PKF on 2D multivariate oscillator

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Abstract

This notebook illustrates the use of sympkf to build and handle the PKF dynamics associated with the harmonic oscillator as 2D fields given by

$$\begin{cases} \partial_t u = v \\ \partial_t v = -u \end{cases}$$

where u and v are functions of t, x, y. For this dynamics, the resulting PKF system is not closed because of the cros-correlation.

1 Definition of the 2D multivariate dynamics

```
[1]: import sympy
sympy.init_printing()
```

Definition of the dynamics from sympy tools

```
[2]: from sympy import Function, Derivative, Eq, symbols from sympkf import SymbolicPKF, t
```

```
[3]: x, y = symbols('x y')
u = Function('u')(t,x,y)
v = Function('v')(t,x,y)
dynamics = [Eq(Derivative(u,t), v), Eq(Derivative(v,t), -u)]
dynamics
```

[3]:
$$\left[\frac{\partial}{\partial t}u(t,x,y)=v(t,x,y),\ \, \frac{\partial}{\partial t}v(t,x,y)=-u(t,x,y)\right]$$

2 Computation of the PKF dynamics by using SymPKF

$$\frac{\sqrt{\mathbf{V}_{\mathbf{u}}(t,x,y)} \mathbb{E}\left(\frac{\partial}{\partial y} \varepsilon_{\mathbf{u}}(t,x,y,\omega) \frac{\partial}{\partial x} \varepsilon_{\mathbf{v}}(t,x,y,\omega)\right)}{\sqrt{\mathbf{V}_{\mathbf{u}}(t,x,y)}} + \frac{\sqrt{\mathbf{V}_{\mathbf{u}}(t,x,y)} \mathbb{E}\left(\varepsilon_{\mathbf{u}}(t,x,y,\omega) \frac{\partial}{\partial y} \varepsilon_{\mathbf{v}}(t,x,y,\omega)\right)}{2 \mathbf{V}_{\mathbf{v}}^{\frac{1}{2}}(t,x,y)} + \frac{2 \mathbf{V}_{\mathbf{v}}^{\frac{1}{2}}(t,x,y)}{2 \mathbf{V}_{\mathbf{v}}^{\frac{1}{2}}(t,x,y)} + \frac{2 \mathbf{V}_{\mathbf{v}}^{\frac{1}{2}}(t,x,y)}{2 \mathbf{V}_{\mathbf{v}}^{\frac{1}{2}}(t,x,y)} + \frac{2 \mathbf{V}_{\mathbf{v}}^{\frac{1}{2}}(t,x,y)}{2 \mathbf{V}_{\mathbf{v}}^{\frac{1}{2}}(t,x,y)} + \frac{2 \mathbf{V}_{\mathbf{v}}^{\frac{1}{2}}(t,x,y)}{2 \mathbf{V}_{\mathbf{v}}(t,x,y)} \frac{\partial}{\partial y} \mathbf{V}_{\mathbf{v}}(t,x,y)}{2 \mathbf{V}_{\mathbf{v}}(t,x,y)} + \frac{2 \mathbf{V}_{\mathbf{v}}$$

```
\sqrt{\mathbf{V}_{\mathbf{v}}\left(t,x,y\right)}\,\mathbf{s}_{\mathbf{u},\mathbf{x}\mathbf{x}}\left(t,x,y\right)\mathbf{s}_{\mathbf{u},\mathbf{x}\mathbf{y}}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\,\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial y}\,\mathbf{V}_{\mathbf{u}}\left(t,x,y\right)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       +
   \sqrt{\mathbf{V}_{\mathbf{v}}\left(t,x,y\right)}\,\mathbf{s}_{\mathbf{u},\mathbf{x}\mathbf{x}}\left(t,x,y\right)\mathbf{s}_{\mathbf{u},\mathbf{x}\mathbf{y}}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\,\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}\,\mathbf{V}_{\mathbf{u}}\left(t,x,y\right)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        +
   \sqrt{\mathbf{V}_{\mathbf{v}}\left(t,x,y\right)}\,\mathbf{s}_{\mathbf{u},\mathbf{x}\mathbf{y}}^{2}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\,\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial y}\,\mathbf{V}_{\mathbf{u}}\left(t,x,y\right)
\frac{\partial}{\partial t} \mathbf{s}_{\mathrm{u,xy}}\left(t,x,y\right) = \frac{2 \mathbf{V}_{\mathrm{uv}}\left(t,x,y\right) \mathbf{s}_{\mathrm{u,xy}}\left(t,x,y\right)}{\mathbf{V}_{\mathrm{u}}\left(t,x,y\right)} - \frac{2 \sqrt{\mathbf{V}_{\mathrm{v}}\left(t,x,y\right)} \mathbf{s}_{\mathrm{u,xx}}\left(t,x,y\right) \mathbf{s}_{\mathrm{u,xy}}\left(t,x,y\right) \mathbb{E}\left(\frac{\partial}{\partial x} \, \varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right) \frac{\partial}{\partial x} \, \varepsilon_{\mathrm{v}}\left(t,x,y\right) \mathbf{s}_{\mathrm{u,xy}}\left(t,x,y\right) \mathbf{s}_{\mathrm{u,xy}
    \sqrt{\mathbf{V}_{\mathbf{v}}\left(t,x,y\right)}\,\mathbf{s}_{\mathbf{u},\mathbf{xx}}\left(t,x,y\right)\mathbf{s}_{\mathbf{u},\mathbf{yy}}\left(t,x,y\right)\mathbb{E}\left(\frac{\partial}{\partial x}\,\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\,\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\right)
                                                                                                                                                                                                   \sqrt{V_{11}(t,x,y)}
   \sqrt{\mathbf{V}_{\mathbf{v}}\left(t,x,y\right)}\,\mathbf{s}_{\mathbf{u},\mathbf{xx}}\left(t,x,y\right)\mathbf{s}_{\mathbf{u},\mathbf{yy}}\left(t,x,y\right)\mathbb{E}\left(\frac{\partial}{\partial y}\,\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\,\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\right)
  \frac{\sqrt{\mathbf{V}_{\mathbf{v}}\left(t,x,y\right)}\mathbf{s}_{\mathbf{u},\mathbf{xy}}^{2}\left(t,x,y\right)\mathbb{E}\left(\frac{\partial}{\partial x}\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\right)}{\sqrt{\mathbf{V}_{\mathbf{u}}\left(t,x,y,\omega\right)}}-\frac{\sqrt{\mathbf{V}_{\mathbf{v}}\left(t,x,y\right)}\mathbf{s}_{\mathbf{u},\mathbf{xy}}^{2}\left(t,x,y\right)\mathbb{E}\left(\frac{\partial}{\partial y}\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\right)}{\sqrt{\mathbf{V}_{\mathbf{u}}\left(t,x,y,\omega\right)}}
 2\sqrt{\mathbf{V}_{\mathbf{v}}\left(t,x,y\right)}\,\mathbf{s}_{\mathbf{u},\mathbf{x}\mathbf{y}}\left(t,x,y\right)\mathbf{s}_{\mathbf{u},\mathbf{y}\mathbf{y}}\left(t,x,y\right)\mathbb{E}\left(\frac{\partial}{\partial y}\,\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\,\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\right)
                                                                                                                                                                                                      \overline{\sqrt{\mathrm{V_{u}}\left(t,x,y\right)}}
  \mathbf{s}_{\mathrm{u,xx}}\left(t,x,y\right)\mathbf{s}_{\mathrm{u,xy}}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}V_{\mathrm{v}}\left(t,x,y\right)
                                                                                                                                                  \sqrt{V_{\rm H}(t,x,y)}\sqrt{V_{\rm V}(t,x,y)}
  \mathbf{s}_{\mathrm{u,xx}}\left(t,x,y\right)\mathbf{s}_{\mathrm{u,yy}}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial y}V_{\mathrm{v}}\left(t,x,y\right)
                                                                                                                                            2\sqrt{V_{11}(t,x,y)}\sqrt{V_{V}(t,x,y)}
  s_{u,xx}\left(t,x,y\right)s_{u,yy}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{v}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\,\varepsilon_{u}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}\,V_{v}\left(t,x,y\right)
                                                                                                                                               2\sqrt{V_{11}(t,x,y)}\sqrt{V_{12}(t,x,y)}
\frac{\mathbf{s_{u,xy}}^{2}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial y}\mathbf{V_{\mathbf{v}}}\left(t,x,y\right)}{2\sqrt{\mathbf{V_{\mathbf{u}}}\left(t,x,y\right)}\sqrt{\mathbf{V_{\mathbf{v}}}\left(t,x,y\right)}}-\frac{\mathbf{s_{u,xy}}^{2}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}\mathbf{V_{\mathbf{v}}}\left(t,x,y,\omega\right)}{2\sqrt{\mathbf{V_{\mathbf{u}}}\left(t,x,y\right)}\sqrt{\mathbf{V_{\mathbf{v}}}\left(t,x,y\right)}}
  \mathbf{s}_{\mathrm{u},\mathrm{xy}}\left(t,x,y\right)\mathbf{s}_{\mathrm{u},\mathrm{yy}}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\,\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial y}\,\mathbf{V}_{\mathrm{v}}\left(t,x,y\right)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       +
                                                                                                                                                     \sqrt{\mathrm{V_{u}}(t,x,y)}\sqrt{\mathrm{V_{v}}(t,x,y)}
