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Operating policies in multi-warehouse drone delivery systems

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Drones are increasingly used to deliver packages with high efficiency in several areas. We study a multi-warehouse drone delivery system, considering the allocation rule that all warehouses share the drones and the allocation rule that each warehouse owns its drones. Both plug-in charge and battery swap strategies are investigated for battery management. We examine the random and closest drone to warehouse assignment rules, and design a heuristic to improve the drone to warehouse assignment rule. A closed queueing network is built to estimate the maximum throughput capacity and a cost minimisation model is developed for cost analysis. We validate the analytical model by simulation and conduct numerical experiments to analyse the operating polices. The results show that the closest drone to warehouse assignment rule outperforms the random drone to warehouse assignment rule when the number of drones is not large, and our heuristic can improve the throughput capacity by about 13.31%. The battery swap strategy provides a better throughput capacity than the plug-in charge strategy in most cases, while it needs more investment. Moreover, the shared allocation rule gives a larger throughput capacity than the dedicated allocation rule, and it reduces the operating cost by about 30.70%.

Keywords: logistics; drone delivery system; performance analysis; queueing networks

1. Introduction

In the recent decade, drones are increasingly used to deliver packages with safety and extremely high speed in the logistics of e-commerce industry. As the pioneer, Amazon started developing and providing a drone delivery service since 2013, called Prime Air (Figure 1). It enhances the service level of the e-commerce giant by promising safe and efficient delivery within 30 minutes (Amazon 2013). With rapid delivery speed and excellent flexibility of throughput capacity, a drone delivery system has seen several implementations in the fulfillment centers of e-commerce companies, including Google, FedEx, Alibaba, etc. Generally, a drone delivery system consists of warehouses, unmanned aerial vehicles (UAVs), packages, and delivery points (Grippa et al. 2019). A drone loads the target package in a warehouse, flies to the designated destination for dropping and then returns to the warehouse. Regarded as a future delivery system, it can also be applied in other areas, such as healthcare (Kim et al. 2017) and metrology (Franceschini, Mastrogiacomo, and Pralio 2010). Compared with a traditional truck or vehicle delivery system, a drone delivery system has the following advantages:

(1) High delivery efficiency: with about 80 km/h flying speed (Shavarani et al. 2018), a drone can rapidly deliver a package to the destination that is more than 10 km away from the warehouse.
(2) Competitive advantages on operational cost and future benefits: much less manpower is required; with the openness of regulation rules, the benefits and potential revenues of the drone delivery far outweigh the operational and start-up costs of the system (Welch 2015).
(3) Flexible throughput capacity: the throughput capacity can be easily adjusted by adding or retracting drones.

There are also several disadvantages, such as the high investment on drones and complex control system to operate a fleet of drones with safety and stability. A good understanding on the system performance is essential before the implementation of a drone delivery system. The throughput capacity and operating cost of a drone delivery system may be affected by many factors, including the number of drones used, the drone to warehouse assignment rule, the battery management strategy and so on. Thereby, an efficient and accurate performance estimation tool is required for the drone delivery system, that can handle the system design and operating policies analysis.

This paper considers a multi-warehouse drone delivery system that uses a fleet of drones to transport items from several warehouses to destinations. We consider the allocation rule that all drones are shared by all warehouses and the allocation

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(a) (b)

Figure 1. Amazon Prime Air drone (Amazon 2013).

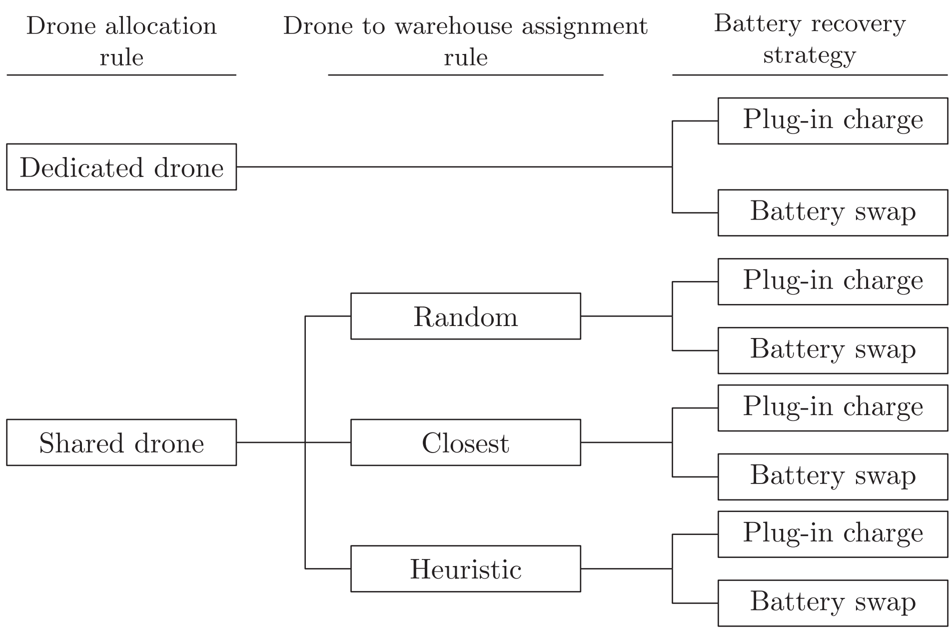


Figure 2. Research scenarios in the multi-warehouse drone delivery system.

rule that each warehouse owns its drones. Under the shared allocation rule, we examine the random and closest assignment rules of drone to warehouse, and design a heuristic to improve the performance of the drone to warehouse assignment rule. The battery management problem is another issue that may significantly affect the throughput capacity and operating cost of the multi-warehouse drone delivery system. We examine both the plug-in charge strategy and the battery swap strategy in both terms of throughput capacity and operating cost. This study explores the research scenarios depicted in Figure 2.

We aim to answer the following research questions:

- How can we build a performance estimation model for a multi-warehouse drone delivery system?
- What drone to warehouse assignment rule provides better throughput capacity?
- Which battery recovery strategy performs better in terms of throughput capacity and operating cost, plug-in charge or battery swap?

To answer above research questions, we model each warehouse of the system with dedicated drones as an independent closed queueing network (CQN), and the system with the shared drones as a CQN. The CQN models the battery charge process as a single service station with multiple servers and the battery swap process as a nested semi-open queueing network (SOQN). The drone to warehouse assignment process is captured by the visit probabilities of drone to warehouse. We design a battery recovery time based heuristic to improve the performance of the drone to warehouse assignment rule. We analyse above operating policies in terms of maximum throughput capacity, and develop a cost minimisation model with a requirement on the maximum system throughput capacity to optimise the system design and operating policies in terms of operating cost.

We validate the analytical models by simulation. The results support that our analytical model can estimate the performance of the drone delivery system with accuracy and effectiveness. We conduct numerical experiments to analyse the operating policies in terms of both maximum throughput capacity and operating cost. Interesting results found within this paper include:

- The comparison among assignment rules of drone to warehouse shows that the closest assignment rule outperforms the random assignment rule in terms of maximum throughput capacity when the number of drones used in the system is smaller than a critical point, otherwise, the random assignment rule is better.
- Our heuristic can improve the maximum system throughput capacity by about 13.31%. The battery swap strategy benefits the throughput capacity more than the plug-in charge strategy, while it requires more investment on spare batteries.
- The shared allocation rule provides larger throughput capacity than the dedicated allocation rule, and it reduces the operating cost by about 30.70%.

This paper is structured as follows: Section 2 presents a review of related works. A brief description of the delivery system is provided in Section 3. Section 4 describes the performance estimation model and the heuristic. Section 5 presents the cost minimisation model. Simulation validation and some numerical experiments are included in Section 6. Section 7 gives the conclusions and future work.

2. Literature review

Drone delivery system is a relatively new research topic of facility logistics. This paper focuses on the system performance evaluation and operating policies analysis in a drone delivery system. We first review the literature on drone delivery systems and then present the studies that investigate the performance estimation of automated warehouse systems using autonomous vehicles, including the autonomous vehicle storage and retrieval system (AVS/RS) and the robotic mobile fulfillment system (RMFS).

