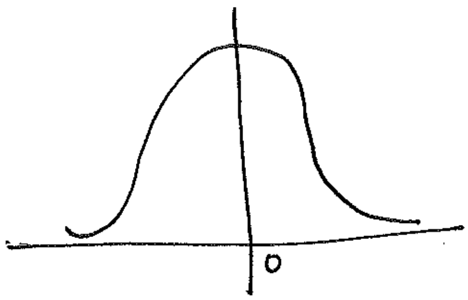
Problem-set 5
(1 Pm class)

① $F(y) = \begin{cases} 0 & y < -4 \\ 0.2 + 0.05y & -4 \leq y < 0 \\ 0.4 + 0.05y & 0 \leq y < 12 \\ 1 & y \geq 12 \end{cases}$

a) X is a continuous variable since the value of X is always in a range



b) Median of X

$0.4 + 0.05y = 0.5$

$0.05y = 0.1$

$y = 0.1/0.05$

Hence $\boxed{\text{Median} = 2}$

c) Expected value $EX = \int_{-\infty}^{\infty} x f(x)$

$= \begin{cases} 0.5 & -4 \leq y < 0 \\ 0.05 & 0 \leq y < 12 \end{cases}$

$$\int_{-4}^{0} 0.05x + \int_{0}^{12} 0.05x$$

$$\frac{0.05x^2}{2}\Big|_{-4}^{0} + \frac{0.05x^2}{2}\Big|_{0}^{12}$$

$$\frac{0.05(0^2-4^2)}{2} + \frac{0.05(12^2-0^2)}{2}$$

$$EX = -0.4 + 3.6 = \boxed{3.2}$$

2) $f(x) = \begin{cases} 0.3 & 0 \le x < 1 \\ 0.7 & 1 \le x < 2 \\ 0 & \text{otherwise.} \end{cases}$

$$F(y) = \begin{cases} 0.3y & 0 \le x < 1 \\ 0.7y - 0.4 & 1 \le x \le 2 \\ 1 & x > 2 \end{cases}$$

a) $E|X-c| = q_2$ & $E|X-c| = 0.5$

Hence $0.7y - 0.4 = 0.5$

$0.7y = 0.9$

$y = 1.28$

Hence $\boxed{a = 1.28}$

b) $E(X-c)^2$

$$\int_0^1 0.3x + \int_1^2 0.7x - 0.4x$$

$$= 0.3\frac{x^2}{2}\Big|_0^1 + 0.7\frac{x^2}{2}\Big|_1^2$$

$$= \frac{0.3}{2}(1-0) + \frac{0.7}{2}(4-1)$$

$$= \frac{0.3}{2} + \frac{3}{2}\times 0.7$$

$$= 1.2$$

3) $g : R \to R$

$$g(x) = \begin{cases} 0 & x < 0 \\ x & x \in (0,1] \\ 1 & x \in [1,2] \\ 3-x & x \in [2,3] \\ 0 & x > 3 \end{cases}$$

$f(x) = cg(x)$, c is undermind constant

a) $$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_0^1 cx\,dx + \int_1^2 c\,dx + \int_2^3 c(3-x) = 1$$

$$\Rightarrow c\frac{x^2}{2}\Big|_0^1 + \cancel{\frac{x^2}{2}}\, cx\Big|_1^2 + c\left\{3x - \frac{x^2}{2}\right\}\Big|_2^3 = 1$$

$$\Rightarrow \frac{c}{2} + c + c\left\{3 - \frac{(9-4)}{2}\right\} = 1$$

$$\Rightarrow 4c + \frac{c}{2} - \frac{5c}{2} = 1$$

$$\Rightarrow 2c = 1$$

$$\boxed{c = 1/2}$$

b) $P(1.5 < x < 2.5)$

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{2} & x \in [0,1] \\ \frac{x^4}{2} - \frac{1}{2} & x \in [1,2] \\ \frac{1}{2}\left(3x - \frac{x^2}{2}\right) - \frac{5}{4} & x \in [2,3) \\ 1 & x > 0 \end{cases}$$

