Strategic Design for Delivery with Trucks and Drones

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Abstract

Home delivery by drones as an alternative or complement to traditional delivery by trucks is attracting considerable attention from major retailers and service providers (Amazon, UPS, Google, DHL, Wal-mart, etc.), as well as several startups. While drone delivery may offer considerable economic savings, the fundamental issues of how best to deploy drones for home delivery are not well understood. Our research provides a strategic analysis for the design of hybrid truck-drone delivery systems using continuous approximation modeling techniques to derive general insights. We formulate and optimize models of hybrid truck-drone delivery, where truck-based drones make deliveries simultaneously with trucks, and compare their performance to truck-only delivery. Our results suggest that truck-drone delivery can be very advantageous economically in many settings, especially with multiple drones per truck, but that the benefits depend strongly on the relative operating costs and marginal stop costs.

1. Introduction

Home delivery by drones is being promoted and researched by a growing number of firms, including Amazon, UPS, Google DHL and Wal-mart, as a possible alternative or complement to traditional delivery by trucks. Though many of the details about drone delivery remain uncertain, recent reports suggest drones might make small package deliveries for only $1 (Bloomberg Business 2015, Keeney 2015). Further interest stems from reports that 86% of Amazon's orders are less than 5 pounds (ideal for drone delivery) (60 Minutes 2013), and that 82% of customers are willing to pay for drone delivery (Snow 2014). Thus, drones provide new opportunities to improve home delivery services, and thus require research on how best to deploy drones for home delivery.

Proposals for drone delivery vary widely, with drones being used independently or in conjunction with delivery by trucks. Delivery drones may be launched and recovered at fixed facilities (e.g., DCs or

retail store locations), from relocatable facilities (e.g., platforms that that can be moved to different locations), or from trucks themselves. In this paper we address hybrid truck-drone delivery, where drones are autonomous vehicles (i.e. unmanned) that make deliveries on their own with each drone departing from, and returning to, a truck. In related work (Campbell et al. 2017) we examine drone deliveries from fixed depots (e.g., a DC) in conjunction with hybrid truck-drone routes and truck-only delivery. While our main focus is on aerial drones that generally have a small payload capacity (e.g., 5 lbs.), our modeling can equally apply to ground-based drones that can carry considerably larger loads (e.g., Pettitt 2015, Pymnts, 2015). While widespread use of delivery drones requires overcoming a variety of economic, regulatory and technological obstacles (Lee et al. 2016), there is a need for strategic analyses of truck-drone delivery systems to help identify promising delivery system designs. Note that our research is oriented towards home delivery systems, not military or surveillance applications, though there are some obvious similarities. There are also many related healthcare applications, including medicine and vaccine delivery (Markoff 2016, Thiels et al. 2015) and organ transport (Francisco 2016), but our focus is on delivery of small parcels to homes and businesses (i.e. "last-mile delivery" as currently done by UPS, FedEx, US Postal Service, etc.), using hybrid truck-drone delivery systems, as exemplified by HorseFly™ (Workhorse 2016), the partnership between Matternet and Mercedes-Benz (Sloat and Kopplin 2016, Matternet 2017) and UPS (UPS 2017). Note that while much attention has been on the use of drones to allow very high service levels (e.g., 30 minute delivery), in this paper our focus is on minimizing expected costs for deliveries across a region, such as by replacing long (e.g., 8-hour) truck-only delivery routes by hybrid truck-drone routes.

☆ 2 main issue

The fundamental research questions we address are: (1) How can hybrid truck-drone routes best be used to serve a region? and (2) How do hybrid truck-drone routes compare with truck–only delivery? Because many different types of delivery drones are being proposed, our flexible modeling framework allows us to analyze a range of drone types and delivery strategies. As an illustration, Figure 1(a) shows a hybrid truck-drone delivery route where the truck and drone alternate deliveries while traveling through the service region. The truck route is shown with rectilinear (L1 metric) travel between the truck

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deliveries indicated with squares, while the drone uses straight-line travel (L2 metric) to deliver to the customers indicated by circles. Fig. 1(b) illustrates hybrid truck-drone delivery where a truck route has three delivery stops, and from each of those there are four single-stop drone routes.

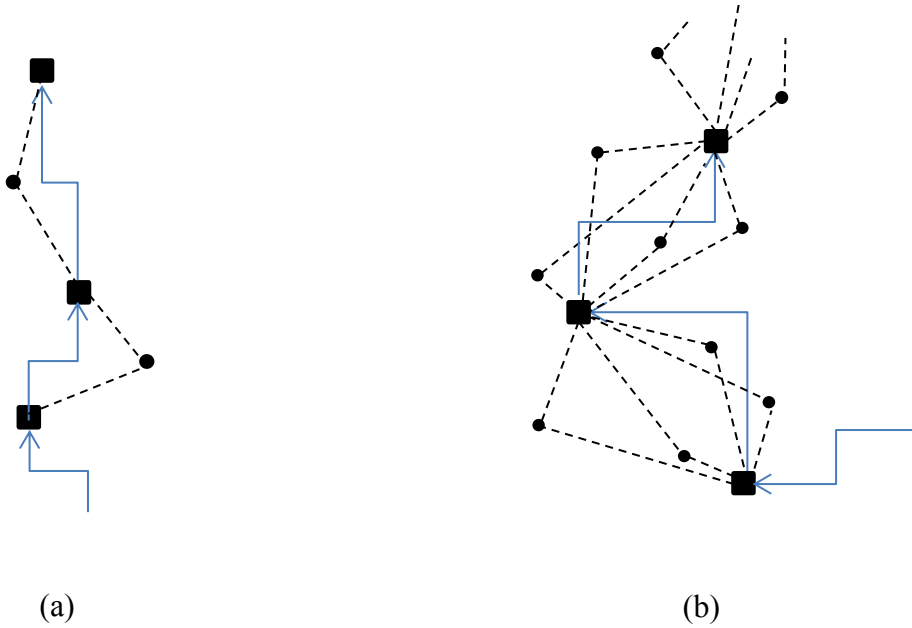(a)   (b)

Figure 1. Hybrid truck drone delivery with (a) alternating deliveries and (b) multiple drone deliveries per truck delivery.

For strategic analyses we formulate continuous approximation (CA) models of truck-drone delivery, where trucks transport drones from a depot to the vicinity of customers, and then both drones and trucks make deliveries simultaneously with the drones launched from and returning to the truck (to pick up more items for delivery). We also model truck-only delivery on multi-stop tours for comparison. In this work we consider only one-stop drone routes so that each drone carries a single item for delivery. Ongoing research is addressing designs with high service levels and with multi-stop drone routes (where the drone visits two or more customers on a route). Our approach differs from most research on hybrid truck-drone delivery in using CA models to derive general insights into the circumstances for beneficial uses of drones and hybrid truck-drone delivery (rather than solving particular problem instances), and in exploring the benefits from allowing multiple drones per truck. We employ reasonable parameter values and operating characteristics in the models to evaluate a range of delivery environments (e.g., from very

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rural to suburban regions) and highlight key areas of importance for beneficial drone operations. Our focus is on logistical and operational aspects, and not on the (important) legal, regulatory, and technological issues, including customer acceptance (Lee et al. 2016, Lotz 2105).

The remainder of the paper is organized as follows. Section 2 reviews the relevant literature on truck-drone delivery modeling. Section 3 provides the continuous approximation model formulations and a short illustration. Section 4 describes the development of reasonable values for the drone operating costs. Section 5 provides results for several analyses for a range of settings and Section 6 is the conclusion.

2. Literature Review

Researchers have begun to analyze truck-drone delivery systems by extending traditional discrete Traveling Salesman Problem (TSP) and Vehicle Routing Problem (VRP) models and solution methods to produce routes for a particular set of customers (discrete points). Perhaps the earliest directly relevant research is Lin (2008, 2011) which considers "transportable delivery resources" carried on trucks. In this setting, foot couriers were the transportable resource and both the couriers and trucks could make deliveries, with trucks travelling much faster (20 km/hr vs 5 km/hr for foot couriers). This is in contrast to the recent research on truck-drone delivery which has largely been directed at aerial drones that are typically much faster than trucks.

Murray and Chu (2015) analyzes a TSP variant with a single truck and single drone called the flying sidekick TSP (FSTSP) where the drone and truck start and end at a depot, the drone travels on the truck, and the drone can make deliveries on its own. Each drone delivery is for a single stop, departing from and returning to the truck at sequential truck delivery stops. Thus, the drone cannot launch and recover at the same customer site (truck stop), so only a single drone trip is launched from any stop. The objective is to minimize the total time for both truck and drone to return to the depot, which includes travel time, launch and recover time for the drone, and waiting time for the drone or truck at the stop where recovery occurs. Murray and Chu (2015) also considers a parallel version of the problem where the truck and drone serve different sets of customers concurrently with the drone traveling back-and-forth

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between the depot and customers, while a truck tour visits other customers. For both formulations, Murray and Chu (2015) provides simple construction and improvement heuristics for solution. Ponza (2016) considers the same problem and presents an improved mathematical formulation and a simulated annealing solution approach. Ha et al. (2015) considers a very similar problem, though with a different name (TSP-D) and in this problem, once the drone is launched from either the depot or the truck (at a customer stop), it either returns to the truck at the next stop of the truck, or it returns to the depot. Ha (2015) optimizes an objective that may be the number of drone trips, travel time or travel distance, and solutions are presented for cluster first–route second and route first–cluster second heuristics. Ha (2016) considers a similar setting but with an objective of minimizing the sum of the truck and drone costs, and presents two heuristic solution methods based on GRASP and modifying an optimal TSP tour. Agatz (2015) considers a variant of the truck-drone delivery problem where the drone may launch and recover at the same truck stop, and provides solutions from a route first–cluster second heuristic. Ferrandez et al. (2016) analyzes a different model where the truck follows a TSP tour and from each truck stop one or more drones can perform multiple single-stop deliveries. Thus each truck stop is essentially a hub for multiple drone deliveries. The solution approach uses K-means clustering to find the truck stops, and a genetic algorithm for finding the truck route. This model topology is essentially the same as in Carlsson and Jia (2015), though this latter reference does not specifically refer to drones. Dorling et al. (2016) use a model for drone energy consumption to develop cost and time minimizing models for multi-stop drone deliveries from a depot. A different perspective is explored by Wang et al. (2016), which provides worst-case analyses for several truck-drone delivery settings including use of multiple truck-drone pairs. On a related theme, Hong et al. (2015) present a model for locating drone recharging stations and routing drones among these stations.

In contrast to the discrete TSP-based optimization models described above, our research develops strategic models for the design of truck-drone delivery systems using continuous approximation (CA) modeling techniques, where the demand for deliveries is treated as a continuous spatial density over a service region. For background and applications of CA models, see Campbell (1993), Langevin et al.

