



Conservation of energy:

$$\frac{m_1}{2}\mathbf{v}_1^2 + \frac{m_2}{2}\mathbf{v}_2^2 = \frac{m_1}{2}\mathbf{u}_1^2 + \frac{m_2}{2}\mathbf{u}_2^2 \quad (1)$$

$$\Rightarrow \mathbf{u}_2^2 = \mathbf{v}_2^2 + \frac{m_1}{m_2} \cdot (\mathbf{v}_1^2 - \mathbf{u}_1^2) \quad (2)$$

Conservation of momentum:

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{u}_1 + m_2\mathbf{u}_2 \quad (3)$$

$$\Rightarrow \mathbf{u}_2^2 = \mathbf{v}_2^2 + \left(\frac{m_1}{m_2}\right)^2 (\mathbf{v}_1 - \mathbf{u}_1)^2 + 2\mathbf{v}_2 \cdot \frac{m_1}{m_2}(\mathbf{v}_1 - \mathbf{u}_1) \quad (4)$$

It follows (with $\mu = \frac{m_1}{m_2}$):

$$\Rightarrow 0 = (1 - \mu)\mathbf{u}_1^2 + 2(\mu\mathbf{v}_1 - \mathbf{v}_2)\mathbf{u}_1 + 2\mathbf{v}_2 \cdot \mathbf{v}_1 + (\mu - 1)\mathbf{v}_1^2 \quad (5)$$

Let $a = 1 - \mu$, $b = 2(\mu\mathbf{v}_1 - \mathbf{v}_2)$, and $c = 2\mathbf{v}_2 \cdot \mathbf{v}_1 + (\mu - 1)\mathbf{v}_1^2$.

With $x = |\mathbf{u}_1|$, the solutions are given by

$$x_{\pm} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (6)$$

Energy per particle:

$$E_i = \frac{m}{2} \mathbf{v}_i^2 \quad (7)$$

Total internal energy:

$$U = N \cdot E_i = pV \quad \Rightarrow \quad p = \frac{N \cdot E_i}{V} \quad (8)$$

Momentum transfer per collision of ball with wall:

$$\Delta p = m \cdot \Delta v \quad (9)$$

average force per area (pressure):

$$p = \frac{F}{A} = \frac{\Delta p}{A \Delta t} \quad (10)$$