MATH 54 Spring 2016
Section 2 Name: ______________________________

The most important concept this week is "Linear Independence".

1. You can use matrix row reduction to show a set of vectors are linearly independent'
2. A set of linearly independent vectors can span a whole space;
3. Showing the columns of A are linearly dependent is the same as solving $Ax = 0$.

1.

- Show that the vectors $v_1 = [1, 0, 2], v_2 = [0, 1, 2], v_3 = [0, 3, 0]$ are linearly independent'
- Describe the span of $W = \{v_1, v_2, v_3\}$;
- Can you find another three vectors that have the same span as W, prove it.

2.

- Show that any two vectors chosen from a linearly independent set are linearly independent;
- Show that a set which contains two linearly dependent vectors must be a linearly dependent set.

Ok, now lets turn to some more concrete questions:

3.

Find a solution to $Ax = 0$ without performing row reductions:

$$A = \begin{pmatrix} 2 & 3 & 5 \\ -5 & 1 & -4 \\ -3 & -1 & -4 \\ 1 & 0 & 1 \end{pmatrix}, \quad A = \begin{pmatrix} 4 & 3 & -5 \\ -2 & -2 & 4 \\ -2 & -3 & 7 \end{pmatrix}$$

4.

Find the value h for which the following set of vectors are linearly *dependent*:

$$\begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix}, \quad \begin{bmatrix} -2 \\ -9 \\ 6 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ h \\ -9 \end{bmatrix}$$

5.

Find a parametric equation of the line M through $\mathbf{p}$ and $\mathbf{q}$:

- $p = [3, -3], q = [4, 1]$;
- $p = [-3, 2], q = [0, -3]$.

Here begins the challenges :D

6.

Suppose S and T are two linearly independent sets of vectors and

$$Span(S) \cap Span(T) = \emptyset.$$

Show that $S \cup T$ is a set of linearly independent vectors.