(1996), and Daganzo et al. (2012). Carlsson and Song (2017) use CA models for an asymptotic analysis of a truck-drone delivery system where all deliveries are made by drones that are launched and recovered at trucks as they make a tour through the service region. The objective is to minimize the time of the last delivery and costs are not modeled. In our research, we develop mathematical models for truck-drone delivery using CA techniques by extending the swath model pioneered by Daganzo (1984). We formulate mathematical expressions for the expected travel cost (and time) for the trucks and drones based on the strategy employed for deploying the trucks and drones, the fundamental parameters of the problem setting (e.g., spatial density of customers (demand) and the fraction of demand served by drones) and the operating characteristics of trucks and drones (e.g., operating costs, stop costs, speeds, etc.). The models allow different travel metrics, as truck travel is via roads, but aerial drones may travel (more or less) in straight lines. In contrast to Carlsson and Song (2017), our models allow comparison of expected costs for truck-drone and truck-only delivery, and we evaluate the sensitivity of results to operating costs and the delivery density.

3. Continuous Approximation Models

This section provides a derivation of the continuous approximation truck-drone delivery model. The fundamental idea is to extend the "strip" modeling strategy for approximating the distance of vehicle routes from Daganzo (1984) to allow the truck to launch and recover drones at truck stops (customers). Extensions where the trucks launch and recover the drone at intermediate points between customers are an area of current research. For modeling purposes, let the service region be a compact region of area *A* where delivery stops are randomly and independently distributed in the region with density δ. Consider a truck traveling using the L1 metric along the length of a swath of width *w,* like that shown in Figure 2(a), and visiting all customers in (vertical) order along the swath. The expected horizontal distance between adjacent stops is *w/3* and the expected vertical distance between adjacent stops is *1/wδ.* If a truck is to visit all stops, combining the expected horizontal and vertical truck travel distance between two adjacent stops gives an expected total truck travel (L1) distance per stop of

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$$\frac{w}{3} + \frac{1}{\delta w} \, . \tag{1}$$

Using a truck alone, the optimal swath width that minimizes the expected truck travel distance (or travel time assuming a constant truck speed) is

$$w^* = \sqrt{\frac{3}{\delta}} \; . \tag{2}$$

Let c_t be the truck variable cost per unit distance and s_t be the fixed truck stop cost. Then the minimum expected total truck cost per delivery is

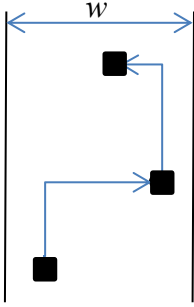$$c_t \frac{2}{\sqrt{3\delta}} + s_t \; . \tag{3}$$

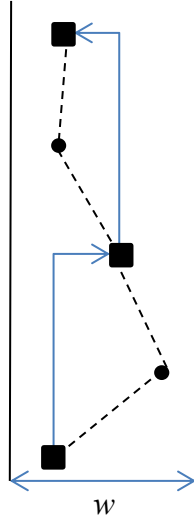Figure 2(a). Three truck stops along a swath of width w.

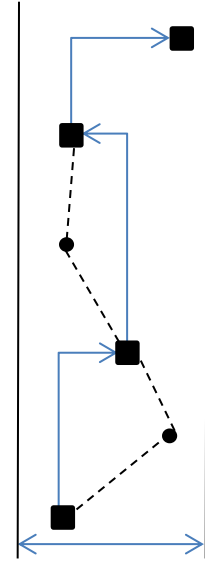Figure 2(b). Truck and drone stops along a swath of width w.

Figure 2(c). Truck and drone delivery for $n = \frac{2}{3}$.

Now suppose that drones can also visit customers to make deliveries in place of the truck and the drones are launched and recovered by the truck. Figure 2(b) shows the truck and drone alternating deliveries over two "cycles" (moving from bottom to top), where the drone travel is indicated by dashed lines and the truck travel has arrows showing direction of travel. If drone travel is via Euclidean distance (the L2 metric) and the drone launches and is recovered at the truck delivery customers adjacent to the drone delivery customer (in the vertical direction), then the expected drone travel distance per point is

$$\sqrt{\left(\frac{w}{3}\right)^2 + \left(\frac{1}{\delta w}\right)^2}. \tag{4}$$

To generalize to situations where the drone and truck do not simply alternate deliveries, we model the truck-drone travel as a connected series of cycles with n representing the ratio of the number of drone deliveries in one cycle to the number of truck deliveries in one cycle. Then we can consider two cases: $0 < n \le 1$ where the drone makes the same ($n = 1$) or fewer stops than the truck per cycle, and $n > 1$ where there are multiple drone stops per truck stop.

3.1 $0 < n \le 1$: One drone per truck

For $n \le 1$, we model the drone launching and returning to the truck at different customers because the drone makes the same ($n = 1$) or fewer stops than the truck. Thus we model travel along the swath such that the drone delivery stop is the first customer after the truck stop where the drone is launched, and the drone is recovered at the first customer (truck stop) immediately after the drone stop. (This is the same setup used in much of the discrete modeling for truck-drone delivery noted earlier.) As an example, Figure 2(c) shows $= \frac{2}{3}$, where the drone stays on the truck between the last two truck stops. The fraction of stops served by the drone is denoted f, and $f = \frac{n}{n+1}$ ($f = 2/5$ in Figure 2(c)).

For $n \le 1$, the expected truck horizontal travel distance per point in one cycle is $\frac{w}{3}\left(1 - \frac{n}{n+1}\right) = \frac{w}{3}\frac{1}{n+1}$ and the expected truck vertical travel distance per point in one cycle is $\frac{1}{\delta w}$ (the same as before because the trucks must pass all stops in the vertical direction). Thus, the expected total truck travel distance per point is

$$\frac{w}{3}\frac{1}{n+1} + \frac{1}{\delta w}. \tag{5}$$

The optimal swath width to minimize the expected truck travel distance or time (assuming a constant truck speed) is

$$w_t{}^* = \sqrt{\frac{3}{\delta}}\sqrt{n+1} \ . \tag{6}$$

Because the fraction of stops made by the drone is $\frac{n}{n+1}$, the expected drone travel distance per point is

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$$2\frac{n}{n+1}\sqrt{\left(\frac{w}{3}\right)^2 + \left(\frac{1}{\delta w}\right)^2},\tag{7}$$

where the square root term provides the expected Euclidean distance for one leg of the drone trip. The optimal swath width to minimize the expected drone travel distance (or travel time assuming a constant drone speed) is the same as for using a truck alone and is given by

$$w_d{}^* = \sqrt{\frac{3}{\delta}}.\tag{8}$$

We combine the truck and drone travel to model the expected total cost of the truck-drone route with $n \leq 1$. Let c_d be the drone variable cost per unit distance, and s_d be the marginal cost for drone delivery (including launch and recovery costs) relative to truck delivery. Thus the delivery stop cost for a drone is given by and $s_d + s_t$ where a negative (positive) value for s_d is the per delivery savings (excess costs) for making a delivery with a drone. (If truck and drone delivery costs are equal, then $s_d = 0$.) Based on the expected distances (5) and (7), the total expected cost per delivery when $n \leq 1$ is

$$c_t\left[\frac{w}{3}\frac{1}{n+1} + \frac{1}{\delta w}\right] + 2c_d\frac{n}{n+1}\sqrt{\left(\frac{w}{3}\right)^2 + \left(\frac{1}{\delta w}\right)^2} + \frac{n}{n+1}s_d + s_t.\tag{9}$$

In terms of the fraction of stops made by drones $f = \frac{n}{n+1}$, the total expected cost per stop is

$$c_t\left[\frac{w}{3}(1-f) + \frac{1}{\delta w}\right] + 2c_d f\sqrt{\left(\frac{w}{3}\right)^2 + \left(\frac{1}{\delta w}\right)^2} + f s_d + s_t,\tag{10}$$

which is linear in f for a fixed swath width. Thus, the optimal fraction of drone deliveries to minimize cost is either: (i) $f = n = 0$, i.e., use truck-only delivery and no drones; or (ii) $f = 0.5$ (when $n = 1$) i.e., use drones for half the deliveries (as much as possible). This latter case (use drones for half the deliveries) is preferred when the ratio of truck to drone operating costs per unit distance is large enough:

$$\frac{c_t}{c_d} > 2\sqrt{1 + \frac{9}{\delta^2 w^4}} + \frac{3}{w}\frac{s_d}{c_d}.\tag{11}$$

This shows how the desirability of using drones depends on their operating cost relative to trucks, the density of deliveries (δ), the magnitude of the marginal drone stop cost relative to the drone operating cost $\left(\frac{s_d}{c_d}\right)$, and the design parameter w (width of the swath). If drone operating costs per mile are many

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times smaller than truck operating costs per mile, then drones should be heavily used (make half the deliveries) unless the marginal drone stop cost is much larger than the drone operating cost. However, note that the delivery characteristics of items being delivered (e.g., heavy weight or location beyond the drone flight range) may preclude the use of drones for half the stops, so in practice n may be less than 1.

The minimum expected total cost per stop is realized with the optimal swath width that minimizes expression (9), but this does not lend itself to a simple closed form solution. However, because both the truck and drone components of the expected travel cost are convex, the optimal swath width w^* will be between the values for using trucks alone or drones alone (i.e., between $\sqrt{\frac{3}{\delta}}$ and $\sqrt{n+1}\sqrt{\frac{3}{\delta}}$; see equations 2 and 8). Extensive numerical experimentation has shown that the weighted average of these swath widths is very nearly optimal (with expected cost exceeding the numerically minimum expected cost by less than 0.02%). Thus, the nearly optimal swath width is

$$w^* \cong \frac{(\sqrt{n+1}\, c_t + 2c_d)}{c_t + 2c_d}\sqrt{\frac{3}{\delta}} = k\sqrt{\frac{3}{\delta}} \quad , \tag{12}$$

where $k = \frac{(\sqrt{n+1}\, c_t + 2c_d)}{c_t + 2c_d}$ and we suppress the dependence of k on n, c_t and c_d. Note that $1 < k \le \sqrt{2}$ because $0 < n \le 1$.

Using k, the expected cost per delivery (expression 9) can be written

$$\frac{1}{\sqrt{3\delta}}\left\{c_t\left[\frac{k}{n+1} + \frac{1}{k}\right] + 2c_d\frac{n}{n+1}\sqrt{k^2 + \frac{1}{k^2}}\right\} + \frac{n}{n+1}s_d + s_t \,, \tag{13}$$

where the term in brackets depends only on n, c_t, c_d and s_d, but not on the density of stops δ. Similarly, expression (11) can now be written

$$\frac{c_t}{c_d} > 2\sqrt{1 + \frac{1}{k^4}} + \sqrt{\delta}\frac{\sqrt{3}}{k}\frac{s_d}{c_d} \,. \tag{14}$$

Because the smallest k value is 1, drones are preferred when $\frac{c_t}{c_d} > 2.83 + 1.74\sqrt{\delta}\frac{s_d}{c_d}$. Thus, when the marginal drone stop cost s_d is zero (drone stop and truck stop costs are equal), using drones will be preferred as long as drone cost per mile is about 35% or less of the truck cost per mile. Drones will be

attractive with higher drone operating costs when the drone marginal stop cost is less than the truck stop cost, especially for large stop densities (as each delivery by a drone saves on the stop cost). Conversely, when the drone marginal stop cost is greater than the truck stop cost ($s_d > 0$), then a high enough density of stops will make drone use unattractive no matter how low the drone operating cost. Equation (14) highlights the key role of the density of stops and the marginal drone stop cost.

