Contents

1 Language

$e :=$	$n \mid C \mid D(e) \mid (e_1, e_2)$	Constructeurs de donnes
	x	Identificateur
	$\lambda x.e \mid \texttt{rec} f.x.e$	Fonctions (rcursive ou non)
	$e_1\ e_2$	Application
	$let\ x\ =\ e_1\ in\ e_2$	Liaison
	$if\ e\ then\ e_1\ else\ e_2$	Expression conditionnelle
	$match\ e\ with\ p \to e_1 \mid x \to e_2$	Filtrage par motif
	$l\ :\ e$	Annotation d'un point d'injection

2 Operational Semantics

2.1 Values/Environments

$$
\begin{array}{lll}
v := & n \mid b \mid C \mid D(v) \mid (v_1, v_2) & \text{Constructeurs de donnes} \\
& < \lambda x.e, \Gamma > & \text{Fermeture} \\
& < \texttt{rec} f.x.e, \Gamma > & \text{Fermeture rcursive}
\end{array}
$$

$$
\Gamma := \quad (x_1, v_1) \oplus \ldots \oplus (x_n, v_n) \quad \text{Environnement}
$$

2.2 Inference Rules

$$
\textsc{op-num}\ \frac{}{\Gamma \vdash n \mapsto n}
\qquad
\textsc{op-ident}\ \frac{v = \Gamma[x]}{\Gamma \vdash x \mapsto v}
\qquad
\textsc{op-abstr}\ \frac{}{\Gamma \vdash \lambda x.e \mapsto < \lambda x.e, \Gamma >}
\qquad
\textsc{op-abstr-rec}\ \frac{}{\Gamma \vdash \texttt{rec} f.x.e \mapsto < \texttt{rec} f.x.e, \Gamma >}
$$

$$
\textsc{op-apply}\ \frac{\begin{array}{c}\Gamma \vdash e_1 \mapsto < \lambda x.e, \Gamma_1 > \\ \Gamma \vdash e_2 \mapsto v_2 \qquad (x, v_2) \oplus \Gamma_1 \vdash e \mapsto v\end{array}}{\Gamma \vdash e_1\ e_2 \mapsto v}
\qquad
\textsc{op-apply-rec}\ \frac{\begin{array}{c}\Gamma \vdash e_1 \mapsto v_1 \qquad v_1 = < \texttt{rec} f.x.e, \Gamma_1 > \\ \Gamma \vdash e_2 \mapsto v_2 \qquad (f, v_1) \oplus (x, v_2) \oplus \Gamma_1 \vdash e \mapsto v\end{array}}{\Gamma \vdash e_1\ e_2 \mapsto v}
$$

$$
\textsc{op-if-true}\ \frac{\Gamma \vdash e \mapsto true \qquad \Gamma \vdash e_1 \mapsto v_1}{\Gamma \vdash if\ \ e\ then\ e_1\ else\ e_2 \mapsto v_1}
\qquad
\textsc{op-if-false}\ \frac{\Gamma \vdash e \mapsto false \qquad \Gamma \vdash e_2 \mapsto v_2}{\Gamma \vdash if\ \ e\ then\ e_1\ else\ e_2 \mapsto v_2}
$$

$$
\textsc{op-constr-0}\ \frac{}{\Gamma \vdash C \mapsto C}
\qquad
\textsc{op-constr-1}\ \frac{\Gamma \vdash e \mapsto v}{\Gamma \vdash D(e) \mapsto D(v)}
\qquad
\textsc{op-couple}\ \frac{\Gamma \vdash e_1 \mapsto v_1 \qquad \Gamma \vdash e_2 \mapsto v_2}{\Gamma \vdash (e_1, e_2) \mapsto (v_1, v_2)}
$$

$$
\textsc{op-match}\ \frac{\begin{array}{c}\Gamma \vdash e \mapsto v \qquad v, p \vdash_p \Gamma_p \\ \Gamma_p \oplus \Gamma \vdash e_1 \mapsto v_1\end{array}}{\Gamma \vdash match\ e\ with\ p \to e_1 \mid x \to e_2 \mapsto v_1}
\qquad
\textsc{op-match-var}\ \frac{\begin{array}{c}\Gamma \vdash e \mapsto v \qquad v, p \vdash_p \bot \\ (x, v) \oplus \Gamma \vdash e_2 \mapsto v_2\end{array}}{\Gamma \vdash match\ e\ with\ p \to e_1 \mid x \to e_2 \mapsto v_2}
$$

$$
\textsc{op-letin}\ \frac{\Gamma \vdash e_1 \mapsto v_1 \qquad (x, v_1) \oplus \Gamma \vdash e_2 \mapsto v_2}{\Gamma \vdash let\ x\ =\ e_1\ in\ e_2 \mapsto v_2}
\qquad
\textsc{op-annot}\ \frac{\Gamma \vdash e \mapsto v}{\Gamma \vdash l\ :\ e \mapsto v}
$$

$$
\textsc{opm-constr-0}\ \frac{}{C, C \vdash_p \{\}}
\qquad
\textsc{opm-constr-1}\ \frac{}{D(v), D(x) \vdash_p \{(x, v)\}}
\qquad
\textsc{opm-couple}\ \frac{}{(v_1, v_2), (x_1, x_2) \vdash_p \{(x_1, v_1); (x_2, v_2)\}}
$$

$$
\textsc{opm-constr-0-not}\ \frac{p \neq C}{C, p \vdash_p \bot}
\qquad
\textsc{opm-constr-1-not}\ \frac{p \neq D'(\_)}{D(v), p \vdash_p \bot}
\qquad
\textsc{opm-couple-not}\ \frac{p \neq (\_, \_)}{(v_1, v_2), p \vdash_p \bot}
$$

3 Operational Semantics with Injection

3.1 Inference Rules

OPINJ-NUM
$$\frac{}{\Gamma \vdash_{l:v_l} n \mapsto n}$$

OPINJ-IDENT
$$\frac{v = \Gamma[x]}{\Gamma \vdash_{l:v_l} x \mapsto v}$$

OPINJ-ABSTR
$$\frac{}{\Gamma \vdash_{l:v_l} \lambda x.e \mapsto < \lambda x.e, \Gamma >}$$

OPINJ-ABSTR-REC
$$\frac{}{\Gamma \vdash_{l:v_l} \texttt{rec} f.x.e \mapsto < \texttt{rec} f.x.e, \Gamma >}$$

OPINJ-APPLY
$$\frac{\begin{array}{c}\Gamma \vdash_{l:v_l} e_1 \mapsto < \lambda x.e, \Gamma_1 > \\ \Gamma \vdash_{l:v_l} e_2 \mapsto v_2 \qquad (x, v_2) \oplus \Gamma_1 \vdash_{l:v_l} e \mapsto v\end{array}}{\Gamma \vdash_{l:v_l} e_1\ e_2 \mapsto v}$$

OPINJ-APPLY-REC
$$\frac{\begin{array}{c}\Gamma \vdash_{l:v_l} e_1 \mapsto v_1 \qquad v_1 = < \texttt{rec} f.x.e, \Gamma_1 > \\ \Gamma \vdash_{l:v_l} e_2 \mapsto v_2 \qquad (f, v_1) \oplus (x, v_2) \oplus \Gamma_1 \vdash_{l:v_l} e \mapsto v\end{array}}{\Gamma \vdash_{l:v_l} e_1\ e_2 \mapsto v}$$

OPINJ-IF-TRUE
$$\frac{\Gamma \vdash_{l:v_l} e \mapsto true \qquad \Gamma \vdash_{l:v_l} e_1 \mapsto v_1}{\Gamma \vdash_{l:v_l} if\ e\ then\ e_1\ else\ e_2 \mapsto v_1}$$

OPINJ-IF-FALSE
$$\frac{\Gamma \vdash_{l:v_l} e \mapsto false \qquad \Gamma \vdash_{l:v_l} e_2 \mapsto v_2}{\Gamma \vdash_{l:v_l} if\ e\ then\ e_1\ else\ e_2 \mapsto v_2}$$

OPINJ-MATCH
$$\frac{\begin{array}{c}\Gamma \vdash_{l:v_l} e \mapsto v \qquad v, p \vdash_p \Gamma_p \\ \Gamma_p \oplus \Gamma \vdash_{l:v_l} e_1 \mapsto v_1\end{array}}{\Gamma \vdash_{l:v_l} match\ e\ with\ p \to e_1 \mid x \to e_2 \mapsto v_1}$$

OPINJ-MATCH-VAR
$$\frac{\begin{array}{c}\Gamma \vdash_{l:v_l} e \mapsto v \qquad v, p \vdash_p \bot \\ (x, v) \oplus \Gamma \vdash_{l:v_l} e_2 \mapsto v_2\end{array}}{\Gamma \vdash_{l:v_l} match\ e\ with\ p \to e_1 \mid x \to e_2 \mapsto v_2}$$

OPINJ-CONSTR-0
$$\frac{}{\Gamma \vdash_{l:v_l} C \mapsto C}$$

OPINJ-CONSTR-1
$$\frac{\Gamma \vdash_{l:v_l} e \mapsto v}{\Gamma \vdash_{l:v_l} D(e) \mapsto D(v)}$$

OPINJ-COUPLE
$$\frac{\Gamma \vdash_{l:v_l} e_1 \mapsto v_1 \qquad \Gamma \vdash_{l:v_l} e_2 \mapsto v_2}{\Gamma \vdash_{l:v_l} (e_1, e_2) \mapsto (v_1, v_2)}$$

