Today

- Prim's Algorithm
- Huffman Encoding

What is a Cut?

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What is a cut in an undirected graph, $G = (V, E)$?

(A) A set of edges whose removal disconnects a graph.

(B) For partition of V, $(S, V - S)$, set of edges across it; $E \cap (S \times V - S)$.

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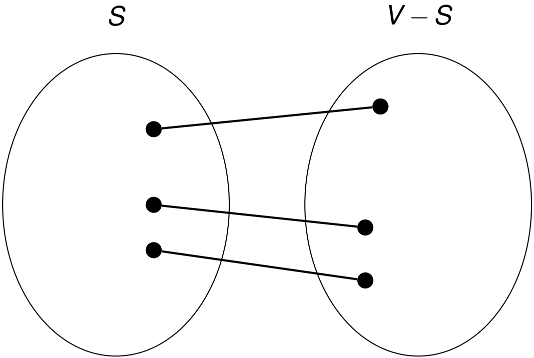

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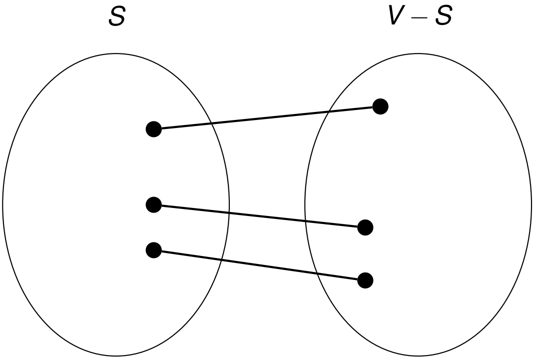

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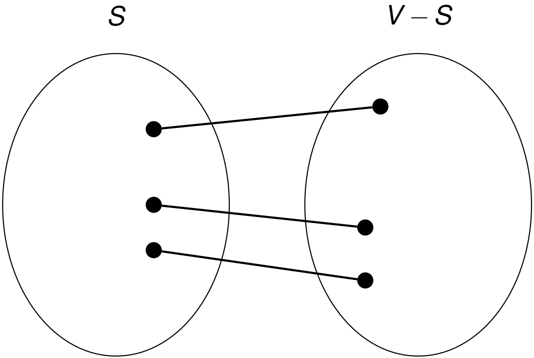

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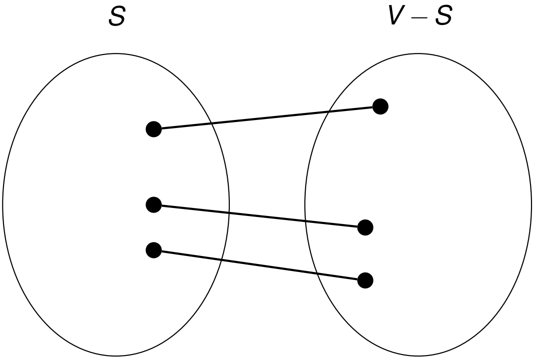

Note:sometimes specified as $(S, V - S)$
..... sometimes explicitly as subset of edges E'.

What is the cut property for MSTs?

(A) Any edge in a cut is in some mst.

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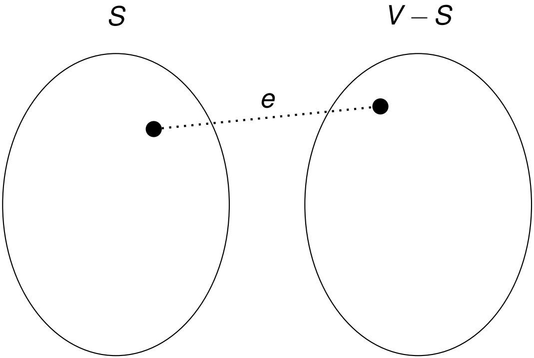

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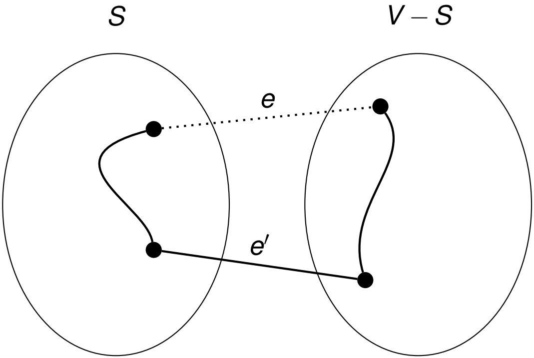

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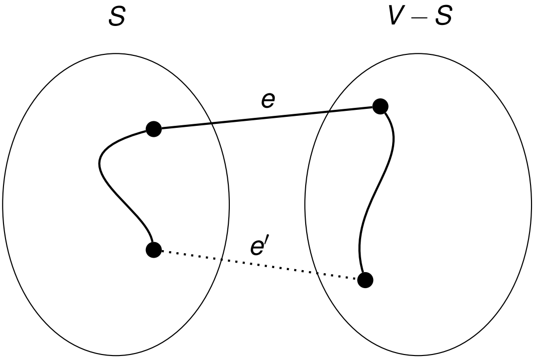

