PKF on 1D multivariate oscillator

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Abstract

This notebook illustrates the use of sympkf to build and handle the PKF dynamics associated with the harmonic oscillator as 2D fields given by

$$\left\{\begin{array}{r}\partial_t u = v\\ \partial_t v = -u\end{array}\right.$$

where u and v are functions of t, x. For this dynamics, the resulting PKF system is not closed because of the cros-correlation.

1 Definition of the 1D multivariate dynamics

[1]:
```
import sympy
sympy.init_printing()
```

Definition of the dynamics from sympy tools

[2]:
```
from sympy import Function, Derivative, Eq, symbols
from sympkf import SymbolicPKF, t
```

[3]:
```
x  = symbols('x')
u = Function('u')(t,x)
v = Function('v')(t,x)
dynamics = [Eq(Derivative(u,t), v), Eq(Derivative(v,t), -u)]
dynamics
```

[3]:

$$\left[\frac{\partial}{\partial t}u(t,x) = v(t,x),\ \frac{\partial}{\partial t}v(t,x) = -u(t,x)\right]$$

2 Computation of the PKF dynamics by using SymPKF

[4]:
```
pkf_dynamics = SymbolicPKF(dynamics)
```

[5]:
```
for equation in pkf_dynamics.in_metric:    display(equation)
```

$$\frac{\partial}{\partial t}u(t,x) = v(t,x)$$

$$\frac{\partial}{\partial t}v(t,x) = -u(t,x)$$

$$\frac{\partial}{\partial t}\operatorname{V_{u}}(t,x) = 2\operatorname{V_{uv}}(t,x)$$

$$\frac{\partial}{\partial t}\operatorname{V_{v}}(t,x) = -2\operatorname{V_{uv}}(t,x)$$

$$\frac{\partial}{\partial t}\operatorname{V_{uv}}(t,x) = -\operatorname{V_{u}}(t,x) + \operatorname{V_{v}}(t,x)$$

$$\frac{\partial}{\partial t}\operatorname{g_{u,xx}}(t,x) = -\frac{2\operatorname{V_{uv}}(t,x)\operatorname{g_{u,xx}}(t,x)}{\operatorname{V_{u}}(t,x)} + \frac{2\sqrt{\operatorname{V_{v}}(t,x)}\mathbb{E}\left(\frac{\partial}{\partial x}\varepsilon_{u}(t,x,\omega)\frac{\partial}{\partial x}\varepsilon_{v}(t,x,\omega)\right)}{\sqrt{\operatorname{V_{u}}(t,x)}} + \frac{\mathbb{E}\left(\varepsilon_{v}(t,x,\omega)\frac{\partial}{\partial x}\varepsilon_{u}(t,x,\omega)\right)\frac{\partial}{\partial x}\operatorname{V_{v}}(t,x)}{\sqrt{\operatorname{V_{u}}(t,x)}\sqrt{\operatorname{V_{v}}(t,x)}} - \frac{\sqrt{\operatorname{V_{v}}(t,x)}\mathbb{E}\left(\varepsilon_{v}(t,x,\omega)\frac{\partial}{\partial x}\varepsilon_{u}(t,x,\omega)\right)\frac{\partial}{\partial x}\operatorname{V_{u}}(t,x)}{\operatorname{V_{u}}^{\frac{3}{2}}(t,x)}$$

$$\frac{\partial}{\partial t}\operatorname{g_{v,xx}}(t,x) = \frac{2\operatorname{V_{uv}}(t,x)\operatorname{g_{v,xx}}(t,x)}{\operatorname{V_{v}}(t,x)} - \frac{2\sqrt{\operatorname{V_{u}}(t,x)}\mathbb{E}\left(\frac{\partial}{\partial x}\varepsilon_{u}(t,x,\omega)\frac{\partial}{\partial x}\varepsilon_{v}(t,x,\omega)\right)}{\sqrt{\operatorname{V_{v}}(t,x)}} + \frac{\sqrt{\operatorname{V_{u}}(t,x)}\mathbb{E}\left(\varepsilon_{u}(t,x,\omega)\frac{\partial}{\partial x}\varepsilon_{v}(t,x,\omega)\right)\frac{\partial}{\partial x}\operatorname{V_{v}}(t,x)}{\operatorname{V_{v}}^{\frac{3}{2}}(t,x)} - \frac{\mathbb{E}\left(\varepsilon_{u}(t,x,\omega)\frac{\partial}{\partial x}\varepsilon_{v}(t,x,\omega)\right)\frac{\partial}{\partial x}\operatorname{V_{u}}(t,x)}{\sqrt{\operatorname{V_{u}}(t,x)}\sqrt{\operatorname{V_{v}}(t,x)}}$$

