Lecture 9

Fitting and Matching

- Problem formulation
- Least square methods
- RANSAC
- Hough transforms
- Multi-model fitting
- Fitting helps matching!

Reading:
[HZ] Chapter: 4 "Estimation – 2D projective transformation"
Chapter: 11 "Computation of the fundamental matrix F"
[FP] Chapter:10 "Grouping and model fitting"

Some slides of this lectures are courtesy of profs. S. Lazebnik & K. Grauman

Fitting

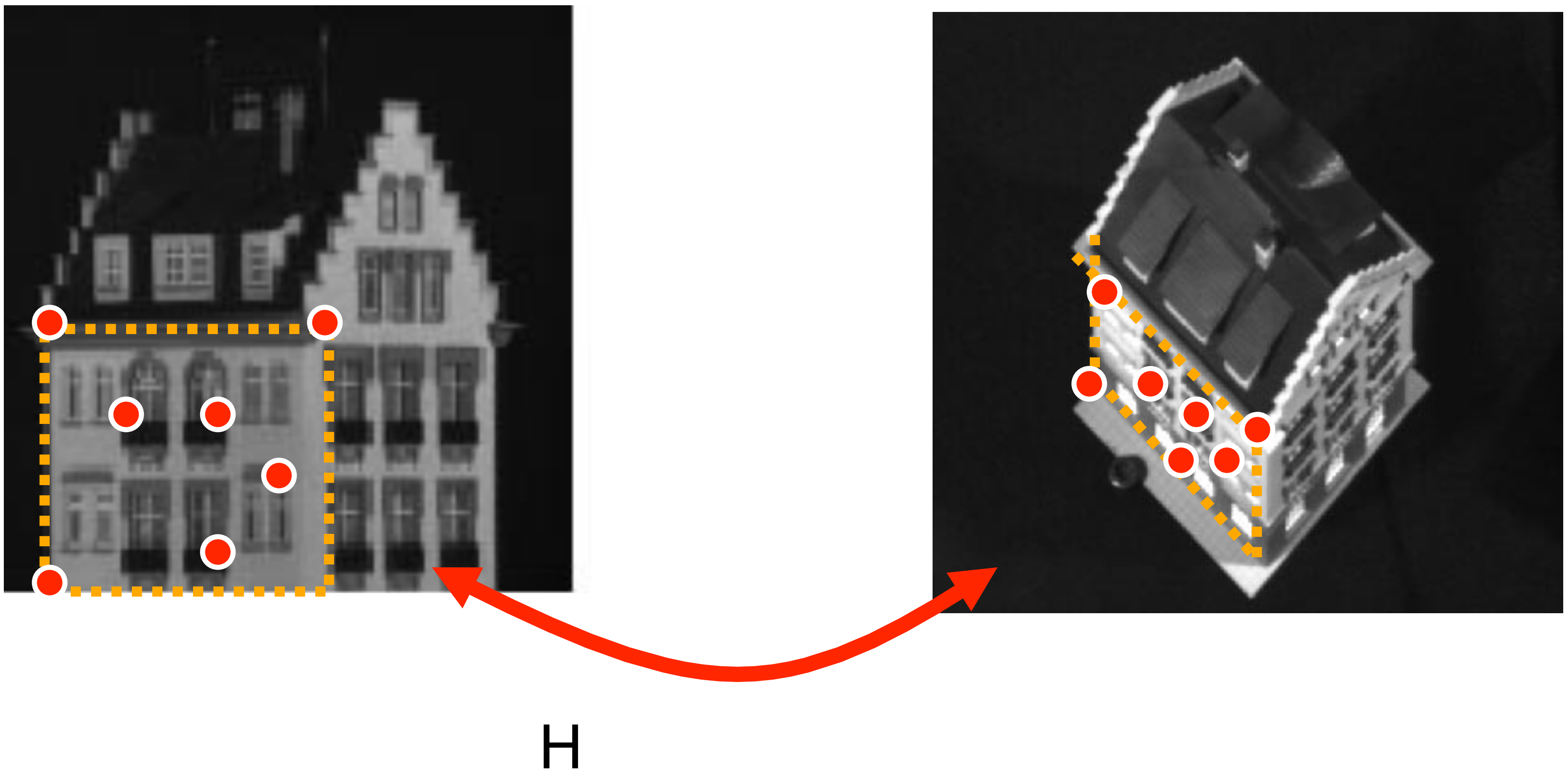
Goals:

- Choose a parametric model to fit a certain quantity from data
- Estimate model parameters

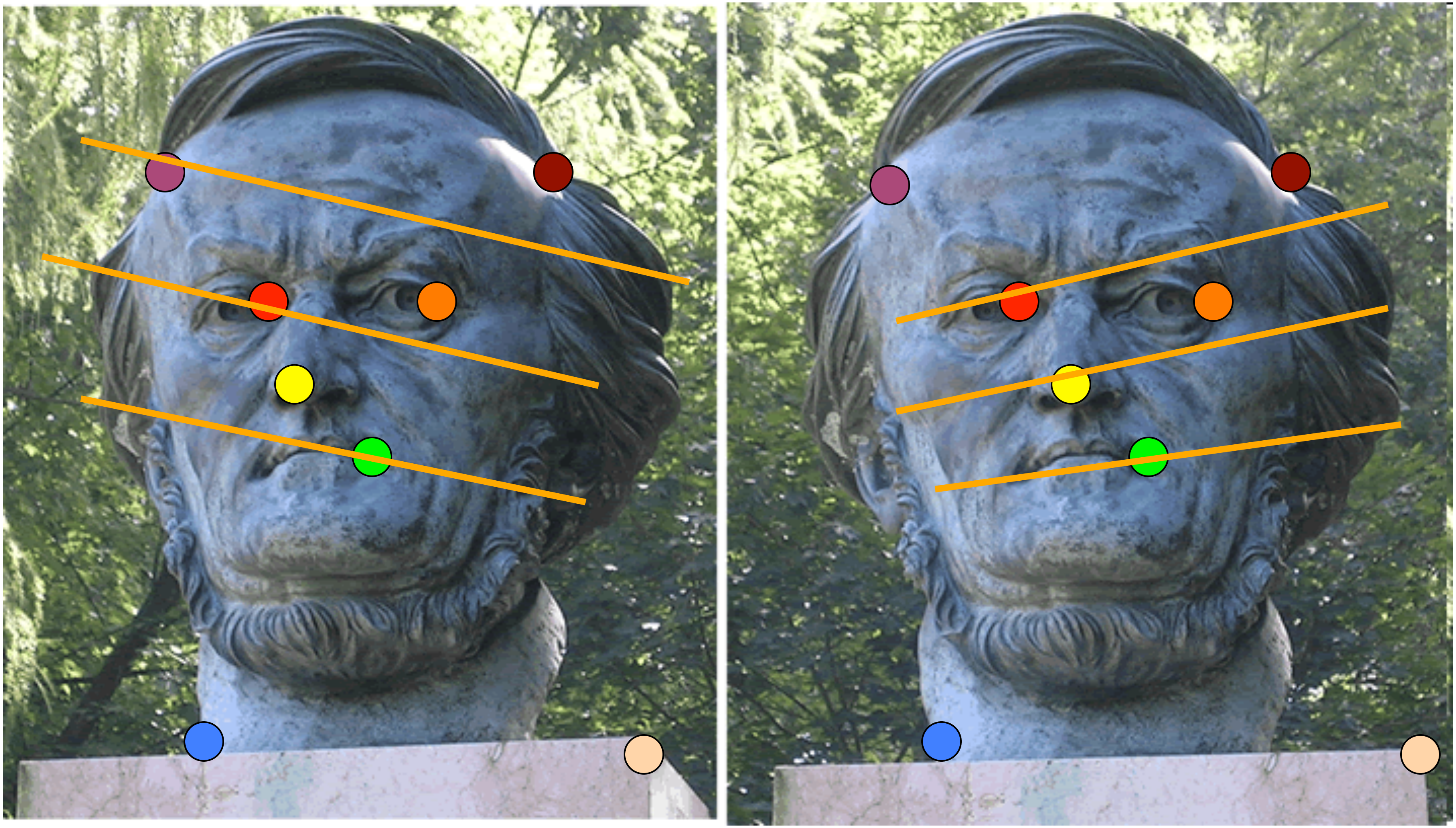
- Lines
- Curves
- Homographic transformations
- Fundamental matrices
- Shape models

Example: fitting lines
(for computing vanishing points)

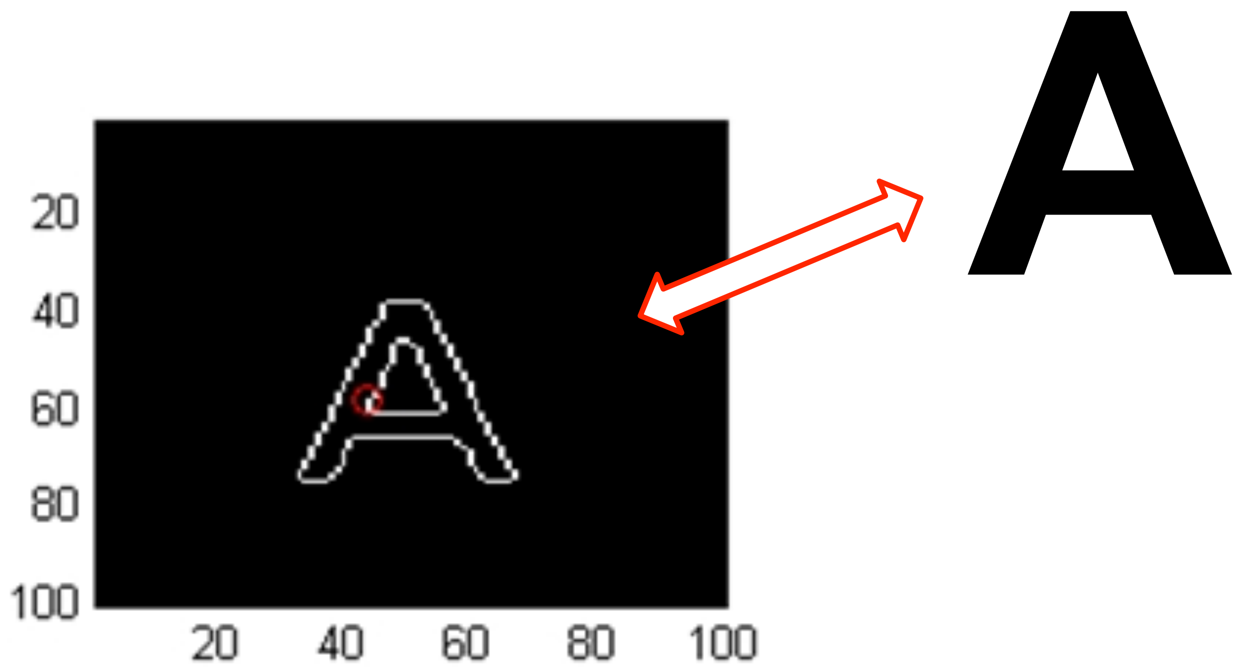
Example: Estimating an homographic transformation



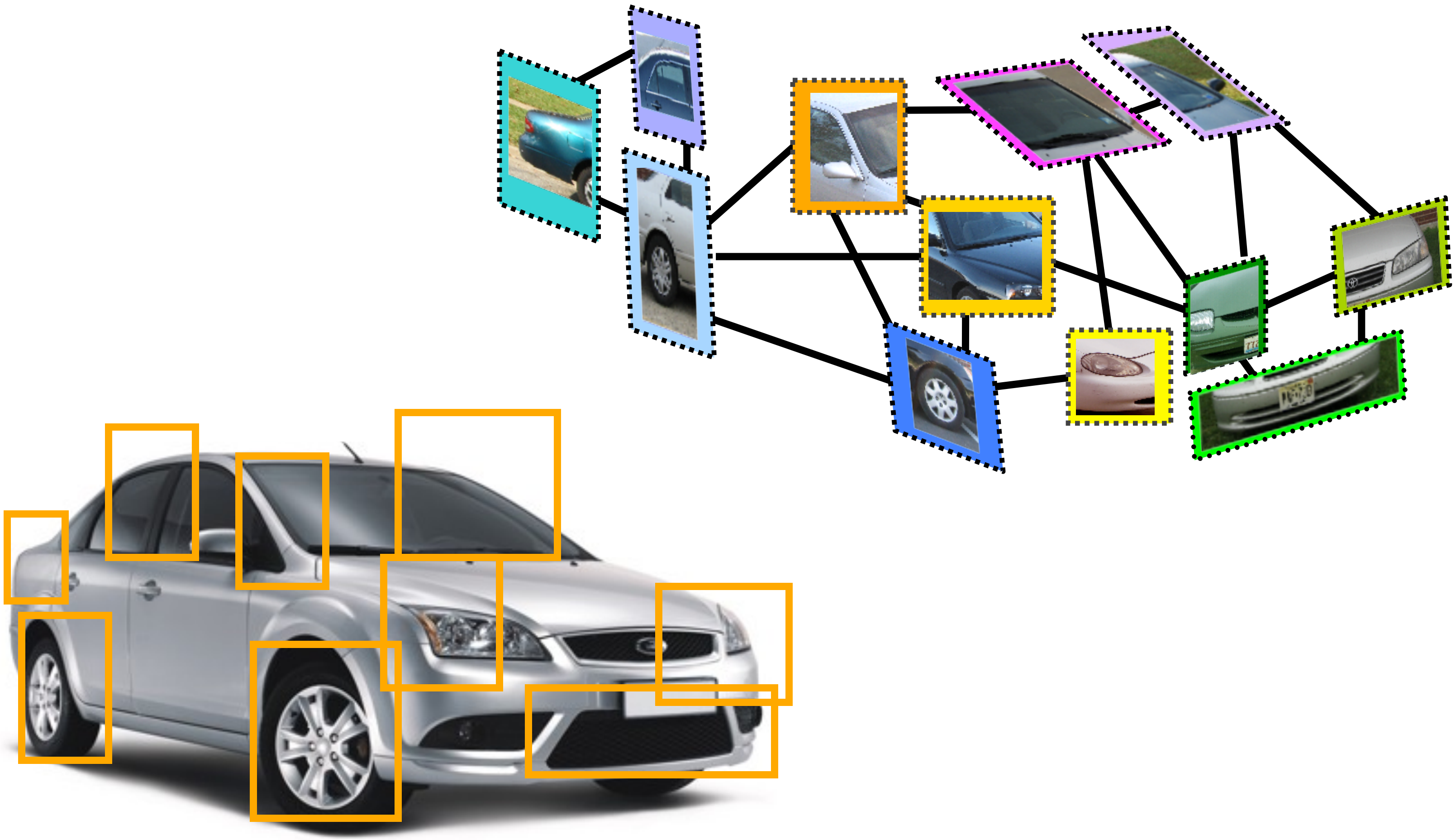
Example: Estimating F



Example: fitting a 2D shape template



Example: fitting a 3D object model



Fitting, matching and recognition are interconnected problems

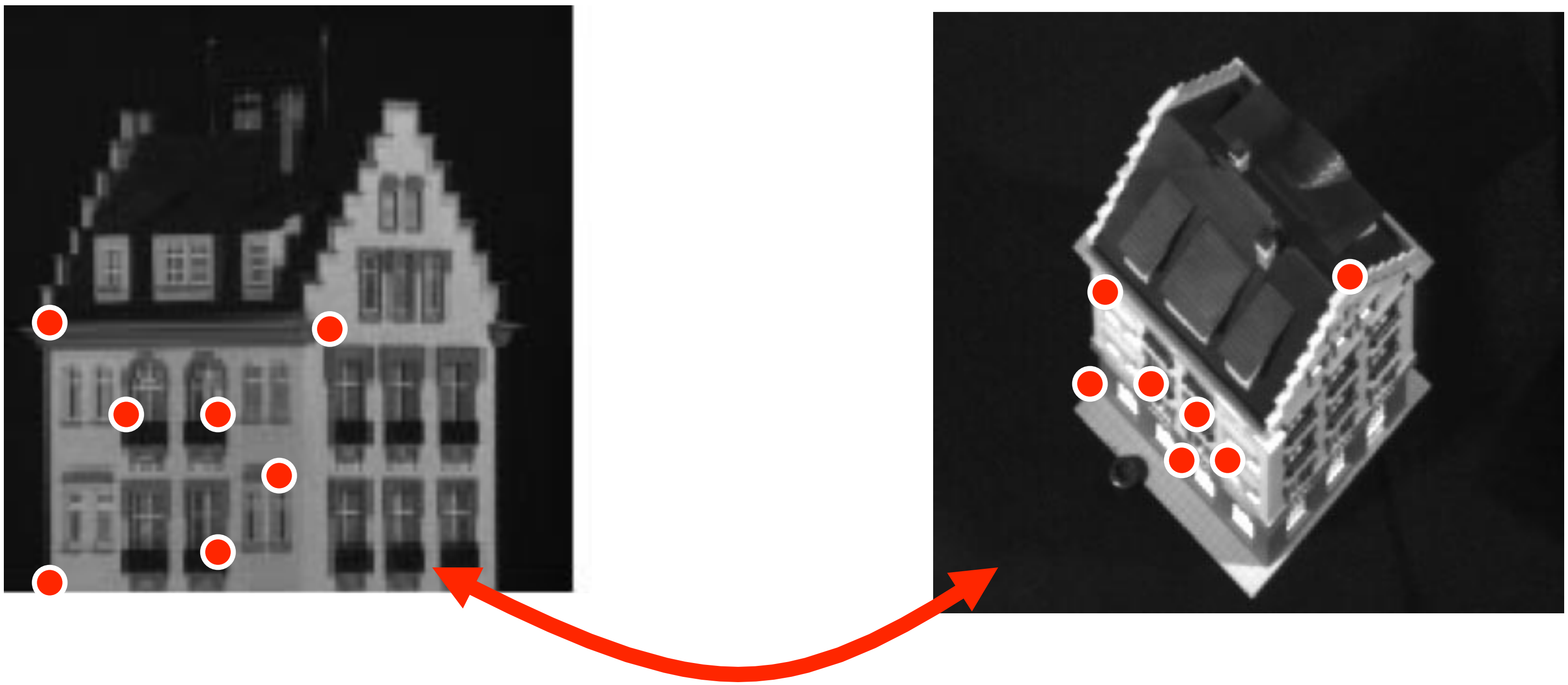
Fitting

Critical issues:

- noisy data
- outliers
- missing data
- Intra-class variation

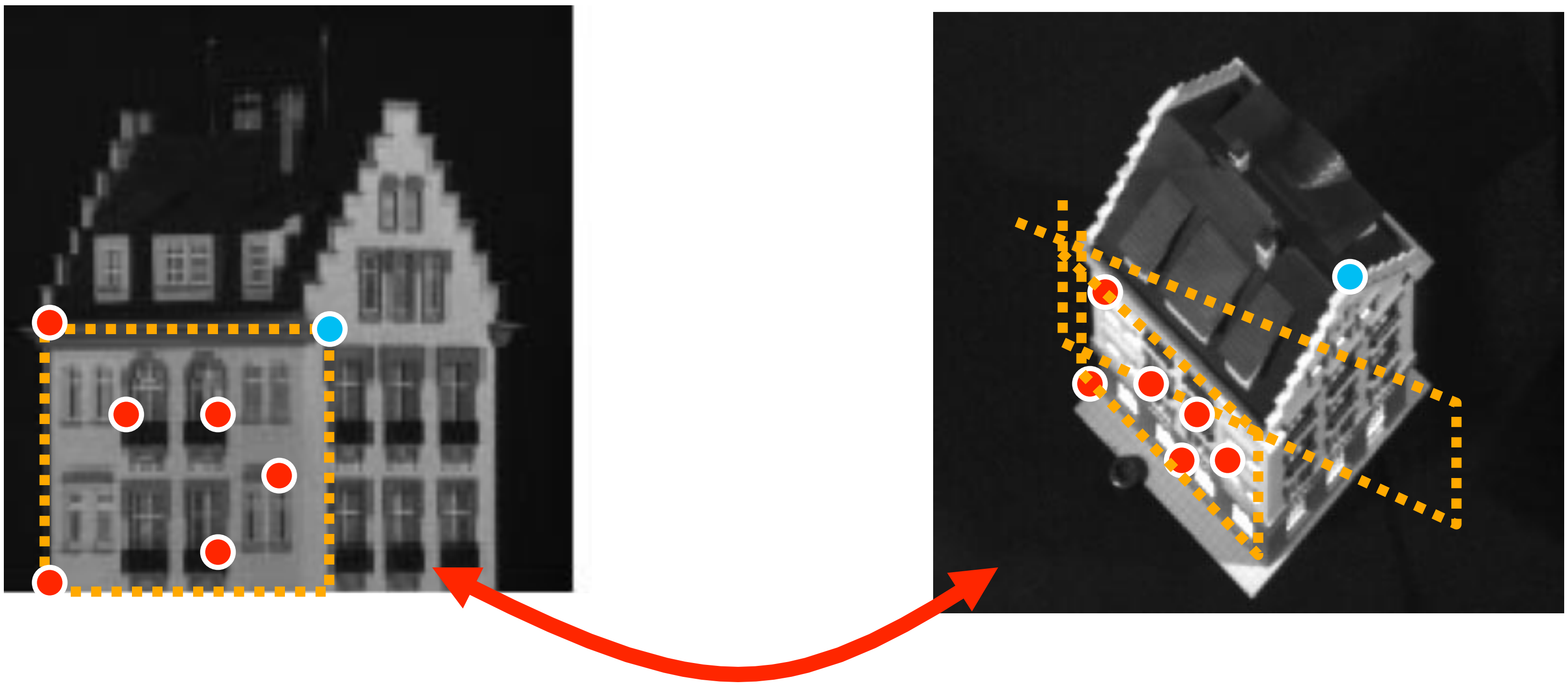
Critical issues: noisy data

Critical issues: outliers



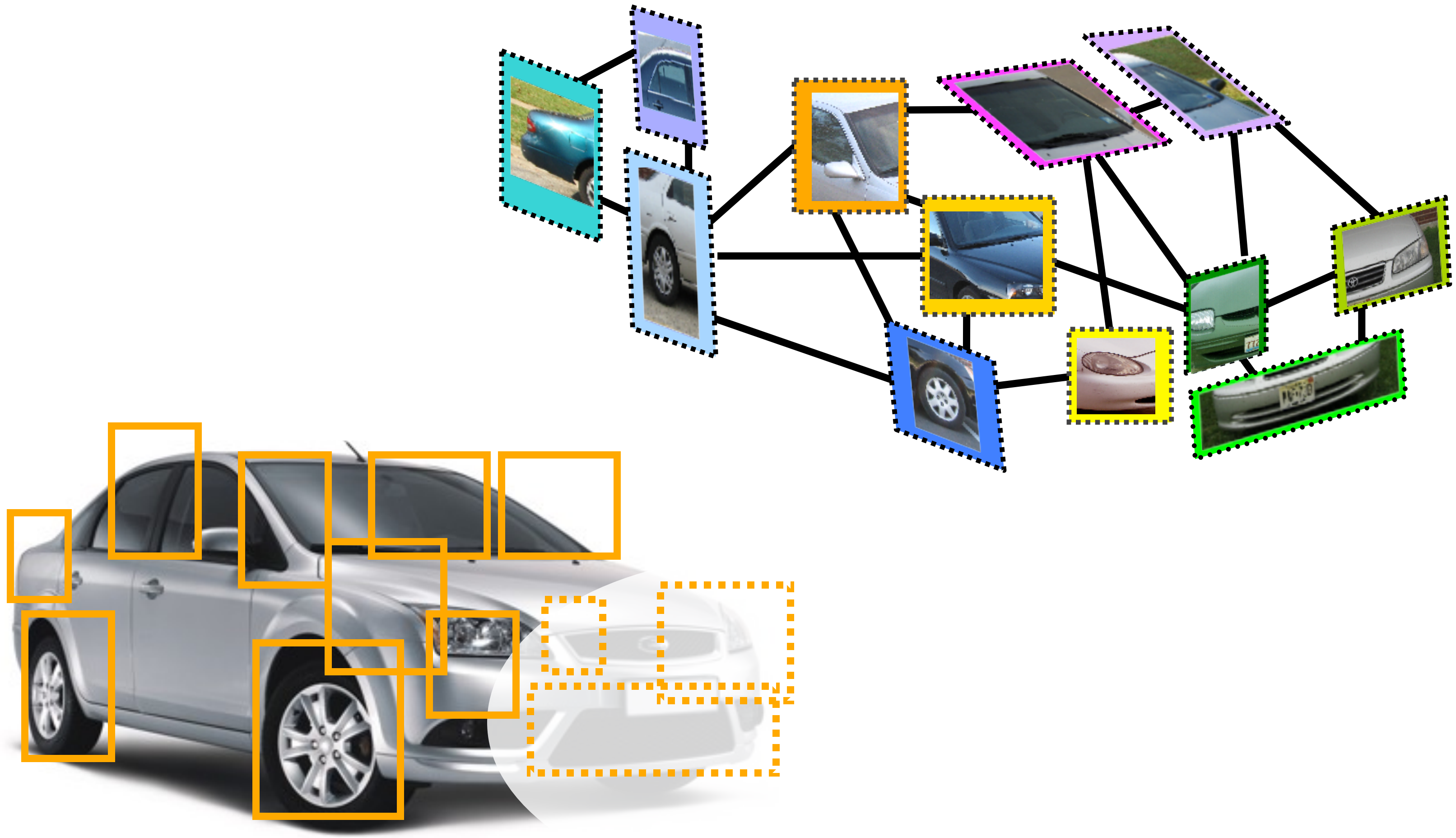
What’s H?

Critical issues: outliers

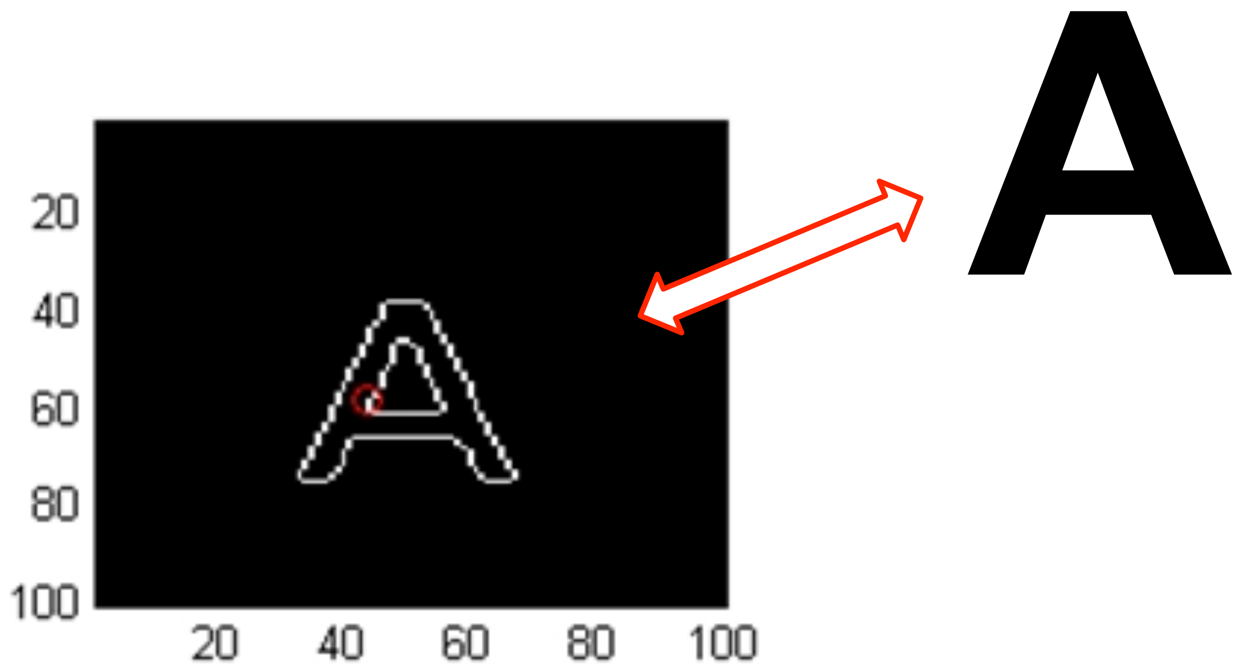


What's H? ☹

Critical issues: missing data (occlusions)



Critical issues: noisy data (intra-class variability)



Fitting

Goal: Choose a parametric model to fit a certain quantity from data

Techniques:

- Least square methods
- RANSAC
- Hough transform
- EM (Expectation Maximization) [not covered]

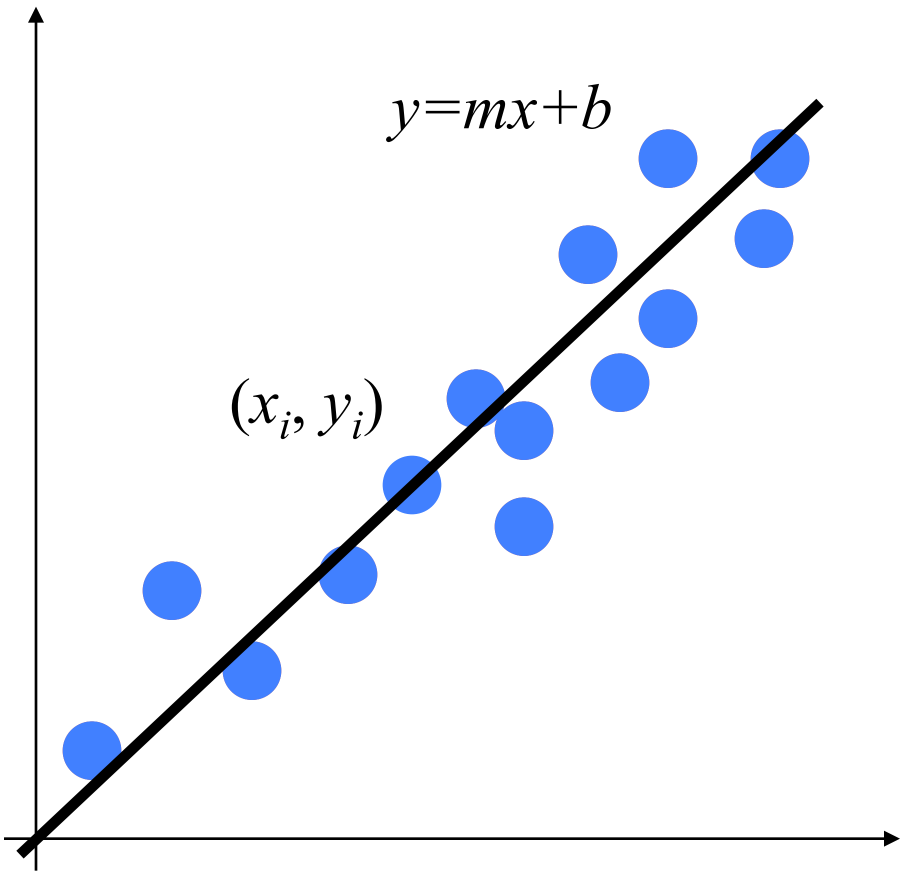
Least squares methods
- fitting a line -

- Data: $(x_1, y_1), \ldots, (x_n, y_n)$
- Model of the line:

$$y_i - m x_i - b = 0 \qquad \text{[Eq. 1]}$$

- Parameters: *m, b*
- Find (m, b) to minimize fitting error (**residual**):

$$\boxed{E = \sum_{i=1}^{n} (y_i - mx_i - b)^2} \qquad \text{[Eq. 2]}$$



Least squares methods
- fitting a line -

$$\boxed{E = \sum_{i=1}^{n} (y_i - m x_i - b)^2}$$ [Eq. 2]

$$E = \sum_{i=1}^{n} \left(y_i - \begin{bmatrix} x_i & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} \right)^2 = \left\| \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} - \begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} \right\|^2 = \| Y - Xh \|^2$$ [Eq. 3]

$$= (Y - Xh)^T (Y - Xh) = Y^T Y - 2(Xh)^T Y + (Xh)^T (Xh)$$ [Eq. 4]

Find $h = [m, b]^T$ that minimizes E

$$\frac{dE}{dh} = -2X^T Y + 2X^T X h = 0$$ [Eq. 5]

$$X^T X h = X^T Y$$ [Eq. 7]

Normal equation

$$\boxed{h = \left(X^T X \right)^{-1} X^T Y}$$ [Eq. 6]

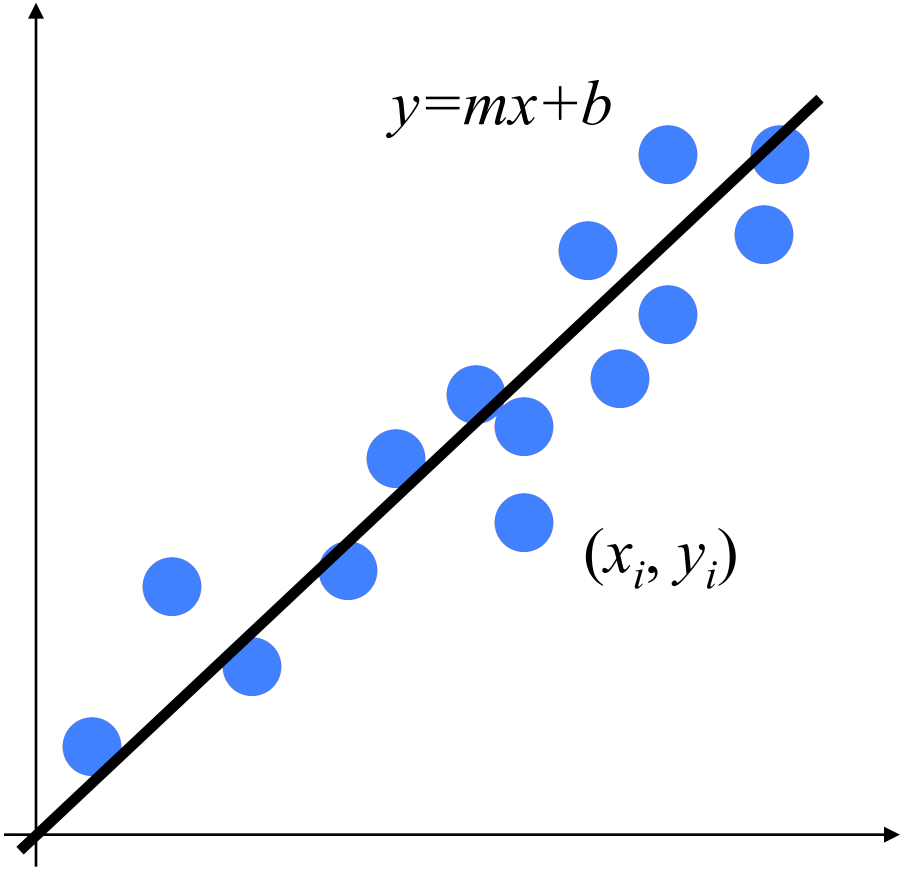
Least squares methods

- fitting a line -

$$E = \sum_{i=1}^{n} (y_i - mx_i - b)^2$$

$$\boxed{h = \left(X^T X\right)^{-1} X^T Y} \qquad h = \begin{bmatrix} m \\ b \end{bmatrix}$$

[Eq. 6]



Issues?

- Fails completely for vertical lines

Least squares methods

- fitting a line -

- Distance between point $(x_i, y_i, 1)$ and line (a, b, d)

- Find (a, b, d) to minimize the sum of squared perpendicular distances

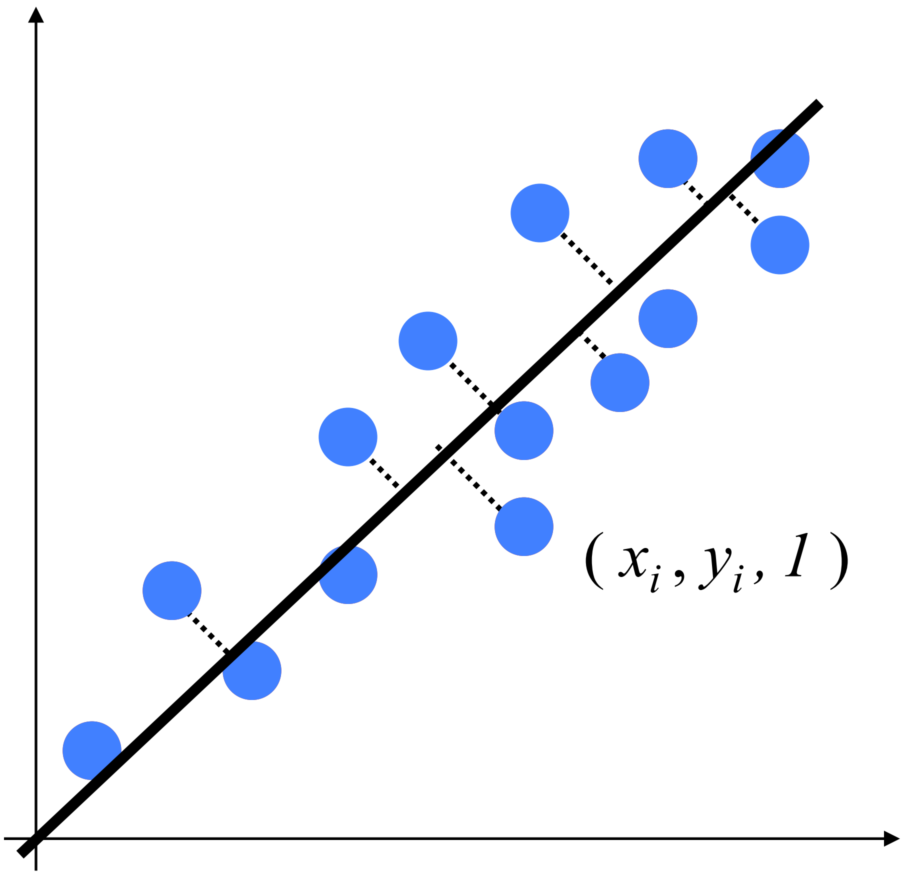
[Eq. 8]

$$\boxed{E = \sum_{i=1}^{n} (a x_i + b y_i + d)^2}$$

$$\boxed{A}\,\boxed{h} = 0$$

[Eq. 9]