   \sqrt{\mathbf{V}_{\mathbf{v}}\left(t,x,y\right)}\,\mathbf{s}_{\mathbf{u},\mathbf{xx}}\left(t,x,y\right)\mathbf{s}_{\mathbf{u},\mathbf{xy}}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\,\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}\,\mathbf{V}_{\mathbf{u}}\left(t,x,y\right)
                                                                                                                                                                                                                                       V_{\mathfrak{n}}^{\frac{3}{2}}(t,x,y)
    \sqrt{\mathbf{V}_{\mathbf{v}}\left(t,x,y\right)}\,\mathbf{s}_{\mathbf{u},\mathbf{xx}}\left(t,x,y\right)\mathbf{s}_{\mathbf{u},\mathbf{yy}}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\,\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial y}\,\mathbf{V}_{\mathbf{u}}\left(t,x,y\right)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       +
                                                                                                                                                                                                                                  2 \operatorname{V}_{11}^{\frac{3}{2}} (t, x, y)
   \sqrt{\mathbf{V}_{\mathbf{v}}\left(t,x,y\right)}\,\mathbf{s}_{\mathbf{u},\mathbf{xx}}\left(t,x,y\right)\mathbf{s}_{\mathbf{u},\mathbf{yy}}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\,\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}\,\mathbf{V}_{\mathbf{u}}\left(t,x,y\right)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        +
  \frac{2 \operatorname{V_{u}}^{\frac{3}{2}}(t, x, y)}{\sqrt{\operatorname{V_{v}}(t, x, y)} \operatorname{s_{u,xy}}^{2}(t, x, y) \mathbb{E}\left(\varepsilon_{v}(t, x, y, \omega) \frac{\partial}{\partial x} \varepsilon_{u}(t, x, y, \omega)\right) \frac{\partial}{\partial y} \operatorname{V_{u}}(t, x, y)}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       +
   \sqrt{\mathbf{V}_{\mathbf{v}}\left(t,x,y\right)}\,\mathbf{s}_{\mathbf{u},\mathbf{x}\mathbf{y}}^{2}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\,\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}\,\mathbf{V}_{\mathbf{u}}\left(t,x,y\right)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       +
                                                                                                                                                                                               2 V_{11}^{\frac{3}{2}} (t, x, y)
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\sqrt{\mathbf{V}_{\mathbf{v}}\left(t,x,y\right)}\,\mathbf{s}_{\mathbf{u},\mathbf{x}\mathbf{y}}\left(t,x,y\right)\mathbf{s}_{\mathbf{u},\mathbf{y}\mathbf{y}}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\,\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial y}\,\mathbf{V}_{\mathbf{u}}\left(t,x,y\right)
                                                                                                                                                                                                                  V_{n}^{\frac{3}{2}}(t, x, y)
\frac{\partial}{\partial t} s_{\text{u,yy}}\left(t,x,y\right) = \frac{2 \operatorname{V}_{\text{uv}}\left(t,x,y\right) s_{\text{u,yy}}\left(t,x,y\right)}{\operatorname{V}_{\text{u}}\left(t,x,y\right)} - \frac{2 \sqrt{\operatorname{V}_{\text{v}}\left(t,x,y\right)} \operatorname{s}_{\text{u,xy}}^{2}\left(t,x,y\right) \mathbb{E}\left(\frac{\partial}{\partial x} \varepsilon_{\text{u}}\left(t,x,y,\omega\right) \frac{\partial}{\partial x} \varepsilon_{\text{v}}\left(t,x,y,\omega\right)\right)}{\sqrt{\operatorname{V}_{\text{u}}\left(t,x,y\right)}}
  2\sqrt{\mathbf{V}_{\mathbf{v}}\left(t,x,y\right)}\,\mathbf{s}_{\mathbf{u},\mathbf{x}\mathbf{y}}\left(t,x,y\right)\mathbf{s}_{\mathbf{u},\mathbf{y}\mathbf{y}}\left(t,x,y\right)\mathbb{E}\left(\frac{\partial}{\partial x}\,\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\,\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\right)
\frac{2\sqrt{\mathbf{V}_{\mathbf{v}}\left(t,x,y\right)}\,\mathbf{s}_{\mathbf{u},\mathbf{xy}}\left(t,x,y\right)\mathbf{s}_{\mathbf{u},\mathbf{yy}}\left(t,x,y\right)\mathbb{E}\left(\frac{\partial}{\partial y}\,\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\,\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\right)}{\sqrt{\mathbf{V}_{\mathbf{u}}\left(t,x,y\right)}}
\frac{2\sqrt{\mathbf{V_{v}}\left(t,x,y\right)}\,\mathbf{s_{u,yy}}^{2}\left(t,x,y\right)\mathbb{E}\left(\frac{\partial}{\partial y}\,\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\,\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\right)}{\sqrt{\mathbf{V_{v}}\left(t,x,y,\omega\right)}}-\frac{\mathbf{s_{u,xy}}^{2}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\,\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\right)}{\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\sqrt{\mathbf{V_{v}}\left(t,x,y\right)}}
\frac{\mathbf{s}_{\mathrm{u,xy}}\left(t,x,y\right)\mathbf{s}_{\mathrm{u,yy}}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial y}V_{\mathrm{v}}\left(t,x,y\right)}{\sqrt{V_{\mathrm{u}}\left(t,x,y\right)}\sqrt{V_{\mathrm{v}}\left(t,x,y\right)}}
\mathbf{s}_{\mathrm{u,xy}}\left(t,x,y\right)\mathbf{s}_{\mathrm{u,yy}}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}V_{\mathrm{v}}\left(t,x,y\right)
                                                                                                                                     \sqrt{V_{\rm H}(t,x,y)}\sqrt{V_{\rm V}(t,x,y)}
\frac{\mathbf{s_{u,yy}}^{2}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\,\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial y}\,\mathbf{V_{\mathbf{v}}}\left(t,x,y\right)}{\sqrt{\mathbf{V_{\mathbf{u}}}\left(t,x,y\right)}\sqrt{\mathbf{V_{\mathbf{v}}}\left(t,x,y\right)}}+\frac{\sqrt{\mathbf{V_{\mathbf{v}}}\left(t,x,y\right)}\,\mathbf{s_{u,xy}}^{2}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\,\varepsilon_{\mathbf{u}}\left(t,x,y\right)\right)}{\mathbf{V_{\mathbf{u}}}^{\frac{3}{2}}\left(t,x,y\right)}
  \sqrt{\mathbf{V}_{\mathbf{v}}\left(t,x,y\right)}\,\mathbf{s}_{\mathbf{u},\mathbf{xy}}\left(t,x,y\right)\mathbf{s}_{\mathbf{u},\mathbf{yy}}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\,\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial y}\,\mathbf{V}_{\mathbf{u}}\left(t,x,y\right)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            +
                                                                                                                                                                                                                \overline{\mathbf{V_{u}}^{\frac{3}{2}}\left(t,x,y\right)}
  \sqrt{\mathbf{V}_{\mathbf{v}}\left(t,x,y\right)}\,\mathbf{s}_{\mathbf{u},\mathbf{x}\mathbf{y}}\left(t,x,y\right)\mathbf{s}_{\mathbf{u},\mathbf{y}\mathbf{y}}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\,\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}\,\mathbf{V}_{\mathbf{u}}\left(t,x,y\right)
  \sqrt{\mathbf{V}_{\mathbf{v}}\left(t,x,y\right)}\,\mathbf{s}_{\mathbf{u},\mathbf{y}\mathbf{y}}^{2}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\,\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial y}\,\mathbf{V}_{\mathbf{u}}\left(t,x,y\right)
\frac{\partial}{\partial t} \operatorname{s_{v,xx}}\left(t,x,y\right) = -\frac{2\operatorname{V_{uv}}\left(t,x,y\right) \sqrt{\operatorname{V_{u}}\left(t,x,y\right)} \operatorname{V_{v}}^{\frac{5}{2}}\left(t,x,y\right) \operatorname{s_{v,xx}}^{2}\left(t,x,y\right) \operatorname{s_{v,yy}}\left(t,x,y\right)}{\sqrt{\operatorname{V_{u}}\left(t,x,y\right)} \operatorname{V_{v}}^{\frac{7}{2}}\left(t,x,y\right) \operatorname{s_{v,xx}}\left(t,x,y\right) \operatorname{s_{v,yy}}\left(t,x,y\right) - \sqrt{\operatorname{V_{u}}\left(t,x,y\right)} \operatorname{V_{v}}^{\frac{7}{2}}\left(t,x,y\right) \operatorname{s_{v,xy}}^{2}\left(t,x,y\right) 