The studies on drone delivery systems mainly focus on the routing problem. Murray and Chu (2015) investigate the optimisation problem of drone routing and scheduling, under a scenario in which drones work together with a truck to delivery parcels and a drone can only deliver a parcel in one trip. There are two kinds of settings when drones work together with trucks. One is that drones and traditional delivery trucks depart from a common warehouse and serve different customers. Under this setting, Saleu et al. (2018) propose a heuristic solution method to minimise completion time, and Ulmer and Thomas (2018) present a dynamic vehicle routing problem and a policy function approximation to decide whether an order is delivered by a drone or by a truck, aiming to maximise the expected number of same–day deliveries. The other is that drones have to pick up parcels from trucks and return to the trucks after a delivery. Boysen et al. (2018) explore diverse scheduling problems of drone delivery, such as whether one or multiple drones are placed on a truck and whether or not take–off and landing stops have to be identical, and compare potential benefits under different schedules. Agatz, Bouman, and Schmidt (2018) regard the routing problem as a new variant of the travelling salesman problem, then model it as an integer program. Based on this, Bouman, Agatz, and Schmidt (2018) provide exact solution approaches for the problem based on dynamic programming and give an experimental comparison among these approaches. In addition, Dorling et al. (2016) consider a scenario in which drones are solely used to make deliveries and also take into account drone capacity, battery weight, and changing payload weight. They propose an energy consumption model and explore multi-trip vehicle routing problems for drone delivery to reduce cost and delivery time. Limited works consider drone delivery at the system level. Hong, Kuby, and Murray (2017) design a new coverage model to optimise recharging station locations of a delivery drone system. Shavarani et al. (2018) propose an optimisation method to determine the number and locations of launch and recharge stations in a drone delivery system and examine it with a real case in San Francisco. Lee (2017) suggests to modularise the system to obtain operational benefits and presents an optimal operating method for modular delivery drones.

While many studies on drone delivery systems focus on the optimisation of routing path and battery management, we can hardly find research works that explore the performance estimation and operating policies analysis on the systematic level of a drone delivery system. Queueing networks are widely used to estimate system performance and analyse operating policies (Epp, Wiedemann, and Furmans 2017; Askin and Hanumantha 2018; Kennedy and Philbin 2019; Mohammadi, Dauzère-Pérès, and Dauzère-Pérès 2019). Therefore, we next concentrate on studies that investigate the performance estimation and operating policies analysis by queueing models in other systems, including AVS/RS and RMFS.

An AVS/RS is an autonomous storage and retrieval system that uses rail-guided vehicles to transport unit-loads stored in high-bay racks. Queueing models have been widely used for performance estimation in an AVS/RS, including open, closed and semi-open queueing networks (SOQN). Heragu et al. (2011) model AVS/RS as an open queueing network and analyse the system performance to provide a guidance for system design. Marchet et al. (2012) formulate an open queuing network to estimate the system response time in a tier-captive AVS/RS, considering the acceleration and deceleration of lifts and vehicles. Roy et al. (2012) build a multi-class SOQN with class switching to estimate the system performance of a single-tier AVS/RS. Ekren et al. (2014) develop a matrix-geometric method (MGM) for the analysis of a SOQN built for tier-to-tier AVS/RS. Cai, Heragu, and Liu (2014) model a tier-to-tier AVS/RS as a multi-class multi-stage SOQN, and use MGM to analyse it. Roy et al. (2015) investigate the effect of the position of the load/unload point, POSC dwell point policies and the location of cross-aisles in a single-tier AVS/RS. Tappia et al. (2016) study a shuttle-based compact storage system (an AVS/RS with multi-deep storage lanes), considering both specialised and generic shuttles and both discrete and continuous lifts. They build multi-class SOQNs for both single and multi-tier systems and use matrix-geometric methods to analyse the SOQNs. Zou et al. (2016) investigate parallel movement of the lift and vehicles in a tier-captive AVS/RS, using a fork-join queueing network model. They find that the parallel policy outperforms the sequenced policy in terms of retrieval throughput time when the retrieval transactions arrival rate is low. Roy et al. (2017) also study a tier-to-tier AVS/RS and model individual tiers as semi-open queues, the vertical transfer units as open queues, then design a linking approach to solve models.

An RMFS is an autonomous storage and retrieval system that stores products on inventory pods and use central controlled robots to transport pods between storage area and workstations. Lamballais, Roy, and De Koster (2017) establish performance estimation models for an RMFS with both single-line and multiple-line orders via SOQN, and find that order throughput of the system can be affected by the storage area shape and workstation locations. Sequentially, Lamballais, Roy, and De Koster (2019) introduce a cross-class matching multi-class SOQN and explore inventory allocation problem for an RMFS. Yuan and Gong (2017) utilise open queueing networks to describe an RMFS and examine two policies of sharing robots for pickers. Zou et al. (2017) examine the assignment rule of robots to workstations in an RMFS. They design a heuristic to improve the performance of assignment rule of robots to workstations. Zou et al. (2018) analyse the battery management by SOQN in a RMFS, considering plug-in charge, inductive charge and battery swap strategies. The results show that the inductive charge strategy can provide the best throughput time, but the plug-in charge strategy is the cheapest choice.

This study contributes by developing an accurate performance estimation model in a multi-warehouse drone delivery system and investigating the drone allocation rule, the drone to warehouse assignment rule and battery management in both terms of throughput capacity and operating cost. This paper studies a newly emerged application of drones, i.e. using drones to serve home deliveries instead of using drones for militaries or disasters management. We investigate operating policies and operating cost of a drone delivery system which can be applied for logistics systems. With a good understanding on the system, we can provide support of design-related decisions for e-commerce companies to build an efficient drone delivery system with minimum investment and operation cost.

3. System description and modelling preparations

We consider a multi-warehouse drone delivery system that consists of m warehouses and n delivery points with order arrival rates $\lambda_j,\ j = 1, 2, \dots, n$. N_D drones are used to deliver packages. The drone releases the item at the destination and then returns to a designated warehouse, depending on the drone allocation rule. Under the shared drone allocation rule, a drone may return to any warehouse depending on the drone to warehouse assignment rule. Under the random assignment rule, a drone returns to warehouse w_i with probability $p^r_{w_i} = 1/m,\ i = 1, 2, \dots, m$. Under the closest assignment rule, a drone returns to warehouse w_i with probability $p^c_{w_i} = \sum_{k\in \mathbf{C}_{\mathbf{w}_i}} \lambda_k / \sum_{j=1}^{n} \lambda_j$, where $\mathbf{C}_{\mathbf{w}_i}$ is the set of delivery points whose closest warehouse is w_i. We will also design a battery recovery time based heuristic to improve the performance of the drone to warehouse assignment rule. Under the dedicated drone allocation rule, each warehouse owns its drones that may go to any destination and returns to the dedicated warehouse.

With a navigation capacity of about 100 km and a maximum velocity of 80 km/h, a fully charged drone can serve several trips of delivery with about 10 kms distance. The battery management determines how fast the depleted batteries can recover and how many drones are available, that may significantly affect the system performance and operating cost. We consider the plug-in charge strategy under which a depleted drone will be fully charged with a plug-in charger, and the battery swap strategy under which a depleted battery will be swapped with a fully charged one (if there is one available). We will investigate the above operating policies in the multi-warehouse drone delivery system in terms of both maximum throughput capacity and operating cost.

The main notations used in this paper are included in Table 1.

We make the following assumptions:

- The customer orders arrive following a Poisson distribution with an arrival rate $\lambda_j,\ j = 1, 2, \dots, n$.
- The customer order arrival rates are large enough to make all drones busy.

Table 1. Main notations.

Notation	Description
m, n	number of warehouses and delivery points.
$N_D, N_D^{w_i}$	total number of drones, and the number of drones dedicated to each warehouse $i = 1, 2, \ldots, m$, such that $N_D = \sum_{i=1}^{m} N_D^{w_i}$.
$n_c^{w_i}, n_b^{w_i}$	number of chargers and spare batteries in warehouse w_i, $i = 1, 2, \ldots, m$.
λ_j	customer demand rate of the jth delivery point (per hour), $j = 1, 2, \ldots, n$.
p_{d_j}	probability that a drone flies to delivery point j, such that $p_{d_j} = \frac{\lambda_j}{\sum_{k=1}^{n} \lambda_k}$, $j = 1, 2, \ldots, n$.
$p_{w_i}^r, p_{w_i}^c, p_{w_i}^h$	probability that a drone returns to warehouse w_i under the random, closest and heuristic-based assignment rule of drone to warehouse, $i = 1, 2, \ldots, m$.
$d_{i,j}$	distance between warehouse i and delivery point j (m), $i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$.
v	flying speed of a drone (m/s).
t_{lu}	the time that a drone loads/unloads an item (s).
t_c	time that charges a depleted battery (s).
t_s	time that swaps a depleted battery (s).
TC	maximum throughput capacity of the system.

