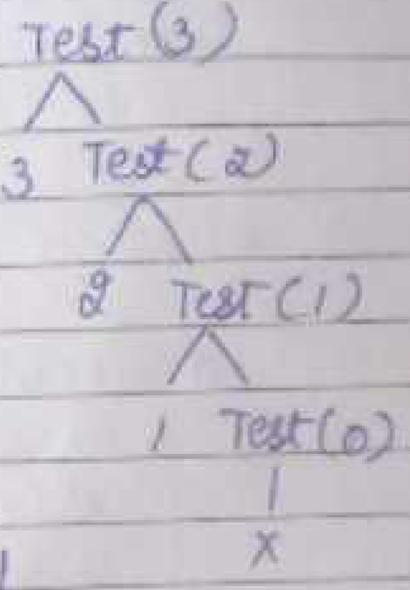
→ Recurrence Relation $(T(n) = T(n-1) + 1)$

void Test(int n)

T(n) Tracing tree

```
void Test(int n)
{
    if (n>0)
    {
        printf("%d", n);     - 1
        Test(n-1);           - T(n-1)
    }
}
```




$f(n) = n+1$

$O(n)$

$T(n) = T(n-1) + 1$

Recurrence Table

$$T(n) = \begin{cases} 1 & n=0 \\ T(n-1)+1 & n>0 \end{cases}$$

$T(n) = T(n-1) + 1$

$\therefore T(n) = T(n-1) + 1$

$T(n-1) = T(n-2) + 1$

substitute $T(n-1)$

$T(n) = [T(n-2)+1] + 1$

$T(n) = T(n-2) + 2$

$T(n) = [T(n-3)+1] + 2$

$T(n) = T(n-3) + 3$

continue k times

$T(n) = T(n-k) + k$

Assume $n-k=0$

$\therefore n=k$

$T(n) = T(n-n) + n$

$T(n) = T(0) + n$

$T(n) = 1 + n$

$\Theta(n)$

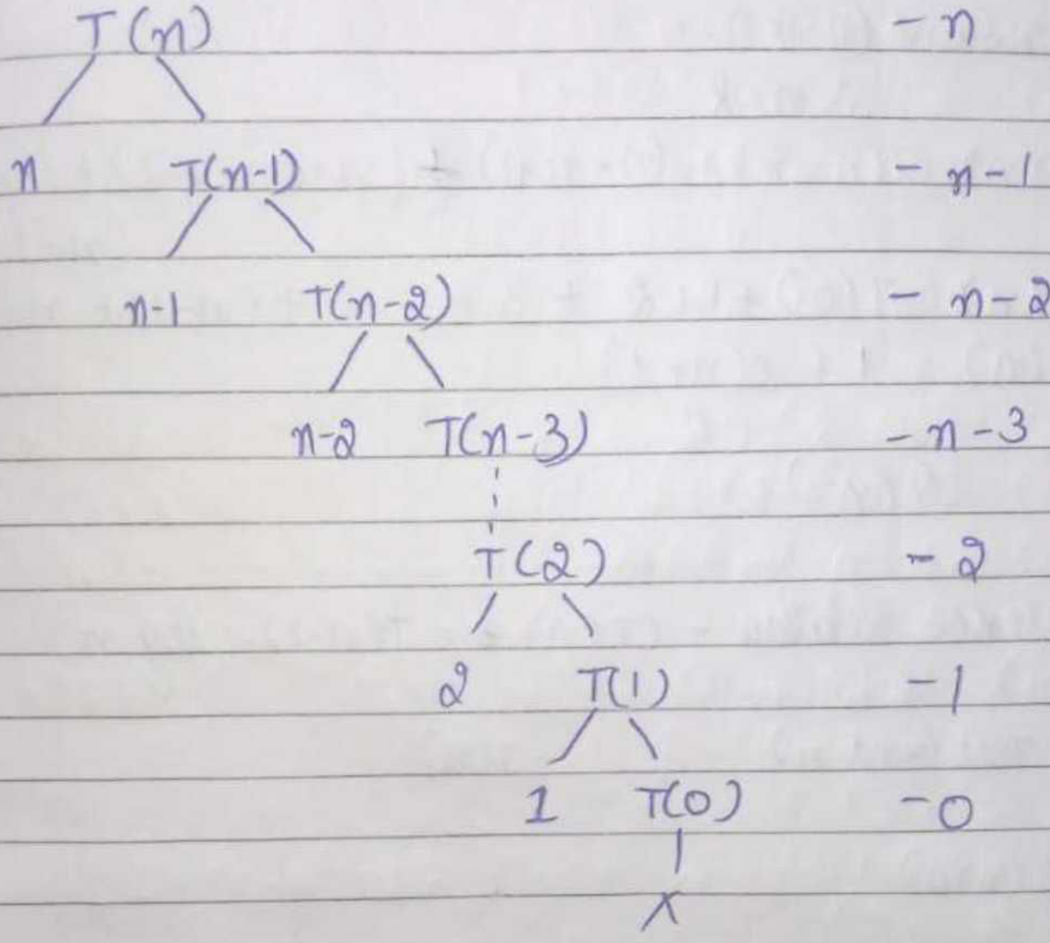
---

→ Recurrence Relation $(T(n) = T(n-1) + n)$

```
void Test(int n)                 -T(n)
{ if (n>0)                       -1
  { for(i=0; i<n; i++){          -n+1
      printf("%d", n); }         -n
  Test(n-1);                     -T(n-1)
  }
}
```

