Problem Set 8

Question 1:

2) Umberto theorizes that living with a dog diminishes depression in the elderly, here defined as more than 70 years of age. To investigate his theory, he recruits 15 single elderly men who own dogs and 15 single elderly men who do not own any pets. The Hamilton instrument for measuring depressive tendency is administered to each subject. High scores indicate depression. How might Umberto use the resulting data to test his theory? (Respond to (a)–(e) above. Especially (d).)

What is the experimental unit?

Answer: Experimental unit here is the elderly men.

(b) From how many populations were the experimental units drawn? Identify the population(s). How many units were drawn from each population? Is this a 1- or a 2-sample problem?

Answer: The experimental units were drawn from one of two populations: Elderly men who own dogs, Elderly men who do not own any pets. 15 units each are drawn from the two populations. This is a 2- sample problem.

c) How many measurements were taken on each experimental unit? Identify them.

Answer: The Hamilton instrument is administered to each subject once. The measure is the depressive tendency.

d) Define the parameter(s) of interest for this problem. For 1- sample problems, this should be μ; for 2-sample problems, this should be .

= $\mu_1 - \mu_2$ where,

μ_1 = mean of depressive tendency as measured by the Hamilton instrument in elderly men who own dogs

μ_2 = mean of depressive tendency as measured by the Hamilton instrument in elderly men who do not own dogs

(e) State appropriate null and alternative hypotheses.

H0: > 0 which is the null hypothesis

H1: <= 0 which is the alternate hypothesis

3) Irmina, a professional massage/physical therapist and ski instructor, decides to moonlight as an aerobics instructor. Her supervisor recommends that she begin each class with 10 minutes of static stretching, but Irmina believes that static stretching is detrimental to athletic performance. She devises an experiment, for which she recruits 20 aerobics students, that consists of two protocols. In protocol S, a participant walks for 5 minutes, then does 10 minutes of static stretches of the hamstring, quadricep, and calf muscles, then rides a stationary bike for 30 minutes. Protocol D replaces static stretches with dynamic stretches. Each bike is equipped with a heart monitor and the ability to measure watts of power expended. To equalize level of exertion, each participant is asked to maintain a constant training heart rate calculated using the Karvonen formula5 with an intensity of 0.80. The study participants perform protocol D one week and protocol S the following week. Irmina records the number of watts expended during each 30-minute ride. How might she use the resulting data to persuade her supervisor that dynamic stretching is superior to static stretching? (Respond to (a)–(e) above.)

What is the experimental unit?

Answer: Experimental unit here is the aerobic students.

(b) From how many populations were the experimental units drawn? Identify the population(s). How many units were drawn from each population? Is this a 1- or a 2-sample problem?

Answer: The experimental units were drawn from one population: The aerobic students. 20 units were drawn from the population. This is a 1-sample problem.

c) How many measurements were taken on each experimental unit? Identify them.

Answer: Total 14 measurements (Watts of power expended) are taken on each person. 7 are taken during the first week (one each day) when they follow Protocol S, and the other 7 are taken during the second week (one each day) when they follow Protocol D.

d) Define the parameter(s) of interest for this problem. For 1- sample problems, this should be μ; for 2-sample problems, this should be .

= μ1 - μ2 where,

μ1 = mean of measurements of all participants taken during Protocol S μ2 = mean of measurements of all participants taken during Protocol D

(e) State appropriate null and alternative hypotheses.

H0: > 0 which is the null hypothesis

H1: <= 0 which is the alternate hypothesis

Question 2:

1. Assume that n = 400 observations are independently drawn from a normal distribution with unknown population mean μ and unknown population variance $\sigma 2$. The resulting sample, _x, is used to test $H0 : \mu \leq 0$ versus $H1 : \mu > 0$ at significance level $\alpha = 0.05$.

(a) What test should be used in this situation? If we observe _x that results in $\overline{x}$ = 3.194887 and $s2$ = 104.0118, then what is the value of the test statistic?

One sample t-test :

```
t = (3.194887)/((104.0118/400)^0.5)

t

[1] 6.265333
```

So t = 6.265

(b) If we observe _x that results in a test statistic value of 1.253067, then which of the following R expressions best approximates the significance probability?

2*pnorm(1.253067)

2*pnorm(-1.253067)

1-pnorm(1.253067)

1-pt(1.253067,df=399)

v. pt(1.253067,df=399)

(c) True or False: if we observe $\bar{x}$ that results in a significance probability of **p** = 0.03044555, then we should reject the null hypothesis.

As pvalue < alpha we should reject the null hypothesis. ***True***

Question 3:

1. Show that this problem can be formulated as a 1-sample location problem.

To do so, you should:

a) Identify the experimental units and the measurement(s) taken on each unit.

Answer: The experimental unit here is pair of seedlings. One measurement (final height) is taken on each member of the pair

b) Define appropriate random variables $X1, \ldots, Xn \sim P$. Remember that the statistical procedures that we will employ assume that these random variables are independent and identically distributed.

Answer: Lets take $Xi = Ai - Bi$ Where Ai is the height of the cross-fertilized plant of pair i, and Bi is the height of the self-fertilized plant of pair i.

(c) Let θ denote the location parameter (measure of centrality) of interest. Depending on which statistical procedure we decide touse, either $\theta = EXi = \mu$ or $\theta = q2(Xi)$. State appropriate null and alternative hypotheses about θ.