- The warehouses follow the First-Come-First-Service (FCFS) service rule.
- The time that charges a depleted battery follows a uniform distribution $[\underline{t_c}, \overline{t_c}](s)$.
- The time that swaps a depleted battery with a fully charged one, i.e. t_s, is constant.

Next, we build closed queueing networks to estimate the throughput capacity. This model can investigate the drone to warehouse assignment rule and battery recovery strategy.

4. Closed queueing network for the multi-warehouse drone delivery system

In this section, we first build a closed queueing network to estimate the maximum throughput capacity of the system. Then, we present the solution approach and design a heuristic to improve the performance of the drone to warehouse assignment rule.

4.1. Closed queueing network

We build a closed queueing network to estimate the maximum throughput capacity of the multi-warehouse drone delivery system with shared drones (Figure 3) and the multi-warehouse drone delivery system with dedicated drones (Figure 4). The CQN can examine both the plug-in charge and battery swap strategies by changing the service node μ_b (see Figure 5(a,b)).

In the system with the shared drones, a drone goes to the warehouse w_i, $i = 1, 2, \ldots, m$ to receive an order with the probability p_{w_i}. Before the drone starts delivering the order, the warehouse checks the battery level of the drone to determine whether it needs to recharge or swap its battery. The battery of the drone is depleted with the probability p_d (see Appendices for the calculation of p_d), then the drone goes to the battery recovery station $\mu_b^{w_i}$. With probability $1 - p_d$, the drone loads the item to be delivered at service node μ_l. Since no waiting is required, we model this node as an infinite service node (IS). Then, the drone flies from the warehouse w_i to the destination d_j with probability p_{d_j} and releases the item at the destination (μ_u). At last, the drone returns to warehouse w_i with probability p_{w_i}. The drone to warehouse assignment rule is captured by the visit probabilities of drone to warehouse, i.e. p_{w_i}, $i = 1, 2, \ldots, m$. p_{w_i} equals to $p_{w_i}^r$ and $p_{w_i}^c$ under the random and closest assignment rule, respectively. Moreover, we will design a heuristic to improve the performance of the drone to warehouse assignment rule. In the warehouse that uses the plug-in charge strategy (Figure 5(a)), the charge process is modelled as a single service node with n_c servers. In the warehouse with the battery swap strategy, the battery swap process is modelled as a nested SOQN (Figure 5(b)).

In the multi-warehouse drone delivery system with dedicated drones, warehouse w_i uses $N_D^{w_i}$ drones to deliver orders assigned to this warehouse. $N_D^{w_i}$ is initially determined based on the demand of each warehouse, then optimised by a heuristic based on the throughput capacity of each warehouse. Other service processes are the same as that in Figure 3.

4.2. Solution approach for the CQN

In this section, we design a decomposition method to analyse the CQN for the multi-warehouse drone delivery system. For the system that uses the plug-in charge strategy, we apply the approximate mean value method to analyse the maximum throughput capacity $\mathrm{TC}(N_D)$.

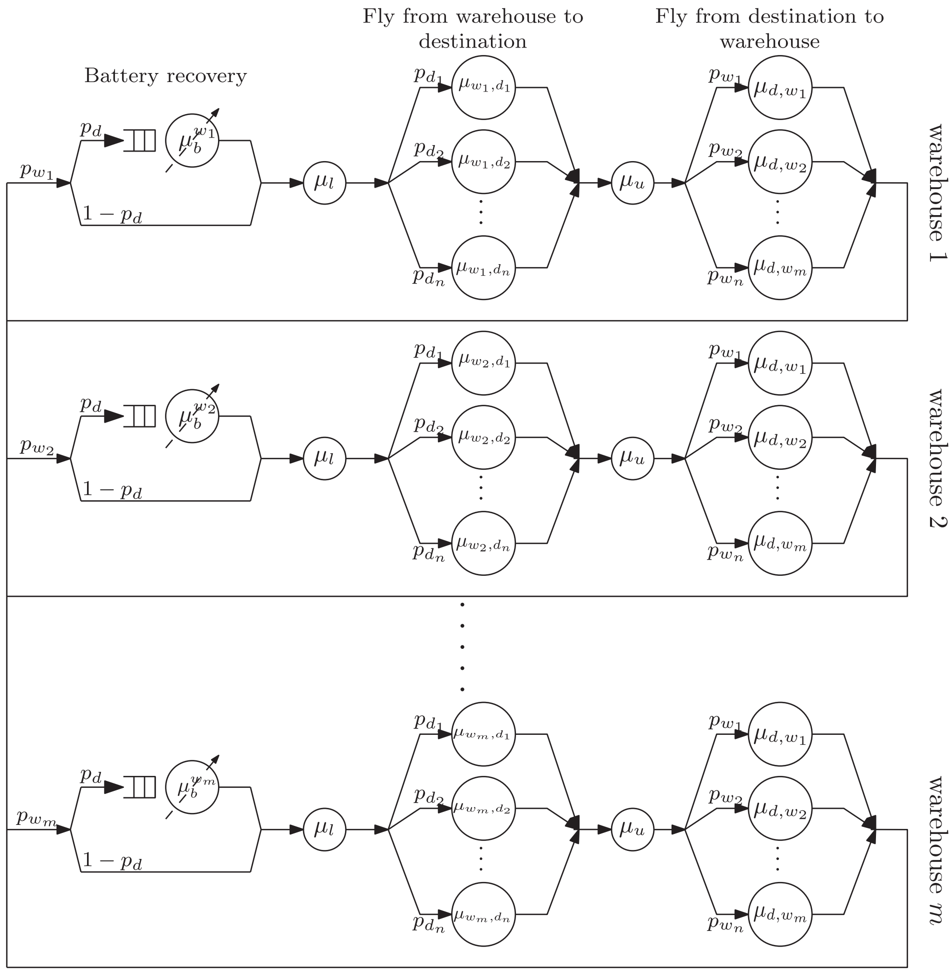


Figure 3. CQN for the multi-warehouse drone delivery system with shared drones.

For the system that uses the battery swap strategy, we use the following decomposition method to analyse the CQN:

(1) Short-cut the SOQN of battery swap process as a CQN with k customers (see Figure 5(c)) and derive its throughput capacity $\mathrm{TC}_{bs}(k)$.
(2) Replace the SOQN by a load-dependent service node $\mu_b^{w_i} = \mathrm{TC}_{bs}(k),\ i = 1, 2, \ldots, m$.
(3) Analyse the CQN by the approximate mean value method to analyse the maximum throughput capacity $\mathrm{TC}(N_D)$.

Assuming that k drones are in the SOQN of the battery swap process. We define the state variable of the SOQN as (N_s, N_c), where N_s is the number of drones in the swapping node μ_s and N_c is the number of batteries in the charging node, respectively.

If the number of drones in the system k is smaller than the number of spare batteries n_b, the underlying Markov chain process of the SOQN can be depicted as Figure 6(a). Otherwise, it can be depicted as Figure 6(b).

Solving the underlying Markov chain process, we can get the state probabilities $\pi(N_s, N_c)$ and the load-dependent throughput of the SOQN by Equation (1)

$$\mathrm{TC}_{bs}(k) = \sum_{N_c=1}^{n_b} \pi(N_s, N_c) \cdot \overline{N_c} \cdot \mu_c, \tag{1}$$

where $\overline{N_c} = \overline{n_b - i} = \min(n_b - i, n_c)$ is the number of batteries that are charging.

After we obtain the load-dependent throughput of the SOQN that models the battery swap process $\mathrm{TC}_{bs}(k)$, we replace the SOQN by a service node with service rate $\mu_b = \mathrm{TC}_{bs}(k),\ k = 1, 2, \ldots, N_D$. Then, we substitute it into the CQN and analyse the CQN by the AMVA (Buitenhek, van Houtum, and Zijm 2000) to obtain the throughput capacity $\mathrm{TC}(N_D)$.

