jun              0.011525   0.043564   0.265 0.791791
## Jul              0.220215   0.043567   5.055 1.47e-06 ***
## Aug              0.228804   0.043574   5.251 6.18e-07 ***
## Sep             -0.003657   0.043586  -0.084 0.933274
## Oct             -0.113620   0.043602  -2.606 0.010260 *
## Nov             -0.053197   0.044472  -1.196 0.233849
## Dec                    NA         NA      NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1043 on 127 degrees of freedom
## Multiple R-squared:  0.9327, Adjusted R-squared:  0.9253
## F-statistic: 125.7 on 14 and 127 DF,  p-value: < 2.2e-16
```

- Just as we did for “birth”, fit a model using sin and cos to model seasonality and time.

```
# Fitting a model with sin and cos for seasonality and time
lm_model_sin_cos <- lm(log(wine) ~ poly(times,3)+sint + cost + Jan, data =
data.frame(times, sint, cost, Jan, log(wine)))

# Displaying the summary of the model
summary(lm_model_sin_cos)

##
## Call:
## lm(formula = log(wine) ~ poly(times, 3) + sint + cost + Jan,
##     data = data.frame(times, sint, cost, Jan, log(wine)))
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.31228 -0.09852 -0.00531  0.09645  0.46227
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)      7.26388    0.01226 592.512  < 2e-16 ***
## poly(times, 3)1  3.08221    0.13837  22.275  < 2e-16 ***
## poly(times, 3)2 -0.20972    0.13825  -1.517 0.131622
## poly(times, 3)3 -0.51123    0.13879  -3.683 0.000332 ***
## sint            -0.17542    0.01682 -10.428  < 2e-16 ***
## cost            -0.13436    0.01789  -7.508 7.31e-12 ***
## Jan             -0.41833    0.04629  -9.037 1.49e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 0.1381 on 135 degrees of freedom
## Multiple R-squared:  0.8745, Adjusted R-squared:  0.8689
## F-statistic: 156.8 on 6 and 135 DF,  p-value: < 2.2e-16
```

- compute the aic, bic and adjusted r^2 corresponding to both models.

```
# Computing AIC, BIC, and adjusted R^2 for the model with a term for each
month and time
lm_aic_bic_adjr2 <- c(AIC(lm_model), BIC(lm_model),
summary(lm_model)$adj.r.squared)
lm_aic_bic_adjr2

## [1] -222.8709896 -175.5777567    0.9252505

# Computing AIC, BIC, and adjusted R^2 for the model with sin and cos for
seasonality and time
lm_sin_cos_aic_bic_adjr2 <- c(AIC(lm_model_sin_cos), BIC(lm_model_sin_cos),
summary(lm_model_sin_cos)$adj.r.squared)
lm_sin_cos_aic_bic_adjr2

## [1] -150.4375221 -126.7909057    0.8689176
```

- Use auto.arima() to obtain the arima model.

```
library(forecast)

# Using auto.arima to obtain the ARIMA model
arima_model <- auto.arima(log(wine))

# Displaying the obtained ARIMA model
arima_model

## Series: log(wine)
## ARIMA(1,1,1)
##
## Coefficients:
##          ar1      ma1
##       0.5214  -0.9277
## s.e.  0.0821   0.0268
##
## sigma^2 = 0.06157:  log likelihood = -3.02
## AIC=12.04   AICc=12.21   BIC=20.88
```

- compare the aic and bic of the arima model to the previous 2 models.

