MATH 54 Spring 2016
Section 3 Name: ______________________

Linear transformations are functions that preserve linearity:

$$T(ax+by) = aT(x) + bT(y).$$

The space of linear transformations $T : \mathbb{R}^m \to \mathbb{R}^n$ is the same as the space of $m \times n$ matrices. And their relation is defined by standard matrices.

You can define the standard matrix of a linear transformation T by:

$$A = [T(e_1), T(e_2), \dots, T(e_n)],$$

where $T(e_i)$ is the image of the ith column of the identity under T.

1. Try to explain to your group mates why the standard matrix is defined as above.

2. Let $A = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$, $u = \begin{bmatrix} 3 \\ 6 \\ -9 \end{bmatrix}$, $v = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$. Define $T : \mathbb{R}^3 \to \mathbb{R}^3$ by $T(x) = Ax$, find $T(u)$ and $T(v)$.

3. Find the vectors that are mapped to zero by the transformation $T(x) = Ax$:

$$A = \begin{bmatrix} 3 & 2 & 10 & -6 \\ 1 & 0 & 2 & -4 \\ 0 & 1 & 2 & 3 \\ 1 & 4 & 10 & 8 \end{bmatrix}$$

4. Let $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $v_1 = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$, and $v_2 = \begin{bmatrix} 7 \\ -2 \end{bmatrix}$, and let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that maps x into $x_1v_1 + x_2v_2$. Find the matrix A for that transformation.

5. Find the standard matrix of $T : \mathbb{R}^2 \to \mathbb{R}^2$ that first reflects points through the horizontal x_1-axis and then rotates points $-\pi/2$ radians.

6. Find the standard matrix of $T : \mathbb{R}^2 \to \mathbb{R}^2$ that first does a horizontal shear transforming e_2 to $e_2 + 2e_1$(leaving e_1 unchanged and then reflects points through the line $x_2 = -x_1$.

Here begins the challenge:

7. Let P_n be the set of polynomials of order at most n, consider the differential operator $T = \frac{d}{dx}$ on P_n.

- Show that T is a linear transformation;
- What is the standard matrix for d?