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The formation of an eccentric gap in a gas disc by a planet in an eccentric orbit

A. Pasha Hosseinbor,★ Richard G. Edgar,★ Alice C. Quillen★ and Amanda LaPage
Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627, USA

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ABSTRACT

We investigate the effect of a planet on an eccentric orbit on a two-dimensional low-mass gaseous disc. At a planet eccentricity above the planet's Hill radius divided by its semimajor axis, we find that the disc morphology differs from that exhibited by a disc containing a planet in a circular orbit. An eccentric gap is created with eccentricity that can exceed the planet's eccentricity and precesses with respect to the planet's orbit. We find that a more massive planet is required to open a gap when the planet is on an eccentric orbit. We attribute this behaviour to spiral density waves excited at corotation resonances by the eccentric planet. These act to increase the disc's eccentricity and exert a torque opposite in sign to that exerted by the Lindblad resonances. The reduced torque makes it more difficult for waves driven by the planet to overcome viscous inflow in the disc.

Key words: planetary systems: protoplanetary discs.

1 INTRODUCTION

The recently discovered extrasolar planets include Jovian mass planets with short-period orbits and isolated planets with large eccentricities.[1] Explanations for the 'hot Jupiters' include orbital migration models (e.g. Papaloizou & Terquem 2006). Explanations for the latter include eccentricity growth via planet–disc interactions (e.g. Goldreich & Sari 2003; Sari & Goldreich 2004; Kley & Dirksen 2006), or planet–planet interactions (e.g. Masset & Snellgrove 2001; Papaloizou 2003; Thommes & Lissauer 2003; Kley, Peitz & Bryden 2004). Protoplanets could also be formed with moderate eccentricity due to interactions and waves in the disc (e.g. Cresswell & Nelson 2006). Because protoplanets embedded in discs might be in eccentric orbits, we are prompted to study gap formation by an eccentric planet.

So-called 'transitional discs' (systems with an accretion disc signature, where the accretion disc does not appear to extend to the star) have been observed for a number of years (Marsh & Mahoney 1992; Jensen & Mathieu 1997; Rice et al. 2003; Bergin et al. 2004). With the launch of the *Spitzer Space Telescope*, observers have been able to obtain high-resolution spectra of these systems (previously, only broadband colours were available), dramatically improving our understanding of these systems. In particular, *Spitzer* has established that the inner discs are very empty, and that the edges are quite sharp (Calvet et al. 2005; D'Alessio et al. 2005). These objects, such as CoKuTau/4, likely harbour massive planets residing just interior to the disc edge. Interactions between the disc and planet and multiple planet interactions could cause this outer planet to be on an eccentric orbit.

In this paper using two-dimensional hydrodynamic simulations, we investigate the ability of a planet on an eccentric orbit to open a gap in a low-mass viscous disc. We also investigate possible differences in disc morphology that are peculiar to planets on eccentric orbits and so would allow an observer to differentiate between a disc perturbed by an eccentric planet and one in a circular orbit.

In Section 2, our simulations are described. In Section 3, we discuss how we identify a gap from the numerical simulations. We discuss comparison numerical simulations of gaps opened by planets on circular orbits. In Section 4, we discuss how a disc responds to a planet on an eccentric orbit. In Section 5, we present a modified gap-opening criterion, based on our simulations, for planets on eccentric orbits. A summary and discussion follows.

2 NUMERICAL EXPERIMENTS

To investigate the evolution of gaps opened by a single protoplanet, we carried out a set of two-dimensional hydrodynamical calculations using the FARGO code developed by Masset (2000a,b). FARGO is an Eulerian polar grid code with a staggered mesh and an artificial second-order viscous pressure to stabilize the shocks. The code allows tidal interaction between one or more planets and a two-dimensional non-self-gravitating gaseous disc. It owes its name to the FARGO advection algorithm that removes the average azimuthal velocity for the Courant time-step limit (Masset 2000b). The simulations are performed in the frame rotating with the planet's guiding centre. The outer boundary does not allow either inflow or outflow, so it must be located sufficiently far from the planet to ensure that

★E-mail: aph5k@virginia.edu (APH); rge21@pas.rochester.edu (RGE); aquillen@pas.rochester.edu (ACQ)
[1] http://exoplanet.eu/.

spiral density waves are damped before they reach it. This is facilitated by adopting a logarithmic grid in radius. The grid inner boundary only allows material to escape so that the disc material may be accreted on to the primary star. While the gas disc feels a gravitational perturbation from the planet, the planet itself was not allowed to feel the disc. This choice allows us to investigate the role of the planet's eccentricity in perturbing the disc without the additional complication of a varying planet eccentricity or inward migration.