```
# AIC and BIC of the model with a term for each month and time
lm_aic_bic <- c(AIC(lm_model), BIC(lm_model))

# AIC and BIC of the model with sin and cos for seasonality and time
lm_sin_cos_aic_bic <- c(AIC(lm_model_sin_cos), BIC(lm_model_sin_cos))

# Displaying all AIC and BIC values for comparison
comparison <- data.frame(
```

```
  Models = c("lm_model", "lm_model_sin_cos", "arima_model"),
  AIC = c(lm_aic_bic[1], lm_sin_cos_aic_bic[1], arima_model$aic),
  BIC = c(lm_aic_bic[2], lm_sin_cos_aic_bic[2], arima_model$bic)
)
comparison
```

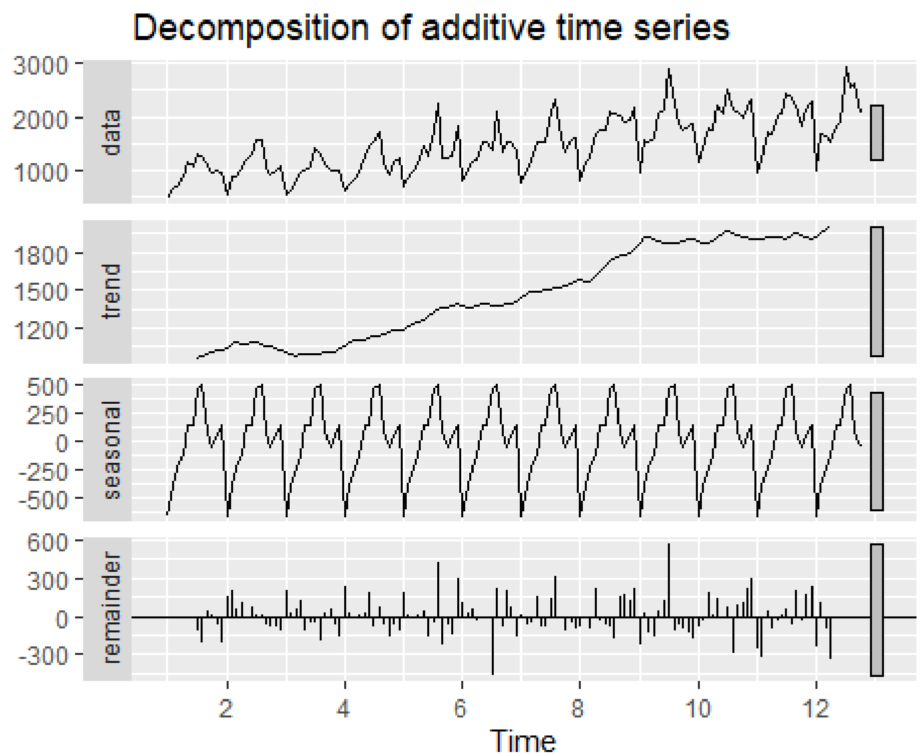
```
##              Models        AIC       BIC
## 1          lm_model -222.87099 -175.5778
## 2 lm_model_sin_cos -150.43752 -126.7909
## 3       arima_model   12.03603   20.8823
```

- Write down the equation corresponding to the arima model.

:(1-Ø1B)(1-B)(yt – yt-1)=(1+Ɵ1B)wt - Ø1 is the autoregressive parameter, - B is the backshift operator (used for differencing), - yt is the observed time series, - yt-1 is the lagged value of the time series, - Ɵ1 is the moving average parameter, - wt is the white noise series. The estimated values for Ø1 and Ɵ1 are 0.5214 and -0.9277, respectively, based on your model summary. You can substitute these values into the equation to get the specific form for your ARIMA(1,1,1) model.

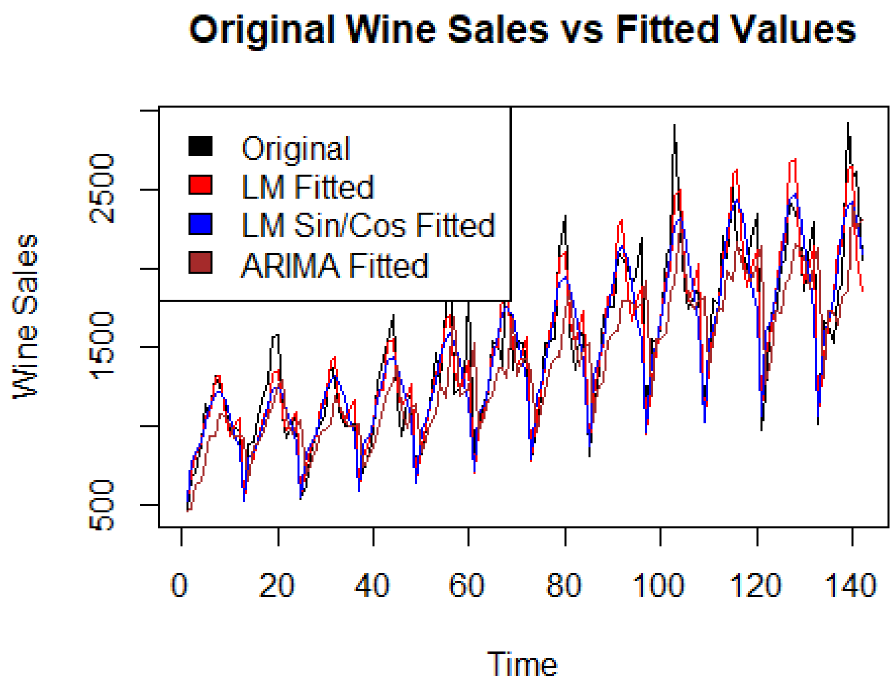
- Plot the decomposition of the series.

