Linear Algebra and Differential Equations(Math 54): Lecture 8

Ruochen Liang

June 29, 2016

Recap

Last time we studied the properties of determinant and proved the important theorem about invertibility and determinant of a matrix.

Vector spaces

We have studied Rn, a collection of vectors and we have been adding two vectors, multiplying a scaler and a vector.

It is natural to abstract this idea and define a new name for it. That is vector space.

Vector spaces

Definition: A vector space over R is a set V with a a distinguished element 0 and two maps:

$$+ : V \times V \to V$$

$$\cdot : \mathbb{R} \times V \to V$$

Such that the addition has the properties of vector addition and multiplication has the properties of scaler-vector multiplication. Moreover:

$$0 \cdot v = 0, 0 + v = v$$

The vector space $\mathbb{R}^n$

The previous definition, the collection of vectors with n entries, is not a vector space. But we can make it a vector space by introducing the addition and scaler multiplication.

$$\mathbf{0} = (0, 0, \ldots, 0)$$

$$(v_1, v_2, \ldots, v_n) + (u_1, u_2, \ldots, u_n) = (v_1 + u_1, v_2 + u_2, \ldots, v_n + u_n)$$

$$c \cdot (v_1, v_2, \ldots, v_n) = (cv_1, cv_2, \ldots, cv_n)$$

Examples of vector spaces

1. The Euclidean plane in geometry;
2. The space of functions;

For example, if S = $\{1, 2, \ldots, n\}$, then all the functions from S to R will be "more or less the same" as $\mathbb{R}^n$

This is also the same relation between different lines, planes and spaces. We will define it precisely later.

Subspace

Definition: If V is a vector space, we say a subset W of V is a subspace if:

1. Zero is in W
2. The sum of any two elements in W is also in W
3. A scaler multiple of an element is still in W

Subspaces

Fact, if W is a subspace of V, itself is also a vector space.

This is because the axioms are also satisfied when restricted to the subspace W.

Examples

Is it a subspace of R3?

1. Point zero
2. A single point other than zero
3. A line
4. A union of lines through the origin
5. A circle through the origin
6. A plane through the origin
7. The intersection of planes through the origin

Examples

Is it a subspace of the function space from R to R?

1. Zero function
2. A single function other than zero
3. Functions which are zero at all integers
4. Functions $f(x) = f(x+1)$
5. Functions $f''(x) = xf(x)$
6. Functions $f(x) = x + 1$
7. Functions equal to their own square

Span

Is the span of n vectors a linear subspace? Yes.