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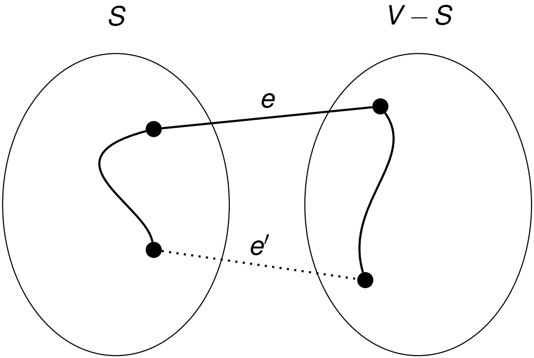

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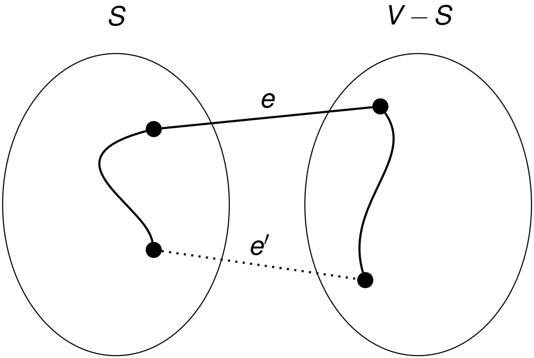

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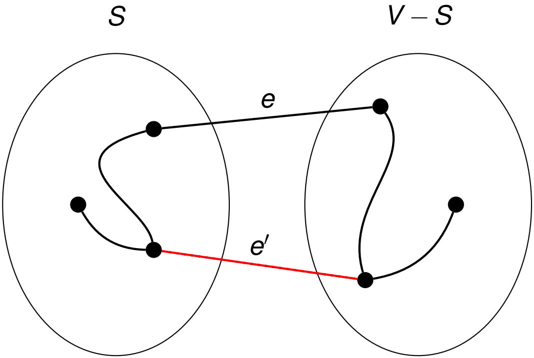

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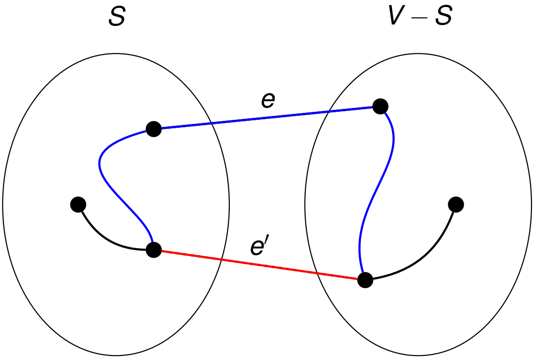

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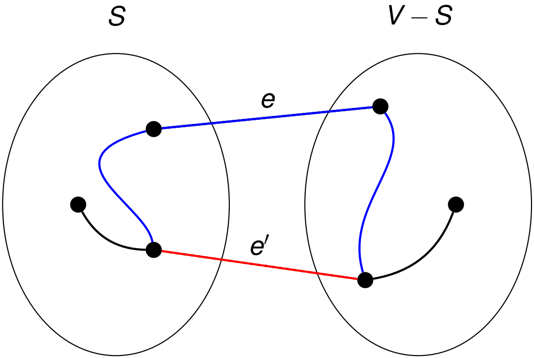

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Proof: replace, $n-1$ edges still connected and as cheap.

Prim's algorithm

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Break ties for smallest edge according to lowest neighbors.

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Actually... use a priority queue to keep "closest" node.

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Prim(G,s)

foreach $v \in V$: $c(v) = \infty, prev(v) = nil$

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With Fibonacci Heaps: $O(m+n\log n)$.

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No codeword is prefix of another.

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Letters: A,B,C,D.

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Can decode!

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$(A:0)$,

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C: .1 100
T: .2

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Prefix free codes and binary trees.

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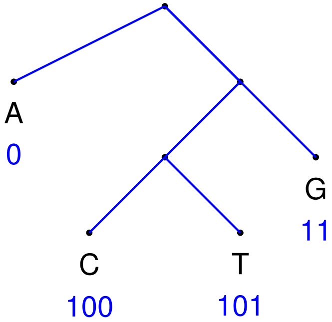

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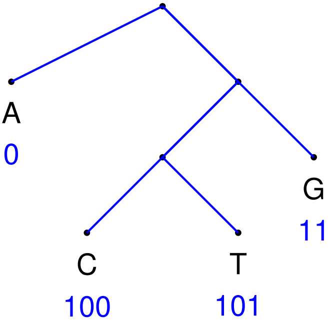

011101100

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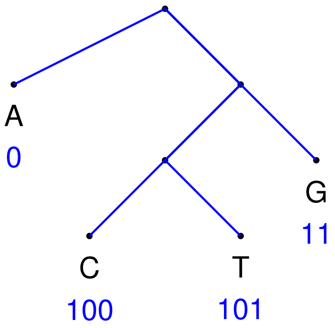