The code uses units such that $G = M_* = a_p = 1$, where M_* is the mass of the central star, and a_p is the planet's (initial) semimajor axis. The planet mass, M_p, is described in terms of the ratio of the planet mass to the stellar mass, q/M_*. The grid extends between $r_{min} = 0.2$ and $r_{max} = 5.0$, and a full circle in azimuth. We used 384 equally spaced cells in the azimuthal direction, and 200 logarithmically spaced cells in the radial direction. This choice allows the grid cells to be nearly square and so minimizes truncation errors. The planet is initially placed at the apocentre.

We initially adopted a grid identical to that of the hydrocode comparison of de Val-Borro et al. (2006), but found that spiral density waves were spuriously reflected by the outer boundary when the planet was on an eccentric orbit. We increased the maximum radius to that listed above and verified that our numerical experiments were not sensitive to the location of the outer boundary.

Our disc viscosity is constant over the entire disc. We typically choose $\nu = 10^{-5}$ in the system of units outlined above (although we have performed runs with other values). Once the viscosity is set, we may calculate the Reynolds number of the disc:

$$\mathcal{R} \equiv \frac{r^2 \Omega}{\nu}. \tag{1}$$

In our units, $\mathcal{R} = \nu^{-1}$ at the radius of the planet's orbit (note that $\mathcal{R}$ is a function of radius).

We use a constant aspect ratio disc, with $h/r = 0.05$, where h is the density scaleheight. The local sound speed is set from this. For our initial conditions, we took a flat surface density profile ($\Sigma(r) = \Sigma_0$). Since the planet was not allowed to migrate, Σ_0 was only important numerically; we verified that it was sufficiently small to avoid spurious numerical effects.

3 GAP OPENING IN THE CIRCULAR CASE

Planets above a certain mass are expected to open a gap in their natal discs. In this section, we will discuss the formation of gaps by planets on circular orbits. Although this has been done by numerous authors, it is important that we first characterize the behaviour of FARGO for planets in a circular orbit, before we proceed to the eccentric case.

Previous studies have shown theoretically and numerically that a gap should be opened in a disc when the planet mass ratio exceeds

$$q > 40\mathcal{R}^{-1} \tag{2}$$

(Lin & Papaloizou 1993; Bryden et al. 1999). This condition can be derived by balancing the viscous torques, $T_\nu = 3\pi\Sigma\nu r^2\Omega$, against those from the spiral density waves the planet excites at its Lindblad resonances. There is an alternative, tidal condition

$$q > 3\left(\frac{h}{r}\right)^3,$$

which compares the disc scaleheight to the planet's Hill sphere. In this work, we will not examine possible variations with the disc aspect ratio (it does not even enter indirectly via $\mathcal{R}$, since we are not using an α viscosity). However, for thick discs, we would expect the *h*/*r* value to affect our results.

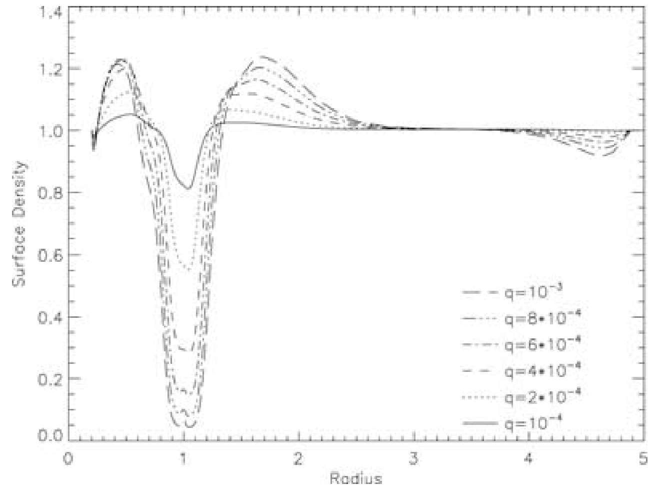


Figure 1. The azimuthally averaged surface density of the disc after 500 orbits for planet mass ratios $10^{-4} < q < 10^{-3}$. All runs had the planet on a fixed circular orbit, and the disc viscosity was $\nu = 10^{-5}$.

In Fig. 1, we show the azimuthally averaged surface density after 500 orbits for planet mass ratios ranging from $q = 10^{-4}$ to 10^{-3} on fixed circular orbits. These runs all have $\nu = 10^{-5}$ for their viscosity. We see that the depth of the gap varies smoothly with q, so determining whether a gap has formed is somewhat arbitrary. Evaluating equation (2), we find that $q = 4 \times 10^{-4}$ is the expected threshold mass ratio for gap formation for this set of runs. Fig. 1 shows that mass ratios larger than this have density contrasts $\Sigma_{gap}/\Sigma_0 < 0.2$ at the bottom of their induced gaps.