data model parameters



Least squares methods

- fitting a line -

$$A\ h = 0 \qquad \text{A is rank deficient}$$

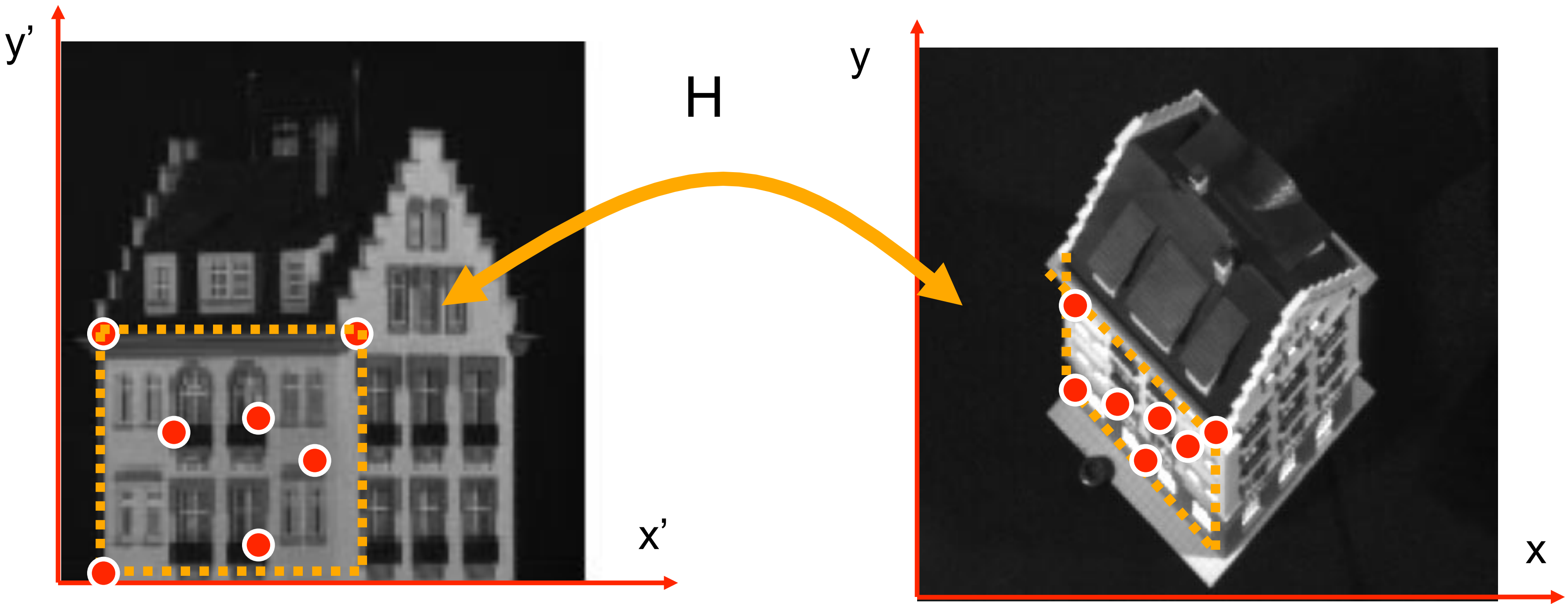
$$\text{Minimize} \ \| A\ h \| \quad \text{subject to} \quad \| h \| = 1$$

$$A = UDV^T$$

$$h = \text{last column of } V$$

See [HZ], sec. A5.3 - page 593

Least squares methods
- fitting an homography -

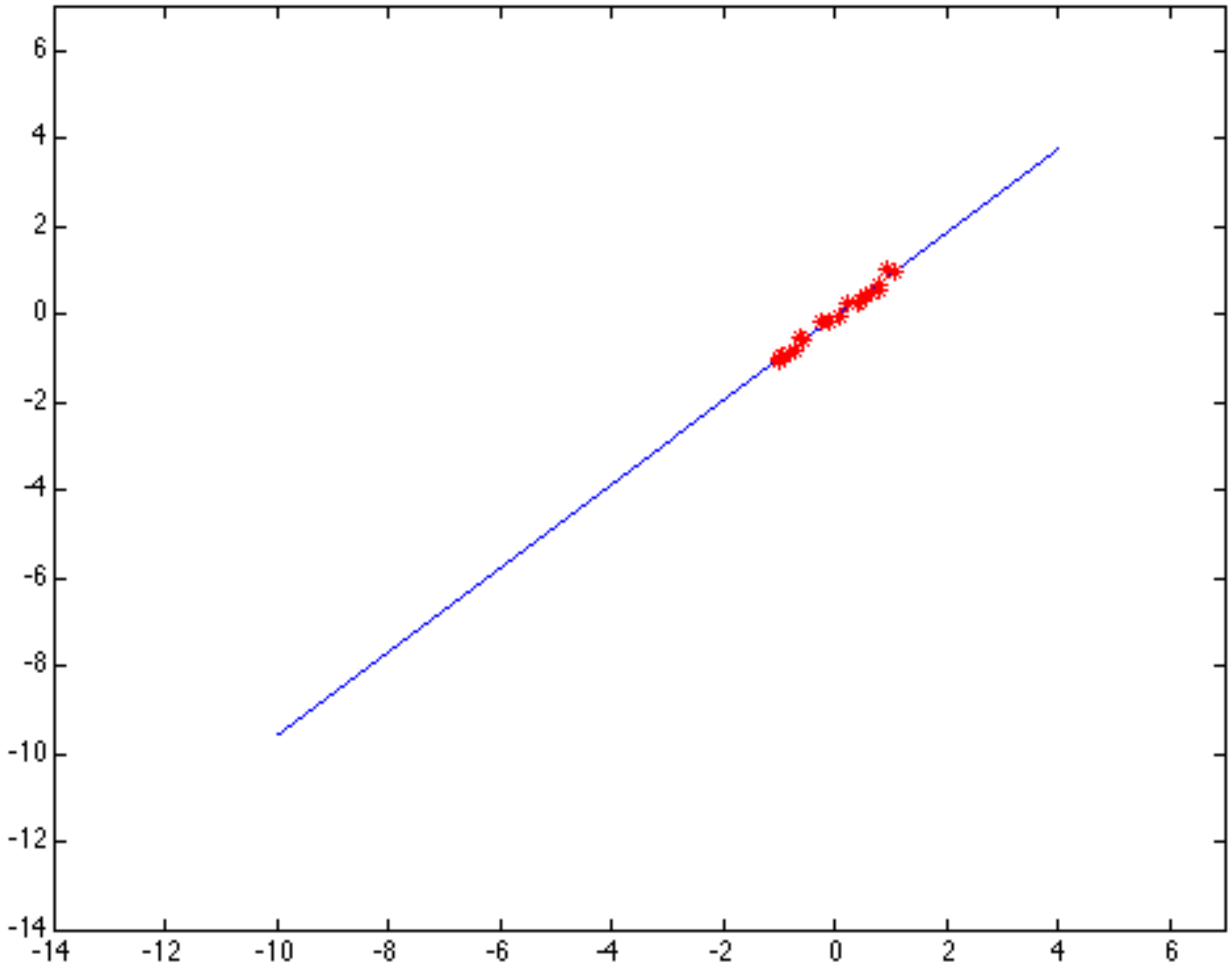


$$\boxed{A}\,\boxed{h} = 0 \qquad \text{[Eq. 10]}$$

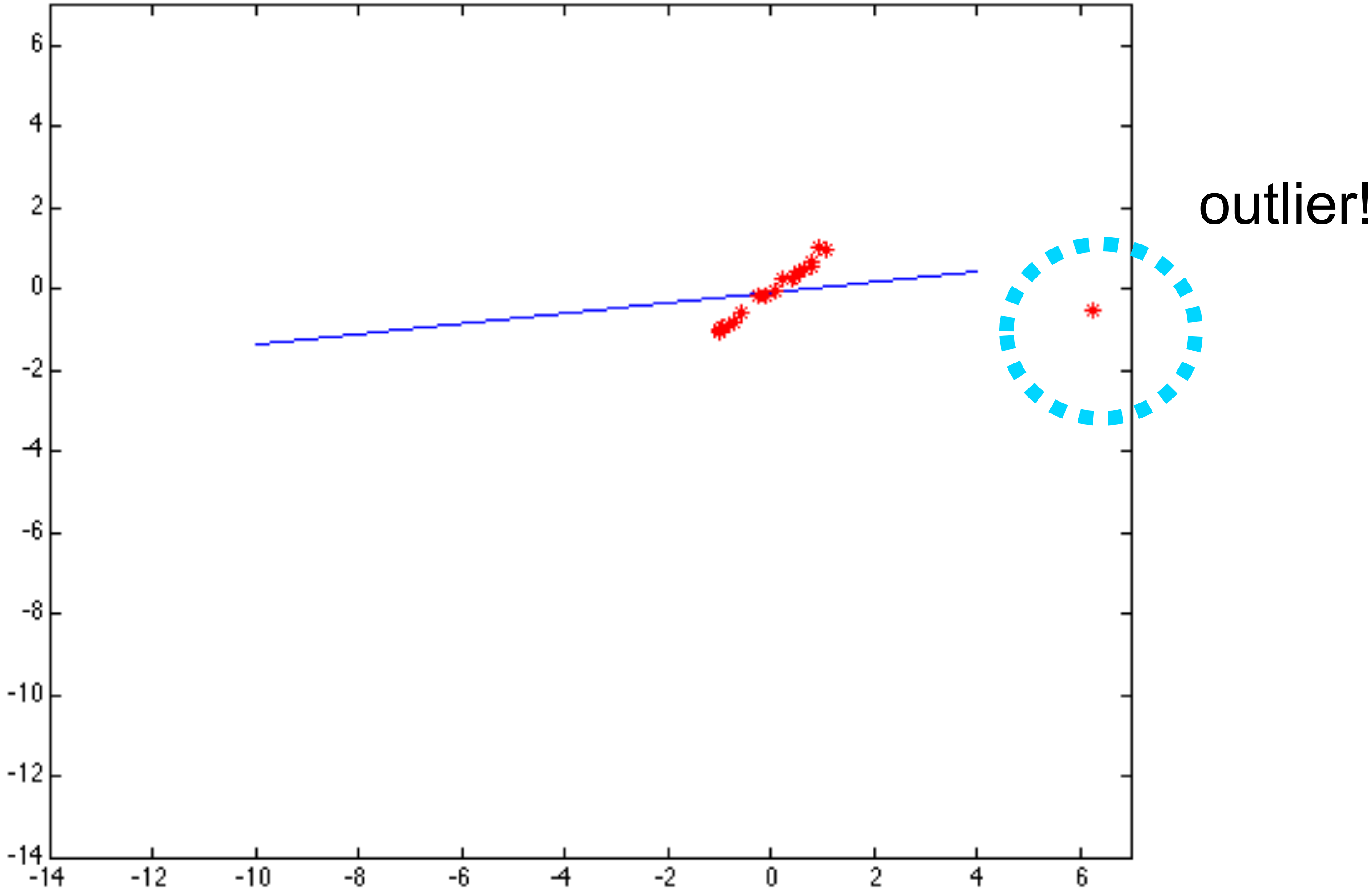
data model parameters

See HZ
- Sec 4.1 for details (DLT algorithm)
- Sec 4.1.2 (or APPENDIX)

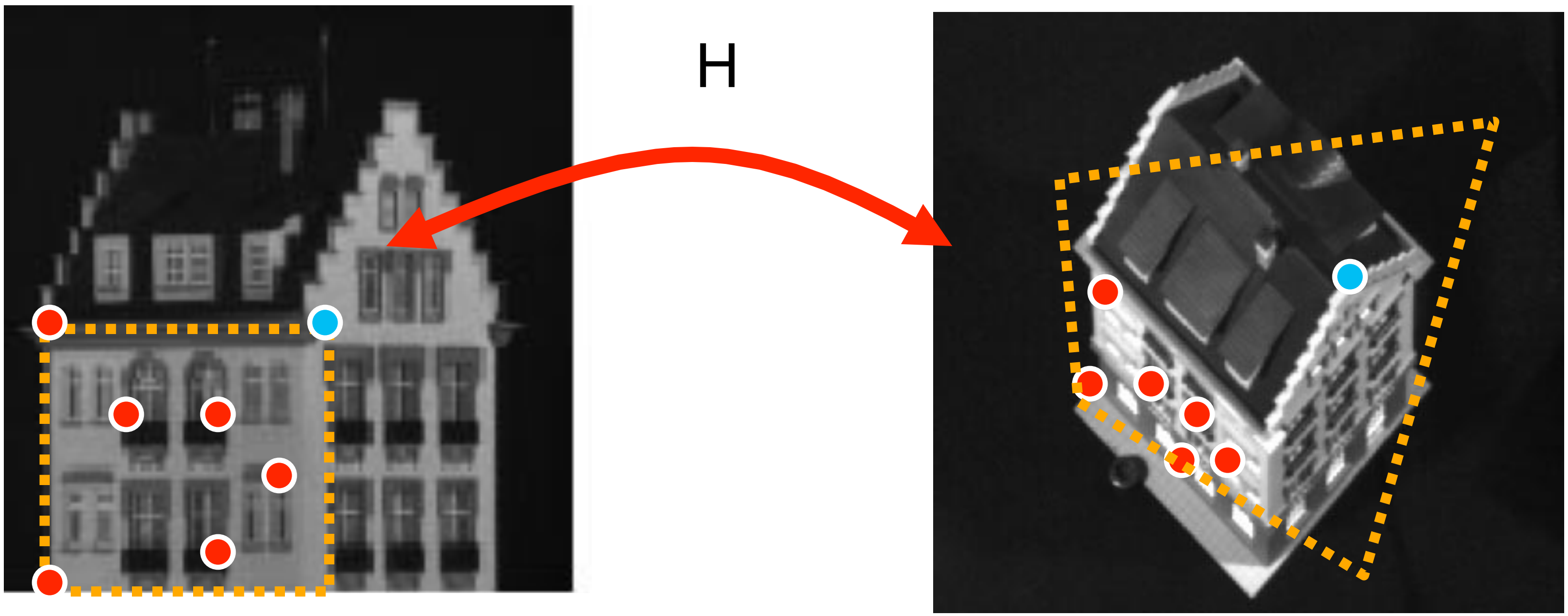
Least squares: Robustness to noise



Least squares: Robustness to noise



Critical issues: outliers



CONCLUSION: Least square is not robust w.r.t. outliers

Least squares: Robust estimators

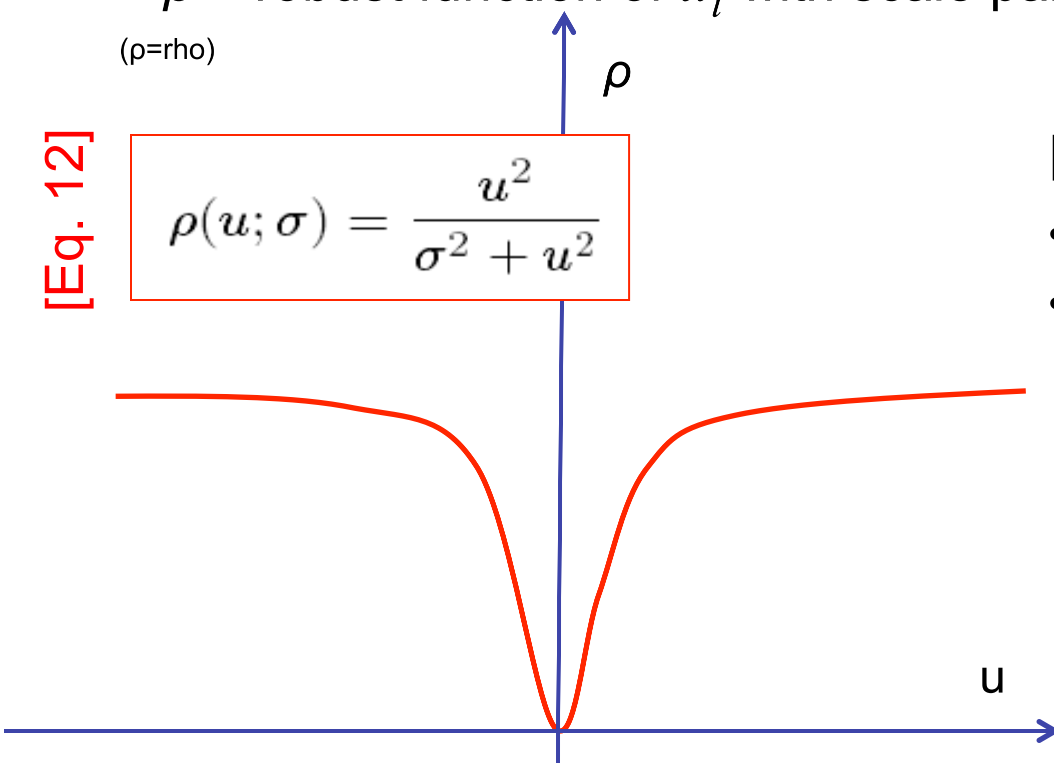
Instead of minimizing $E = \sum_{i=1}^{n} (ax_i + by_i - d)^2$ [Eq. 8]

We minimize $$\boxed{E = \sum_i \rho(u_i; \sigma)}$$ [Eq. 11] $$u_i = ax_i + by_i - d$$

- u_i = error (residual) of i^{th} point w.r.t. model parameters $h = $ (a,b,d)
- ρ = robust function of u_i with scale parameter σ

(ρ=rho)

[Eq. 12] $$\boxed{\rho(u;\sigma) = \frac{u^2}{\sigma^2 + u^2}}$$



Robust function ρ:

- When u is large, ρ saturates to 1
- When u is small, ρ is a function of u^2

In conclusion:

- Favors a configuration with small residuals
- Penalizes large residuals

Least squares: Robust estimators

Instead of minimizing $E = \sum_{i=1}^{n} (a x_i + b y_i - d)^2$ [Eq. 8]

We minimize $\boxed{E = \sum_i \rho(u_i ; \sigma)}$ [Eq. 11] $u_i = a x_i + b y_i - d$

