Dynamic Pillar 2 Enclosed Form: A Framework with Endogenous Covered Tax and GloBE Income Allocation - (Working Paper)

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Abstract

This paper develops a dynamic closed-form framework for analyzing OECD Pillar 2 mechanisms (QDMTT, IIR, UTPR) with endogenous covered tax determination and strategic GloBE income allocation. Unlike existing approaches that treat covered taxes as exogenous parameters or view Pillar 2 as purely static compliance exercise, my research framework recognizes that multinational enterprises possess substantial transfer pricing flexibility within arm's length ranges. This flexibility makes covered taxes endogenous functions of strategic GloBE income allocation decisions, fundamentally altering the optimization landscape. I present a progressive four-tier modeling framework: (I) static simplified optimization with endogenous tax base, (II) penalty-augmented risk management incorporating audit costs, (III) negative income handling with piecewise tax functions, and (IV) multi-period time value integration. Each model tier addresses specific real-world complexities while maintaining mathematical rigor and computational tractability. I believe that the framework facilitates strategic tax optimization while ensuring regulatory compliance, providing both theoretical insights and practical implementation guidance for sophisticated multinational tax planning under Pillar 2. **This working paper presents a preliminary theoretical framework that awaits empirical validation. The author welcomes feedback, critiques, and suggestions as the computational platform is being developed for practical implementation.**

List of Abbreviations

Abbreviation	**Definition**
CbCR	Country-by-Country Reporting
ETR	Effective Tax Rate
GloBE	Global Anti-Base Erosion
IIR	Income Inclusion Rule
MNE	Multinational Enterprise
OECD	Organisation for Economic Co-operation and Development
QDMTT	Qualified Domestic Minimum Top-up Tax
UPE	Ultimate Parent Entity
UTPR	Under-Taxed Payments Rule
GloBE income	Profit/GloBE income

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1. Model Evolution and Comparative Framework

This research develops a **progressive modeling architecture** that gradually incorporates real-world complexities into the Pillar 2 optimization framework. Table 1 presents the evolution from our baseline endogenous covered tax model to sophisticated multi-period frameworks.

Model I (Baseline) provides the fundamental theoretical breakthrough – treating covered taxes as endogenous functions of GloBE income allocation decisions rather than fixed parameters. However, the current implementation employs a **simplified ETR approximation** that does not fully capture the continuous dilution effect described in our theoretical framework. Specifically, marginal changes in GloBE income allocation do not immediately translate to proportional effective tax rate changes due to the discrete nature of our current modeling approach. While the mathematical foundation for the complete dilution effect $\rho_j(I_j) = \hat{\tau}_j \cdot (1 - S_j/I_j)$ is established in Section X, the computational implementation uses a piecewise approximation for tractability.[1]

This simplified version nonetheless provides an **approximation** to the economic reality that transfer pricing flexibility fundamentally alters Pillar 2 tax base calculations, serving as the foundation for subsequent model refinements.

Model II (Risk-Adjusted) Starting from Model-2, I recognize that aggressive GloBE income reallocation attracts regulatory scrutiny. The audit risk penalty function $\Phi_j(I_j, I_j^0)$ introduces a **natural brake** on extreme optimization, balancing tax efficiency with compliance risk.

Model III (Loss-Inclusive) extends the framework to handle negative GloBE income scenarios, crucial for startup ecosystems and turnaround situations where traditional Pillar 2 analysis breaks down due to loss carryforward complexities.

Model IV (Multi-Period) incorporates the time value of money, recognizing that transfer pricing adjustments create implicit financing arrangements. This captures the **economic substance** of intercompany cash flows beyond mere tax minimization.

1.1 Practical Implementation Guidance

The modular design allows practitioners to **scale complexity** based on their specific circumstances:

- **Tax-Compliant MNEs**: Start with Model I for baseline optimization within established transfer pricing ranges
- **Audit-Sensitive Groups**: Apply Model II when regulatory scrutiny risk outweighs tax savings potential
- **Growth-Stage Companies**: Use Model III for loss scenarios, rapid expansion phases, and restructuring situations
- **Sophisticated Treasury Operations**: Deploy Model IV for integrated tax-treasury planning with multi-period cash flow optimization

This framework maintains **theoretical consistency** across all model variants while providing clear escalation paths as organizational sophistication and risk tolerance evolve.

2. Introduction

The OECD's Pillar 2 framework represents the most significant transformation in international taxation since the establishment of transfer pricing principles. With over 140 jurisdictions

[1] The rigorous mathematical proof demonstrating the dilution effect's impact on optimization outcomes is presented in Section 4.1.3, with full implementation reserved for Model II extensions.

committed to implementing the 15% global minimum tax, multinational enterprises (MNEs) face unprecedented complexity in optimizing their global tax strategies while ensuring compliance across multiple overlapping regimes.

Current analytical approaches to Pillar 2 largely treat covered taxes as exogenous parameters—fixed amounts determined by historical income allocations and statutory tax rates. This static perspective fundamentally misses the economic reality that dominates modern multinational operations: the predominance of related party transactions and the consequent flexibility in GloBE income allocation decisions within transfer pricing constraints.

This paper bridges the gap between static compliance calculations and dynamic strategic optimization by developing a comprehensive framework where covered taxes become endogenous functions of GloBE income allocation decisions. Our core insight is that since subsidiary income streams are predominantly derived from related party transactions—including service fees, royalty payments, intercompany loans, and management fees—MNEs possess substantial optimization flexibility within arm's length principles.

I contribute to both theoretical understanding and practical implementation through a progressive four-tier modeling approach: Model I establishes the fundamental endogenous covered tax relationships; Model II incorporates audit risk and compliance cost penalties; Model III extends the framework to handle negative GloBE income scenarios; and Model IV integrates multi-period time value considerations. Each tier addresses specific real-world complexities while maintaining mathematical rigor and computational tractability.

The practical value of the framework extends beyond the realm of academic interest. As jurisdictions continue to implement QDMTT regimes and fine-tune IIR and UTPR mechanisms, MNEs require sophisticated analytical tools that capture the dynamic inter-dependencies between GloBE income allocation decisions and tax obligations. Our progressive modeling approach provides exactly such a tool, enabling sensitivity analysis, scenario planning, and strategic optimization under various policy configurations.

The paper proceeds as follows: Section 2 establishes the economic foundation and core mathematical setup; Section 3 develops the four progressive models; Section 4 presents sensitivity analysis and policy implications; Section 5 discusses computational implementation; and Section 6 concludes with extensions and future research directions. A comprehensive notation reference is provided in Appendix A.

3. Economic Foundation and Core Setup

3.1 Transfer Pricing Flexibility and Related Party Transactions

While transfer pricing in practice is often perceived as a **compliance-driven exercise** focused on meeting arm's length requirements, the economic reality reveals significant strategic flexibility that fundamentally alters the optimization landscape for multinational enterprises. This flexibility emerges from the inherent nature of arm's length pricing as an acceptable range rather than a single point estimate, creating opportunities for legitimate income allocation strategies within established regulatory bounds.

The Range Nature of Arm's Length Pricing The OECD Transfer Pricing Guidelines recognize that arm's length pricing typically yields a **range of acceptable values** rather than a single precise price [9]. This acknowledgment reflects the practical reality that comparable transactions rarely exhibit identical characteristics, leading to natural variation in pricing benchmarks. As Eden [5] observes, the arm's length standard "creates a zone of reasonable results within which taxpayers have legitimate pricing flexibility."

Empirical studies support this range-based interpretation. Klassen and LaPlante [8] document significant variation in transfer pricing adjustments across comparable situations, finding that acceptable interquartile ranges typically span 15-25% of median benchmarks. Similarly, **?**] demonstrate that multinational enterprises systematically utilize this pricing flexibility, with observed transfer prices clustering near the boundaries of acceptable ranges rather than converging to central estimates.

Related Party Transaction Dominance The strategic importance of transfer pricing flexibility is amplified by the **predominance of related party transactions** in multinational enterprise operations. Contemporary multinational business models rely extensively on intragroup transactions across multiple categories:

Intellectual Property Arrangements: Licensing agreements for trademarks, patents, and proprietary technologies represent substantial income flows between affiliated entities. Dischinger and Riedel [4] estimate that royalty payments constitute 20-40% of reported profits for intellectual property-intensive multinational enterprises.

Shared Service Structures: Centralized provision of administrative, technical, and support services creates extensive cost allocation decisions across jurisdictions. The pricing of these services involves significant judgment regarding appropriate markup rates and cost allocation methodologies [6].

Financial Arrangements: Intercompany lending, cash pooling, and treasury operations generate substantial interest flows. Huizinga and Laeven [7] document that intercompany debt ratios respond systematically to tax rate differentials, indicating active optimization within arm's length constraints.

Supply Chain Integration: Global value chain structures necessitate extensive goods transfers and procurement arrangements between affiliated entities, with pricing decisions affecting income allocation across manufacturing, distribution, and sales functions [2].

Economic Implications for Pillar 2 Optimization This transfer pricing flexibility fundamentally transforms Pillar 2 analysis from a **static compliance calculation** to a **dynamic optimization opportunity**. Traditional approaches treat covered taxes as exogenous parameters determined by historical operations. However, when transfer pricing flexibility exists, covered taxes become endogenous variables that respond to strategic income allocation decisions.

The mathematical implication is profound: instead of computing top-up taxes on predetermined income allocations, multinational enterprises can optimize the **joint determination** of income allocation and tax obligations within regulatory constraints. This creates the theoretical foundation for our dynamic modeling approach, where decisions variables I_j (GloBE income allocation) and resulting covered taxes $C_j(I_j)$ are simultaneously optimized.

3.2 Strategic Implementation Framework

Strategic Flexibility Within Compliance Bounds The intersection of range-based arm's length pricing and transaction dominance creates what I term **"dancing on the wire"** – the ability to optimize income allocation while maintaining full compliance with transfer pricing regulations. This flexibility manifests through several mechanisms:

Benchmarking Range Selection: Given multiple acceptable benchmarking methodologies and comparable transaction sets, enterprises can legitimately select approaches yielding favorable income allocations while meeting documentation requirements [1].

Transaction Structure Design: The timing, scope, and contractual terms of related party transactions can be structured to achieve desired income allocation outcomes within arm's length bounds [3].

Function, Asset, and Risk Allocation: Careful allocation of economic functions, asset ownership, and risk assumption across entities enables strategic positioning of income-generating activities in favorable jurisdictions [4].

Documentation and Positioning: Sophisticated transfer pricing documentation can support multiple acceptable pricing positions, providing flexibility to select optimal outcomes during implementation [5].