Having tested the behaviour of the code as q varies, we now turn our attention to the variation with viscosity. Fig. 2 is similar to Fig. 1, except the planet mass ratio is fixed at $q = 10^{-3}$, and we varied the disc viscosity. Equation (2) predicts that for a protoplanet with $q = 10^{-3}$, a disc with Reynolds number at the planet above $\mathcal{R} = 8 \times 10^4$ should cause a gap to open in the disc. This corresponds to a viscosity of $\nu < 2.5 \times 10^{-5}$. The runs satisfying this criterion all show $\Sigma_{gap}/\Sigma_0 < 0.2$ for their gaps.

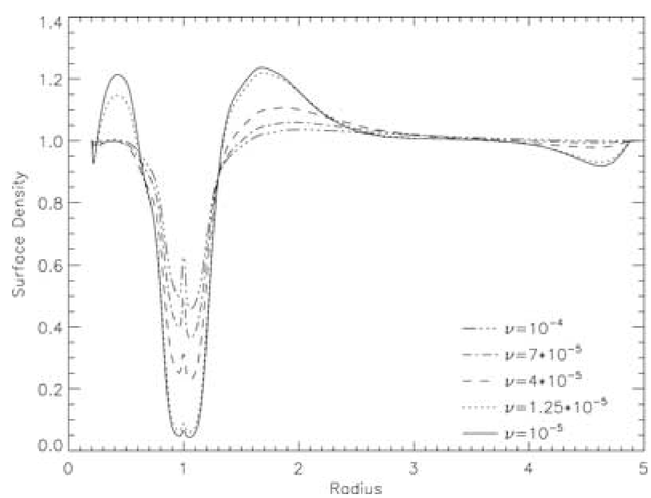


Figure 2. The azimuthally averaged surface density of the disc as the viscosity varied between $\nu = 10^{-5}$ and 10^{-4}. The deeper gaps correspond to the lower viscosity discs. All simulations are shown after 500 orbits, for a $q = 10^{-3}$ planet on a circular orbit.

Based on the surface density profiles shown in Figs 1 and 2, we adopt the following criterion to identify formation of a gap in our numerical experiments: the surface density in the gap must be less than 20 per cent of the unperturbed surface density. We have chosen this criterion so that it is consistent with equation (2). We adopt this criterion during our discussion of the effects of eccentric planets in subsequent sections. However, as Figs 1 and 2 show, the precise definition of 'gap' is arbitrary, since the variation with q and ν is relatively smooth.

4 DISC RESPONSE TO AN ECCENTRIC PLANET

In this section, we discuss how a disc responds to a planet on an eccentric orbit. We concentrate on planets with mass ratio $q = 6 \times 10^{-4}$ and 10^{-3} in discs with viscosity $\nu = 10^{-5}$.

4.1 Azimuthally averaged density profiles

In Fig. 3, we show how the surface density profiles vary with planetary eccentricity, $e_{\rm p}$ for a $q = 6 \times 10^{-4}$ planet. We see that the shape of the gap is similar to the circular case for planet eccentricity below $e < 0.06$. The planet's Hill radius is given by

$$r_{\rm Hill} = \left(\frac{q}{3}\right)^{1/3} a_{\rm p}, \qquad (3)$$

which measures the distance over which the planet's gravity dominates (the Hill sphere is the size of the Roche lobe in the low q limit). In our units, we note that $r_{\rm Hill} = 0.058$ for these runs. Above this eccentricity, the gap as seen in the azimuthally averaged density profiles is shallower and wider. By the 20 per cent criterion discussed in the previous section, the planet no longer induces a gap for $e_{\rm p} > 0.1$.

Fig. 4 is similar to Fig. 3 but for a $q = 10^{-3}$ planet. At eccentricities of $e_{\rm p} < 0.065$, the shape of the gap is nearly identical to that of the circular case. Above this threshold, the gap becomes shallower and wider, and increasingly deviates from the circular case. We note that the planet's Hill radius is $0.069a_{\rm p}$. This suggests that the planet's eccentricity has little effect on the density profile unless the planet's orbit takes it outside the sum of its semimajor axis and Hill radius. As spiral density waves are driven from resonances located at radii all the way up to the planet's Hill radius, this limiting eccentricity is not unexpected.