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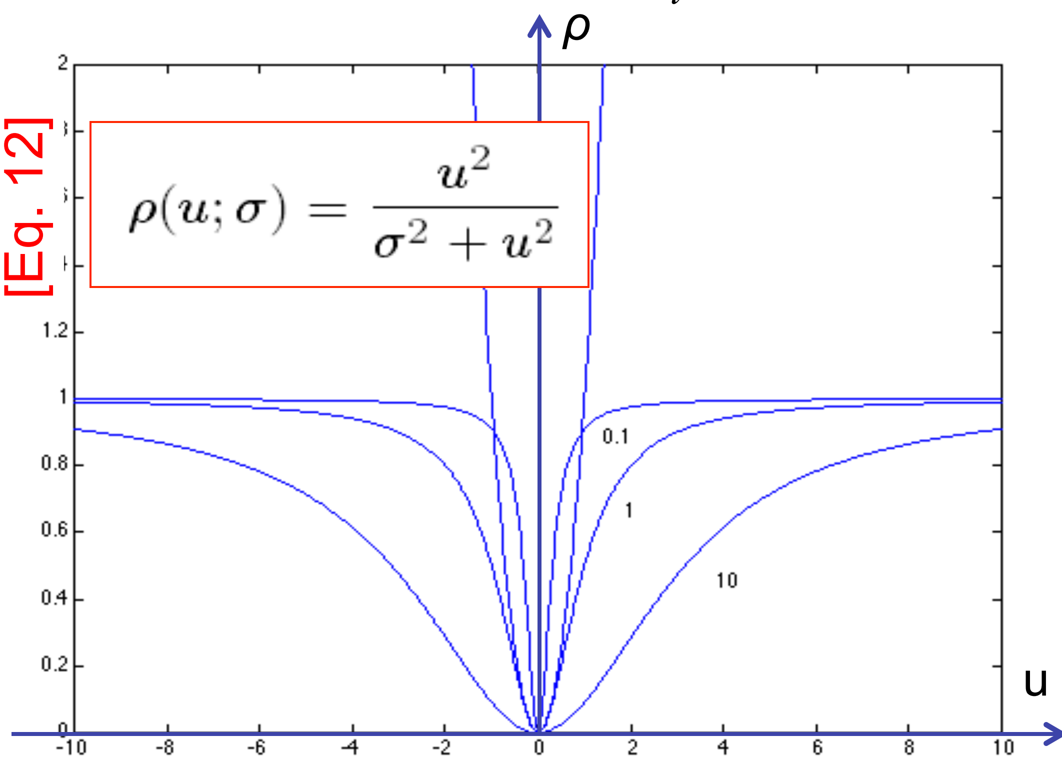
- ρ = robust function of u_i with scale parameter σ

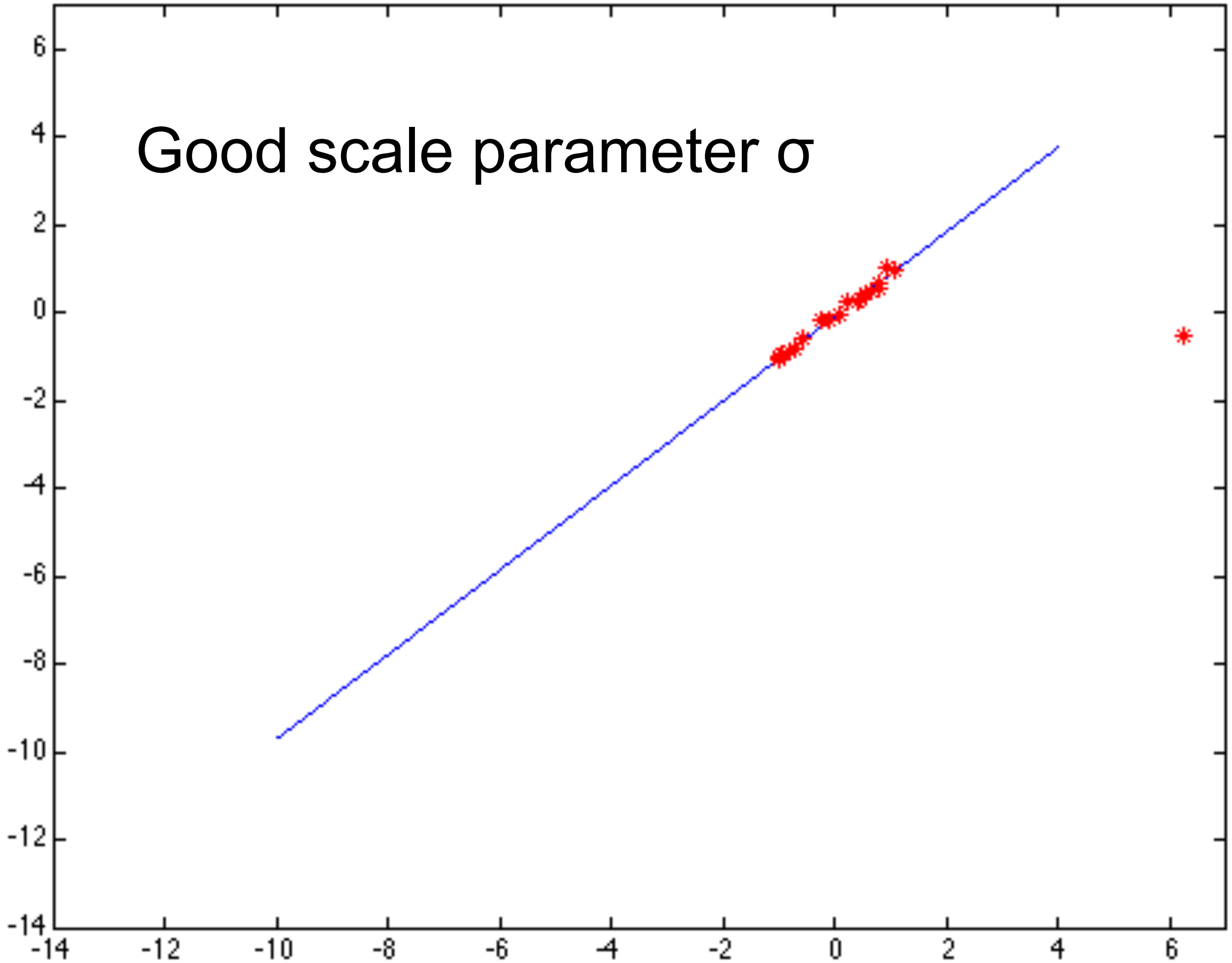
[Eq. 12]

$$\rho(u;\sigma) = \frac{u^2}{\sigma^2 + u^2}$$



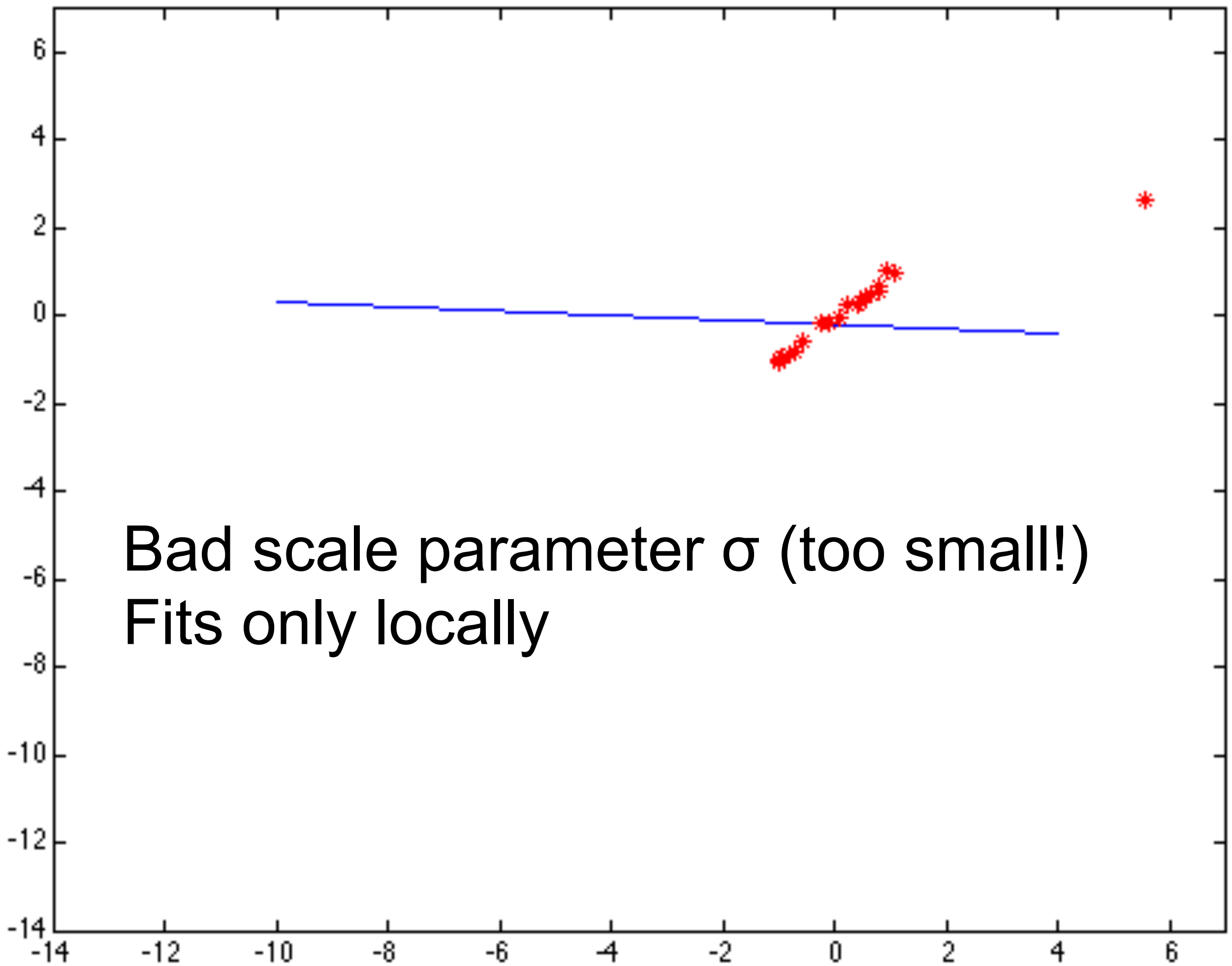
- Small sigma → highly penalize large residuals
- Large sigma → mildly penalize large residual (like LSQR)

Least squares: Robust estimators

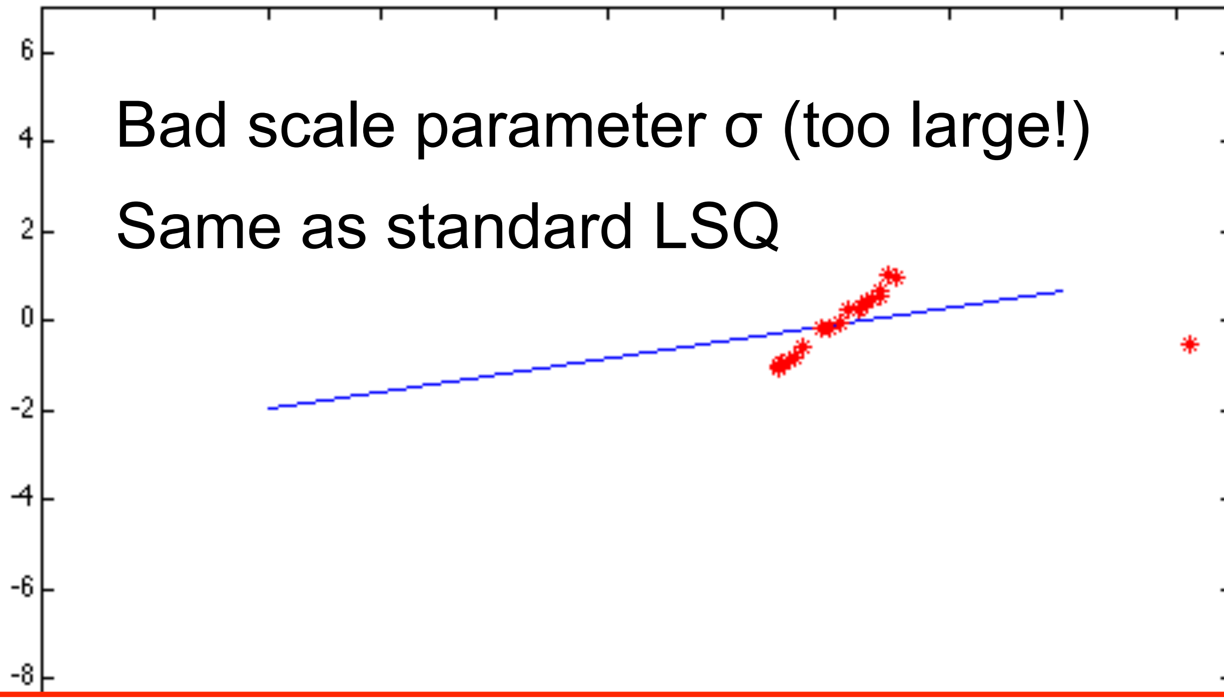


The effect of the outlier is eliminated

Least squares: Robust estimators



Least squares: Robust estimators



•CONCLUSION: Robust estimator useful if prior info about the distribution of points is known

•Robust fitting is a nonlinear optimization problem (iterative solution)

•Least squares solution provides good initial condition

Fitting

Goal: Choose a parametric model to fit a certain quantity from data

Techniques:

- Least square methods
- RANSAC
- Hough transform

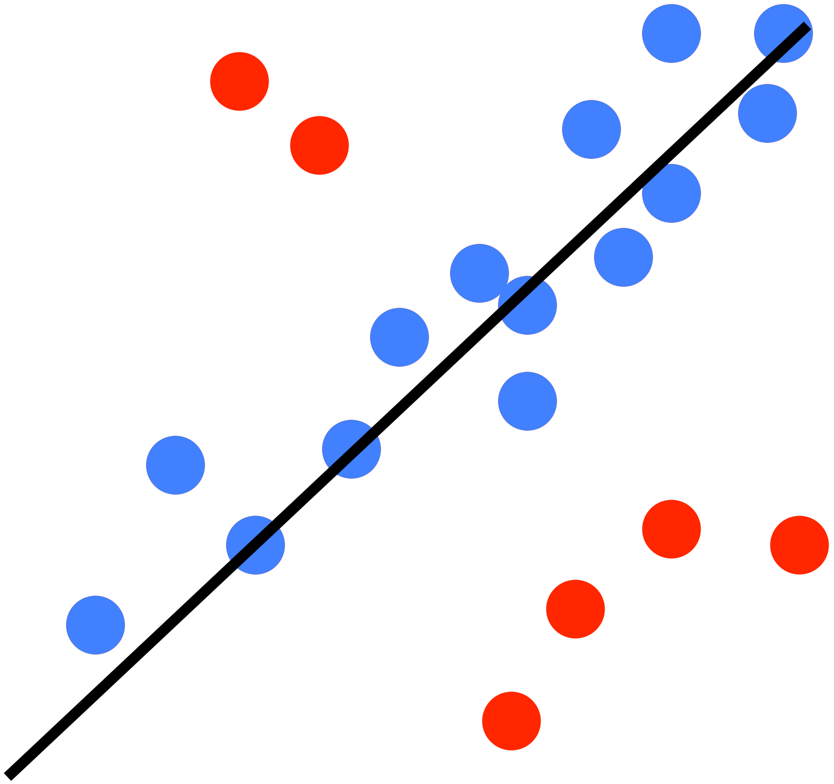
Basic philosophy
(voting scheme)

- Data elements are used to vote for one (or multiple) models

- Robust to outliers and missing data

- Assumption 1: Noisy data points will not vote consistently for any single model   ("few" outliers)
- Assumption 2: There are enough data points to agree on a good model ("few" missing data)

Example: Line fitting

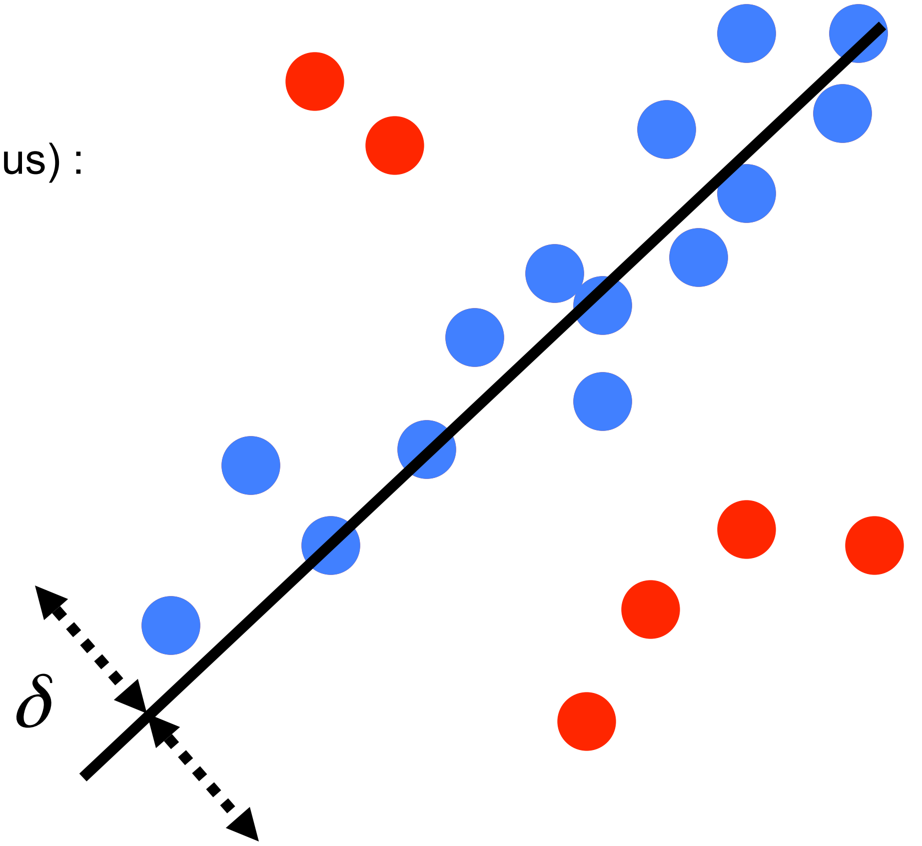


- Enough “good” data points supporting the line model in presence of noise
- “Few” outliers compared to the “good” data points – these few outliers won’t “consistently” vote for a line model

RANSAC

(RANdom SAmple Consensus) :

Fischler & Bolles in '81.



$$\pi : \mathbf{P} \rightarrow \{\mathbf{I}, \mathbf{O}\} \qquad \min_{\pi} |\boldsymbol{O}|$$

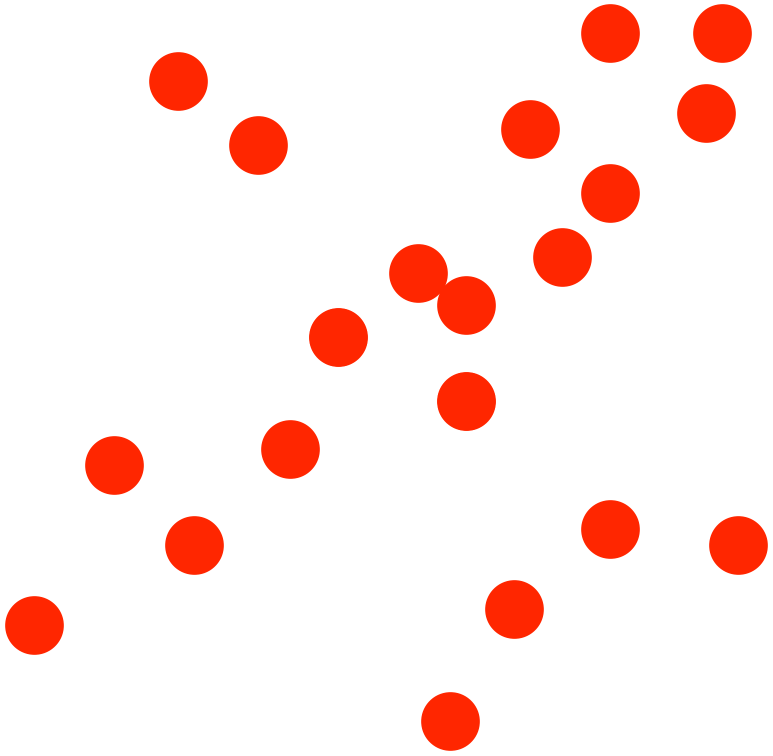
Model parameters a,b,d

such that:

$$r(I, h) < \delta, \quad \forall I \in \mathbf{I} \qquad r(I, h) = residual \boxed{= \sum_{i=1}^{n} (ax_i + by_i + d)^2}$$