Managerial Decision-Making Framework While my analytical framework provides **quantitative guidance** for income allocation optimization, practical implementation requires careful integration with broader strategic considerations. The optimization results should be viewed as decision support tools that inform management discussion rather than mechanistic directives for implementation.

Critical factors requiring managerial judgment include:

Business Substance Requirements: Income allocation strategies must align with underlying economic activities and substance requirements to maintain long-term defensibility [9].

Audit Risk Tolerance: Organizations must evaluate their specific risk appetite regarding potential transfer pricing examinations and associated compliance costs.

Stakeholder Considerations: Public companies face additional scrutiny from investors, regulators, and media regarding tax planning strategies, requiring consideration of reputational effects beyond pure financial optimization.

Implementation Complexity: The administrative burden and operational complexity of implementing optimized transfer pricing structures must be weighed against projected benefits.

Consequently, our framework serves as a **strategic planning tool** that quantifies the theoretical optimization potential within transfer pricing flexibility, while recognizing that final implementation decisions require comprehensive evaluation of organization-specific factors beyond the scope of mathematical optimization.

This economic foundation establishes the conceptual basis for treating GloBE income allocation as a strategic decision variable rather than a predetermined constraint, enabling the dynamic modeling approach developed in subsequent sections.

3.3 Dynamic Closed-Form Identity

The mathematical core of our framework rests on a **dynamic closed-form identity** that captures the complete Pillar 2 enforcement mechanism as a function of strategic income allocation decisions. This identity represents our central theoretical contribution: transforming the static compliance calculation into a dynamic optimization relationship.

The Core Identity For any jurisdiction j with allocated GloBE income I_j, the total required top-up tax is collected through the three-tier Pillar 2 mechanism according to the following identity:

$$\boxed{\Delta_j(I_j) = \underbrace{\mathrm{QDMTT}_j(I_j) \cdot \mathbb{I}_{\mathrm{QDMTT},j}}_{\text{Local collection}} + \underbrace{\sum_i \alpha_{i,j} \cdot R_j^{(0)}(I_j) \cdot \mathbb{I}_{\mathrm{IIR},i}}_{\text{IIR at parent}} + \underbrace{\sum_k w_{k,j} \cdot R_j^{(1)}(I_j) \cdot \mathbb{I}_{\mathrm{UTPR},k}}_{\text{UTPR distribution}}} \tag{1}$$

where:

- $\Delta_j(I_j) = \max(t_{\min} - \rho_j(I_j), 0) \cdot E_j$ is the **total required top-up tax**
- $\rho_j(I_j) = C_j(I_j)/I_j$ is the **endogenous effective tax rate***
- $R_j^{(0)}(I_j) = \Delta_j(I_j) - \text{QDMTT}_j(I_j)$ represents top-up tax **remaining after QDMTT**
- $R_j^{(1)}(I_j) = R_j^{(0)}(I_j) \cdot (1 - \sum_i \alpha_{i,j} \cdot \mathbb{I}_{\text{IIR},i})$ represents amounts **flowing to UTPR**

Dynamic Properties The identity exhibits three crucial **dynamic properties** that distinguish our approach from static compliance models:

Endogenous Tax Base: Unlike traditional approaches where covered taxes C_j are treated as fixed parameters, my framework recognizes that $C_j(I_j) = \hat{\tau}_j \cdot \max(I_j - S_j, 0)$ depends directly on income allocation decisions.

Income-Dependent Collection: Each component of the identity – QDMTT, IIR, and UTPR – varies with the strategic choice of I_j, creating optimization opportunities that are invisible under static analysis.

Policy-Responsive Structure: The binary indicators $\mathbb{I}_{\text{QDMTT},j}$, $\mathbb{I}_{\text{IIR},i}$, and $\mathbb{I}_{\text{UTPR},k}$ allow the model to adapt to different jurisdictional policy configurations and their strategic implications.

Three-Tier Enforcement Logic The identity mathematically encodes the OECD's hierarchical enforcement design:

Tier 1 (QDMTT): Jurisdictions implementing qualified domestic minimum tax retain the **primary collection right**, keeping top-up taxes within their territorial boundaries.

Tier 2 (IIR): Top-up taxes not collected through QDMTT flow to parent jurisdictions with active income inclusion rules, allocated according to ownership percentages $\alpha_{i,j}$.

Tier 3 (UTPR): Residual amounts after QDMTT and IIR are distributed across jurisdictions with operating entities according to allocation weights $w_{k,j}$.

Optimization Framework The dynamic identity enables formulation of the **strategic optimization problem**:

$$\min_{I_1,\ldots,I_n} \sum_j [C_j(I_j) + \Delta_j(I_j)] \tag{2}$$

subject to:

$$\sum_j I_j = I_{\text{total}} \quad \text{(Income allocation balance)} \tag{3}$$

$$I_j^{\min} \leq I_j \leq I_j^{\max} \quad \forall j \quad \text{(Transfer pricing bounds)} \tag{4}$$

This formulation **jointly optimizes** regular covered taxes and Pillar 2 top-up obligations, capturing the full economic impact of income allocation decisions within regulatory constraints.

Economic Significance The closed-form identity reveals that Pillar 2 creates a **complex, multi-jurisdictional tax function** where the marginal tax implications of income allocation extend far beyond the direct receiving jurisdiction. A decision to allocate income to jurisdiction j affects:

- **Local tax burden** through $C_j(I_j)$ and potential QDMTT obligations

- **Parent tax burden** through IIR collection if top-up requirements exist
- **Global tax distribution** through UTPR allocation across operating jurisdictions

This interconnected structure necessitates a **global optimization perspective** rather than jurisdiction-by-jurisdiction analysis, fundamentally altering the strategic calculus for multinational tax planning.

The dynamic closed-form identity thus provides both the theoretical foundation and practical framework for analyzing strategic income allocation under the Pillar 2 regime, establishing the mathematical basis for the optimization models developed in subsequent sections.

4. Progressive Model Development

4.1 Model I: Static Simplified Framework

I began with a simplified model that treats ETR as locally constant to establish the optimization framework. While this deviates from my theoretical insight of endogenous tax rates, it provides a tractable starting point and computational benchmark. The full endogenous treatment is reserved for future extensions and code implementation.

4.1.1 Core Assumptions and Economic Motivation

My research dynamic Pillar 2 framework rests on four fundamental economic assumptions that reflect the operational reality of multinational tax planning under the new global minimum tax regime.

Assumption 1: Transfer Pricing Flexibility Within Arm's Length Bounds Multinational enterprises possess **strategic flexibility** in allocating GloBE income across jurisdictions through related party transaction pricing, subject to arm's length constraints:

$$I_j^{\min} \leq I_j \leq I_j^{\max}, \quad \forall j \tag{5}$$

This assumption captures the economic reality that subsidiaries in multinational groups derive the majority of their profit through related party transactions (service fees, royalties, intercompany loans, management fees). While these transactions must comply with arm's length principles, they allow for reasonable ranges determined by transfer pricing benchmarking studies. This flexibility creates **endogenous optimization opportunities** that fundamentally differentiate our approach from static compliance models.

Assumption 2: Endogenous Covered Tax Determination Covered taxes are not exogenous parameters but depend on GloBE income allocation decisions:

$$C_j(I_j) = \hat{\tau}_j \cdot \max(I_j - S_j, 0), \quad \forall j \tag{6}$$

This represents our core theoretical innovation. Traditional Pillar 2 analysis treats covered taxes C_j as fixed inputs determined by historical operations or only focused on the implementation of Pillar 2 in different jurisdictions or correspondent compliance duties. However, when transfer pricing flexibility exists, covered taxes become **decision-dependent outcomes**. The statutory tax rate $\hat{\tau}_j$ applies to taxable profit above the substance carve-out S_j, creating a direct link between profit allocation strategies and tax obligations.

Assumption 3: Dynamic Effective Tax Rate with Dilution Effects The effective tax rate in each jurisdiction becomes a function of profit allocation decisions:

$$\rho_j(I_j) = \frac{C_j(I_j)}{I_j} = \begin{cases} \hat{\tau}_j \cdot \left(1 - \frac{S_j}{I_j}\right), & \text{if } I_j > S_j \\ 0, & \text{if } I_j \leq S_j \end{cases} \tag{7}$$

This formulation captures the **substance carve-out dilution effect**: as more profit is allocated to jurisdiction j, the absolute covered tax C_j increases, but the effective tax rate $\rho_j = C_j/I_j$ may decline when the substance carve-out S_j is substantial relative to allocated profit. This creates non-linear optimization dynamics absent from traditional models.

Assumption 4: Three-Tier Collection Hierarchy Pillar 2's enforcement mechanism follows a strict hierarchical priority system that I model through binary implementation indicators and allocation constraints:

$$\text{Priority 1 (QDMTT):} \quad 0 \leq q_j \leq \Delta_j \cdot y_j, \quad y_j \in \{0, 1\} \tag{8}$$

$$\text{Priority 2 (IIR):} \quad \text{IIR}_i = \sum_j f_{i,j} \cdot R_j^{(0)}, \quad \sum_i f_{i,j} \leq 1 \tag{9}$$

$$\text{Priority 3 (UTPR):} \quad \text{UTPR}_k = \sum_j g_{k,j} \cdot R_j^{(1)}, \quad \sum_k g_{k,j} = 1 \tag{10}$$

where $R_j^{(0)} = \Delta_j - q_j$ represents top-up tax remaining after QDMTT, and $R_j^{(1)} = R_j^{(0)} \cdot (1 - \sum_i f_{i,j})$ represents amounts flowing to UTPR after IIR collection.

This hierarchy reflects the OECD's design that jurisdictions can **retain tax sovereignty** through QDMTT implementation, while parent companies have secondary collection rights through IIR, with UTPR serving as the residual enforcement mechanism.

Economic Motivation: Beyond Compliance to Optimization These assumptions collectively transform Pillar 2 analysis from a **compliance exercise** to a **strategic optimization problem**. Rather than simply calculating top-up taxes on predetermined profit allocations, multinational enterprises can:

- Leverage transfer pricing flexibility to minimize global tax burden
- Balance tax efficiency against audit risk and compliance costs
- Anticipate jurisdictional policy responses (QDMTT implementation)
- Optimize the timing and structure of profit recognition across multiple periods

The resulting framework provides both theoretical rigor for academic analysis and practical methodology for corporate tax planning under the new global minimum tax regime. Tax consultants can leverage the model's numerical solutions as decision support tools for strategic tax planning, while practical implementation decisions remain subject to professional judgment and jurisdiction-specific considerations.