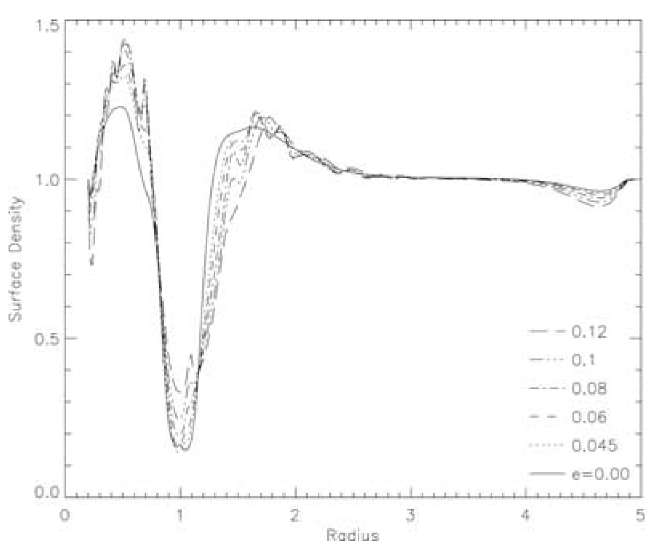


Figure 3. The azimuthally averaged surface density of the disc after 500 orbits for planet eccentricities between $0 < e_{\rm p} < 0.12$. The planets had $q = 6 \times 10^{-4}$ and the disc viscosity was $\nu = 10^{-5}$.

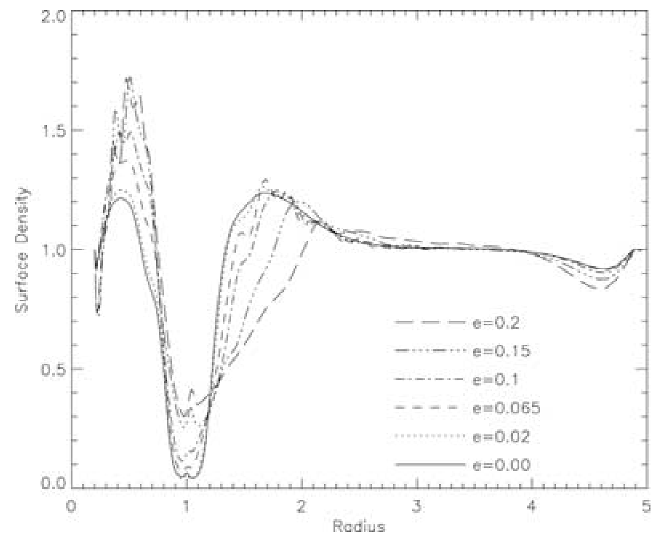


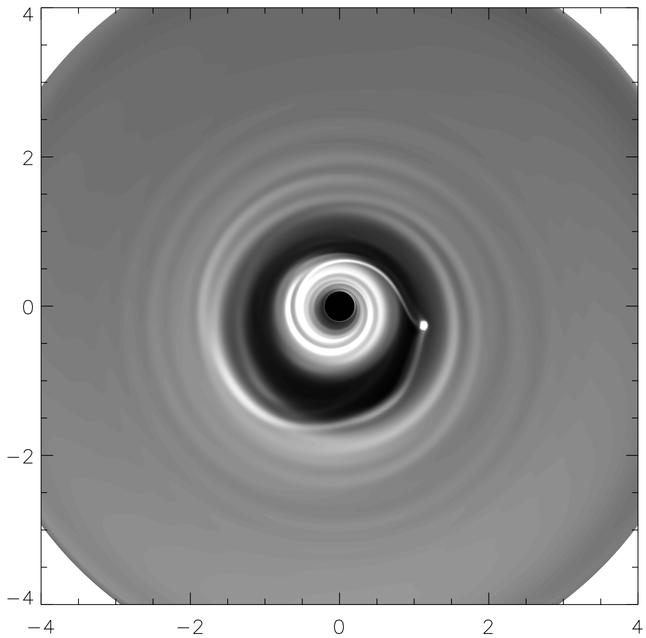
Figure 4. Similar to Fig. 3, but the planets had mass ratio $q = 10^{-3}$ and the eccentricity range is $0 < e_{\rm p} < 0.2$.

Figs 3 and 4 show that as planet eccentricity increases, the azimuthally averaged gap density profile becomes wider and shallower. It becomes harder for the protoplanet to open up a deep gap in the disc. Because the density in the gap is higher at higher planet eccentricity, a more massive planet is required to open a gap in the disc. We will discuss possible explanations for this behaviour after we describe the disc morphology seen in these simulations.