                                                                                 2 V_{\text{uv}}(t, x, y) \sqrt{V_{\text{u}}(t, x, y)} V_{\text{v}}^{\frac{5}{2}}(t, x, y) s_{\text{v,xx}}(t, x, y) s_{\text{v,xy}}^{2}(t, x, y)
  \sqrt{V_{\mathrm{u}}(t,x,y)} V_{\mathrm{v}}^{\frac{7}{2}}(t,x,y) s_{\mathrm{v,xx}}(t,x,y) s_{\mathrm{v,vy}}(t,x,y) - \sqrt{V_{\mathrm{u}}(t,x,y)} V_{\mathrm{v}}^{\frac{7}{2}}(t,x,y) s_{\mathrm{v,xy}}^{2}(t,x,y)
                       2\operatorname{V}_{\mathrm{u}}\left(t,x,y\right)\operatorname{V}_{\mathrm{v}}^{3}\left(t,x,y\right)\operatorname{s}_{\mathrm{v},\mathrm{xx}}^{3}\left(t,x,y\right)\operatorname{s}_{\mathrm{v},\mathrm{yy}}\left(t,x,y\right)\mathbb{E}\left(\frac{\partial}{\partial x}\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)
  \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s}_{\rm v,xx}\left(t,x,y\right){\rm s}_{\rm v,yy}\left(t,x,y\right) - \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s}_{\rm v,xv}^{2}\left(t,x,y\right)
                  2 \operatorname{V}_{\mathrm{u}}(t,x,y) \operatorname{V}_{\mathrm{v}}^{3}(t,x,y) \operatorname{s}_{\mathrm{v},\mathrm{xx}}^{2}(t,x,y) \operatorname{s}_{\mathrm{v},\mathrm{xy}}^{2}(t,x,y) \mathbb{E}\left(\frac{\partial}{\partial x} \varepsilon_{\mathrm{u}}(t,x,y,\omega) \frac{\partial}{\partial x} \varepsilon_{\mathrm{v}}(t,x,y,\omega)\right)
  \sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xx}}\left(t,x,y\right)\mathbf{s_{v,yy}}\left(t,x,y\right) - \sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xv}}^{2}\left(t,x,y\right)
 2\operatorname{V}_{\mathrm{u}}\left(t,x,y\right)\operatorname{V}_{\mathrm{v}}^{3}\left(t,x,y\right)\operatorname{s}_{\mathrm{v},\mathrm{xx}}^{2}\left(t,x,y\right)\operatorname{s}_{\mathrm{v},\mathrm{xy}}\left(t,x,y\right)\operatorname{s}_{\mathrm{v},\mathrm{yy}}\left(t,x,y\right)\mathbb{E}\left(\frac{\partial}{\partial x}\,\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\,\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)
                     \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xx}}(t,x,y)\,\mathbf{s_{v,yy}}(t,x,y) - \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xy}}^{2}(t,x,y)
 2 \operatorname{V}_{\mathrm{u}}\left(t, x, y\right) \operatorname{V}_{\mathrm{v}}^{3}\left(t, x, y\right) \operatorname{s}_{\mathrm{v}, \mathrm{xx}}^{2}\left(t, x, y\right) \operatorname{s}_{\mathrm{v}, \mathrm{xy}}\left(t, x, y\right) \operatorname{s}_{\mathrm{v}, \mathrm{yy}}\left(t, x, y\right) \mathbb{E}\left(\frac{\partial}{\partial y} \varepsilon_{\mathrm{u}}\left(t, x, y, \omega\right) \frac{\partial}{\partial x} \varepsilon_{\mathrm{v}}\left(t, x, y, \omega\right)\right)
                     \sqrt{V_{\rm u}(t,x,y)} \, V_{\rm v}^{\frac{7}{2}}(t,x,y) \, s_{\rm v,xx}(t,x,y) \, s_{\rm v,vv}(t,x,y) - \sqrt{V_{\rm u}(t,x,y)} \, V_{\rm v}^{\frac{7}{2}}(t,x,y) \, s_{\rm v,xv}^{2}(t,x,y)
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```
2 V_{\mathrm{u}}(t, x, y) V_{\mathrm{v}}^{3}(t, x, y) s_{\mathrm{v,xx}}(t, x, y) s_{\mathrm{v,xy}}^{3}(t, x, y) \mathbb{E}\left(\frac{\partial}{\partial x} \varepsilon_{\mathrm{u}}(t, x, y, \omega) \frac{\partial}{\partial y} \varepsilon_{\mathrm{v}}(t, x, y, \omega)\right)
\frac{\sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xx}}(t,x,y)\,\mathbf{s_{v,yy}}(t,x,y) - \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xy}}^{2}(t,x,y)}}{\sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xy}}^{\frac{7}{2}}(t,x,y)}}
          2 V_{\mathrm{u}}(t, x, y) V_{\mathrm{v}}^{3}(t, x, y) s_{\mathrm{v,xx}}(t, x, y) s_{\mathrm{v,xy}}^{3}(t, x, y) \mathbb{E}\left(\frac{\partial}{\partial y} \varepsilon_{\mathrm{u}}(t, x, y, \omega) \frac{\partial}{\partial x} \varepsilon_{\mathrm{v}}(t, x, y, \omega)\right)
\overline{\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xx}}\left(t,x,y\right)\mathbf{s_{v,yy}}\left(t,x,y\right) - \sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xy}}^{2}\left(t,x,y\right)}
 2 \operatorname{V}_{\mathrm{u}}\left(t,x,y\right) \operatorname{V}_{\mathrm{v}}^{3}\left(t,x,y\right) \operatorname{s}_{\mathrm{v,xx}}\left(t,x,y\right) \operatorname{s}_{\mathrm{v,xy}}^{2}\left(t,x,y\right) \operatorname{s}_{\mathrm{v,yy}}\left(t,x,y\right) \mathbb{E}\left(\frac{\partial}{\partial y} \varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right) \frac{\partial}{\partial y} \varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)
          \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xx}}(t,x,y)\,\mathbf{s_{v,yy}}(t,x,y) - \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xv}}^{2}(t,x,y)
                             2 \operatorname{V}_{\mathrm{u}}(t, x, y) \operatorname{V}_{\mathrm{v}}^{3}(t, x, y) \operatorname{s}_{\mathrm{v}, \mathrm{xy}}^{4}(t, x, y) \mathbb{E}\left(\frac{\partial}{\partial y} \varepsilon_{\mathrm{u}}(t, x, y, \omega) \frac{\partial}{\partial y} \varepsilon_{\mathrm{v}}(t, x, y, \omega)\right)
 \frac{1}{\sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xx}}(t,x,y)\,\mathbf{s_{v,yy}}(t,x,y) - \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xy}}^{2}(t,x,y)}}
 \mathbf{V}_{\mathrm{u}}\left(t,x,y\right)\mathbf{V}_{\mathrm{v}}^{2}\left(t,x,y\right)\mathbf{s}_{\mathrm{v,xx}}^{3}\left(t,x,y\right)\mathbf{s}_{\mathrm{v,yy}}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}\mathbf{V}_{\mathrm{v}}\left(t,x,y,\omega\right)
    \sqrt{V_{\mathrm{u}}(t,x,y)} V_{\mathrm{v}}^{\frac{7}{2}}(t,x,y) s_{\mathrm{v,xx}}(t,x,y) s_{\mathrm{v,vy}}(t,x,y) - \sqrt{V_{\mathrm{u}}(t,x,y)} V_{\mathrm{v}}^{\frac{7}{2}}(t,x,y) s_{\mathrm{v,xy}}^{2}(t,x,y)
V_{\mathrm{u}}\left(t,x,y\right)V_{\mathrm{v}}^{2}\left(t,x,y\right)\mathrm{s_{v,\mathrm{xx}}}^{2}\left(t,x,y\right)\mathrm{s_{v,\mathrm{xy}}}^{2}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}V_{\mathrm{v}}\left(t,x,y\right)
      \frac{1}{\sqrt{\mathrm{V_{u}}\left(t,x,y\right)}\,\mathrm{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathrm{s_{v,xx}}\left(t,x,y\right)\mathrm{s_{v,yy}}\left(t,x,y\right)-\sqrt{\mathrm{V_{u}}\left(t,x,y\right)}\,\mathrm{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathrm{s_{v,xv}}^{2}\left(t,x,y\right)}
V_{\mathrm{u}}\left(t,x,y\right)V_{\mathrm{v}}^{2}\left(t,x,y\right)\mathrm{s_{v,xx}}^{2}\left(t,x,y\right)\mathrm{s_{v,xy}}\left(t,x,y\right)\mathrm{s_{v,yy}}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\,\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial y}\,V_{\mathrm{v}}\left(t,x,y\right)
                        \sqrt{V_{\mathrm{u}}(t,x,y)} V_{\mathrm{v}}^{\frac{7}{2}}(t,x,y) s_{\mathrm{v,xx}}(t,x,y) s_{\mathrm{v,vy}}(t,x,y) - \sqrt{V_{\mathrm{u}}(t,x,y)} V_{\mathrm{v}}^{\frac{7}{2}}(t,x,y) s_{\mathrm{v,xy}}^{2}(t,x,y)
V_{\mathrm{u}}\left(t,x,y\right)V_{\mathrm{v}}^{2}\left(t,x,y\right)\mathrm{s_{v,\mathrm{xx}}}^{2}\left(t,x,y\right)\mathrm{s_{v,\mathrm{xy}}}\left(t,x,y\right)\mathrm{s_{v,\mathrm{yy}}}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial u}\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}V_{\mathrm{v}}\left(t,x,y\right)
                          \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xx}}(t,x,y)\,\mathbf{s_{v,yy}}(t,x,y) - \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xy}}^{2}(t,x,y)
V_{\mathrm{u}}\left(t,x,y\right)V_{\mathrm{v}}^{2}\left(t,x,y\right)\mathrm{s}_{\mathrm{v,xx}}\left(t,x,y\right)\mathrm{s}_{\mathrm{v,xy}}^{3}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial u}V_{\mathrm{v}}\left(t,x,y\right)
