CS170 Discussion Section 10: *4/5*

Duality

Consider the following linear program:

$$\begin{aligned} \max\ & 4x_1 + 7x_2 \\ x_1 + 2x_2 &\leq 10 \\ 3x_1 + x_2 &\leq 14 \\ 2x_1 + 3x_2 &\leq 11 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Construct the dual of the above linear program.

Zero-sum Games

Consider a two-player, zero-sum game with the following pay-off matrix (by the column player to the row player):

	A	B
a	4	-2
b	-3	1

1. Assume that you are the row player, and play strategies a and b with probabilities x_1 and x_2 respectively, where x_1 and x_2 are known to the column player. What is your optimal return? Formulate this as a linear program.

2. Assume that you are the column player, and play strategies A and B with probabilities y_1 and y_2 respectively, where y_1 and y_2 are known to the row player. What is your optimal return? Formulate this as a linear program.

Max of min

Argue that these two problems have the same optimal value:

$$\begin{aligned} \max \quad & \min\{a+b, b+c\} \\ \text{s.t.} \quad & a + 10b + 5c \leq 100 \\ & a \geq 0 \quad b \geq 0 \quad c \geq 0 \end{aligned}$$

$$\begin{aligned} \max \quad & s \\ \text{s.t.} \quad & s \leq a+b \quad s \leq b+d \\ & a + 10b + 5c \leq 100 \\ & a \geq 0 \quad b \geq 0 \quad c \geq 0 \end{aligned}$$

Bipartite Vertex Cover

A vertex cover of an undirected graph $G = (V, E)$ is a subset of the vertices which touches every edge. In other words, a subset $S \subset V$ such that for each edge $\{u, v\} \in E$, one or both of u, v are in S.

Show that the problem of finding the minimum vertex cover in a bipartite graph reduces to maximum flow. Prove that your reduction is correct.