```
library(forecast)
# Convert the y to a time series (replace 'frequency = 12' with your actual
frequency if different)
wine_ts <- ts(wine, frequency = 12)
# Try decomposing the time series
decomposed <- try(decompose(wine_ts))
# Plot the decomposition if successful
if (class(decomposed) != "try-error") {
autoplot(decomposed)
} else {
cat("Unable to decompose the time series.")
}
```



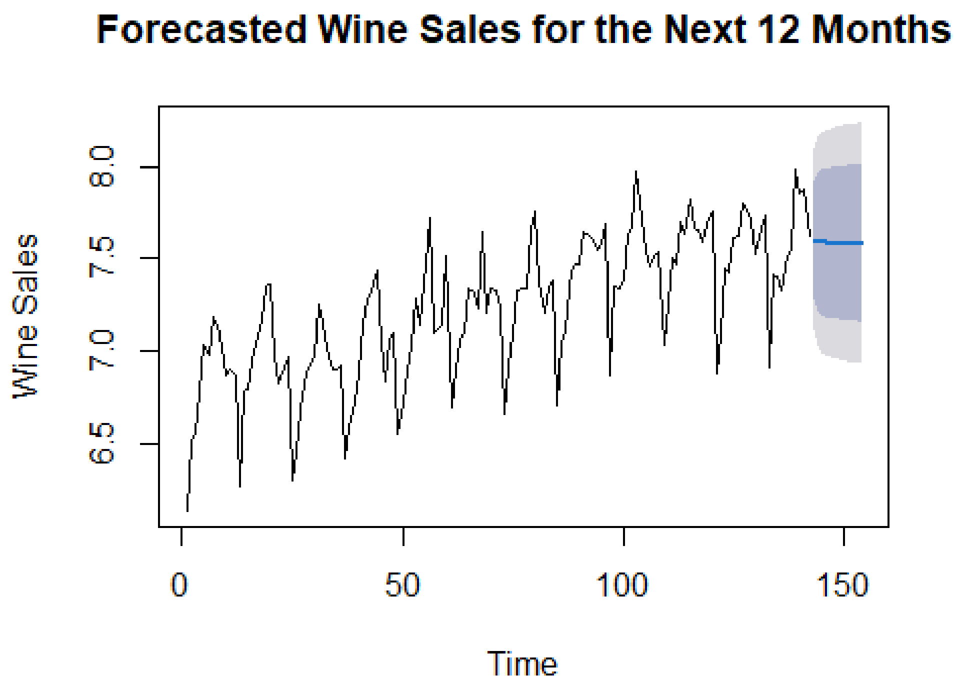
* Plot the fitted values of all 3 models over the values of wine. Remember that your models were for log(wine) but you are plotting wine, so you need to adjust your fitted values.