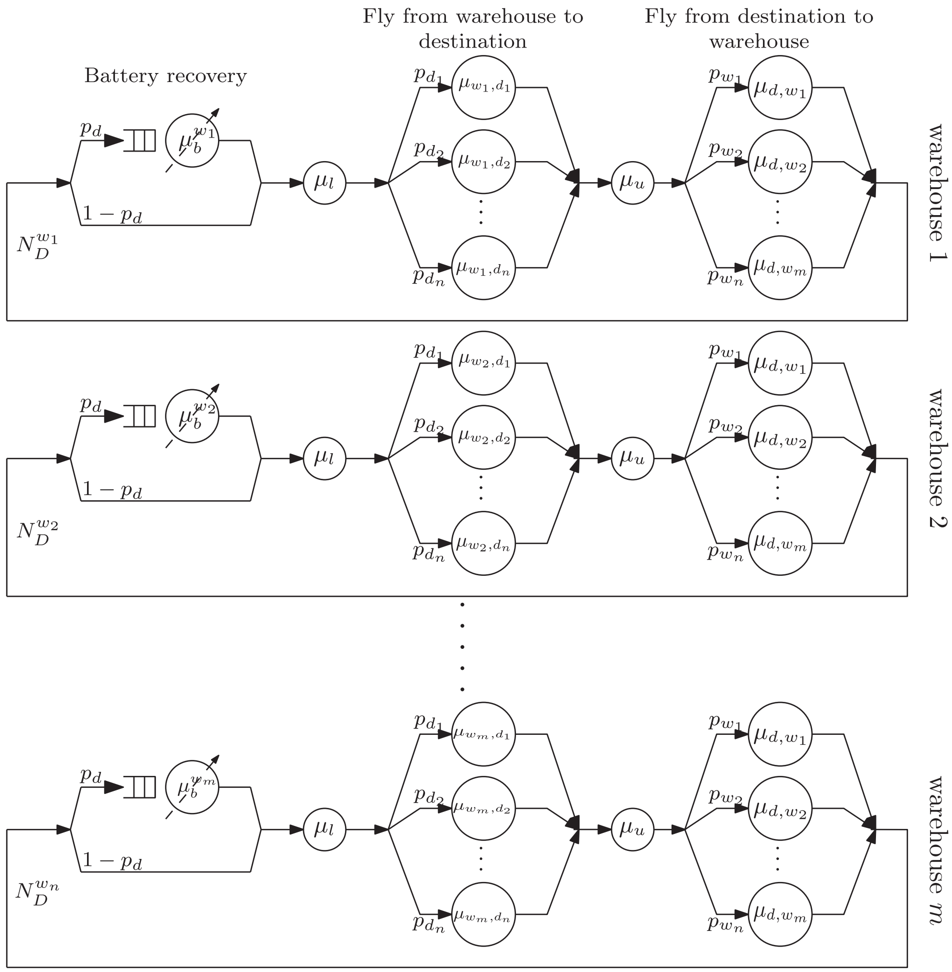


Figure 4. CQN for the multi-warehouse drone delivery system with dedicated drones.

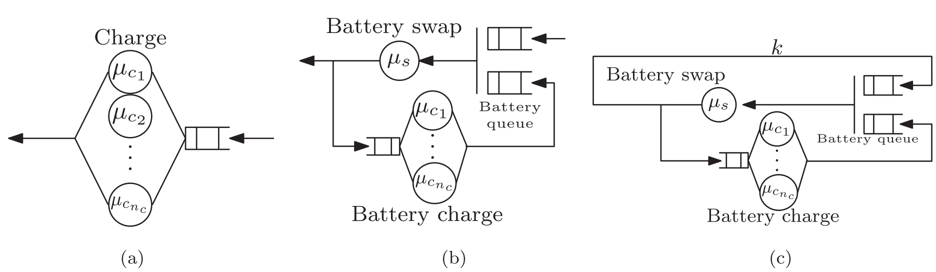


Figure 5. Queueing models for the battery charge process, SOQN of battery swap process, and CQN for SOQN of battery swap process. (a) Battery charge. (b) Battery swap and (c) CQN for SOQN of battery swap process.

4.3. A battery recovery time based heuristic for the drone to warehouse assignment rule

In this section, we design a heuristic based on the battery recovery time of each warehouse to improve the throughput capacity in the multi-warehouse drone delivery system with shared drones (Algorithm 1). The idea of the heuristic is to keep increasing the visit probability of the warehouse with the smallest battery recovery time and decreasing the visit probability of the warehouse with the largest battery recovery time correspondingly, until the maximum throughput capacity cannot be improved for a given number of iterations MAX_{ite}.

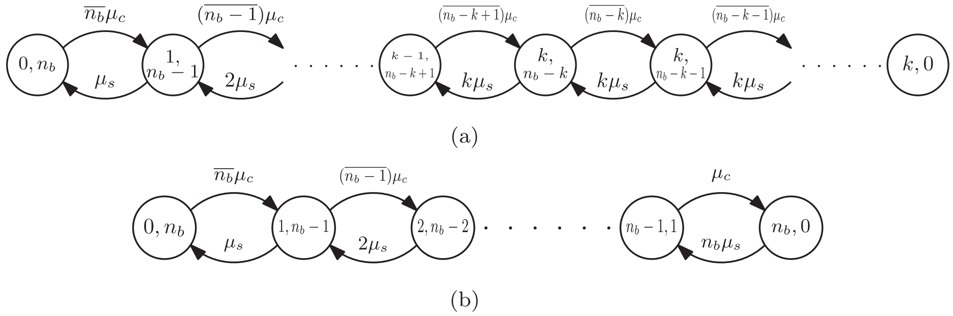


Figure 6. State transition process of the SOQN. (a) $k \leq n_b$ and (b) $k > n_b$.

Algorithm 1 The drone to warehouse assignment rule improve heuristic

Initialisation:
Let the assignment probability of drone to warehouse be $\boldsymbol{p_w} = \{p^c_{w_1}, p^c_{w_2}, \cdots, p^c_{w_m}\}$.
Input $\boldsymbol{p_w}$ into the CQN and get the maximum throughput capacity $TC(\boldsymbol{p_w})$ by AMVA.
Based on the result of AMVA, get the expected battery recovery time at each warehouse, i.e., $ET_b^{w_i}(N_D)$.
Initialise iterations by letting $N_{ite} = 1$ and set the maximum number of iterations to MAX_{ite}. Let the adjustment percentage of assignment probability be ε.
Iteration:
while $N_{ite} \leq MAX_{ite}$ **do**
  Find the warehouse with the largest battery recovery time $\max_i ET_b^{w_i}(N_D)$ and let its visit probability be $p^{\max}_{w_i}$. Find the warehouse with the smallest battery recovery time $\min_i ET_b^{w_i}(N_D)$ and let its visit probability be $p^{\min}_{w_i}$.
  Let $p^{\max}_{w_i} = p^{\max}_{w_i} - \varepsilon\% \cdot p^{\min}_{w_i}$ and $p^{\min}_{w_i} = p^{\min}_{w_i} + \varepsilon\% \cdot p^{\min}_{w_i}$ and denote the updated vector of assignment probabilities as $\overline{\boldsymbol{p}}_{\boldsymbol{w}}$. Input $\overline{\boldsymbol{p}}_{\boldsymbol{w}}$ into the CQN to calculate the maximum throughput capacity $TC(\overline{\boldsymbol{p}}_{\boldsymbol{w}})$.
  if $TC(\overline{\boldsymbol{p}}_{\boldsymbol{w}}) > TC(\boldsymbol{p_w})$ **then**
    Let $\boldsymbol{p_w} = \overline{\boldsymbol{p}}_{\boldsymbol{w}}$, $TC(\boldsymbol{p_w}) = TC(\overline{\boldsymbol{p}}_{\boldsymbol{w}})$ and $N_{ite} = 1$.
  else
    Let $N_{ite} = N_{ite} + 1$.
  end if
end while
End

5. Cost minimisation model

In this section, we analyse the system design and operating policies in a multi-warehouse drone delivery system in terms of operating cost. We first build the total annual cost minimisation model (M.1).

$$\begin{aligned}
&\min\ \text{Cost}\quad (N_D^{w_i}, n_c^{w_i}, n_b^{w_i}, v) = \sum_{i=1}^{m}(C_d N_D^{w_i} + C_c n_c^{w_i} + C_b n_b^{w_i}) + p_e \cdot (\text{WD} \cdot H \cdot \text{TC} \cdot E_o) \\
&\text{s.t.}\quad \begin{cases} \text{TC} \geq \text{TC}_{\min} \\ m, n, \lambda_j, \quad j = 1, 2, \ldots, n \text{ are given} \end{cases}
\end{aligned} \tag{M.1}$$

where C_d is the annualised cost per drone, C_c is the annual cost per plug-in charger, C_b is the annual cost per spare battery, p_e is the price of electricity, WD is the working days per year, H is the working hours per day and E_o is the expected energy consumption per order.

