Signals & Systems

Mathematics of Signals & Systems

D. Polynomials

Polynomial Functions

- Polynomial functions are of the form:

$$P(s) = p_N s^N + p_{N-1} s^{N-1} + p_{N-2} s^{N-2} + \cdots + p_2 s^2 + p_1 s + p_0$$

- Typically, the polynomial coefficients are normalized such that $p_N = 1$

- Example: $P(s) = s^4 + 6s^3 + 13s^2 + 4s - 24$

Roots

- An N^{th} degree polynomial will have N **roots** $\lambda_n, n = 1, \cdots N$, which satisfy:

$$P(s = \lambda_n) = 0$$

- An N^{th} degree polynomial can be expressed in terms of its roots (assuming $p_N = 1$):

$$P(s) = \prod_{n=1}^{N} (s - \lambda_n)$$

An Example

- Find the roots of the 4th degree polynomial:

$$P(s) = s^4 + 6s^3 + 13s^2 + 4s\ - 24$$

- Note that the polynomial can be expressed in terms of its roots:

$$P(s) = (s+3)(s+2-j2)(s+2+j2)\ (s-1)$$

Rational Functions

- **Rational functions** are the ratio of two polynomial functions:

$$\frac{q_M s^M + q_{M-1} s^{M-1} + q_{M-2} s^{M-2} + \cdots + q_2 s^2 + q_1 s + q_0}{s^N + p_{N-1} s^{N-1} + p_{N-2} s^{N-2} + \cdots + p_2 s^2 + p_1 s + p_0}$$

- The roots of the numerator polynomial are called **zeros**
- The roots of the denominator polynomial are called **poles**