MATH 1B Fall 2015
Section 7 Name: ________________________________

Power series is a series of the following form: $\sum_{n=1}^{\infty} a_n(x-a)^n$.

For power series, we only care about absolute convergence. Hence the values of x for it to converge should be a certain interval centered at a. Also notice that if $x = a$, the series is convergent.

Warm Up Questions (Find radius and interval of convergence):

1. $\sum_{n=1}^{\infty} \frac{(-1)^n n}{4^n}(x+3)^n$

Comment: Use root test on the absolute value, interval of convergence is $(-7, 1)$.

2. $\sum_{n=1}^{\infty} \frac{(2^n)!}{n}(4x-8)^n$

Comment: Use ratio test and notice that the limit of the ratio is infinity if $4x - 8 \neq 0$. So the radius of convergence is 0 and the only point that this is convergent is $x = 2$.

3. $\sum_{n=1}^{\infty} n!(2x+1)^n$

Comment: Similar to the above one, convergent only when $x = -1/2$.
Intermediate questions:) (Still radius and interval of convergence)

4. $\sum_{n=1}^{\infty} \frac{(x-6)^n}{n^n}$.

Comment: Use root test and the limit of the root is always zero regardless of x, so radius is ∞.

5. $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!}x^{2n+1}$.

Comment: You can use the ratio test for this, but latter you will know that this is just the series expansion for $\sin x$ so the radius of convergence is ∞ and the interval is $\mathbb{R}$.

6. $\sum_{n=1}^{\infty} \frac{x^{2n}}{(-3)^n}$.

Comment: Use root test on the absolute value, and you get $x^2 < 3$. So the radius of convergence is $\sqrt{3}$, and notice that the series is divergent at the boundaries so the interval of convergence is $(-\sqrt{3}, \sqrt{3})$.

7. $\sum_{n=1}^{\infty} (2^n + 3^n)x^n$.

Comment: Use ratio test and divide both the top and the bottom of the ratio by 3^{n+1} you can get the limit of the ratio to be $3x$, so the radius of convergence is $1/3$, but the series is divergent at both boundaries, so the interval of convergence is $(-1/3, 1/3)$.

8. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{x^{2n}}{2^{2n}(n!)^2}$.

Comment: a small trick is to convert everything to power n, i.e $x^{2n} = (x^2)^n, 2^{2n} = 4^n$. Then you can use ratio test and find that the radius of convergence is actually infinity.