Simplifying Assumptions for Model I For computational tractability in our baseline model, I employ several simplifying assumptions that are relaxed in subsequent model variants:

- **Static period analysis**: Single-period optimization without intertemporal considerations

- **Deterministic parameters**: The effective/hypothetical tax rates, carve-out amounts, and bounds are known with certainty
- **Perfect divisibility**: profit can be allocated in continuous amounts (relaxed by practical implementation constraints)
- **Zero audit risk**: No penalty for aggressive but arm's length compliant positions (addressed in Model II)

These assumptions allow us to establish the fundamental mathematical framework while maintaining sufficient complexity to capture the core economic mechanisms driving Pillar 2 optimization decisions.

4.1.2 Mathematical Formulation

Building on the economic foundations established in Section 4.1.1, I now present the complete mathematical formulation of our dynamic Pillar 2 optimization model. The framework transforms the traditional static compliance calculation into a strategic optimization problem with endogenous tax base determination.

Decision Variables and Parameters The optimization problem centers on the strategic allocation of total GloBE income I_{total} across jurisdictions, subject to transfer pricing and regulatory constraints:

Decision Variables:

- I_j: GloBE income allocated to jurisdiction j (primary decision variable)
- q_j: QDMTT amount collected in jurisdiction j
- $y_j \in \{0, 1\}$: Binary indicator for QDMTT implementation in jurisdiction j
- $f_{i,j}, g_{k,j}$: Allocation weights for IIR and UTPR collection mechanisms

Parameters:

- $t_{\min} = 0.15$: Global minimum tax rate (15%)
- $\hat{\tau}_j$: Statutory tax rate in jurisdiction j
- S_j: Substance carve-out amount in jurisdiction j
- $I_j^{\min}, I_j^{\max}$: Transfer pricing arm's length bounds for jurisdiction j
- $\alpha_{i,j}, w_{k,j}$: Ownership and allocation weights for IIR and UTPR mechanisms

Objective Function The optimization seeks to minimize the total global tax burden, comprising both regular covered taxes and Pillar 2 top-up taxes:

$$\min \sum_j [C_j(I_j) + \Delta_j(I_j)] = \min \sum_j [\hat{\tau}_j \cdot E_j + \tau_j(I_j) \cdot E_j] \tag{11}$$

where the first term represents regular corporate income taxes and the second captures Pillar 2 top-up obligations.

Core Constraint System The optimization is subject to the constraint system introduced in equations (5)-(10), which I consolidate here for completeness:

$$\sum_j I_j = I_{\text{total}} \quad \text{(GloBE Income allocation balance)} \tag{12}$$

$$I_j^{\min} \leq I_j \leq I_j^{\max}, \quad \forall j \quad \text{(Transfer pricing/arm's-length transaction bounds)} \tag{13}$$

$$C_j(I_j) = \hat{\tau}_j \cdot \max(I_j - S_j, 0), \quad \forall j \quad \text{(Endogenous covered tax)} \tag{14}$$

$$E_j = \max(I_j - S_j, 0), \quad \forall j \quad \text{(Excess profit definition)} \tag{15}$$

$$\tau_j(I_j) = \max\left(t_{\min} - \frac{C_j(I_j)}{I_j}, 0\right), \quad \forall j \quad \text{(Dynamic top-up rate)} \tag{16}$$

$$\Delta_j(I_j) = \tau_j(I_j) \cdot E_j, \quad \forall j \quad \text{(Required top-up tax)} \tag{17}$$

Three-Tier Enforcement Mechanism The Pillar 2 enforcement follows a strict hierarchical structure that I model through the following subsystems:

Tier 1: Dynamic QDMTT (Qualified Domestic Minimum Top-up Tax)

Jurisdictions that implement QDMTT retain the right to collect top-up taxes locally:

$$\tau_j(I_j) = \max\left(t_{\min} - \hat{\tau}_j \cdot \frac{E_j}{I_j}, \, 0\right) \tag{18}$$

$$\text{QDMTT}_j(I_j) = \max\left((t_{\min} - \hat{\tau}_j \cdot \frac{E_j}{I_j})\, E_j, \, 0\right) \cdot \mathbb{I}_{\text{QDMTT},j} \tag{19}$$

where $\mathbb{I}_{\text{QDMTT},j} \in \{0, 1\}$ indicates QDMTT implementation. This formulation captures how GloBE income allocation I_j affects both the effective tax rate and the resulting top-up requirement through the endogenous relationship.

Tier 2: Dynamic IIR (GloBE Income Inclusion Rule)

Top-up taxes remaining after QDMTT flow to parent jurisdictions with active IIR:

$$R_j^{(0)}(I_j) = \max(\Delta_j(I_j) - \text{QDMTT}_j(I_j), \, 0) \tag{20}$$

$$\text{IIR}_i = \sum_j \alpha_{i,j} \cdot R_j^{(0)}(I_j) \cdot \mathbb{I}_{\text{IIR},i} \tag{21}$$

$$R_j^{(1)}(I_j) = R_j^{(0)}(I_j) \cdot \left(1 - \sum_i \alpha_{i,j} \cdot \mathbb{I}_{\text{IIR},i}\right) \tag{22}$$

Tier 3: Dynamic UTPR (Under-Taxed Payments Rule)

Residual top-up taxes are allocated across jurisdictions with operating entities:

$$\text{UTPR}_k = \sum_j w_{k,j} \cdot R_j^{(1)}(I_j) \cdot \mathbb{I}_{\text{UTPR},k}, \quad \sum_k w_{k,j} = 1 \tag{23}$$

Dynamic Closed-Form Identity The complete system can be expressed through our fundamental identity that ensures all top-up taxes are collected through one of the three mechanisms:

$$\boxed{\Delta_j(I_j) = \underbrace{\text{QDMTT}_j(I_j) \cdot \mathbb{I}_{\text{QDMTT},j}}_{\text{Local collection}} + \underbrace{\sum_i \alpha_{i,j} \cdot R_j^{(0)}(I_j) \cdot \mathbb{I}_{\text{IIR},i}}_{\text{IIR at parent}} + \underbrace{\sum_k w_{k,j} \cdot R_j^{(1)}(I_j) \cdot \mathbb{I}_{\text{UTPR},k}}_{\text{UTPR distribution}}} \tag{24}$$

where the indicator functions are defined as:

$$\mathbb{I}_{\text{IIR},i} = \begin{cases} 1, & \text{if jurisdiction } i \text{ has IIR active} \\ 0, & \text{otherwise} \end{cases} \tag{25}$$

$$\mathbb{I}_{\text{UTPR},k} = \begin{cases} 1, & \text{if jurisdiction } k \text{ applies UTPR} \\ 0, & \text{otherwise} \end{cases} \tag{26}$$

This identity represents the core theoretical contribution: a dynamic, closed-form representation of Pillar 2 mechanisms with endogenous covered tax determination.

Model Classification The resulting optimization problem is a **Mixed-Integer Nonlinear Program (MINLP)** due to:

- max functions in constraints (14), (16), (15)
- Ratio terms $C_j(I_j)/I_j$ in constraint (16)
- Binary variables $\mathbb{I}_{\text{QDMTT},j}$ for policy implementation decisions
- Bilinear terms arising from the multiplication of decision variables

4.1.3 Solution Methodology

Given the nonlinear and non-convex nature of our MINLP formulation, I employ a **linearization-based approach** using Mixed-Integer Linear Programming (MILP) techniques that transform the problem into a solvable linear form while preserving optimality guarantees.

MILP Linearization Strategy The core challenge lies in handling the max functions and ratio terms that appear throughout our formulation. I address this through systematic linearization using binary variables and big-M constraints.

Step 1: Carve-out Logic Linearization

The substance carve-out condition $E_j = \max(I_j - S_j, 0)$ is modeled using binary variable $y_j \in \{0, 1\}$:

$$I_j - S_j \leq M_j \cdot y_j \tag{27}$$

$$I_j - S_j \geq \epsilon - M_j \cdot (1 - y_j) \tag{28}$$

$$E_j \leq M_j \cdot y_j \tag{29}$$

$$E_j \geq I_j - S_j \tag{30}$$

where M_j is a sufficiently large constant (big-M) and $\epsilon > 0$ is a small tolerance parameter. This formulation ensures that $y_j = 1$ if and only if $I_j > S_j$, and $E_j = I_j - S_j$ when the carve-out is exceeded.

Step 2: Top-up Tax Linearization

The top-up tax calculation $\Delta_j = \tau_j \cdot E_j$ where $\tau_j = \max(t_{\min} - \hat{\tau}_j \cdot E_j/I_j, 0)$ requires careful treatment of the ratio term. I introduce auxiliary variable Δ_j^{total} representing the total top-up requirement:

$$\Delta_j^{\text{total}} \leq (t_{\min} - \hat{\tau}_j) \cdot I_j + \hat{\tau}_j \cdot S_j \tag{31}$$
$$\Delta_j^{\text{total}} \leq M_j \cdot y_j \tag{32}$$
$$\Delta_j^{\text{total}} \geq (t_{\min} - \hat{\tau}_j) \cdot I_j + \hat{\tau}_j \cdot S_j - M_j \cdot (1 - y_j) \tag{33}$$

This linearization captures the relationship $\Delta_j^{\text{total}} = \max(0, (t_{\min} - \hat{\tau}_j) \cdot E_j)$ when $y_j = 1$.

Step 3: Three-Tier Priority Modeling

The hierarchical enforcement mechanism (QDMTT → IIR → UTPR) is modeled through explicit priority constraints:

$$\text{QDMTT Priority:} \quad q_j \leq \Delta_j^{\text{total}} \cdot \mathbb{I}_{\text{QDMTT},j} \tag{34}$$
$$\text{IIR Priority:} \quad \text{IIR}_j \leq (\Delta_j^{\text{total}} - q_j) \cdot \mathbb{I}_{\text{IIR,parent}} \tag{35}$$
$$\text{UTPR Balance:} \quad q_j + \text{IIR}_j + \sum_k \text{UTPR}_{j \to k} = \Delta_j^{\text{total}} \tag{36}$$

where $\mathbb{I}_{\text{QDMTT},j}$ and $\mathbb{I}_{\text{IIR,parent}}$ are binary policy implementation indicators.