OPINJ-LETIN
$$\frac{\Gamma \vdash_{l:v_l} e_1 \mapsto v_1 \qquad (x, v_1) \oplus \Gamma \vdash_{l:v_l} e_2 \mapsto v_2}{\Gamma \vdash_{l:v_l} let\ x\ =\ e_1\ in\ e_2 \mapsto v_2}$$

OPINJ-ANNOT-SAME
$$\frac{}{\Gamma \vdash_{l:v_l} l\ :\ e \mapsto v_l}$$

OPINJ-ANNOT-OTHER
$$\frac{\Gamma \vdash_{l:v_l} e \mapsto v \qquad l \neq l'}{\Gamma \vdash_{l:v_l} l'\ :\ e \mapsto v}$$

4 Injection Semantics

4.1 Inference Rules

INJ-NUM
$$\frac{}{t\Gamma^{oi} \vdash_{l:v_l} n \mapsto n}$$

INJ-LETIN
$$\frac{t\Gamma^{oi} \vdash_{l:v_l} e_1 \mapsto v_1 \qquad (x, \uparrow^{toi}(v_1)) \oplus t\Gamma^{oi} \vdash_{l:v_l} e_2 \mapsto v_2}{t\Gamma^{oi} \vdash_{l:v_l} let\ x\ =\ e_1\ in\ e_2 \mapsto v_2}$$

INJ-APPLY
$$\frac{\begin{array}{c}t\Gamma^{oi} \vdash_{l:v_l} e_1 \mapsto < \lambda x.e, \Gamma_1 > \\ t\Gamma^{oi} \vdash_{l:v_l} e_2 \mapsto v_2 \\ (x, \uparrow^{toi}(v_2)) \oplus \uparrow^{toi}(\Gamma_1) \vdash_{l:v_l} e \mapsto v\end{array}}{t\Gamma^{oi} \vdash_{l:v_l} e_1\ e_2 \mapsto v}$$

INJ-APPLY-REC
$$\frac{\begin{array}{c}t\Gamma^{oi} \vdash_{l:v_l} e_1 \mapsto v_1 \qquad v_1 = < \texttt{rec} f.x.e, \Gamma_1 > \\ t\Gamma^{oi} \vdash_{l:v_l} e_2 \mapsto v_2 \\ (f, \uparrow^{toi}(v_1)) \oplus (x, \uparrow^{toi}(v_2)) \oplus \uparrow^{toi}(\Gamma_1) \vdash_{l:v_l} e \mapsto v\end{array}}{t\Gamma^{oi} \vdash_{l:v_l} e_1\ e_2 \mapsto v}$$

$$\frac{t\Gamma^{oi} \vdash_{l:v_l} e \mapsto true \qquad t\Gamma^{oi} \vdash_{l:v_l} e_1 \mapsto v_1}{t\Gamma^{oi} \vdash_{l:v_l} if\ e\ then\ e_1\ else\ e_2 \mapsto v_1}\ \text{INJ-IF-TRUE}$$

$$\frac{t\Gamma^{oi} \vdash_{l:v_l} e \mapsto false \qquad t\Gamma^{oi} \vdash_{l:v_l} e_2 \mapsto v_2}{t\Gamma^{oi} \vdash_{l:v_l} if\ e\ then\ e_1\ else\ e_2 \mapsto v_2}\ \text{INJ-IF-FALSE}$$

$$\frac{\begin{array}{c} t\Gamma^{oi} \vdash_{l:v_l} e \mapsto v \qquad v, p \vdash_p \Gamma_p \\ \uparrow^{toi}(\Gamma_p) \oplus t\Gamma^{oi} \vdash_{l:v_l} e_1 \mapsto v_1 \end{array}}{t\Gamma^{oi} \vdash_{l:v_l} match\ e\ with\ p \to e_1 \mid x \to e_2 \mapsto v_1}\ \text{INJ-MATCH}$$

$$\frac{\begin{array}{c} t\Gamma^{oi} \vdash_{l:v_l} e \mapsto v \qquad v, p \vdash_p \bot \\ (x, \uparrow^{toi}(v)) \oplus t\Gamma^{oi} \vdash_{l:v_l} e_2 \mapsto v_2 \end{array}}{t\Gamma^{oi} \vdash_{l:v_l} match\ e\ with\ p \to e_1 \mid x \to e_2 \mapsto v_2}\ \text{INJ-MATCH-VAR}$$

$$\frac{}{t\Gamma^{oi} \vdash_{l:v_l} C \mapsto C}\ \text{INJ-CONSTR-0}$$

$$\frac{t\Gamma^{oi} \vdash_{l:v_l} e \mapsto v}{t\Gamma^{oi} \vdash_{l:v_l} D(e) \mapsto D(v)}\ \text{INJ-CONSTR-1}$$

$$\frac{t\Gamma^{oi} \vdash_{l:v_l} e_1 \mapsto v_1 \qquad t\Gamma^{oi} \vdash_{l:v_l} e_2 \mapsto v_2}{t\Gamma^{oi} \vdash_{l:v_l} (e_1, e_2) \mapsto (v_1, v_2)}\ \text{INJ-COUPLE}$$

$$\frac{v = \downarrow^{l:v_l}(t\Gamma^{oi}[x])}{t\Gamma^{oi} \vdash_{l:v_l} x \mapsto v}\ \text{INJ-IDENT}$$

$$\frac{}{t\Gamma^{oi} \vdash_{l:v_l} l\ :\ e \mapsto v_l}\ \text{INJ-ANNOT-SAME}$$

$$\frac{t\Gamma^{oi} \vdash_{l:v_l} e \mapsto v \qquad l \neq l'}{t\Gamma^{oi} \vdash_{l:v_l} l'\ :\ e \mapsto v}\ \text{INJ-ANNOT-OTHER}$$

$$\frac{}{t\Gamma^{oi} \vdash_{l:v_l} \lambda x.e \mapsto < \lambda x.e, \downarrow^{l:v_l}(t\Gamma^{oi}) >}\ \text{INJ-ABSTR}$$

$$\frac{}{t\Gamma^{oi} \vdash_{l:v_l} \texttt{rec} f.x.e \mapsto < \texttt{rec} f.x.e, \downarrow^{l:v_l}(t\Gamma^{oi}) >}\ \text{INJ-ABSTR-REC}$$

5 Over-Instrumented Semantics

5.1 Values/Environments

$$tu^{oi} := \quad [\ td^{oi} \ \mid\ u^{oi}\ ] \quad \text{Valeur avec annotation de } t\text{-dpendances}$$

$$u^{oi} := \quad [\ d^{oi} \ \mid\ v^{oi}\ ] \quad \text{Valeur avec annotation de } v\text{-dpendances}$$

$$\begin{array}{rll} v^{oi} := & n \mid b \mid C \mid D(u^{oi}) \mid (u_1^{oi}, u_2^{oi}) & \text{Constructeurs de donnes} \\ & < \lambda x.e, \Gamma^{oi} > & \text{Fermeture} \\ & < \texttt{rec} f.x.e, \Gamma^{oi} > & \text{Fermeture rcursive} \end{array}$$

$$\begin{array}{rll} td^{oi} := & (l_1, tf_1); \ldots (l_n, tf_n) & t\text{-dpendances} \\ tf : & v \to bool & t\text{-fonction d'impact} \end{array}$$

$$\begin{array}{rll} d^{oi} := & (l_1, f_1); \ldots (l_n, f_n) & v\text{-dpendances} \\ f : & v \to v & v\text{-fonction d'impact} \end{array}$$

$$t\Gamma^{oi} := \quad (x_1, tu_1^{oi}); \ldots; (x_n, tu_n^{oi}) \quad t\text{-environnement sur-instrument}$$

$$\Gamma^{oi} := \quad (x_1, u_1^{oi}); \ldots; (x_n, u_n^{oi}) \quad v\text{-environnement sur-instrument}$$

5.2 Inference Rules

OI-NUM

$$\frac{}{t\Gamma^{oi} \vdash^{oi} n \mapsto \ [\ \emptyset \ \mid \ \ [\ \emptyset \ \mid \ n\ ]\ ]}$$

OI-ABSTR

$$\frac{}{t\Gamma^{oi} \vdash^{oi} \lambda x.e \mapsto \ [\ \emptyset \ \mid \ \ [\ \emptyset \ \mid < \lambda x.e, \uparrow^{oi}_{toi}(t\Gamma^{oi}) >\ ]\ ]}$$

OI-IDENT

$$\frac{tu^{oi} = t\Gamma^{oi}[x]}{t\Gamma^{oi} \vdash^{oi} x \mapsto tu^{oi}}$$

OI-ABSTR-REC

$$\frac{}{t\Gamma^{oi} \vdash^{oi} \mathtt{rec} f.x.e \mapsto \ [\ \emptyset \ \mid \ \ [\ \emptyset \ \mid < \mathtt{rec} f.x.e, \uparrow^{oi}_{toi}(t\Gamma^{oi}) >\ ]\ ]}$$

OI-APPLY

$$\frac{\begin{array}{c} t\Gamma^{oi} \vdash^{oi} e_1 \mapsto \ [\ td_1^{oi} \ \mid \ \ [\ d_1^{oi} \ \mid < \lambda x.e, \Gamma_1^{oi} >\ ]\ ] \\ t\Gamma^{oi} \vdash^{oi} e_2 \mapsto tu_2^{oi} \qquad tu_2^{oi} = \ [\ td_2^{oi} \ \mid \ u_2^{oi}\ ] \\ (x, tu_2^{oi}) \oplus \uparrow^{toi}_{oi}(\Gamma_1^{oi}) \vdash^{oi} e \mapsto \ [\ td^{oi} \ \mid \ \ [\ d^{oi} \ \mid \ v^{oi}\ ]\ ] \\ deps\_spec\_apply(tu_2^{oi}, d_1^{oi}) = (td'^{oi}, d'^{oi}) \end{array}}{t\Gamma^{oi} \vdash^{oi} e_1\ e_2 \mapsto \ [\ td_1^{oi} \cup td_2^{oi} \cup td^{oi} \cup td'^{oi} \ \mid \ \ [\ d'^{oi} \cup d^{oi} \ \mid \ v^{oi}\ ]\ ]}$$