$$\begin{aligned} P(1.5 < X < 2.5) &= P(X < 2.5) - P(X < 1.5) \\ &= F(2.5) - F(1.5) \\ &= \frac{1}{2}\left(3\cdot(2.5) - \frac{(2.5)^2}{2}\right) - \frac{5}{4} - \frac{(1.5)^4}{2} - \frac{1}{2} \\ &= 0.437 \end{aligned}$$

c) $$\begin{aligned} EX &= \int_{-\infty}^{\infty} x f(x) \\ &= \int_0^1 \frac{x^2}{2} + \int_1^2 \frac{x}{2} + \int_2^3 \frac{3x - x^2}{2} \\ &= \frac{x^3}{6}\Big|_0^1 + \frac{x^2}{4}\Big|_1^2 + \left[\frac{\frac{3x^2}{2} - \frac{x^3}{3}}{2}\right]_2^3 \\ &= \frac{1}{6} + \frac{3}{4} - \frac{19}{4} \\ &= \frac{11 + 81 - 54 - 36 + 16}{12} \\ &= \frac{18}{12} = 3/2 = 1.5 \end{aligned}$$

d) $F(1)$ is given by $\frac{x^2}{4}$

Hence $F(1) = \frac{1}{4}$

e) $\left(3x - \frac{x^2}{2}\right)$

$= (0.9 + 1.25)2$

$3x - \frac{x^2}{2} - 4.3 = 0$

Roots are 3.632 & 2.367

Hence $\boxed{2.367}$

4) $f(x) = \begin{cases} 0 & x < 0 \\ x & x \in (0,1) \\ \frac{3-x}{4} & x \in (1,3) \\ 0 & x > 3 \end{cases}$

a) q_2

By integration, $F(y) = \frac{6y - y^2 - 5}{8} + \frac{1}{2}$

$q_2 = 1/2$

Hence $\frac{6y - y^2 - 5}{8} + \frac{1}{2} = 1/2$

$6y - y^2 - 5 = 0$

two roots are 5 & 1

Hence Median $= 1$

b) which is greater $q_\alpha(x)$ or Ex

$q_\alpha(x) = 1$

$E\,x = \int_{-\infty}^{\infty} x f(x)$ which is little as 1

$$\boxed{Ex > q_\alpha(x)}$$

c) $P(0.5 < x < 1.5)$

$F(1.5) - F(0.5)$

$= \dfrac{6(1.5) - (1.5)^2}{8} + \dfrac{1}{2} - \dfrac{6(0.5) - (0.5)^2}{8} + \dfrac{1}{2}$