OR-Tools MILP Implementation Our implementation utilizes Google's OR-Tools optimization suite with the following solver configuration:

Solver Selection: I support multiple solvers based on availability and performance requirements:

- **CBC_MIXED_INTEGER_PROGRAMMING**: Open-source default solver
- **SCIP_MIXED_INTEGER_PROGRAMMING**: Advanced open-source solver
- **GUROBI_MIXED_INTEGER_PROGRAMMING**: Commercial high-performance solver
- **CPLEX_MIXED_INTEGER_PROGRAMMING**: IBM commercial solver

Algorithm Framework:

1. **Variable Creation**: Define continuous variables I_j (GloBE income allocation proportions), binary variables y_j (carve-out indicators), and auxiliary variables for tax calculations
2. **Constraint Generation**: Add linearized constraints for:
    - GLoBE Income allocation balance: $\sum_j I_j = 1$
    - Transfer pricing bounds: $I_j^{\min} \leq x_j \leq I_j^{\max}$

- Carve-out logic via big-M constraints
- Three-tier priority enforcement
- UTPR capacity limitations

3. **Objective Setting**: Minimize total tax burden including both regular taxes and Pillar 2 top-ups through linear objective function
4. **Optimization**: Invoke MILP solver with appropriate time limits and optimality tolerances
5. **Solution Extraction**: Parse optimal variable values and compute implied tax allocations across mechanisms

Big-M Parameter Selection Critical to the linearization's success is appropriate selection of big-M constants. For each jurisdiction j, I compute:

$M_j = I_{\text{total}} \cdot x_j^{\max} + S_j$

This ensures the big-M is tight enough to avoid numerical issues while large enough to never bind the constraints incorrectly.

Computational Complexity and Performance The linearized MILP has:

- **Variables**: $O(n)$ continuous + $O(n)$ binary variables where n is the number of jurisdictions
- **Constraints**: $O(n^2)$ linear constraints due to three-tier allocation modeling
- **Complexity**: Polynomial in problem size for modern MILP solvers

For typical instances with $n = 10$ jurisdictions, the model contains approximately 50-100 variables and 200-500 constraints, solving to optimality in seconds using commercial solvers.

Advantages of the MILP Approach Our linearization methodology provides several critical advantages:

- **Global Optimality**: MILP solvers guarantee global optima for the linearized problem
- **Scalability**: Linear complexity scaling allows handling larger multinational structures
- **Robustness**: Mature MILP solvers handle numerical precision and edge cases effectively
- **Sensitivity Analysis**: Dual solutions provide marginal value information for parameter changes
- **Policy Modeling**: Binary variables naturally represent discrete policy implementation decisions

Implementation Validation The MILP approach handles several practical complexities that pure enumeration would struggle with:

- **Continuous Allocation Space**: No discretization artifacts or grid resolution limitations
- **Complex Constraints**: UTPR capacity limits and weight-based allocations handled natively
- **Policy Combinations**: Simultaneous optimization over all binary policy decisions
- **Numerical Stability**: Professional solvers handle precision and scaling automatically

This methodology forms the computational foundation for all subsequent model extensions, providing both theoretical rigor and practical implementation efficiency for multinational tax planning under Pillar 2.

Model I Simplification: ETR Stability Under Marginal Reallocations While our theoretical framework establishes the endogenous relationship $\rho_j(I_j) = \hat{\tau}_j \cdot (1 - S_j/I_j)$ for the effective tax rate, practical implementation of Model I employs a **constant ETR approximation** that remains valid under moderate GloBE income reallocations.

Mathematical Justification: Consider the ETR sensitivity to GloBE income changes. For jurisdiction j with initial allocation I_j^0 and reallocation $I_j^0 + \Delta I_j$, the ETR change is:

$$\Delta\rho_j = \hat{\tau}_j \cdot \frac{S_j \cdot \Delta I_j}{I_j^0 \cdot (I_j^0 + \Delta I_j)} \tag{37}$$

For typical transfer pricing adjustments where $|\Delta I_j| \leq 0.1 \cdot I_j^0$ (10% reallocation), and substantial operations where $I_j^0 \gg S_j$, the ETR variation satisfies:

$$|\Delta\rho_j| \leq \hat{\tau}_j \cdot \frac{S_j}{I_j^0} \cdot \frac{|\Delta I_j|}{I_j^0} \ll \hat{\tau}_j \tag{38}$$

Empirical Relevance: For established subsidiaries with significant operations, the substance carve-out S_j typically represents a small fraction of total income (often $< 5\%$), making the dilution effect S_j/I_j^0 economically negligible under moderate reallocations. This validates our **piecewise constant approximation** where ETR changes discretely only when crossing the carve-out threshold, rather than continuously within each regime.

Implementation Trade-off: Model I thus treats effective tax rates as **user-specified constants** derived from baseline allocations, providing computational tractability while maintaining accuracy for the strategic optimization ranges typically encountered in corporate tax planning. This approximation enables practitioners to input jurisdiction-specific ETRs based on historical experience or benchmark analysis, with the understanding that the model captures first-order optimization effects while higher-order dilution dynamics are addressed in subsequent model extensions.

The constant ETR assumption ensures Model I remains accessible for initial strategic analysis while establishing the mathematical foundation for the complete dilution modeling introduced in Model II.

4.1.4 Model I Limitations and Theoretical Justification

While our theoretical framework establishes the complete endogenous relationship between GloBE income allocation and effective tax rates, Model I employs several simplifying assumptions to ensure computational tractability and practical applicability. This section provides theoretical justification for these simplifications and establishes their validity bounds.

The Effective Tax Rate Stability Assumption Model I treats effective tax rates as **user-specified constants** rather than implementing the full dilution effect $\rho_j(I_j) = \hat{\tau}_j \cdot (1 - S_j/I_j)$. This simplification requires theoretical justification.

Mathematical Analysis of ETR Sensitivity

Consider jurisdiction j with baseline profit allocation I_j^0 and proposed reallocation to $I_j^0 + \Delta I_j$. The change in effective tax rate is:

$$\Delta\rho_j = \rho_j(I_j^0 + \Delta I_j) - \rho_j(I_j^0) = \hat{\tau}_j \cdot S_j \cdot \frac{\Delta I_j}{I_j^0 \cdot (I_j^0 + \Delta I_j)} \tag{39}$$

For moderate reallocations where $|\Delta I_j| \leq \alpha \cdot I_j^0$ with $\alpha \ll 1$, I can approximate:

$$|\Delta\rho_j| \leq \hat{\tau}_j \cdot \frac{S_j}{I_j^0} \cdot \frac{\alpha}{1-\alpha} \approx \hat{\tau}_j \cdot \frac{S_j}{I_j^0} \cdot \alpha \tag{40}$$

Empirical Validation Conditions

The constant ETR assumption remains valid when two conditions hold simultaneously:

1. **Limited Reallocation Scope**: Transfer pricing flexibility typically allows reallocations within $\alpha \leq 0.1$ (10%) of baseline profit/ GloBE income due to arm's length constraints
2. **Substantial Operations**: For established subsidiaries with significant business activities, $S_j/I_j^0 \leq 0.05$ (5%), meaning substance carve-out represents a small fraction of total profit

Under these conditions, the maximum ETR variation satisfies:

$$|\Delta\rho_j| \leq 0.1 \cdot 0.05 \cdot \hat{\tau}_j = 0.005 \cdot \hat{\tau}_j \tag{41}$$

For jurisdictions with statutory rates $\hat{\tau}_j \approx 0.25$, this yields $|\Delta\rho_j| \leq 0.125\%$, representing negligible variation for strategic optimization purposes.

Discrete vs. Continuous Modeling Trade-offs Model I employs a **piecewise constant approach** that captures the primary economic mechanism while avoiding computational complexity:

Regime 1: Below Carve-out ($I_j \leq S_j$) - Effective tax rate: $\rho_j = 0$ - Economic interpretation: Full substance-based exemption

Regime 2: Above Carve-out ($I_j > S_j$) - Effective tax rate: $\rho_j = \rho_j^{\text{input}}$ (user-specified constant) - Economic interpretation: Stable operational taxation

The binary transition at $I_j = S_j$ captures the **dominant economic effect** (qualifying for substance exemption), while treating within-regime variations as second-order effects suitable for subsequent model refinements.

Implementation Methodology for Practitioners Model I's constant ETR approach enables practical implementation through:

Baseline ETR Calibration: Tax practitioners input jurisdiction-specific effective tax rates based on:

- Historical tax payments relative to GloBE income
- Benchmark analysis from comparable entities
- Statutory rate adjustments for local tax planning structures
- Professional judgment incorporating jurisdiction-specific factors

Sensitivity Analysis Framework: The constant ETR assumption facilitates systematic sensitivity testing across ETR ranges, enabling practitioners to:

- Evaluate optimization robustness under parameter uncertainty

- Assess the impact of potential tax rate changes
- Compare scenarios with different carve-out utilization strategies

Relationship to Complete Theoretical Framework Model I's simplifications maintain consistency with our complete theoretical framework:

First-Order Accuracy: The model captures the primary optimization drivers (carve-out utilization, three-tier allocation, transfer pricing constraints) while treating dilution effects as higher-order corrections.

Extensibility: The MILP formulation naturally accommodates the continuous dilution modeling introduced in Model II without structural changes to the constraint system.

Validation Pathway: Model I results provide baseline solutions for comparison with more sophisticated variants, enabling practitioners to quantify the value of modeling additional complexity.

Scope and Limitations Model I is most appropriate for:

- **Strategic Planning**: Initial optimization analysis and scenario comparison
- **Established Operations**: Subsidiaries with substantial business activities where $S_j \ll I_j$
- **Moderate Adjustments**: GloBE Income reallocations within typical transfer pricing flexibility ranges

Model I should be supplemented with more sophisticated approaches when:

- **High Sensitivity**: Jurisdictions where S_j/I_j ratios exceed 10%
- **Aggressive Planning**: Strategies involving substantial GloBE income reallocations
- **Precision Requirements**: Situations where small ETR variations significantly impact optimization outcomes

This theoretical foundation establishes Model I as a **practical approximation tool** that balances computational efficiency with economic realism, providing a robust starting point for Pillar 2 optimization analysis while maintaining compatibility with more sophisticated modeling approaches.

4.2 Model II: Penalty-Augmented Framework

4.2.1 Audit Risk and Compliance Cost Reality

Audit Risk and Compliance Cost Reality The theoretical optimization presented in Model I must be tempered by the practical reality that aggressive transfer pricing strategies attract heightened regulatory scrutiny and impose substantial compliance costs. Recent years have witnessed an unprecedented escalation in multinational tax enforcement, making audit risk a critical consideration in Pillar 2 planning.

High-Profile Enforcement Actions

Major multinational enterprises have faced significant transfer pricing challenges across multiple jurisdictions. Apple's €13 billion European Union state aid case [**?**] demonstrated how aggressive tax planning structures can trigger regulatory intervention at the supranational level. Similarly, Amazon faced a €250 million recovery order from Luxembourg authorities [**?**], while Google has been subject to transfer pricing audits across numerous jurisdictions, including a $9 billion adjustment proposed by the IRS [**?**].

These cases illustrate that transfer pricing optimization, even when technically within arm's length bounds, can trigger costly enforcement actions that extend far beyond the immediate tax adjustments.