3.2 Linehaul travel

The above results are appropriate for the truck or truck-drone pair traveling along the swath making deliveries, starting and ending at a depot. Figure 4(a) presents an example, where the entire service area can be served with 5 routes from a centrally located depot, with each route serving a wedge-shaped sector. However, when each route serves only a small portion of the service region, then the total expected travel distance should include a "linehaul" portion from the depot to the region where the deliveries on a route begin. Figure 4(b) shows the start of two of the many routes in the service region when each route serves only a small part of the region, and linehaul travel is needed from the depot to the area of delivery stops for each route.
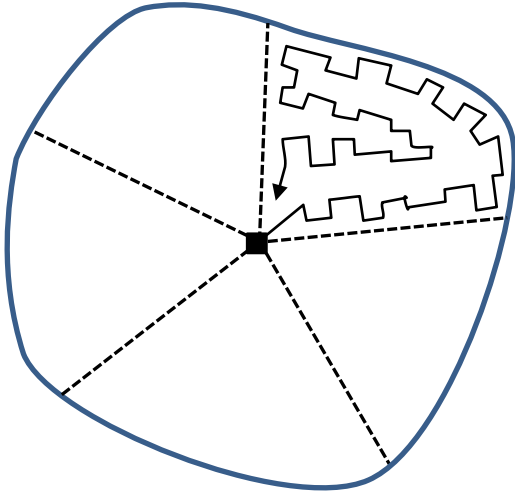

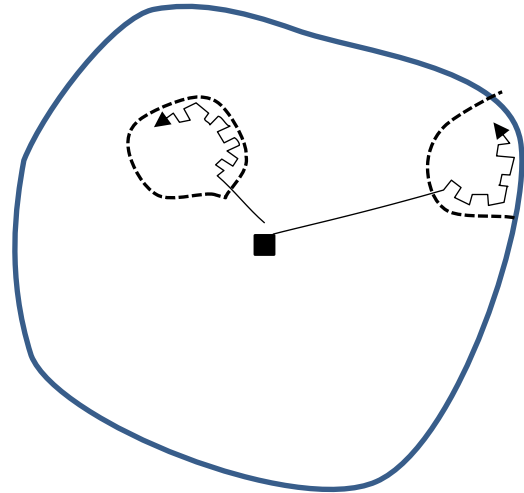Figure 3(a). No linehaul travel when few routes cover the service region.

Figure 3(b). Linehaul travel when many routes are in the service region.

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To formulate the expected linehaul travel, suppose the truck has a delivery capacity of serving m stops (total number of deliveries by trucks and drones = m). If the depot is centrally located in a compact service region of area A and the starting points for truck-drone delivery routes are randomly located in the service region, then the expected one-way straight-line linehaul distance can be approximated by $\frac{2}{3}\sqrt{A/\pi}$. The expected roundtrip linehaul travel cost per stop, with a 20% circuity factor to account for truck travel longer than the straight-line distance, is

$$c_t \frac{1.2 \times 4}{3m}\sqrt{A/\pi} = c_t \frac{0.9027}{m}\sqrt{A}. \tag{15}$$

When linehaul travel is required, then the expected linehaul cost from equation (15) is added to the earlier costs per stop (e.g., equation 3 or 13). This gives costs a minimum expected cost per delivery for truck-only travel, denoted E_{to}, of

$$E_{to} = c_t \left\{ \frac{2}{\sqrt{3\delta}} + \frac{0.9027}{m_{to}}\sqrt{A} \right\} + s_t \tag{16}$$

and a minimum expected cost per delivery stop for hybrid truck-drone travel, denoted $E_{td}(n)$, of

$$E_{td}(n) = \frac{1}{\sqrt{3\delta}} \left\{ c_t \left[\frac{k^*}{n+1} + \frac{1}{k^*} \right] + 2c_d \frac{n}{n+1}\sqrt{k^{*2} + \frac{1}{k^{*2}}} \right\} + \frac{n}{n+1}s_d + s_t, \quad \text{or} \tag{17a}$$

$$E_{td}(n) = \frac{1}{\sqrt{3\delta}} \left\{ c_t \left[\frac{k^*}{n+1} + \frac{1}{k^*} \right] + 2c_d \frac{n}{n+1}\sqrt{k^{*2} + \frac{1}{k^{*2}}} \right\} + c_t \frac{0.9027}{m_{td}}\sqrt{A} + \frac{n}{n+1}s_d + s_t, \tag{17b}$$

where k^* provides the optimal swath width for truck-drone travel. Separate m vales are used in equations (16) and (17b) as the number of stops per route may differ with and without drones. Though not the focus in the present paper, note that the linehaul travel will be more important when high service levels (quick deliveries) dictate short routes with few stops on each, so more routes are needed to cover the service region. (This is the subject of current research.)

3.3 $n > 1$: Multiple drone deliveries per truck delivery

For $n > 1$, there are more drone deliveries than truck deliveries in a cycle, so unlike with $n \leq 1$, there cannot be a single drone trip departure and return at each truck delivery. There are several operational possibilities associated with $n > 1$, including having multiple drones per truck each making a

single delivery per cycle, having a single drone per truck making multiple deliveries per cycle, or some combination of these. Our interest here is in exploring the benefits of multiple drones per truck, so we consider the situation where *n* drones are launched from the truck at each truck delivery stop, these drones make one delivery each visiting the next *n* stops along the strip, and then all these drones recovered at the $n+1^{st}$ stop which is made by the truck. Figure 4 is an illustration with $n = 3$. Thus, there is a single truck delivery in each cycle and n is the number of drones per truck, as well as the ratio of drone deliveries to truck deliveries per cycle. Other options with $n > 1$, such as having a single drone launch and return to the same truck stop repeatedly, are an area of ongoing research.
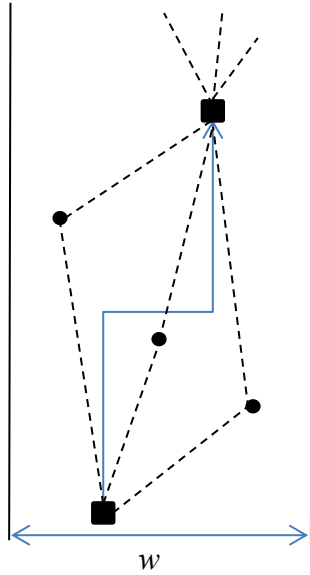

Figure 4. Truck and drone delivery for $n = 3$.

The expected horizontal distance between adjacent truck deliveries is *w/3* and the expected vertical truck travel distance between adjacent truck deliveries is $(n+1)/w\delta$. Combining the expected horizontal and vertical truck travel distance between two adjacent deliveries gives an expected total truck travel (L1) distance per delivery of $\left(\frac{w}{3}\frac{1}{n+1} + \frac{1}{\delta w}\right)$, which is the same as for $0 < n \leq 1$ (See expression 5). Considering the truck travel alone, the optimal swath width is given by equation (6).

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With drone travel via Euclidean distance (the L2 metric) and all n drones launched from a single truck stop and recovered at the $n + 1^{st}$ truck stop, then the total expected drone travel distance for the n delivery stops in a cycle is

$$\sum_{i=1}^{n} \left(\sqrt{\left(\frac{w}{3}\right)^2 + \left(\frac{i}{\delta w}\right)^2} + \sqrt{\left(\frac{w}{3}\right)^2 + \left(\frac{n+1-i}{\delta w}\right)^2} \right). \tag{18}$$

To simplify equation (18) we can approximate the distance for a drone delivery by the distance for the average drone delivery, which is halfway between the truck stops, or a vertical distance $\frac{n+1}{2}\frac{1}{\delta w}$ from each truck stop. This is a very good approximation (errors < 2%) unless the drone delivery is very close to the truck delivery. Replacing each square root in the summation by a distance $\sqrt{\left(\frac{w}{3}\right)^2 + \left(\frac{n+1}{2\delta w}\right)^2}$ yields the expected approximate drone distance per delivery of

$$\frac{2n}{n+1} \sqrt{\left(\frac{w}{3}\right)^2 + \left(\frac{n+1}{2\delta w}\right)^2} \quad . \tag{19}$$

The optimal swath width that minimizes the expected total drone travel distance is then approximately

$$w^* = \sqrt{\frac{n+1}{2}} \sqrt{\frac{3}{\delta}} \quad . \tag{20}$$

The expected total cost per delivery for the hybrid truck-drone delivery with $n \geq 1$, including truck travel, drone travel and the stop cost, can then be approximated as

$$E_{td}(n) = c_t \left[\frac{1}{n+1}\frac{w}{3} + \frac{1}{\delta w} \right] + c_d \frac{2n}{n+1} \sqrt{\left(\frac{w}{3}\right)^2 + \left(\frac{n+1}{2\delta w}\right)^2} + \frac{n}{n+1} s_d + s_t \quad \text{or} \tag{21a}$$

$$E_{td}(n) = c_t \left[\frac{1}{n+1}\frac{w}{3} + \frac{1}{\delta w} + \frac{0.9027}{m_{td}} \sqrt{A} \right] + c_d \frac{2n}{n+1} \sqrt{\left(\frac{w}{3}\right)^2 + \left(\frac{n+1}{2\delta w}\right)^2} + \frac{n}{n+1} s_d + s_t \quad , \tag{21b}$$

where (21a) is with the linehaul truck travel.

Again, the optimal swath width w^* is not easy to derive analytically from equation (21a) or (21b), though it can be found numerically. But because the expected total cost is the weighted sum of two convex functions of w, then both equations (21a) and (21b) are convex in w and the swath widths that minimize (21a) and (21b) is well approximated by the weighted average of the optimal widths for truck delivery component (6) and the drone delivery component (20):

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$$w^* \cong \frac{c_t\sqrt{n+1}\sqrt{\frac{3}{\delta}}+2nc_d\sqrt{\frac{n+1}{2}}\sqrt{\frac{3}{\delta}}}{c_t+2nc_d} = \sqrt{n+1}\sqrt{\frac{3}{\delta}}\left[\frac{c_t+\sqrt{2}nc_d}{c_t+2nc_d}\right] \quad . \tag{22}$$

Extensive computational experiments verify that equation (22) is a good approximation for the optimal swath width (see Appendix A).

Note that as with $n \le 1$, the optimal swath width can be written as $k'\sqrt{\frac{3}{\delta}}$, where k' is a function of n, c_t and c_d:

$$k' = \sqrt{n+1}\left[\frac{c_t+\sqrt{2}nc_d}{c_t+2nc_d}\right] = \sqrt{n+1}\left[\frac{1+\sqrt{2}n\frac{c_d}{c_t}}{1+2n\frac{c_d}{c_t}}\right] \quad , \tag{23}$$

which has a maximum of $0.724\sqrt{n+1} < k' \le \sqrt{n+1}$ for reasonable values of $n \le 8$ (8 drones per truck) and $c_d \le c_t$. The expected cost in equation (21a)-(21b) can then be written

$$E_{td}(n) = \frac{1}{\sqrt{3\delta}}\left\{c_t\left[\frac{k'}{n+1}+\frac{1}{k'}\right] + c_d\frac{2n}{n+1}\sqrt{k'^2+\left(\frac{n+1}{2}\right)^2\frac{1}{k'^2}}\right\} + c_t\frac{0.9027}{m_{td}}\sqrt{A} + \frac{n}{n+1}s_d + s_t \quad \text{or} \tag{24a}$$

$$E_{td}(n) = \frac{1}{\sqrt{3\delta}}\left\{c_t\left[\frac{k'}{n+1}+\frac{1}{k'}\right] + c_d\frac{2n}{n+1}\sqrt{k'^2+\left(\frac{n+1}{2}\right)^2\frac{1}{k'^2}}\right\} + \frac{n}{n+1}s_d + s_t \quad , \tag{24b}$$

where the term in brackets depends only on n, c_t, c_d and s_d, but not on the density of stops δ.