OI-REC-APPLY

$$\frac{\begin{array}{c} t\Gamma^{oi} \vdash^{oi} e_1 \mapsto tu_1^{oi} \qquad tu_1^{oi} = \ [\ td_1^{oi} \ \mid \ \ [\ d_1^{oi} \ \mid < \mathtt{rec} f.x.e, \Gamma_1^{oi} >\ ]\ ] \\ t\Gamma^{oi} \vdash^{oi} e_2 \mapsto tu_2^{oi} \qquad tu_2^{oi} = \ [\ td_2^{oi} \ \mid \ u_2^{oi}\ ] \\ (f, tu_1^{oi}) \oplus (x, tu_2^{oi}) \oplus \uparrow^{toi}_{oi}(\Gamma_1^{oi}) \vdash^{oi} e \mapsto \ [\ td^{oi} \ \mid \ \ [\ d^{oi} \ \mid \ v^{oi}\ ]\ ] \\ deps\_spec\_apply(tu_2^{oi}, d_1^{oi}) = (td'^{oi}, d'^{oi}) \end{array}}{t\Gamma^{oi} \vdash^{oi} e_1\ e_2 \mapsto \ [\ td_1^{oi} \cup td_2^{oi} \cup td^{oi} \cup td'^{oi} \ \mid \ \ [\ d'^{oi} \cup d^{oi} \ \mid \ v^{oi}\ ]\ ]}$$

OI-LETIN

$$\frac{\begin{array}{c} t\Gamma^{oi} \vdash^{oi} e_1 \mapsto tu_1^{oi} \qquad tu_1^{oi} = \ [\ td_1^{oi} \ \mid \ u_1^{oi}\ ] \\ (x, tu_1^{oi}) \oplus t\Gamma^{oi} \vdash^{oi} e_2 \mapsto \ [\ td_2^{oi} \ \mid \ u_2^{oi}\ ] \end{array}}{t\Gamma^{oi} \vdash^{oi} let\ x\ =\ e_1\ in\ e_2 \mapsto \ [\ td_1^{oi} \cup td_2^{oi} \ \mid \ u_2^{oi}\ ]}$$

OI-IF-TRUE

$$\frac{\begin{array}{c} t\Gamma^{oi} \vdash^{oi} e \mapsto \ [\ td^{oi} \ \mid \ \ [\ d^{oi} \ \mid \ true\ ]\ ] \\ t\Gamma^{oi} \vdash^{oi} e_1 \mapsto \ [\ td_1^{oi} \ \mid \ \ [\ d_1^{oi} \ \mid \ v_1^{oi}\ ]\ ] \\ deps\_spec\_if(t\Gamma^{oi}, e_1, e_2, d^{oi}) = (td'^{oi}, d'^{oi}) \end{array}}{t\Gamma^{oi} \vdash^{oi} if\ e\ then\ e_1\ else\ e_2 \mapsto \ [\ td'^{oi} \cup td^{oi} \cup td_1^{oi} \ \mid \ \ [\ d'^{oi} \cup d_1^{oi} \ \mid \ v_1^{oi}\ ]\ ]}$$

OI-IF-FALSE

$$\frac{\begin{array}{c} t\Gamma^{oi} \vdash^{oi} e \mapsto \ [\ td^{oi} \ \mid \ \ [\ d^{oi} \ \mid \ false\ ]\ ] \\ t\Gamma^{oi} \vdash^{oi} e_2 \mapsto \ [\ td_2^{oi} \ \mid \ \ [\ d_2^{oi} \ \mid \ v_2^{oi}\ ]\ ] \\ deps\_spec\_if(t\Gamma^{oi}, e_1, e_2, d^{oi}) = (td'^{oi}, d'^{oi}) \end{array}}{t\Gamma^{oi} \vdash^{oi} if\ e\ then\ e_1\ else\ e_2 \mapsto \ [\ td'^{oi} \cup td^{oi} \cup td_2^{oi} \ \mid \ \ [\ d'^{oi} \cup d_2^{oi} \ \mid \ v_2^{oi}\ ]\ ]}$$

OI-MATCH

$$\frac{\begin{array}{c} t\Gamma^{oi} \vdash^{oi} e \mapsto tu^{oi} \qquad tu^{oi} = \ [\ td^{oi} \ \mid \ \ [\ d^{oi} \ \mid \ v^{oi}\ ]\ ] \\ tu^{oi}, p \vdash^{oi}_{p} t\Gamma^{oi}_{p} \\ t\Gamma^{oi}_{p} \oplus t\Gamma^{oi} \vdash^{oi} e_1 \mapsto \ [\ td_1^{oi} \ \mid \ \ [\ d_1^{oi} \ \mid \ v_1^{oi}\ ]\ ] \\ deps\_spec\_match(t\Gamma^{oi}, p, x, e_1, e_2, d^{oi}) = (td'^{oi}, d'^{oi}) \end{array}}{t\Gamma^{oi} \vdash^{oi} match\ e\ with\ p \rightarrow e_1 \mid x \rightarrow e_2 \mapsto \ [\ td'^{oi} \cup td^{oi} \cup td_1^{oi} \ \mid \ \ [\ d'^{oi} \cup d_1^{oi} \ \mid \ v_1^{oi}\ ]\ ]}$$

OI-MATCH-VAR

$$\frac{t\Gamma^{oi} \vdash^{oi} e \mapsto tu^{oi} \qquad tu^{oi} = [\ td^{oi}\ \mid\ [\ d^{oi}\ \mid\ v^{oi}\ ]\ ] \quad tu^{oi}, p \vdash_p^{oi} \bot \quad (x, tu^{oi}) \oplus t\Gamma^{oi} \vdash^{oi} e_2 \mapsto [\ td_2^{oi}\ \mid\ [\ d_2^{oi}\ \mid\ v_2^{oi}\ ]\ ] \quad deps\_spec\_match(t\Gamma^{oi}, p, x, e_1, e_2, d^{oi}) = (td'^{oi}, d'^{oi})}{t\Gamma^{oi} \vdash^{oi} match\ e\ with\ p \rightarrow e_1 \mid x \rightarrow e_2 \mapsto [\ td'^{oi} \cup td^{oi} \cup td_2^{oi}\ \mid\ [\ d'^{oi} \cup d_2^{oi}\ \mid\ v_2^{oi}\ ]\ ]}$$

OI-CONSTR-0

$$\frac{}{t\Gamma^{oi} \vdash^{oi} C \mapsto [\ \emptyset\ \mid\ [\ \emptyset\ \mid\ C\ ]\ ]}$$

OI-CONSTR-1

$$\frac{t\Gamma^{oi} \vdash^{oi} e \mapsto [\ td^{oi}\ \mid\ u^{oi}\ ]}{t\Gamma^{oi} \vdash^{oi} D(e) \mapsto [\ td^{oi}\ \mid\ [\ \emptyset\ \mid\ D(u^{oi})\ ]\ ]}$$

OI-COUPLE

$$\frac{t\Gamma^{oi} \vdash^{oi} e_1 \mapsto [\ td_1^{oi}\ \mid\ u_1^{oi}\ ] \qquad t\Gamma^{oi} \vdash^{oi} e_2 \mapsto [\ td_2^{oi}\ \mid\ u_2^{oi}\ ]}{t\Gamma^{oi} \vdash^{oi} (e_1, e_2) \mapsto [\ td_1^{oi} \cup td_2^{oi}\ \mid\ [\ \emptyset\ \mid\ (u_1^{oi}, u_2^{oi})\ ]\ ]}$$

OI-ANNOT

$$\frac{t\Gamma^{oi} \vdash^{oi} e \mapsto [\ td^{oi}\ \mid\ [\ d^{oi}\ \mid\ v^{oi}\ ]\ ]}{t\Gamma^{oi} \vdash^{oi} l\ :\ e \mapsto [\ td^{oi}\ \mid\ [\ (l, fun\ x \Rightarrow x); d^{oi}\ \mid\ v^{oi}\ ]\ ]}$$

OIM-CONSTR-0

$$\frac{}{[\ td^{oi}\ \mid\ [\ d^{oi}\ \mid\ C\ ]\ ], C \vdash_p^{oi} \{\}}$$

OIM-CONSTR-1

$$\frac{}{[\ td^{oi}\ \mid\ [\ d^{oi}\ \mid\ D(u^{oi})\ ]\ ], D(x) \vdash_p^{oi} \{(x,\ [\ \emptyset\ \mid\ u^{oi}\ ])\}}$$

OIM-COUPLE

$$\frac{}{(\ [\ td_1^{oi}\ \mid\ [\ d_1^{oi}\ \mid\ u_1^{oi}\ ]\ ],\ [\ td_2^{oi}\ \mid\ [\ d_2^{oi}\ \mid\ u_2^{oi}\ ]\ ]), (x_1, x_2) \vdash_p^{oi} \{(x_1,\ [\ \emptyset\ \mid\ u_1^{oi}\ ]); (x_2,\ [\ \emptyset\ \mid\ u_2^{oi}\ ])\}}$$

OIM-CONSTR-0-NOT

$$\frac{p \neq C}{[\ td^{oi}\ \mid\ [\ d^{oi}\ \mid\ C\ ]\ ], p \vdash_p^{oi} \bot}$$

