MSIA400_HW3

Ziwen Wang

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```
web = read.table("markov100.txt", header=F)
P=as.matrix(web,nrow=100,ncol=100,byrow=TRUE)
head(P)
```

```
##          V1     V2     V3     V4     V5     V6     V7     V8     V9    V10
## [1,] 0.1868 0.2687 0.1046 0.4399 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
## [2,] 0.2357 0.1195 0.0518 0.0615 0.5315 0.0000 0.0000 0.0000 0.0000 0.0000
## [3,] 0.1624 0.1848 0.1845 0.0984 0.0842 0.2857 0.0000 0.0000 0.0000 0.0000
## [4,] 0.0000 0.0000 0.2794 0.0276 0.0414 0.0081 0.0406 0.0491 0.0480 0.5058
## [5,] 0.0000 0.0000 0.5724 0.0211 0.0586 0.0966 0.0424 0.0428 0.0910 0.0145
## [6,] 0.0000 0.0000 0.2514 0.2514 0.0961 0.0771 0.0925 0.0721 0.0408 0.0862
##         V11 V12 V13 V14 V15 V16 V17 V18 V19 V20 V21 V22 V23 V24 V25 V26
## [1,] 0.0000   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [2,] 0.0000   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [3,] 0.0000   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [4,] 0.0000   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [5,] 0.0606   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [6,] 0.0324   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
##      V27 V28 V29 V30 V31 V32 V33 V34 V35 V36 V37 V38 V39 V40 V41 V42 V43
## [1,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [2,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [3,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [4,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [5,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [6,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
##      V44 V45 V46 V47 V48 V49 V50 V51 V52 V53 V54 V55 V56 V57 V58 V59 V60
## [1,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [2,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [3,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [4,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [5,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [6,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
##      V61 V62 V63 V64 V65 V66 V67 V68 V69 V70 V71 V72 V73 V74 V75 V76 V77
## [1,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [2,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [3,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [4,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [5,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [6,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
##      V78 V79 V80 V81 V82 V83 V84 V85 V86 V87 V88 V89 V90 V91 V92 V93 V94
## [1,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [2,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [3,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [4,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [5,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
## [6,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
##      V95 V96 V97 V98 V99 V100
```

```
## [1,]   0   0   0   0   0    0
## [2,]   0   0   0   0   0    0
## [3,]   0   0   0   0   0    0
## [4,]   0   0   0   0   0    0
## [5,]   0   0   0   0   0    0
## [6,]   0   0   0   0   0    0
```

```
#Q1a
a=c(1,rep(0,99))
library(expm)
```

```
## Loading required package: Matrix
```

```
##
## Attaching package: 'expm'
```

```
## The following object is masked from 'package:Matrix':
##
##     expm
```

```
q1<-a %*% (P %^%10)
q1
```

```
##              V1         V2         V3         V4       V5       V6
## [1,] 0.03210252 0.03315294 0.09941539 0.06462136 0.045091 0.048171
##              V7         V8        V9        V10       V11        V12
## [1,] 0.02981315 0.04957693 0.0678567 0.08126983 0.1194159 0.02849685
##             V13        V14        V15        V16        V17         V18
## [1,] 0.03397078 0.09048926 0.07560999 0.01825296 0.02909586 0.01210839
##             V19          V20         V21          V22         V23
## [1,] 0.01702716 0.0004803007 0.001104914 0.009981056 0.001970024
##              V24         V25         V26          V27         V28
## [1,] 0.003182653 0.001596568 0.005224745 0.0009182241 1.76305e-06
##               V29 V30 V31 V32 V33 V34 V35 V36 V37 V38 V39 V40 V41 V42 V43
## [1,] 1.761169e-06   0   0   0   0   0   0   0   0   0   0   0   0   0   0
##      V44 V45 V46 V47 V48 V49 V50 V51 V52 V53 V54 V55 V56 V57 V58 V59 V60
## [1,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
##      V61 V62 V63 V64 V65 V66 V67 V68 V69 V70 V71 V72 V73 V74 V75 V76 V77
## [1,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
##      V78 V79 V80 V81 V82 V83 V84 V85 V86 V87 V88 V89 V90 V91 V92 V93 V94
## [1,]   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
##      V95 V96 V97 V98 V99 V100
## [1,]   0   0   0   0   0    0
```

