1 Introduction

Start by building the model from the ground up, then extending in both time and space dimensions.

2 Model

2.1 Within district, one time period

A simple linear model for performance in Dallas ISD, in 2006:

$$\begin{aligned}\text{Gr5.Avg}_i = \beta_0 &+ \beta_1\text{Gr4.Avg.Lag1}_i + \beta_2\text{Gr3.Avg.Lag2}_i \\ &+ \beta_3\text{Per.Pupil.Exp}_i + \beta_4\text{Econ.Disadv.Per}_i \\ &+ \beta_5\text{T.Avg.Sal}_i + \beta_6\text{T.Avg.Exp}_i \\ &+ \beta_7\text{Gr5.Class.Size}_i + \epsilon_i\end{aligned}$$

where ϵ_j is a normally-distributed random deviation with mean 0 and variance σ^2; that is,

$$\epsilon_j \sim \mathcal{N}(0, \sigma^2) \;\text{ for all } j.$$

This model can be written alternatively in matrix form. Let

$$\boldsymbol{Y} = \begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{pmatrix}, \quad \boldsymbol{X} = \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{pmatrix},$$

$\boldsymbol{\beta} = (\beta_0, \beta_1)^T$, and $\boldsymbol{\epsilon} = (\epsilon_1, \epsilon_2, \ldots, \epsilon_n)^T$. Then (**??**) can be expressed more concisely as

$$\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}. \tag{1}$$

Thus, (1) implies that $\boldsymbol{Y} \sim \mathcal{N}(\boldsymbol{X}\boldsymbol{\beta}, \sigma^2\boldsymbol{I})$, where $\boldsymbol{I}$ is an $(n \times n)$ identity matrix.