MATH 1B Fall 2015
Section 9 Name: ______________________________

Power series is a series of the following form: $\sum_{n=1}^{\infty} a_n(x-a)^n$.

For Taylor series and Maclaurin series, you should be aware of the following issues:

1. The power series expansion of a function is unique, so if there are two, they must be equal;
2. Taylor series has a center and this center can be tricky (could be a letter);
3. Memorize Taylor series remainder term $R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}x^{n+1}$;
4. Use series expansion to find limits;
5. Use series expansion to find some hard sums;
6. Memorize the table of some common Taylor series expansions.

Warm Up Questions :

1. If p is an $n-$th degree polynomial, show that

$$p(x+1) = \sum_{i=0}^{n} \frac{p^{(i)}(x)}{i!}$$

Comment: This is the taylor series expansion of $p(x+1)$ at $a = x$. Try to understand the Taylor series expansion by this example.

2. Find the Maclaurin series of $f(x) = \sin(\pi x)$
Comment: This is a canonical expansion problem, just plug in πx for x in the expansion of $\sin x$.

Intermediate questions:)

3. Find the sum of $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n!)}$.
Comment: This is $\cos \pi/6$.

4. Find the sum of $\frac{1}{1\cdot 2} - \frac{1}{3\cdot 2^3} + \frac{1}{5\cdot 2^5} - \frac{1}{7\cdot 2^7} + \ldots$
Comment: This is $\arctan 1/2$.

5. If $f(x) = (1+x^3)^{30}$, what is $f^{(58)}(0)$?Comment: This is zero like I did in class.

6. Use series to find the limit:

$$\lim_{x\to 0} \frac{x - \ln(1+x)}{x^2}.$$

Comment: The version you have has a typo, the limit is evaluated when x goes to zero. You should expand $\ln(1+x)$ as a Taylor series at 0 then divide the top by x^2 and plug in zero. Then you can use LH's rule to check your answer.