3.4 Truck travel time and number of stops per route

The expected travel time of a route can be calculated using the expected distance relations derived earlier. Let c_t' be the reciprocal of the average truck speed while delivering, c_{tl}' be the reciprocal of the average truck linehaul speed, c_d' be the reciprocal of the average drone speed, s_t' be the average stop time per truck delivery at a customer and s_d' be the marginal stop time per drone delivery, relative to the truck stop time per delivery. Thus if $s_t' = 1$ minute and the average drone stop time for a drone delivery is 0.6 minutes, then $s_d' = -0.4$ minutes. With m_{td} total truck and drone stops per route, the number of drone delivery stops per route is $\frac{m_{td}\times n}{n+1}$ and the number of truck delivery stops per route is $\frac{m_{td}}{n+1}$. The expected travel time for a truck on a truck-drone route with m total deliveries (using the expected truck travel distance from equation 9 or 21b) is

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$$m_{td} \times \left[c_t' \left(\frac{1}{n+1} \frac{w}{3} + \frac{1}{\delta w} \right) + \frac{1}{n+1} s_t' \right] + 0.9027 c_{tl}' \sqrt{A} \,.$$

This expected time assumes the truck does not need to wait for the drone (as in the illustrations that

follow). With an available time for the route of T hours, the number of delivery stops for a truck-drone

route is given by

$$m_{td} = \frac{T - 0.9027 c_{tl}' \sqrt{A}}{c_t' \left[\frac{1}{n+1} \frac{w}{3} + \frac{1}{\delta w} \right] + \frac{1}{n+1} s_t'} \qquad \text{or} \qquad (25a)$$

$$m_{td} = \frac{T}{c_t' \left[\frac{1}{n+1} \frac{w}{3} + \frac{1}{\delta w} \right] + \frac{1}{n+1} s_t'} \,, \qquad (25b)$$

depending on whether or not linehaul travel is needed. The number of delivery stops for a truck-only

route can be derived similarly as

$$m_{to} = \frac{T - 0.9027 c_{tl}' \sqrt{A}}{c_t' \left[\frac{2}{\sqrt{3\delta}} \right] + s_t'} \qquad \text{or} \qquad (26a)$$

$$m_{to} = \frac{T}{c_t' \left[\frac{2}{\sqrt{3\delta}} \right] + s_t'} \,. \qquad (26b)$$

3.5 Illustration

To illustrate the use of the continuous approximation equations, consider a service region of area

1250 square miles with c_t = \$1.25/mile, c_d = \$0.05/mile and $s_d = 0$. Table 1 presents results of the

models for two selected delivery stop densities, $\delta = 0.1$ per square mile and $\delta = 50$ per square mile, and

fours levels of drone use: $n = 0, 0.5, 1$ and 2. The density $\delta = 0.1$ per square mile corresponds to delivery

to all addresses in a rather rural area, and $\delta = 50$ per square mile can be viewed as delivery to all

addresses in a suburban area. But note that the delivery density may be very different than the density of

households, as only a fraction of households will receive a delivery on a typical day. We use "rural" and

"suburban" to refer to the density of deliveries, not the density of households. The approximately optimal

swath widths for the truck-drone delivery routes are given by equation (12) for $n \leq 1$ and by equation

(23) for $n > 1$ and are shown in row 3. Recall that $n = 0$ corresponds to truck-only delivery. With

$\delta = 0.1$ there are only 125 total deliveries in the region, so we do not include the linehaul travel as there

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are only a few delivery routes; but with $\delta = 50$ there are 62,500 deliveries served by many delivery routes in the region, so we do include linehaul travel.

Table 1. Illustration of truck-drones delivery routes with two densities of stops.

	$\delta = 0.1$/square mile				$\delta = 50$/square mile			
n	0	0.5	1	2	0	0.5	1	2
Swath width (miles)	5.5	6.6	7.6	9.1	0.24	0.30	0.34	0.41
Number of truck deliveries per route	37.6	31.2	27.3	22.5	269.9	251.3	237.5	217.5
Number of drone deliveries per route	0	15.6	27.3	45.0	0	125.7	237.5	434.9
Average truck route length (miles)	137.5	139.6	140.9	142.5	76.0	82.2	86.8	93.5
Average drone travel distance per drone delivery (miles)	-	5.3	5.7	6.9	-	0.24	0.26	0.31
Number of routes as a % of truck only routes	-	80.4%	69.0%	55.7%	-	71.6%	56.8%	41.4%

Suppose the average truck speed while delivering is 20 mph ($c_t' = 1/20$), the average truck linehaul speed is 40 mph (c_{tl}'=1/40), the stop time is 1 minute per delivery for truck or drone (i.e., $s_t' = 1$ minute and $s_d' = 0$) and the available time for a route is $T = 7.5$ hours. Then the average number of truck and drone deliveries per truck-drone route are shown in rows 4 and 5 of Table 1, respectively. Note that for $n = 2$ each of the two drones makes the same number of deliveries as the truck. The expected truck route length is shown in row 6 of Table 1. These route lengths are about 140 miles when $\delta = 0.1$ and considerably shorter (76-94 miles) when $\delta = 50$. Row 7 in Table 1 provides the expected drone travel distance per drone delivery (distance from the truck to the drone delivery stop and back to the truck), which varies from about one-quarter mile to 7 miles, as the optimal swath width changes. For reasonable drone speeds (e.g., 40 mph) the average drone travel time for each delivery is less than the average truck travel time between the stops where the drone is launched and recovered, which suggests the drones could

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be operated so that the truck does not need to wait for the drone(s). However, details of the timing for drone/truck meetings are beyond the scope of this research.

The use of truck-drone routes allows fewer routes to cover a given demand compared to using trucks alone, as there are more deliveries per truck-drone route (see rows 4-5). The last row of Table 1 (the ratio of deliveries per route using trucks alone to deliveries per route for truck-drone routes) highlights this benefit and shows that truck-drone routes can reduce the number of routes (and drivers) required substantially (from about 20% to almost 60% in the table), depending on the delivery density and number of drones per truck. This illustration shows how the models produce realistic routes and how the optimal swath width and the number of deliveries per route depend strongly on the density of deliveries δ and the ratio of drone deliveries to truck deliveries n.

4. Drone Operational and Cost Data

Evaluation of potential benefits from truck-drone delivery systems using the expected cost formulae requires reasonable values of the relevant parameters, especially drone costs. Unfortunately, we cannot collect such data in the field as drone-based delivery systems are not currently in operation. Further, the firms experimenting with drone delivery have not published operational and performance data from an actual implementation, and any future deployment of drone delivery systems may well incur costs different than those of today's drones. To further complicate matters, there is a wide range of data from academic, practitioner and industry sources (in part due to the various types of drones envisioned). Following Amazon's announcement in 2013 of plans for drone delivery from fixed depots, there were several efforts to estimate drone operating costs for such a system. Menon (2013) estimated costs for the drones alone as only $0.04 - $0.18 per delivery, while Raffaello (2014) estimated only the battery and electricity costs at about $0.01 per km. Both of these analyses ignored labor costs for drone operators, as well as infrastructure costs for the drone delivery system. Lewis (2014) suggested a much larger cost of $10-$17 per delivery, which included "fulfillment center costs, insurances, drone fees and service", along with large labor costs resulting from having one operator (pilot) per drone (as specified in current US law).

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One of the best attempts to analyze drone system costs and operating parameters at a strategic level is by ARK Invest (Keeney 2105). ARK Invest suggested that a large scale implementation (based on Amazon) could serve 400 million deliveries/year in the US, which includes packages under 5 lbs and within 10 miles of a depot. They consider system costs for drone delivery from fixed depots where each drone makes an average of 30 deliveries per day in two eight-hour shifts and each drone operator is responsible for 10-12 drones. They estimate annual capital costs for infrastructure, drones and batteries, and annual operating costs for labor (drone operators), data bandwidth, maintenance and electricity. This results in a total annual cost of $350 million, and a suggested price of $1 per delivery (assuming a 15% discount rate). With an average 7 mile (one way) drone delivery distance, the $1 cost per drone delivery equates to about $0.07/mile. Labor costs are the large majority of operating costs, even with the assumption that each operator controls 10-12 drones at a time, which does not follow current US law.

In this paper, to explore a range of operating costs and the sensitivity of the model results to various operational parameter values, we consider $n = 1$ to 8 drones per truck, drone operating costs per mile from c_d = $0.01 to $0.625, and marginal drone stop costs from s_d = -$0.20 to $0.10. We also vary the density of deliveries widely to explore the benefits of truck-drone delivery in very different settings. All models employ a baseline setting of a service region of A= 1250 square miles, time limit T= 8 hours per route, truck operating cost c_t = $1.25 per mile, and truck stop cost of $0.40. We limit the truck capacity to a maximum of 500 delivery stops to reflect the size of delivery trucks currently in use. In all instances, we use a truck linehaul speed of 40 mph (c'_{tl}=1/40), truck delivery speed of 20 mph ($c'_t = 1/20$), and a stop time of 1 minute per delivery for truck and for the drone (i.e., $s'_t = 1$ minute and $s'_d = 0$).

5. Results

For a particular problem instance (value of $\delta, n, c_d,$ and s_d) we calculate the swath width (w) from equation (12) or (22) and the number of truck and drone stops per route from equation (25) or (26), limited to a maximum of 500 deliveries per route. The total expected costs per delivery are E_{to} and $E_{td}(n)$. The % savings per delivery from truck-drone delivery relative to truck-only delivery is

$$PSAV = \frac{E_{to} - E_{td}(n)}{E_{to}} \ . \tag{27}$$

Positive values of $PSAV$ indicate savings from truck-drone delivery relative to truck-only delivery, and negative values indicate that truck-only delivery has lower expected cost. The following results highlight the potential benefits of truck-drone delivery and the impact of four key parameters of interest: the number of drones per truck n, the density of deliveries δ, the drone operating cost per mile c_d, and the drone marginal stop cost s_d.