OIM-CONSTR-1-NOT

$$\frac{p \neq D'(\_)}{[\ td^{oi}\ \mid\ [\ d^{oi}\ \mid\ D(u^{oi})\ ]\ ], p \vdash_p^{oi} \bot}$$

OIM-COUPLE-NOT

$$\frac{p \neq (\_, \_)}{(\ [\ td_1^{oi}\ \mid\ [\ d_1^{oi}\ \mid\ u_1^{oi}\ ]\ ],\ [\ td_2^{oi}\ \mid\ [\ d_2^{oi}\ \mid\ u_2^{oi}\ ]\ ]), p \vdash_p^{oi} \bot}$$

6 Instrumented Semantics

6.1 Values/Environments

$tu^i :=\ [\ td^i\ \mid\ u^i\ ]$ Valeur avec annotation de t-dpendance

$u^i :=\ [\ d^i\ \mid\ v^i\ ]$ Valeur avec annotation de v-dpendance

$v^i :=$	$n \mid b \mid C \mid D(u^i) \mid (u_1^i, u_2^i)$	Constructeurs de donnes
	$< \lambda x.e, \Gamma^i >$	Fermeture
	$< \texttt{rec} f.x.e, \Gamma^i >$	Fermeture rcursive

$td^i :=\ \{l_1; \ldots; l_n\}$ t-dpendances

$d^i :=\ \{l_1; \ldots; l_m\}$ v-dpendances

$$t\Gamma^i := \quad (x_1, tu_1^i); \ldots; (x_n, tu_n^i) \quad t\text{-environnement instrument}$$

$$\Gamma^i := \quad (x_1, u_1^i); \ldots; (x_n, u_n^i) \quad v\text{-environnement instrument}$$

6.2 Inference Rules

I-NUM
$$\frac{}{t\Gamma^i \vdash^i n \mapsto \ [\ \emptyset \ \mid \ \ [\ \emptyset \ \mid \ n\ ]\ ]}$$

I-IDENT
$$\frac{tu^i = t\Gamma^i[x]}{t\Gamma^i \vdash^i x \mapsto tu^i}$$

I-LETIN
$$\frac{t\Gamma^i \vdash^i e_1 \mapsto tu_1^i \qquad tu_1^i = \ [\ td_1^i \ \mid \ u_1^i\ ] \qquad (x, tu_1^i) \oplus t\Gamma^i \vdash^i e_2 \mapsto \ [\ td_2^i \ \mid \ u_2^i\ ]}{t\Gamma^i \vdash^i let\ x \ = \ e_1\ in\ e_2 \mapsto \ [\ td_1^i \cup td_2^i \ \mid \ u_2^i\ ]}$$

I-ABSTR
$$\frac{}{t\Gamma^i \vdash^i \lambda x.e \mapsto \ [\ \emptyset \ \mid \ \ [\ \emptyset \ \mid < \lambda x.e, \uparrow_{ti}^i(t\Gamma^i) >\ ]\ ]}$$

I-ABSTR-REC
$$\frac{}{t\Gamma^i \vdash^i \mathtt{rec} f.x.e \mapsto \ [\ \emptyset \ \mid \ \ [\ \emptyset \ \mid < \mathtt{rec} f.x.e, \uparrow_{ti}^i(t\Gamma^i) >\ ]\ ]}$$

I-APPLY
$$\frac{\begin{array}{c} t\Gamma^i \vdash^i e_1 \mapsto \ [\ td_1^i \ \mid \ \ [\ d_1^i \ \mid < \lambda x.e, \Gamma_1^i >\ ]\ ] \\ t\Gamma^i \vdash^i e_2 \mapsto tu_2^i \qquad tu_2^i = \ [\ td_2^i \ \mid \ u_2^i\ ] \\ (x, tu_2^i) \oplus \uparrow_i^{ti}(\Gamma_1^i) \vdash^i e \mapsto \ [\ td^i \ \mid \ \ [\ d^i \ \mid \ v^i\ ]\ ] \end{array}}{t\Gamma^i \vdash^i e_1\ e_2 \mapsto \ [\ td_1^i \cup td_2^i \cup td^i \cup d_1^i \ \mid \ \ [\ d_1^i \cup d^i \ \mid \ v^i\ ]\ ]}$$

I-APPLY-REC
$$\frac{\begin{array}{c} t\Gamma^i \vdash^i e_1 \mapsto tu_1^i \qquad tu_1^i = \ [\ td_1^i \ \mid \ \ [\ d_1^i \ \mid < \mathtt{rec} f.x.e, \Gamma_1^i >\ ]\ ] \\ t\Gamma^i \vdash^i e_2 \mapsto tu_2^i \qquad tu_2^i = \ [\ td_2^i \ \mid \ u_2^i\ ] \\ (f, tu_1^i) \oplus (x, tu_2^i) \oplus \uparrow_i^{ti}(\Gamma_1^i) \vdash^i e \mapsto \ [\ td^i \ \mid \ \ [\ d^i \ \mid \ v^i\ ]\ ] \end{array}}{t\Gamma^i \vdash^i e_1\ e_2 \mapsto \ [\ td_1^i \cup td_2^i \cup td^i \cup d_1^i \ \mid \ \ [\ d_1^i \cup d^i \ \mid \ v^i\ ]\ ]}$$

I-IF-TRUE
$$\frac{t\Gamma^i \vdash^i e \mapsto \ [\ td^i \ \mid \ \ [\ d^i \ \mid \ true\ ]\ ] \qquad t\Gamma^i \vdash^i e_1 \mapsto \ [\ td_1^i \ \mid \ \ [\ d_1^i \ \mid \ v_1^i\ ]\ ]}{t\Gamma^i \vdash^i if\ e\ then\ e_1\ else\ e_2 \mapsto \ [\ d^i \cup td^i \cup td_1^i \ \mid \ \ [\ d^i \cup d_1^i \ \mid \ v_1^i\ ]\ ]}$$

I-IF-FALSE
$$\frac{t\Gamma^i \vdash^i e \mapsto \ [\ td^i \ \mid \ \ [\ d^i \ \mid \ false\ ]\ ] \qquad t\Gamma^i \vdash^i e_2 \mapsto \ [\ td_2^i \ \mid \ \ [\ d_2^i \ \mid \ v_2^i\ ]\ ]}{t\Gamma^i \vdash^i if\ e\ then\ e_1\ else\ e_2 \mapsto \ [\ d^i \cup td^i \cup td_2^i \ \mid \ \ [\ d^i \cup d_2^i \ \mid \ v_2^i\ ]\ ]}$$

I-MATCH
$$\frac{\begin{array}{c} t\Gamma^i \vdash^i e \mapsto tu^i \qquad tu^i = \ [\ td^i \ \mid \ \ [\ d^i \ \mid \ v^i\ ]\ ] \qquad tu^i, p \vdash_p^i t\Gamma_p^i \\ t\Gamma_p^i \oplus t\Gamma^i \vdash^i e_1 \mapsto \ [\ td_1^i \ \mid \ \ [\ d_1^i \ \mid \ v_1^i\ ]\ ] \end{array}}{t\Gamma^i \vdash^i match\ e\ with\ p \to e_1 \mid x \to e_2 \mapsto \ [\ d^i \cup td^i \cup td_1^i \ \mid \ \ [\ d^i \cup d_1^i \ \mid \ v_1^i\ ]\ ]}$$

I-MATCH-VAR
$$\frac{\begin{array}{c} t\Gamma^i \vdash^i e \mapsto tu^i \qquad tu^i = \ [\ td^i \ \mid \ \ [\ d^i \ \mid \ v^i\ ]\ ] \qquad tu^i, p \vdash_p^i \bot \\ (x, tu^i) \oplus t\Gamma^i \vdash^i e_2 \mapsto \ [\ td_2^i \ \mid \ \ [\ d_2^i \ \mid \ v_2^i\ ]\ ] \end{array}}{t\Gamma^i \vdash^i match\ e\ with\ p \to e_1 \mid x \to e_2 \mapsto \ [\ d^i \cup td^i \cup td_2^i \ \mid \ \ [\ d^i \cup d_2^i \ \mid \ v_2^i\ ]\ ]}$$

$$\textsc{i-constr-0}\quad \frac{}{t\Gamma^i \vdash^i C \mapsto [\ \emptyset \ \mid\ [\ \emptyset \ \mid\ C\ ]\ ]}$$

$$\textsc{i-constr-1}\quad \frac{t\Gamma^i \vdash^i e \mapsto [\ td^i \ \mid\ u^i\ ]}{t\Gamma^i \vdash^i D(e) \mapsto [\ td^i \ \mid\ [\ \emptyset \ \mid\ D(u^i)\ ]\ ]}$$

$$\textsc{i-couple}\quad \frac{t\Gamma^i \vdash^i e_1 \mapsto [\ td^i_1 \ \mid\ u^i_1\ ] \qquad t\Gamma^i \vdash^i e_2 \mapsto [\ td^i_2 \ \mid\ u^i_2\ ]}{t\Gamma^i \vdash^i (e_1, e_2) \mapsto [\ td^i_1 \cup td^i_2 \ \mid\ [\ \emptyset \ \mid\ (u^i_1, u^i_2)\ ]\ ]}$$

$$\textsc{i-annot}\quad \frac{t\Gamma^i \vdash^i e \mapsto [\ td^i \ \mid\ [\ d^i \ \mid\ v^i\ ]\ ]}{t\Gamma^i \vdash^i l\ :\ e \mapsto [\ td^i \ \mid\ [\ l; d^i \ \mid\ v^i\ ]\ ]}$$

