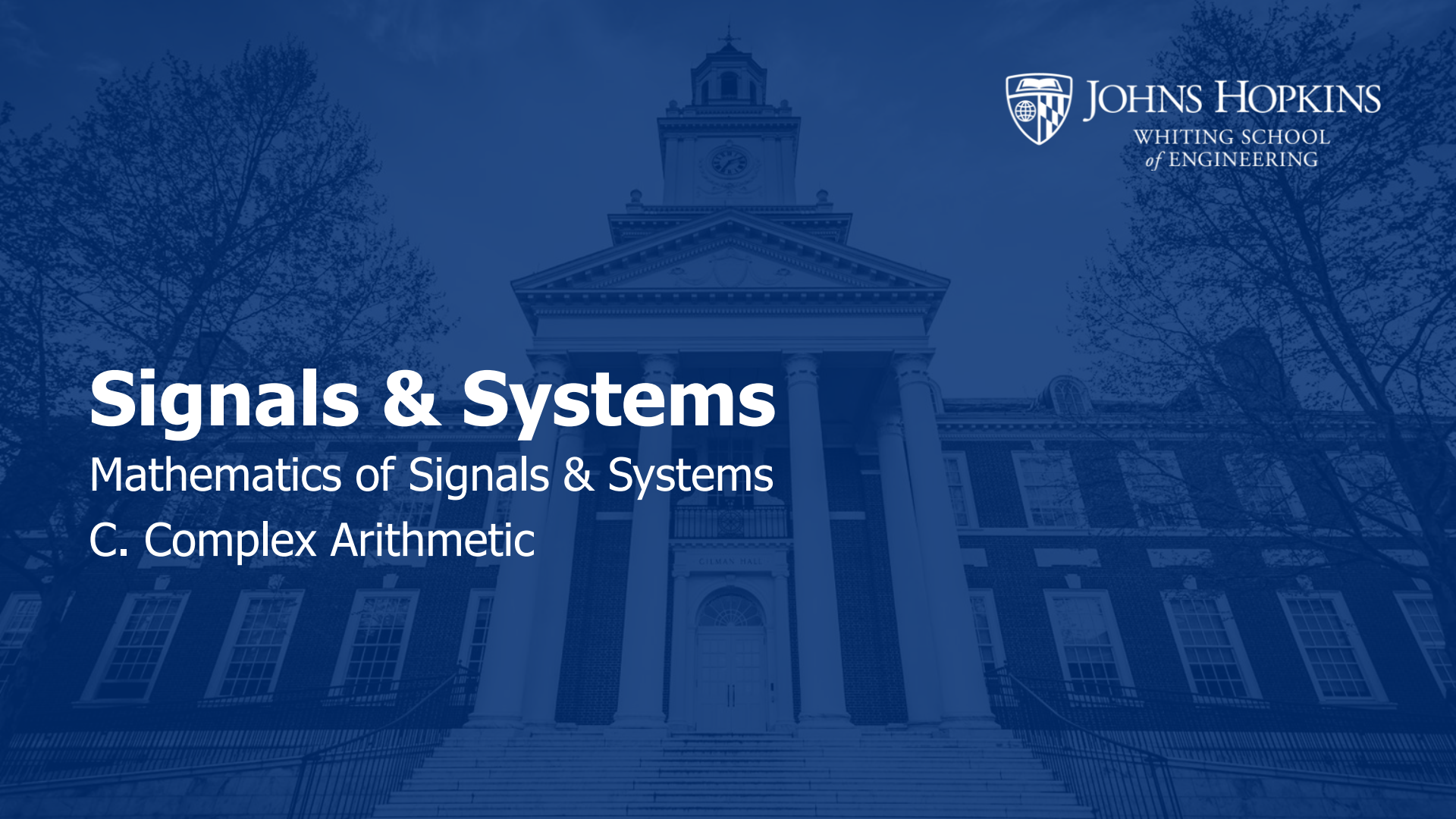


Signals & Systems

Mathematics of Signals & Systems

C. Complex Arithmetic

Addition of Complex Numbers

$$z_0 = x_0 + jy_0$$

$$z_1 = x_1 + jy_1$$

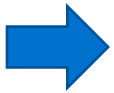
$$z_0 + z_1 = x_0 + jy_0 + x_1 + jy_1$$

$$z_0 + z_1 = (x_0 + x_1) + j(y_0 + y_1)$$

Multiplication of Complex Numbers

$$z_0 = x_0 + jy_0$$

$$z_1 = x_1 + jy_1$$



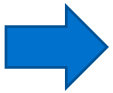
$$z_0 \times z_1 = (x_0 + jy_0) \times (x_1 + jy_1)$$

$$z_0 \times z_1 = (x_0x_1 - y_0y_1) + j\,(x_0y_1 + y_0x_1)$$

Multiplication of Complex Numbers Continued

$$z_0 = x_0 + jy_0 = r_0\, e^{j\theta_0}$$

$$z_1 = x_1 + jy_1 = r_1\, e^{j\theta_1}$$



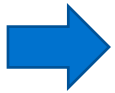
$$z_0 \times z_1 = r_0\, e^{j\theta_0} \times r_1\, e^{j\theta_1}$$