5.1 Benefits from multiple drones per truck

To illustrate the benefits from using multiple drones per truck, Figure 5 shows $PSAV$ with 1-8 drones as a function of the drone operating cost c_d, where the marginal drone stop cost is zero and the delivery density is $\delta = 10$ per square mile. The three dashed straight lines show the savings for a given number of drones per truck: $n = 1$, 2 or 5. The solid curve shows the maximum savings from allowing the number of drones per truck to vary with the drone operating cost (i.e., the upper envelope of the savings with 1-8 drones per truck). With low drone operating costs ($c_d < 0.12$), eight drones per truck provides the largest savings (with a maximum of nearly 40%). Fewer drones per truck are used as c_d increases and there is no savings for $c_d > 0.81$. With 1 drone per truck the maximum savings (for very small drone operating cost) is about 21%, and there are marginally decreasing additional savings from using more drones per truck. The maximum savings is 28.3% with 2 drones per truck, 32.6% with 3 drones per truck, 34.9% with 4 drones per truck and 39.8% with 8 drones per truck. The solid line in Figure 6 shows substantial benefits from multiple drones per truck when $c_d < 0.40$ (about one-third of the truck operating cost per mile), which seem to be very reasonable values for drone operating cost.
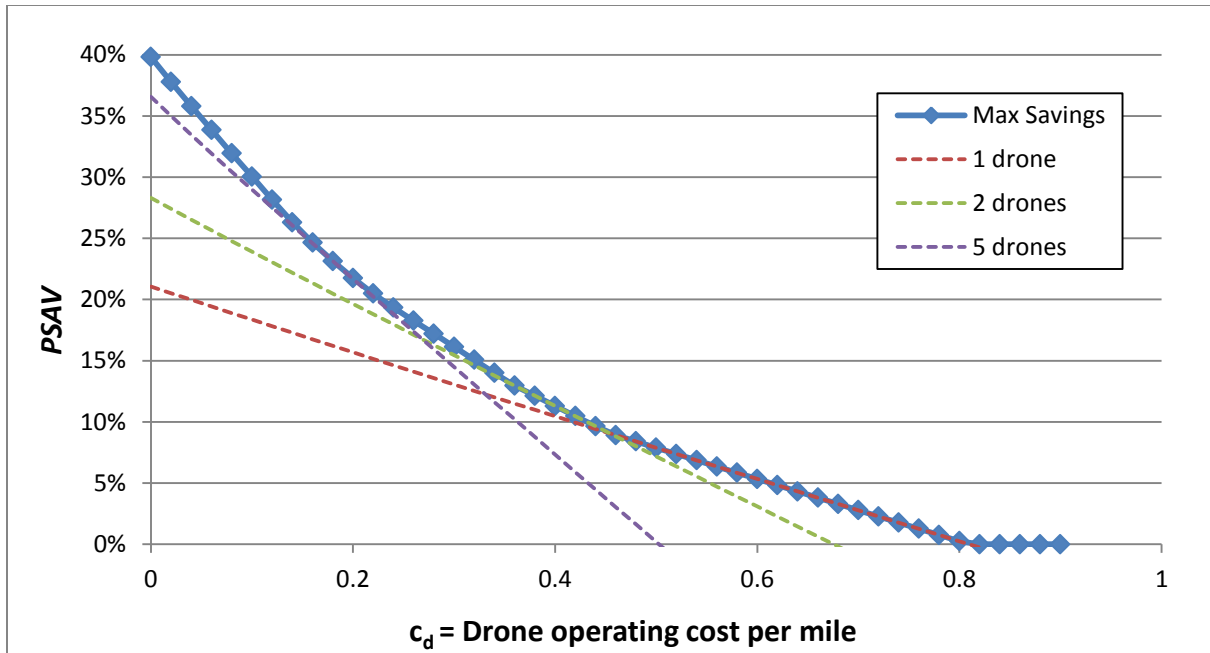
Figure 5. Percentage savings with 1-8 drones with increasing drone operating cost for $\delta = 10$ and $s_d = 0$.
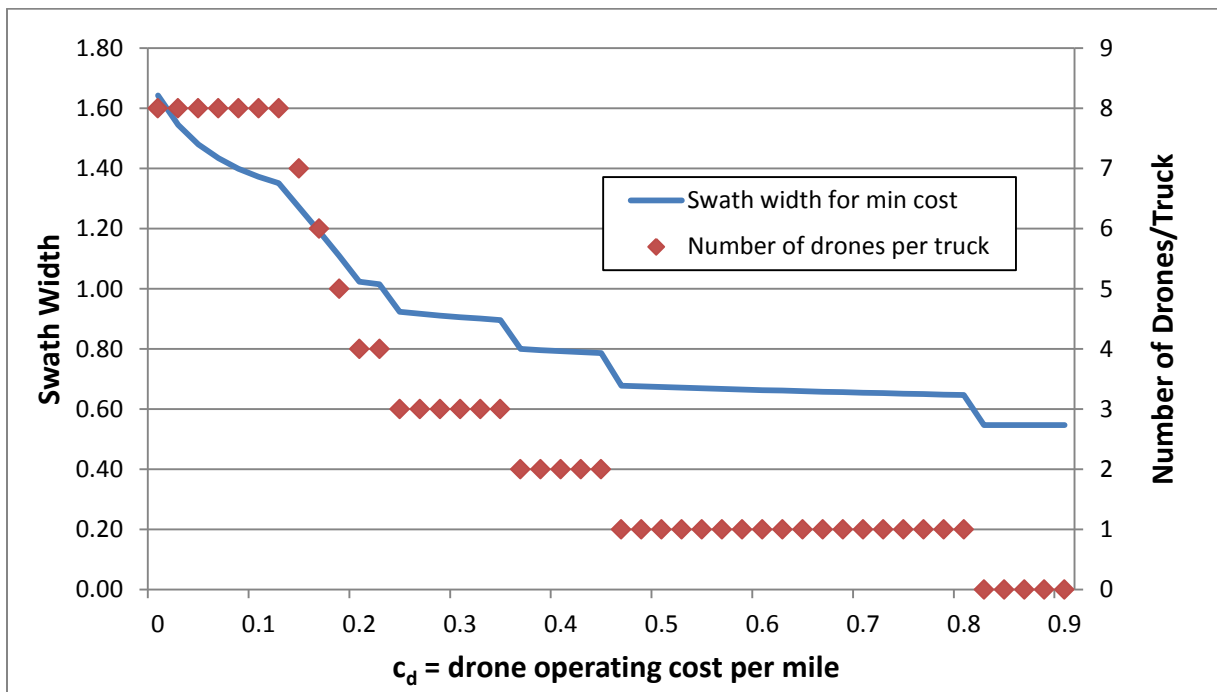

Figure 6. Swath width and number of drones per trucks with 1-8 drones with increasing drone operating cost for $\delta = 10$ and $s_d = 0$.

Figure 6 is a companion to Figure 5 that shows how the swath width and number of drones per truck decrease as the drone operating cost rises. This illustrates that as drones become more expensive to operate, then fewer drones per truck should be used and the swath width should decrease to reduce travel in the direction perpendicular to the general direction of travel along the swath. Table 2 shows for selected values of c_d the percentage savings per delivery and the number of drones per truck that provide the largest savings with $s_d = 0$ (as in Figures 6 and 7), and also for $s_d = -0.2$ (drone delivery stops cost $0.20 less than truck delivery stops). This shows how the greater savings in the drone stop cost with $s_d = -0.2$ leads to larger overall savings with a maximum of 56.7% (vs. 39.8% for $s_d = 0$), and greater use of multiple drones per truck even with relatively large drone operating costs. Similar results to Figures 5-6 and Table 2 are produced using other delivery densities, as higher drone operating costs per mile lead to fewer drones per truck and lower savings per stop.

Table 2. Percentage savings ($PSAV$) and number of drones per truck (n) that produces the lowest cost per stop with increasing drone operating cost for $\delta = 10$.

	$s_d = 0$		$s_d = -0.2$	
c_d	$PSAV$	n	$PSAV$	n
0.0	39.8%	8	56.7%	8
0.1	30.0%	8	47.0%	8
0.14	26.3%	7	43.2%	8
0.16	24.6%	6	41.3%	8
0.18	23.1%	5	39.5%	7
0.2	21.8%	4	37.8%	6
0.24	19.3%	3	34.7%	5
0.3	16.1%	3	30.7%	4
0.4	11.3%	2	25.1%	3
0.5	7.9%	1	19.9%	2
0.6	5.3%	1	15.8%	2
0.7	2.8%	1	12.3%	1
0.8	0.3%	1	9.8%	1
0.9	-2.3%	0	7.3%	1

5.2 Impact of density of deliveries

This section explores the impact of the density of deliveries in the region. As a baseline we consider $s_d = -0.1$ (the drone stop cost is 10 cents *less* than the truck stop cost). Figure 7 shows how truck-drone delivery savings per delivery ($PSAV$) decrease at a decreasing rate as customer density increases for selected values of the drone operating cost. Each solid curve for the different drone operating costs allows using up to 8 drones per truck, whatever number provides the lowest cost, and the two dashed curves for 1 and 2 drones per truck with $c_d = 0.1$ highlight the benefits from allowing multiple drones per truck. In this figure, the left most density values can be viewed as sparsely populated rural areas ($\delta = 0.01$ is one delivery per 100 square miles), with the central and right-most values more typical of suburban regions in the US. (The average housing unit density in the US is 32.8 per square mile overall, and 128.7 per square mile in metropolitan areas (Census-Charts.com, 2017).) With very low drone operating costs ($c_d = 0.01$; the upper curve in Figure 7), truck-drone delivery provides savings up to 63.2% per delivery for very low densities of deliveries. Savings decrease with the drone operating costs, but even for densities of 500 per square mile, the savings range from 20-27.5% for $0.01 \leq c_d \leq 0.3$. For the solid lines in this figure, the number of drones per truck is the value that provides the lowest cost (and thus greatest $PSAV$ values). This is always 8 drones per truck for $c_d = 0.01$ and $c_d = 0.1$, but it varies from 2-7 drones per truck for $c_d = 0.3$ depending on the delivery density. Limiting the number of drones per truck reduces $PSAV$, and with $c_d = 0.1$ allowing only one drone per truck produces savings per delivery of only 22.8% for $\delta = 0.01$ and 16.1% for $\delta = 500$; versus 43.6% and 25.3%, respectively, when allowing up to 8 drones per truck.

