JOHNS HOPKINS
WHITING SCHOOL of ENGINEERING

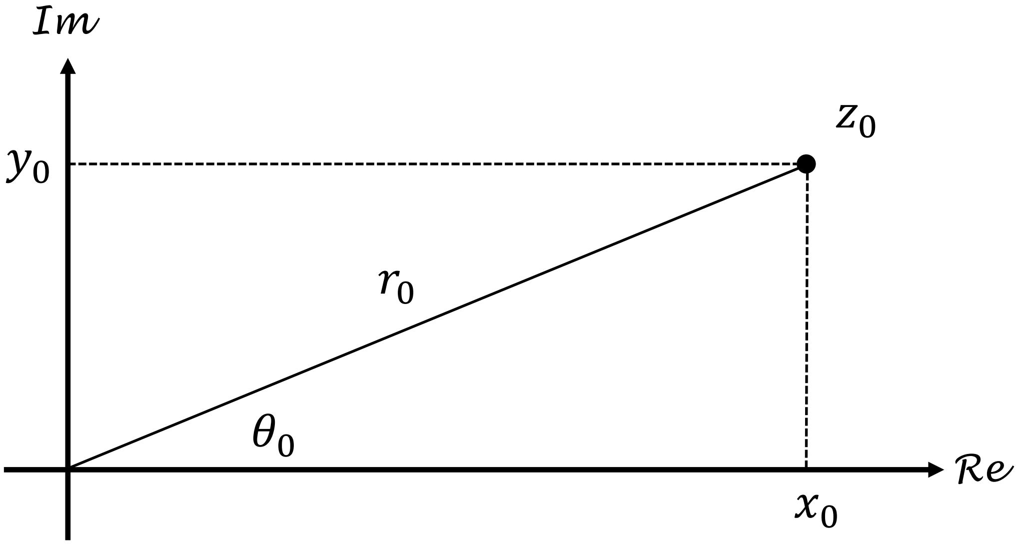
# Signals & Systems

## Mathematics of Signals & Systems

## B. Complex Numbers

# Complex Numbers

- Consider the complex plane, with the imaginary unit $j$ defined by $j^2 = -1$




**Rectangular Form**

$$z_0 = x_0 + jy_0$$

$$x_0 = Re\{z_0\}$$

$$y_0 = Im\{z_0\}$$

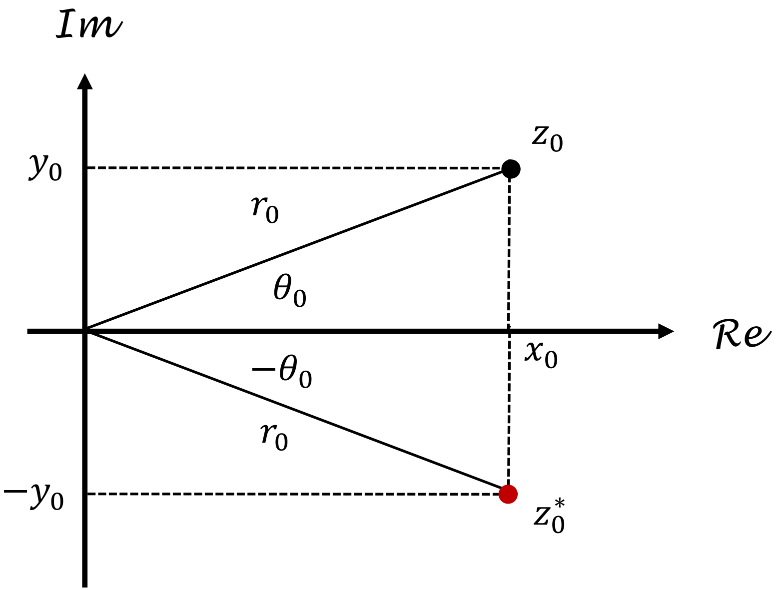
**Polar Form**

$$z_0 = r_0 e^{j\theta_0}$$

$$r_0 = |z_0|$$

$$\theta_0 = \angle z_0$$

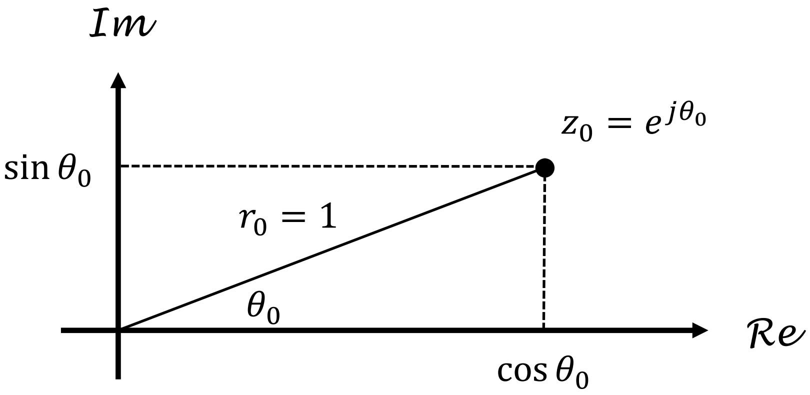
# Complex Conjugate




$$z_0^* = x_0 - jy_0$$

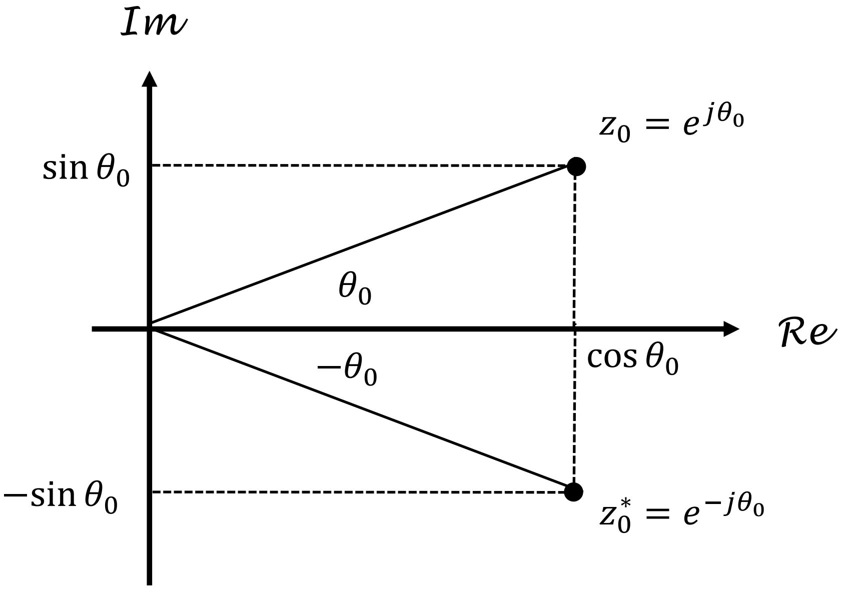
$$z_0^* = r_0 e^{-j\theta_0}$$

# Euler's Identity




$$\boxed{e^{j\theta_0} = \cos\theta_0 + j\sin\theta_0}$$

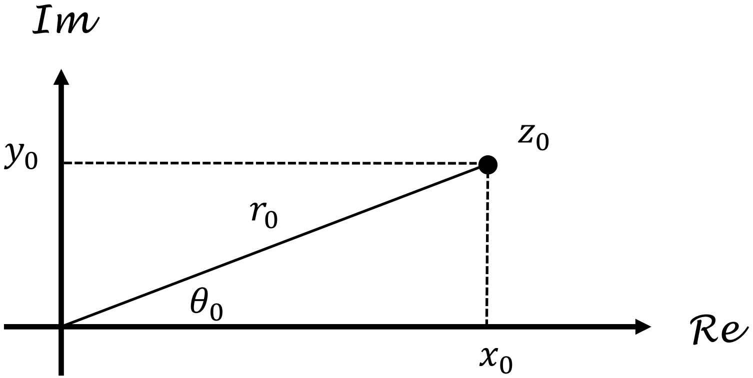
