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EN 525.202

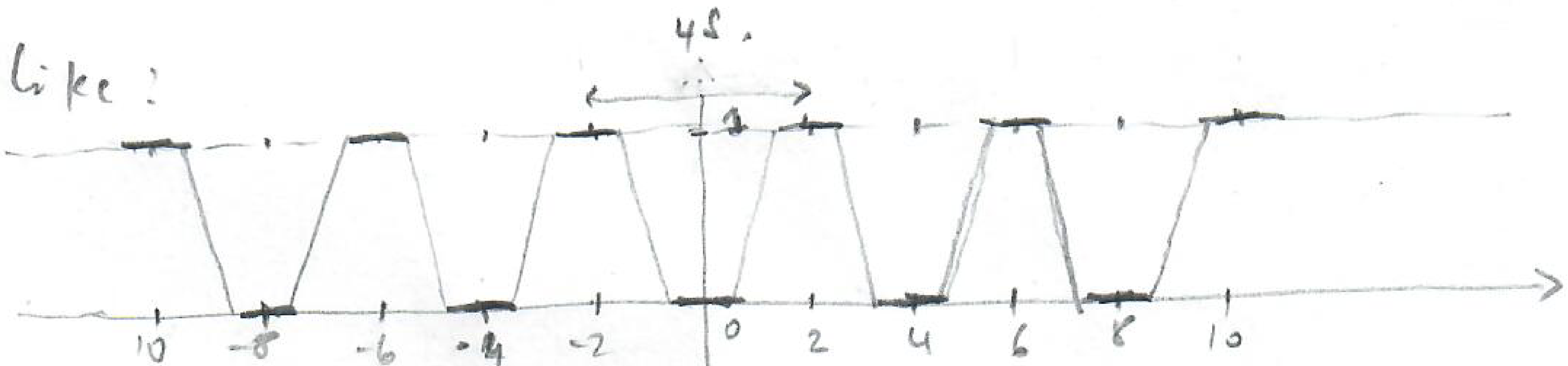
Module 3

Problem 1: $x(t) = x(t+T)$

$$x(t) = \sum_{k=-\infty}^{+\infty} X_k e^{j\frac{2\pi k t}{T}} \quad \text{with} \quad \begin{cases} X_0 = \frac{1}{2} \\ X_k = \begin{cases} 0, & k \text{ even} \\ \frac{6}{\pi^2 k^2}\sin\left(\frac{\pi k}{2}\right)\sin\left(\frac{\pi k}{6}\right), & k \text{ odd} \end{cases} \end{cases}$$

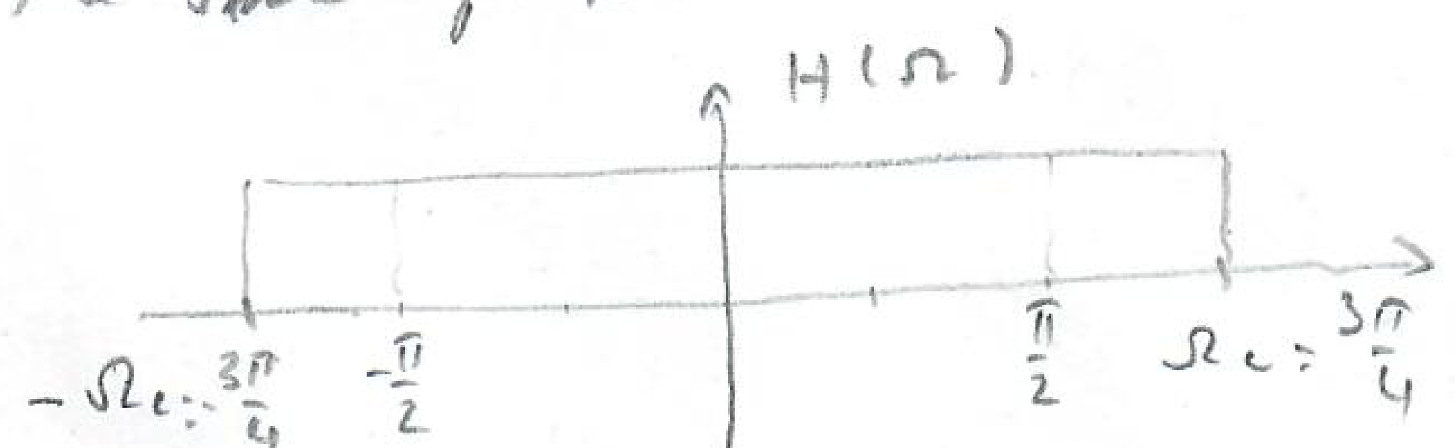
(a) Find & Plot $z(t)$ given $Z_k = X_k e^{j\pi k}$.

