Linear Algebra and Differential Equations(Math 54): Lecture 1

Ruochen Liang

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Welcome

My name: Ruochen Liang(Vincent)

Office hours:
MWF 11:00 - 12:00 940 Evans

I will also be the GSI this term, so come and ask questions!

Administria

Enrollment issues:Thomas Brown in 965 Evans Hall

Text Book:

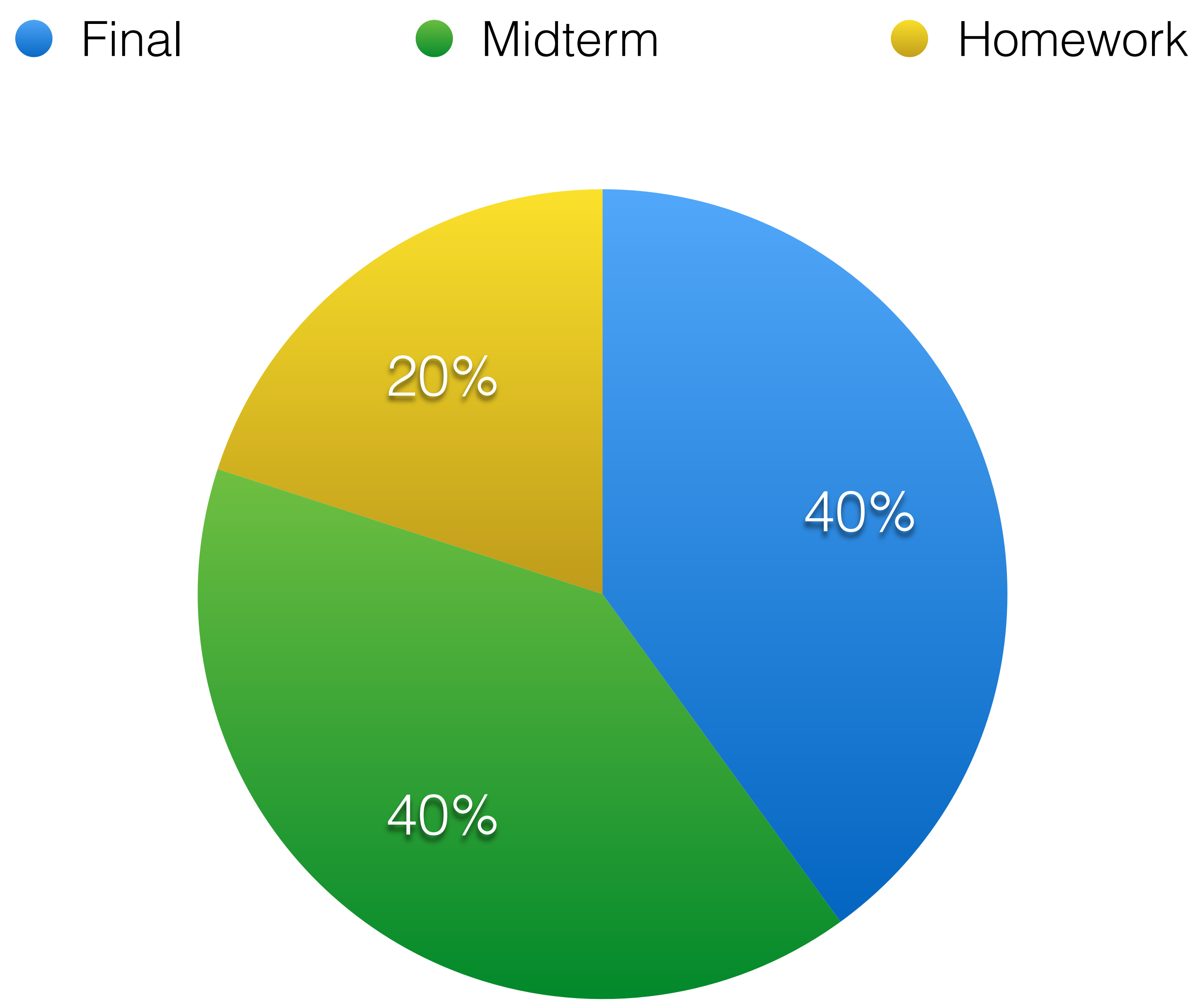
Linear algebra and Differential equations, Berkeley custom edition

Prerequisites:

Math 1B or equivalent.

(Remark: 1B covers more than single variable calculus, it involves 1st and 2nd order differential equations as well

Grading



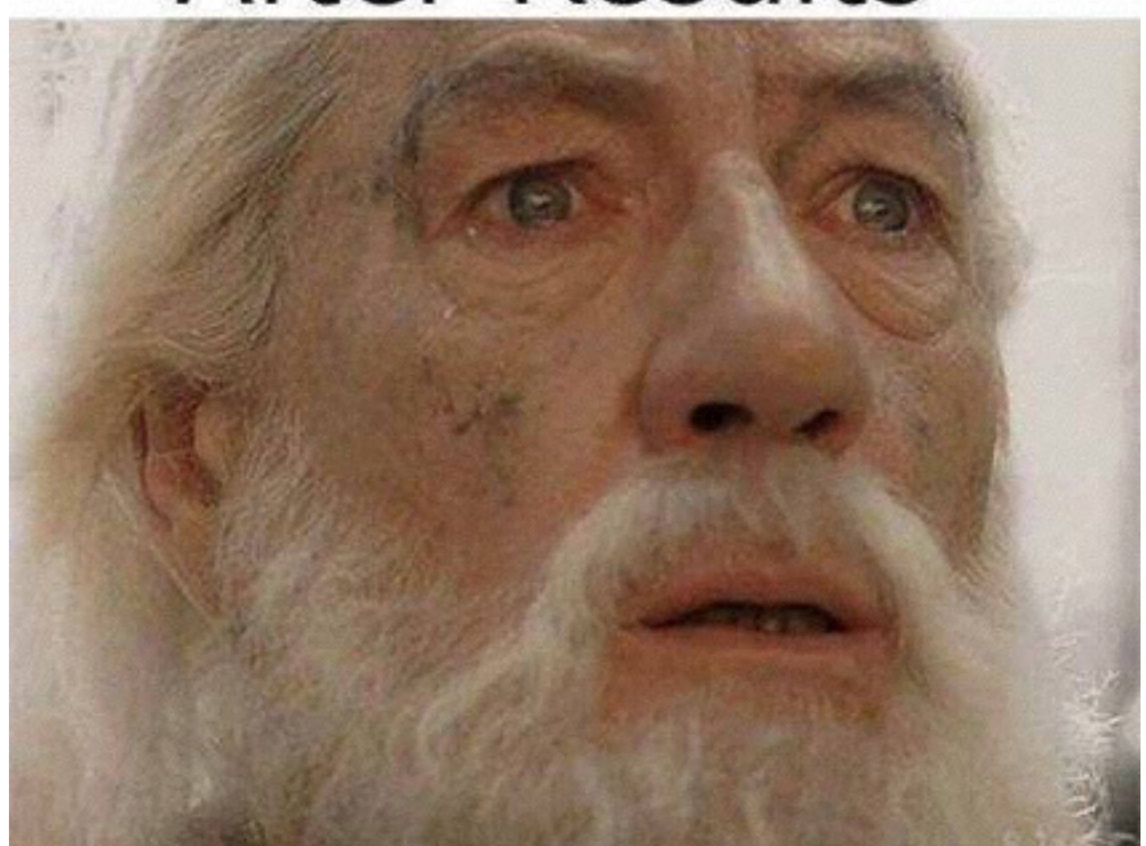
Grading

Homework: Tue & Thu, about 10 questions each submission is through bCourse.

Exams: midterm on Friday the 5th week, Final Friday of the last week

After Exams

After Results



Makeup Policy

There will be no makeup in any situations.

Instead:

- The lowest homework and quiz score will be dropped
- If you missed the midterm, 10% grade will be deducted and final will count 70%
- If you miss the final, you automatically fail the course

Incompletes can be offered, but only if a medical emergency causes you to miss the final and your performance before final is satisfactory.

Online Resources

Course website:

https://bcourses.berkeley.edu/courses/1451941

Archived exams:

https://math.berkeley.edu/courses/archives/exams

Homework template:

https://www.overleaf.com/5507617qpdcny#/17668641/

Linear algebra first look

More broadly, linear algebra is the study of transformations between linear space and linear space that transform lines to lines.

We will talk about what is a space and what is a line later.

Linear algebra first look

Linear algebra originated from study of linear systems, but now it has 3 major problems:

Solving linear systems: $Ax = b$

Solving linear least squares problem: $\min_x ||Ax - b||^2$

Solving eigenvalue problem: $Ax = \lambda x$

We will spend the first 5 weeks talking about the above three problems in order

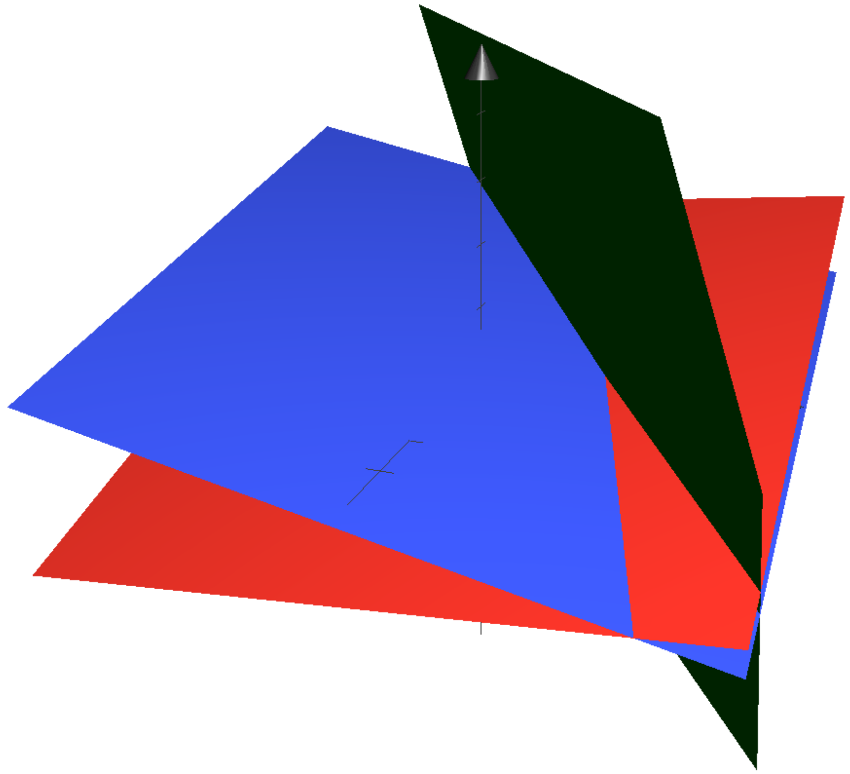
Linear Systems

$$3x + 2y + 3z = 6$$

$$x - y + z = 1$$

$$2x + 3y + 4z = 9$$

Linear Systems



Geometrically, each such equation represents a plane in three-dimensional space. And the so the solution to the equations is the intersection of the planes.

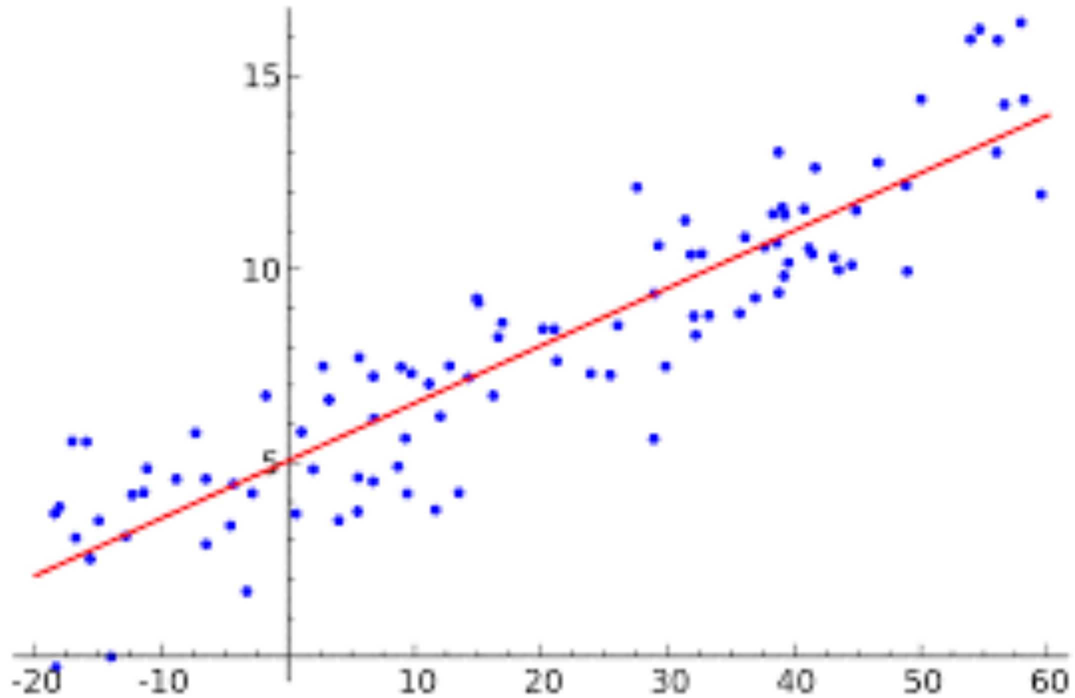
Linear least squares

Trying to find a line that fits the data points.

Want to minimize the sum of square errors of all points:

$$\{(x_i, y_i), i = 1, 2, \ldots, n\}$$

$$\min_{a,b} \sum_{i=1}^{n} (y_i - ax_i - b)^2$$



Eigenvalue problem

We will leave that till later because it requires understanding of matrices

What you will learn from me

1. Concrete algorithms to solve the above three problem.

2. Abstract reasoning of linear transformations in high dimension.

3. Real world linear algebra that happens in your Matlab/Numpy, etc.

4. Application of linear algebra in practical problems.

Linear equation

Definition: An equation in $x1, x2, \ldots, xn$ is said to be linear if it can be written in the following form:

$$a1x1 + a2x2 + \ldots + anxn = b$$

where ai are independent of xi, usually they are just numbers

Example:
1. $x + 2y + 3z = 0$ is linear in x, y, z
2. $x^2 + 2x + 1 = 0$ is not linear in x
3. $xy = 1$ is “linear in x” and “linear in y” but not “linear in x and y

Linear systems

Definition: A linear system in x1, x2, …, xn is a collection of linear equations in x1, x2, …, xn.

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &= b_m \end{aligned}$$

We say that this is a system of m equations in n variables.

Solutions

Definition: A solution to a linear system in x1, x2, …, xn is a the set of all tuples (s1, s2, …, sn) such that substituting xi by si will satisfy all the equations.

Definition: A linear system is called consistent if it has solutions, otherwise it is inconsistent.

What is the possible number of solutions?

Solutions

A linear system can have $0, 1, \infty$ solutions.

How can we determine the number of solutions?

Think about the geometric pictures, but what about higher dimension?

Solving linear system: example

$$x + 2y + 3z = 6$$
$$x - y + z = 1$$
$$2x + 3y + 4z = 9$$

$$\Rightarrow$$

$$3y + 2z = 5$$
$$x - y + z = 1$$
$$5y + 2z = 7$$

Solving linear system: example

$$
\begin{aligned}
3y + 2z &= 5 \\
x - y + z &= 1 \\
5y + 2z &= 7
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
3y + 2z &= 5 \\
x - y + z &= 1 \\
2y &= 2
\end{aligned}
$$

Summary

We added a multiple of one equation to another, which does not change the set of solutions. Then we multiplied an equation by a constant, which does not change the solution either.

Obviously, exchange two equations won't change the solutions.

These three are elementary transformations we use to transform one system to another simpler one with the same set of solutions.