Note that both the decision variables and objective function depend on the battery management strategy used in the system. In the system using the plug-in charge strategy, the decision variables include the number of drones, the number of plug-in chargers and the velocity of drone. Large velocity of drone provides large throughput capacity, while it also means more energy consumption. We consider the following relationship between the energy consumption per unit distance and the velocity of drone (Equation (2), see Stolaroff et al. 2018)

$$E(v) = \frac{mg + F_{\text{drag}}}{\eta}(v \sin\alpha + \hat{v}) \tag{2}$$

Where $m = 2.57$ (kg) is the average mass of a drone with and without packages, g is the gravitational constant, $\eta = 0.7$ is the overall power efficiency of the drone, F_{drag} is the total drag force which is estimated according to air speed, density of air, drag coefficient, etc. Referring to Stolaroff et al. (2018), we assume $F_{\text{drag}} = 0.889$ (N). The pitch angle $\alpha = \tan^{-1}(F_{\text{drag}}/mg)$. And $\hat{v}$ is the induced velocity required for a given drone and a drag force. $\hat{v}$ is the solution of the following equation

$$\hat{v} = \frac{2(mg + F_{\text{drag}})}{0.305\pi\sqrt{(v\cos\alpha)^2 + (v\sin\alpha + \hat{v})^2}} \tag{3}$$

With $E(v)$, we can calculate the expected energy consumption per order by $E_o = E(v) \cdot (\text{ED}_w^d + \text{ED}_d^w)$, where ED_w^d and ED_d^w is the expected distance between the warehouse and the delivery point that can be obtained by the AMVA method.

The objective function consists of the drone cost per year $C_d N_D^{w_i}$, the plug-in charger cost per year $C_c n_c^{w_i}$ and the energy cost per year $p_e \cdot (\text{WD} \cdot H \cdot TC \cdot E_o)$. In the system using the battery swap strategy, the decision variable adds the number of spare batteries and the total cost adds the spare battery cost per year $C_b n_b^{w_i}$. The constraint is to ensure that the throughput capacity is larger than a required level $\text{TC}_{\min}$. Depending on the allocation rule of drones among warehouses (either shared or dedicated), we need to determine the number of drones dedicated to each warehouse or the total number of drones shared by all warehouses. We use our neighbourhood search procedure to improve the performance of the drone to warehouse assignment rule under the shared allocation rule of drones among warehouses. We design a grid search procedure to analyse model (M.1).

Algorithm 2 Solution approach of model (M.1)

1: **Initialisation**: Give the price of a drone p_d, the price of a plug-in charge facility p_c, the price of a spare battery p_b, and the electricity price p_e.
2: **Initial solution**: Find a feasible solution $(N_D^{w_i}, n_c^{w_i}, n_b^{w_i}, v)$ by enumerating all variables from a large value. For the shared allocation rule, optimise the drone to warehouse assignment rule by Algorithm 1. Record the throughput capacity of the initial solution as the current best TC_{CB}.
3: **while** $TC_{CB} > TC_{\min}$ **do**
4: Reduce the number of drones by one, i.e., $N_D = N_D - 1$. Optimise the drone to warehouse assignment rule and get the throughput capacity TC.
5: **while** $TC \geq TC_{\min}$ **do**
6: Reduce both n_b and n_c by one and find the optimal velocity of drone v.
7: Improve the drone to warehouse assignment rule under the shared allocation rule by Algorithm 1. Get the throughput capacity TC.
8: Let $TC_{CB} = TC$.
9: **end while**
10: **end while**
11: **Output**: The optimal throughput capacity TC_{CB}, the optimal solution $(N_D^{w_i}, n_c^{w_i}, n_b^{w_i}, v)$.

6. Simulation validation and numerical experiments

In this section, we first validate the analytical models by simulation and then conduct some numerical experiments to analyse the operating policies examined in this paper in both terms of maximum throughput capacity and operating cost.

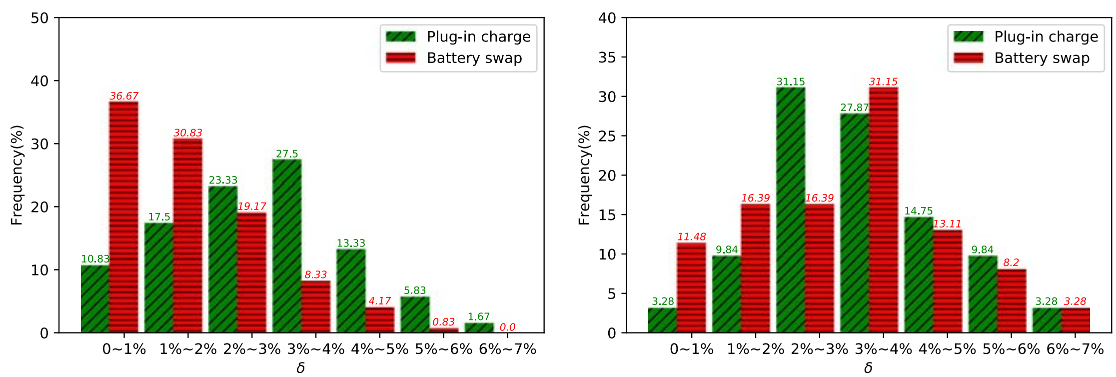
6.1. Simulation validation

We consider two scenarios for the drone delivery system: $m = 5$, $n = 100$ and $m = 7$, $n = 140$. In the system scenarios with shared drones rule, we consider both the random and closest assignment rule of drone to warehouse. Both the plug-in charge and battery swap strategies are examined in each scenario. Thereby, we have eight cases for the system with shared drones rule and four cases for the system with dedicated drones rule. The system parameters are presented in Table 2 and the simulation model is built by Arena. To make all drones always busy (to estimate the maximum throughput capacity), we set the order arrival rates large enough to make the drones utilisation $\rho_D > 99\%$. For each scenario, we first run a 24 hours warming up period to eliminate the initial bias, and then run 30 replications of 240 hours simulation. This leads to a 95% confidence interval of maximum throughput capacity with the half width smaller than 2% of the average. We calculate the

Table 2. Simulation validation results.

Drone to warehouse allocation rule	(m, n)	Assignment rule	Battery recovery	TC_S(h)	TC_A(h)	δ_{TC}(%)
Shared rule	(5, 100)	Random	Plug-in charge	199.62	194.21	2.71
			Battery swap	227.52	223.83	1.62
		Closest	Plug-in charge	212.86	208.07	2.25
			Battery swap	218.92	215.42	1.60
	(7, 140)	Random	Plug-in charge	205.49	197.99	3.65
			Battery swap	266.12	264.39	0.65
		Closest	Plug-in charge	229.92	224.88	2.19
			Battery swap	281.23	278.33	1.03
Dedicated rule	(5, 100)	–	Plug-in charge	202.95	197.25	2.81
		–	Battery swap	219.90	212.62	3.31
	(7, 140)	–	Plug-in charge	217.28	209.89	3.40
		–	Battery swap	208.05	201.91	2.95

Other system parameters: $d_{i,j} \in U[5000, 10{,}000]$ m, $\lambda_{d_j} \in U[1, 10]$ (per hour), $t_{lu} = 120$ seconds, $N_D = 80, n_c = 10, n_b = 10, t_c \in U[1, 2]h$ and $t_s = 300$ seconds.