    \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xx}}(t,x,y)\,\mathbf{s_{v,yy}}(t,x,y) - \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xy}}^{2}(t,x,y)
V_{\mathrm{u}}\left(t,x,y\right)V_{\mathrm{v}}^{2}\left(t,x,y\right)s_{\mathrm{v,xx}}\left(t,x,y\right)s_{\mathrm{v,xy}}^{3}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}V_{\mathrm{v}}\left(t,x,y\right)
      \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s_{\rm v,xx}}\left(t,x,y\right){\rm s_{\rm v,yy}}\left(t,x,y\right) - \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s_{\rm v,xy}}^{2}\left(t,x,y\right)
V_{\mathrm{u}}\left(t,x,y\right)V_{\mathrm{v}}^{2}\left(t,x,y\right)\mathrm{s}_{\mathrm{v,xx}}\left(t,x,y\right)\mathrm{s}_{\mathrm{v,xy}}^{2}\left(t,x,y\right)\mathrm{s}_{\mathrm{v,yy}}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\,\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial y}\,V_{\mathrm{v}}\left(t,x,y,\omega\right)
                         \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xx}}(t,x,y)\,\mathbf{s_{v,yy}}(t,x,y) - \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xv}}^{2}(t,x,y)
              V_{\mathrm{u}}(t,x,y) V_{\mathrm{v}}^{2}(t,x,y) s_{\mathrm{v,xy}}^{4}(t,x,y) \mathbb{E}\left(\varepsilon_{\mathrm{u}}(t,x,y,\omega) \frac{\partial}{\partial y} \varepsilon_{\mathrm{v}}(t,x,y,\omega)\right) \frac{\partial}{\partial y} V_{\mathrm{v}}(t,x,y)
                                                                                                                                                                                                                                                                                                                        +
 \overline{\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xx}}\left(t,x,y\right)\mathbf{s_{v,yy}}\left(t,x,y\right) - \sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xv}}^{2}\left(t,x,y\right)}
             V_{v}^{3}(t, x, y) s_{v,xx}^{3}(t, x, y) s_{v,yy}(t, x, y) \mathbb{E} \left( \varepsilon_{u}(t, x, y, \omega) \frac{\partial}{\partial x} \varepsilon_{v}(t, x, y, \omega) \right) \frac{\partial}{\partial x} V_{u}(t, x, y)
 \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xx}}(t,x,y)\,\mathbf{s_{v,yy}}(t,x,y) - \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xy}}^{2}(t,x,y)
           \mathbf{V_{v}}^{3}\left(t,x,y\right)\mathbf{s_{v,xx}}^{2}\left(t,x,y\right)\mathbf{s_{v,xy}}^{2}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}\mathbf{V_{\mathbf{u}}}\left(t,x,y\right)
 \sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xx}}\left(t,x,y\right)\mathbf{s_{v,yy}}\left(t,x,y\right) - \sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xv}}^{2}\left(t,x,y\right)
V_{v}^{3}(t, x, y) s_{v,xx}^{2}(t, x, y) s_{v,xy}(t, x, y) s_{v,yy}(t, x, y) \mathbb{E}\left(\varepsilon_{u}(t, x, y, \omega) \frac{\partial}{\partial x} \varepsilon_{v}(t, x, y, \omega)\right) \frac{\partial}{\partial y} V_{u}(t, x, y)
       \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s}_{\rm v,xx}\left(t,x,y\right){\rm s}_{\rm v,yy}\left(t,x,y\right) - \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s}_{\rm v,xy}^{2}\left(t,x,y\right)
V_{v}^{3}\left(t,x,y\right)s_{v,xx}^{2}\left(t,x,y\right)s_{v,xy}\left(t,x,y\right)s_{v,yy}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{u}\left(t,x,y,\omega\right)\frac{\partial}{\partial u}\varepsilon_{v}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}V_{u}\left(t,x,y\right)
        \sqrt{\mathbf{V}_{\mathrm{u}}\left(t,x,y\right)}\,\mathbf{V}_{\mathrm{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s}_{\mathrm{v,xx}}\left(t,x,y\right)\mathbf{s}_{\mathrm{v,yy}}\left(t,x,y\right) - \sqrt{\mathbf{V}_{\mathrm{u}}\left(t,x,y\right)}\,\mathbf{V}_{\mathrm{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s}_{\mathrm{v,xy}}^{2}\left(t,x,y\right)
             V_{\mathbf{v}}^{3}\left(t,x,y\right)\mathbf{s}_{\mathbf{v},\mathbf{xx}}\left(t,x,y\right)\mathbf{s}_{\mathbf{v},\mathbf{xx}}^{3}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial y}V_{\mathbf{u}}\left(t,x,y\right)
\frac{\sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xx}}(t,x,y)\,\mathbf{s_{v,yy}}(t,x,y) - \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xy}}^{2}(t,x,y)}}{\sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xy}}^{2}(t,x,y)}}
            V_{v}^{3}(t, x, y) s_{v,xx}(t, x, y) s_{v,xy}^{3}(t, x, y) \mathbb{E}\left(\varepsilon_{u}(t, x, y, \omega) \frac{\partial}{\partial y} \varepsilon_{v}(t, x, y, \omega)\right) \frac{\partial}{\partial x} V_{u}(t, x, y)
                                                                                                                                                                                                                                                                                                                        +
\sqrt{V_{11}(t,x,y)} V_{V_{1}}^{\frac{7}{2}}(t,x,y) s_{V,XX}(t,x,y) s_{V,YY}(t,x,y) - \sqrt{V_{11}(t,x,y)} V_{V_{1}}^{\frac{7}{2}}(t,x,y) s_{V,XY}^{2}(t,x,y)
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V_{v}^{3}(t, x, y) s_{v,xx}(t, x, y) s_{v,xy}^{2}(t, x, y) s_{v,yy}(t, x, y) \mathbb{E}\left(\varepsilon_{u}(t, x, y, \omega) \frac{\partial}{\partial y} \varepsilon_{v}(t, x, y, \omega)\right) \frac{\partial}{\partial y} V_{u}(t, x, y)
                      \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xx}}(t,x,y)\,\mathbf{s_{v,yy}}(t,x,y) - \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xy}}^{2}(t,x,y)
                                                                             V_{v}^{3}(t, x, y) s_{v,xy}^{4}(t, x, y) \mathbb{E}\left(\varepsilon_{u}(t, x, y, \omega) \frac{\partial}{\partial y} \varepsilon_{v}(t, x, y, \omega)\right) \frac{\partial}{\partial y} V_{u}(t, x, y)
   \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s}_{\rm v,xx}\left(t,x,y\right){\rm s}_{\rm v,yy}\left(t,x,y\right) - \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s}_{\rm v,xy}^{2}\left(t,x,y\right)
\frac{\partial}{\partial t} \operatorname{s_{v,xy}}\left(t,x,y\right) = -\frac{4\operatorname{V_{uv}}\left(t,x,y\right)\sqrt{\operatorname{V_{u}}\left(t,x,y\right)}\operatorname{V_{v}}^{\frac{5}{2}}\left(t,x,y\right)\operatorname{s_{v,xx}}\left(t,x,y\right)\operatorname{s_{v,xy}}\left(t,x,y\right)\operatorname{s_{v,yy}}\left(t,x,y\right)}{2\sqrt{\operatorname{V_{u}}\left(t,x,y\right)}\operatorname{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\operatorname{s_{v,xx}}\left(t,x,y\right)\operatorname{s_{v,yy}}\left(t,x,y\right) - 2\sqrt{\operatorname{V_{u}}\left(t,x,y\right)}\operatorname{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\operatorname{s_{v,xy}}^{2}\left(t,x,y\right)}
                                                                                                                                                           4 V_{uv}(t, x, y) \sqrt{V_{u}(t, x, y)} V_{v}^{\frac{5}{2}}(t, x, y) s_{v,xv}^{3}(t, x, y)
   2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xx}}\left(t,x,y\right)\mathbf{s_{v,yy}}\left(t,x,y\right) - 2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xy}}^{2}\left(t,x,y\right)
   4 \operatorname{V}_{\mathrm{u}}\left(\underline{t}, x, y\right) \operatorname{V}_{\mathrm{v}}^{3}\left(t, x, y\right) \operatorname{s}_{\mathrm{v}, \mathrm{xx}}^{2}\left(t, x, y\right) \operatorname{s}_{\mathrm{v}, \mathrm{xy}}\left(t, x, y\right) \operatorname{s}_{\mathrm{v}, \mathrm{yy}}\left(t, x, y\right) \operatorname{\mathbb{E}}\left(\frac{\partial}{\partial x} \varepsilon_{\mathrm{u}}\left(t, x, y, \omega\right) \frac{\partial}{\partial x} \varepsilon_{\mathrm{v}}\left(t, x, y, \omega\right)\right) + \frac{1}{2} \operatorname{V}_{\mathrm{u}}\left(t, x, y\right) \operatorname{S}_{\mathrm{v}, \mathrm{xx}}^{2}\left(t, x, y\right) \operatorname{S}_{\mathrm{v}, \mathrm{xy}}^{2}\left(t, x, y\right) \operatorname{S}_{\mathrm{v}, \mathrm{xy}
