

→ Best, Worst and Average case analysis

1. Linear search

2. Binary Search Tree

→ Linear search

$$B(n) = O(1)$$

$$W(n) = O(n)$$

$$A(n) = n+1$$

A	8	6	12	5	19	7	14	3	16	18	↓
	0	1	2	3	4	5	6	7	8	9	2

key = 7 → Total Comparison = 6

key = 20 → Total comparison = 10

- Best element → searching key element present at first index.

• Best case time → 1  $O(1)$

$$B(n) = O(1)$$

• Worst case → searching a key at last index

• Worst case time → n

$$W(n) = n$$

$$O(n)$$

Average case - all possible case time  
no. of cases

Average case time -  $\frac{1+2+3+\dots+n}{n} = \frac{n(n+1)/2}{n} = \frac{n+1}{2}$

→ Asymptotic Notation of Best, Worst and Average Case Time:

• Best case time:

$$B(n) = 1$$

$$B(n) = O(1)$$

$$B(n) = \Omega(1)$$

$$B(n) = \Theta(1)$$

• Worst case time:

$$w(n) = n$$

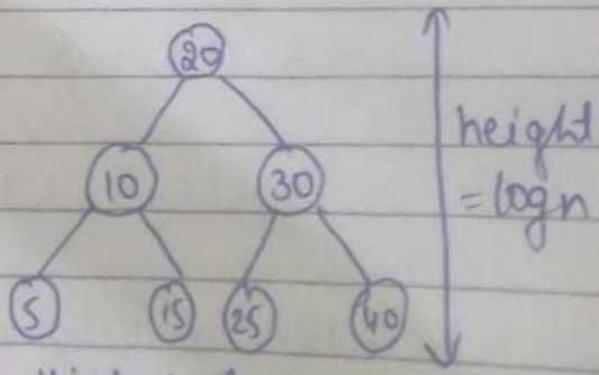
$$w(n) = O(n)$$

$$w(n) = \Omega(n)$$

$$w(n) = \Theta(n)$$

→ Binary Search Tree

key = 15 - Total comparison = 3  
 $\log 8 = 3$

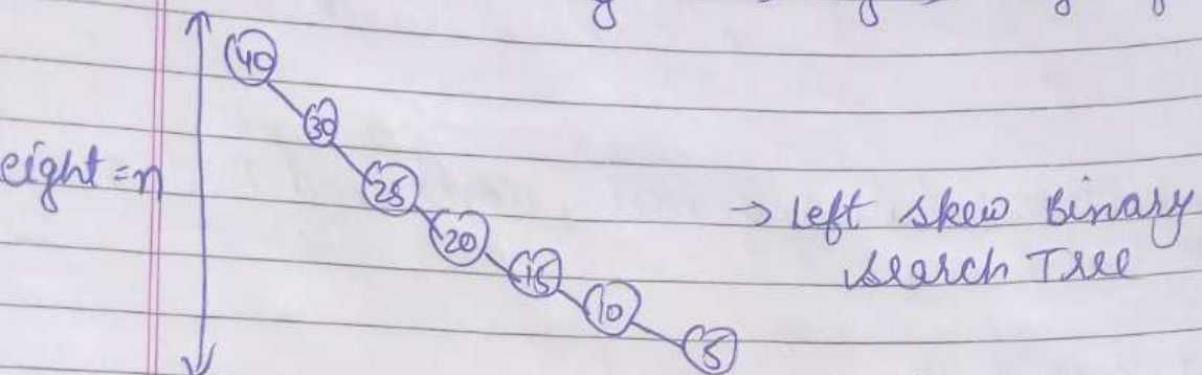


High-balanced  
Binary Search Tree

Best case - searching root element  
 Best case time -  $B(n) = 1$

Worst case - searching for leaf element

eg. 5, 15, 25, 40  
 Worst case time -  $W(n) = \log(n) = \text{height of tree}$

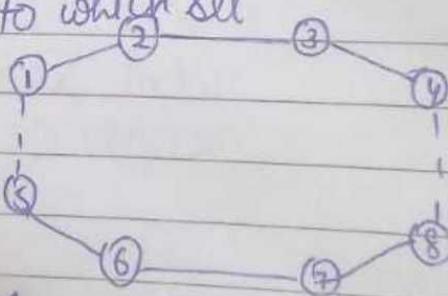


$$\therefore \min W(n) = \log n \\ \max W(n) = n$$

→ Disjoint sets Data Structure

1. find → element belongs to which set
2. Union

$$S_1 = \{1, 2, 3, 4\}$$



$$S_2 = \{5, 6, 7, 8\}$$

$$(4, 8) \quad (1, 5)$$