(a) (b)

Figure 7. Relative errors between analytical results and simulation results. (a) Shared drones and (b) Dedicated drones.

relative error of the analytical throughput capacity TC_A to the simulation throughput capacity TC_S by Equation (4),

$$\delta_{\text{TC}} = \frac{|\overline{\text{TC}}_S - \text{TC}_A|}{\overline{\text{TC}}_S} \times 100\%, \tag{4}$$

where $\overline{\text{TC}}_S$ is the average throughput capacity of multiple simulation replications.

We present the frequency of the relative errors in Figure 7, and include the average value of the relative errors in Table 2. The results show that the average relative error is 2.35%, and the maximum relative error is smaller than 7%. The error mainly comes from the estimation of the waiting time of drones at warehouses. This suggests that our analytical models can accurately estimate the maximum throughput capacity of the multi-warehouse drone delivery system. Moreover, the results in Table 2 show that the shared rule outperforms the dedicated rule, the battery swap strategy outperforms the plug-in charge strategy, and the closest assignment rule outperforms the random assignment rule in terms of maximum throughput capacity in the simulation system scenarios. Next, we conduct numerical experiments to analyse the drone allocation policy (shared or dedicated), the drone to warehouse assignment rule (random, closest and heuristic) and the battery management strategy (plug-in charge or battery swap), in terms of both maximum throughput capacity and cost.

6.2. Comparison among random, closest and heuristic drone to warehouse assignment rules

In this section, we conduct some numerical experiments to compare the drone to warehouse assignment rules examined in this paper in terms of the maximum throughput capacity TC. We consider the system scenarios examined in simulation validation, i.e. $m = 5, n = 100$ and $m = 7, n = 140$, examining both the plug-in charge and battery swap strategies. $n_b = 10$ spare batteries (with $n_c = 10$ chargers) are used in the battery swapping station, $n_c = 20$ chargers are used in the plug-in

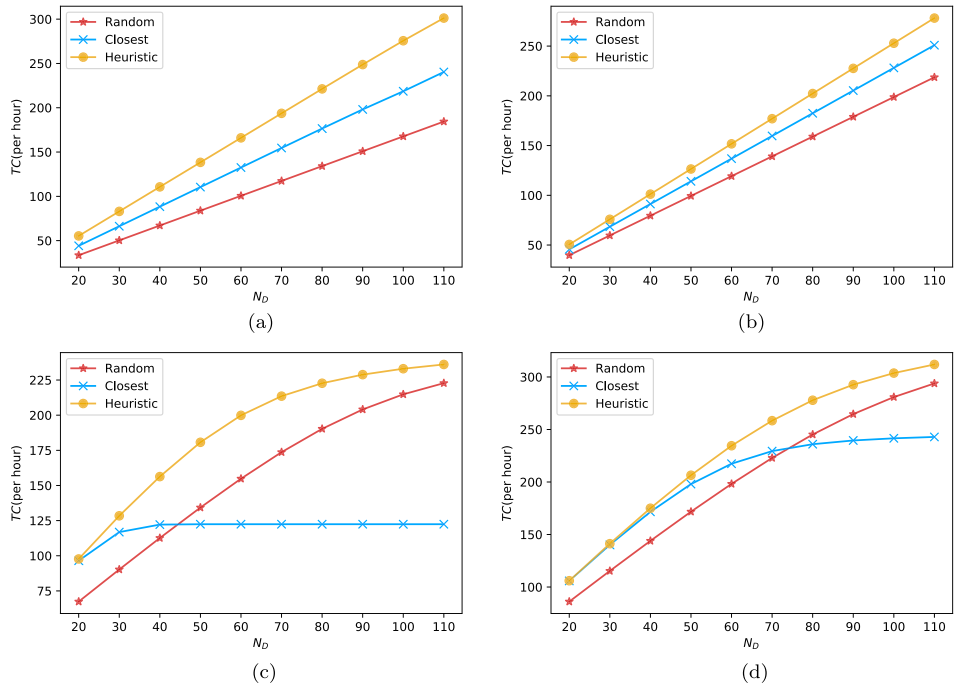


Figure 8. Throughput capacity comparison among different drone to warehouse assignment rules. (a) $m = 5$, $n = 100$ with plug-in charge. (b) $m = 7$, $n = 140$ with plug-in charge. (c) $m = 5$, $n = 100$ with battery swap and (d) $m = 7$, $n = 140$ with battery swap.

charge station, and other system parameters come from Table 2. We vary the number of drones N_D from 50 to 110 with a step size of 10, and collect the throughput capacity (per hour) of each scenario.

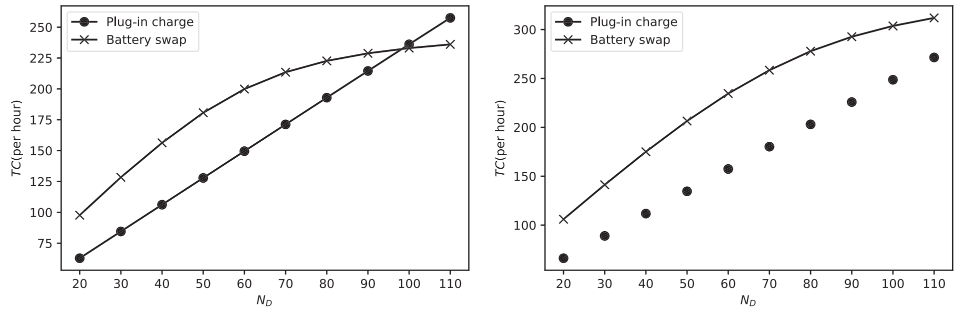
The comparison results of drone to warehouse assignment rules are depicted in Figure 8. We have the following observations based on the results:

(1) In the system using the plug-in charge strategy, the closest drone to warehouse assignment rule outperforms the random drone to warehouse assignment rule, and the heuristic performs the best in terms of maximum throughput capacity. This can be explained as that the closest assignment rule provides smaller flying time from delivery point back to warehouse than that provided by the random assignment rule.
(2) In the system using the battery swap strategy, there may exist an intersection between the throughput capacity curve of the random drone to warehouse assignment rule and that of the closest drone to warehouse assignment rule. This suggests that when the number of drones used in the system is smaller than the intersection, the random assignment rule of drone to warehouse performs better in terms of maximum throughput capacity, otherwise, the closest assignment rule is better. It can be explained as that while the fly time of a drone from delivery point back to warehouse can be saved on some degree under the closest drone to warehouse assignment rule, it may wait longer for the battery recovery process in the warehouse without enough spare batteries that are relatively expensive.
(3) Our heuristic can further improve the performance of the drone to warehouse assignment rule by balancing the battery recovery time of each warehouse.

6.3. Analysis of battery management strategies and drone allocation rules

In this section, we analyse the battery management strategies and drone allocation rules in terms of maximum throughput capacity. We first compare the plug-in charge strategy and the battery swap strategy, considering two system scenarios, i.e. $m = 5, n = 100$ and $m = 7, n = 140$. $n_c = 20$ chargers are used in the warehouse with plug-in charge strategy, and $n_b = 10$ (with $n_c = 10$ chargers) spare batteries are used in the warehouse with battery swap strategy. We consider the shared drone allocation rule and the heuristic-based drone to warehouse assignment rule. Other system parameters come from Table 2. We vary the number of drones used in the system from $N_D = 20$ to $N_D = 110$ with a stepsize of 10. We estimate the maximum throughput capacity of each scenario by our analytical models and present the results in Figure 9.

The result suggests that the battery swap strategy outperforms the plug-in charge strategy in terms of maximum throughput capacity in most cases. When the number of drones used is large, the plug-in charge strategy may perform better than the



(a) (b)

Figure 9. Throughput capacity comparison between plug-in charge strategy and battery swap strategy. (a) $m = 5$, $n = 100$ and (b) $m = 7, n = 140$.

Table 3. Throughput capacity comparison between shared drone allocation rule and dedicated drone allocation rule.