$T(n) = T(n-1) + 2n + 2$

$$T(n) = \begin{cases} 1 & n = 0 \\ T(n-1) + n & n > 0 \end{cases}$$




$$0 + 1 + 2 + \cdots + (n-1) + n = \frac{n(n+1)}{2}$$

$$T(n) = \frac{n(n+1)}{2}$$

$$\Theta = (n^2)$$

Another method

$T(n) = T(n-1) + n \quad -①$ $\quad \because T(n) = T(n-1) + n$

$T(n) = [T(n-2) + n-1] + n$ $\quad \therefore T(n-1) = T(n-2) + n-1$

$T(n) = T(n-2) + (n-1) + n \quad -②$ $\quad \because T(n-2) = T(n-3) + n-2$

$T(n) = [T(n-3) + n-2] + (n-1) + n$

$T(n) = T(n-3) + (n-2) + (n-1) + n$

$$T(n) = T(n-k) + (n-(k-1)) + (n-(k-2)) + \cdots + (n-1) + n$$

Assume $(n-k) = 0$

$\therefore n = k$

$$T(n) = T(n-n) + (n-n+1) + (n-n+2) + \cdots + (n-1) + n$$

$$T(n) = T(0) + 1 + 2 + 3 + \cdots + (n-1) + n$$

$$T(n) = 1 + \frac{n(n+1)}{2}$$

$$\Theta(n^2)$$

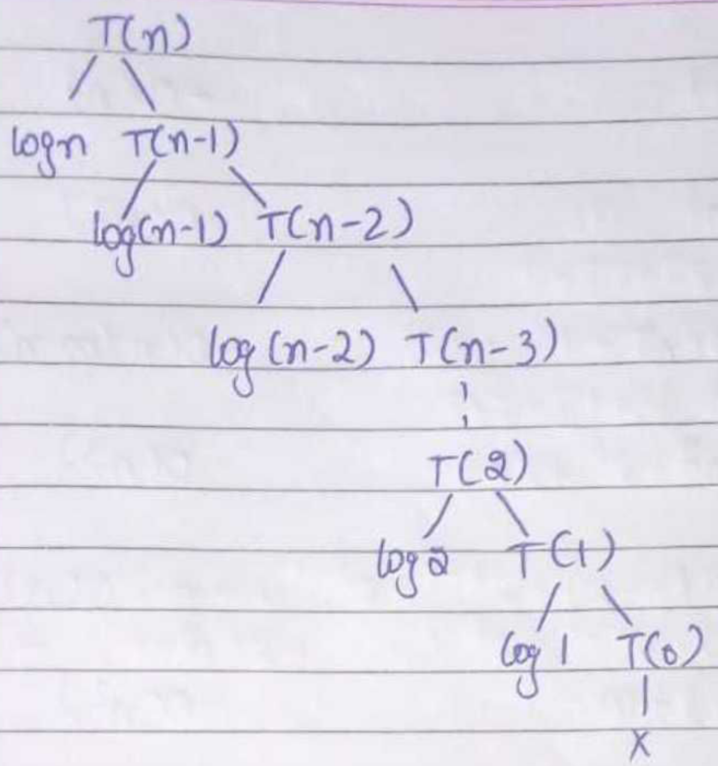
---

→ Recurrence Relation - ($T(n) = T(n-1) + \log n$)

```
void Test(int n)                  - T(n)
{
  if (n>0)
  {
    for (i=1; i<n; i=i*2)
    {
      printf("%d", i);            - log(n)
    }
    Test(n-1);                    - T(n-1)
  }
}
```

$T(n) = T(n-1) + \log(n)$

$$T(n) = \begin{cases} 1 & n = 0 \\ T(n-1) + \log(n) & n > 0 \end{cases}$$




$\log n + \log(n-1) + \log(n-2) + \cdots + \log 2 + \log 1$

$= \log [n \times (n-1) \times \cdots \times 2 \times 1]$

$= \log n!$

$O(n \log n)$

Substitution Method

$T(n) = T(n-1) + \log n$ — (1)

$T(n) = [T(n-2) + \log(n-1)] + \log n$

$T(n) = T(n-2) + \log(n-1) + \log n$ — (2)

$T(n) = [T(n-3) + \log(n-2)] + \log(n-1) + \log n$

$T(n) = T(n-3) + \log(n-2) + \log(n-1) + \log n$ — (3)