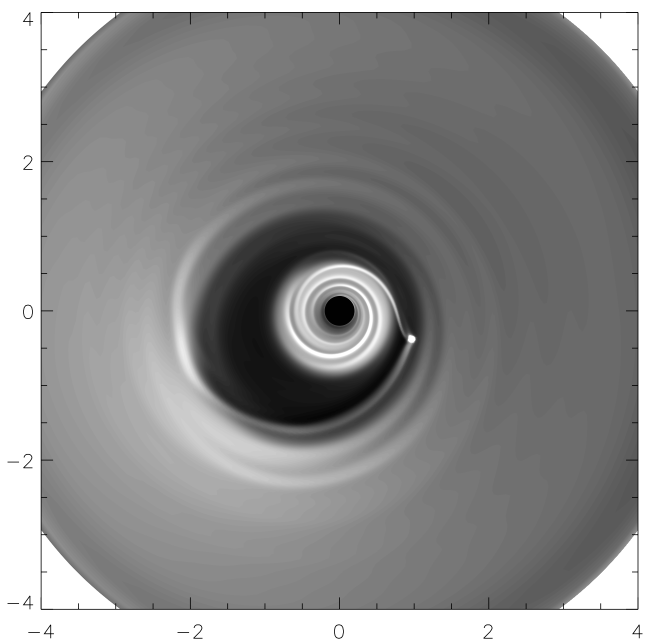
4.2 Eccentric gaps

In Fig. 5, we show the two-dimensional surface density of the disc after 250 and 500 orbits for planet eccentricity $e_{\rm p} = 0.2$ and mass ratio $q = 10^{-3}$. Examining the surface density as a function of time, we see the planet crossing between the inner and outer gap edges, producing one-armed spirals as it encounters each edge. By $t = 500$ orbits, the planet has formed an eccentric gap. The eccentricity of the gap increases to a maximum of about twice that of the planet at this time. The gap also slowly precesses. At $t = 500$ orbits the gap's apocentre is about 180° from that of the planet.

Because the gap precesses with respect to the planet's orbit, the planet approaches the disc edge at different longitudes. Close approaches allow the planet to clear out additional material from the disc edge. On longer time-scales, of several thousands of orbits, the gap slowly widens and eventually circularizes. However, the gap is much wider than the gap produced by a planet on a circular orbit, extending (roughly) between the planet's peri- and apcentre. During the long circularization period accretion on to the planet could exceed that of a similar mass planet on a circular orbit. In the case of a planet on a circular orbit, the formation of a gap can significantly reduce the accretion rate on to the planet (D'Angelo, Kley & Henning 2003). Recently, Kley & Dirksen (2006) looked at the excitation of disc eccentricity by massive planets (up to $q = 5 \times 10^{-3}$), and suggested that the induced eccentricity could aid accretion by the planet. Although we did not permit accretion in our numerical experiments, we would expect behaviour of this nature.



(a)



(b)

Figure 5. Disc surface densities after (a) 250 and (b) 500 orbits for a $q = 10^{-3}$ planet on a orbit with eccentricy fixed at $e_{\mathrm{p}} = 0.2$. The gas viscosity was $\nu = 10^{-5}$.

Fig. 4 showed the azimuthally averaged density profile was shallower for gaps opened by eccentric planets. The smoothness or shallow gradient of the azimuthally averaged profile in the gap is due to the eccentricity of the gap edge, rather than a smooth change in the disc density with radius. The gap itself has sharp edges. The apparent shallow profile in the azimuthally averaged profiles is caused by averaging the density profile at different longitudes.

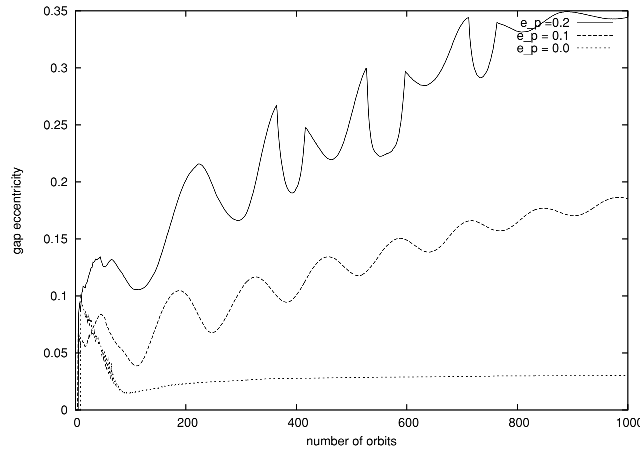


Figure 6. The eccentricity of the gap opened in the disc as a function of time for a $q = 10^{-3}$ planet, for $e_{\mathrm{p}} = 0$, 0.1 and 0.2, and a disc viscosity of $\nu = 10^{-5}$. We computed the eccentricity by locating all the cells satisfying $\Sigma/\Sigma_0 < 0.2$, treating each as a free particle, and taking the mean of the resulting eccentricities.

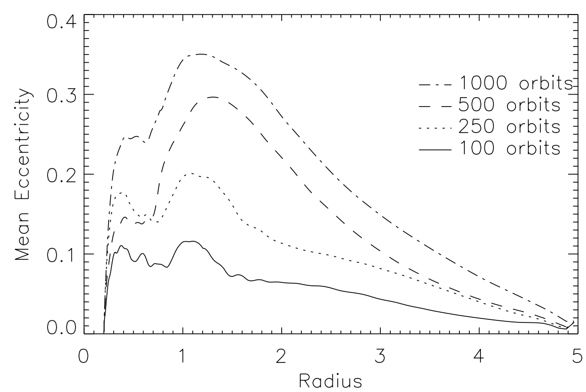


Figure 7. Eccentricity as a function of radius for a $q = 10^{-3}$ planet on an $e_{\mathrm{p}} = 0.2$ orbit and a disc viscosity of $\nu = 10^{-5}$. The eccentricities were computed by treating each cell as a free particle.