$$\textsc{im-constr-0}\quad \frac{}{[\ td^i \ \mid\ [\ d^i \ \mid\ C\ ]\ ], C \vdash^i_p \{\}}$$

$$\textsc{im-constr-1}\quad \frac{}{[\ td^i \ \mid\ [\ d^i \ \mid\ D(u^i)\ ]\ ], D(x) \vdash^i_p \{(x,\ [\ \emptyset \ \mid\ u^i\ ])\}}$$

$$\textsc{im-couple}\quad \frac{}{[\ td^i \ \mid\ [\ d^i \ \mid\ (u^i_1, u^i_2)\ ]\ ], (x_1, x_2) \vdash^i_p \{(x_1,\ [\ \emptyset \ \mid\ u^i_1\ ]); (x_2,\ [\ \emptyset \ \mid\ u^i_2\ ])\}}$$

$$\textsc{im-constr-0-not}\quad \frac{p \neq C}{[\ td^i \ \mid\ [\ d^i \ \mid\ C\ ]\ ], p \vdash^i_p \bot}$$

$$\textsc{im-constr-1-not}\quad \frac{p \neq D(\_)}{[\ td^i \ \mid\ [\ d^i \ \mid\ D(u^i)\ ]\ ], p \vdash^i_p \bot}$$

$$\textsc{im-couple-not}\quad \frac{p \neq (\_, \_)}{[\ td^i \ \mid\ [\ d^i \ \mid\ (u^i_1, u^i_2)\ ]\ ], p \vdash^i_p \bot}$$

7 Multiple Instrumented Semantics

7.1 Inference Rules

$$\textsc{i-multiple}\quad \frac{v^{im} = \{tu^i \mid \exists t\Gamma^i \in t\Gamma^{im}.\ t\Gamma^i \vdash^i e \mapsto tu^i\}}{t\Gamma^{im} \vdash^{im} e \mapsto v^{im}}$$

8 Collecting Semantics

8.1 Values/Environments

$$tu^c := \quad [\ td^c \ \mid\ d^c \ \mid\ vs^i\ ] \quad \text{Valeur collectrice}$$

$$\begin{aligned} td^c &:= \quad \{l_1; \ldots; l_n\} \quad t\text{-dpendances} \\ d^c &:= \quad \{l_1; \ldots; l_m\} \quad v\text{-dpendances} \end{aligned}$$

$$t\Gamma^c := \quad (x_1, tu^c_1); \ldots; (x_n, tu^c_n) \quad t\text{-environnement collecteur}$$

8.2 Inference Rules

C-NUM
$$\frac{}{t\Gamma^c \vdash^c n \mapsto [\ \emptyset \ \mid \ \emptyset \ \mid \ \{n\}\ ]}$$

C-IDENT
$$\frac{tu^c = t\Gamma^c[x]}{t\Gamma^c \vdash^c x \mapsto tu^c}$$

C-IDENT-EMPTY
$$\frac{x \notin support(t\Gamma^c)}{t\Gamma^c \vdash^c x \mapsto [\ \emptyset \ \mid \ \emptyset \ \mid \ \emptyset\ ]}$$

C-ABSTR
$$\frac{vs^i = \{< \lambda x.e, \Gamma^i > \ \mid \Gamma^i \in \uparrow_c^{im}(t\Gamma^c)\}}{t\Gamma^c \vdash^c \lambda x.e \mapsto [\ \emptyset \ \mid \ \emptyset \ \mid \ vs^i\ ]}$$

C-ABSTR-REC
$$\frac{vs^i = \{< \mathtt{rec} f.x.e, \Gamma^i > \ \mid \Gamma^i \in \uparrow_c^{im}(t\Gamma^c)\}}{t\Gamma^c \vdash^c \mathtt{rec} f.x.e \mapsto [\ \emptyset \ \mid \ \emptyset \ \mid \ vs^i\ ]}$$

C-APPLY
$$\frac{\begin{array}{c} t\Gamma^c \vdash^c e_1 \mapsto [\ td_1^c \ \mid \ d_1^c \ \mid \ vs_1^i\ ] \qquad t\Gamma^c \vdash^c e_2 \mapsto [\ td_2^c \ \mid \ d_2^c \ \mid \ vs_2^i\ ] \\ v^{im} = multiple\_instrumented\_application(td_1^c, d_1^c, vs_1^i, td_2^c, d_2^c, vs_2^i) \\ (\forall l.\ l \in td^c \Leftrightarrow (\exists (td^i, d^i, v^i).\ l \in td^i \wedge [\ td^i \ \mid \ [\ d^i \ \mid \ v^i\ ]\ ] \in v^{im})) \\ (\forall l.\ l \in d^c \Leftrightarrow (\exists (td^i, d^i, v^i).\ l \in d^i \wedge [\ td^i \ \mid \ [\ d^i \ \mid \ v^i\ ]\ ] \in v^{im})) \\ vs^i = \{v^i \mid \exists (td^i, d^i).\ [\ td^i \ \mid \ [\ d^i \ \mid \ v^i\ ]\ ] \in v^{im}\} \end{array}}{t\Gamma^c \vdash^c e_1\ e_2 \mapsto [\ td^c \ \mid \ d^c \ \mid \ vs^i\ ]}$$

C-IF-TRUE
$$\frac{\begin{array}{c} t\Gamma^c \vdash^c e \mapsto [\ td^c \ \mid \ d^c \ \mid \ vs^i\ ] \qquad true \in vs^i \qquad false \notin vs^i \\ t\Gamma^c \vdash^c e_1 \mapsto [\ td_1^c \ \mid \ d_1^c \ \mid \ vs_1^i\ ] \qquad tu'^c = [\ d^c \cup td^c \cup td_1^c \ \mid \ d^c \cup d_1^c \ \mid \ vs_1^i\ ] \end{array}}{t\Gamma^c \vdash^c if\ e\ then\ e_1\ else\ e_2 \mapsto tu'^c}$$

C-IF-FALSE
$$\frac{\begin{array}{c} t\Gamma^c \vdash^c e \mapsto [\ td^c \ \mid \ d^c \ \mid \ vs^i\ ] \qquad false \in vs^i \qquad true \notin vs^i \\ t\Gamma^c \vdash^c e_2 \mapsto [\ td_2^c \ \mid \ d_2^c \ \mid \ vs_2^i\ ] \qquad tu'^c = [\ d^c \cup td^c \cup td_2^c \ \mid \ d^c \cup d_2^c \ \mid \ vs_2^i\ ] \end{array}}{t\Gamma^c \vdash^c if\ e\ then\ e_1\ else\ e_2 \mapsto tu'^c}$$

C-IF-UNKNOWN
$$\frac{\begin{array}{c} t\Gamma^c \vdash^c e \mapsto [\ td^c \ \mid \ d^c \ \mid \ vs^i\ ] \qquad true \in vs^i \qquad false \in vs^i \\ t\Gamma^c \vdash^c e_1 \mapsto [\ td_1^c \ \mid \ d_1^c \ \mid \ vs_1^i\ ] \qquad t\Gamma^c \vdash^c e_2 \mapsto [\ td_2^c \ \mid \ d_2^c \ \mid \ vs_2^i\ ] \\ tu'^c = [\ d^c \cup td^c \cup td_1^c \cup td_2^c \ \mid \ d^c \cup d_1^c \cup d_2^c \ \mid \ vs_1^i \cup vs_2^i\ ] \end{array}}{t\Gamma^c \vdash^c if\ e\ then\ e_1\ else\ e_2 \mapsto tu'^c}$$

C-IF-EMPTY
$$\frac{t\Gamma^c \vdash^c e \mapsto [\ td^c \ \mid \ d^c \ \mid \ vs^i\ ] \qquad true \notin vs^i \qquad false \notin vs^i}{t\Gamma^c \vdash^c if\ e\ then\ e_1\ else\ e_2 \mapsto [\ \emptyset \ \mid \ \emptyset \ \mid \ \emptyset\ ]}$$

C-MATCH
$$\frac{\begin{array}{c} t\Gamma^c \vdash^c e \mapsto tu^c \qquad tu^c = [\ td^c \ \mid \ d^c \ \mid \ vs^i\ ] \qquad vs^i \cap vs_{matchable}^i \neq \emptyset \\ tu^c, p \vdash_p^c t\Gamma_p^c \qquad t\Gamma_p^c \oplus t\Gamma^c \vdash^c e_1 \mapsto [\ td_1^c \ \mid \ d_1^c \ \mid \ vs_1^i\ ] \end{array}}{t\Gamma^c \vdash^c match\ e\ with\ p \to e_1 \mid x \to e_2 \mapsto [\ d^c \cup td^c \cup td_1^c \ \mid \ d^c \cup d_1^c \ \mid \ vs_1^i\ ]}$$

C-MATCH-VAR
$$\frac{\begin{array}{c} t\Gamma^c \vdash^c e \mapsto tu^c \qquad tu^c = [\ td^c \ \mid \ d^c \ \mid \ vs^i\ ] \qquad vs^i \cap vs_{matchable}^i \neq \emptyset \\ tu^c, p \vdash_p^c \perp \qquad (x, tu^c) \oplus t\Gamma^c \vdash^c e_2 \mapsto [\ td_2^c \ \mid \ d_2^c \ \mid \ vs_2^i\ ] \end{array}}{t\Gamma^c \vdash^c match\ e\ with\ p \to e_1 \mid x \to e_2 \mapsto [\ d^c \cup td^c \cup td_2^c \ \mid \ d^c \cup d_2^c \ \mid \ vs_2^i\ ]}$$