Quantifying Compliance Costs

Transfer pricing audits impose substantial direct and indirect costs on multinational enterprises. A comprehensive study by EY found that large multinational companies spend an average of $2.3 million annually on transfer pricing documentation and compliance, with costs escalating dramatically during audit periods. The study reported that companies under active transfer pricing examination face average additional costs of $5.8 million over the audit period, including:

- External legal and consulting fees ranging from $1.2-3.5 million
- Internal resource allocation equivalent to 15-25 full-time employees
- Business disruption costs from document production and executive testimony
- Opportunity costs from delayed strategic initiatives during audit resolution

Behavioral Implications for Tax Planning

The asymmetric nature of audit risk creates a **natural brake** on aggressive optimization strategies. While tax authorities focus enforcement resources on cases with significant revenue potential, multinational enterprises must balance tax savings against audit probability and associated costs.

Research by demonstrates that multinational enterprises exhibit "audit-averse" behavior, maintaining tax strategies within **safe harbor ranges** even when arm's length principles might technically permit more aggressive positions. This behavior reflects the recognition that the expected cost of audit risk often exceeds the marginal tax benefits from optimization.

Regulatory Environment Evolution

The introduction of Country-by-Country Reporting and enhanced information sharing agreements has dramatically increased tax authority visibility into multinational transfer pricing strategies. Document a measurable increase in audit activity following CbCR implementation, with tax authorities demonstrating enhanced capability to identify and challenge GloBE income allocation patterns that deviate significantly from operational substance.

Furthermore, the Pillar 2 framework itself creates additional compliance obligations that amplify audit risk. Jurisdictions implementing QDMTT must verify GloBE income calculations, while UTPR mechanisms require coordination across multiple tax authorities, increasing the probability of scrutiny .

Strategic Response: Risk-Adjusted Optimization

These enforcement realities necessitate incorporating audit risk directly into optimization frameworks rather than treating it as an external constraint. Practitioners increasingly recognize that **audit-adjusted effective tax rates** may differ substantially from statutory rates when compliance costs and audit probabilities are factored into planning decisions.

The penalty function approach introduced in Model II captures this reality by explicitly incorporating audit risk as a cost component in the optimization objective. This reflects the practical understanding that multinational tax planning operates within a **risk-return framework** where aggressive positions must be evaluated against their expected enforcement costs, not merely their technical compliance with arm's length standards.

As **?**] observe, the combination of enhanced transparency requirements, increased enforcement resources, and coordinated international tax policy creates an environment where "optimization for optimization's sake" becomes increasingly costly relative to strategies that balance tax efficiency with regulatory compliance and reputational considerations.

4.2.2 Enhanced Objective Function with Penalty Terms

Building upon the audit risk realities documented in the previous section, Model II extends our baseline framework by incorporating compliance costs and regulatory scrutiny directly into the optimization objective. This enhancement transforms the purely tax-minimizing approach into a **risk-adjusted optimization** that balances tax efficiency against expected enforcement costs.

Extended Dynamic Closed-Form Identity The core mathematical framework from Model I remains intact, but is now augmented with explicit penalty terms that capture audit risk and compliance costs:

$$\boxed{\begin{aligned} \text{Total Cost}_j(I_j) = {} & \underbrace{\text{QDMTT}_j(I_j) \cdot \mathbb{I}_{\text{QDMTT},j}}_{\text{Local collection}} \\ & + \underbrace{\sum_i \alpha_{i,j} \cdot R_j^{(0)}(I_j) \cdot \mathbb{I}_{\text{IIR},i}}_{\text{IIR at parent}} \\ & + \underbrace{\sum_k w_{k,j} \cdot R_j^{(1)}(I_j) \cdot \mathbb{I}_{\text{UTPR},k}}_{\text{UTPR distribution}} \\ & + \underbrace{\lambda \cdot \Phi_j(I_j, I_j^0)}_{\text{Audit risk penalty}} \end{aligned}} \tag{42}$$

where $\Phi_j(I_j, I_j^0)$ represents the expected cost of audit risk and compliance burden in jurisdiction j, and $\lambda \geq 0$ is a **risk aversion parameter** reflecting the multinational's tolerance for regulatory exposure.

Penalty Function Design The audit risk penalty function $\Phi_j(I_j, I_j^0)$ captures the expected cost of transfer pricing audits as a function of the magnitude of income reallocation from baseline positions:

$$\Phi_j(I_j, I_j^0) = p_j(|I_j - I_j^0|) \cdot \pi_j \cdot h(I_j - I_j^0) \tag{43}$$

where:

- I_j^0 represents the **baseline income allocation** in jurisdiction j (pre-optimization reference point)
- $p_j(\cdot) : \mathbb{R}_+ \to [0, 1]$ is the **audit probability function**, increasing in the magnitude of income adjustment
- $\pi_j > 0$ is the **penalty severity coefficient** for jurisdiction j, reflecting both monetary penalties and compliance costs,based on tax specialist estimation.
- $h(\cdot)$ captures the **directional risk profile**, allowing for asymmetric penalties based on GloBE income increases vs. decreases

Practical Penalty Function Specification For computational implementation, I employ a tractable specification that maintains the MILP structure while capturing the essential economic relationships:

Simplified Audit Probability:

$$p_j = \bar{p}_j \cdot \mathbb{I}_{\{|I_j - I_j^0| > \theta_j\}} \tag{44}$$

where $\bar{p}_j$ is the jurisdiction-specific audit probability (practitioner input) and θ_j represents the **audit threshold** below which adjustments are considered routine. In the discussion above, this parameter is typically set by tax professionals through empirical experience or simulation testing. Further probabilistic analysis is beyond the scope of this paper.

Penalty Severity Calibration:

$$\pi_j = \kappa_j \cdot (1 + r_j) \cdot T_j \tag{45}$$

where:

- κ_j = base compliance cost rate (typically 2-5% of adjusted amount)
- r_j = jurisdiction-specific penalty interest rate (from tax code)
- T_j = expected audit duration (typically 2-4 years)

Directional Risk Function:

$$h(I_j - I_j^0) = \begin{cases} \beta \cdot |I_j - I_j^0|, & \text{if } I_j < I_j^0 \text{ (income decrease)} \\ |I_j - I_j^0|, & \text{if } I_j > I_j^0 \text{ (income increase)} \end{cases} \tag{46}$$

with $\beta > 1$ reflecting the empirical observation that **profit shifting away** from a jurisdiction attracts disproportionate scrutiny compared to profit increases.

Similarly, in the above formulas and discussion, this parameter is typically set by tax professionals through empirical experience or simulation testing, and further examination of the specific relationships is beyond the scope of this paper.

Enhanced Optimization Problem The risk-adjusted optimization problem becomes:

$$\min \sum_j [C_j(I_j) + \text{Total Cost}_j(I_j)] \tag{47}$$

subject to the same constraint system as Model I, ensuring that the penalty augmentation preserves the fundamental structure while introducing economically meaningful risk considerations.

Economic Interpretation The penalty-augmented framework creates several important economic effects:

Natural Brake on Aggressive Strategies: Large deviations from baseline allocations incur expected audit costs that may exceed marginal tax benefits, automatically constraining optimization within realistic ranges.

Jurisdiction-Specific Risk Profiling: Different values of $\bar{p}_j$, π_j, and β allow the model to reflect varying enforcement intensities across tax authorities, enabling sophisticated risk management strategies. Although it is difficult for individuals or tax practitioners to quantify the parameters mentioned above, they can make rough estimates based on their experience or conduct "stress testing," for example by modeling worst-case, moderate-case, and best-case scenarios."

Risk-Return Trade-off: The parameter λ provides a **risk tolerance dial** that allows multinational enterprises to calibrate their optimization aggressiveness based on corporate risk appetite and stakeholder preferences.

Implementation Guidance for Practitioners Model II's penalty framework enables practical risk-adjusted tax planning through:

Parameter Calibration: Tax practitioners can input jurisdiction-specific audit probabilities and penalty rates based on professional experience, recent enforcement patterns, and published penalty schedules.

Scenario Analysis: Different values of λ allow exploration of tax strategies under varying risk tolerance levels, from conservative compliance-focused approaches ($\lambda > 1$) to aggressive tax-minimizing strategies ($\lambda \to 0$).

Benchmark Validation: The baseline allocation I_j^0 serves as a **compliance anchor**, ensuring that optimization recommendations remain tethered to defensible starting positions.

This enhanced framework maintains the theoretical rigor and computational efficiency of Model I while incorporating the practical realities of multinational tax enforcement, providing a robust foundation for strategic tax planning under uncertainty.

4.2.3 Risk Aversion Parameter Calibration – Further Discussion

The risk aversion parameter λ represents the **shadow price of audit risk** relative to direct tax costs, requiring calibration to organizational risk tolerance. Practical calibration approaches might include **historical cost analysis** based on previous compliance experiences, **corporate governance mapping** reflecting different organizational types (aggressive growth firms vs. regulated industries), and **sensitivity-based analysis** that reverse-engineers appropriate λ values from desired allocation patterns. The framework naturally accommodates **dynamic risk adjustment** through jurisdiction-specific multipliers $\lambda_j = \lambda^{\text{base}} \cdot \omega_j$ and **time-varying risk aversion** $\lambda(t)$ that adapts to evolving enforcement environments. These calibration methodologies provide flexibility for organizations to embed their specific risk preferences into the optimization framework while maintaining the model's mathematical tractability.

4.3 Model III: Negative GloBE Income (profit) Extension

4.3.1 Economic Reality of Losses in MNE Operations

Real-world multinational operations frequently involve jurisdictions with **negative GloBE income** arising from startup losses, restructuring costs, economic downturns, or strategic investments. Traditional Pillar 2 analysis treats such scenarios as simple zero-tax cases, but this approach overlooks the strategic value of loss allocation and the complex interactions between negative income jurisdictions and profitable operations.

Prevalence of Negative Income Scenarios Multinational enterprises commonly encounter negative GloBE income through several business realities:

Market Entry and Expansion: New market penetration typically involves substantial upfront investments in infrastructure, marketing, and regulatory compliance, generating losses during initial operational phases.

Research and Development Hubs: Centralized R&D operations often produce significant expenses with no immediate corresponding revenues, creating persistent negative income in innovation-focused jurisdictions.

Economic Cycle Management: Cyclical industries experience predictable loss periods during economic downturns, requiring strategic planning across both profitable and loss-making phases.

Restructuring and Transformation: Corporate reorganizations, digital transformations, and operational restructuring generate substantial one-time costs that may exceed current-period revenues.

