1 The 2-body problem: Orbit of a planet around the Sun

Consider the problem of a star of mass $M_* = M_\odot$ (where $M_\odot = 1.99 \times 10^{30}$ kg surrounded by a planet of $M_p = 10^{-3} M_\odot$. At time $t = 0$ the planet is located at coordinates $(1, 0, 0)$ in units of AU= 1.496×10^{11} m. The planet's velocity is $(0, 0.5, 0)$ in units of the Kepler velocity at the that location, which is $v_K(1\text{ AU}) = 2.98 \times 10^4\,\mathrm{m\,s^{-1}}$

Solve the Kepler orbit of this planet using numerical integration with the leapfrog algorithm. Find an appropriate time step. Plot the result for the first few orbits.

Now integrate for 100 orbits and see how the orbit behaves.

Repeat this with the RK2 and RK4 algorithms and see how the system behaves. Discuss the difference to the leapfrog algorithm. For this, also plot the time evolution of the relative error of the total energy and the time evolution of the total kinetic energy of the system.

2 N-body problem

Let us now consider a N-body problem, with $N > 2$ gravitationally interacting objects. We will solve this system with the *direct summation method*. First, however, let us cast the problem in dimensionless units. The N-body equation reads

$$\frac{d\vec{x}_i}{dt} = \vec{v}_i \tag{1}$$

$$m_i \frac{d\vec{v}_i}{dt} = G m_i \sum_{k \neq i} m_l \frac{\vec{x}_k - \vec{x}_i}{|\vec{x}_k - \vec{x}_i|^3} \tag{2}$$