→ Masters Theorem in Algorithms for Dividing Function

$T(n) = aT(n/b) + f(n)$ ① $\log_b a$

$a \geq 1$ $f(n) = \Theta(n^k \log^p n)$ ② k

$b > 1$

Case 1:

if $\log_b a > k$ then $\Theta(n^{\log_b a})$

Case 2:

if $\log_b a = k$

Case(i)

if $p > -1$ $\Theta(n^k \log^{p+1} n)$

Case(ii)

if $p = -1$ $\Theta(n^k \log \log n)$

Case(iii) if $p < -1$ $\Theta(n^k)$

Case 3:

if $\log_b a < k$

Case(i)

if $p \geq 0$ $\Theta(n^k \log^p n)$

Case(ii)

if $p < 0$ $O(n^k)$

→ Example 1:

$T(n) = 2T(n/2) + 1$

$a = 2$

$b = 2$

$f(n) = \theta(1)$

$= \theta(n^0 \log^0 n)$

$\therefore k = 0$

$p = 0$

Case 1 →

$\theta(n^1)$

$[\log_2 2 = 1 ,\ k = 0]$
Case 1

→ Example 2:

$T(n) = 4(T n/2) + n$

$\log_2 4 = 2 > k = 1 \quad p = 0$

$\theta(n^2) = $ Case 1

→ Example 3:

$T(n) = 8T(n/2) + n$

$\log_2 8 = 3 > k = 1 \quad p = 0$

Case 1

$\theta(n^3)$

→ Example 4:

$T(n) = 9T(n/3) + 1$

$\log_3 9 = 2 > k = 0$

Case 1

$\theta(n^2)$

→ Example 5:

$T(n) = 9T(n/3) + n^2$

$\log_3 9 = 2 \quad = k = 2$

$\Theta(n^2)$

Case 2

→ Example 6

$T(n) = 2T(n/2) + n$

$\log_2 2 = 1 \qquad k = 1 \qquad p = 0$

$\Theta(n \log n)$

→ Example 7

$T(n) = 4T(n/2) + n^2$

$\log_2 4 = 2 \qquad k = 2$

$\Theta(n^2 \log n)$

→ Example 8

$T(n) = 4T(n/2) + n^2 \log n$

$\log_2 4 = 2 \qquad k = 2$

$\Theta(n^2 \log^2 n)$

→ Example - 9

$T(n) = 2T(n/2) + \frac{n}{\log n} = 2T(n/2) + n\log^{-1} n$

$\log_2^2 = 1 \quad k = 1 \quad p = -1$

$\theta(n \log \log n)$

→ Example - 10

$T(n) = 2T(n/2) + n\log^{-2} n = 2T(n/2) + \frac{n}{\log^2 n}$

$\log_2^2 = 1 \quad k = 1 \quad P = -2$

$\theta(n)$

→ Example 11

$T(n) = T(n/2) + n^2$

$\log_2^1 = 0 < k = 2$

$\theta(n^2)$

→ Example 12

$T(n) = 2T(n/2) + n^2 \log^2 n$

$\log_2^2 = 1 < k = 2$

$\Theta(n^2 \log^2 n)$

→ Example 13

$T(n) = 4T(n/2) + \frac{n^3}{\log n}$

$\log_2^2 = 2 < k = 3$

$\Theta(n^3)$

→ Some more examples

- Case I:

1. $T(n) = 2T(n/2) + 1$ — $\theta(n^1)$

2. $T(n) = 4T(n/2) + 1$ — $\theta(n^2)$

3. $T(n) = 4T(n/2) + n^1$ — $\theta(n^2)$

4. $T(n) = 8T(n/2) + n^2$ — $\theta(n^3)$

5. $T(n) = 16T(n/2) + n^2$ — $\theta(n^4)$

- Case III:

1. $T(n) = T(n/2) + n$ — $\theta(n)$

2. $T(n) = 2T(n/2) + n^2$ — $\theta(n^2)$

3. $T(n) = 2T(n/2) + n^2 \log n$ — $\theta(n^2 \log n)$

4. $T(n) = 4T(n/2) + n^3 \log^2 n$ — $\theta(n^3 \log^2 n)$

5. $T(n) = 2T(n/2) + \frac{n^2}{\log n}$ — $\theta(n^2)$

· Case II:

1. $T(n) = T(n/2) + 1$ $\quad - \theta(\log n)$

2. $T(n) = 2T(n/2) + n$ $\quad - \theta(n \log n)$

3. $T(n) = 2T(n/2) + n \log n$ $\quad - \theta(n \log^2 n)$

4. $T(n) = 4T(n/2) + n^2$ $\quad - \theta(n^2 \log n)$

5. $T(n) = 4T(n/2) + (n \log n)^2$ $\quad - \theta(n^2 \log^2 n)$

6. $T(n) = 2T(n/2) + \dfrac{n}{\log n}$ $\quad - \theta(n \log \log n)$

7. $T(n) = 2T(n/2) + \dfrac{n}{\log^2 n}$ $\quad - \theta(n)$

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→ Root function

$$T(n) = \begin{cases} 1 & n = 2 \\ T(\sqrt{n}) + 1 & n > 2 \end{cases}$$

```
T(n) — void Test(int n)
       {
          if (n>2)
          {
     1  —    stmt;
T(√n)   —    Test(√n);
          }
       }
```

$T(n) = T(\sqrt{n}) + 1$

$T(n) = T(\sqrt{n}) + 1$

$T(n) = T(n^{1/2}) + 1 \quad - (1)$

$T(n) = T(n^{1/2^2}) + 1 + 1$

$T(n) = T(n^{1/2^2}) + 2 \quad - (2)$

$T(n) = T(n^{1/2^3}) + 3 \quad - (3)$

$\vdots$

$T(n) = T(n^{1/2^k}) + k \quad - (4)$

Assume n is in power 2

$\therefore \quad n = 2^m$

$T(2^m) = T(2^{m/2^k}) + k$

Assume $T(2^{m/2^k}) = T(2)$

$\therefore \frac{m}{2^k} = 1$

$m = 2^k$ and $k = \log_2 m$

since $n = 2^m$ and $m = \log_2 n$

$\therefore K = \log \log_2 n$

$\underline{O(\log \log_2 n)}$