               2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xx}}\left(t,x,y\right)\mathbf{s_{v,yy}}\left(t,x,y\right) - 2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xy}}^{2}\left(t,x,y\right)
                             2 \operatorname{V}_{\mathrm{u}}\left(t, x, y\right) \operatorname{V}_{\mathrm{v}}^{3}\left(t, x, y\right) \operatorname{s}_{\mathrm{v}, \mathrm{xx}}^{2}\left(t, x, y\right) \operatorname{s}_{\mathrm{v}, \mathrm{yy}}^{2}\left(t, x, y\right) \mathbb{E}\left(\frac{\partial}{\partial x} \varepsilon_{\mathrm{u}}\left(t, x, y, \omega\right) \frac{\partial}{\partial u} \varepsilon_{\mathrm{v}}\left(t, x, y, \omega\right)\right)
  \frac{1}{2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xx}}\left(t,x,y\right)\mathbf{s_{v,yy}}\left(t,x,y\right)-2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xy}}^{2}\left(t,x,y\right)}
                             2 \operatorname{V}_{\mathrm{u}}(t,x,y) \operatorname{V}_{\mathrm{v}}^{3}(t,x,y) \operatorname{s}_{\mathrm{v.xx}}^{2}(t,x,y) \operatorname{s}_{\mathrm{v.vv}}^{2}(t,x,y) \mathbb{E}\left(\frac{\partial}{\partial u} \varepsilon_{\mathrm{u}}(t,x,y,\omega) \frac{\partial}{\partial x} \varepsilon_{\mathrm{v}}(t,x,y,\omega)\right)
  \frac{2\sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s_{\rm v.xx}}\left(t,x,y\right){\rm s_{\rm v.vv}}\left(t,x,y\right)-2\sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s_{\rm v.xv}}^{2}\left(t,x,y\right)}
                                    4 V_{\mathrm{u}}\left(t, x, y\right) V_{\mathrm{v}}^{3}\left(t, x, y\right) \mathbf{s}_{\mathrm{v,xx}}\left(t, x, y\right) \mathbf{s}_{\mathrm{v,xy}}^{3}\left(t, x, y\right) \mathbb{E}\left(\frac{\partial}{\partial x} \varepsilon_{\mathrm{u}}\left(t, x, y, \omega\right) \frac{\partial}{\partial x} \varepsilon_{\mathrm{v}}\left(t, x, y, \omega\right)\right)
  2\sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right)s_{\rm v,xx}\left(t,x,y\right)s_{\rm v,yy}\left(t,x,y\right) - 2\sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right)s_{\rm v,xy}^{2}\left(t,x,y\right)
  4 \operatorname{V}_{\mathrm{u}}\left(t, x, y\right) \operatorname{V}_{\mathrm{v}}^{3}\left(t, x, y\right) \operatorname{s}_{\mathrm{v}, \mathrm{xx}}\left(t, x, y\right) \operatorname{s}_{\mathrm{v}, \mathrm{xy}}\left(t, x, y\right) \operatorname{s}_{\mathrm{v}, \mathrm{yy}}^{2}\left(t, x, y\right) \mathbb{E}\left(\frac{\partial}{\partial y} \varepsilon_{\mathrm{u}}\left(t, x, y, \omega\right) \frac{\partial}{\partial y} \varepsilon_{\mathrm{v}}\left(t, x, y, \omega\right)\right)
                2\sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s_{\rm v,xx}}\left(t,x,y\right){\rm s_{\rm v,yy}}\left(t,x,y\right) - 2\sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s_{\rm v,xy}}^{2}\left(t,x,y\right)
                                                                               2 \operatorname{V}_{\mathrm{u}}(t, x, y) \operatorname{V}_{\mathrm{v}}^{3}(t, x, y) \operatorname{s}_{\mathrm{v}, \mathrm{xy}}^{4}(t, x, y) \mathbb{E}\left(\frac{\partial}{\partial x} \varepsilon_{\mathrm{u}}(t, x, y, \omega) \frac{\partial}{\partial y} \varepsilon_{\mathrm{v}}(t, x, y, \omega)\right)
  \frac{1}{2\sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s_{\rm v,xx}}\left(t,x,y\right){\rm s_{\rm v,vy}}\left(t,x,y\right)-2\sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s_{\rm v,xy}}^{2}\left(t,x,y\right)}
                                                                             2\operatorname{V}_{\mathrm{u}}\left(t,x,y\right)\operatorname{V}_{\mathrm{v}}^{3}\left(t,x,y\right)\operatorname{s}_{\mathrm{v},\mathrm{xy}}^{4}\left(t,x,y\right)\mathbb{E}\left(\frac{\partial}{\partial y}\,\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\,\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)
  \frac{2\sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s_{\rm v,xx}}\left(t,x,y\right){\rm s_{\rm v,vy}}\left(t,x,y\right)-2\sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s_{\rm v,xy}}^{2}\left(t,x,y\right)}
                                 4 \operatorname{V}_{\mathrm{u}}\left(t, x, y\right) \operatorname{V}_{\mathrm{v}}^{3}\left(t, x, y\right) \operatorname{s}_{\mathrm{v}, \mathrm{xy}}^{3}\left(t, x, y\right) \operatorname{s}_{\mathrm{v}, \mathrm{yy}}\left(t, x, y\right) \mathbb{E}\left(\frac{\partial}{\partial y} \varepsilon_{\mathrm{u}}\left(t, x, y, \omega\right) \frac{\partial}{\partial y} \varepsilon_{\mathrm{v}}\left(t, x, y, \omega\right)\right)
   2\sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s_{\rm v,xx}}\left(t,x,y\right){\rm s_{\rm v,yy}}\left(t,x,y\right) - 2\sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s_{\rm v,xy}}^{2}\left(t,x,y\right)
   2 \overset{\cdot}{\mathbf{V}}_{\mathbf{u}}(t,x,y) \overset{\cdot}{\mathbf{V}}_{\mathbf{v}}^{2}(t,x,y) \overset{\cdot}{\mathbf{s}}_{\mathbf{v},\mathbf{xx}}^{2}(t,x,y) \overset{\cdot}{\mathbf{s}}_{\mathbf{v},\mathbf{xy}}(t,x,y) \overset{\cdot}{\mathbf{s}}_{\mathbf{v},\mathbf{yy}}(t,x,y) \overset{\cdot}{\mathbb{E}} \left( \varepsilon_{\mathbf{u}}(t,x,y,\omega) \frac{\partial}{\partial x} \varepsilon_{\mathbf{v}}(t,x,y,\omega) \right) \frac{\partial}{\partial x} \overset{\cdot}{\mathbf{V}}_{\mathbf{v}}(t,x,y) \overset{\cdot}{\mathbf{v}}_{\mathbf{v},\mathbf{y}}^{2}(t,x,y) \overset{\cdot}{\mathbf{v}}_{\mathbf{v},\mathbf{y}}^{2
                                                       2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xx}}\left(t,x,y\right)\mathbf{s_{v,yy}}\left(t,x,y\right) - 2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xy}}^{2}\left(t,x,y\right)
   V_{\mathrm{u}}\left(t,x,y\right)V_{\mathrm{v}}^{2}\left(t,x,y\right)\mathrm{s_{\mathrm{v,xx}}}^{2}\left(t,x,y\right)\mathrm{s_{\mathrm{v,yy}}}^{2}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial y}V_{\mathrm{v}}\left(t,x,y\right)
       2\sqrt{\mathrm{V_{u}}\left(t,x,y\right)}\,\mathrm{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathrm{s_{v,xx}}\left(t,x,y\right)\mathrm{s_{v,yy}}\left(t,x,y\right) - 2\sqrt{\mathrm{V_{u}}\left(t,x,y\right)}\,\mathrm{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathrm{s_{v,xy}}^{2}\left(t,x,y\right)
   V_{\mathrm{u}}\left(t,x,y\right)V_{\mathrm{v}}^{2}\left(t,x,y\right)\mathrm{s_{v,xx}}^{2}\left(t,x,y\right)\mathrm{s_{v,yy}}^{2}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}V_{\mathrm{v}}\left(t,x,y\right)
         2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xx}}\left(t,x,y\right)\mathbf{s_{v,yy}}\left(t,x,y\right) - 2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xv}}^{2}\left(t,x,y\right)
   2 \operatorname{V}_{\mathrm{u}}\left(t, x, y\right) \operatorname{V}_{\mathrm{v}}^{2}\left(t, x, y\right) \operatorname{s}_{\mathrm{v, xx}}\left(t, x, y\right) \operatorname{s}_{\mathrm{v, xy}}^{3}\left(t, x, y\right) \mathbb{E}\left(\varepsilon_{\mathrm{u}}\left(t, x, y, \omega\right) \frac{\partial}{\partial x} \varepsilon_{\mathrm{v}}\left(t, x, y, \omega\right)\right) \frac{\partial}{\partial x} \operatorname{V}_{\mathrm{v}}\left(t, x, y\right)
         2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xx}}\left(t,x,y\right)\mathbf{s_{v,yy}}\left(t,x,y\right) - 2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xy}}^{2}\left(t,x,y\right)
