Assignment 1: Signals & Systems Fundamentals

Signals & Systems

Complete the following problems. Please submit your solutions to these problems in a single PDF document to the course site. Use the *lastname_firstname_PS01.pdf* format for your file name.

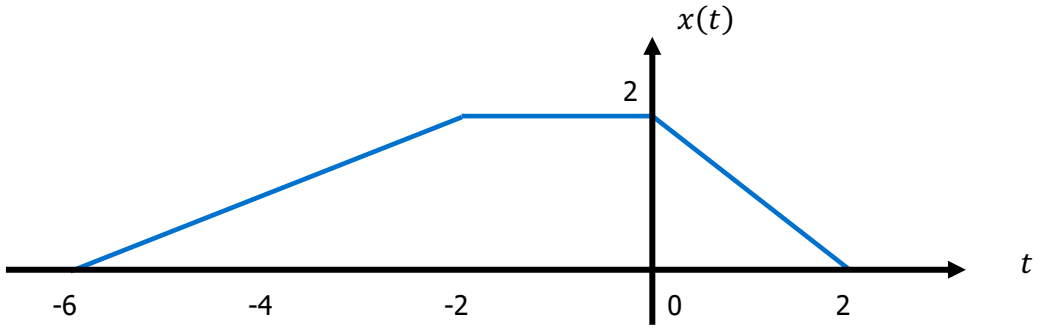
1. Consider the real-valued two-sided discrete-time signal $x[n] = a^{|n|}$, where a is a real number satisfying $0 < a < 1$, and $|\cdot|$ denotes the *absolute value*.

   a. Plot $x[n]$ for $a = \frac{2}{3}$ and $n = -10, \ldots, 10$.

   b. Find a *closed form expression* for the energy E of $x[n]$ as a function of a.

   c. Numerically compute the partial sum energy over $n = -10, \ldots, 10$, for $a = \frac{2}{3}$. Specifically, compute $\sum_{n=-10}^{10} |x[n]|^2$. How does this partial sum energy compare with your closed form expression from part b.?

2. Consider the continuous-time signal $x(t)$ plotted below. Sketch $y(t) = 0.5\, x(-2t + 4)$.



3. Consider the discrete-time sinusoid $x[n] = sin\left(\frac{\pi n}{6}\right)$.

   a. Plot $x[n]$ for $n = 0, \ldots, 48$.

   b. Is this discrete-time sinusoid periodic? If so, what is the period?

   c. Find and plot another discrete-time sinusoid $y[n] = sin(\omega_1 n)$ with a *different* frequency ($\omega_1 \neq \frac{\pi}{6}$) that yields *exactly* the same values as $x[n]$.

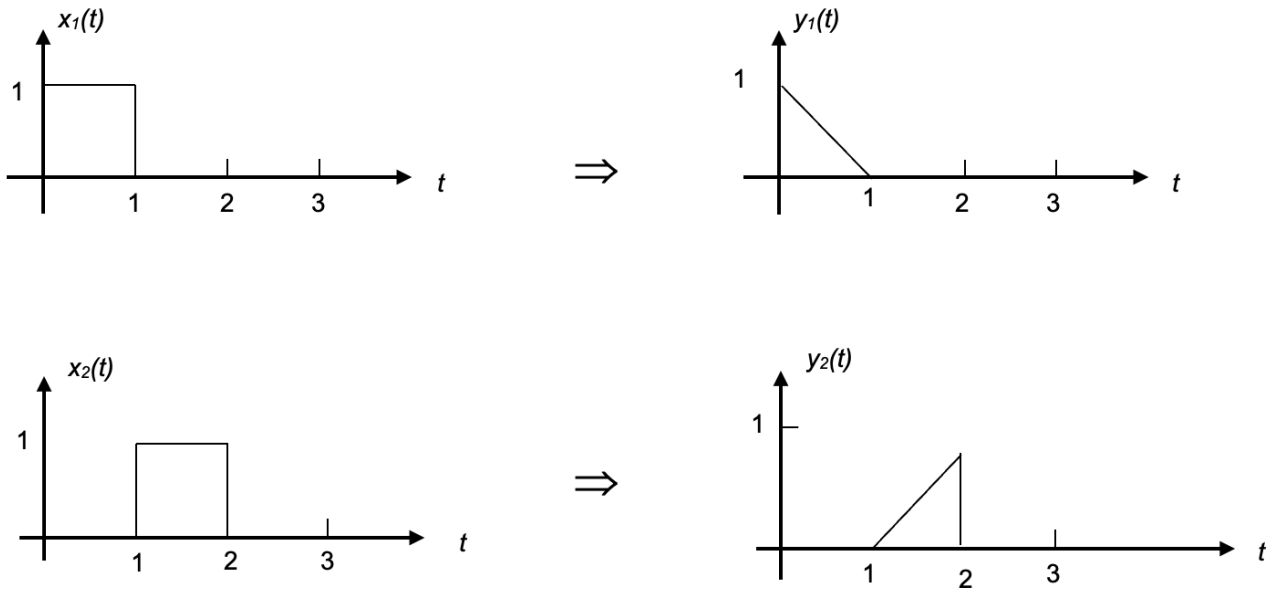
4. This problem will investigate a linear combination of continuous-time sinusoids.

   a. What is the period of $x_1(t) = \sin\left(\frac{\pi t}{2} + \frac{\pi}{4}\right)$?

   b. What is the period of $x_2(t) = \cos\left(\frac{\pi t}{3} - \frac{\pi}{6}\right)$?

   c. Is $y(t) = x_1(t) + x_2(t) = \sin\left(\frac{\pi t}{2} + \frac{\pi}{4}\right) + \cos\left(\frac{\pi t}{3} - \frac{\pi}{6}\right)$ periodic? If yes, what is the period of $y(t)$?

5. A *linear* continuous-time system is observed to have the following input-output pairs.



a. Could the system be time-invariant?

b. Find the output signal if the input signal is $x_3(t)$ shown below: