MATH 1B Fall 2015
Section 5 Name: ______________________________

The best way to prepare for series tests is to do more problems :D
Two important limits to know:

$$\lim_{x\to 0}\frac{\sin x}{x}=1,\quad \lim_{n\to\infty}(1+\frac{1}{n})^n=e.$$

Warm Up Questions:

1. $\sum_{n=1}^{\infty}(-1)^n(\sqrt{n+1}-\sqrt{n})=\sum_{n=1}\frac{(-1)^n}{\sqrt{n+1}+\sqrt{n}}$.
Then use alternating series test on it.

2. $\sum_{n=1}^{\infty}\frac{(2n)!}{(n!)^2}$
Comment: Use ratio test, should be convergent.

3. $\sum_{n=1}^{\infty}\frac{1}{n+n\cos^2 n}$
Comment: $(1+\cos^2 n)\leq 2$, so $\frac{1}{n(1+\cos^2 n)}\geq\frac{1}{2n}$, which is divergent.
Intermediate questions:)

4. Determine absolute convergence and convergence: $\sum_{n=1}^{\infty}(-1)^n\frac{n}{\sqrt{n^3+2}}$.
Comment: This is an alternating series, you have to prove $\frac{n}{\sqrt{n^3+3}}$ is decreasing and goes to zero. An easy way of doing it is to convert to a function $f(x)=\frac{x}{\sqrt{x^3+3}}$ and show it is decreasing by taking derivative. Then show the limit is zero by LH.

5. Determine absolute convergence and convergence: $\sum_{n=1}^{\infty}\frac{\cos n\pi/3}{n!}$.
Comment: First notice that this is not a positive series, so DO NOT use comparison test. However, you can take the absolute value of it and $|\cos(n\pi/3)|\leq 1$, so the series is smaller than $\sum_{n=1}^{\infty}\frac{1}{n!}$, which is convergent by ratio test.

6. $\sum_{n=1}^{\infty}(\sqrt[n]{2}-1)^n$
Comment: Just use root test and notice $\lim_{n\to\infty}\sqrt[n]{2}=1$.
Ultimate challenges :D

7. $\sum_{n=1}^{\infty}(\sqrt[n]{2}-1)$ Comment: Limit compare it to $\frac{1}{n}$.

8. $\sum_{n=1}^{\infty}\frac{1}{(\ln n)^{\ln n}}$ Comment: This one is a bit complicated, please refer to what I did in section and make sure you know that $\ln\ln n>2$ for large enough n.