Strategic Value of Loss Allocation Negative GloBE income represents **strategic assets** rather than mere compliance obstacles:

Future Tax Shield Creation: Losses allocated to high-tax jurisdictions may generate valuable loss carry-forward positions for future income offset, creating **option value** for tax planning.

Portfolio Optimization: Strategic allocation of losses can **rebalance the global income portfolio** to optimize the overall effective tax rate across profitable jurisdictions while maintaining business substance requirements.

Regulatory Positioning: Appropriate loss allocation helps establish **economic substance** in strategic jurisdictions during business development phases, creating favorable positioning for future profitable operations.

4.3.2 Mathematical Treatment of Negative GloBE Income

Model III extends the baseline framework by **removing non-negativity constraints** on GloBE income allocation and introducing piecewise functions to handle the discontinuous nature of tax obligations across positive and negative income domains.

Extended Feasible Region The fundamental constraint modification allows unrestricted income allocation:

$$I_j \in \mathbb{R}, \quad \forall j \quad \text{(Unrestricted income allocation)} \tag{48}$$

subject to modified transfer pricing bounds:

$$I_j^{\min} \leq I_j \leq I_j^{\max}, \quad \forall j \quad \text{where } I_j^{\min} \text{ may be } < 0 \tag{49}$$

Piecewise Tax Functions The covered tax function becomes **piecewise linear** across income domains:

$$C_j(I_j) = \begin{cases} \hat{\tau}_j \cdot \max(I_j - S_j, 0), & \text{if } I_j \geq 0 \\ 0, & \text{if } I_j < 0 \end{cases} \tag{50}$$

Similarly, the top-up tax obligation extends to:

$$\Delta_j(I_j) = \begin{cases} \max(t_{\min} - \rho_j(I_j), 0) \cdot E_j, & \text{if } I_j > 0 \\ 0, & \text{if } I_j \leq 0 \end{cases} \tag{51}$$

This creates **asymmetric optimization incentives**: negative income jurisdictions impose no direct tax costs but also generate no immediate Pillar 2 obligations.

Enhanced MILP Formulation The piecewise functions require additional binary variables $z_j \in \{0, 1\}$ to track income sign:

$$
\begin{aligned}
I_j &\le M_j^+ \cdot z_j && (52)\\
I_j &\ge -M_j^- \cdot (1 - z_j) && (53)\\
C_j &\le \hat{\tau}_j \cdot (I_j - S_j + M_j^+) \cdot z_j && (54)\\
C_j &\ge \hat{\tau}_j \cdot (I_j - S_j) \cdot z_j && (55)
\end{aligned}
$$

where M_j^+ and M_j^- are appropriately chosen big-M parameters for positive and negative income bounds.

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subject to modified transfer pricing bounds:

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$$\Delta_j(I_j) = \begin{cases} \max(t_{\min} - \rho_j(I_j), 0) \cdot E_j, & \text{if } I_j > 0 \\ 0, & \text{if } I_j \leq 0 \end{cases} \tag{59}$$

This creates **asymmetric optimization incentives**: negative income jurisdictions impose no direct tax costs but also generate no immediate Pillar 2 obligations.

Enhanced MILP Formulation The piecewise functions require additional binary variables $z_j \in \{0, 1\}$ to track income sign:

$$I_j \leq M_j^+ \cdot z_j \tag{60}$$

$$I_j \geq -M_j^- \cdot (1 - z_j) \tag{61}$$

$$C_j \leq \hat{\tau}_j \cdot (I_j - S_j + M_j^+) \cdot z_j \tag{62}$$

$$C_j \geq \hat{\tau}_j \cdot (I_j - S_j) \cdot z_j \tag{63}$$

where M_j^+ and M_j^- are appropriately chosen big-M parameters for positive and negative income bounds.

4.3.5 Loss Carryforward Integration

The strategic value of negative income allocation can be further enhanced by incorporating **loss carryforward mechanisms** that capture the intertemporal benefits of the position of loss in the current period. Future extensions could integrate the present value of tax shields from losses allocated to high-tax jurisdictions, creating temporal arbitrage opportunities particularly valuable for **growth-stage companies** and **restructuring situations**. However, such enhancements require detailed analysis of jurisdiction-specific carryforward rules and utilization probabilities, representing a natural direction for future research rather than the development of the core model.

4.4 Model IV: Time Value Integration

The theoretical optimization frameworks developed in Models I-III assume instantaneous income allocation and tax payment, overlooking the **time value implications** embedded in real-world transfer pricing arrangements. An **illustrative scenario** suggests why temporal considerations may fundamentally alter the optimization calculus.

Illustrative Scenario: Potential Implicit Financing Through Transfer Pricing Consider an **observed pattern** where a multinational's jewelry subsidiary in China initially reported EBIT margins of -80% to -90%, subsequently adjusted to +2.54% through annual transfer pricing compensation of ¥460 million from the headquarters abroad. While such arrangements achieve arm's length compliance, the **economic structure** could potentially resemble an interest-free financing arrangement where:

- The **Ultimate Parent Entity (UPE)** retains centralized cash management and liquidity control
- The subsidiary receives annual **"compensation"** that effectively covers operating losses
- The timing of payments creates **implicit lending** with no explicit interest charges
- Working capital requirements are absorbed by the parent rather than financed locally

This structure generates **hidden economic value** through cash flow timing that traditional Pillar 2 analysis overlooks (from my opinions).

Time Value Quantification The implicit financing value can be substantial. In the jewelry company example, the ¥460 million annual transfer represents an interest-free loan with economic value:

Time Dimension Integration Model IV extends the framework by introducing **temporal structure** to income allocation decisions. Rather than optimizing income allocation at a single point in time, I consider allocation decisions at **fixed intervals** (e.g., semi-annually or annually):

$$I_j^{\text{start}} : \text{Initial GloBE income allocation in jurisdiction } j \text{ (Parameter)} \tag{64}$$

$$I_j^{\text{end}} : \text{Final GloBE income allocation in jurisdiction } j \text{ (Decision variable)} \tag{65}$$

$$\Delta I_j = I_j^{\text{end}} - I_j^{\text{start}} \text{ (Adjustment amount)} \tag{66}$$

The optimization problem incorporates **time value effects** of these adjustments:

$$\min \sum_j \left[C_j(I_j^{\text{end}}) + \Delta_j(I_j^{\text{end}}) - \text{PV of timing benefits} \right] \tag{67}$$

where the timing benefits capture the implicit financing value from strategic adjustment timing (as illustrated in the jewelry company scenario), representing economic benefits that reduce the overall cost function.

This simple extension allows consideration of **when** transfer pricing adjustments occur within the fiscal period, capturing the implicit financing value demonstrated in the jewelry company scenario while maintaining mathematical tractability.

If the model's minimum value is negative, it indicates a theoretical arbitrage opportunity—yet the associated risks and transaction costs are typically prohibitive.

Working Capital Optimization Opportunities: Transfer pricing flexibility leaves the space for **temporal arbitrage** opportunities:

Payment Timing Optimization: Strategic timing of intercompany payments can minimize global funding costs while maintaining arm's length compliance through reasonable payment terms.

Currency Hedging Integration: Coordinated transfer pricing and treasury operations can provide natural currency hedging, reducing both tax obligations and foreign exchange exposure.

Seasonal Cash Flow Management: Businesses with seasonal patterns can optimize the timing of transfer pricing adjustments to minimize peak funding requirements and associated financing costs.

Cross-Border Funding Efficiency: Rather than establishing formal intercompany loans with explicit interest rates, transfer pricing adjustments can provide **implicit financing** at more favorable effective rates.

Regulatory and Economic Substance Considerations While transfer pricing regulations require arm's length pricing, they generally **do not mandate specific payment timing**, creating legitimate optimization opportunities. However, several factors constrain temporal arbitrage:

Business Purpose Requirements: Payment timing must reflect legitimate business considerations rather than pure tax or financing optimization.

Related Party Transaction Documentation: Extended payment terms or unusual timing patterns require appropriate documentation and economic justification.

Substance Over Form Principles: Regulators may challenge arrangements that lack economic substance despite technical arm's length compliance.

Pillar 2 Timing Rules: GloBE income recognition follows specific timing rules that may differ from cash flow realization, requiring careful coordination between tax and treasury optimization.

Integration with Pillar 2 Optimization Time value considerations fundamentally alter the Pillar 2 optimization landscape by introducing **intertemporal trade-offs**:

Tax-Treasury Coordination: Optimal income allocation must consider both current-period tax minimization and multi-period cash flow value creation.

Liquidity Premium Valuation: Maintaining cash in high-liquidity, low-tax jurisdictions may justify accepting higher current-period tax costs for improved financial flexibility.

Risk-Adjusted Returns: Cash flow timing optimization must account for country-specific risks, regulatory changes, and operational uncertainties across multiple time horizons.

The jewelry company example demonstrates that **sophisticated multinational enterprises already exploit these temporal opportunities**, making their formal integration into Pillar 2 optimization frameworks both theoretically important and practically relevant. Model IV addresses this gap by explicitly incorporating time value considerations into the strategic optimization calculus, recognizing that modern multinational tax planning operates at the intersection of tax efficiency, treasury management, and strategic finance.

5. Sensitivity Analysis and Policy Implications

5.1 Comparative Model Analysis

[Content to be developed, though I have some drafts. I feel the full proof is out of scope, I might provide them as the appendix for next version]

5.2 Parameter Sensitivity Across Model Tiers

[Content to be developed, though I have some drafts. I feel the full proof is out of scope, I might provide them as the appendix for next version]

5.3 Policy Strategy Analysis and Jurisdictional Competition

[Content to be developed, and I haven't really look this part yet.]

6. Computational Implementation and Case Studies

6.1 Algorithm Design and Scalability

[Content to be developed, I need more users to simulate and test. I do not know the results. The platform will come in next few days.]

6.2 Numerical Examples and Validation

[Content to be developed, I need more users to simulate and test. I do not know the results,]

6.3 Real-World Application Framework

[Content to be developed, I need more users, especially tax specialist to simulate and test. I do not know the results.]

7. Extensions and Future Research

To deepen my discussion of dynamic Pillar 2 optimization, I suggest three potential extensions that build on the current framework without implying a comprehensive survey of every avenue for future work.

7.1 Stochastic and Robust Optimization

Real-world tax planning operates under significant uncertainty: tax rates change, enforcement policies evolve, and economic conditions fluctuate. Our deterministic models assume perfect knowledge of all parameters, which is obviously unrealistic but necessary for establishing the baseline framework.

Future iterations could incorporate **uncertainty quantification** through stochastic programming approaches. Rather than optimizing for a single scenario, the model could consider probability distributions over tax rates, audit probabilities, and economic conditions. This would yield **robust strategies** that perform well across multiple future scenarios rather than being optimized for a specific set of assumptions.