  2 \operatorname{V}_{\mathrm{u}}\left(t, x, y\right) \operatorname{V}_{\mathrm{v}}^{2}\left(t, x, y\right) \operatorname{s}_{\mathrm{v, xx}}\left(t, x, y\right) \operatorname{s}_{\mathrm{v, xy}}\left(t, x, y\right) \operatorname{s}_{\mathrm{v, yy}}^{2}\left(t, x, y\right) \mathbb{E}\left(\varepsilon_{\mathrm{u}}\left(t, x, y, \omega\right) \frac{\partial}{\partial y} \varepsilon_{\mathrm{v}}\left(t, x, y, \omega\right)\right) \frac{\partial}{\partial y} \operatorname{V}_{\mathrm{v}}\left(t, x, y\right) \operatorname{S}_{\mathrm{v, xy}}^{2}\left(t, x, y\right) \operatorname{S}_{\mathrm{v, yy}}^{2}\left(t, x, y\right) \mathbb{E}\left(\varepsilon_{\mathrm{u}}\left(t, x, y, \omega\right) \frac{\partial}{\partial y} \varepsilon_{\mathrm{v}}\left(t, x, y, \omega\right)\right) \frac{\partial}{\partial y} \operatorname{V}_{\mathrm{v}}\left(t, x, y\right) \operatorname{S}_{\mathrm{v, xy}}^{2}\left(t, x, y\right) \operatorname{S}_{\mathrm{v, xy}}^{2}\left(t, x, y\right) \operatorname{S}_{\mathrm{v, xy}}^{2}\left(t, x, y\right) \mathbb{E}\left(\varepsilon_{\mathrm{u}}\left(t, x, y, \omega\right) \frac{\partial}{\partial y} \varepsilon_{\mathrm{v}}\left(t, x, y, \omega\right)\right) \frac{\partial}{\partial y} \operatorname{V}_{\mathrm{v}}\left(t, x, y\right) \operatorname{S}_{\mathrm{v, xy}}^{2}\left(t, x, y\right) \operatorname{S}_{\mathrm{v, xy}}^{2}\left(t, x, y\right) \operatorname{S}_{\mathrm{v, xy}}^{2}\left(t, x, y\right) \operatorname{E}\left(\varepsilon_{\mathrm{u}}\left(t, x, y, \omega\right) \frac{\partial}{\partial y} \varepsilon_{\mathrm{v}}\left(t, x, y, \omega\right)\right) \frac{\partial}{\partial y} \operatorname{E}\left(\varepsilon_{\mathrm{u}}\left(t, x, y, \omega\right) \operatorname{E}\left(t, x,
                                                        2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xx}}\left(t,x,y\right)\mathbf{s_{v,yy}}\left(t,x,y\right) - 2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xy}}^{2}\left(t,x,y\right)
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V_{\mathrm{u}}\left(t,x,y\right)V_{\mathrm{v}}^{2}\left(t,x,y\right)s_{\mathrm{v},\mathrm{xy}}^{4}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial y}V_{\mathrm{v}}\left(t,x,y\right)
 \frac{2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xx}}\left(t,x,y\right)\mathbf{s_{v,yy}}\left(t,x,y\right)-2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xv}}^{2}\left(t,x,y\right)}
                           V_{\mathrm{u}}\left(t,x,y\right)V_{\mathrm{v}}^{2}\left(t,x,y\right)\mathrm{s_{v,xy}}^{4}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\,\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}\,V_{\mathrm{v}}\left(t,x,y\right)
 \frac{1}{2\sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s_{\rm v,xx}}\left(t,x,y\right){\rm s_{\rm v,xy}}\left(t,x,y\right)-2\sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s_{\rm v,xv}}^{2}\left(t,x,y\right)}
 2\operatorname{V}_{\mathrm{u}}\left(t,x,y\right)\operatorname{V}_{\mathrm{v}}^{2}\left(t,x,y\right)\operatorname{s}_{\mathrm{v},\mathrm{xy}}^{3}\left(t,x,y\right)\operatorname{s}_{\mathrm{v},\mathrm{yy}}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\,\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial y}\operatorname{V}_{\mathrm{v}}\left(t,x,y\right)
      2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xx}}\left(t,x,y\right)\mathbf{s_{v,yy}}\left(t,x,y\right) - 2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xy}}^{2}\left(t,x,y\right)
 2\operatorname{V_{v}}^{3}\left(t,x,y\right)\operatorname{s_{v,xx}}^{2}\left(t,x,y\right)\operatorname{s_{v,xy}}\left(t,x,y\right)\operatorname{\mathbb{E}}\left(\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}\operatorname{V}_{\mathrm{u}}\left(t,x,y\right)
         2\sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right)s_{\rm v,xx}\left(t,x,y\right)s_{\rm v,yy}\left(t,x,y\right) - 2\sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right)s_{\rm v,xy}^{2}\left(t,x,y\right)
                       V_{v}^{3}(t, x, y) s_{v,xx}^{2}(t, x, y) s_{v,yy}^{2}(t, x, y) \mathbb{E}\left(\varepsilon_{u}(t, x, y, \omega) \frac{\partial}{\partial x} \varepsilon_{v}(t, x, y, \omega)\right) \frac{\partial}{\partial u} V_{u}(t, x, y)
                                                                                                                                                                                                                                                                                                                                                                                                                                                           +
 2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xx}}\left(t,x,y\right)\mathbf{s_{v,yy}}\left(t,x,y\right) - 2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xy}}^{2}\left(t,x,y\right)
                     V_{v}^{3}(t, x, y) s_{v,xx}^{2}(t, x, y) s_{v,yy}^{2}(t, x, y) \mathbb{E}\left(\varepsilon_{u}(t, x, y, \omega) \frac{\partial}{\partial u} \varepsilon_{v}(t, x, y, \omega)\right) \frac{\partial}{\partial x} V_{u}(t, x, y)
 \frac{2\sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s_{\rm v.xx}}\left(t,x,y\right){\rm s_{\rm v.xy}}\left(t,x,y\right)-2\sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s_{\rm v.xy}}^{2}\left(t,x,y\right)}
                      2\operatorname{V}_{\mathbf{v}}^{3}\left(t,x,y\right)\operatorname{s}_{\mathbf{v},\mathbf{xx}}\left(t,x,y\right)\operatorname{s}_{\mathbf{v},\mathbf{xy}}^{3}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}\operatorname{V}_{\mathbf{u}}\left(t,x,y\right)
 2\sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right)s_{\rm v,xx}\left(t,x,y\right)s_{\rm v,yy}\left(t,x,y\right) - 2\sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right)s_{\rm v,xy}^{2}\left(t,x,y\right)
2 \operatorname{V_{v}}^{3}(t, x, y) \operatorname{s_{v,xx}}(t, x, y) \operatorname{s_{v,xy}}(t, x, y) \operatorname{s_{v,yy}}^{2}(t, x, y) \mathbb{E}\left(\varepsilon_{\mathrm{u}}(t, x, y, \omega) \frac{\partial}{\partial y} \varepsilon_{\mathrm{v}}(t, x, y, \omega)\right) \frac{\partial}{\partial y} \operatorname{V_{u}}(t, x, y)
          2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xx}}\left(t,x,y\right)\mathbf{s_{v,yy}}\left(t,x,y\right) - 2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xy}}^{2}\left(t,x,y\right)
                                                     V_{\mathbf{v}}^{3}(t,x,y) \mathbf{s_{\mathbf{v},\mathbf{xy}}}^{4}(t,x,y) \mathbb{E}\left(\varepsilon_{\mathbf{u}}(t,x,y,\omega) \frac{\partial}{\partial x} \varepsilon_{\mathbf{v}}(t,x,y,\omega)\right) \frac{\partial}{\partial y} V_{\mathbf{u}}(t,x,y)
 \overline{2\sqrt{\mathrm{V_{u}}\left(t,x,y\right)}\,\mathrm{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathrm{s_{v,xx}}\left(t,x,y\right)\mathrm{s_{v,yy}}\left(t,x,y\right)-2\sqrt{\mathrm{V_{u}}\left(t,x,y\right)}\,\mathrm{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathrm{s_{v,xy}}^{2}\left(t,x,y\right)}
                                                    \mathbf{V_{v}}^{3}\left(t,x,y\right)\mathbf{s_{v,xy}}^{4}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\,\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}\,\mathbf{V_{u}}\left(t,x,y\right)
 2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xx}}\left(t,x,y\right)\mathbf{s_{v,yy}}\left(t,x,y\right) - 2\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xv}}^{2}\left(t,x,y\right)
                    2 \operatorname{V_v}^3(t, x, y) \operatorname{s_{v,xv}}^3(t, x, y) \operatorname{s_{v,vv}}(t, x, y) \mathbb{E}\left(\varepsilon_{\mathrm{u}}(t, x, y, \omega) \frac{\partial}{\partial u} \varepsilon_{\mathrm{v}}(t, x, y, \omega)\right) \frac{\partial}{\partial u} \operatorname{V_u}(t, x, y)