Answer: We can choose $\theta = q2(Xi)$, we cannot assume normality on such small sample. So, we can formulate our hypotheses as follows:

H0: $\theta <= 0$

H1: $\theta > 0$

3. Assume that $X1, \ldots, Xn$ are normally distributed and let $\theta = EXi = \mu$.

(a) Test the null hypothesis derived above using Student's 1-sample *t*-test. What is the significance probability? If we adopt a significance level of $\alpha = 0.05$, should we reject the null hypothesis?

```
>seeds.a=c(23.5,12.0,21.0,22.0,19.1,21.5,22.1,20.4,18.3,21.6,23.3,21.0,22.1,23.0,12.0)
>seeds.b=c(17.4,20.4,20.0,20.0,18.4,18.6,18.6,15.3,16.5,18.0,16.3,18.0,12.8,15.5,18.0)
> length(seeds.a)
[1] 15
> me=mean(seeds.a-seeds.b)
> me
[1] 2.606667
> std=sd(seeds.a-seeds.b)
> std
[1] 4.712819
> answer=(me-0)/(std/(15^0.5))
> answer
[1] 2.142152
> p=1-pt(answer,15-1)
> p
[1] 0.02512287
```

Hence P value is 0.02512 and hence we can reject the null hypothesis.

(b) Construct a (2-sided) confidence interval for θ with a confidence coefficient of approximately 0.90.

```
> li=mean(seeds.a-seeds.b)-qt(0.95,df=14)*std/sqrt(15)
> ui=mean(seeds.a-seeds.b)+qt(0.95,df=14)*std/sqrt(15)
> li
[1] 0.4634257
> ui
[1] 4.749908
```

Hence 90% confidence interval ranges from .46 to 4.74

5(a)

```
> plus=sum(seeds.a>seeds.b)
> plus
[1] 13
> p=1-pbinom(plus-1,15,0.5)
> p
[1] 0.003692627
```

Since the p value is less, we can reject null hypothesis.

Question: 4

“Glycemic index” is a measure of how quickly blood sugar level rises after eating a particular food. (Glucose has a glycemic index of 100, while water has a glycemic index of 0.) A group of researchers wished to study glycemic index when dates and coffee were consumed together by individuals with type 2 diabetes. They performed a study on 10subjects with diabetes. Firstly, they

measured glycemic index for each patient after consuming dates without coffee. The mean was 53 with standard deviation 19. Then (several days later) they measured glycemic index for each patient after consuming dates with coffee. The mean was 41.5 with standard deviation 17. The differences between the measurements (“without coffee” minus “with coffee”) had mean 11.5 with standard deviation 21.

a) What is the experimental unit? What measurements are taken on the experimental units? Is this a problem with one or two independent samples?

Answer: The experimental unit here is an individual with type-2 diabetes. The glycemic index is the measurement taken. This problem has one independent sample. The two samples in the study are paired**.**

b) Give null and alternative hypotheses for an appropriate two-tailed t-test, and calculate the t-statistic.

Answer: Let Ci be the measurement taken on the population after consuming dates without coffee and Di be the measurement taken on population after consuming dates with coffee. Let Xi = Ci - Di

Let μ = EXi

The hypotheses can be given as follows: H0: μ = 0 H1: μ ≠ 0

We can calculate t-stats using R:

```
exp = 11.5
```

```
sd = 21
l=10
t = (exp/(sd/sqrt(l)))
t
[1] 1.731723
```

(c) The P-value (significance probability) was calculated to be 0.12, so the null hypothesis was not rejected. From this and the other information given, is it correct to conclude that we are sure that on average, dates have the same glycemic index with or without coffee? Explain.

Answer: We cannot be so sure of the fact as the sample size is small, some significant amount of data can clarify more. There could be some factors that might be influencing the glycemic index and not taken into account.

Question 5:

(5 points.) Crustaceologists counted the number of horseshoe crabs on 25 beaches in Delaware Bay in both of the 2011 and 2012 breeding seasons. They wish to know whether changes in the number of crabs from one year to the next can be attributed to chance. On the following page is a table of the 2011 and 2012 counts and the changes from year-to-year at each location.

(a) What is the experimental unit in this study? What measurements are taken on theexperimental units? How many independent samples are there?

Answer: The experimental unit is the beach. The number of crustacean crabs on the beach is the measurement taken. There is one independent sample here.

(b) One researcher uses R to perform a paired t-test on the data, which results in a P-value of

0.045. Draw a graph and convince the researcher that a paired t-test may not be the best method for this data.

```
> carbs=c(-13468,-275850,-32415,-18855,56212,-4080,70146,-12417,4864,36020,-48960,-27873,-22776,-28260,-26700,5432,-80720,19324,-149130,-48254,-38390,-24420,-22080,-24120,1135)
```

Clearly the qq plot is not a straight line so we can infer that this data does not come from a normal distribution. Paired t-test doesn’t really looks like a really good option as it assumes normality which is clearly not the case here.

(c) Another researcher suggests performing a sign test on the data. Find the P-value for this test, and explain carefully what this P-value means.

The hypotheses can be given as follows: H0: v0 = 0 H1: v ≠ 0

```
exp = -28225
sd = 66823
l =25
t = (exp)/(sd/sqrt(l))
t
[1] -2.111923
```

Its observable that 7 values are above the hypothesized median which is < n/2

Calculating p value :

```
> 2 * pbinom(7, l, 0.5) [1] 0.04328525
```

Assuming α=0.05, we see that p-value< α, as 0.04329. So, we can reject the null hypothesis pvalue says the median is not zero. So the data is not equally distributed across zero, a median of zero might have suggested this case.