Fig. 6 plots the eccentricity evolution of the gap for three values of e_{p}. To determine the eccentricity of the gap, we selected cells based on the $\Sigma/\Sigma_0 < 0.2$ criterion introduced in Section 3. We then computed the cell eccentricity by treating each as a free particle, and took the mean of these. Because of the rather arbitrary gap criterion used, the time-evolution shows some large oscillations.[2] The circular ($e_{\mathrm{p}} = 0$) case shows a brief initial spike of eccentricity in the gap, before decaying to a low value. For the two runs with an eccentric planet, we see a steady growth in gap eccentricity, approaching 2 e_{p} after 1000 orbits.

In Fig. 7, we trace the evolution of the radial eccentricity profile of the disc surrounding the $e_{\mathrm{p}} = 0.2$ planet. We see a steady rise in the disc eccentricity with time, with even the outer portions of the disc substantially eccentric. Indeed it seems likely that the final disc eccentricity is being limited by the (circular) outer boundary of the grid. Note also that the highest-eccentricity region is just outside the

[2] We could produce a weighted mean, but the appropriate weighting is not obvious, so we prefer to show the simple average.

planet's orbit. Fig. 7 should be compared to fig. 2 of Kley & Dirksen (2006). We can see that an eccentric planet is far more efficient at exciting disc eccentricity than a planet on a circular orbit. Their eccentricities peak at around 0.2 for a $q = 5 \times 10^{-3}$ planet after 2500 orbits.

As a final check, we replotted Fig. 7 for the same on a circular orbit We found that this planet did not cause the equivalent curves to exceed $e = 0.05$, and the eccentricity was fully evolved after 100 orbits. This is practically identical to the result shown by Kley & Dirksen for such a planet.

5 GAP OPENING AS A FUNCTION OF PLANET MASS AND ECCENTRICITY

We performed a series of runs, all with $\nu = 10^{-5}$, and with different planet masses and eccentricities. For each run we measured the depth of the density deficit near the planet and labelled the simulation as 'gap opening' if the density near the planet was lower than 20 per cent of the unperturbed value. These measurements were taken after 500 orbits of the planet. Fig. 8 shows the result of these runs. We distinguish between three behaviours: no gap opened (by the 20 per cent criterion introduced above); a gap formed which is significantly different from the circular case; and a gap formed which is essentially identical to the circular case. Determining 'essentially identical' is slightly subjective. Consider Fig. 4. The curve for $e = 0.02$ traces the $e = 0$ curve almost exactly. We also judged the $e = 0.065$ curve to be sufficiently close to the $e = 0$ case to qualify as 'essentially identical.' However, the $e = 0.1$ line has a gap surface density over twice that of the circular case, and the density does not rise to the unperturbed value so quickly.

From Fig. 8, we see that simulations similar to the circular case are found at the bottom of the plot. The division between density profiles resembling the circular case and those differing is not a

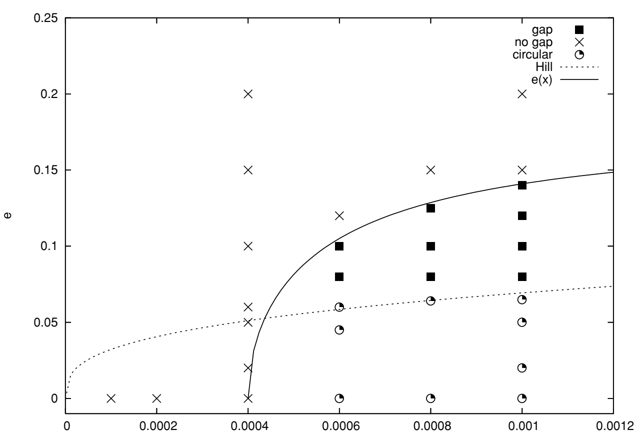


Figure 8. Gap formation as a function of planet mass ratio and eccentricity. For these runs the disc viscosity was $\nu = 10^{-5}$ so a gap is expected for $q > 4 \times 10^{-4}$. The shaded square points refer to runs in which a gap was opened in the disc. The circular points represent runs that closely followed their respective circular cases when a gap was formed. The '×' symbol indicates a run lacking a gap. Each point was assigned after 500 orbits of the planet. The dotted line shows the planet's Hill radius divided by the planet's semimajor axis. Simulations below this line closely resembled their circular orbit counterparts. At large planet eccentricities, the planet could not open a gap. We have empirically fitted a function, shown as a solid line to mark the division between gap-opening simulations and those lacking a gap. This function is in the form (equation 5) expected if corotation resonances reduce the total torque on the disc due to density waves driven by the planet.

