

After some practice, you might notice that the basic idea of integration by part is to “make a part of the integral go away while maintaining the easy part”. Some general rules for IP is that

Functions that can go away: $\ln x, \arctan x, x, x^2, \dots$;

Functions that will not go away: $e^2, \sin x, \cos x, \dots$

Warm Up Questions:

$$1. \quad \int_{-\pi}^{\pi} x \sin x \, dx =$$

$$2. \quad \int x^3 \sin x^2 \, dx =$$

Combination of substitution and IP:)

$$3. \quad \int \cos \sqrt{x} \, dx =$$

$$4. \quad \int \sin(\ln x) \, dx =$$

$$5. \quad \int_1^{\sqrt{3}} \arctan(1/x) \, dx =$$

Derivation questions, try to prove the following (challenging) :D

$$6. \quad \int (\ln x)^n \, dx = x(\ln x)^n - n \int (\ln x)^{n-1} \, dx$$

$$7. \quad \int \sec^n x \, dx = \frac{\tan x \sec^{n-1} x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx$$

$$8. \quad \int_0^{\frac{\pi}{2}} \sin^n x \, dx = \frac{n-1}{n} \int_0^{\frac{\pi}{2}} \sin^{n-2} x \, dx.$$

Then use the above formula to prove: $\int_0^{\frac{\pi}{2}} \sin^{2n+1} x \, dx = \frac{2 \cdot 4 \cdot 6 \cdot \dots \cdot 2n}{3 \cdot 5 \cdot 7 \cdot \dots \cdot (2n+1)}$