[Eq. 12]

RANSAC



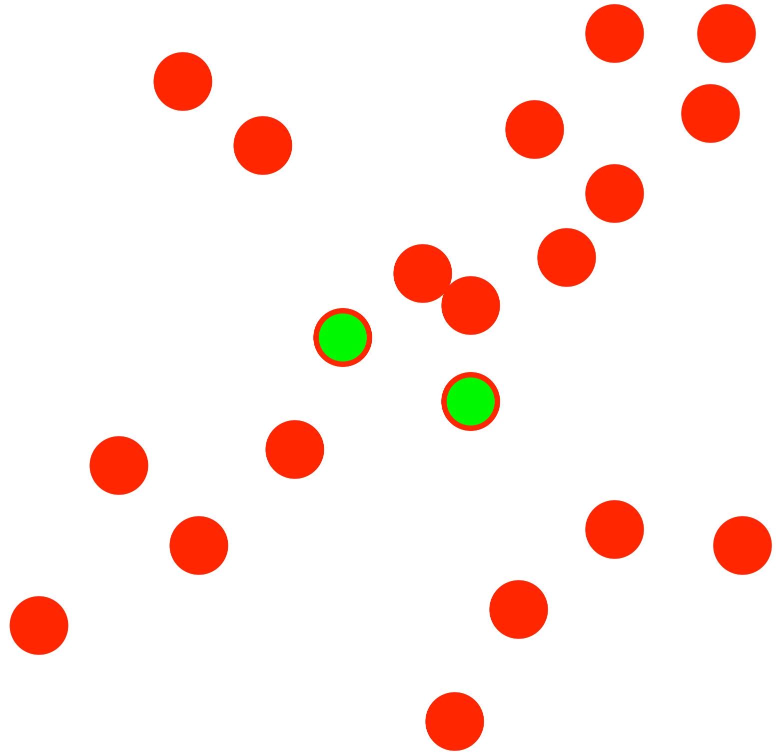
P = Sample set = set of points in 2D

Algorithm:

1. Select random sample of minimum required size to fit model
2. Compute a putative model from sample set
3. Compute the set of inliers to this model from whole data set

Repeat 1-3 until model with the most inliers over all samples is found

RANSAC



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Algorithm:

1. Select random sample of minimum required size to fit model [?]
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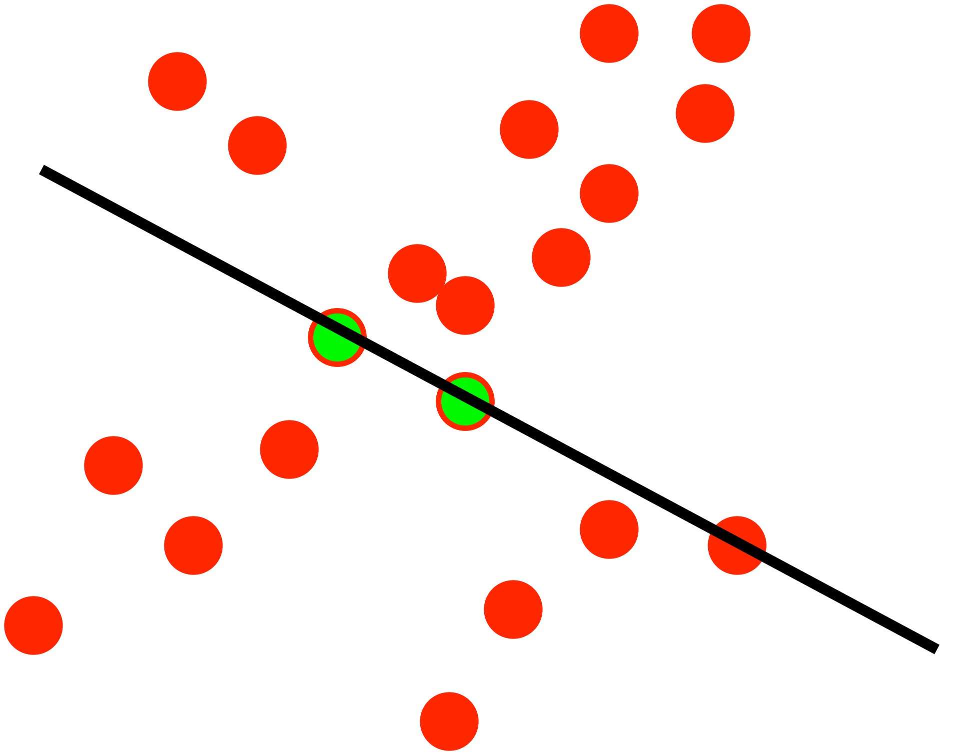
RANSAC



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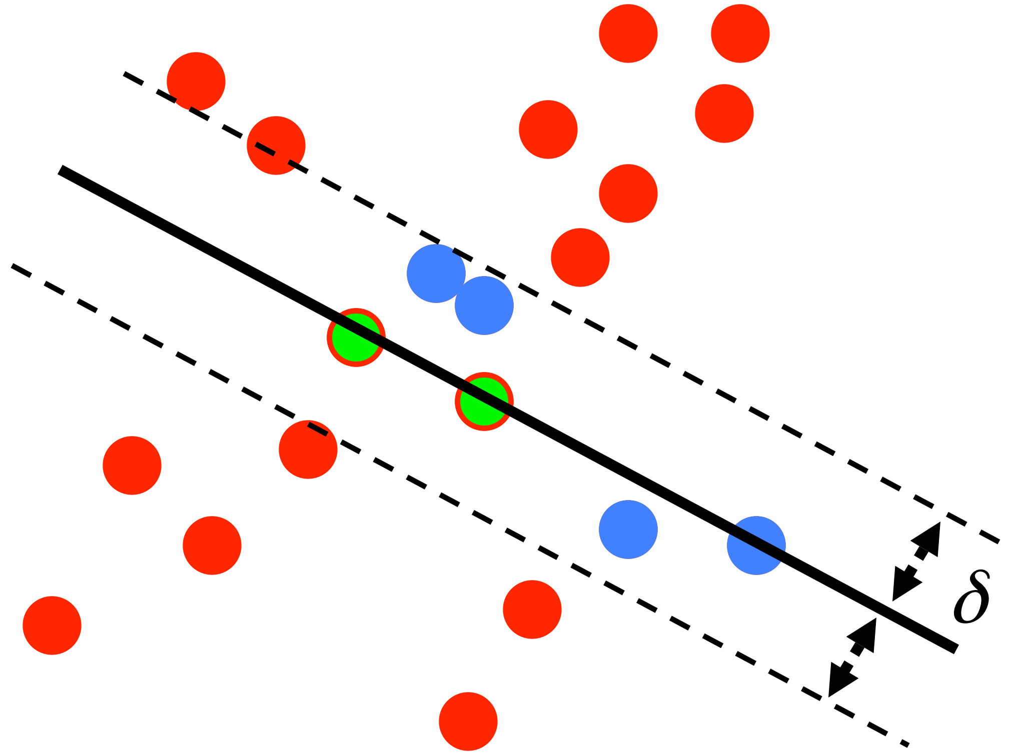
RANSAC



P = Sample set = set of points in 2D

$$|\mathbf{O}| = ? \;= 14$$

$$|\mathbf{I}| = ? \;= 6$$

Algorithm:

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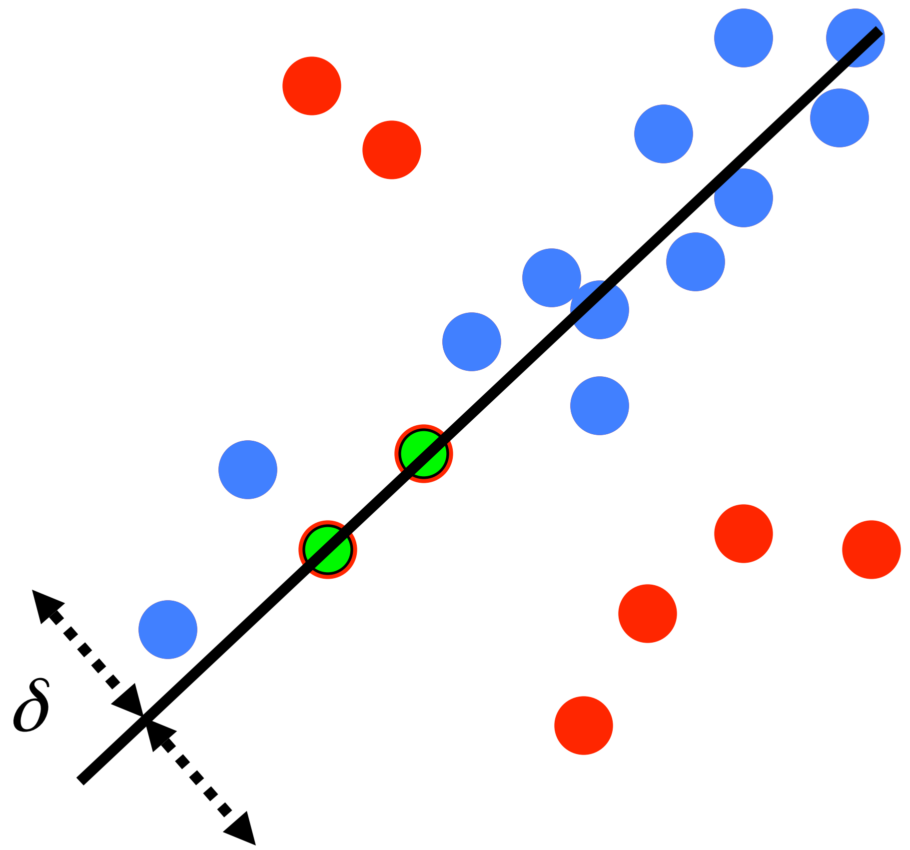
RANSAC



$$|\mathbf{O}| = 6$$

$$|\mathbf{I}| = 14$$

Algorithm:

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Repeat 1-3 until model with the most inliers over all samples is found

How many samples?

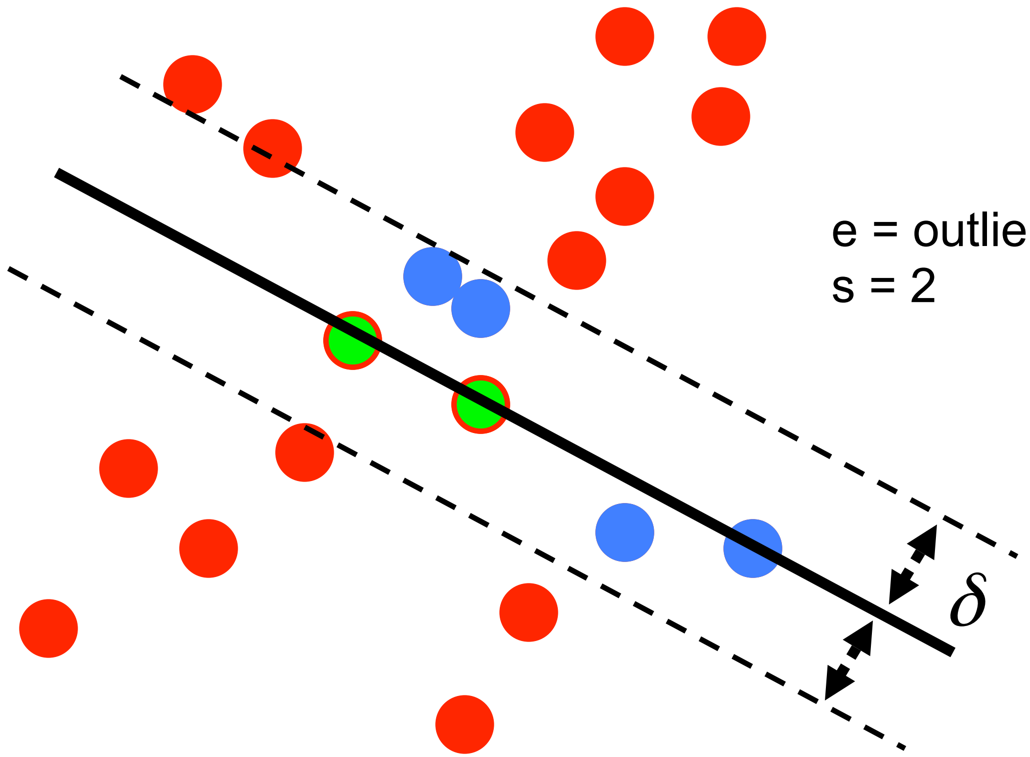
- Computationally unnecessary (and infeasible) to explore the entire sample space

- **N samples are sufficient**
- N = number of samples required to ensure, with a probability p, that at least one random sample produces an inlier set that is free from “real” outliers

- Function of *s* and *e*:
  - *e* = outlier ratio
  - s = minimum number of data points needed to fit the model

- Usually, p=0.99

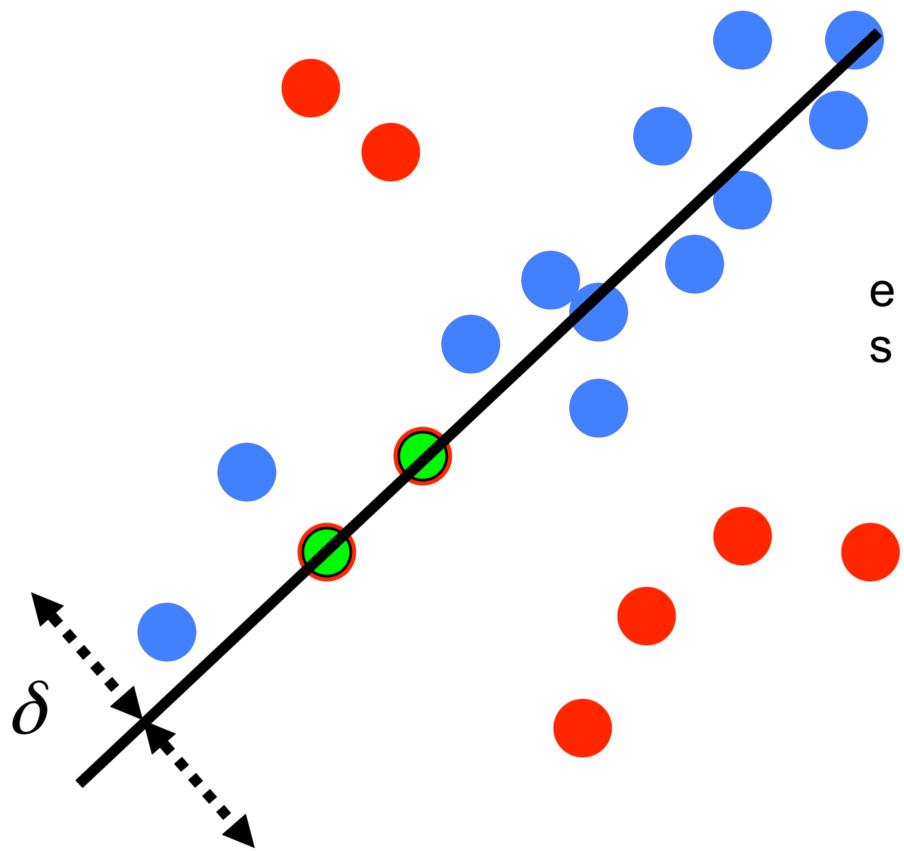
Example



e = outlier ratio is 6/20 = 30%
s = 2

- Here a random sample is given by two green points
- The estimated inlier set is given by the green+blue points
- How many “real” outliers we have here? 2

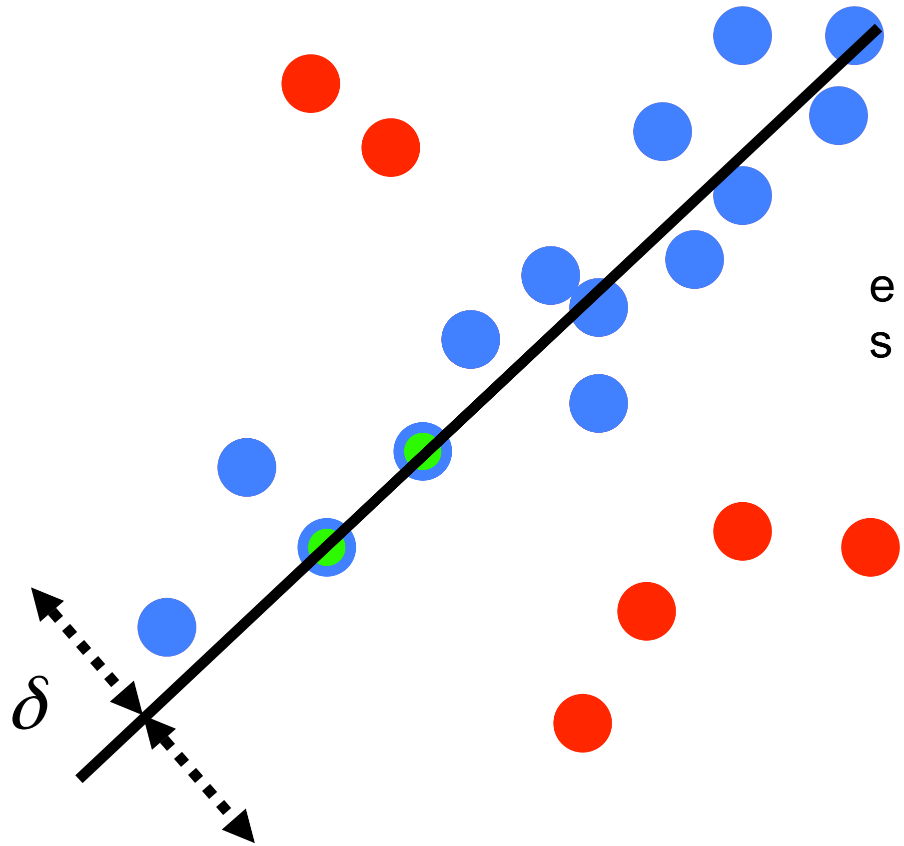
Example



e = outlier ratio is 6/20 = 30%
s = 2

- Random sample is given by two green points
- The estimated inlier set is given by the green+blue points
- How many “real” outliers we have here? 0

Example



e = outlier ratio is 6/20 = 30%
s = 2

N is the number of times we need to sample my data (and thus repeat the steps 1-3 in the previous slides) before I find the configuration above with probability *p*. Again this is function of *e* and *s* as well.

How many samples?