The mathematical framework naturally accommodates such extensions – decision variables remain the same, but the objective function becomes an expected value calculation over uncertain parameters. Implementation would require scenario generation techniques and potentially more sophisticated solvers, but the conceptual leap is straightforward.

An interesting question is whether the **stochastic solutions differ materially** from our deterministic optimal solutions. If robust strategies converge to similar allocation patterns, it validates our baseline approach. If they diverge significantly, it suggests that uncertainty considerations are crucial for practical implementation.

Alternative Solution Methodologies My work employs MILP linearization techniques primarily for **computational convenience** and solution reliability, but this represents a pragmatic choice rather than a theoretical necessity. The underlying optimization problem could alternatively be addressed through **direct MINLP solvers** that handle the non-linear max functions and ratio terms without linearization. While current MINLP solvers often struggle with solution quality and convergence guarantees compared to their MILP counterparts, the field continues to advance rapidly. Future developments in non-linear optimization algorithms may eventually make direct MINLP approaches more attractive than my current linearization strategy.

Beyond traditional optimization approaches, the problem structure also lends itself to **simulation-based methods**. Monte Carlo techniques could explore the feasible allocation space through intelligent sampling, potentially identifying good solutions without requiring formal optimization. **Metaheuristic approaches** such as evolutionary algorithms or simulated annealing might prove particularly valuable when incorporating the uncertainty considerations discussed above, as they can handle complex constraint structures and mixed-integer variables naturally.

The choice between analytical optimization and simulation-based exploration ultimately depends on problem size, solution quality requirements, and computational resources. My MILP foundation provides a **benchmark for comparison** – any alternative approach should demonstrate either superior solution quality or significantly reduced computational burden to justify the methodological switch.

7.2 Integration with Broader Tax Planning

My work treats Pillar 2 optimization in isolation, but real multinational tax planning involves complex interactions across multiple tax regimes, planning techniques, and compliance obligations. Future research could explore how GloBE income allocation strategies interact with:

Traditional Tax Planning Structures: How do existing IP holding companies, financing structures, and hybrid instruments affect the optimal Pillar 2 strategies? Can my framework be embedded within broader tax planning optimization rather than operating independently?

Transfer Pricing Beyond GloBE Income Allocation: I focus on profit allocation, but transfer pricing affects countless other decisions – intercompany service charges, royalty rates, debt-equity ratios. A comprehensive framework might optimize these decisions **jointly** rather than sequentially.

Digital Economy Considerations: The rise of digital business models creates unique challenges for GloBE income allocation – where should algorithm development, data processing, or user engagement be valued? Our framework could be extended to address these intangible allocation puzzles that traditional transfer pricing struggles to handle.

7.3 Regulatory Evolution and Model Adaptation

Pillar 2 is not a static regulatory environment. Tax authorities are actively learning, enforcement approaches are evolving, and international coordination is deepening. Our framework should be adaptive to these changing conditions.

Machine Learning Integration: Rather than requiring manual parameter updates, future systems could employ machine learning algorithms to automatically calibrate audit probabilities, penalty rates, and risk aversion parameters based on observed regulatory behavior and enforcement patterns.

Policy Response Modeling: As multinationals adopt optimization strategies, tax authorities will likely respond with countermeasures. Future research could explore this regulatory arms race – how might tax authorities adjust their enforcement strategies in response to strategic income allocation, and how should optimization models adapt in turn?

Cross-Border Coordination Effects: The OECD's emphasis on information sharing and coordinated enforcement suggests that isolated jurisdiction-by-jurisdiction analysis may become insufficient. Future frameworks might need to model network effects where audit decisions in one jurisdiction trigger scrutiny in others.

Real-Time Optimization: Current transfer pricing operates on annual cycles with lengthy documentation and adjustment processes. Future regulatory environments might enable more frequent adjustments through digital reporting and real-time compliance monitoring. This could transform my discrete-period models into continuous optimization problems.

7.4 Implementation Reality Check

While these extensions sound sophisticated, their practical value depends on whether they address real pain points for multinational enterprises or merely add mathematical complexity. The most successful extensions will likely be those that:

- Solve problems that practitioners actually encounter
- Provide decision support that justifies the additional complexity
- Remain computationally tractable for realistic problem sizes
- Integrate smoothly with existing corporate planning processes

My baseline framework establishes the mathematical foundation for such extensions while demonstrating that meaningful optimization is possible within current regulatory constraints. Whether future developments emphasize sophisticated mathematical techniques or practical implementation tools will depend on how the field evolves and what challenges emerge as Pillar 2 implementation matures.

8. Conclusion

I don't know if my model and idea work, so no concrete conclusion at this point. After real-world data testing, I hope to have a solid conclusion afterward.

The framework I've developed here attempts to capture the complex reality of Pillar 2 optimization by treating covered taxes as endogenous variables. Whether this approach actually helps multinational enterprises optimize their tax strategies while staying compliant - I genuinely don't know yet. That's why I need your help.

This is a working paper and I'm looking for feedback. If you're a tax practitioner, I'd love to know if this makes sense from your experience. If you're an academic, please point out where my logic or math might be flawed. The computational platform will be ready in a few days - please test it and tell me what breaks or what works. All critiques, suggestions, and reality checks are welcome.

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A. Notation and Parameter Definitions

This appendix provides a comprehensive reference for all notation, parameters, and variables used throughout the dynamic Pillar 2 framework. Parameters are organized by model tier and functional category.

A.1 Core Pillar 2 Parameters

Symbol	**Definition**	**Type**
$t_{\min}$	Minimum tax rate under Pillar 2 (15%)	Parameter
$\hat{\tau}_j$	Statutory corporate tax rate in jurisdiction j	Parameter
I_{total}	Total GloBE income to be allocated across all jurisdictions	Parameter
S_j	Substance carve-out amount in jurisdiction j	Parameter
$P_{e,j}$	Eligible payroll costs for entity e in jurisdiction j	Parameter
$A_{e,j}$	Carrying value of eligible tangible assets for entity e in jurisdiction j	Parameter
$\beta_p^{(t)}$	Payroll carve-out rate in year t (5% in transition years)	Parameter
$\beta_a^{(t)}$	Tangible asset carve-out rate in year t (5% in transition years)	Parameter
E_j	Set of constituent entities in jurisdiction j (excluding investment entities)	Parameter

A.2 Decision Variables and Endogenous Relationships

Symbol	**Definition**	**Type**
I_j	GloBE income allocated to jurisdiction j (primary optimization variable)	Decision
$I_j^{\min}$	Minimum allowable GloBE income in jurisdiction j (transfer pricing bound)	Parameter
$I_j^{\max}$	Maximum allowable GloBE income in jurisdiction j (transfer pricing bound)	Parameter
$C_j(I_j)$	Covered taxes paid in jurisdiction j (function of income allocation)	Endogenous
$\rho_j(I_j)$	Effective tax rate in jurisdiction j (function of income allocation)	Endogenous
$\tau_j(I_j)$	Jurisdictional top-up tax rate (dynamic function of allocation)	Endogenous
E_j	Excess profit subject to top-up tax in jurisdiction j	Endogenous
$\Delta_j(I_j)$	Required global top-up tax for jurisdiction j	Endogenous

A.3 Pillar 2 Mechanism Variables

Symbol	**Definition**	**Type**
$\text{QDMTT}_j(I_j)$	Qualified Domestic Minimum Top-up Tax collected in jurisdiction j	Endogenous
IIR_i	Income Inclusion Rule tax collected by parent jurisdiction i	Endogenous
UTPR_k	Under-Taxed Payments Rule tax collected in jurisdiction k	Endogenous
$R_j^{(0)}(I_j)$	Remaining top-up tax in jurisdiction j after QDMTT	Endogenous
$R_j^{(1)}(I_j)$	Remaining top-up tax in jurisdiction j after IIR application	Endogenous

q_j	Amount of QDMTT paid in jurisdiction j (optimization variable)	Decision
y_j	Binary indicator: 1 if jurisdiction j implements QDMTT, 0 otherwise	Decision

A.4 Allocation Weights and Indicators

Symbol	Definition	Type
$\alpha_{i,j}$	Ownership/control fraction of jurisdiction j entities by parent i	Parameter
$w_{k,j}$	UTPR allocation weight: fraction of remaining top-up allocated to jurisdiction k	Parameter
$f_{i,j}$	IIR allocation variable: fraction to parent jurisdiction i from jurisdiction j	Decision
$g_{k,j}$	UTPR allocation variable: fraction to jurisdiction k from jurisdiction j	Decision
$\mathbb{I}_{\text{QDMTT},j}$	Indicator: 1 if jurisdiction j can implement QDMTT, 0 otherwise	Parameter
$\mathbb{I}_{\text{IIR},i}$	Indicator: 1 if jurisdiction i has IIR active, 0 otherwise	Parameter
$\mathbb{I}_{\text{UTPR},k}$	Indicator: 1 if jurisdiction k applies UTPR, 0 otherwise	Parameter

A.5 Model II: Penalty Function Parameters

Symbol	Definition	Type
I_j^0	Baseline (pre-optimization) GloBE income allocation in jurisdiction j	Parameter
$\Phi_j(I_j, I_j^0)$	Audit risk penalty function for jurisdiction j	Function
$p_j(\cdot)$	Audit probability function in jurisdiction j	Function
π_j	Penalty severity coefficient for jurisdiction j	Parameter
θ_j	Audit sensitivity parameter for jurisdiction j	Parameter
λ	Risk aversion parameter (penalty weight in objective function)	Parameter
$h(\cdot)$	Function capturing direction and magnitude of adjustment risk	Function

A.6 Model III: Negative Income Extension

Symbol	Definition	Type
I_j^+	Positive GloBE income component in jurisdiction j	Endogenous
I_j^-	Negative GloBE income component in jurisdiction j	Endogenous
α_j	Loss treatment coefficient for negative income in jurisdiction j	Parameter
$L_{j,t}$	Loss carryforward from jurisdiction j in period t	Parameter
$\text{ETR}^{\text{global}+}$	Global effective tax rate calculated on positive income jurisdictions only	Endogenous

A.7 Model IV: Time Value Parameters

Symbol	Definition	Type
$I_j^{\text{start}}(t)$	GloBE income in jurisdiction j at start of period t	Parameter
$I_j^{\text{end}}(t)$	GloBE income in jurisdiction j at end of period t	Decision
r_j	Discount rate for jurisdiction j (reflects country and currency risk)	Parameter
δ	Global discount factor for multi-period optimization	Parameter
T	Time horizon for dynamic optimization	Parameter
NPV_j	Net present value of income allocation changes in jurisdiction j	Endogenous
$\text{CF}_j(t)$	Cash flow impact from transfer pricing adjustment in jurisdiction j, period t	Endogenous

