MATH 108A Review Sheet

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1. Vector Spaces

1.A

1. A complex number is an ordered pair (a, b) of real numbers $a, b \in \mathbb{R}$ that we write ____________

2. Suppose we have $\alpha, \beta \in \mathbb{C}$ such that

$$\alpha := a + bi$$

$$\beta := c + di$$

   What is $\alpha \cdot \beta$?

3. If $a \in \mathbb{R}$ then if $b \in \mathbb{R}$ is the multiplicative inverse of a, $a \cdot b =$ ____________

4. Two lists of vectors are considered the same if they have the same length and ____________

1.B

1. A ____________ is a set with two operations ________________________ and ________________________ such that the operations are commutative, ________________________, and distributive and both have identity and inverse elements.

2. A ________________________ over a field $\mathbb{F}$ is a set V with two operations
   - Vector addition $+$
   - Scalar multiplication $\cdot$

3. In general, is there a such thing as multiplication between vectors?

4. An element of a vector space is called a ____________

5. What is the trivial vector space?

6. The set F^∞ is the set of __________________

7. The set $\mathbb{R}^\mathbb{R}$ is the set of ________________________

8. What are the sets $\mathbb{R}^n, \mathbb{C}^n$, and $\mathbb{F}^n$?

1.C

1. Subspace Test
   (a) In English, explain the _______ conditions of the Subspace Test.
   (b) Write out the Subspace Test using logical symbols.
2. Suppose that $U_1, U_2, ...U_m$ are subspaces of V such that
   $$0 = u_1 + u_2 + ... + u_m$$
   where $u_1 \in U_1, ..., u_m \in U_m$, has a unique solution.
   Then we can say that ____________________________ (not talking about linear independence).
   (a) And what specifically is the value of these u_i?
3. If $U + W$ are subspaces of V, then $U + W$ is a direct sum if and only if ______________
4. Let $v_1, v_2, ..., v_m$ be a list of vectors in V. Then, $\text{span}(v_1, v_2, ..., v_m)$ is a ______________ of V.
5. Let U_1, U_2, U_3 be subspaces of V such that $U_1 \cap U_2 \cap U_3 = \{0\}$ and that $U_1 + U_2 + U_3 = V$. Can we conclude that $U_1 \oplus U_2 \oplus U_3 = V$?

2. Finite Dimensional Vector Spaces

2.A

1. The set $\mathscr{P}(\mathbb{F})$ is ______________-dimensional while $\mathscr{P}_m(\mathbb{F})$ is ______________-dimensional.
2. Two functions p, q are the **same** if for all $z \in \mathbb{F}$ ____________________________.
3. Suppose for there exists a $v_j \in \text{span}(v_1, v_2, ..., v_n)$. Then, we know that $v_1, v_2, ..., v_j, ..., v_n$ is ____________________________.
4. True or False. If $a_1v_1 + a_2v_2 + ... + a_mv_m = 0$, given that $a_1 = a_2 = ... = a_m = 0$, then $v_1, v_2, ..., v_m$ are linearly independent.

2.B

1. If a list $v_1, v_2, ..., v_n \in V$ is both linearly independent and spanning, then it is a ______________.
2. If a list is spanning but not a basis, then it is ______________________.
3. The standard basis for $\mathbb{F}^n$ is the list ____________________________
4. If a list $v_1, v_2, ..., v_n$ (of length n) is a basis for V, then $\dim V =$ ______________.
5. If $v_1, v_2, ..., v_j$ is linearly independent in V, and $v_1, v_2, ..., v_k$ spans V, then j ____ k.

2.C

...

3. Linear Maps

3.A

1. A linear map (linear transformation) from V to W is a function $T : V \to W$ such that
   - __________________________
   - __________________________

3.B

1. Suppose $v_1, v_2, ..., v_n$ is linearly independent in V. Do we know that $v_1, v_2, ..., v_n$ can be extended to be a basis for V?
2. Let $T \in \mathscr{L}(V, W)$. Then,

   dim V = __________ + __________.
3. Let $T : L \to W$, be a linear map. Then the ____________________ of T is the set

$$\{v \in V | Tv = 0\}$$

4. Does the null space contain the zero vector?
5. A linear map $T : V \to W$ if injective if whenever $u \neq w$, then _____________
6. Prove that T is injective, if and only if null $T = \{\vec{0}\}$.
7. If a linear map $T : V \to W$ is such that range $T = W$ then we say T is ____________________.

3.C

1. $\mathbb{F}^{m,n}$ is the set of all _____________.
2. Suppose $T \in \mathscr{L}(V)$ is such that with respect to the basis v_1, v_2, v_3

$$\mathscr{M}(T) = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

   What are Tv_1, Tv_2, Tv_3 equal to? Source: Last lecture

3.D

1. Suppose $T \in \mathscr{L}(V, W)$ is such that null $T = \{\vec{0}\}$ and range $T = W$. Then, we know that T is ______________.
2. Let $T \in L(V, W)$ be such there are $R, S \in L(W, V)$ such that R, S are inverses of T. Is $R = S$?
3. Suppose $T \in L(V, W)$ is invertible. Then, we can say V and W are ______________ and T is ______________.
4. Let V, W be finite-dimensional and isomorphic. Then,

$$\underline{\hspace{3cm}} = \underline{\hspace{3cm}}$$

5. Suppose T is a linear function from $\mathbb{R}^3$ to $\mathbb{R}^3$. Then we can say, T is a(n) ____________________________ on $\mathbb{R}^3$.

4. Polynomials

1. Let $p(z) = 0$ for all $z \in \mathbb{F}$, i.e. the zero polynomial. Then, $\deg(p) =$ _______.
2. Let $p(z) = a_0 + a_1 z + a_2 z^2 + ... + a_m z^m$ for all $z \in \mathbb{F}$. Then, $\deg(p) =$ _______.

5. Eigenvalues, Eigenvectors, and Invariant Subspaces

5.A

1. Suppose U is a subspace of V and $T \in \mathscr{L}(V)$. If for any $u \in U$, Tu is also in U, we say U is...
2. Suppose T is a linear operator on V. If for some $v \in V$, there is some scalar $\lambda \in \mathbb{F}$ is an eigenvalue (Def 5.5) for v, then $Tv = ...$
3. Is null T invariant under T? Why?
4. Is range T invariant under T? Why?
5. Suppose that U is a one-dimensional subspace invariant under T and v is a basis for U. Then v is a(n) ______ for T and there exists a __ $\in \mathbb{F}$ such that __ $=$ __.
6. Suppose that $T \in \mathscr{L}(V)$, when applied to some basis of V, gives the following matrix

$$\begin{bmatrix} 1 & 5 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

   Is T invertible?

7. Suppose $T \in \mathscr{L}(V)$ with distinct eigenvalues $\lambda_1, \lambda_2, ..., \lambda_m$. Then, we know that the corresponding eigenvectors $v_1, v_2, ... v_m$ are __________________________.