C-MATCH-UNKNOWN
$$\frac{\begin{array}{c} t\Gamma^c \vdash^c e \mapsto tu^c \qquad tu^c = [\ td^c \ \mid \ d^c \ \mid \ vs^i\ ] \qquad vs^i \cap vs_{matchable}^i \neq \emptyset \\ tu^c, p \vdash_p^c ?\ t\Gamma_p^c \qquad t\Gamma_p^c \oplus t\Gamma^c \vdash^c e_1 \mapsto [\ td_1^c \ \mid \ d_1^c \ \mid \ vs_1^i\ ] \\ (x, tu^c) \oplus t\Gamma^c \vdash^c e_2 \mapsto [\ td_2^c \ \mid \ d_2^c \ \mid \ vs_2^i\ ] \end{array}}{t\Gamma^c \vdash^c match\ e\ with\ p \to e_1 \mid x \to e_2 \mapsto [\ d^c \cup td^c \cup td_1^c \cup td_2^c \ \mid \ d^c \cup d_1^c \cup d_2^c \ \mid \ vs_1^i \cup vs_2^i\ ]}$$

$$\frac{t\Gamma^c \vdash^c e \mapsto tu^c \qquad tu^c = [\ td^c \ \mid\ d^c \ \mid\ vs^i\ ] \qquad vs^i \cap vs^i_{matchable} = \emptyset}{t\Gamma^c \vdash^c match\ e\ with\ p \to e_1 \mid x \to e_2 \mapsto\ [\ \emptyset \ \mid\ \emptyset \ \mid\ \emptyset\ ]}\ \textsc{C-Match-Empty}$$

$$\frac{}{t\Gamma^c \vdash^c C \mapsto\ [\ \emptyset \ \mid\ \emptyset \ \mid\ \{C\}\ ]}\ \textsc{C-Constr-0}$$

$$\frac{t\Gamma^c \vdash^c e \mapsto\ [\ td^c \ \mid\ d^c \ \mid\ vs^i\ ]}{t\Gamma^c \vdash^c D(e) \mapsto\ [\ td^c \ \mid\ \emptyset \ \mid\ \{D(\ [\ d^c \ \mid\ v^i\ ]) \mid v^i \in vs^i\}\ ]}\ \textsc{C-Constr-1}$$

$$\frac{t\Gamma^c \vdash^c e_1 \mapsto\ [\ td^c_1 \ \mid\ d^c_1 \ \mid\ vs^i_1\ ] \qquad t\Gamma^c \vdash^c e_2 \mapsto\ [\ td^c_2 \ \mid\ d^c_2 \ \mid\ vs^i_2\ ]}{t\Gamma^c \vdash^c (e_1, e_2) \mapsto\ [\ td^c_1 \cup td^c_2 \ \mid\ \emptyset \ \mid\ \{(\ [\ d^c_1 \ \mid\ v^i_1\ ],\ [\ d^c_2 \ \mid\ v^i_2\ ]) \mid v^i_1 \in vs^i_1 \wedge v^i_2 \in vs^i_2\}\ ]}\ \textsc{C-Couple}$$

$$\frac{\begin{array}{c} t\Gamma^c \vdash^c e_1 \mapsto tu^c_1 \qquad tu^c_1 = [\ td^c_1 \ \mid\ d^c_1 \ \mid\ vs^i_1\ ] \\ vs^i_1 \neq \emptyset \qquad (x, tu^c_1) \oplus t\Gamma^c \vdash^c e_2 \mapsto\ [\ td^c_2 \ \mid\ d^c_2 \ \mid\ vs^i_2\ ] \end{array}}{t\Gamma^c \vdash^c let\ x\ =\ e_1\ in\ e_2 \mapsto\ [\ td^c_1 \cup td^c_2 \ \mid\ d^c_2 \ \mid\ vs^i_2\ ]}\ \textsc{C-LetIn}$$

$$\frac{t\Gamma^c \vdash^c e_1 \mapsto\ [\ td^c_1 \ \mid\ d^c_1 \ \mid\ \emptyset\ ]}{t\Gamma^c \vdash^c let\ x\ =\ e_1\ in\ e_2 \mapsto\ [\ \emptyset \ \mid\ \emptyset \ \mid\ \emptyset\ ]}\ \textsc{C-LetIn-Empty}$$

$$\frac{t\Gamma^c \vdash^c e \mapsto\ [\ td^c \ \mid\ d^c \ \mid\ vs^i\ ]}{t\Gamma^c \vdash^c l\ :\ e \mapsto\ [\ td^c \ \mid\ \{l\} \cup d^c \ \mid\ vs^i\ ]}\ \textsc{C-Annot}$$

$$vs^i_{matchable} = \{C \mid \forall C \in Constr^0\} \cup \{D(u^i) \mid \forall D \in Constr^1, \forall u^i\} \cup \{(u^i_1, u^i_2) \mid \forall u^i_1, \forall u^i_2\}$$

$$\frac{vs^i \cap vs^i_{matchable} \subseteq \{C \mid \forall C \in Constr^0\}}{[\ td^c \ \mid\ d^c \ \mid\ vs^i\ ], C \vdash^c_p \{\}}\ \textsc{CM-Constr-0}$$

$$\frac{\begin{array}{c} vs^i \cap vs^i_{matchable} \subseteq \{D(u^i) \mid \forall D \in Constr^1, \forall u^i\} \\ vs'^i = \{v^i \mid \exists d^i. D(\ [\ d^i \ \mid\ v^i\ ]) \in vs^i\} \\ \forall l. l \in d'^c \Leftrightarrow \exists d^i. l \in d^i \wedge \exists v^i. D(\ [\ d^i \ \mid\ v^i\ ]) \in vs^i \end{array}}{[\ td^c \ \mid\ d^c \ \mid\ vs^i\ ], D(x) \vdash^c_p \{(x,\ [\ \emptyset \ \mid\ d'^c \ \mid\ vs'^i\ ])\}}\ \textsc{CM-Constr-1}$$

$$\frac{\begin{array}{c} vs^i \cap vs^i_{matchable} \subseteq \{(u^i_1, u^i_2) \mid \forall u^i_1, \forall u^i_2\} \\ vs^i_1 = \{v^i_1 \mid \exists (d^i_1, d^i_2, v^i_2).(\ [\ d^i_1 \ \mid\ v^i_1\ ],\ [\ d^i_2 \ \mid\ v^i_2\ ]) \in vs^i\} \\ \forall l. l \in d^c_1 \Leftrightarrow \exists d^i_1. l \in d^i_1 \wedge \exists (v^i_1, d^i_2, v^i_2).(\ [\ d^i_1 \ \mid\ v^i_1\ ],\ [\ d^i_2 \ \mid\ v^i_2\ ]) \in vs^i \\ vs^i_2 = \{v^i_2 \mid \exists (d^i_1, v^i_1, d^i_2).(\ [\ d^i_1 \ \mid\ v^i_1\ ],\ [\ d^i_2 \ \mid\ v^i_2\ ]) \in vs^i\} \\ \forall l. l \in d^c_2 \Leftrightarrow \exists d^i_2. l \in d^i_2 \wedge \exists (d^i_1, v^i_1, v^i_2).(\ [\ d^i_1 \ \mid\ v^i_1\ ],\ [\ d^i_2 \ \mid\ v^i_2\ ]) \in vs^i \end{array}}{[\ td^c \ \mid\ d^c \ \mid\ vs^i\ ], (x_1, x_2) \vdash^c_p \{(x_1,\ [\ \emptyset \ \mid\ d^c_1 \ \mid\ vs^i_1\ ]); (x_2,\ [\ \emptyset \ \mid\ d^c_2 \ \mid\ vs^i_2\ ])\}}\ \textsc{CM-Couple}$$

$$\frac{vs^i \cap \{C \mid \forall C \in Constr^0\} = \emptyset}{[\ td^c \ \mid\ d^c \ \mid\ vs^i\ ], C \vdash^c_p \bot}\ \textsc{CM-Constr-0-Not}$$

$$\frac{vs^i \cap \{D(u^i) \mid \forall D \in Constr^1, \forall u^i\} = \emptyset}{[\ td^c \ \mid\ d^c \ \mid\ vs^i\ ], D(x) \vdash^c_p \bot}\ \textsc{CM-Constr-1-Not}$$

$$\frac{vs^i \cap \{(u^i_1, u^i_2) \mid \forall u^i_1, \forall u^i_2\} = \emptyset}{[\ td^c \ \mid\ d^c \ \mid\ vs^i\ ], (x_1, x_2) \vdash^c_p \bot}\ \textsc{CM-Couple-Not}$$

CM-CONSTR-0-UNKNOWN

$$\frac{vs^i \cap vs^i_{matchable} \not\subseteq \{C | \forall C \in Constr^0\} \qquad vs^i \cap \{C | \forall C \in Constr^0\} \neq \emptyset}{[\ td^c \ | \ d^c \ | \ vs^i \ ], C \vdash^c_p ?\ \{\}}$$

CM-CONSTR-1-UNKNOWN

$$\frac{\begin{array}{c} vs^i \cap vs^i_{matchable} \not\subseteq \{D(u^i) | \forall D \in Constr^1, \forall u^i\} \\ vs^i \cap \{D(u^i) | \forall D \in Constr^1, \forall u^i\} \neq \emptyset \\ vs'^i = \{v^i | \exists d^i . D(\ [\ d^i \ | \ v^i \ ]) \in vs^i \cap \{D(u^i) | \forall D \in Constr^1, \forall u^i\}\} \\ \forall l. l \in d'^c \Leftrightarrow \exists d^i . l \in d^i \wedge \exists v^i . D(\ [\ d^i \ | \ v^i \ ]) \in vs^i \cap \{D(u^i) | \forall D \in Constr^1, \forall u^i\} \end{array}}{[\ td^c \ | \ d^c \ | \ vs^i \ ], D(x) \vdash^c_p ?\ \{(x,\ [\ \emptyset \ | \ d'^c \ | \ vs'^i \ ])\}}$$

