MATH 1B Fall 2015
Section 3 Name: ______________________

After some practice, you might notice that the basic idea of integration by part is to "make a part of the integral go away while maintaining the easy part". Some general rules for IP is that
Functions that can go away: $\ln x, \arctan x, x, x^2, \ldots;$
Functions that will not go away: $e^2, \sin x, \cos x, \ldots$.

Warm Up Questions:

1. $\displaystyle\int_{-\pi}^{\pi} x \sin x \, dx = 2\pi$

2. $\displaystyle\int x^3 \sin x^2 \, dx = \frac{1}{2}(-x^2 \cos x^2 + \sin x^2) + C$

Combination of substitution and IP:)

3. $\displaystyle\int \cos\sqrt{x} \, dx = 2(\sqrt{x}\sin\sqrt{x} - \cos\sqrt{x}) + C$

Comment: Use $t = \sqrt{x}$, so $dx = 2tdt$.

4. $\displaystyle\int \sin(\ln x) \, dx = \frac{x \sin \ln x - x \cos \ln x}{2} + C$

Comment: Substitution $t = \ln x$ or integration by part directly.

5. $\displaystyle\int_1^{\sqrt{3}} \arctan(1/x) \, dx = \int_{\frac{1}{\sqrt{3}}}^{1} 1/t^2 \arctan t \, dt = (-\frac{1}{t}\arctan t + \ln t - \frac{1}{2}\ln t^2 + 1)|_{\frac{1}{\sqrt{3}}}^{1}.$

Comment: $t = 1/x$, and notice that you have to use partial fraction: $\frac{1}{t(t^2+1)} = \frac{1}{t} - \frac{t}{t^2+1}$.

Derivation questions, try to prove the following (challenging) :D

6. $\displaystyle\int (\ln x)^n \, dx = x(\ln x)^n - n\int (\ln x)^{n-1} \, dx$

Comment: Just integration by part $dv = 1, u = \ln x$.

7. $\displaystyle\int \sec^n x \, dx = \frac{\tan x \sec^{n-1} x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x \, dx$

Comment: Integration by part, but notice that $dv = \sec^2 x$, so $v = \tan x$.

8. $\displaystyle\int_0^{\frac{\pi}{2}} \sin^n x \, dx = \frac{n-1}{n}\int_0^{\frac{\pi}{2}} \sin^{n-2} x \, dx.$

Then use the above formula to prove: $\displaystyle\int_0^{\frac{\pi}{2}} \sin^{2n+1} x \, dx = \frac{2 \cdot 4 \cdot 6 \cdot \ldots \cdot 2n}{3 \cdot 5 \cdot 7 \cdot \ldots \cdot (2n+1)}$

Comment: Integration by part for the first part. Then use **Mathematical induction** for the second part.