How to solve computational questions in exams

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- A little about myself: I have been teaching calculus and linear algebra for 2.5 years, both as a teaching assistant and an instructor. As this career is hopefully coming to an end, I would like to share some of my suggestions to help students conquer computational questions in math exams.
- Who should read this: Students who are going to take any course that involves computations and of course other interested people.

1 Overview

Computational mistake is the second largest contributor to point losses in exams. I have heard a lot of people complaining about making "silly" computational mistakes as well as requests to get regrades or "more partial credits".

I would like to first share my principals as a grader. Usually, the first and most important part of a computational question is the answer. This is obvious even when you leave university: *people care about whether your thing "works", then probably how much effort you have put into it.*

When I was grading, if the answer is wrong and not because you did $1 * 2 = 3$ in the last step, I would take at least half of the points off, then check your overall solution. And very often, one would get only a small fraction of the question. On the other hand, if your answer is right, I would very quickly skim through your solution and give you all the points.

All that said, the point I want to make is the emphasis on correctness rather than partial credits. Of course, the purpose of this article is to help you avoid making mistakes and get the correct answer.

2 Methods

In this section I am going to share my 5 steps to solve mathematical questions, and you should try following them when you are doing homework, sample exams, or just practicing. You will establish a systematic way to solve such questions after enough practice.

1. Translate the question into a mathematical form. In reality, this is probably the most important step of problem solving, and the same goes for exams. This involves formulating a circuit question into a differential equation or a fluid statics question into an integration.
2. List all the algorithms you can think of to solve this question. This is the main part where your knowledge comes in and also the step where you know to give up the question, if you have no algorithms.
3. Estimate your algorithm complexity and choose the most efficient one, then predict possible mistakes. This is a step that a lot people have overlooked. Blindly proceed with "a" method you know can cause to a dead end due to the complexity of it, which will waste your time, and cause anxiety in exams. Also, predicting mistakes will make you more careful in certain steps and avoid mistakes.
4. Solve the question using the algorithm you pick and be careful with possible mistakes.

5. Double check your answer. This is another important step ignored by a lot people. However, this is an easy and very useful step. Most computational questions are similar to combinatorial search, meaning that they are in **NP**, that is, the solution if given can be efficiently verified.

3 Example

Now in this section, I will illustrate the above ideas using an concrete example. For this particular example, you are supposed to have some basic knowledge about differential equations.

Example .1. In a circuit with a capacitor and a resistor, under constant voltage, the following equation holds by Kirchhoff's voltage law:

$$Ri + \frac{1}{C}\int i\,dt = V,$$

where R is the resistance, C is the capacity and i being current.

Now I will solve this problem, following the above five steps. The first step will translate this real-world problem into a math one. Actually, it is partially done as it is already in an integral equation form. However, there is no good way to solve an integral equation, so we need to transform it into a differential equation, by differentiating with respect to t.

$$R\frac{di}{dt} + \frac{1}{C}i = 0.$$

The second step is to come up with a list of algorithms to solve this differential equation. It can be first noted that this is a first order constant coefficient ODE, so we can use integrating factors. In addition, this is a separable equation whereas the is can be moved to one end of the equation and the constants to the other side.

For illustration purpose I will demonstrate both methods here:

$$e^{1/RCt}(i' + 1/RCi) = 0 \Rightarrow (e^{1/RCt}i)' = 0 \Rightarrow i = C_0e^{-t/RC},$$

where C_0 is a constant that can be determined by the previous integral equation. Another way is to separate the variables:

$$i'/i = -1/RC \Rightarrow (ln(i))' = -1/RC \Rightarrow i = C_0e^{-1/RCt},$$

where C_0 is a constant that can be determined by the previous integral equation.

Although both algorithms are fairly efficient, I would personally prefer the second one because the computation of the "integrating factor"($e^{1/RC}$) in the first method needs extra computation and multiplying it does too. The possible mistake for the second method is the integration in the last step where $\int i'/i = |i| = i$ because the current is always positive.

The last step is to double check the correctness of your answer. In our example, this is an easy step because you only have to differentiate your solution and see if it satisfies the original equation.

4 Summary

I hope that this article can help you avoid making mistakes and build your own system of problem solving! May everyone succeed in exams.