 \frac{2\sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xx}}(t,x,y)\,\mathbf{s_{v,yy}}(t,x,y)-2\sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xy}}^{2}(t,x,y)}
\frac{\partial}{\partial t} \mathbf{s}_{\text{v,yy}}\left(t,x,y\right) = -\frac{2 \mathbf{V}_{\text{uv}}\left(t,x,y\right) \sqrt{\mathbf{V}_{\text{u}}\left(t,x,y\right)} \, \mathbf{V}_{\text{v}}^{\frac{5}{2}}\left(t,x,y\right) \mathbf{s}_{\text{v,xx}}\left(t,x,y\right) \mathbf{s}_{\text{v,yy}}^{2}\left(t,x,y\right)}{\sqrt{\mathbf{V}_{\text{u}}\left(t,x,y\right)} \, \mathbf{V}_{\text{v}}^{\frac{7}{2}}\left(t,x,y\right) \mathbf{s}_{\text{v,xx}}\left(t,x,y\right) \mathbf{s}_{\text{v,yy}}\left(t,x,y\right) - \sqrt{\mathbf{V}_{\text{u}}\left(t,x,y\right)} \, \mathbf{V}_{\text{v}}^{\frac{7}{2}}\left(t,x,y\right) \mathbf{s}_{\text{v,xy}}^{2}\left(t,x,y\right)}
                                                              2 V_{uv}(t, x, y) \sqrt{V_{u}(t, x, y)} V_{v}^{\frac{5}{2}}(t, x, y) s_{v,v}^{2}(t, x, y) s_{v,v}(t, x, y)
                                                                                                                                                                                                                                                                                                                                                                                                                                                           +
  \sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xx}}\left(t,x,y\right)\mathbf{s_{v,yy}}\left(t,x,y\right) - \sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xy}}^{2}\left(t,x,y\right)
  2\operatorname{V}_{\mathrm{u}}\left(t,x,y\right)\operatorname{V}_{\mathrm{v}}^{3}\left(t,x,y\right)\operatorname{s}_{\mathrm{v,xx}}\left(t,x,y\right)\operatorname{s}_{\mathrm{v,xy}}^{2}\left(t,x,y\right)\operatorname{s}_{\mathrm{v,yy}}\left(t,x,y\right)\mathbb{E}\left(\frac{\partial}{\partial x}\,\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\check{\delta}}{\partial x}\,\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)+
               \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s}_{\rm v,xx}\left(t,x,y\right){\rm s}_{\rm v,vy}\left(t,x,y\right) - \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s}_{\rm v.xv}^{2}\left(t,x,y\right)
 2 \operatorname{V}_{\mathrm{u}}(t,x,y) \operatorname{V}_{\mathrm{v}}^{3}(t,x,y) \operatorname{s}_{\mathrm{v},\mathrm{xx}}(t,x,y) \operatorname{s}_{\mathrm{v},\mathrm{xy}}(t,x,y) \operatorname{s}_{\mathrm{v},\mathrm{yy}}^{2}(t,x,y) \mathbb{E}\left(\frac{\partial}{\partial x} \varepsilon_{\mathrm{u}}(t,x,y,\omega) \frac{\partial}{\partial y} \varepsilon_{\mathrm{v}}(t,x,y,\omega)\right) + \frac{1}{2} \operatorname{V}_{\mathrm{u}}(t,x,y) \operatorname{V}_{\mathrm{v}}^{3}(t,x,y) \operatorname{S}_{\mathrm{v},\mathrm{xx}}(t,x,y) \operatorname{S}_{\mathrm{v},\mathrm{xy}}(t,x,y) \operatorname{S}_{\mathrm{v},\mathrm{yy}}^{2}(t,x,y) \mathbb{E}\left(\frac{\partial}{\partial x} \varepsilon_{\mathrm{u}}(t,x,y,\omega) \frac{\partial}{\partial y} \varepsilon_{\mathrm{v}}(t,x,y,\omega)\right) + \frac{1}{2} \operatorname{V}_{\mathrm{u}}(t,x,y) \operatorname{V}_{\mathrm{v}}^{3}(t,x,y) \operatorname{S}_{\mathrm{v},\mathrm{xx}}(t,x,y) \operatorname{S}_{\mathrm{v},\mathrm{xy}}(t,x,y) \operatorname{S}_{\mathrm{v}
                \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s}_{\rm v,xx}\left(t,x,y\right){\rm s}_{\rm v,yy}\left(t,x,y\right) - \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s}_{\rm v,xy}^{2}\left(t,x,y\right)
 2 V_{\rm u}\left(t,x,y\right) V_{\rm v}^{3}\left(t,x,y\right) s_{\rm v,xx}\left(t,x,y\right) s_{\rm v,xy}\left(t,x,y\right) s_{\rm v,yy}^{2}\left(t,x,y\right) \mathbb{E}\left(\frac{\partial}{\partial y} \varepsilon_{\rm u}\left(t,x,y,\omega\right) \frac{\partial}{\partial x} \varepsilon_{\rm v}\left(t,x,y,\omega\right)\right)
                \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xx}}(t,x,y)\,\mathbf{s_{v,yy}}(t,x,y) - \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xv}}^{2}(t,x,y)
               2 V_{\mathrm{u}}(t, x, y) V_{\mathrm{v}}^{3}(t, x, y) s_{\mathrm{v,xx}}(t, x, y) s_{\mathrm{v,yy}}^{3}(t, x, y) \mathbb{E}\left(\frac{\partial}{\partial y} \varepsilon_{\mathrm{u}}(t, x, y, \omega) \frac{\partial}{\partial y} \varepsilon_{\mathrm{v}}(t, x, y, \omega)\right)
  \sqrt{\mathbf{V}_{\mathrm{u}}\left(t,x,y\right)}\,\mathbf{V}_{\mathrm{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s}_{\mathrm{v,xx}}\left(t,x,y\right)\mathbf{s}_{\mathrm{v,yy}}\left(t,x,y\right) - \sqrt{\mathbf{V}_{\mathrm{u}}\left(t,x,y\right)}\,\mathbf{V}_{\mathrm{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s}_{\mathrm{v,xy}}^{2}\left(t,x,y\right)
                                             2 \operatorname{V}_{\mathrm{u}}\left(t, x, y\right) \operatorname{V}_{\mathrm{v}}^{3}\left(t, x, y\right) \operatorname{s}_{\mathrm{v}, \mathrm{xy}}^{4}\left(t, x, y\right) \mathbb{E}\left(\frac{\partial}{\partial x} \varepsilon_{\mathrm{u}}\left(t, x, y, \omega\right) \frac{\partial}{\partial x} \varepsilon_{\mathrm{v}}\left(t, x, y, \omega\right)\right)
  \sqrt{V_{11}(t,x,y)} V_{v}^{\frac{7}{2}}(t,x,y) s_{v,xx}(t,x,y) s_{v,yy}(t,x,y) - \sqrt{V_{11}(t,x,y)} V_{v}^{\frac{7}{2}}(t,x,y) s_{v,xy}^{2}(t,x,y)
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2 V_{\mathrm{u}}\left(t,x,y\right) V_{\mathrm{v}}^{3}\left(t,x,y\right) \mathrm{s_{v,xy}}^{3}\left(t,x,y\right) \mathrm{s_{v,yy}}\left(t,x,y\right) \mathbb{E}\left(\frac{\partial}{\partial x} \varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right) \frac{\partial}{\partial y} \varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)
 \frac{\sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xx}}(t,x,y)\,\mathbf{s_{v,yy}}(t,x,y) - \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xy}}^{2}(t,x,y)}}{\sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xy}}^{\frac{7}{2}}(t,x,y)}}
          2 V_{\mathrm{u}}(t, x, y) V_{\mathrm{v}}^{3}(t, x, y) s_{\mathrm{v,xy}}^{3}(t, x, y) s_{\mathrm{v,yy}}(t, x, y) \mathbb{E}\left(\frac{\partial}{\partial y} \varepsilon_{\mathrm{u}}(t, x, y, \omega) \frac{\partial}{\partial x} \varepsilon_{\mathrm{v}}(t, x, y, \omega)\right)
 \frac{1}{\sqrt{V_{\rm u}(t,x,y)}\,V_{\rm v}^{\frac{7}{2}}(t,x,y)\,s_{\rm v,xx}(t,x,y)\,s_{\rm v,vv}(t,x,y)} - \sqrt{V_{\rm u}(t,x,y)}\,V_{\rm v}^{\frac{7}{2}}(t,x,y)\,s_{\rm v,xv}^{2}(t,x,y)}
         2\operatorname{V}_{\mathrm{u}}\left(t,x,y\right)\operatorname{V}_{\mathrm{v}}^{3}\left(t,x,y\right)\operatorname{s}_{\mathrm{v},\mathrm{xy}}^{2}\left(t,x,y\right)\operatorname{s}_{\mathrm{v},\mathrm{yy}}^{2}\left(t,x,y\right)\mathbb{E}\left(\frac{\partial}{\partial y}\,\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\,\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)
 \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s}_{\rm v,xx}\left(t,x,y\right){\rm s}_{\rm v,yy}\left(t,x,y\right) - \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s}_{\rm v,xy}^{2}\left(t,x,y\right)
V_{\rm u}\left(t,x,y\right)V_{\rm v}^{2}\left(t,x,y\right)s_{\rm v,xx}\left(t,x,y\right)s_{\rm v,xy}^{2}\left(t,x,y\right)s_{\rm v,yy}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\rm u}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\varepsilon_{\rm v}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}V_{\rm v}\left(t,x,y\right)
                         \sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xx}}\left(t,x,y\right)\mathbf{s_{v,yy}}\left(t,x,y\right) - \sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xy}}^{2}\left(t,x,y\right)
V_{\mathrm{u}}\left(t,x,y\right)V_{\mathrm{v}}^{2}\left(t,x,y\right)\mathrm{s}_{\mathrm{v,xx}}\left(t,x,y\right)\mathrm{s}_{\mathrm{v,xy}}\left(t,x,y\right)\mathrm{s}_{\mathrm{v,yy}}^{2}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial y}V_{\mathrm{v}}\left(t,x,y\right)