- Number N of samples required to ensure, with a probability p, that at least one random sample produces an inlier set that is free from "real" outliers for a given s and e.
- E.g., *p*=0.99

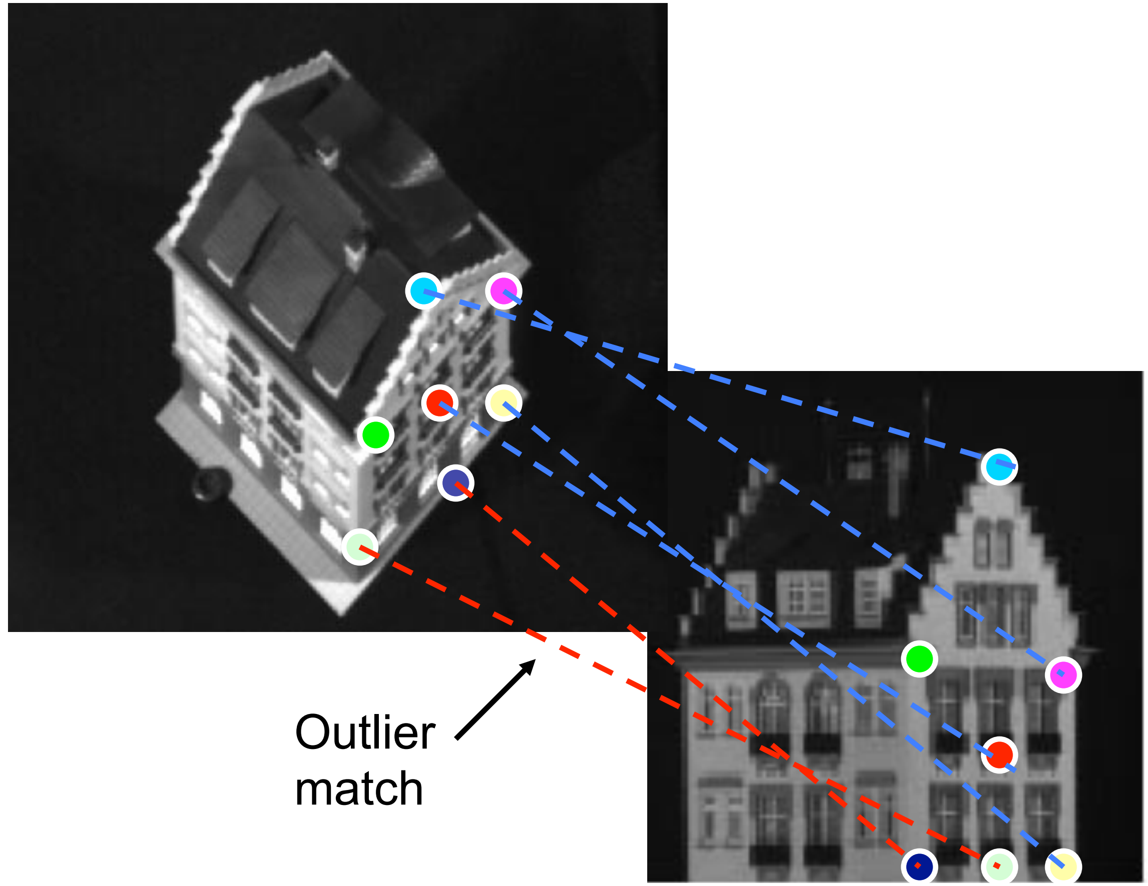
$$N = \log(1-p)/\log\left(1-(1-e)^s\right)$$ [Eq. 13]

	proportion of outliers e						
s	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	$\boxed{7}$	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

e = outlier ratio
s = minimum number needed to fit the model

Note: this table assumes "negligible" measurement noise

Estimating H by RANSAC



- H → 8 DOF
- Need 4 correspondences

P = Sample set = set of matches between 2 images

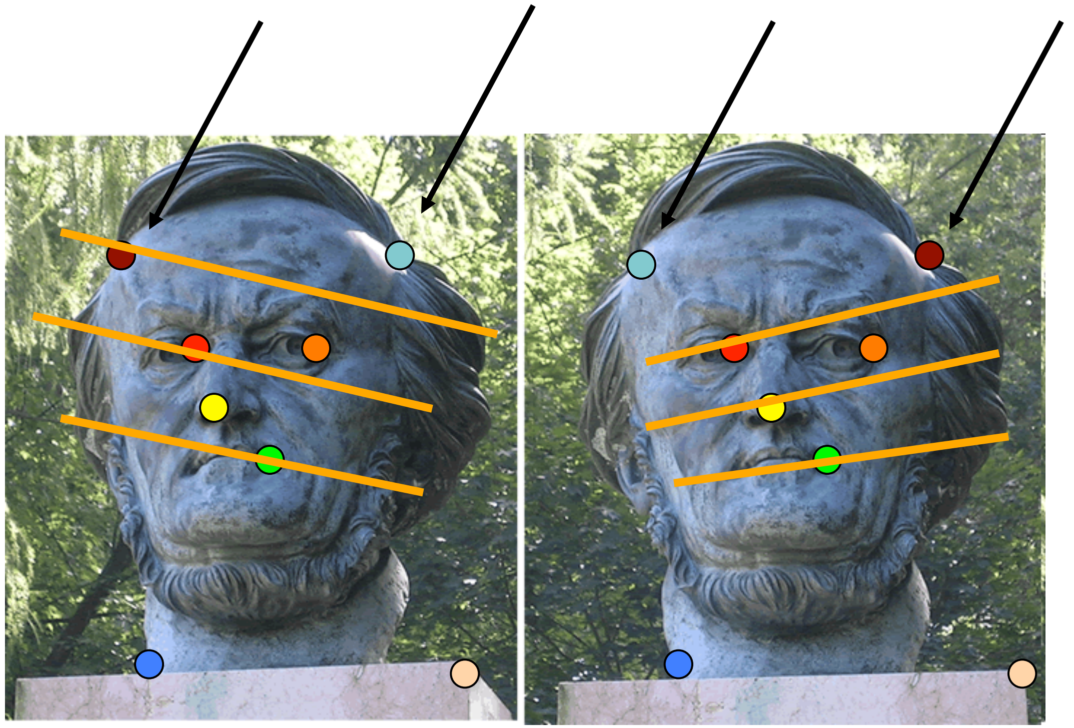
Algorithm:

1. Select a random sample of minimum required size [?]
2. Compute a putative model from these
3. Compute the set of inliers to this model from whole sample space

Repeat 1-3 until model with the most inliers over all samples is found

Estimating F by RANSAC

Outlier matches



•F → 7 DOF
•Need 7 (8) correspondences

P = Sample set = set of matches between 2 images

Algorithm:

1. Select a random sample of minimum required size [?]
2. Compute a putative model from these
3. Compute the set of inliers to this model from whole sample space

Repeat 1-3 until model with the most inliers over all samples is found

RANSAC - conclusions

Good:

- Simple and easily implementable
- Successful in different contexts

Bad:

- Many parameters to tune
- Trade-off accuracy-vs-time
- Cannot be used if ratio inliers/outliers is too small

Fitting

Goal: Choose a parametric model to fit a certain quantity from data

Techniques:

- Least square methods
- RANSAC
- Hough transform

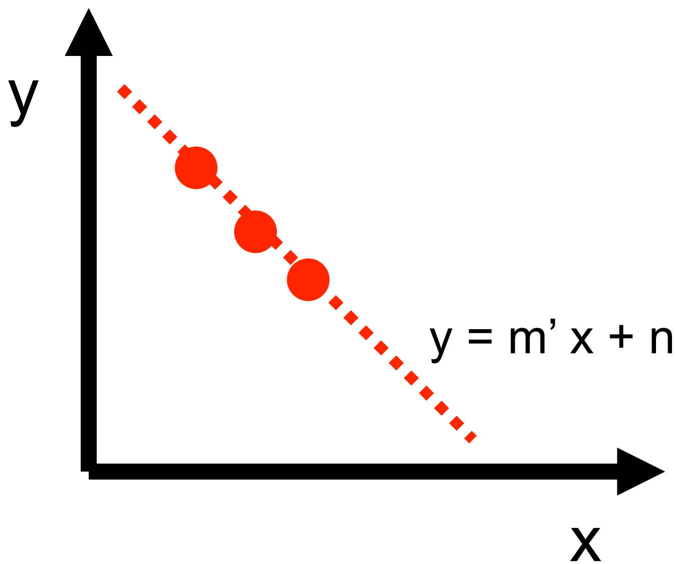
Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

Given a set of points, find the line parameterized by *m,n* that explains the data points best: that is, *m = m'* and *n = n'*



Hough transform

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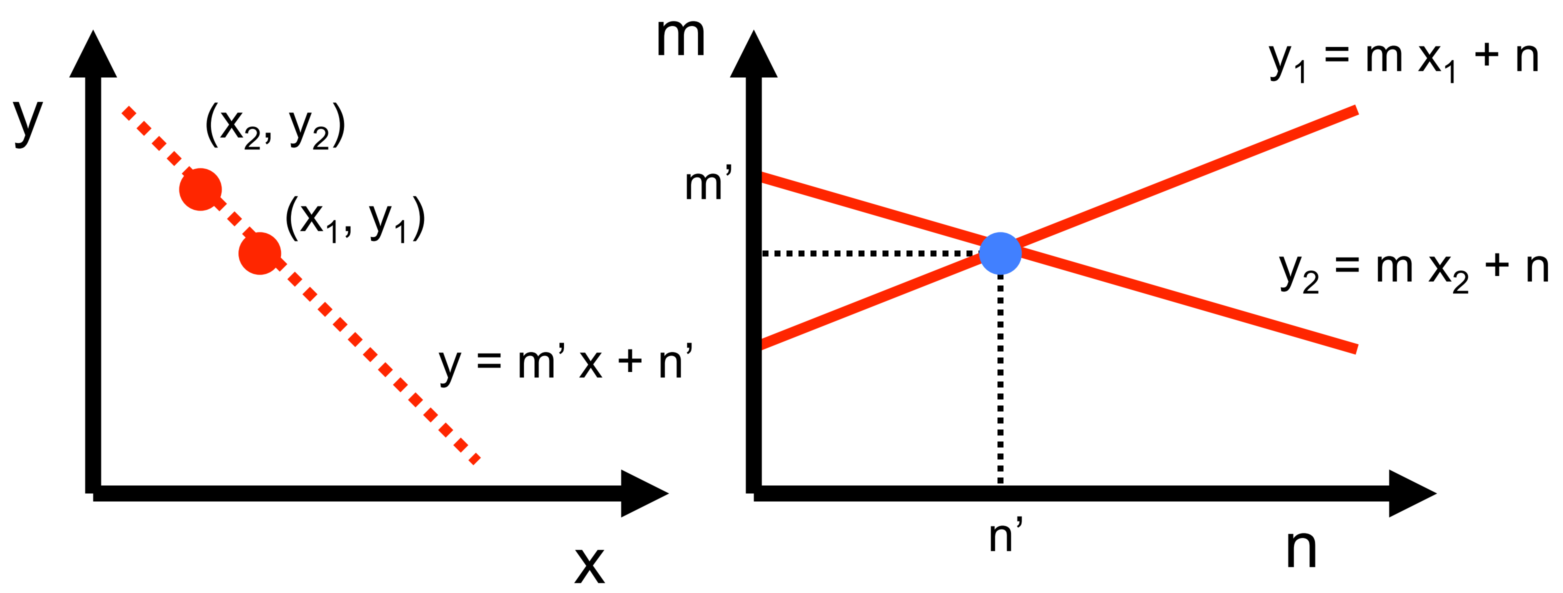


Original space where the data points are

Hough space defined by the parameters of the model we want to fit (i.e., m, n)

Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

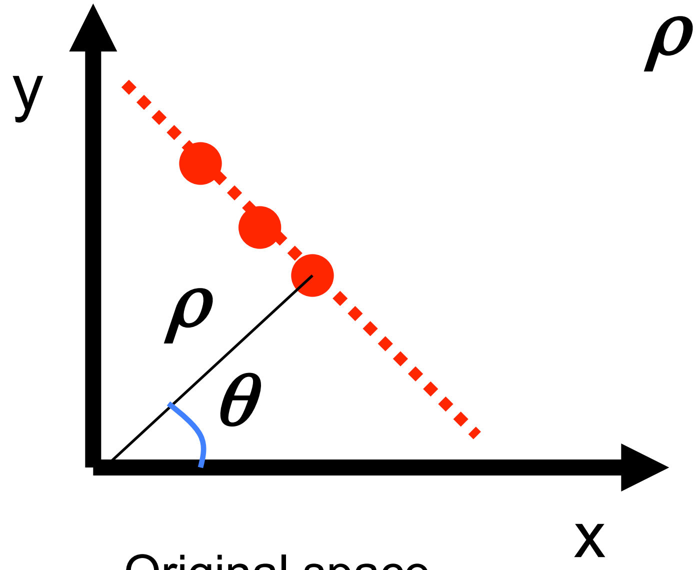
Any Issue? The parameter space [m,n] is unbounded…

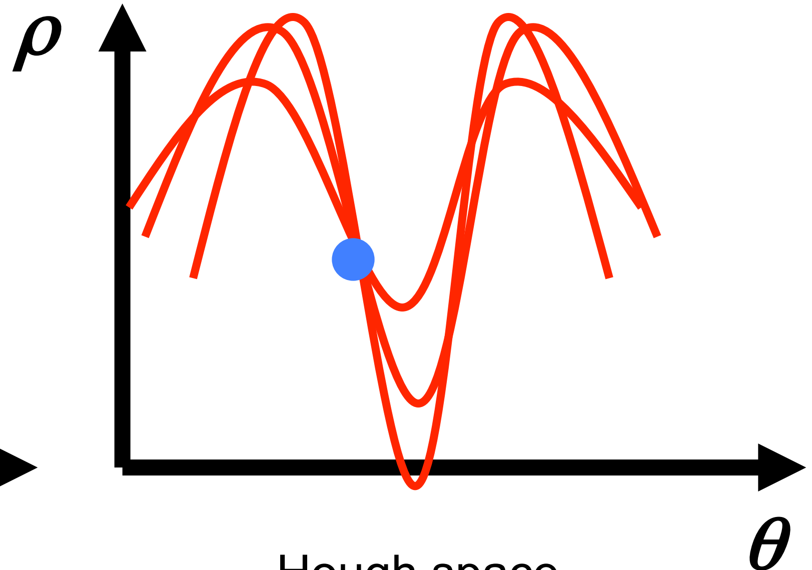
Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

Any Issue? The parameter space [m,n] is unbounded…

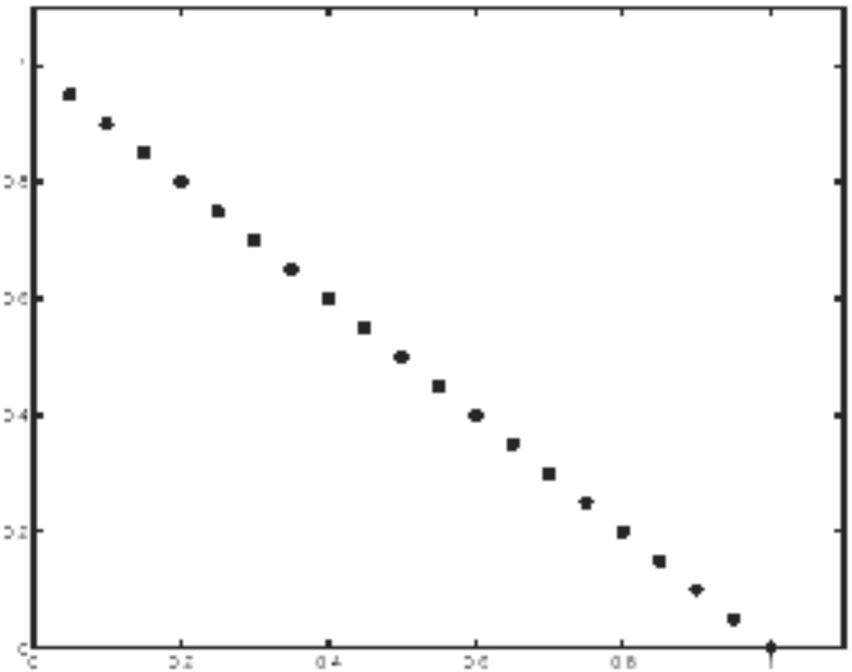
Use a polar representation for the parameter space



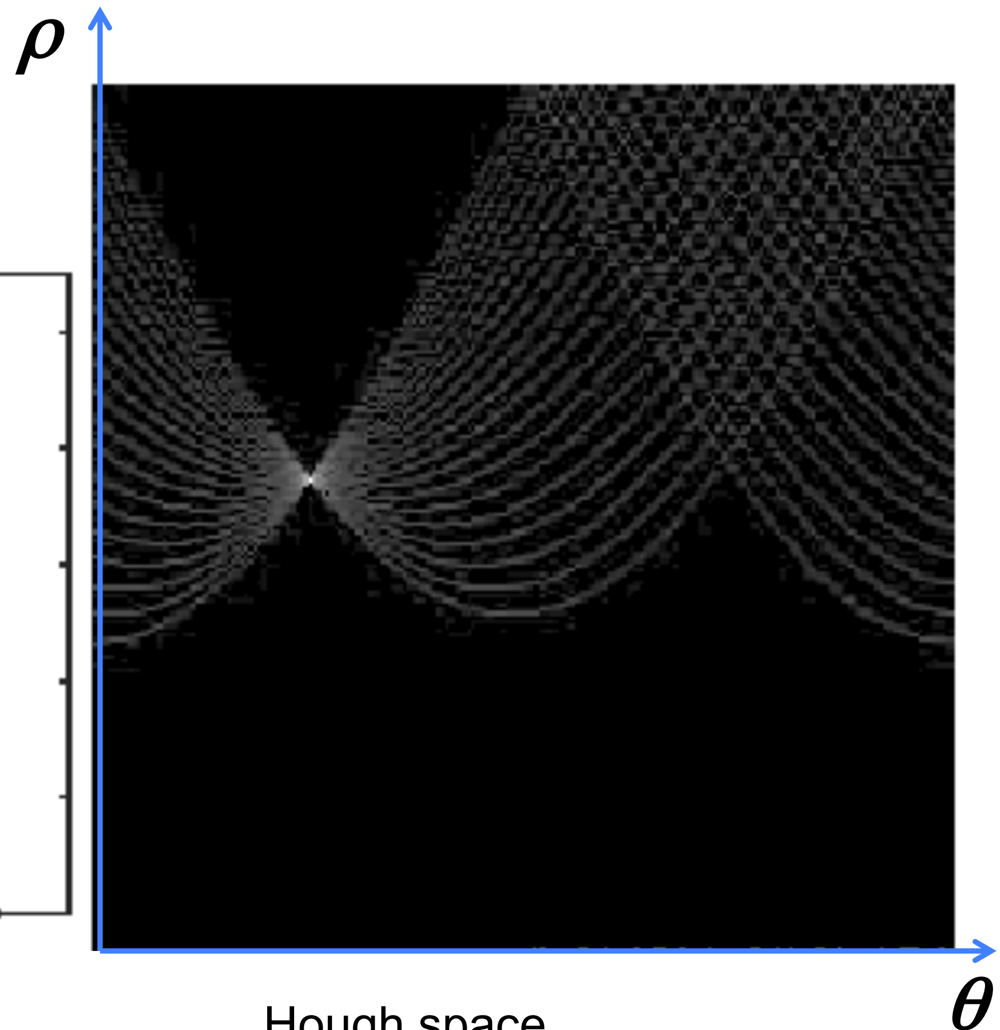


$$x\cos\theta + y\sin\theta = \rho$$ [Eq. 13]

