→ Comparison of functions

Comparison of n^2 and n^3 by simple method

n	n^2	$<$	n^3
2	$(2)^2 = 4$		$(2)^3 = 8$
3	$(3)^2 = 9$		$(3)^3 = 27$
4	$(4)^2 = 16$		$(4)^3 = 64$

Log Method

n^2 $\quad$ n^3

Apply log on Both sides

$\log n^2$ $\quad$ $\log n^3$

$2\log n$ $\quad < \quad$ $3\log n$

Log formulas:

$\log ab = \log a + \log b$

$\log \frac{a}{b} = \log a - \log b$

$\log a^b = b \log a$

$a^{\log_c b} = b^{\log_c a}$

$a^b = n$ then $b = \log_a n$

- Compare $f(n) = n^2 \log n$ and $g(n) = n(\log n)^{10}$

Apply log

$\log [n^2 \log n]$ $\quad$ $\log [n(\log n)^{10}]$

$\log n^2 + \log \log n$ $\quad$ $\log n + \log [\log n]^{10}$

$2 \log n + \log \log n$ $>$ $\log n + 10 \log \log n$

- Compare $f(n) = 3n^{\sqrt{n}}$ and $g(n) = 2^{\sqrt{n} \log n}$

$3n^{\sqrt{n}}$ $\quad$ $2^{\sqrt{n} \log n}$

Applying property of log on right side

$3n^{\sqrt{n}}$ $\quad$ ~~$(n^{\sqrt{n}}) \log$~~ $2^{\log_2 n^{\sqrt{n}}}$

$3n^{\sqrt{n}}$ $\quad$ $(n^{\sqrt{n}})^{\log_2 2}$ $\quad [\because \log_2 2 = 1]$

$3n^{\sqrt{n}}$ $>$ $n^{\sqrt{n}}$ $\quad [\because a^{\log_c b} = b^{\log_c a}]$

$3n^{\sqrt{n}}$ $\geq$ $n^{\sqrt{n}}$

→ Value wise this is greater
Asymptotically they are equal.

- Compare $f(n) = n^{\log n}$ $g(n) = 2^{\sqrt{n}}$

  Apply log on both sides

  $\log n^{\log n}$ $\log 2^{\sqrt{n}}$

  $\log n \log n$ $\sqrt{n} \log 2$

  $\log^2 n$ $\sqrt{n}$

  Again, log on Both sides

  $2 \log\log n < \frac{1}{2} \log n$

- Compare $f(n) = 2^{\log n}$ $g(n) = n^{\sqrt{n}}$

  Apply log

  $\log n \times \log_2 2$ $\sqrt{n} \log n$

  $\log n < \sqrt{n} \log n$

- Compare $f(n) = 2n$ $g(n) = 3n$

  Both are equal, Asymptotically

- Compare $f(n) = 2^n$ and $g(n) = 2^{2n}$

$\log 2^n$ $\quad$ $\log 2^{2n}$

$n \log 2$ $\quad$ $2n \log 2$

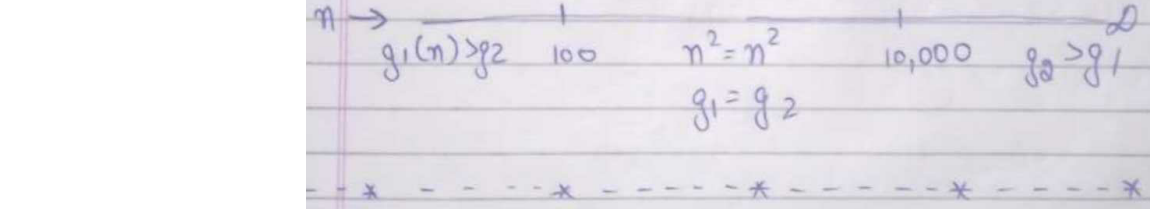
$n \quad < \quad 2n$

As we have applied log they cannot be equal.

- Compare

$$g_1(n) = \begin{cases} n^3 & n < 100 \\ n^2 & n \geq 100 \end{cases}$$

$$g_2(n) = \begin{cases} n^2 & n < 10,000 \\ n^3 & n \geq 10,000 \end{cases}$$



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True or False

1. $(n+k)^m = \Theta(n^m)$

eg. $(n+3)^2 = \Theta(n^2)$

True

2. $2^{n+1} = O(2^n)$

2×2^n

True

$2^{2n} = O(2^n)$

$4^n > 2^n$

False

4 $\sqrt{\log n} = O(\log \log n)$

2 $\sqrt{\log n} > \log \log n$

False

5 $n^{\log n} = O(2^n)$

2 $\log n \log n = n \log 2$

~~$2 \log n$~~

Ynd