                         \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s_{\rm v,xx}}\left(t,x,y\right){\rm s_{\rm v,yy}}\left(t,x,y\right) - \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s_{\rm v,xy}}^{2}\left(t,x,y\right)
V_{\mathrm{u}}\left(t,x,y\right)V_{\mathrm{v}}^{2}\left(t,x,y\right)\mathrm{s}_{\mathrm{v,xx}}\left(t,x,y\right)\mathrm{s}_{\mathrm{v,xy}}\left(t,x,y\right)\mathrm{s}_{\mathrm{v,yy}}^{2}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}V_{\mathrm{v}}\left(t,x,y\right)
                         \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s}_{\rm v,xx}\left(t,x,y\right){\rm s}_{\rm v,yy}\left(t,x,y\right) - \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s}_{\rm v,xv}^{2}\left(t,x,y\right)
V_{\mathrm{u}}\left(t,x,y\right)V_{\mathrm{v}}^{2}\left(t,x,y\right)s_{\mathrm{v,xx}}\left(t,x,y\right)s_{\mathrm{v,yy}}^{3}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial y}V_{\mathrm{v}}\left(t,x,y\right)
      \sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xx}}\left(t,x,y\right)\mathbf{s_{v,yy}}\left(t,x,y\right) - \sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xy}}^{2}\left(t,x,y\right)
                V_{\mathrm{u}}\left(t,x,y\right)V_{\mathrm{v}}^{2}\left(t,x,y\right)\mathrm{s_{v,xy}}^{4}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}V_{\mathrm{v}}\left(t,x,y\right)
                                                                                                                                                                                                                                                                                                                  +
 \sqrt{V_{\rm u}(t,x,y)} \, V_{\rm v}^{\frac{7}{2}}(t,x,y) \, s_{\rm v,xx}(t,x,y) \, s_{\rm v,vv}(t,x,y) - \sqrt{V_{\rm u}(t,x,y)} \, V_{\rm v}^{\frac{7}{2}}(t,x,y) \, s_{\rm v,xv}^{2}(t,x,y)
 V_{\mathrm{u}}\left(t,x,y\right)V_{\mathrm{v}}^{2}\left(t,x,y\right)s_{\mathrm{v},\mathrm{xy}}^{3}\left(t,x,y\right)s_{\mathrm{v},\mathrm{yy}}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\,\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial y}\,V_{\mathrm{v}}\left(t,x,y\right)
    \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xx}}(t,x,y)\,\mathbf{s_{v,yy}}(t,x,y) - \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xy}}^{2}(t,x,y)
V_{\mathrm{u}}\left(t,x,y\right)V_{\mathrm{v}}^{2}\left(t,x,y\right)s_{\mathrm{v},\mathrm{xy}}^{3}\left(t,x,y\right)s_{\mathrm{v},\mathrm{yy}}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial x}V_{\mathrm{v}}\left(t,x,y\right)
                                                                                                                                                                                                                                                                                                                  +
      \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xx}}(t,x,y)\,\mathbf{s_{v,yy}}(t,x,y) - \sqrt{\mathbf{V_{u}}(t,x,y)}\,\mathbf{V_{v}}^{\frac{7}{2}}(t,x,y)\,\mathbf{s_{v,xv}}^{2}(t,x,y)
V_{\mathrm{u}}\left(t,x,y\right)V_{\mathrm{v}}^{2}\left(t,x,y\right)s_{\mathrm{v},\mathrm{xy}}^{2}\left(t,x,y\right)s_{\mathrm{v},\mathrm{yy}}^{2}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathrm{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\varepsilon_{\mathrm{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial y}V_{\mathrm{v}}\left(t,x,y,\omega\right)
        \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s}_{\rm v,xx}\left(t,x,y\right){\rm s}_{\rm v,yy}\left(t,x,y\right) - \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s}_{\rm v,xy}^{2}\left(t,x,y\right)
V_{v}^{3}(t, x, y) s_{v,xx}(t, x, y) s_{v,xy}^{2}(t, x, y) s_{v,yy}(t, x, y) \mathbb{E}\left(\varepsilon_{u}(t, x, y, \omega) \frac{\partial}{\partial x} \varepsilon_{v}(t, x, y, \omega)\right) \frac{\partial}{\partial x} V_{u}(t, x, y)
        \sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xx}}\left(t,x,y\right)\mathbf{s_{v,yy}}\left(t,x,y\right) - \sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xy}}^{2}\left(t,x,y\right)
\mathbf{V_{v}}^{3}\left(t,x,y\right)\mathbf{s_{v,xx}}\left(t,x,y\right)\mathbf{s_{v,xy}}\left(t,x,y\right)\mathbf{s_{v,xy}}^{2}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial x}\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial y}\mathbf{V_{u}}\left(t,x,y\right)
        \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right)s_{\rm v,xx}\left(t,x,y\right)s_{\rm v,yy}\left(t,x,y\right) - \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right)s_{\rm v,xy}^{2}\left(t,x,y\right)
V_{v}^{3}(t, x, y) s_{v,xx}(t, x, y) s_{v,xy}(t, x, y) s_{v,yy}^{2}(t, x, y) \mathbb{E}\left(\varepsilon_{u}(t, x, y, \omega) \frac{\partial}{\partial y} \varepsilon_{v}(t, x, y, \omega)\right) \frac{\partial}{\partial x} V_{u}(t, x, y)
         \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s}_{\rm v,xx}\left(t,x,y\right){\rm s}_{\rm v,yy}\left(t,x,y\right) - \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s}_{\rm v,xy}^{2}\left(t,x,y\right)
            V_{v}^{3}(t, x, y) s_{v,xx}(t, x, y) s_{v,yy}^{3}(t, x, y) \mathbb{E}\left(\varepsilon_{u}(t, x, y, \omega) \frac{\partial}{\partial y} \varepsilon_{v}(t, x, y, \omega)\right) \frac{\partial}{\partial y} V_{u}(t, x, y)
 \frac{\sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s_{\rm v,xx}}\left(t,x,y\right){\rm s_{\rm v,yy}}\left(t,x,y\right)-\sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s_{\rm v,xy}}^{2}\left(t,x,y\right)}
                                 V_{\mathbf{v}}^{3}(t, x, y) \mathbf{s}_{\mathbf{v}, \mathbf{x}\mathbf{y}}^{4}(t, x, y) \mathbb{E}\left(\varepsilon_{\mathbf{u}}(t, x, y, \omega) \frac{\partial}{\partial x} \varepsilon_{\mathbf{v}}(t, x, y, \omega)\right) \frac{\partial}{\partial x} V_{\mathbf{u}}(t, x, y)
 \overline{\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xx}}\left(t,x,y\right)\mathbf{s_{v,yy}}\left(t,x,y\right) - \sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xy}}^{2}\left(t,x,y\right)}
             V_{v}^{3}(t, x, y) s_{v,xy}^{3}(t, x, y) s_{v,yy}(t, x, y) \mathbb{E}\left(\varepsilon_{u}(t, x, y, \omega) \frac{\partial}{\partial x} \varepsilon_{v}(t, x, y, \omega)\right) \frac{\partial}{\partial y} V_{u}(t, x, y)
 \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s}_{\rm v,xx}\left(t,x,y\right){\rm s}_{\rm v,yy}\left(t,x,y\right) - \sqrt{V_{\rm u}\left(t,x,y\right)}\,V_{\rm v}^{\frac{7}{2}}\left(t,x,y\right){\rm s}_{\rm v,xv}^{2}\left(t,x,y\right)
             V_{v}^{3}(t, x, y) s_{v,xy}^{3}(t, x, y) s_{v,yy}(t, x, y) \mathbb{E}\left(\varepsilon_{u}(t, x, y, \omega) \frac{\partial}{\partial y} \varepsilon_{v}(t, x, y, \omega)\right) \frac{\partial}{\partial x} V_{u}(t, x, y)
 \sqrt{V_{11}(t,x,y)} V_{V_{1}}^{\frac{7}{2}}(t,x,y) s_{V,XX}(t,x,y) s_{V,YY}(t,x,y) - \sqrt{V_{11}(t,x,y)} V_{V_{1}}^{\frac{7}{2}}(t,x,y) s_{V,XY}^{2}(t,x,y)
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$$\frac{\mathbf{V_{v}}^{3}\left(t,x,y\right)\mathbf{s_{v,xy}}^{2}\left(t,x,y\right)\mathbf{s_{v,yy}}^{2}\left(t,x,y\right)\mathbb{E}\left(\varepsilon_{\mathbf{u}}\left(t,x,y,\omega\right)\frac{\partial}{\partial y}\,\varepsilon_{\mathbf{v}}\left(t,x,y,\omega\right)\right)\frac{\partial}{\partial y}\,\mathbf{V_{u}}\left(t,x,y\right)}{\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xx}}\left(t,x,y\right)\mathbf{s_{v,yy}}\left(t,x,y\right)-\sqrt{\mathbf{V_{u}}\left(t,x,y\right)}\,\mathbf{V_{v}}^{\frac{7}{2}}\left(t,x,y\right)\mathbf{s_{v,xy}}^{2}\left(t,x,y\right)}$$