strong function of eccentricity. The border between the two regions is well described by

$$e_{\mathrm{threshold}} \approx \left(\frac{q}{3}\right)^{1/3}. \tag{4}$$

This is essentially equation (3), and compares the variation in peristron and apastron to the planet's Hill sphere. This suggests that the planet's eccentricity has little effect on the density profile unless the planet's orbit takes it outside the sum of its semimajor axis and Hill radius. Spiral density waves are driven by resonances up to the edge of the planet's Hill sphere (this assumption is used to derive equation 2), so it is not unsurprising that once the planet strays over these resonances (effectively destroying them) the behaviour changes. Since our determination of whether the gap profile was 'essentially identical' is somewhat subjective, there may be a constant of the order of unity in front of equation (4). However, the general behaviour should be robust.

Fig. 8 shows that at high planet eccentricity a gap is not opened in the disc even when equation (2) would predict one. We now discuss possible reasons for this. We expect that a gap is opened when torque due to damping of density waves exceeds that due to viscous flow. We first consider the accretion rate through a viscous and eccentric disc. Syer & Clarke (1992, 1993) showed that the torque due to a viscous accretion varies with eccentricity according to $T_\nu \propto (1 - e^2)$ (see their equation 2.16). However, this reduces the viscous torque rather than increasing it, and would make it easier for an eccentric planet to open a gap rather than harder as we see from Fig. 8.

The solution lies in the other reseonances present in the system. Because of perturbations from the planet, spiral density waves are excited at both Lindblad and corotation resonances (a planet on a circular orbit would only excite Lindblad waves). Lindblad resonances are expected to increase the eccentricity of the planet, and hence would decrease the eccentricity of the disc (e.g. Sari & Goldreich 2004). Corotation resonances, however, when in the presence of a density gradient, damp the eccentricity of the planet and hence are expected to increase the eccentricity of the disc. First-order terms in the potential, proportional to the planet eccentricity, can drive corotation resonances into the disc at the same location as the traditional Lindblad resonances. Notably the corotation resonances are also expected to have torque with the opposite sign as the Lindblad resonances. Waves driven at these resonances and damped in the disc do not push the disc away from the planet, but rather pull the disc towards the planet. Consequently, the corotation resonances reduce the torque on the disc due to spiral density waves driven by the planet. Combining equations (8), (9) and (14) of Goldreich & Tremaine (1980) at leading order (see also Tanaka, Takeuchi & Ward 2002), we find that

$$T_{\mathrm{CR}} \propto \frac{\mathrm{d}\Sigma}{\mathrm{d}r} q^2 e_{\mathrm{p}}^2.$$

(Note that these torques vanish for a planet on a circular orbit, as per the comments above.) We therefore expect a modified gap-opening criterion of the following form:

$$q\left(1 - \chi e_{\mathrm{p}}^2\right) = 40\mathcal{R}^{-1}, \tag{5}$$

where χ depends on the density gradient and the ratio between the sum of torques from the corotation resonances and the sum of torques from the Lindblad resonances. Unfortunately, χ is dependent on the density profile and is not easy to calculate. The parameter χ may be large since a sharp gradient in the density could cause a large torque even if the gap is not deep. Since the shape of the gap is certain to be a function of viscosity (cf. Fig. 2), we expect χ to vary with viscosity too. However, we did not vary viscosity in this set of runs.

We have fitted a function in the form of equation (5) consistent with the gap/no-gap line seen in Fig. 8. The gap/no-gap curve is consistent with $\chi \approx 30.2$. This suggests that the corotation resonances are primarily important in regions of strong density gradients. Only a planet massive enough to cause a moderate density gradient in the disc would be able to increase the disc eccentricity.