Hough transform - experiments



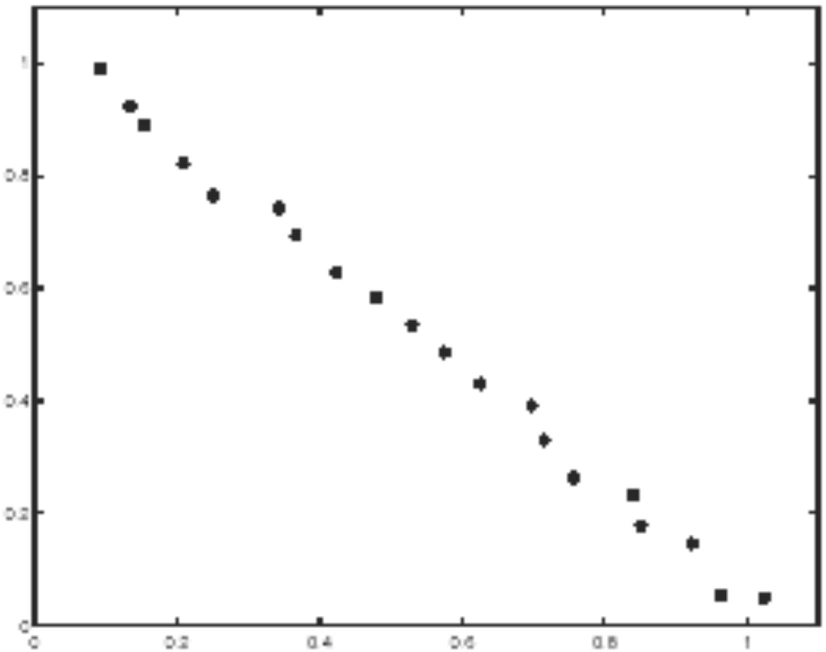
Original space



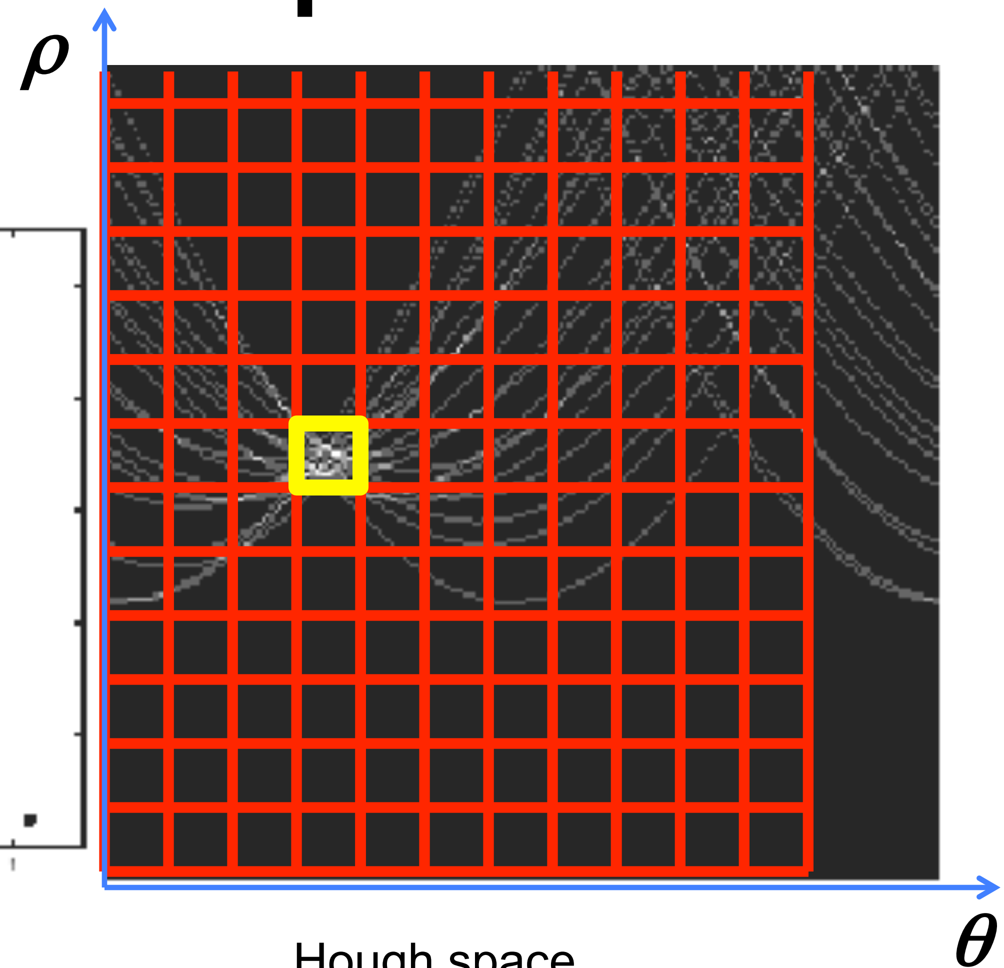
Hough space

Hough transform - experiments

Noisy data



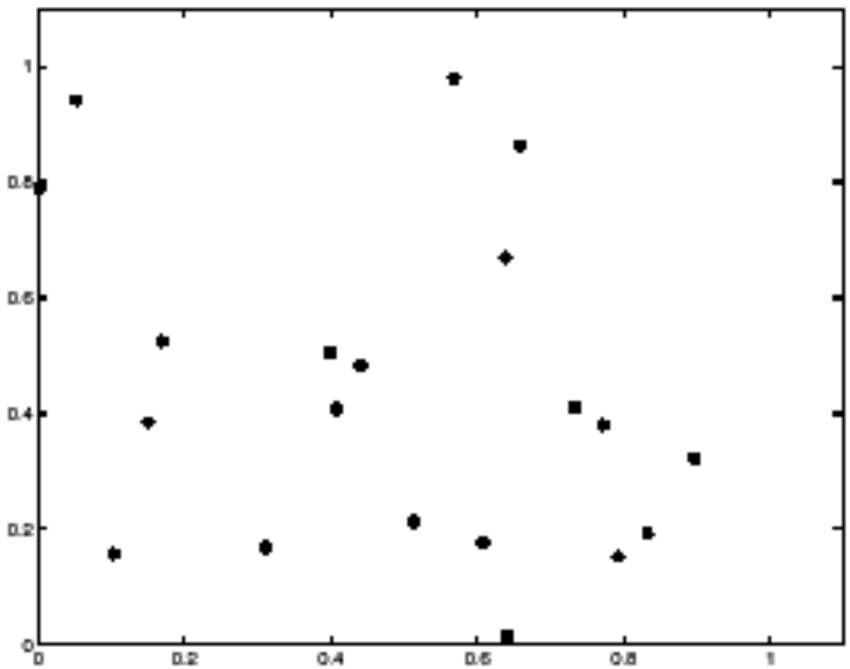
Original space



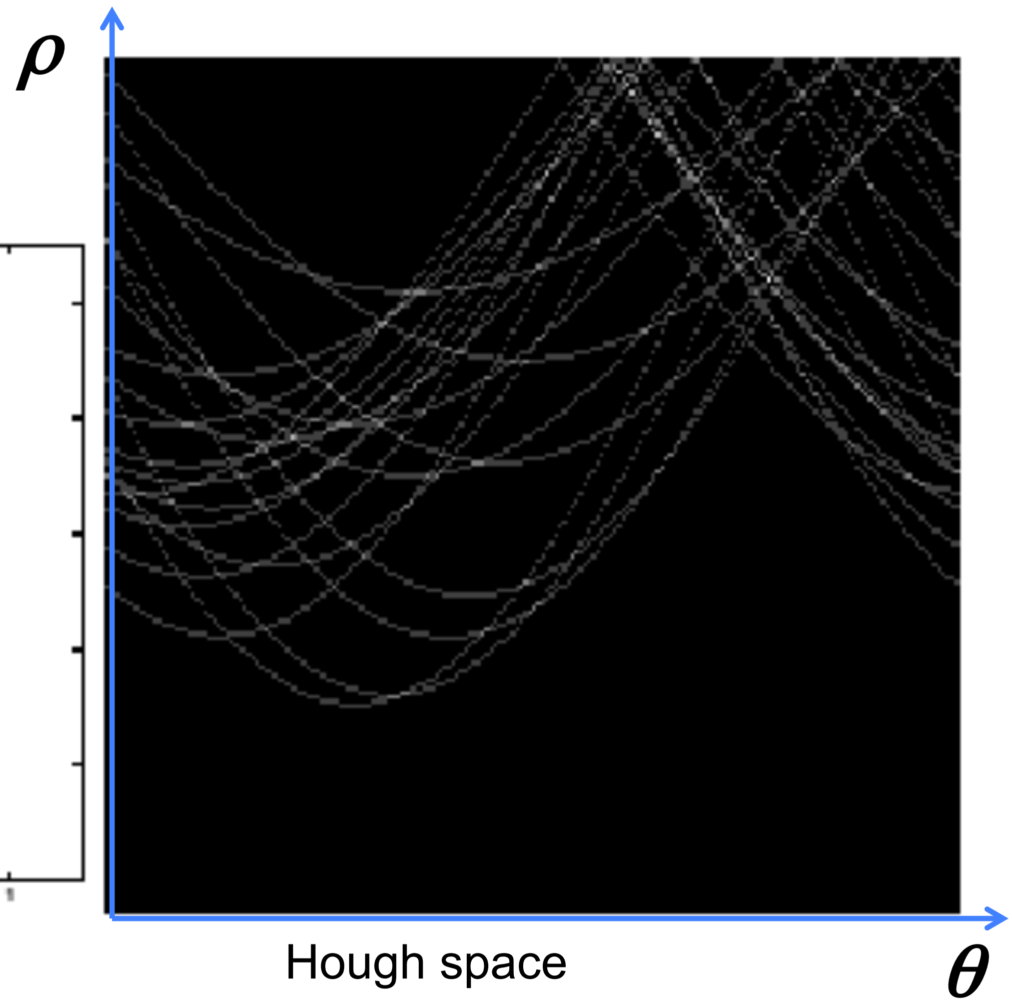
Hough space

How to compute the intersection point? In presence of noise!
IDEA: introduce a grid a count intersection points in each cell
Issue: Grid size needs to be adjusted…

Hough transform - experiments



Original space



Hough space

Issue: spurious peaks due to uniform noise

Hough transform - conclusions

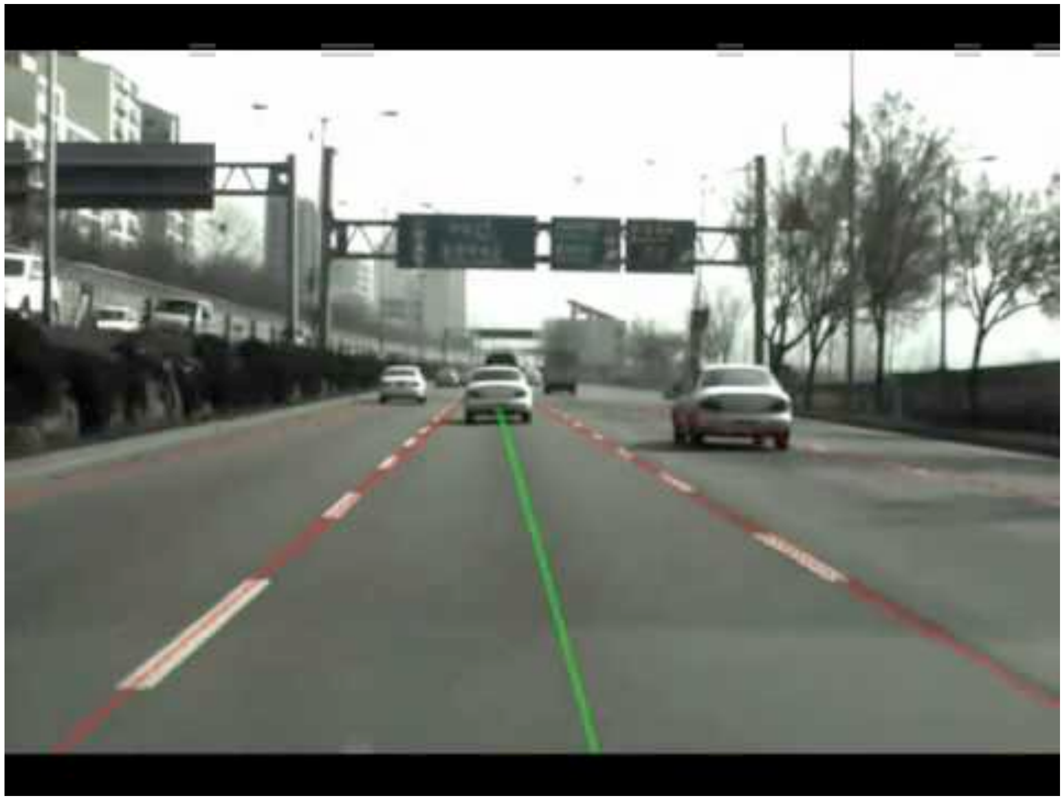
Good:

- All points are processed independently, so can cope with occlusion/outliers
- Some robustness to noise: noise points unlikely to contribute consistently to any single cell

Bad:

- Spurious peaks due to uniform noise
- Trade-off noise-grid size (hard to find sweet point)
- Doesn’t handle well high dimensional models

Applications – lane detection



Courtesy of Minchae Lee

Applications – computing vanishing points

Generalized Hough transform

[more on forthcoming lectures]

D. Ballard, Generalizing the Hough Transform to Detect Arbitrary Shapes, Pattern Recognition 13(2), 1981

- Parameterize a shape by measuring the location of its parts and shape centroid
- Given a set of measurements, cast a vote in the Hough (parameter) space
- Used in object recognition! (the implicit shape model)

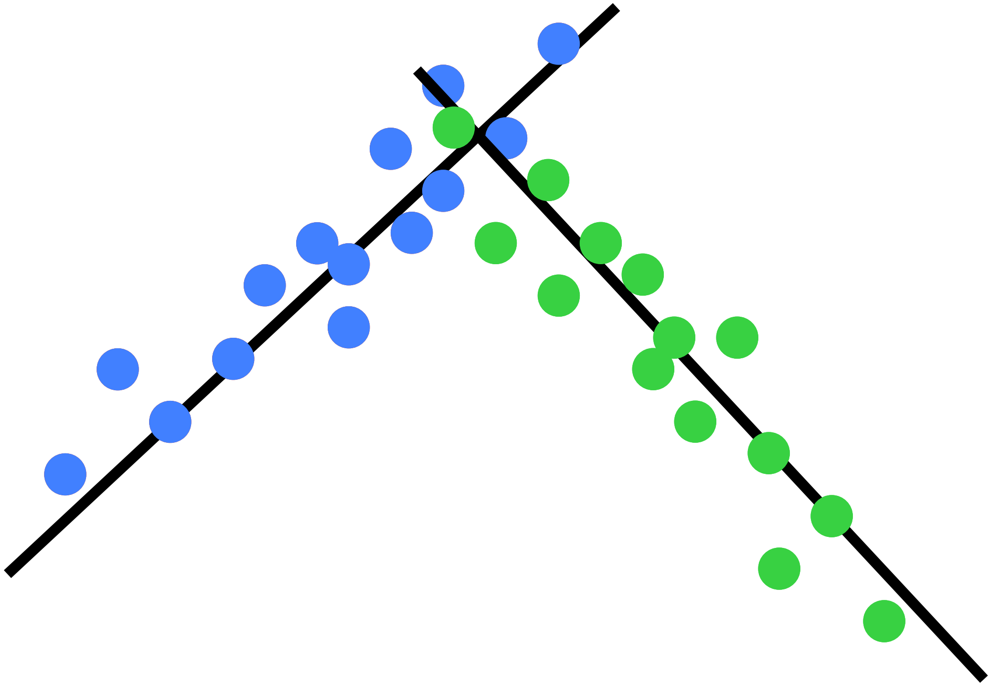
B. Leibe, A. Leonardis, and B. Schiele, Combined Object Categorization and Segmentation with an Implicit Shape Model, ECCV Workshop on Statistical Learning in Computer Vision 2004

Lecture 9

Fitting and Matching

- Problem formulation
- Least square methods
- RANSAC
- Hough transforms
- Multi-model fitting
- Fitting helps matching!

Fitting multiple models



- Incremental fitting
- E.M. (probabilistic fitting)
- Hough transform

Incremental line fitting

Scan data point sequentially (using locality constraints)

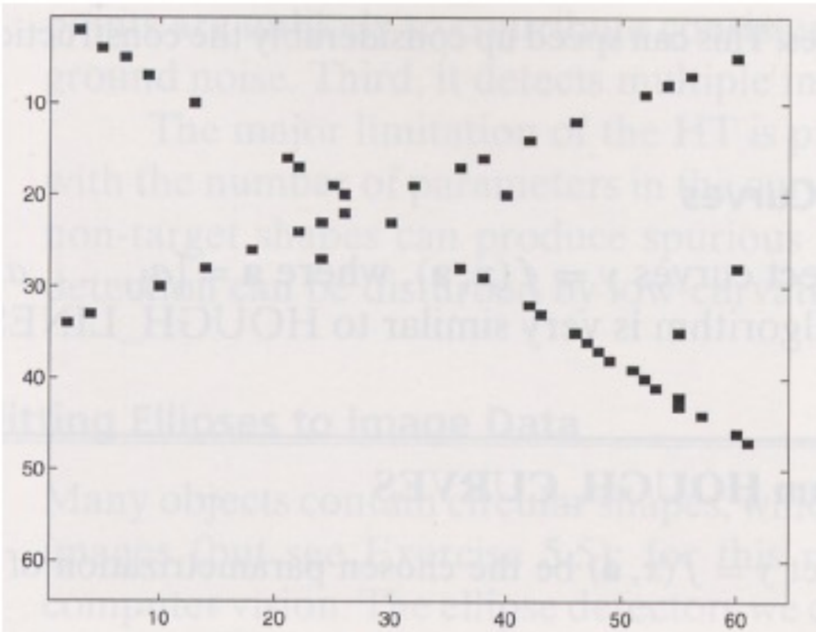
Perform following loop:

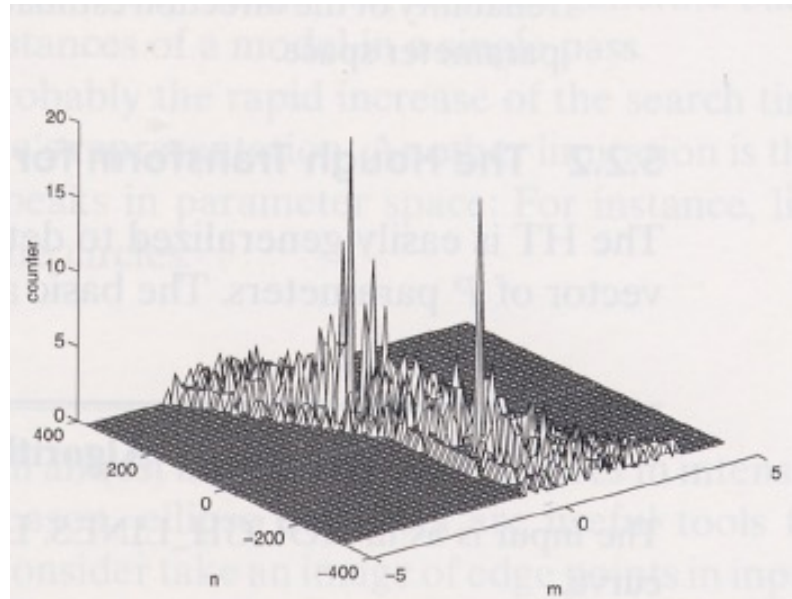
1. Select N point and fit line to N points
2. Compute residual R_N
3. Add a new point, re-fit line and re-compute R_{N+1}
4. Continue while line fitting residual is small enough,