$= \dfrac{1}{2}$

d) $iqr(x)$

$q_1 = \dfrac{6y - y^2 - 5}{8} + 0.5 = 0.25$

$q_3 = \dfrac{6y - y^2 - 5}{8} + 0.5 = 0.75$

$q_1 = 0.55$

$q_3 = 1.58$

$iqr(x) = q_3 - q_1$

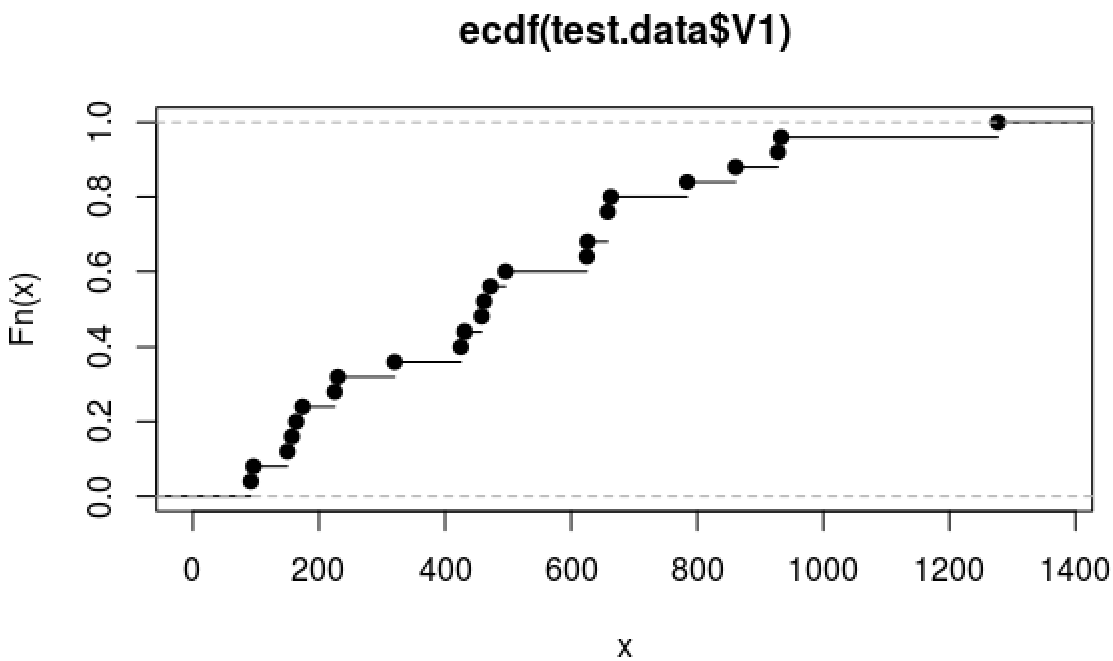
$= 1.58 - 0.55$

$= 1.03$

5) Let x denote the following sample from an unknown population:
462 472 92 225 458 425
658 230 150 933 164 658
96 320 861 784 625 663
928 626 1277 496 157
174 431

(a) Graph the empirical cdf of x .

```
test.data<-read.csv("data1.csv",header = FALSE)
plot(ecdf(test.data$V1))
```



(b) Compute the plug-in estimates of the population mean and variance.

```
mean(test.data$V1)
494.6

mean(test.data$V1^2)-( mean(test.data$V1))^2
91078.72
```

(c) Compute the plug-in estimates of the population median and interquartile range.

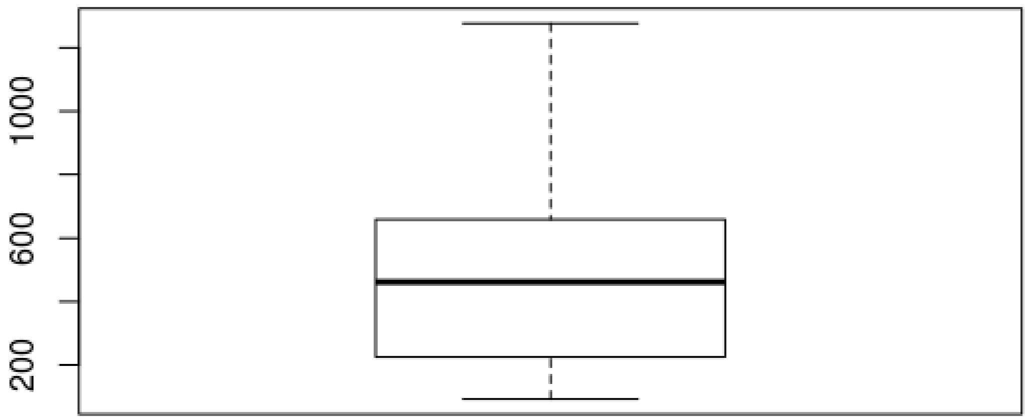
```
sort=sort(test.data$V1)
median(sort)
462
```