Given the phase is $\pi = 2\pi f \Rightarrow t = 2s$ and the period of $x(t)$ is $T = 4s$, we deduce the time shift of $+2s$. Then $z(t)$ would look something like:



(b) Since both input $x(t)$ & $y(t)$ have the same period:

$Y_k = X_k H(k\Omega_0) = X_k H\left(\frac{2\pi}{T_0}k\right)$



$\frac{2\pi}{T_0} = \frac{2\pi}{4} = \frac{\pi}{2}$

$Y_0 = X_0 H(0) = X_0 = \frac{1}{2}$

$Y_1 = X_1 H\left(\frac{2\pi}{T_0}\right) = X_1$

$Y_{-1} = X_{-1} H\left(-\frac{2\pi}{T_0}\right) = X_{-1}$

Since the cut-off freq. $\Omega_c = \frac{3\pi}{4}$ then $y(t)$ only includes Y_0, Y_1, and Y_{-1} since the 2nd, 3rd, ... fall outside the cutoff freq. Ω_c: $Y_{\pm 2}, Y_{\pm 3} = 0$
0 (by definition)

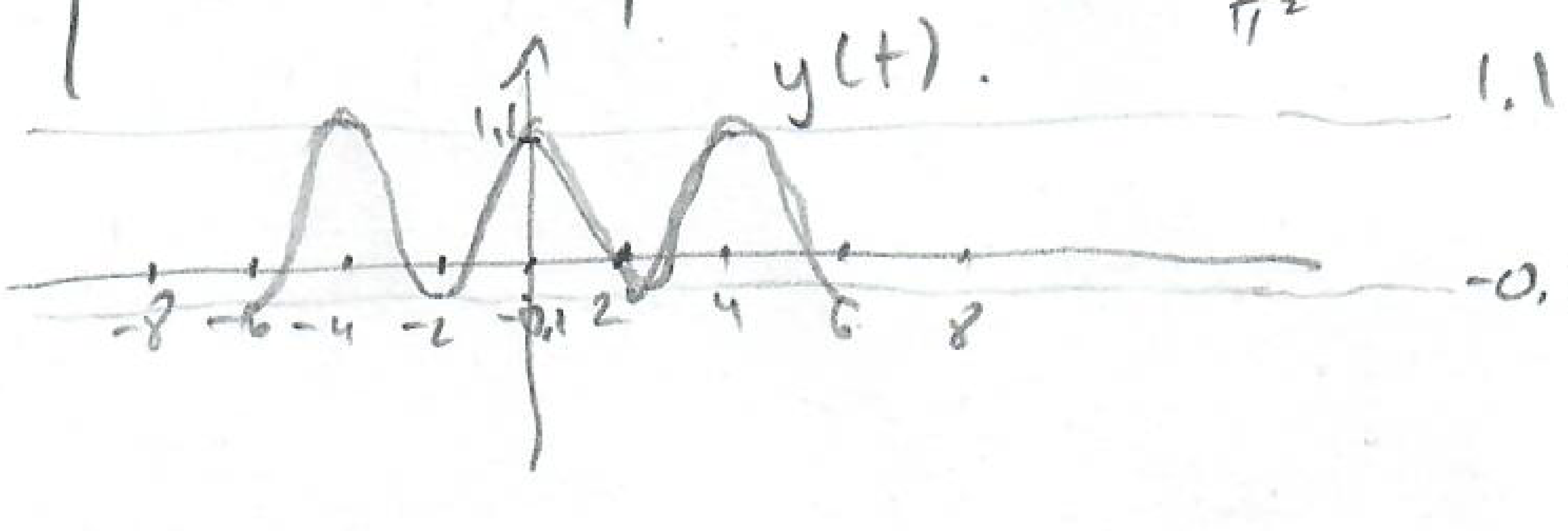
$$\Rightarrow y(t) = \frac{1}{2} + X_1 e^{j\frac{\pi t}{2}} + X_{-1} e^{-j\frac{\pi t}{2}}$$

$$= \frac{1}{2} + \frac{6}{\pi^2}\underbrace{\sin\left(\frac{\pi}{2}\right)}_{1}\underbrace{\sin\left(\frac{\pi}{6}\right)}_{1/2}\left\{2\cos\left(\frac{\pi}{2}t\right)\right\}$$