The large percentage savings per delivery ($PSAV$) with low delivery densities suggest a high value from using drones for delivery in rural areas; however, the low density of deliveries in rural areas means that these large savings apply to relatively few deliveries. The savings per square mile, $SPSM$, combines $PSAV$ with the density of deliveries and the expected cost per delivery (which varies with delivery density):

$$SPSM = PSAV \times \delta \times E_{to} = [E_{to} - E_{td}(n)] \times \delta . \tag{28}$$

$SPSM$ is the spatial density of total delivery savings and it provides a very different perspective than the savings per delivery ($PSAV$). Figure 8 provides $SPSM$ for the same situations depicted in Figure 7 and shows how the intensity of savings increases with the delivery density. This figure better shows the important and growing benefits from using multiple drones per truck with large delivery densities. For example, with $c_d = 0.1$, $SPSM$ is \$45.9/mile^2 with one drone per truck, \$57.1/mile^2 with 2 drones per truck and \$72.1/mile^2 with 8 drones per truck.
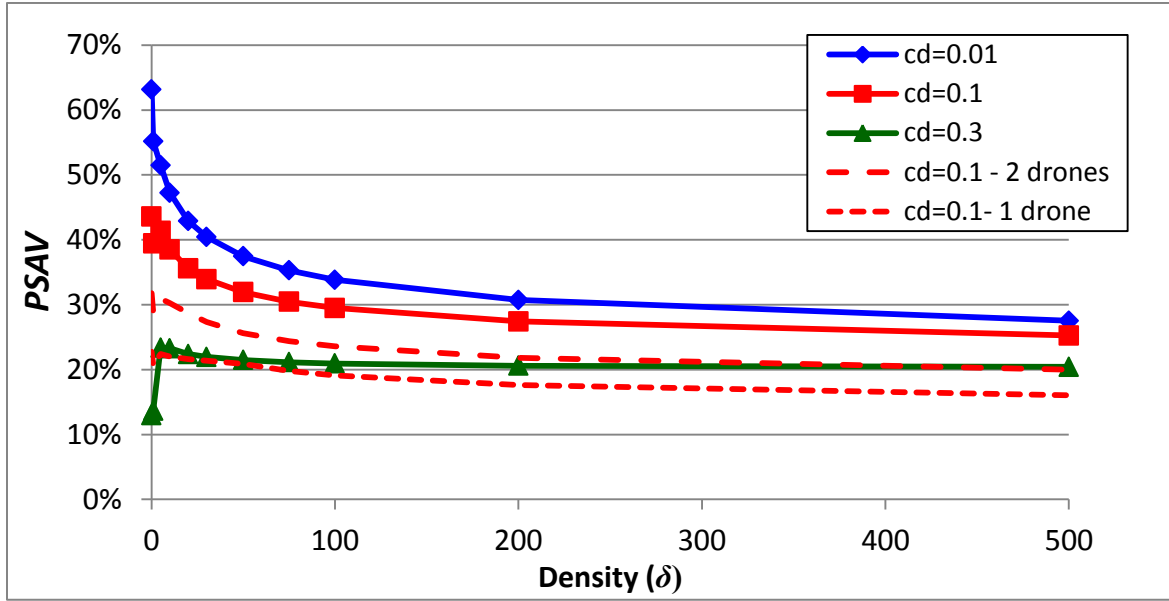

Figure 7. Percentage savings with 1-8 drones per truck with increasing delivery density for $s_d = -0.1$.
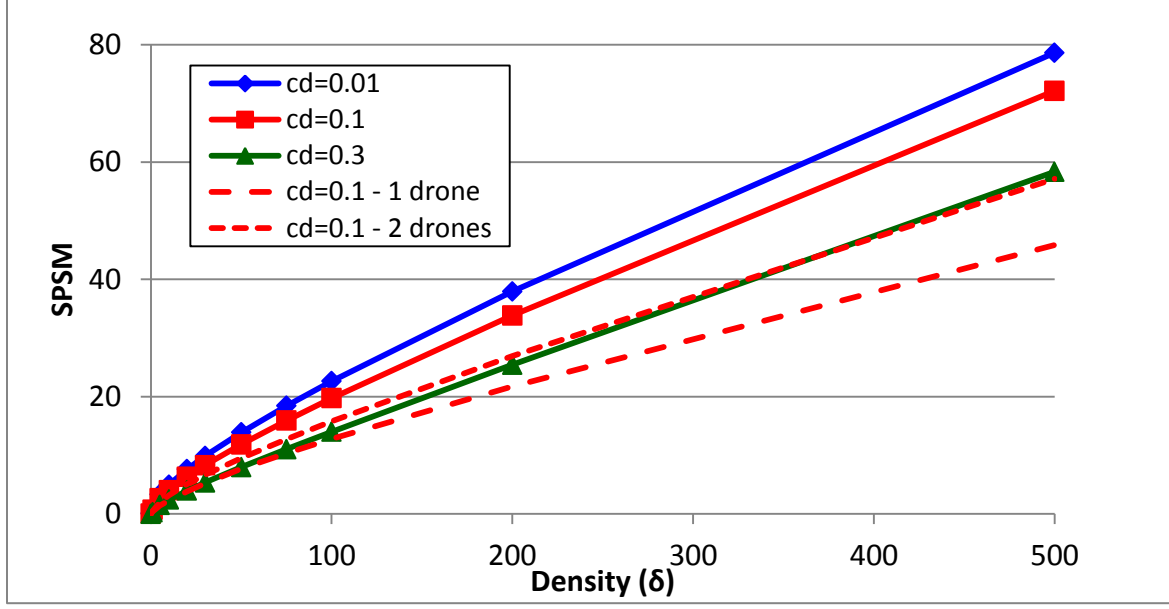
Figure 8. Savings per square mile with 1-8 drones per truck with increasing delivery density for $s_d = -0.1$.

Even with expensive drones where $c_d = 0.625$ (half the cost per mile for trucks), when $s_d = -0.1$ $SPSM$ is \$35.8/mile^2 for $\delta = 500$ and \$15.4/mile^2 for $\delta = 200$, though only \$0.42/mile^2 for $\delta = 5$. With this large drone operating cost and even lower delivery densities, $SPSM$ becomes negative as truck-only delivery provides lower cost due to the differences in total distance traveled for truck-drone and truck-only delivery, and the interplay between linehaul costs and the number of deliveries per route. Because hybrid truck-drone routes require longer total travel distance for the truck and drone together compared to truck-only routes, (though less distance for the truck travel - e.g., see Figure 2), a high drone operating cost per mile (relative to the truck cost per mile) causes truck-drone delivery to have higher total costs than truck-only delivery. However, with higher delivery densities there are more deliveries per route (as illustrated in Table 1) and linehaul travel becomes an important component of the total cost per delivery. This has a greater impact for truck-only travel as those routes have fewer deliveries than truck-drone routes (see Table 1, where $n = 0$ is truck-only delivery).

We limit our presentation of results to a maximum of $\delta = 500$, though the models apply for even greater densities. However, a very high delivery density is likely to occur in a dense urban region where

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practical considerations reduce the appropriateness of the model for travel and for delivery. For example, tall buildings and infrastructure may preclude direct drone flights, and apartment buildings, crowded sidewalks, and limited open space for delivery are not accounted for in the model. Further, as shown in Appendix B, $PSAV$ has a lower limit as density increases that is typically only a few percent below the value for $\delta = 500$.

While Figures 7 and 8 are constructed for a marginal drone stop cost of $s_d = -0.1$, similar results for $PSAV$ and $SPSM$ with $s_d = 0$ are shown in Figures 9 and 10. With very low delivery densities, the results for $s_d = 0$ are very similar to those with for $s_d = -0.1$ because there are so few deliveries that the stop cost s_d is a small component of total costs. However, with increasing delivery density the stop cost becomes important and the stop cost savings with $s_d = -0.1$ makes drone use more attractive. For example, in Figure 7 with $s_d = -0.1$ and $\delta = 500$, $PSAV$ ranges from 20.0-27.5% for $0.01 \le c_d \le 0.3$, while in Figure 9 with $s_d = 0$, $PSAV$ ranges from 6.1-12.0%. Even one drone per truck provides much greater savings when $s_d = -0.1$, as the savings with $\delta = 500$ and $c_d = 0.1$ are 16.1% vs. 7.3% for $s_d = 0$. The benefits from the reduction in marginal drone stop cost are also clear from comparing Figures 8 and 10, where the savings intensity is much larger with $s_d = -0.1$. Further, the relative benefits of $s_d = -0.1$ (Figure 8) versus $s_d = 0$ (Figure 10) increase with the delivery density. The results with $s_d = 0$ reflect using 8 drones per truck for low values of drone operating cost ($c_d \le 0.1$), but only 2 drones per truck for $c_d = 0.3$, unlike the results with $s_d = -0.1$ where the number of drones per truck increase with the delivery density. Such an increase does not happen with $s_d = 0$ because drone use is not as attractive as when there are drone stop cost savings.
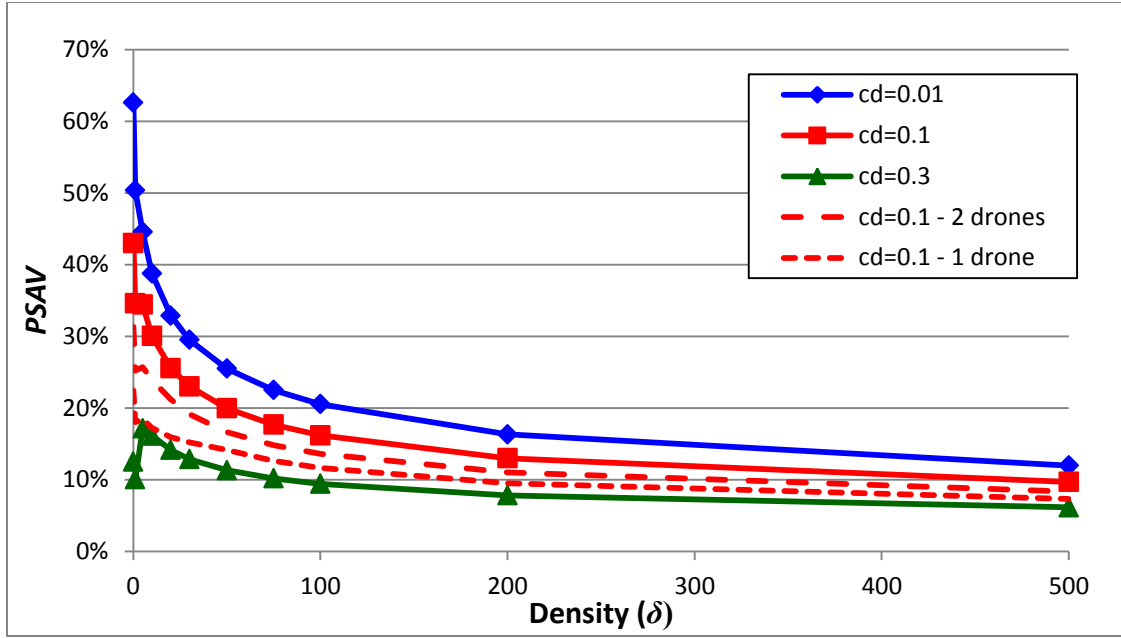
Figure 9. Percentage savings per delivery for 1-8 drones per truck with increasing delivery density for $s_d = 0$.
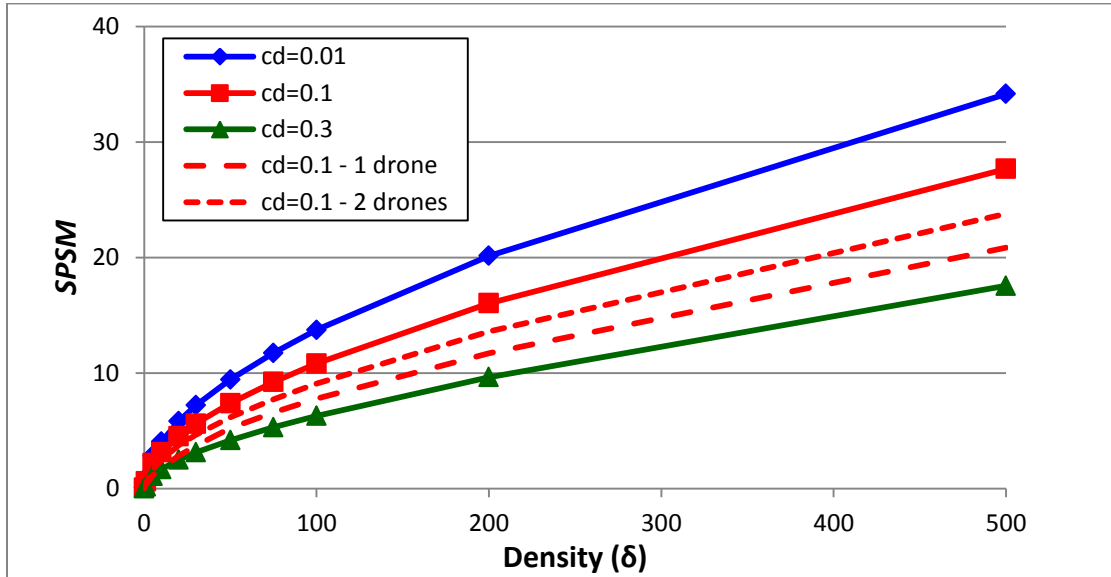

Figure 10. Savings per square mile with 1-8 drones per truck with increasing delivery density for $s_d = 0$.