IQR(sort)
433

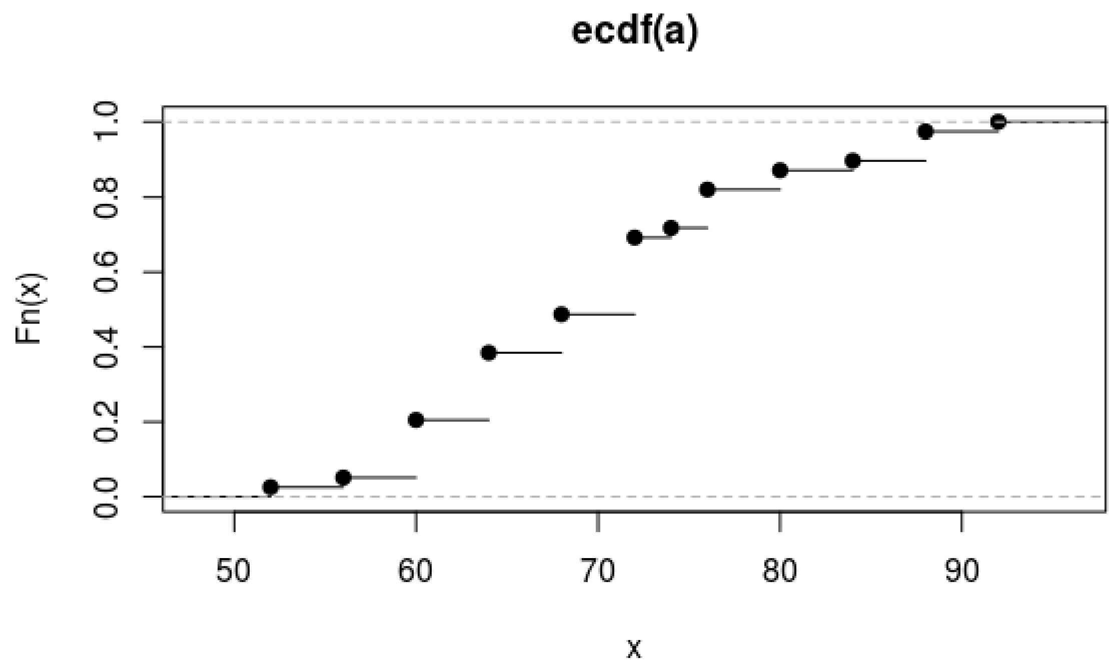
(d) Compute the ratio of the plug-in estimate of the interquartile range to the square root of the plug-in estimate of the variance.

IQR(sort)/sqrt(mean(test.data$V1^2)-(mean(test.data$V1))^2)
1.434761

(e) Construct a boxplot



6)
a. plot(ecdf(a))



b. mean(a)

[1] 70.30769

mean(a^2)-(mean(a))^2

[1] 87.90533

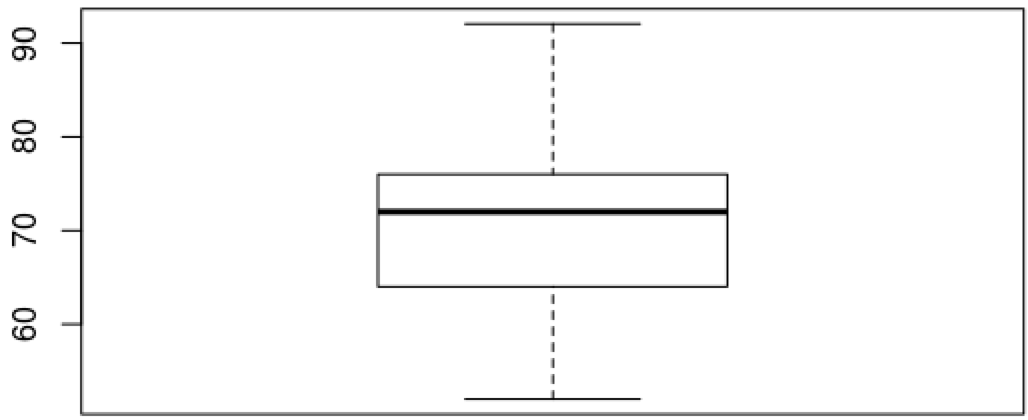
c. median(a)
72

IQR(a)
12

d. IQR(a)/sqrt(mean(a^2)-(mean(a))^2)

1.279893

e. boxplot



90
80
70
60