```
q1_a<-q1[5]
q1_a
```

```
## [1] 0.045091
```

```
#Q1b
a<-c(1/3,1/3,1/3,rep(0,97))
library(expm)
q2<-a %*% (P %^%10)
q2_a<-q2[10]
q2_a
```

```
## [1] 0.08268901
```

```r
#Q1c
Q = t(P) - diag(100)
Q[100,] = rep(1,100)
rhs = c(rep(0,99),1)
Pi = solve(Q) %*% rhs
Pi[1]
```

```
## [1] 0.01256589
```

```r
#Q1d
B =P[1:99,1:99]
Q = diag(99) - B
e = rep(1,99)
m = solve(Q) %*% e
m[1]
```

```
## [1] 254.9395
```

```r
#Q2a
web = read.table("webtraffic.txt", header=T)
#View(web)
web1<-colSums(web)
P=matrix(web1,nrow=9,ncol=9,byrow=TRUE)
P
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
##  [1,]    0  447  553    0    0    0    0    0    0
##  [2,]    0   23  230  321    0    0    0    0   63
##  [3,]    0  167   43  520    0    0    0    0   96
##  [4,]    0    0    0   44  158  312  247    0  124
##  [5,]    0    0    0    0   22   52   90  127  218
##  [6,]    0    0    0    0   67   21    0  294   97
##  [7,]    0    0    0    0    0   94    7  185   58
##  [8,]    0    0    0    0  262    0    0   30  344
##  [9,]    0    0    0    0    0    0    0    0    0
```

```r
#Q2b
Traffic<-P
Traffic[9,1]<-1000
sum1<-rowSums(Traffic)
P2<-Traffic/sum1
P2[is.na(P2)]<-0
P_Q2<-P2
P_Q2
```

```
##       [,1]       [,2]       [,3]       [,4]      [,5]       [,6]
##  [1,]    0 0.44700000 0.55300000 0.00000000 0.0000000 0.00000000
##  [2,]    0 0.03610675 0.36106750 0.50392465 0.0000000 0.00000000
##  [3,]    0 0.20217918 0.05205811 0.62953995 0.0000000 0.00000000
##  [4,]    0 0.00000000 0.00000000 0.04971751 0.1785311 0.35254237
##  [5,]    0 0.00000000 0.00000000 0.00000000 0.0432220 0.10216110
##  [6,]    0 0.00000000 0.00000000 0.00000000 0.1398747 0.04384134
##  [7,]    0 0.00000000 0.00000000 0.00000000 0.0000000 0.27325581
##  [8,]    0 0.00000000 0.00000000 0.00000000 0.4119497 0.00000000
##  [9,]    1 0.00000000 0.00000000 0.00000000 0.0000000 0.00000000
##             [,7]       [,8]      [,9]
##  [1,] 0.00000000 0.00000000 0.0000000
```

```
##  [2,] 0.00000000 0.00000000 0.0989011
##  [3,] 0.00000000 0.00000000 0.1162228
##  [4,] 0.27909605 0.00000000 0.1401130
##  [5,] 0.17681729 0.24950884 0.4282908
##  [6,] 0.00000000 0.61377871 0.2025052
##  [7,] 0.02034884 0.53779070 0.1686047
##  [8,] 0.00000000 0.04716981 0.5408805
##  [9,] 0.00000000 0.00000000 0.0000000
```

```
#Q2c
Q = t(P_Q2) - diag(9)
Q[9,] = rep(1,9)
rhs = c(rep(0,8),1)
Pi = solve(Q) %*% rhs
Pi
```