5.3 Impact of marginal drone delivery cost

To illustrate the impact of the marginal drone delivery cost on the savings from truck-drone delivery, Figure 11 shows $PSAV$ with up to 8 drones per truck for $c_d = 0.1$ and four selected values of

the drone marginal delivery cost s_d = -0.2, -0.1, 0 and 0.1. With a large drone marginal stop cost savings

of $s_d = -0.2$, truck-drone delivery provides large savings per delivery (about 40-50%) that are relatively

insensitive to the delivery density beyond the smallest densities. Conversely, when the drone stops are

more expensive than truck stops ($s_d = 0.1$), then the benefits per delivery of truck-drone delivery are

very sensitive to the delivery density and decrease dramatically, even becoming negative (i.e. truck-only

delivery is better) for densities larger than about 300 per square mile. Table 3 summarizes $PSAV$ results

with 1, 2 and up to 8 drones per truck for $c_d = 0.1$ and $c_d = 0.01$ with four levels of marginal drone stop

cost in both "rural" regions (i.e., $\delta \le 10$ per square mile) and "suburban" regions (i.e., $30 \le \delta \le$

500 per square mile). The first two columns provide the number of drones per truck and the marginal

drone stop cost. The next four columns provide the range of savings ($PSAV$) in the rural and suburban

regions for the two values of the drone operating cost per mile. This table shows greater savings as the

drone marginal stop cost decreases (as expected), and the benefits of using multiple drones per truck.

With a positive marginal drone stop cost of $s_d = 0.1$ (drone stop cost is 10 cents *more* than the truck stop

cost), drones are naturally less attractive and $PSAV$ may become negative (i.e. truck only delivery is

preferred) with high delivery densities. However, Table 3 shows that in rural areas the savings from

hybrid truck-drone delivery are positive even with a positive marginal drone stop cost of $s_d = 0.1$,

showing that truck-drone delivery is preferred over truck-only delivery at these low densities.

Figure 12 is a companion to Figure 11 that presents the savings per square mile ($SPSM$) for the

situations depicted in Figure 11. Given the relatively flat curves in Figure 11 for moderately large

delivery densities, Figure 12 shows nearly linear curves reflecting the greater benefits in regions of higher

density. The strong advantages of reduced marginal stop costs are more evident in the growing separation

of the curves in Figure 12 versus the nearly parallel lines in Figure 11.   Figure 12 also show the low

savings (or small losses) from truck-drone delivery when drone deliveries are more expensive than truck
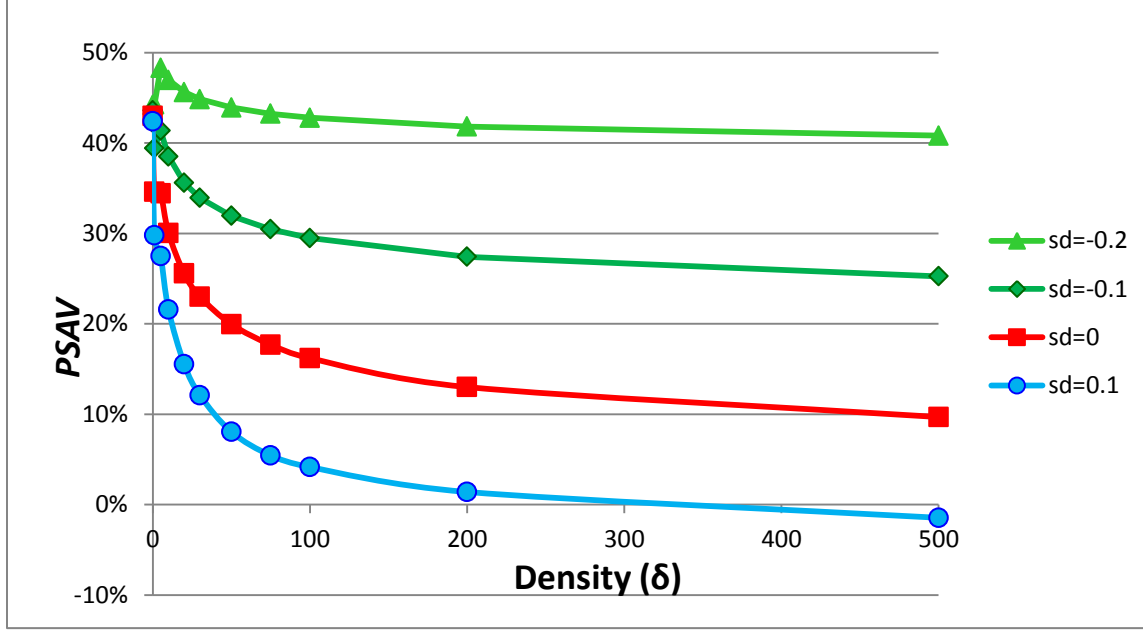
deliveries.

Figure 11. Percentage savings per delivery with up to 8 drones per truck with increasing delivery density for $c_d = 0.1$.

Table 3. Percentage savings ($PSAV$) and number of drones per truck (n) with different drone marginal stop costs for $c_d = 0.1$.

| n	s_d	$c_d = 0.1$		$c_d = 0.01$	
		Rural $\delta \leq 10$	Suburban $\delta = 30 - 500$	Rural $\delta \leq 10$	Suburban $\delta = 30 - 500$
1	-0.2	$23.1 - 26.8\%$	$24.8 - 27.5\%$	$28.6 - 29.2\%$	$25.4 - 29.3\%$
	-0.1	$22.0 - 22.8\%$	$16.1 - 21.4\%$	$24.4 - 28.2\%$	$16.7 - 23.2\%$
	0	$17.2 - 22.4\%$	$7.3 - 15.2\%$	$19.7 - 27.9\%$	$7.9 - 17.0\%$
	0.1	$12.5 - 22.1\%$	$-1.5 - 9.1\%$	$14.9 - 27.5\%$	$-0.8 - 10.9\%$
2	-0.2	$32.2 - 36.6\%$	$31.7 - 35.5\%$	$40.6 - 41.0\%$	$32.7 - 38.4\%$
	-0.1	$30.3 - 31.8\%$	$20.0 - 27.3\%$	$34.2 - 40.6\%$	$21.0 - 30.2\%$
	0	$23.9 - 31.3\%$	$8.3 - 19.1\%$	$27.9 - 40.1\%$	$9.4 - 22.1\%$
	0.1	$17.6 - 30.9\%$	$-3.3 - 10.9\%$	$21.5 - 39.7\%$	$-2.3 - 13.9\%$
1-8	-0.2	$44.2 - 47.0\%$	$40.8 - 44.8\%$	$55.7 - 63.8\%$	$43.1 - 53.4\%$
	-0.1	$38.5 - 43.6\%$	$25.3 - 33.9\%$	$47.3 - 63.2\%$	$27.5 - 40.4\%$
	0	$30.0 - 43.0\%$	$9.7 - 23.0\%$	$38.8 - 62.6\%$	$12.0 - 29.5\%$
	0.1	$21.6 - 42.4\%$	$-1.5 - 12.1\%$	$30.0 - 62.0\%$	$-0.8 - 18.6\%$
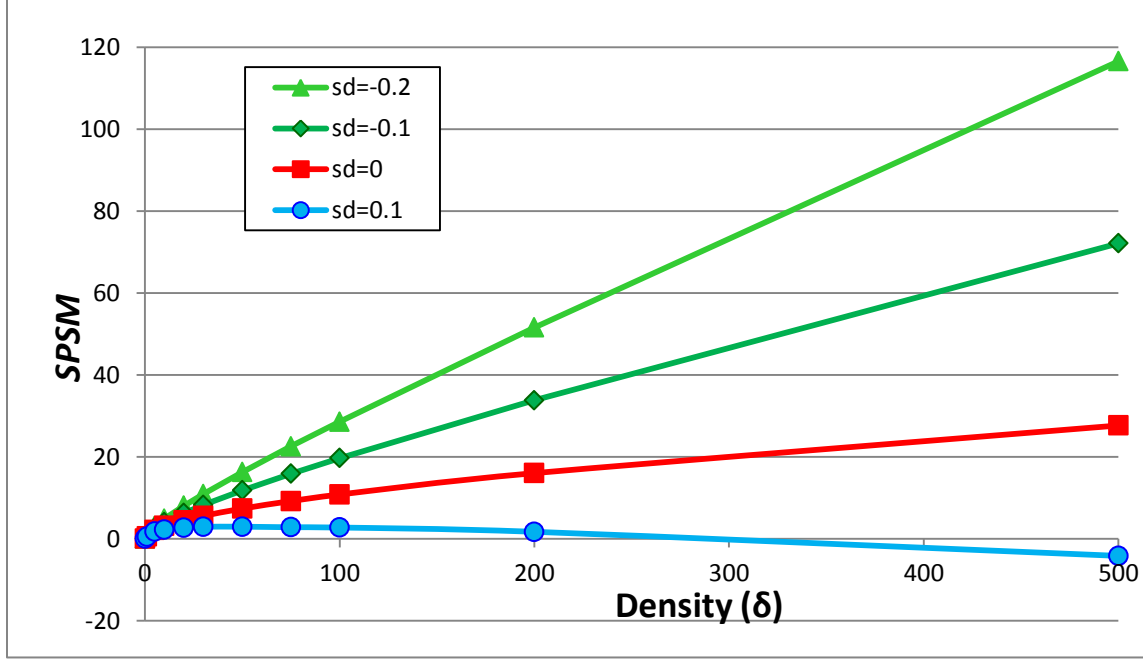
Figure 12. Savings per square mile with 1-8 drones per truck with increasing delivery density for $c_d = 0.1$.

Collectively, the results highlight that truck-drone delivery appears to have strong potential to reduce delivery costs. Regions of low delivery density often provide the greatest percentage savings per stop, but these savings apply to relatively few deliveries. The net savings per square mile generally increase with the delivery density so regions of greater delivery density, which may have a lower savings per delivery, actually often generate much greater total savings. However, the complex interplay of the drone marginal stop cost and the drone operating cost strongly influence the results.