```
# Creating a dataframe with original and fitted values in the original scale
# Creating a data frame with original and fitted values
df_fitted <- data.frame(
  times = 1:length(wine),
  wine = wine,
  lm_fitted = exp(predict(lm_model)),
  lm_sin_cos_fitted = exp(predict(lm_model_sin_cos)),
  arima_fitted = exp(fitted(arima_model))
)

# Plotting the original wine values and the fitted values from the models
plot(df_fitted$times, df_fitted$wine, type = "l", xlab = "Time", ylab = "Wine
Sales",
     main = "Original Wine Sales vs Fitted Values")
lines(df_fitted$times, df_fitted$lm_fitted, col = "red")
lines(df_fitted$times, df_fitted$lm_sin_cos_fitted, col = "blue")
lines(df_fitted$times, df_fitted$arima_fitted, col = "brown")
legend("topleft", legend = c("Original", "LM Fitted", "LM Sin/Cos Fitted",
"ARIMA Fitted"),
       fill  = c("black", "red", "blue", "brown"))
```



* plot the predicted values for the next 12 months.

```
# Generating forecasts for the next 12 months
forecast_values <- forecast(arima_model, h = 12)

# Plotting the forecasted values
plot(forecast_values, xlab = "Time", ylab = "Wine Sales",
     main = "Forecasted Wine Sales for the Next 12 Months")
```



- auto.arima does not work with covariates. But we can use the structure it developed to add one or several covarites. Consider the models:
  - Arima(y, order = c(1,1,1), xreg = X) and
  - Arima(y, order = c(1,0,1), xreg = X) where X is the data frame with times and the monthly dummy variables

```
# Fitting the ARIMA model with covariates and differencing
arima_model_xreg_diff <- Arima(log(wine), order = c(1, 1, 1), xreg = X)

# Displaying the model summary
summary(arima_model_xreg_diff)

## Series: log(wine)
## Regression with ARIMA(1,1,1) errors
##
## Coefficients:
##          ar1      ma1   times     Feb     Mar     Apr     May     Jun
Jul
##       0.0986  -0.8350  0.0058  0.2939  0.4195  0.5300  0.6984  0.6958
0.9033
## s.e.  0.1093   0.0668  0.0016  0.0362  0.0382  0.0386  0.0387  0.0389
0.0389
##          Aug     Sep     Oct     Nov     Dec
##       0.9107  0.6771  0.5659  0.6322  0.6841
## s.e.  0.0389  0.0389  0.0388  0.0391  0.0372
##
## sigma^2 = 0.01128:  log likelihood = 122.98
```

```
## AIC=-215.97   AICc=-212.13   BIC=-171.74
##
## Training set error measures:
##                        ME      RMSE        MAE         MPE     MAPE      MASE
## Training set 0.001359051 0.100425 0.07901908 0.00517242 1.087874 0.4068724
##                      ACF1
## Training set -0.008184978
```

```
# Fitting the ARIMA model with covariates without differencing
arima_model_xreg <- Arima(log(wine), order = c(1, 0, 1), xreg = X)

# Displaying the model summary
summary(arima_model_xreg)
```

```
## Series: log(wine)
## Regression with ARIMA(1,0,1) errors
##
## Coefficients:
##          ar1      ma1  intercept   times     Feb     Mar    Apr     May
Jun
##       0.8694  -0.6558     6.1964  0.0063  0.2936  0.4189  0.529  0.6969
0.6938
## s.e.  0.0891   0.1412     0.0474  0.0005  0.0363  0.0373  0.038  0.0385
0.0388
##          Jul     Aug     Sep     Oct     Nov     Dec
##       0.9009  0.9078  0.6736  0.5619  0.6308  0.6832
## s.e.  0.0390  0.0389  0.0387  0.0383  0.0382  0.0373
##
## sigma^2 = 0.01092:  log likelihood = 127.02
## AIC=-222.05   AICc=-217.7   BIC=-174.75
##
## Training set error measures:
##                         ME       RMSE        MAE         MPE     MAPE
MASE
## Training set 0.0003300068 0.09881831 0.07768568 -0.01376188 1.070394
0.4000067
##                    ACF1
## Training set 0.01067203
```