$$S_1 \cap S_2 = \emptyset \rightarrow \text{Intersection of two set}$$

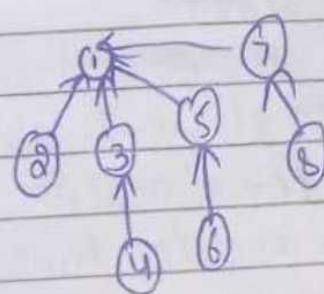
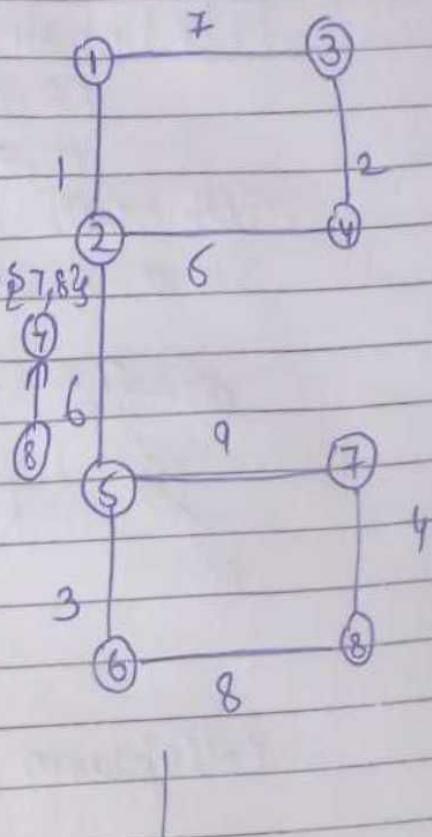
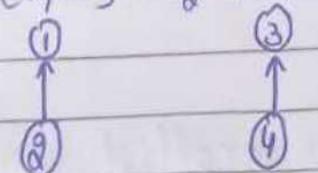
$$S_1 \cup S_2 = \{1, 2, 3, 4, 5, 6, 7, 8\} = S_3$$

Take an edge if both vertices belongs to same set then there is a cycle in a graph.

eg. Graphical representation

$$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$S_1 = \{1, 2\} \quad S_2 = \{3, 4\} \quad S_3 = \{5, 6\} \quad S_4 = \{7, 8\}$$



• Array representation

$$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

↓  
Same graph

~~Parent~~

-1	-1	-1	-1	-1	-1	-1	-1	0
1	2	3	4	5	6	7	8	

Parent

-2	1	-2	3	-2	5	-2	7
(1,2)	(3,4)	(5,6)	(7,8)				

-4	1	1	3	-2	5	-2	7
(2,4)	(6,8)						

-6	1	1	3	1	5	-2	7
(2,6)						(1,3)	

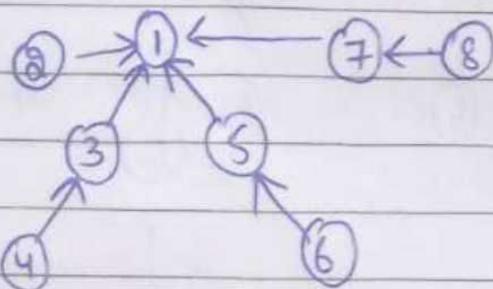
Both parents are same  
This will form a cycle

[8, 1, 1, 1, 3, 1, 5, 1, 7, 7]

(6, 8)

(5, 7)

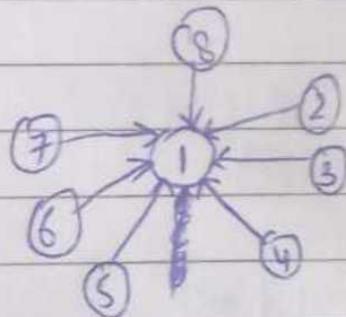
Both exists in same set  
This forms a cycle



→ This is called weighted union

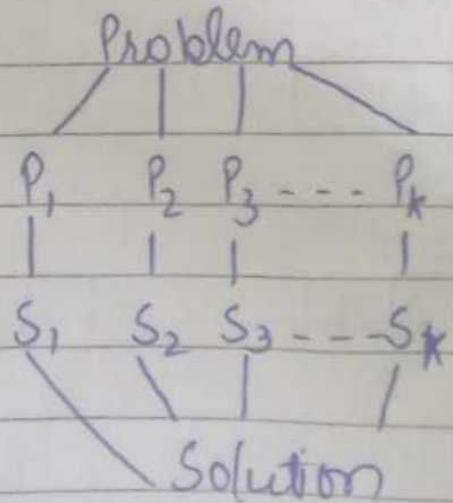
That is set with more elements always become parent

Collapsing find → We can reduce time to find value



→ Divide and Conquer

- Recursive in nature



Size = n

DAC(P)

{

if (small(P))

{

S(P);

}

else

{

divide P into  $P_1, P_2, P_3, \dots, P_k$

Apply DAC( $P_1$ ), DAC( $P_2$ ), ...

Combine (DAC( $P_1$ ), DAC( $P_2$ ), ...)

y

y

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