CM-COUPLE-UNKNOWN

$$\frac{\begin{array}{c} vs^i \cap vs^i_{matchable} \not\subseteq \{(u^i_1, u^i_2) | \forall u^i_1, \forall u^i_2\} \qquad vs^i \cap \{(u^i_1, u^i_2) | \forall u^i_1, \forall u^i_2\} \neq \emptyset \\ vs^i_1 = \{v^i_1 | \exists (d^i_1, d^i_2, v^i_2) . (\ [\ d^i_1 \ | \ v^i_1 \ ],\ [\ d^i_2 \ | \ v^i_2 \ ]) \in vs^i \cap \{(u^i_1, u^i_2) | \forall u^i_1, \forall u^i_2\}\} \\ \forall l. l \in d^c_1 \Leftrightarrow \exists d^i_1 . l \in d^i_1 \wedge \exists (v^i_1, d^i_2, v^i_2) . (\ [\ d^i_1 \ | \ v^i_1 \ ],\ [\ d^i_2 \ | \ v^i_2 \ ]) \in vs^i \cap \{(u^i_1, u^i_2) | \forall u^i_1, \forall u^i_2\} \\ vs^i_2 = \{v^i_2 | \exists (d^i_1, v^i_1, d^i_2) . (\ [\ d^i_1 \ | \ v^i_1 \ ],\ [\ d^i_2 \ | \ v^i_2 \ ]) \in vs^i \cap \{(u^i_1, u^i_2) | \forall u^i_1, \forall u^i_2\}\} \\ \forall l. l \in d^c_2 \Leftrightarrow \exists d^i_2 . l \in d^i_2 \wedge \exists (d^i_1, v^i_1, v^i_2) . (\ [\ d^i_1 \ | \ v^i_1 \ ],\ [\ d^i_2 \ | \ v^i_2 \ ]) \in vs^i \cap \{(u^i_1, u^i_2) | \forall u^i_1, \forall u^i_2\} \end{array}}{[\ td^c \ | \ d^c \ | \ vs^i \ ], (x_1, x_2) \vdash^c_p ?\ \{(x_1,\ [\ \emptyset \ | \ d^c_1 \ | \ vs^i_1 \ ]); (x_2,\ [\ \emptyset \ | \ d^c_2 \ | \ vs^i_2 \ ])\}}$$

9 Abstract Semantics

9.1 Values/Environments

$tu^a :=$ $[\ td^a \ | \ u^a \ ]$ Valeur avec annotation de t-dpendance

$u^a :=$ $[\ d^a \ | \ v^a \ ]$ Valeur avec annotation de v-dpendance

$v^a :=$	$C \mid D(u^a) \mid (u^a_1, u^a_2)$	Constructeurs de donnes
	$< \lambda x.e, \Gamma^a >$	Fermeture
	$< \mathtt{rec} f.x.e, \Gamma^a >$	Fermeture rcursive
	$\bot$	Aucune valeur
	$\top$	Valeur quelconque

$td^a :=$	$\{l_1; \ldots; l_n\}$	t-dpendances
$d^a :=$	$\{l_1; \ldots; l_m\}$	v-dpendances

$t\Gamma^a :=$ $(x_1, tu^a_1); \ldots; (x_n, tu^a_n)$ t-environnement abstrait

$\Gamma^a :=$ $(x_1, u^a_1); \ldots; (x_n, u^a_n)$ v-environnement abstraite

9.2 Inference Rules

A-NUM
$$\frac{}{t\Gamma^a \vdash^a n \mapsto [\ \emptyset \ |\ [\ \emptyset \ |\ \top\ ]\ ]}$$

A-IDENT
$$\frac{tu^a = t\Gamma^a[x]}{t\Gamma^a \vdash^a x \mapsto tu^a}$$

A-IDENT-EMPTY
$$\frac{x \notin support(t\Gamma^a)}{t\Gamma^a \vdash^a x \mapsto [\ \emptyset \ |\ [\ \emptyset \ |\ \bot\ ]\ ]}$$

A-ABSTR
$$\frac{}{t\Gamma^a \vdash^a \lambda x.e \mapsto [\ \emptyset \ |\ [\ \emptyset \ | < \lambda x.e, \uparrow^a_{ta}(t\Gamma^a) >\ ]\ ]}$$

A-ABSTR-REC
$$\frac{}{t\Gamma^a \vdash^a \texttt{rec} f.x.e \mapsto [\ \emptyset \ |\ [\ \emptyset \ | < \texttt{rec} f.x.e, \uparrow^a_{ta}(t\Gamma^a) >\ ]\ ]}$$

A-LETIN
$$\frac{\begin{array}{c} t\Gamma^a \vdash^a e_1 \mapsto tu_1^a \qquad tu_1^a = [\ td_1^a \ |\ u_1^a\ ] \\ (x, tu_1^a) \oplus t\Gamma^a \vdash^a e_2 \mapsto [\ td_2^a \ |\ u_2^a\ ] \end{array}}{t\Gamma^a \vdash^a let\ x\ =\ e_1\ in\ e_2 \mapsto [\ td_1^a \cup td_2^a \ |\ u_2^a\ ]}$$

A-APPLY
$$\frac{\begin{array}{c} t\Gamma^a \vdash^a e_1 \mapsto [\ td_1^a \ |\ [\ d_1^a \ | < \lambda x.e, \Gamma_1^a >\ ]\ ] \\ t\Gamma^a \vdash^a e_2 \mapsto tu_2^a \qquad tu_2^a = [\ td_2^a \ |\ u_2^a\ ] \\ (x, tu_2^a) \oplus \uparrow^{ta}_a(\Gamma_1^a) \vdash^a e \mapsto [\ td^a \ |\ [\ d^a \ |\ v^a\ ]\ ] \end{array}}{t\Gamma^a \vdash^a e_1\ e_2 \mapsto [\ td_1^a \cup td_2^a \cup td^a \cup d_1^a \ |\ [\ d_1^a \cup d^a \ |\ v^a\ ]\ ]}$$

A-APPLY-REC
$$\frac{\begin{array}{c} t\Gamma^a \vdash^a e_1 \mapsto [\ td_1^a \ |\ [\ d_1^a \ | < \texttt{rec} f.x.e, \Gamma_1^a >\ ]\ ] \\ tu_1^a = [\ td_1^a \ |\ [\ d\_of\_freevars(\texttt{rec} f.x.e, t\Gamma^a) \ |\ \top\ ]\ ] \\ t\Gamma^a \vdash^a e_2 \mapsto tu_2^a \qquad tu_2^a = [\ td_2^a \ |\ u_2^a\ ] \\ (f, tu_1^a) \oplus (x, tu_2^a) \oplus \uparrow^{ta}_a(\Gamma_1^a) \vdash^a e \mapsto [\ td^a \ |\ [\ d^a \ |\ v^a\ ]\ ] \end{array}}{t\Gamma^a \vdash^a e_1\ e_2 \mapsto [\ td_1^a \cup td_2^a \cup td^a \cup d_1^a \ |\ [\ d_1^a \cup d^a \ |\ v^a\ ]\ ]}$$

A-APPLY-UNKNOWN
$$\frac{\begin{array}{c} t\Gamma^a \vdash^a e_1 \mapsto [\ td_1^a \ |\ [\ d_1^a \ |\ v_1^a\ ]\ ] \qquad t\Gamma^a \vdash^a e_2 \mapsto [\ td_2^a \ |\ [\ d_2^a \ |\ v_2^a\ ]\ ] \\ \forall (f, x, e, \Gamma_1^a). v_1^a \neq < \lambda x.e, \Gamma_1^a > \wedge v_1^a \neq < \texttt{rec} f.x.e, \Gamma_1^a > \end{array}}{t\Gamma^a \vdash^a e_1\ e_2 \mapsto [\ td_1^a \cup td_2^a \cup d_1^a \cup d_2^a \ |\ [\ d_1^a \cup d_2^a \ |\ \top\ ]\ ]}$$

A-IF
$$\frac{\begin{array}{c} t\Gamma^a \vdash^a e \mapsto [\ td^a \ |\ [\ d^a \ |\ v^a\ ]\ ] \\ t\Gamma^a \vdash^a e_1 \mapsto [\ td_1^a \ |\ [\ d_1^a \ |\ v_1^a\ ]\ ] \qquad t\Gamma^a \vdash^a e_2 \mapsto [\ td_2^a \ |\ [\ d_2^a \ |\ v_2^a\ ]\ ] \end{array}}{t\Gamma^a \vdash^a if\ e\ then\ e_1\ else\ e_2 \mapsto [\ d^a \cup td^a \cup td_1^a \cup td_2^a \ |\ [\ d^a \cup d_1^a \cup d_2^a \ |\ \top\ ]\ ]}$$

A-MATCH
$$\frac{\begin{array}{c} t\Gamma^a \vdash^a e \mapsto tu^a \qquad tu^a = [\ td^a \ |\ [\ d^a \ |\ v^a\ ]\ ] \qquad tu^a, p \vdash^a_p t\Gamma^a_p \\ t\Gamma^a_p \oplus t\Gamma^a \vdash^a e_1 \mapsto [\ td_1^a \ |\ [\ d_1^a \ |\ v_1^a\ ]\ ] \end{array}}{t\Gamma^a \vdash^a match\ e\ with\ p \to e_1 \mid x \to e_2 \mapsto [\ d^a \cup td^a \cup td_1^a \ |\ [\ d^a \cup d_1^a \ |\ v_1^a\ ]\ ]}$$