$0.5 - \frac{6}{\pi^2} \sim -0.1$

$0.5 + \frac{6}{\pi^2} \sim 1.1$

$$\boxed{y(t) = \frac{1}{2} + \frac{6}{\pi^2}\cos\left(\frac{\pi}{2}t\right)}$$



Problem2: $x[n] = \sum_{k=0}^{N_0-1} c_k e^{j\frac{2\pi n k}{N_0}}$, $N_0 = 6$

$$c_k = \left\{ \overset{c_0}{\frac{3}{2}}, \overset{c_1}{-\frac{1}{3}}, \overset{c_2}{-1}, \overset{c_3}{\frac{1}{6}}, \overset{c_4}{-1}, \overset{c_5}{-\frac{1}{3}} \right\}$$

$$x[n] = \sum_{k=0}^{N_0-1} c_k e^{j\frac{2\pi n k}{N_0}} = \sum_{k=0}^{5} c_k e^{j\frac{2\pi n k}{6}} = \sum_{k=0}^{5} c_k e^{j\frac{\pi n k}{3}}$$

$$\Rightarrow \quad x[n] = c_0 \times (1) + c_1 e^{j\frac{\pi n}{3}} + c_2 e^{j\frac{2\pi n}{3}} + c_3 e^{j\pi n} + c_4 e^{j\frac{4\pi n}{3}} + c_5 e^{j\frac{5\pi n}{3}}$$

$$\boxed{x[n] = \frac{3}{2} - \frac{1}{3} e^{j\frac{\pi n}{3}} - e^{j\frac{2\pi n}{3}} + \frac{1}{6} e^{j\pi n} - e^{j\frac{4\pi n}{3}} - \frac{1}{3} e^{j\frac{5\pi n}{3}}}$$

(probably can simplify a little more with sin & cos)

(with some help from Matlab*) –

$$x[0] = \frac{3}{2} - \frac{1}{3} - 1 + \frac{1}{6} - 1 - \frac{1}{3} = \frac{9-2-6+1-6-2}{6} = -1$$

The below results can be computed in a faster manner using

dtfs.m : output = dtfs(ck, 1)

By modifying the flag "1" iteration (first "for"-loop iteration)

from $n = 0:N-1$ to $n = 0:(18-1)$.

$N_0 = 6$, $N_0 = 6$, $N_0 = 6$

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
x[n]	-1	2	3	0	3	2	-1	2	3	0	3	2	-1	2	3	0	3	2

$$x[3] = \frac{3}{2} - \frac{1}{3} e^{j\pi} - e^{j2\pi} + \frac{1}{6} e^{j3\pi} - 1 e^{j4\pi} - \frac{1}{3} e^{j5\pi}$$

$$= \frac{3}{2} + \frac{1}{3} - 1 - \frac{1}{6} - 1 + \frac{1}{3} = \frac{9+2-6-1-6+2}{6} = \frac{13-13}{6}$$

$$\Rightarrow \boxed{x[3] = 0}$$

n3 :   N = 4 samples.

$c_k = \{5, c_1, -1, -1+j\}$

nd c_1 : $c_{-k} = c_{N_0-k} = c_k^*$

using dfts.m routine

$\Rightarrow c_{-1} = c_{N_0-1} = c_3 = -1+j = c_1^*$

$\Rightarrow \boxed{c_1 = c_{-1}^* = -1-j}$ and by inspection $\boxed{c_1 = -1-j}$ ✓

Parseval: $P = \sum_{k=0}^{N_0-1} |c_k|^2 = 5^2 + c_1 \cdot c_1^* + (-1)^2 + c_3 \cdot c_3^*$

$$\boxed{P = 25 + 2 + 1 + 2 = 30}$$