$\vdots$

$T(n) = T(n-k) + \log 1 + \log 2 + \log 3 + \cdots + \log(n-1) + \log n$

$\therefore n-k=0$

$n=k$

$T(n) = T(0) + \log n!$

$T(n) = 1 + \log n!$

~~O(n log n)~~ $O(n \log n)$

→ $T(n) = T(n-1) + 1$ $\quad - O(n)$

$T(n) = T(n-1) + n$ $\quad - O(n^2)$

$T(n) = T(n-1) + \log(n)$ $\quad - O(n \log n)$

$T(n) = T(n-1) + n^2$ $\quad - O(n^3)$

$T(n) = T(n-2) + 1$ $\quad - \frac{n}{2} \quad O(n)$

$T(n) = T(n-100) + n$ $\quad - O(n^2)$

$T(n) = 2[T(n-1) + 1]$ $\quad - O(2^n)$

---

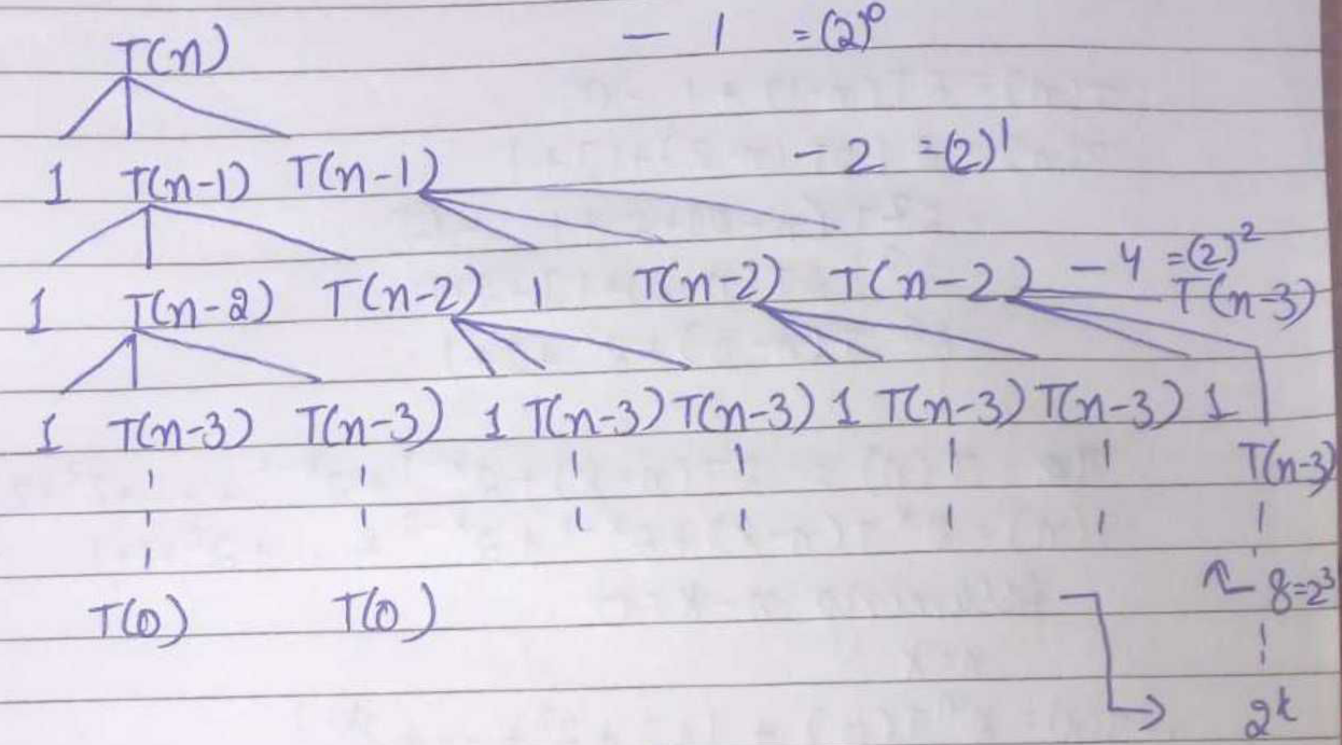
→ Recurrence Relation $T(n) = 2\{T(n-1) + 1\}$

```
Algorithm Test (int n)          - T(n)
{
  if (n>0)
  {
    printf("%d", n);            - 1
    Test (n-1);                 - T(n-1)
    Test (n-1);                 - T(n-1)
  }
}
```

$T(n) = 2\{T(n-1) + 1\}$

$$T(n) = \begin{cases} 1 & n = 0 \\ 2T(n-1) + 1 & n > 0 \end{cases}$$