6 DISCUSSION

We have seen that an eccentric planet can open an eccentric gap in a disc. The eccentricity should be an observable quantity. Although we see eccentricity in images of discs – the eccentric ring around Fomalhaut (Kalas, Graham & Clampin 2005), for example – we would expect eccentric holes of the nature described here to only be a few au from their host stars. Such holes would be too small to be imaged by current observatories. However, a strongly eccentric gap should leave a mark on the spectral energy distribution (SED) of a disc. The surface temperature of a disc illuminated by the star is set by the radial distance $T \propto r^{-1/2}$. Consequently, the eccentric edge of a disc would correspond to a range of dust temperatures in the edge. This gives a very rapidly rising SED, as seen in stars such as CoKuTau/4 and GM Aur and DM Tau (Calvet et al. 2005; D'Alessio et al. 2005). The SED of CoKuTau/4's disc edge was consistent with a single temperature optically thick wall. This in turn implies that the wall is at a particular distance from the star, and hence circular. Such a strong conclusion is possible because the 'blue' side of a black body temperature distribution is exponentially steep. An eccentric hole would produce an SED consistent with a range of dust edge temperatures. Such a spectrum could be either interpreted as due to a smooth radial density gradient or an eccentric hole. If the disc were optically thick, then the second possibility would be more likely than the first. We have found that the disc edge can be more highly eccentric than the planet; consequently, the variation in edge temperature around an eccentric disc edge may be detectable even though the dust temperature is only proportional to the inverse of the square root of the radius. After circularization, evidence would be less clear-cut. As mentioned above, the circularized gap of an eccentric planet is wider than the gap opened by a planet on a circular orbit. Detection of an extremely wide, circular gap might be an indication of an eccentric planet. However, two less-massive planets on circular orbits would be an alternative interpretation.

How might such a system form? In our runs, the planet was on a fixed orbit. Papaloizou, Nelson & Masset (2001) noted that the planet and disc should exchange eccentricity, with the exchange being most efficient when the planet and disc are of comparable mass. A massive disc would also be expected to damp the planet's eccentricity (Artymowicz 1993). It therefore led us to two possible situations in which an eccentric gap might arise. The first is the case of an eccentric planet in an old, dissipating disc. In this case, we expect the planet to be more massive than the disc, and hence little affected by it. How the planet achieved its high eccentricity is then an open question. The second scenario requires a second, inner planet. If locked in resonance with the outer planet, it could 'resupply' eccentricity to the outer planet, even in the face of damping by the outer disc. We might even expect the outer planet to have been pushed into the resonance by conventional planet–disc migration. This is somewhat analagous to the case of GJ 876, as discussed by Kley et al. (2005). However, we would expect the inner planet to be more massive, and hence less affected by the resonant interaction. This is not the case in GJ 876, where the inner planet is believed to be of lower mass, and on a substantially more eccentric orbit. Kley et al. (2005) required rapid dissipation of their gas disc, to prevent the inner planet becoming too eccentric. This would be consistent with the low-mass discs we have considered here.

7 SUMMARY

In this paper, we have examined the effect of an eccentric planet on a gas disc. From our numerical experiments, we propose that the conventional gap-opening criterion should be modified to become

$$q\left(1 - 30.2e_{\mathrm{p}}^2\right) = 40\mathcal{R}^{-1}. \quad (6)$$

The form of equation (6) is justified by the consideration of corotation torques, which are expected to act to oppose the Lindblad torques. However, the extra coefficient had to be determined numerically, due to the sensitivity of the corotation torques to the exact gap profile. We expect its value to depend on the assumed disc viscosity, but we have not parametrized this variation.

We have found that the gap eccentricity can substantially exceed the planet's eccentricity. In the early stages of evolution, as the planet crosses between the inner and outer discs, it induces one-armed spirals in each. Coupled with the precession of the gap, we expect this to lead to accretion by the planet, despite the presence of a gap.

If planet's eccentricity is sufficiently low, then the gap formed is almost identical to the circular case. We have found the threshold eccentricity to be well fitted by

$$e_{\mathrm{p}} < \left(\frac{q}{3}\right)^{1/3}, \quad (7)$$

which compares the pericentre and apocentre of the planet to its Hill sphere. The form of equation (7) is compatible with the standard theory of gap formation. This theory assumes that resonances up to the edge of the Hill sphere contribute to opening the gap (within the Hill sphere, the resonances do not exist, since there material orbits the planet, not the star). Once e_{p} is high enough to let the planet move outside its notional Hill sphere, these resonances will be destroyed, and we would expect the morphology of the gap to change.

We would expect an eccentric gap to be detectable in a disc SED, but the task will not be easy. Without corroborating data, it might be difficult to disentangle eccentricity from an azimuthally symmetric radial density gradient. Also, the gap will only remain eccentric for a few thousand orbits, which is unlikely to be more than a few per cent of the disc lifetime. After circularization, the gap will be wider than that cleared by a planet of similar mass on a circular orbit. However, multiple planets on circular orbits would be a possible alternative explanation.

Future work is needed, allowing the planet to feel the disc. This will allow us to determine the maximum disc mass which does not rapidly circularize the planet. If the scaleheight of the disc is large, then it might have an effect on the gap-opening criterion too (cf. Section 3), a possibility which should be examined in the future. We have also suggested that a high planetary eccentricity might be maintained by a second, massive, inner planet, even in the presence of strong damping by a higher-mass disc. Further calculations are needed to examine this possibility.

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