# Euler's Inverse Identities




$$\boxed{\cos\theta_0 = \frac{e^{j\theta_0} + e^{-j\theta_0}}{2}}$$

$$\boxed{\sin\theta_0 = \frac{e^{j\theta_0} - e^{-j\theta_0}}{2j}}$$

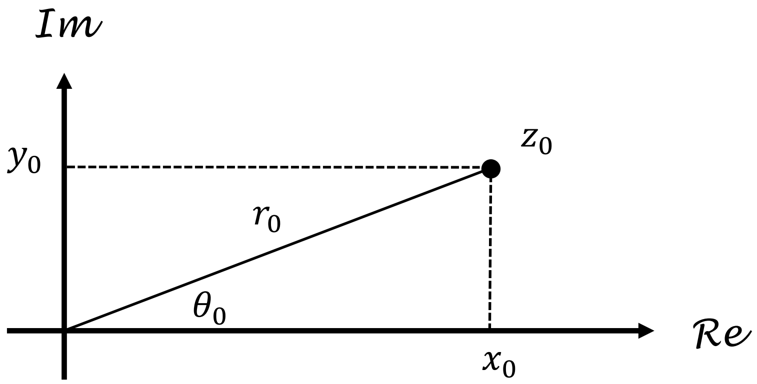
# Conversion From Polar to Rectangular Form




$$z_0 = r_0 e^{j\theta_0} = r_0 \cos\theta_0 + j r_0 \sin\theta_0$$

$$\boxed{\begin{aligned} x_0 &= Re\{z_0\} = r_0 \cos\theta_0 \\ y_0 &= Im\{z_0\} = r_0 \sin\theta_0 \end{aligned}}$$

# Conversion From Rectangular to Polar Form




$$z_0 = x_0 + jy_0$$

$$r_0 = \sqrt{x_0^2 + y_0^2}$$

$$\theta_0 = \tan^{-1}\frac{y_0}{x_0}$$

$$\boxed{z_0 = \sqrt{x_0^2 + y_0^2}\ e^{j\tan^{-1}\left(\frac{y_0}{x_0}\right)}}$$

# An Example

- Consider the complex number $z_0 = 3 + j4$
  - Express the complex number in polar form
  - Find its complex conjugate