[6]:
```
for equation in pkf_dynamics.in_aspect:    display(equation)
```

$$\frac{\partial}{\partial t}u(t,x) = v(t,x)$$

$$\frac{\partial}{\partial t}v(t,x) = -u(t,x)$$

$$\frac{\partial}{\partial t}\operatorname{V_{u}}(t,x) = 2\operatorname{V_{uv}}(t,x)$$

$$\frac{\partial}{\partial t}\operatorname{V_{v}}(t,x) = -2\operatorname{V_{uv}}(t,x)$$

$$\frac{\partial}{\partial t}\operatorname{V_{uv}}(t,x) = -\operatorname{V_{u}}(t,x) + \operatorname{V_{v}}(t,x)$$

$$\frac{\partial}{\partial t}\operatorname{s_{u,xx}}(t,x) = \frac{2\operatorname{V_{uv}}(t,x)\operatorname{s_{u,xx}}(t,x)}{\operatorname{V_{u}}(t,x)} - \frac{2\sqrt{\operatorname{V_{v}}(t,x)}\operatorname{s_{u,xx}}^{2}(t,x)\mathbb{E}\left(\frac{\partial}{\partial x}\varepsilon_{u}(t,x,\omega)\frac{\partial}{\partial x}\varepsilon_{v}(t,x,\omega)\right)}{\sqrt{\operatorname{V_{u}}(t,x)}} - \frac{\operatorname{s_{u,xx}}^{2}(t,x)\mathbb{E}\left(\varepsilon_{v}(t,x,\omega)\frac{\partial}{\partial x}\varepsilon_{u}(t,x,\omega)\right)\frac{\partial}{\partial x}\operatorname{V_{v}}(t,x)}{\sqrt{\operatorname{V_{u}}(t,x)}\sqrt{\operatorname{V_{v}}(t,x)}} + \frac{\sqrt{\operatorname{V_{v}}(t,x)}\operatorname{s_{u,xx}}^{2}(t,x)\mathbb{E}\left(\varepsilon_{v}(t,x,\omega)\frac{\partial}{\partial x}\varepsilon_{u}(t,x,\omega)\right)\frac{\partial}{\partial x}\operatorname{V_{u}}(t,x)}{\operatorname{V_{u}}^{\frac{3}{2}}(t,x)}$$

$$\frac{\partial}{\partial t}\mathrm{s}_{\mathrm{v,xx}}(t,x) = -\frac{2\,\mathrm{V}_{\mathrm{uv}}(t,x)\,\mathrm{s}_{\mathrm{v,xx}}(t,x)}{\mathrm{V}_{\mathrm{v}}(t,x)} + \frac{2\sqrt{\mathrm{V}_{\mathrm{u}}(t,x)}\,{\mathrm{s}_{\mathrm{v,xx}}}^{2}(t,x)\mathbb{E}\left(\frac{\partial}{\partial x}\varepsilon_{\mathrm{u}}(t,x,\omega)\frac{\partial}{\partial x}\varepsilon_{\mathrm{v}}(t,x,\omega)\right)}{\sqrt{\mathrm{V}_{\mathrm{v}}(t,x)}} - \frac{\sqrt{\mathrm{V}_{\mathrm{u}}(t,x)}\,{\mathrm{s}_{\mathrm{v,xx}}}^{2}(t,x)\mathbb{E}\left(\varepsilon_{\mathrm{u}}(t,x,\omega)\frac{\partial}{\partial x}\varepsilon_{\mathrm{v}}(t,x,\omega)\right)\frac{\partial}{\partial x}\mathrm{V}_{\mathrm{v}}(t,x)}{{\mathrm{V}_{\mathrm{v}}}^{\frac{3}{2}}(t,x)} + \frac{{\mathrm{s}_{\mathrm{v,xx}}}^{2}(t,x)\mathbb{E}\left(\varepsilon_{\mathrm{u}}(t,x,\omega)\frac{\partial}{\partial x}\varepsilon_{\mathrm{v}}(t,x,\omega)\right)\frac{\partial}{\partial x}\mathrm{V}_{\mathrm{u}}(t,x)}{\sqrt{\mathrm{V}_{\mathrm{u}}(t,x)}\sqrt{\mathrm{V}_{\mathrm{v}}(t,x)}}$$

[7]:
```
pkf_dynamics.internal_closure
```

[7]:

$$\left\{\mathbb{E}\left(\varepsilon_{\mathrm{u}}(t,x,\omega)\,\varepsilon_{\mathrm{v}}(t,x,\omega)\right) : \frac{\mathrm{V}_{\mathrm{uv}}(t,x)}{\sqrt{\mathrm{V}_{\mathrm{u}}(t,x)}\sqrt{\mathrm{V}_{\mathrm{v}}(t,x)}},\ \mathbb{E}\left(\varepsilon_{\mathrm{u}}(t,x,\omega)\frac{\partial}{\partial x}\varepsilon_{\mathrm{u}}(t,x,\omega)\right) : 0,\ \mathbb{E}\left(\varepsilon_{\mathrm{u}}(t,x,\omega)\frac{\partial^{2}}{\partial x^{2}}\varepsilon_{\mathrm{u}}(t,x,\omega)\right.$$