A-MATCH-VAR
$$\frac{\begin{array}{c} t\Gamma^a \vdash^a e \mapsto tu^a \qquad tu^a = [\ td^a \ |\ [\ d^a \ |\ v^a\ ]\ ] \qquad tu^a, p \vdash^a_p \bot \\ (x, tu^a) \oplus t\Gamma^a \vdash^a e_2 \mapsto [\ td_2^a \ |\ [\ d_2^a \ |\ v_2^a\ ]\ ] \end{array}}{t\Gamma^a \vdash^a match\ e\ with\ p \to e_1 \mid x \to e_2 \mapsto [\ d^a \cup td^a \cup td_2^a \ |\ [\ d^a \cup d_2^a \ |\ v_2^a\ ]\ ]}$$

A-MATCH-UNKNOWN

$$\frac{t\Gamma^a \vdash^a e \mapsto tu^a \qquad tu^a = [\ td^a \ \mid\ [\ d^a \ \mid\ v^a\ ]\ ] \qquad tu^a, p \vdash^a_p ?\ t\Gamma^a_p \\ t\Gamma^a_p \oplus t\Gamma^a \vdash^a e_1 \mapsto [\ td^a_1 \ \mid\ [\ d^a_1 \ \mid\ v^a_1\ ]\ ] \\ (x, tu^a) \oplus t\Gamma^a \vdash^a e_2 \mapsto [\ td^a_2 \ \mid\ [\ d^a_2 \ \mid\ v^a_2\ ]\ ]}{t\Gamma^a \vdash^a match\ e\ with\ p \to e_1 \mid x \to e_2 \mapsto [\ d^a \cup td^a \cup td^a_1 \cup td^a_2 \ \mid\ [\ d^a \cup d^a_1 \cup d^a_2 \ \mid\ \top\ ]\ ]}$$

A-MATCH-ERROR

$$\frac{t\Gamma^a \vdash^a e \mapsto [\ td^a \ \mid\ [\ d^a \ \mid\ v^a\ ]\ ] \qquad tu^a, p \vdash^a_p \times}{t\Gamma^a \vdash^a match\ e\ with\ p \to e_1 \mid x \to e_2 \mapsto [\ \emptyset \ \mid\ [\ \emptyset \ \mid\ \bot\ ]\ ]}$$

A-CONSTR-0

$$\frac{}{t\Gamma^a \vdash^a C \mapsto [\ \emptyset \ \mid\ [\ \emptyset \ \mid\ C\ ]\ ]}$$

A-CONSTR-1

$$\frac{t\Gamma^a \vdash^a e \mapsto [\ td^a \ \mid\ u^a\ ]}{t\Gamma^a \vdash^a D(e) \mapsto [\ td^a \ \mid\ [\ \emptyset \ \mid\ D(u^a)\ ]\ ]}$$

A-COUPLE

$$\frac{t\Gamma^a \vdash^a e_1 \mapsto [\ td^a_1 \ \mid\ u^a_1\ ] \qquad t\Gamma^a \vdash^a e_2 \mapsto [\ td^a_2 \ \mid\ u^a_2\ ]}{t\Gamma^a \vdash^a (e_1, e_2) \mapsto [\ td^a_1 \cup td^a_2 \ \mid\ [\ \emptyset \ \mid\ (u^a_1, u^a_2)\ ]\ ]}$$

A-ANNOT

$$\frac{t\Gamma^a \vdash^a e \mapsto [\ td^a \ \mid\ [\ d^a \ \mid\ v^a\ ]\ ]}{t\Gamma^a \vdash^a l\ :\ e \mapsto [\ td^a \ \mid\ [\ l; d^a \ \mid\ v^a\ ]\ ]}$$

$$d\_of\_freevars(e, t\Gamma^a) := \quad dof\_aux(e, t\Gamma^a, \emptyset, \emptyset)$$

$$\begin{array}{rl}
dof_aux(n, t\Gamma^a, bvars, acc) := & acc \\
dof_aux(C, t\Gamma^a, bvars, acc) := & acc \\
dof_aux(D(e), t\Gamma^a, bvars, acc) := & dof_aux(e, t\Gamma^a, bvars, acc) \\
dof_aux(x, t\Gamma^a, bvars, acc) := & acc \text{ si } x \in bvars \\
dof_aux(x, t\Gamma^a, bvars, acc) := & d^a \cup acc \text{ si } x \notin bvars \wedge t\Gamma^a[x] = [\ td^a \ \mid\ [\ d^a \ \mid\ v^a\ ]\ ] \\
dof_aux(\lambda x.e, t\Gamma^a, bvars, acc) := & dof_aux(e, t\Gamma^a, x; bvars, acc) \\
dof_aux(\texttt{rec} f.x.e, t\Gamma^a, bvars, acc) := & dof_aux(e, t\Gamma^a, f; x; bvars, acc) \\
dof_aux(e_1\ e_2, t\Gamma^a, bvars, acc) := & dof_aux(e_2, t\Gamma^a, bvars, acc_1) \\
 & \text{pour } acc_1 = dof_aux(e_1, t\Gamma^a, bvars, acc) \\
dof_aux(if\ e\ then\ e_1\ else\ e_2, t\Gamma^a, bvars, acc) := & dof_aux(e_2, t\Gamma^a, bvars, acc_1) \\
 & \text{pour } acc_1 = dof_aux(e_1, t\Gamma^a, bvars, acc_0) \\
 & \text{et } \quad acc_0 = dof_aux(e, t\Gamma^a, bvars, acc) \\
dof_aux(match\ e\ with\ p \to e_1 \mid x \to e_2, t\Gamma^a, bvars, acc) := & dof_aux(e_2, t\Gamma^a, x; bvars, acc_1) \\
 & \text{pour } acc_1 = dof_aux(e_1, t\Gamma^a, binders_of(p); bvars, acc_0) \\
 & \text{et } \quad acc_0 = dof_aux(e, t\Gamma^a, bvars, acc) \\
dof_aux((e_1, e_2), t\Gamma^a, bvars, acc) := & dof_aux(e_2, t\Gamma^a, bvars, acc_1) \\
 & \text{pour } acc_1 = dof_aux(e_1, t\Gamma^a, bvars, acc) \\
dof_aux(l\ :\ e, t\Gamma^a, bvars, acc) := & dof_aux(e, t\Gamma^a, bvars, acc) \\
dof_aux(let\ x\ =\ e_1\ in\ e_2, t\Gamma^a, bvars, acc) := & dof_aux(e_2, t\Gamma^a, x; bvars, acc_1) \\
 & \text{pour } acc_1 = dof_aux(e_1, t\Gamma^a, bvars, acc)
\end{array}$$

$$\begin{array}{rl}
binders_of(C) := & \{\} \\
binders_of(D(x)) := & \{x\} \\
binders_of((x, y)) := & \{x; y\}
\end{array}$$

AM-CONSTR-0

$$\frac{}{[\ td^a \ \mid\ [\ d^a \ \mid\ C\ ]\ ], C \vdash^a_p \{\}}$$

AM-CONSTR-1

$$\frac{}{[\ td^a \ \mid\ [\ d^a \ \mid\ D(u^a)\ ]\ ], D(x) \vdash^a_p \{(x,\ [\ \emptyset \ \mid\ u^a\ ])\}}$$

AM-COUPLE

$$\frac{}{[\ td^a \ \mid\ [\ d^a \ \mid\ (u^a_1, u^a_2)\ ]\ ], (x_1, x_2) \vdash^a_p \{(x_1,\ [\ \emptyset \ \mid\ u^a_1\ ]); (x_2,\ [\ \emptyset \ \mid\ u^a_2\ ])\}}$$

AM-CONSTR-0-NOT

$$\frac{p \neq C}{[\ td^a \ \mid\ [\ d^a \ \mid\ C\ ]\ ], p \vdash^a_p \bot}$$

AM-CONSTR-1-NOT

$$\frac{p \neq D(\_)}{[\ td^a \ \mid\ [\ d^a \ \mid\ D(u^a)\ ]\ ], p \vdash^a_p \bot}$$

AM-COUPLE-NOT

$$\frac{p \neq (\_, \_)}{[\ td^a \ \mid\ [\ d^a \ \mid\ (u^a_1, u^a_2)\ ]\ ], p \vdash^a_p \bot}$$

AM-CONSTR-0-UNKNOWN

$$\frac{}{[\ td^a \ \mid \ [\ d^a \ \mid \ \top\ ]\ ], C \vdash_p^a ?\ \{\}}$$

AM-CONSTR-1-UNKNOWN

$$\frac{}{[\ td^a \ \mid \ [\ d^a \ \mid \ \top\ ]\ ], D(x) \vdash_p^a ?\ \{(x,\ [\ \emptyset \ \mid \ [\ \emptyset \ \mid \ \top\ ]\ ])\}}$$

AM-COUPLE-UNKNOWN

$$\frac{}{[\ td^a \ \mid \ [\ d^a \ \mid \ \top\ ]\ ], (x_1, x_2) \vdash_p^a ?\ \{(x_1,\ [\ \emptyset \ \mid \ [\ \emptyset \ \mid \ \top\ ]\ ]); (x_2,\ [\ \emptyset \ \mid \ [\ \emptyset \ \mid \ \top\ ]\ ])\}}$$

AM-ERROR

$$\frac{\forall C. v^a \neq C \qquad \forall D. v^a \neq D(\_) \qquad v^a \neq (\_, \_) \qquad v^a \neq \top}{[\ td^a \ \mid \ [\ d^a \ \mid \ v^a\ ]\ ], p \vdash_p^a \times}$$