	Std = 12			*Std* = 15			*Std* = 17			*Std* = 18		
N_D	$\mathrm{TC}_{sh}(/h)$	$\mathrm{TC}_{de}(/h)$	*Imp*%	$\mathrm{TC}_{sh}(/h)$	$\mathrm{TC}_{de}(/h)$	*Imp*%	$\mathrm{TC}_{sh}(/h)$	$\mathrm{TC}_{de}(/h)$	*Imp*%	$\mathrm{TC}_{sh}(/h)$	$\mathrm{TC}_{de}(/h)$	*Imp*%
10	31.48	21.37	32.10	31.52	20.30	35.58	33.93	22.07	34.96	34.11	23.55	30.96
20	65.66	61.29	6.66	65.73	60.58	7.84	69.58	59.91	13.90	69.68	64.99	6.73
30	97.52	96.95	0.58	97.73	95.04	2.76	100.88	88.59	12.18	101.91	94.66	7.12
40	128.43	124.10	3.37	128.25	118.87	7.31	131.55	108.84	17.26	133.02	116.69	12.27
50	156.81	143.87	8.26	156.50	141.06	9.87	159.22	126.83	20.34	160.85	136.22	15.31
60	181.18	164.94	8.96	180.71	158.83	12.11	183.56	144.63	21.21	185.07	152.56	17.57
70	200.27	184.75	7.75	199.88	177.69	11.10	202.21	161.26	20.25	203.42	169.75	16.55
80	213.83	198.50	7.17	213.54	192.41	9.90	215.23	175.08	18.65	216.09	182.98	15.32
90	222.91	212.25	4.79	222.72	206.45	7.30	223.88	188.74	15.69	224.47	189.39	15.63
100	228.98	219.98	3.93	228.84	216.38	5.45	229.65	196.47	14.45	230.06	199.90	13.11

*Note: TC_{sh} is the maximum throughput capacity of shared allocation rule and TC_{de} is the maximum throughput capacity of dedicated allocation rule. The improvement percentage *Imp* is calculated by $Imp = \frac{\mathrm{TC}_{sh} - \mathrm{TC}_{de}}{\mathrm{TC}_{sh}} \cdot 100\%$.

battery swap strategy (Figure 9(a)). This can be explained as that while the battery swap process can provide shorter battery recovery time than the plug-in charge process, its capacity is limited by the number of spare batteries used that are relatively expensive. When the number of batteries is given, the throughput capacity of the system with battery swap strategy will have an upper bound as the number of drones increase.

Next, we compare the shared drone allocation rule and the dedicated drone allocation rule in terms of maximum throughput capacity, considering the system with $m = 5$ warehouses and $n = 100$ delivery points. We examine four scenarios by varying the distribution of the distance between warehouses and delivery points. The standard deviations of distribution of distance (denoted by *Std*) between warehouses and destinations are *Std* = 12, 15, 17, 18, respectively. In each scenario, we vary the number of drones used in the system from 10 to 100 with a stepsize of 10. For the shared rule, we use the heuristic to improve the drone to warehouse assignment rule. For the dedicated rule, we distribute the drones among warehouses based on the percentage of demands of a specific warehouse to all demands. The result is included in Table 3.

The result shows that the shared allocation rule outperforms the dedicated allocation rule by about 13.31% in terms of maximum throughput capacity in all cases. This can be explained as that while the flying time from the warehouse to the delivery point under the shared allocation rule is larger than that under the dedicated allocation rule, both the flying time from the delivery point back to the warehouse and the battery recovery time under the shared allocation rule can be improved by our heuristic, which is better than that under the dedicated allocation rule. Moreover, we can find that the maximum throughput capacity under the shared drone allocation rule keeps stable, while that under the dedicated drone allocation rule basically keeps decreasing. With the increase of the standard deviation of distribution of distance between warehouses and destinations, some warehouses need to service more demands while other warehouses serve less. Under the shared drone allocation rule, drones are distributed among warehouses based on their working load, i.e. order demands. Thereby, the throughput capacity keeps stable. However, under the dedicated drone allocation rule, the number of drones used by each warehouse is fixed. Some warehouse will serve too many demands with the increase of the deviation. Thereby,

Table 4. Cost comparison between shared allocation rule and dedicated allocation rule.

Allocation rule	N_D	n_c	n_b	v (km/h)	TC (per hour)	$\mathrm{TC_{min}}$ (per hour)	*Cost* ($)
Shared	63	46	46	60	200.04	200	92,313
	80	51	52	60	250.60	250	113,804
	94	61	62	60	300.23	300	134,364
	108	71	72	60	350.04	350	154,930
	122	82	83	60	401.31	400	176,042
	136	92	92	60	450.00	450	196,173
	152	98	99	60	501.27	500	217,184
	165	109	110	60	550.55	550	237,233
	178	121	122	60	600.56	600	257,805
Dedicated	114	67	67	60	200.25	200	150,814
	137	67	67	60	250.32	250	174,608
	145	100	100	65	303.31	300	200,629
	170	100	100	60	350.38	350	225,704
	195	100	102	60	407.88	400	252,414
	215	101	103	65	454.97	450	274,678
	239	100	100	65	500.78	500	298,211
	245	134	134	65	557.76	550	322,234
	250	167	167	75	602.25	600	348,015

the overall throughput capacity decreases and the improvement percentage of TC under the shared allocation rule over that under the dedicated allocation rule increases.

6.4. Cost analysis

In this section, we analyse the operating cost in a drone delivery system with 5 warehouses and 100 delivery points. The system works 250 days with 8 working hours per day. The price of a drone is $p_d = \$10{,}000$ (annualisation in 10 years), the price of a plug-in charger $p_c = \$1000$ (annualisation in 10 years), the price of a spare battery $p_b = \$4000$ (annualisation in 10 years), and the interest rate $\mathrm{IR} = 0.5\%$. Then, we can calculate the annualised cost of a drone, a plug-in charger and a spare battery by the following equation

$$C_d = \frac{\sum_{i=1}^{10} 10,000(1+\mathrm{IR})^{i-1}}{10}, \quad C_c = \frac{\sum_{i=1}^{10} 1000(1+\mathrm{IR})^{i-1}}{10}, \quad C_b = \frac{\sum_{i=1}^{10} 4000(1+\mathrm{IR})^{i-1}}{10}, \tag{5}$$

The operating costs of both the shared and dedicated allocation rules are presented in Table 4. We have the following observations based on the results

- The shared allocation rule outperforms the dedicated allocation rule in terms of operating cost by about 30.70%. The advantage mainly comes from the saving on the resources, namely the drones, the spare batteries and the plug-in chargers. The optimal velocity of drones under the shared allocation rule is relatively smaller than that under the dedicated allocation rule.
- The optimal number of spare batteries basically equals that of plug-in chargers. This can be explained as that the plug-in charger is relatively cheaper than the spare battery. Thereby, we can use as many plug-in charges as needed to charge all spare batteries.

7. Conclusion

In this study, we investigate a multi-warehouse drone delivery system. In such a system, a fleet of drones is used by multiple warehouses to transport items to delivery points. We consider the shared allocation rule under which drones are shared by warehouses and the dedicated allocation rule under which each warehouse has its own drones. We examine the drone to warehouse assignment rules, considering the random and closest assignment rules and design a neighbourhood search algorithm to improve the performance of the drone to warehouse assignment rule. Both the plug-in charge strategy and the battery swap strategy are investigated for battery recovery. We build a closed queueing network to estimate the maximum throughput capacity, in which the drone to warehouse assignment rule is captured by the warehouse visit probabilities. In the CQN of the system with battery swap strategy, the battery swap process is modelled as an SOQN and we design a decomposition method to solve the model.

We validate the accuracy and effectiveness of the analytical model by simulation. Numerical experiments are conducted to compare the performance of different drone to warehouse assignment rules and the performance of the plug-in charge strategy and the battery swap strategy. The results show that there exists a trade-off between the random drone to warehouse assignment rule and the closest drone to warehouse assignment rule. When the number of drones used in the system is smaller than a critical point, the closest assignment rule is better than the random assignment rule in terms of maximum throughput capacity. Otherwise, the random assignment rule is better. Our neighbourhood search algorithm can improve the maximum system throughput capacity by about 13.31%, compared with the random and closest drone to warehouse assignment rules. We find that the battery swap strategy outperforms the plug-in charge strategy, while more investment is required for spare batteries. Moreover, the shared allocation rule outperforms the dedicated allocation rule in terms of maximum throughput capacity, and it can reduce the operating cost by about 30.70% compared with the dedicated allocation rule. Our research can provide design-related decision support for e-commerce companies to build an efficient drone delivery system with the minimum investment and operation cost, and can be applied to handle complex operations of the system.

