MATH231BR HOMEWORK 1 (DUE THURSDAY, 2/5/26)

Exercise 1. Let $\pi\colon S \to X$ be any sheaf and $z\colon X \to S$ the zero section. Then $\mathrm{im}(z) \subseteq S$ is open.

Exercise 2. Show that every morphism between sheaves of abelian groups is a local homeomorphism.

Exercise 3. Let $f\colon S \to S'$ be a morphism of sheaves over X. Verify if $S_x \to S'_x$ is an isomorphism of abelian groups for each $x \in X$ then f is a homeomorphism (i.e. an isomorphism of sheaves).

Exercise 4. Let $f\colon F \to G$ be a morphism of presheaves on X satisfying locality and gluing. Verify if the induced map on stalks $f_x\colon F_x \to G_x$ is an isomorphism for each $x \in X$ then f is a natural isomorphism.