

Before doing problems, let's do a little recap on **Integration by Parts**.

Recall the product rule : $(uv)' = u'v + v'u \Rightarrow \int v'u = uv - \int u'v$.

My secret way of memorizing the formula:

Integrate the first one while keeping the second one, then subtract the integral of the derivative of the first one and the antiderivative of the second one.

Warm Up Questions:

$$1. \quad \int x \ln x = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$$

$$2. \quad \int e^x \cos x = \frac{e^2(\cos x + \sin x)}{2} + C$$

Slightly more complicated :)

$$3. \quad \int 2x \arctan x = x^2 \arctan x - x + \arctan x + C$$

$$4. \quad \int x \sin x \cos x = \frac{1}{2}x \sin^2 x - \frac{1}{4}x + \frac{1}{8} \sin 2x + C$$

$$5. \quad \int \left(\frac{\ln x}{x}\right)^2 = -\frac{(\ln x)^2}{x} - \frac{2 \ln x}{x} - \frac{2}{x} + C$$

Ultimate challenges :D

$$6. \quad \int \sin 3x \cos 5x = -\frac{1}{16} \cos 8x + \frac{1}{4} \cos 2x + C$$

Comment: use integration by part twice, or use trig identity:

$$\sin a + b = \sin a \cos b + \sin b \cos a.$$

$$7. \quad \int \frac{x^2}{\sqrt{1-x^2}} = \frac{1}{2}(\arcsin x - x\sqrt{1-x^2}) + C$$

Comment: use integration by part knowing $((\arcsin x)' = \frac{1}{\sqrt{1-x^2}})$ or use trig substitution. Either way you will end up integrating $\sin^2 x$