# Recursion Tree Method




$2^0 + 2^1 + 2^2 + 2^3 + \cdots + 2^k = 2^{k+1} - 1$

$a = 1 \quad r = 2$ [using formula: $a + ar + ar^2 + ar^3 + \cdots + ar^k = \dfrac{a(r^{k+1} - 1)}{r - 1}$]

$\therefore = \dfrac{1(2^{k+1} - 1)}{2 - 1}$

$= 2^{k+1} - 1$

Assume $n - k = 0$

$n = k$

$\therefore 2^{n+1} - 1 \Rightarrow O(2^n)$

Substitution Method :

$T(n) = 2T(n-1) + 1$ — ①

$T(n) = 2[2T(n-2)+1]+1$

$= 2^2 T(n-2) + 2 + 1$ — ②

$= 2^2[2T(n-3)+1] + 2 + 1$

$= 2^3 T(n-3) + 2^2 + 2 + 1$

$\vdots$

$T(n) = 2T(n-k) + 2^{k-1} + 2^{k-2} + \dots + 2^2 + 2 + 1$

$T(n) = 2^k T(n-k) + 2^{k-1} + 2^{k-2} + \dots + 2^2 + 2 + 1$ — ④

Assuming $n-k=0$

$n=k$

$\therefore T(n) = 2^n T(0) + 1 + 2 + 2^2 + \dots + 2^{k-1}$

$= 2^n \times 1 + 2^k - 1$

$= 2^n + 2^n - 1$

$= 2^{n+1} - 1$

$\Theta(2^n)$

--- * ---- * ---- * ---- * ---- * ---