- ➢ When residual exceeds a threshold, start fitting new model (line)

Hough transform

Courtesy of unknown





Same cons and pros as before…

Lecture 9
Fitting and Matching

- Problem formulation
- Least square methods
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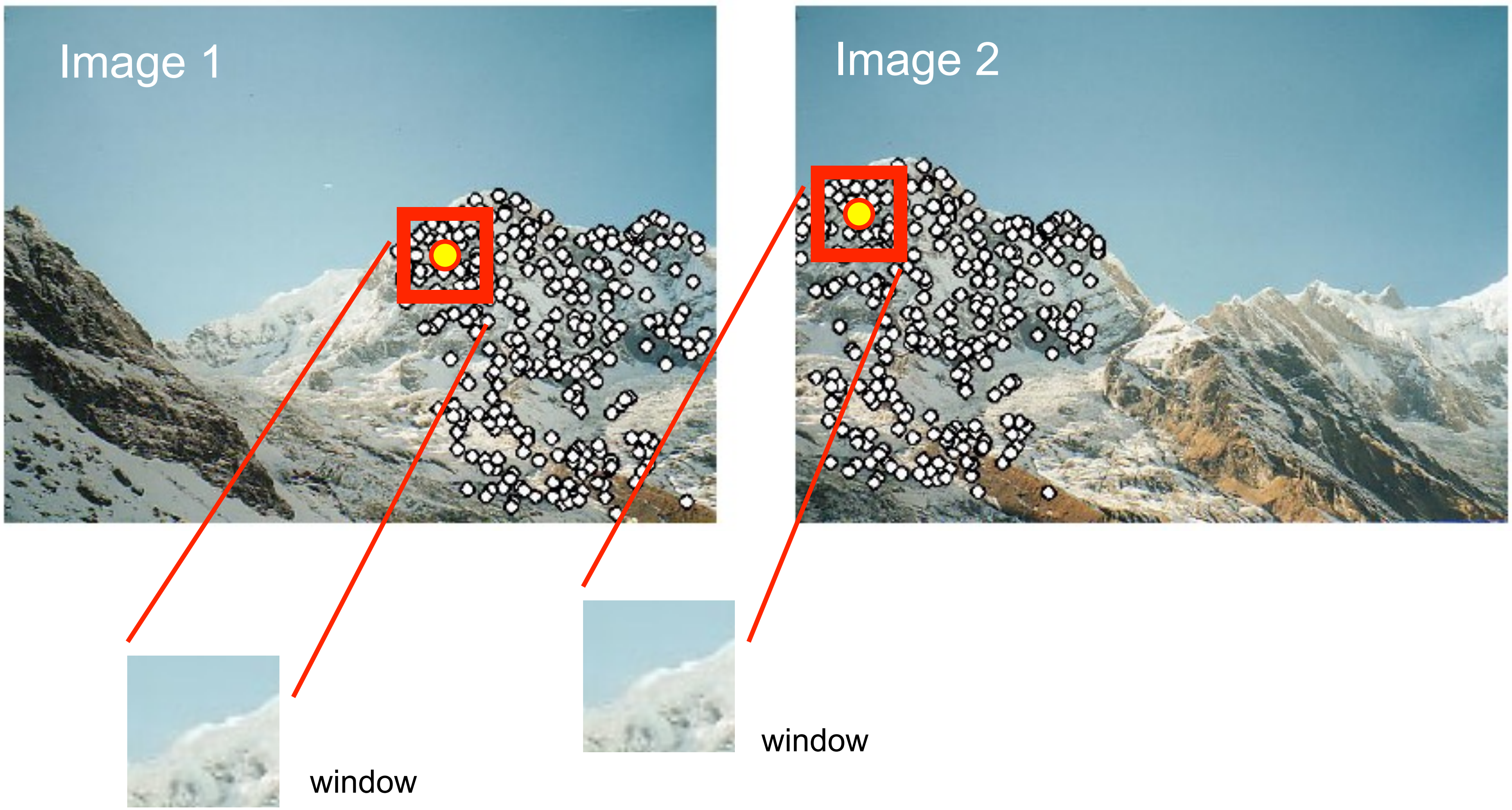
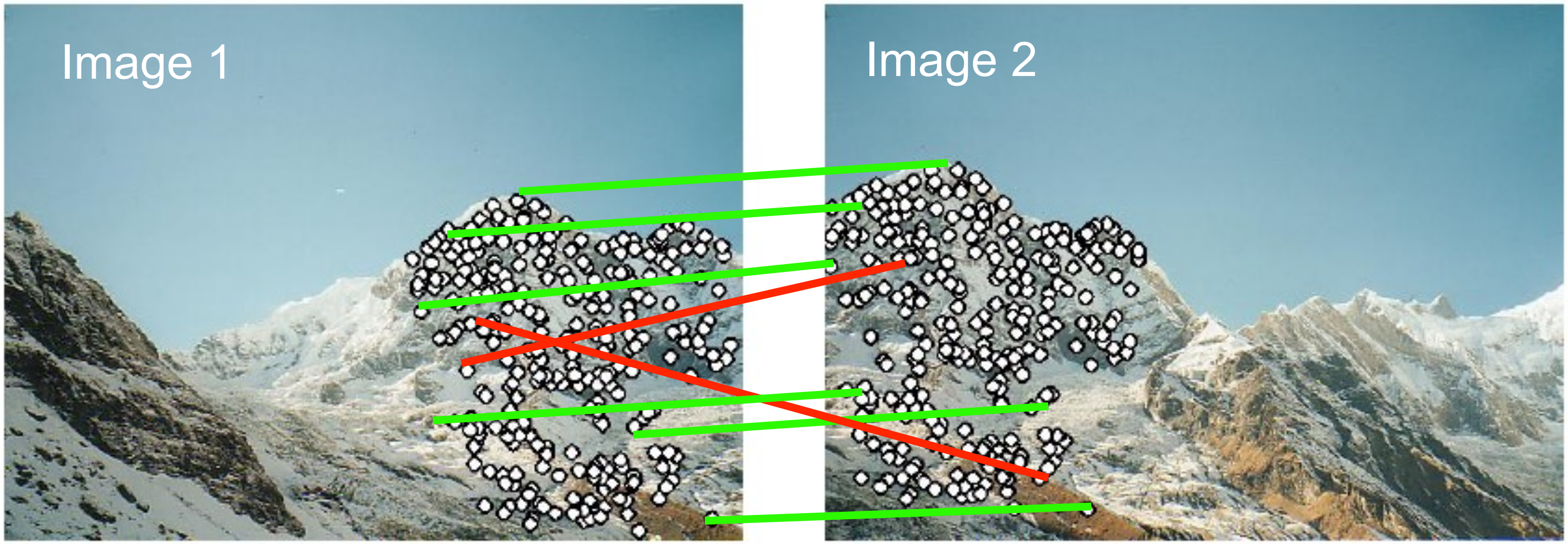
Fitting helps matching!



Features are matched (for instance, based on correlation)

Fitting helps matching!



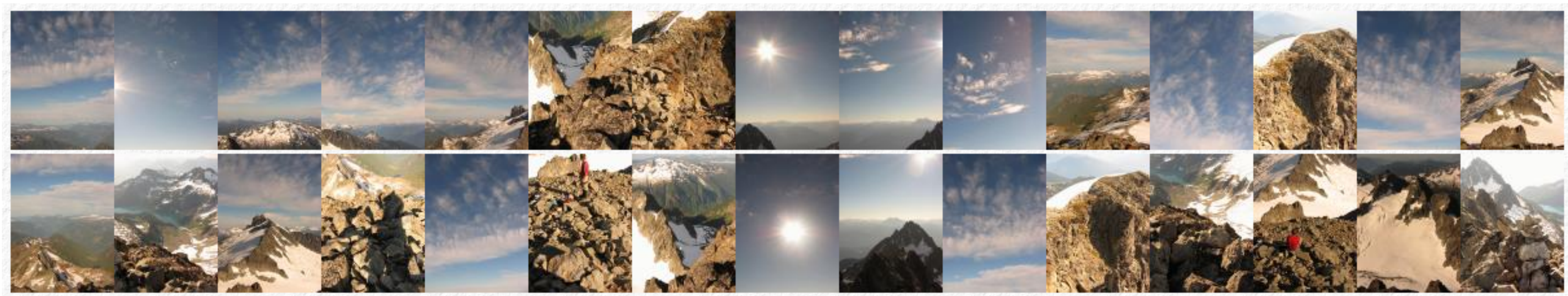
Matches based on appearance only
Green: good matches
Red: bad matches

Idea:
- Fitting an homography H (by RANSAC) mapping features from images 1 to 2
- Bad matches will be labeled as outliers (hence rejected)!

Fitting helps matching!

Recognising Panoramas

M. Brown and D. G. Lowe. Recognising Panoramas. In Proceedings of the *9th International Conference on Computer Vision -- ICCV2003*



Next lecture:
Feature detectors and descriptors

Least squares methods

- fitting a line -

$$Ax = b$$

- More equations than unknowns
- Look for solution which minimizes $\|Ax-b\| = (Ax-b)^T(Ax-b)$
- Solve $\dfrac{\partial (Ax-b)^T (Ax-b)}{\partial x_i} = 0$
- LS solution

$$x = (A^T A)^{-1} A^T b$$

Least squares methods

- fitting a line -

Solving $x = (A^t A)^{-1} A^t b$

$\mathrm{A}^{+} = (\mathrm{A}^{t}\mathrm{A})^{-1}\mathrm{A}^{t}$ = pseudo-inverse of A

$\mathrm{A} = \mathrm{U}\sum \mathrm{V}^{t}$ = SVD decomposition of A

$$A^{-1} = V \sum^{-1} U^{T}$$

$$A^{+} = V \sum^{+} U^{T}$$

with $\sum^{+}$ equal to $\sum^{-1}$ for all nonzero singular values and zero otherwise

Least squares methods

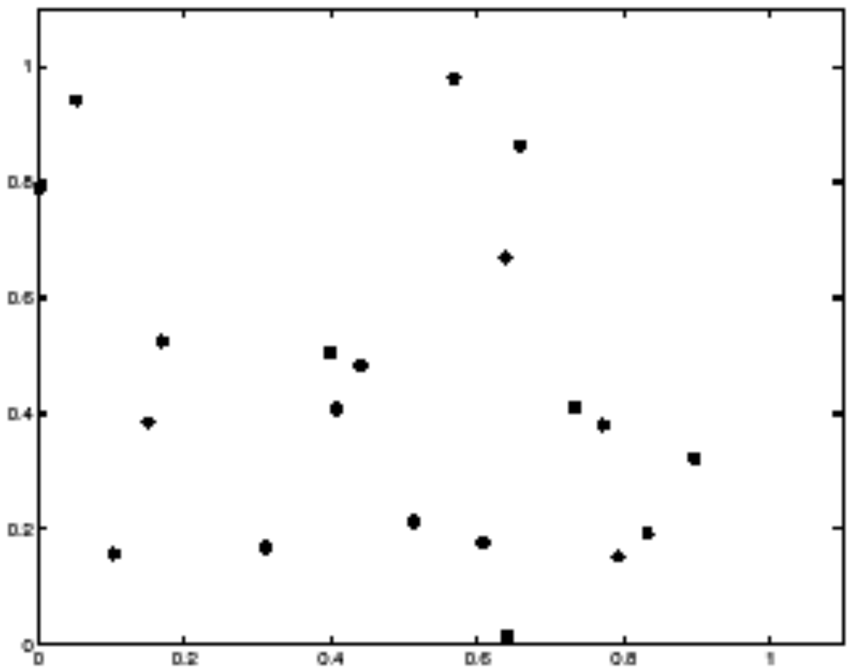
- fitting an homography -

$$
\begin{aligned}
h_{11}x + h_{12}y + h_{13} - h_{31}xx' - h_{32}yx' - x' &= 0 \\
h_{21}x + h_{22}y + h_{23} - h_{31}xy' - h_{32}yy' - y' &= 0
\end{aligned}
$$

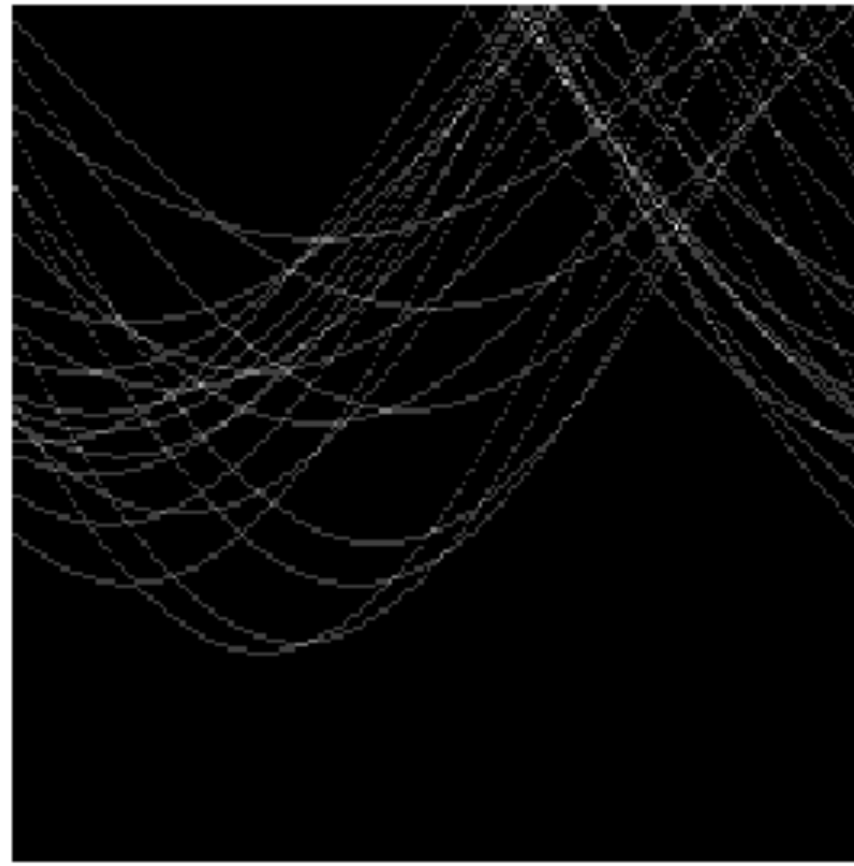
From n>=4 corresponding points: $\boxed{A\,h = 0}$

$$
\begin{pmatrix}
x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' & -x_1' \\
0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' & -y_1' \\
x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2x_2' & -y_2x_2' & -x_2' \\
0 & 0 & 0 & x_2 & y_2 & 1 & -x_2y_2' & -y_2y_2' & -y_2' \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
x_n & y_n & 1 & 0 & 0 & 0 & -x_nx_n' & -y_nx_n' & -x_n' \\
0 & 0 & 0 & x_n & y_n & 1 & -x_ny_n' & -y_ny_n' & -y_n'
\end{pmatrix}
\begin{bmatrix}
h_{1,1} \\ h_{1,2} \\ \vdots \\ h_{3,3}
\end{bmatrix}
= 0
$$

Hough transform - experiments



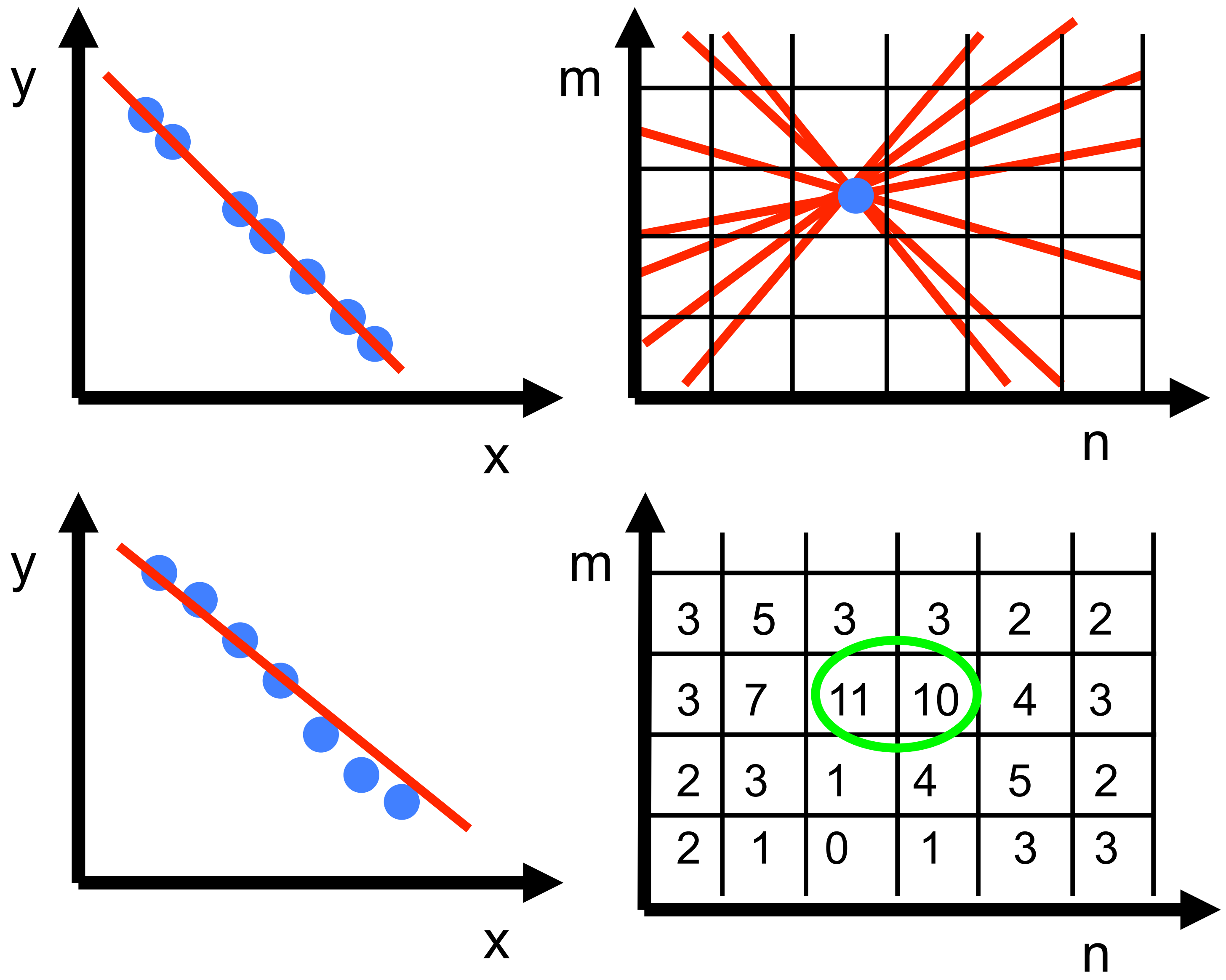
features



votes

Issue: spurious peaks due to uniform noise

Hough transform



Fitting helps matching!

Images courtesy of Brandon Lloyd