```
##             [,1]
##  [1,] 0.15832806
##  [2,] 0.10085497
##  [3,] 0.13077897
##  [4,] 0.14012033
##  [5,] 0.08058898
##  [6,] 0.07583914
##  [7,] 0.05446485
##  [8,] 0.10069664
##  [9,] 0.15832806
```

```
#Q2d
avg<-c(0.1,2,3,5,5,3,3,2,0)
time<-avg * Pi
avg_time<-sum(time)
avg_time
```

```
## [1] 2.305731
```

```
#Q2e
Traffic<-P
Traffic[9,1]<-1000
sum1<-rowSums(Traffic)
P2<-Traffic/sum1
P2[is.na(P2)]<-0
P_Q2<-P2
P_Q3<-P_Q2

P_Q3[2,3]<-P_Q3[2,3]*(1-0.3)
P_Q3[2,6]<-P_Q2[2,3]*(0.3)+P_Q3[2,6]
P_Q3[2,4]<-P_Q3[2,4]*(1-0.2)
P_Q3[2,7]<-P_Q2[2,4]*(0.2)+P_Q3[2,7]

P_final<-P_Q3
P_final
```

```
##       [,1]       [,2]       [,3]       [,4]      [,5]       [,6]
##  [1,]    0 0.44700000 0.55300000 0.00000000 0.0000000 0.00000000
##  [2,]    0 0.03610675 0.25274725 0.40313972 0.0000000 0.10832025
##  [3,]    0 0.20217918 0.05205811 0.62953995 0.0000000 0.00000000
##  [4,]    0 0.00000000 0.00000000 0.04971751 0.1785311 0.35254237
```

```
##  [5,]    0 0.00000000 0.00000000 0.00000000 0.0432220 0.10216110
##  [6,]    0 0.00000000 0.00000000 0.00000000 0.1398747 0.04384134
##  [7,]    0 0.00000000 0.00000000 0.00000000 0.0000000 0.27325581
##  [8,]    0 0.00000000 0.00000000 0.00000000 0.4119497 0.00000000
##  [9,]    1 0.00000000 0.00000000 0.00000000 0.0000000 0.00000000
##             [,7]       [,8]      [,9]
##  [1,] 0.00000000 0.00000000 0.0000000
##  [2,] 0.10078493 0.00000000 0.0989011
##  [3,] 0.00000000 0.00000000 0.1162228
##  [4,] 0.27909605 0.00000000 0.1401130
##  [5,] 0.17681729 0.24950884 0.4282908
##  [6,] 0.00000000 0.61377871 0.2025052
##  [7,] 0.02034884 0.53779070 0.1686047
##  [8,] 0.00000000 0.04716981 0.5408805
##  [9,] 0.00000000 0.00000000 0.0000000
```

```
Q2 = t(P_final) - diag(9)
Q2[9,] = rep(1,9)
rhs = c(rep(0,8),1)
Pi2 = solve(Q2) %*% rhs

#PI2
Pi2
```

```
##             [,1]
##  [1,] 0.16162840
##  [2,] 0.10034341
##  [3,] 0.12104331
##  [4,] 0.12275720
##  [5,] 0.08164613
##  [6,] 0.08250884
##  [7,] 0.06003218
##  [8,] 0.10841213
##  [9,] 0.16162840
```

```
#PI1
Pi
```

```
##             [,1]
##  [1,] 0.15832806
##  [2,] 0.10085497
##  [3,] 0.13077897
##  [4,] 0.14012033
##  [5,] 0.08058898
##  [6,] 0.07583914
##  [7,] 0.05446485
##  [8,] 0.10069664
##  [9,] 0.15832806
```

```
#since the steady state pro of S23 and S24 from PI2 are 0.12104331 and 0.12275720, which are
#lower than the ones in PI1, the link did help balance the traffic.
```

```
#compare the variance of Pi and Pi2
var(Pi)
```

```
##             [,1]
## [1,] 0.001410675
```

```r
var(Pi2)
```

```
##              [,1]
## [1,] 0.001219604
```

```r
#since the Var(P2)<Var(P1), the P2 becomes better
```