5.4 Delivery Region with Varying Densities

As an example of modeling a region with varying densities of deliveries, consider a hypothetical urban region and its outlying areas as a circular region with a radius of 30 miles. Suppose the depot is at the center of the region and this region can be divided into three concentric bands, where the delivery density in band 1, within 10 miles of the depot, is 500/mile^2, the delivery density in band 2, 10-20 miles from the depot, is 200/mile^2, and the delivery density in band 3, 20-30 miles from the depot, is 1/mile^2. This region represents a stylized city and surrounding area where the large outlying area 20-30 miles from

the center has a low delivery density, and the small central area (radius 10) has the highest delivery

density. We model the linehaul distance from the depot to the nearest edge of the band for the two farther

bands and as the expected distance from the depot to a random point in a circle of radius 10 for band 1

(containing the depot). Using parameter values $c_d = 0.1, s_d = -0.1$ and the other values from Section 5

as a baseline, the continuous approximation models provide the truck-drone system design values

presented in Table 4.

Table 4: Example results for a circular service region with 3 delivery densities.

	Band 1 0-10 miles	Band 2 10-20 miles	Band 3 20-30 miles
Delivery density	500	200	1
Linehaul distance (round trip)	13.33	20	40
Number of deliveries	157,079.6	188,495.6	1570.8
m (number of stops per route)	394.8	356.6	91.4
# of routes	397.8	528.5	17.2
# of drones per truck	4	5	7
PSAV	19.5%	19.8%	25.1%
SPSM	50.2	23.2	0.63
Savings ($)	$15,783	$21,878	$984
% of area	11.1%	33.3%	55.6%
% of deliveries	45.2%	54.3%	0.5%
% of total cost	41.6%	56.5%	1.9%
% of total savings	40.8%	56.6%	2.5%

In this stylized setting truck-drone delivery provides lower cost than truck-only delivery in every

band. Table 4 shows that while the majority of the service region is in band 3 (55.6% of the total area),

this band has very few deliveries with only 0.5% of the total deliveries. The vast majority of the

deliveries are in bands 1 and 2 (45.2% and 56.3%, respectively). The distribution of the truck-drone

delivery cost mirrors the percentage of deliveries, though band 3 has a relatively greater percentage of

cost (1.9% of total cost vs. 0.5% of total deliveries) due to the long linehaul travel to band 3 and the

longer distance between deliveries in band 3. The PSAV values are greatest in band 3 (25.1% per stop),

but its low delivery density causes this band to generate a very low SPSM ($0.63/ mile^2) and

consequently only 2.5% of the total savings over the entire region. In contrast, band 1 is smallest with

only 11.1% of the total area, but its high delivery density allows it to generate 40.8% of the total savings. Note also the differential use of drones per truck where band 1 uses 4 drones per truck, band 2 uses 5 drones per truck, and band 3 uses only 2 drones per truck.

6. Conclusions

This research derives continuous approximation models for travel cost and travel time of hybrid truck-drone delivery and truck-only delivery to facilitate a strategic analysis of the design of drone delivery systems. The mathematical expressions in the models are optimized to determine important drone delivery system design parameters, including the optimal number of truck and drone deliveries per route, the optimal number of drones per truck and whether truck-only or hybrid truck-drone travel provides a lower total cost. Travel time constraints and vehicle capacity limits are incorporated in the models to increase their realism. The CA models allow us to assess the best use of truck-drone delivery, the appropriate mix of drones and trucks, and the expected economic performance of different delivery systems. Comparison of the expected delivery cost from the CA models for different delivery systems helps identify when and where implementing truck-drone delivery is likely to be beneficial.

The results highlight some key tradeoffs in the use of the trucks and drones and their dependence on relative operating characteristics, costs and delivery densities.   Key findings are:

- Truck-drone hybrid delivery has the potential to provide substantial cost savings, especially in suburban areas. These savings arise from reduced route costs and from allowing demand to be covered with fewer routes.

- Incorporating multiple drones per truck offers important, but marginally decreasing savings that can be large.

- The benefits from truck-drone delivery depend strongly on the relative operating costs per mile for trucks and drones, the relative stop costs for trucks and drones, and the spatial density of customers.

- Measures of savings per delivery and savings intensity per square mile provide complimentary perspectives that highlight the conditions and regions likely to generate the greatest savings.

The importance of drone operating costs is highlighted in the results and it is likely that these costs will continue to fall as drone technology advances. However, more detailed analysis to identify real values of cost per mile and per stop would be very useful. Our models can be used to identify the critical level of drone operating and stop cost below which drones become an attractive complement to truck delivery. The benefits from a decrease in drone stop costs suggest firms may wish to invest in reducing drone stop costs via automated loading and quick delivery (e.g., parachuting packages rather landing to release a package).

The continuous approximation models provide valuable managerial insights by analyzing a general version of the delivery problem, and a wide range of different settings can be modeled and analyzed by varying the delivery (demand) density and the operating characteristics for drones and trucks. For example, in a region of varying delivery densities, our results show how a combination of differentiated drone use is best, where the number of drones per truck may vary in response to the delivery densities. Some areas of future research are to incorporate drone deliveries directly from depots along with hybrid truck-drone delivery, to explore drone use in high service level circumstances (e.g., 1 hour delivery) and to integrate multiple depot locations and costs for drones and trucks.

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Appendix A: Optimal swath width approximation

Extensive computational experiments verified that equation (22) is a good approximation for the optimal swath width. As an illustration, Figure A1 shows the percentage difference in cost per stop as a function of the ratio c_d/c_t for three possible swath widths, relative to the cost with the optimal swath width found numerically for a single drone ($n = 1$). The cost difference using equation (22) is nearly zero for all drone operating costs. When drones are inexpensive to operate (left side) the optimal swath width is very close to that for trucks alone, since the majority of cost is for truck travel. Conversely when drones are very expensive to operate (right side), then the optimal swath width is more like that for drones alone (since the majority of cost is for drone travel).
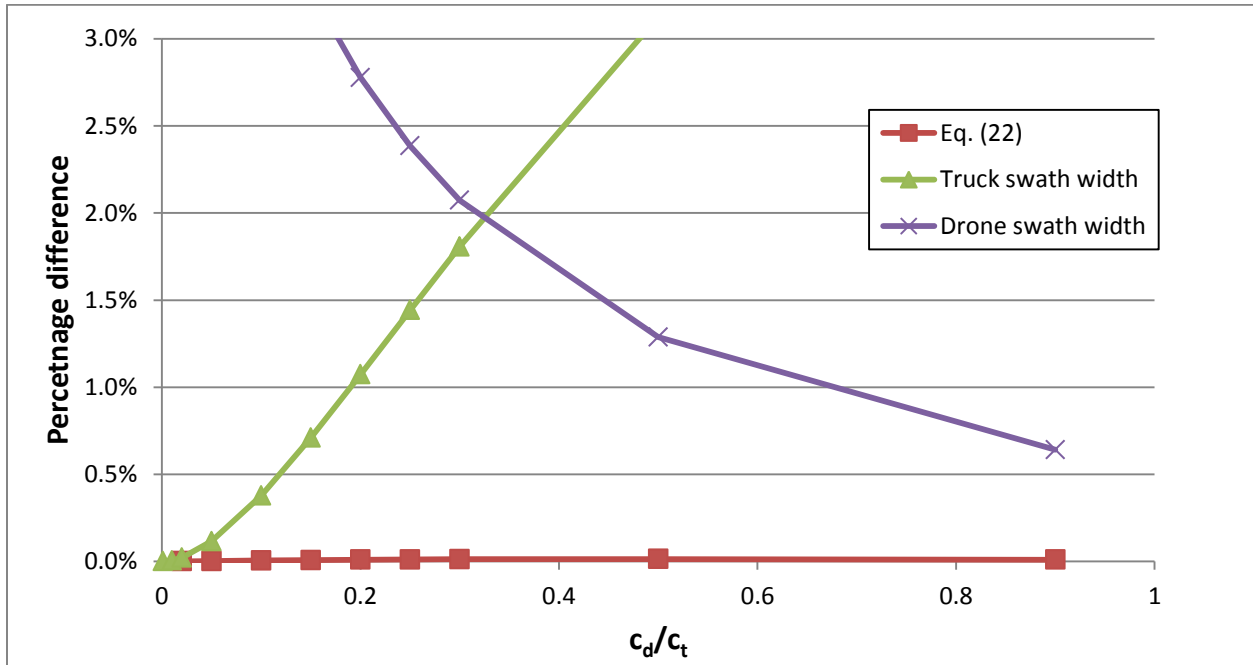

Figure A1. Percentage difference from numerically optimal cost with different swath widths

Appendix B: Limiting values for large delivery densities

This appendix considers limiting values as the density of deliveries δ increases. In the limit as $\delta \to \infty$, the swath width goes to 0, and the local travel components of the truck and drone cost go to 0 as well. For both truck-only and truck-drone delivery, the number of stops per route goes to a constant value as does the expected cost per delivery. For example, with truck-drone delivery the limiting number of stops per route (from equation 25a) is

$$\lim_{\delta \to \infty} m_{td} = m_{td}^{\infty} = \frac{(n+1)}{s_t'} \left[T - 0.9027 c_{tl}' \sqrt{A} \right] \tag{B.1}$$

and the limiting expected cost per stop (from equation 24b) is

$$\lim_{\delta \to \infty} E_{td}(n) = c_t \frac{0.9027}{m_{td}^{\infty}} \sqrt{A} + \frac{n}{n+1} s_d + s_t . \tag{B.2}$$

Similarly, with truck-only delivery the limiting number of stops per route is

$$\lim_{\delta \to \infty} m_{to} = m_{to}^{\infty} = \frac{1}{s_t'} \left[T - 0.9027 c_{tl}' \sqrt{A} \right] \tag{B.3}$$

and the limiting expected cost per stop (from equation 24b) is

$$\lim_{\delta \to \infty} E_{to} = c_t \frac{0.9027}{m_{to}^{\infty}} \sqrt{A} + s_t . \tag{B.4}$$

Equations (B.2) and (B.4), provide the limiting value of $PSAV$ and it is only a few percent lower than the value for $\delta = 500$. For example, in the base case from Section 5 with $c_t = \$1.25/\text{mile}$, $s_d = -\$0.1$ and $A = 1250$ square miles, the limiting values (for $\delta = \infty$) of $PSAV$ for $n = 1, 2$ and up to 8 drones per truck are 12.7%, 16.1% and 20.6%, respectively, while the corresponding values for for $\delta = 500$ are 16.1%, 20.0% and 25.3%, respectively. However, $SPSM$ values will increase with density in a nearly linear fashion (as suggested by Figure 8) proportional to the delivery density (see equation 28) as

$$\lim_{\delta \to \infty} SPSM = \left[0.9027 c_t \sqrt{A} \left(\frac{1}{m_{to}^{\infty}} - \frac{1}{m_{td}^{\infty}} \right) + \frac{n}{n+1} s_d \right] \times \delta . \tag{B.5}$$