For the future study, we are interested in the order to drone assignment rule and the routing problem of drone with multiple items delivery capacity.

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Disclosure statement

No potential conflict of interest was reported by the author(s).

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Appendices

Appendix 1. Estimation of the depletion probability p_d

Let the voltage of the drone’s battery be V_D (V), the battery capacity of the drone be B_D $(A \cdot h)$. A drone will be depleted if its battery has 5% capacity remaining, then, the energy of a fully charged battery is

$$E = 0.95 \times 3600 \cdot V_D \cdot B_D,$$

where the unit of E is joule (J).

To fulfill an order, the drone will operate for a time t_B. We have that the mean value of t_B can be calculated as $\bar{t}_B = 2t_{lu} + \bar{t}_{w,d} + \bar{t}_{d,w}$. Let E_f^l and E_f^u be the energy consumed by the loaded and unloaded drone per second, while flying, respectively. Also, let E_{lu} be the energy consumed by the drone to load/unload an item. In the duration $\bar{t}_B$, the energy of the drone’s battery drops by $E_c = 2E_{lu}t_{lu} + E_f^l\bar{t}_{w,d} + E_f^u\bar{t}_{d,w}$. So, the total number of orders a fully charged drone can carry out is $M_o = E/E_c$ and the depletion probability $p_d = 1/M_o$

Appendix 2. Approximate mean value method

Table A1 presents the notations used in the AMVA (Buitenhek, van Houtum, and Zijm 2000)

Table A1. Notations used in AMVA.

Notation	Description
M	the number of service stations in the closed queueing network.
c_m	the number of servers in service station m.
v_m	the visit ratio of customer at service station m.
n	the number of customers in the closed queueing network, $n = 1, 2, \ldots, N$.
$ES_{rem,m}$	the expected time remaining until the first departure of customer at service station m.
ES_m	the expected service time of service station m (the first moment).
ES_m^2	the second moment of the service time at service station m.
$p_m(l \mid n)$	the probability that there are l customers in service station m when the system contains n customers.
$Q_m(n)$	the probability that all servers at service station m are busy when the system contains n customers.
$EL_m(n)$	the mean number of customers in the queue of service station m (excluding jobs in service) when the system contains n customers.
$ET_m(n)$	the lead time of service station m when the system contains n customers.
$TH(n)$	the system throughput when the system contains n customers.

The AMVA method includes the following steps:

(1) Initialise. Let $p_m(0 \mid 0) = 1, Q_m(0) = 0, EL_m(0) = 0, m = 1, 2, \ldots, M$.
(2) Preprocessing. Enumerate n from 0 to N, do the following procedures:

(a) For $m = 1, 2, \ldots, M$, calculate

$$ET_m(n) = Q_m(n-1)ES_{rem,m} + EL_m(n-1)\frac{ES_m}{c_m} + ES_m,$$

where $ES_{rem,m}$ is given by the following equation

$$ES_{rem,m} = \frac{c_m - 1}{c_m + 1} \cdot \frac{ES_m}{c_m} + \frac{2}{c_m + 1} \cdot \frac{1}{c_m} \cdot \frac{ES_m^2}{2ES_m}.$$

(b) Calculate the load-dependent throughput

$$TH(n) = \frac{n}{\sum_{m=1}^{M} v_m ET_m(n)},$$

(c) For $m = 1, 2, \ldots, M$ and $l = 1, 2, \ldots, min(c_m - 1, n)$, calculate

$$p_m(l \mid n) = \frac{ES_m}{l} v_m TH(n) p_m(l - 1 \mid n - 1).$$

(d) For $m = 1, 2, \ldots, M$, if $n < c_m, Q_m(n) = 0$, otherwise,

$$Q_m(n) = \frac{ES_m}{c_m} v_m TH(n) \cdot [Q_m(n-1) + p_m(c_m - 1 \mid n - 1)].$$

(e) For $m = 1, 2, \ldots, M$, calculate

$$p_m(0 \mid n) = 1 - \sum_{l=1}^{\min(c_m-1,n)} p_m(l \mid n) - Q_m(n).$$

(f) For $m = 1, 2, \ldots, M$ and if $n < c_m$, $EL_m(n) = 0$, otherwise,

$$EL_m(n) = \frac{ES_m}{c_m} v_m TH(n)[EL_m(n-1) + Q_m(n-1)].$$

Appendix 3. Simulation model

We build a simulation model for the multi-warehouse drone delivery system by Arena. The simulation process is depicted in Figure A.1. The system parameters come from Table A2. To obtain the maximum throughput capacity of the system, we set the order arrival rates $\sum_{j=1}^{n} \lambda_{d_j}$ as large enough to make the drones utilization $\rho_D \geq 95\%$.

Table A2. Parameters for simulation validation.

$d_{i,j}$ (meter)	λ_{d_j} (per hour)	t_{lu} (sec)	N_D	n_c	n_b	t_c (hour)	t_s (sec)
$U[5000, 10{,}000]$	$U[1, 10]$	120	80	10	10	$[1, 2]$	300

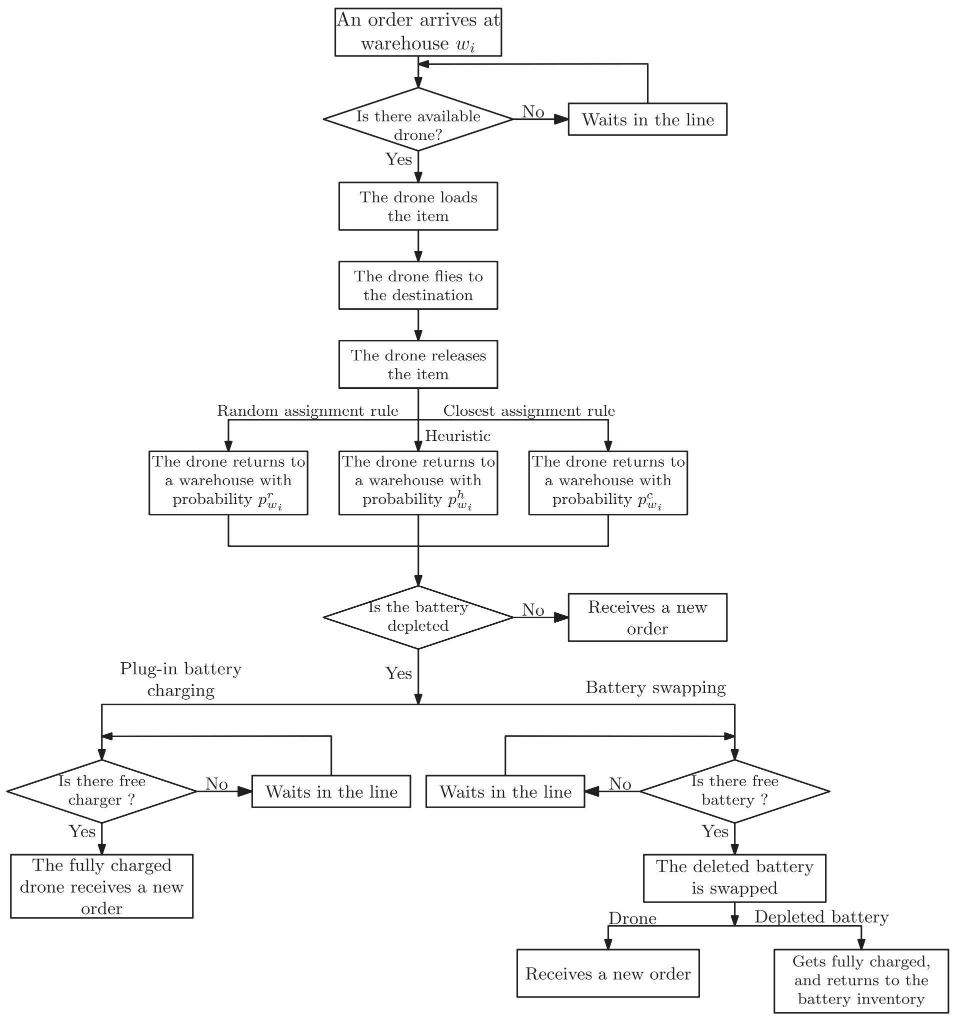

Figure A.1. Simulation process.