Boerner's Theorem

Vincent La

December 25, 2017

True or False If you sort each column of a matrix, then sort each row, the columns are still sorted. Justify your answer.

True.

Proof. Given any matrix, suppose we sort each column and then sort each row. For this case, suppose we sort in ascending order. Now, suppose for a contradiction that the columns are no longer sorted. This implies we have a matrix in the form

$$\begin{bmatrix} * & & \\ & b & \geq b \\ & a & \geq a \end{bmatrix}$$

where b is some number greater than a. Because the rows are sorted, we know that a is greater than or equal to all numbers to the left of it. Furthermore, we know that b was moved during the sort by row operation.

□