$$z_0 \times z_1 = r_0\, r_1\, e^{j(\theta_0+\theta_1)}$$

Division of Complex Numbers

$$z_0 = x_0 + jy_0 = r_0\, e^{j\theta_0}$$

$$z_1 = x_1 + jy_1 = r_1\, e^{j\theta_1}$$



$$z_0 \div z_1 = \frac{z_0}{z_1} = \frac{r_0 e^{j\theta_0}}{r_1 e^{j\theta_1}}$$

$$z_0 \div z_1 = \frac{z_0}{z_1} = \frac{r_0}{r_1}\, e^{j(\theta_0 - \theta_1)}$$

Magnitude-Squared of a Complex Number

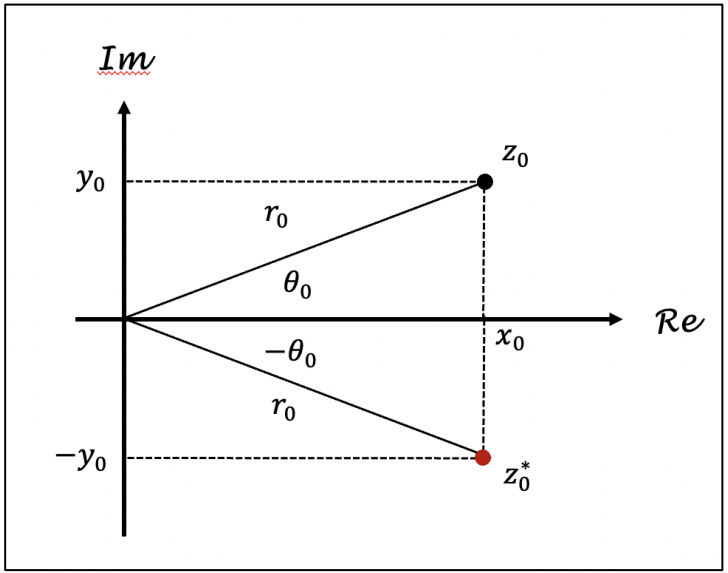
- Find the magnitude-squared of a complex number: $|z_0|^2$

Rectangular Form $z_0 = x_0 + jy_0$

$|z_0|^2 = z_0 \times z_0^*$

$|z_0|^2 = (x_0 + jy_0)(x_0 - jy_0)$

$$|z_0|^2 = x_0^2 + y_0^2$$



Polar Form $z_0 = r_0\, e^{j\theta_0}$

$|z_0|^2 = z_0 \times z_0^*$

$|z_0|^2 = \left(r_0 e^{j\theta_0}\right)\left(r_0 e^{-j\theta_0}\right)$

$$|z_0|^2 = r_0^2$$