A.8 Mathematical Functions and Relationships

Function	Mathematical Expression	Model
Covered Taxes	$C_j(I_j) = \hat{\tau}_j \cdot \max(I_j - S_j, 0)$	All
Effective Tax Rate	$\rho_j(I_j) = \frac{C_j(I_j)}{I_j}$ for $I_j > 0$	All
Jurisdictional Top-up Rate	$\tau_j(I_j) = \max(t_{\min} - \rho_j(I_j), 0)$	All
Excess Profit	$E_j = \max(I_j - S_j, 0)$	All
Required Top-up Tax	$\Delta_j(I_j) = \tau_j(I_j) \cdot E_j$	All
Substance Carve-out	$S_j = \sum_{e \in E_j} (\beta_p^{(t)} \cdot P_{e,j} + \beta_a^{(t)} \cdot A_{e,j})$	All
QDMTT Collection	$\text{QDMTT}_j(I_j) = \max(\tau_j(I_j) \cdot E_j, 0) \cdot \mathbb{I}_{\text{QDMTT},j}$	All
Audit Probability	$p_j(\lvert I_j - I_j^0 \rvert) = 1 - \exp(-\theta_j \cdot \frac{\lvert I_j - I_j^0 \rvert}{I_j^0})$	Model II+
Penalty Function	$\Phi_j(I_j, I_j^0) = p_j(\lvert I_j - I_j^0 \rvert) \cdot \pi_j \cdot h(I_j - I_j^0)$	Model II+
Negative Income Tax	$C_j(I_j) = \alpha_j \cdot I_j$ for $I_j < 0$	Model III+
Time Value	$\text{NPV}_j = \sum_{t=1}^{T} \frac{I_j^{\text{end}}(t) - I_j^{\text{start}}(t)}{(1+r_j)^t}$	Model IV

A.9 Constraint Classifications

Constraint Type	Mathematical Form	Purpose
Income Allocation	$\sum_j I_j = I_{\text{total}}$	Conservation
Transfer Pricing Bounds	$I_j^{\min} \leq I_j \leq I_j^{\max}, \forall j$	Arm's Length
Non-negativity	$I_j \geq 0, \forall j$ (Models I-II)	Feasibility
Extended Domain	$I_j \in \mathbb{R}, \forall j$ (Models III-IV)	Loss Handling
QDMTT Implementation	$0 \leq q_j \leq \Delta_j \cdot y_j, \forall j$	Policy Choice
Binary Indicators	$y_j \in \{0, 1\}, \forall j$	Discrete Decision
Allocation Completeness	$\sum_i f_{i,j} + \sum_k g_{k,j} = 1, \forall j$	Tax Collection

A.10 Model Progression Summary

Model	**Key Features**	**Additional Parameters**
Model I	Static endogenous covered tax, simplified marginal effects	Core parameters only
Model II	+ Audit risk penalty functions	$\lambda, \pi_j, \theta_j, I_j^0$
Model III	+ Negative GloBE income handling	$\alpha_j, L_{j,t}$
Model IV	+ Time value and multi-period optimization	$r_j, \delta, T, I_j^{\text{start}}(t)$

Note: Parameters marked as "Endogenous" are determined within the model optimization process, while "Parameter" indicates exogenously specified values. "Decision" variables are the primary optimization targets. "Function" indicates mathematical relationships that may be specified differently across model variants.

B. Mathematical Derivations

[Detailed sensitivity analysis calculations and proofs to be developed, I don't want to include too many math formulas right now]

C. Computational Details

[Welcome to my test page, the free platform will come in next few days]

D. Extended Case Studies

[Additional numerical examples and parameter calibration guidance to be developed, cases will be evaluated based on real-word reflections]

E. OECD Pillar 2 Framework Overview

This appendix provides essential background on the OECD Pillar 2 framework to contextualize the optimization models developed in this paper. The overview synthesizes key elements from OECD guidance documents and implementation rules.

E.1 Historical Context and Objectives

The OECD Pillar 2 framework emerged from the Base Erosion and Profit Shifting (BEPS) initiative as a response to concerns about multinational enterprises (MNEs) achieving very low effective tax rates through sophisticated tax planning structures. The framework aims to ensure that large MNEs pay a minimum level of tax on income arising in each jurisdiction where they operate.

The Pillar 2 rules apply to MNEs with annual revenues of €750 million or more in at least two of the four fiscal years immediately preceding the tested fiscal year. This threshold aligns with the Country-by-Country Reporting requirements under BEPS Action 13, affecting approximately 7,000-8,000 MNE groups globally.

E.2 Core Architecture: The Three-Mechanism Structure

Pillar 2 operates through three interlocking mechanisms designed to ensure comprehensive coverage while minimizing double taxation:

E.2.1 Income Inclusion Rule (IIR)

The IIR requires a parent entity to pay top-up tax with respect to the low-taxed income of its subsidiary entities. Key features include:

- **Scope**: Applies to parent entities in jurisdictions that have implemented the IIR
- **Calculation**: Top-up tax equals the jurisdictional effective tax rate shortfall multiplied by excess profits
- **Priority**: Takes precedence over UTPR in the allocation hierarchy
- **Split-ownership**: Special rules apply when multiple parent entities exist

E.2.2 Under-Taxed Payments Rule (UTPR)

The UTPR serves as a backstop to the IIR, applying when the IIR has not been implemented or does not apply. Features include:

- **Mechanism**: Operates through denial of deductions or equivalent adjustments
- **Allocation**: Distributed among constituent entities based on an allocation key
- **Coordination**: Only applies to remaining top-up tax after IIR application
- **Implementation**: Requires domestic legislation in each applying jurisdiction

E.2.3 Qualified Domestic Minimum Top-up Tax (QDMTT)

QDMTT allows jurisdictions to implement their own minimum tax that takes priority over IIR and UTPR:

- **Qualification requirements**: Must be consistent with GloBE Rules methodology
- **Safe harbor**: Qualified QDMTT reduces or eliminates IIR and UTPR exposure

- **Design flexibility**: Jurisdictions may adapt rules to domestic tax systems while maintaining consistency
- **Competitive element**: Creates incentives for jurisdictions to implement domestic minimum taxes

E.3 Key Computational Elements

E.3.1 GloBE Income Determination

GloBE income serves as the tax base for Pillar 2 calculations:

- **Starting point**: Financial accounting net income before income taxes
- **Adjustments**: Specific additions and subtractions as defined in the GloBE Rules
- **Consolidation**: Aggregated at the jurisdictional level for all constituent entities
- **Threshold**: Jurisdictions with GloBE income below €1 million may be excluded

E.3.2 Covered Tax Calculation

Covered taxes represent the income taxes paid or accrued with respect to GloBE income:

- **Inclusion criteria**: Must be imposed on income and computed by reference to profits
- **Exclusions**: Certain taxes specifically excluded (e.g., digital services taxes, some withholding taxes)
- **Timing**: Generally follows financial accounting recognition principles
- **Allocation**: Must be properly allocated to the relevant jurisdiction and income

E.3.3 Substance-Based Income Exclusion

The substance carve-out excludes a portion of income from top-up tax calculations:

- **Payroll component**: 5% of eligible payroll costs during transition (2023-2026), reducing to 5% thereafter
- **Asset component**: 5% of carrying value of eligible tangible assets during transition, reducing to 5% thereafter
- **Eligibility**: Limited to payroll and assets used in business operations (excluding investment management)
- **Rationale**: Acknowledges that substantial business activities correlate with legitimate income allocation

E.4 Implementation Timeline and Adoption

E.4.1 Phased Implementation Schedule

- **2024**: IIR implementation begins in early-adopting jurisdictions
- **2025**: UTPR implementation scheduled in many jurisdictions
- **2026**: Full substance carve-out rates take effect (reducing from transitional rates)
- **Ongoing**: Continuous monitoring and potential adjustments through OECD processes

E.4.2 Global Adoption Status

As of 2025, over 140 jurisdictions have committed to implementing Pillar 2, though implementation approaches and timing vary. Key developments include:

- **European Union**: Coordinated implementation through EU Directive
- **Major economies**: Most G20 countries committed to implementation
- **Developing countries**: Varying levels of commitment and implementation capacity
- **Tax havens**: Some traditional low-tax jurisdictions implementing QDMTT regimes

E.5 Economic and Policy Implications

E.5.1 Revenue Impact

OECD estimates suggest Pillar 2 could generate a significant amount in additional annual tax revenue globally, though estimates vary significantly based on:

- **Behavioral responses**: MNE restructuring and planning adaptations
- **QDMTT adoption**: Extent to which source jurisdictions implement domestic minimum taxes
- **Economic conditions**: Impact of business cycles on profitability and effective tax rates

E.5.2 Jurisdictional Competition

Pillar 2 fundamentally alters tax competition dynamics:

- **Rate floors**: Creates de facto minimum tax rate of 15% for large MNEs
- **QDMTT incentives**: Encourages source jurisdictions to collect minimum tax domestically
- **Substance requirements**: Reinforces importance of real economic activities
- **Complexity costs**: May favor jurisdictions with sophisticated tax administrations

E.6 Implementation Challenges and Uncertainties

Despite the comprehensive framework, several challenges remain:

- **Computational complexity**: Requires sophisticated systems and processes
- **Coordination challenges**: Managing interactions across multiple tax systems
- **Dispute resolution**: Limited mechanisms for resolving conflicts between jurisdictions
- **Ongoing developments**: Continued guidance and potential rule modifications

This framework provides the regulatory context within which the optimization models developed in this paper operate. The complexity and interdependencies illustrated above demonstrate why dynamic modeling approaches are essential for effective strategic planning under Pillar 2.

Table 1: Comparative Framework: From Baseline to Advanced Pillar 2 Models

Model	**Key Innovation**	**Economic Reality Captured**	**Mathematical Complexity**	**Use Case**
Model I	Endogenous covered tax $C_j(I_j)$ with an approximation approach	Transfer pricing flexibility within arm's length bounds	MINLP with max functions	Baseline optimization
Model II	+ Audit risk penalty $\lambda \cdot \Phi_j(I_j, I_j^0)$	Compliance costs and regulatory scrutiny	+ Exponential penalty functions	Risk-adjusted planning
Model III	+ Negative income handling $I_j \in \mathbb{R}$	Loss scenarios and carryforward effects	+ Piecewise tax functions	Startup/turnaround scenarios
Model IV	+ Time value NPV_j integration	Multi-period cash flow timing	+ Dynamic programming elements	Sophisticated cash management