Homework 4
Information Theory for Complex Systems
Februari - 2023

There are 9 possible probabilities, which through symmetries can be reduced down 4.

	A	B	C
A	$P(AA) = p_1$	$P(AB) = p_2$	$P(AC) = p_2$
B	$P(BA) = p_3$	$P(BB) = p_4$	$P(BC) = p_4$
C	$P(CA) = p_3$	$P(CB) = p_4$	$P(CC) = p_4$

Since energy is only determined by interactions in pairs we can conclude that finding a maximum of ΔS_2 should be enough to determine p_1, p_2, p_3 and p_4.

Attempt a Lagrange optimization

$$
\begin{aligned}
L(p_1, p_2, p_3, p_4, \beta, \mu) = \Delta S_2 &+ \beta(u - J(2p_2 + 2p_3 - p_1)) \\
&+ \mu(1 - p_1 - 2p_2 - 2p_3 - 4p_4)
\end{aligned}
\tag{1}
$$

(is $p_3 = p_2$?)

U is the expectation value of the internal energy.

$$
S[P] = \sum_i p_i \ln \frac{1}{p_i} \tag{2}
$$

$$
\sum_i p_i h(i) = U \tag{3}
$$

$$
\sum_i p_i f_k(i) = n_k, \, (k = 1, \ldots, M) \tag{4}
$$

$$
\sum_i p_i = 1 \tag{5}
$$