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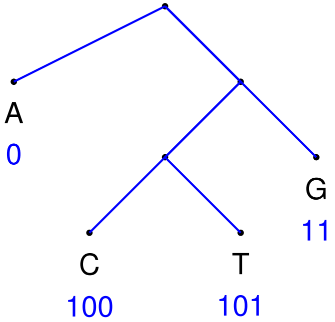

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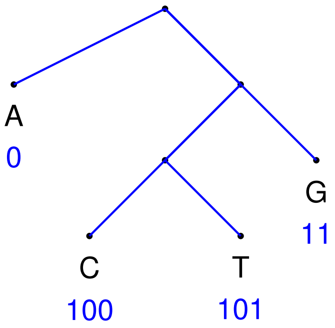

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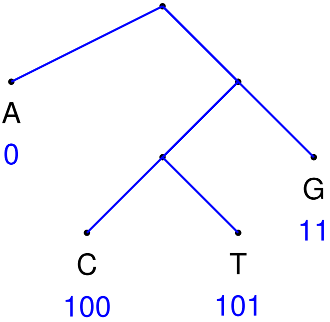

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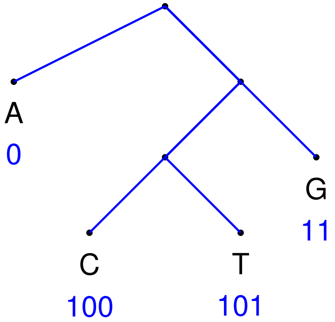

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A G C T

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Given symbol frequencies $f_1, \ldots, f_n$, find "best" prefix code.

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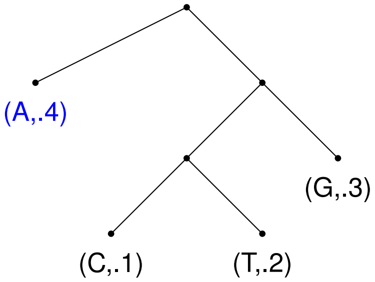

Cost: $.4 * 1$

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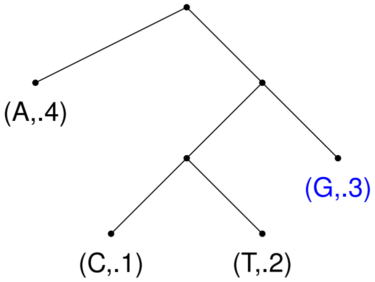

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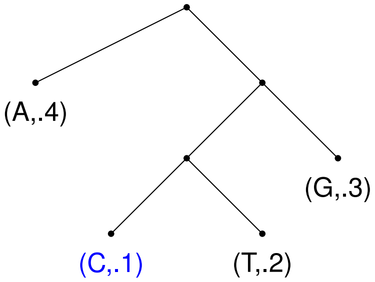

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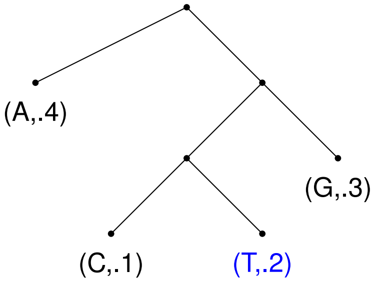

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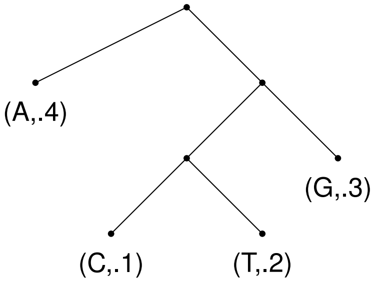

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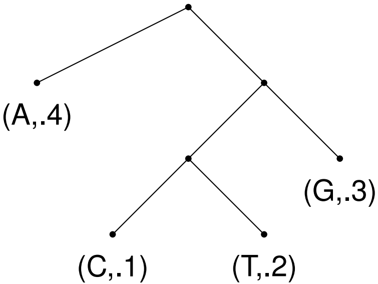
Another view of cost.



Sum over all nodes, except root, of their frequency.

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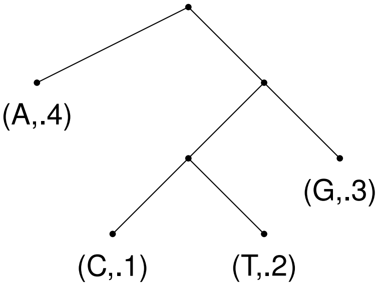

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.4

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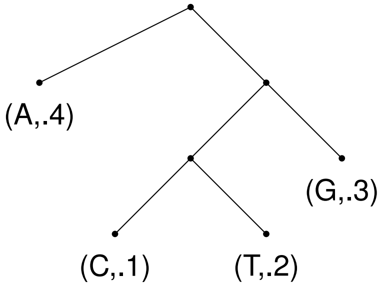

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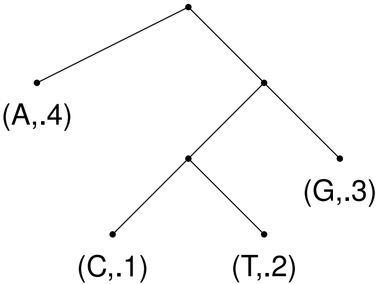

Sum over all nodes, except root, of their frequency.

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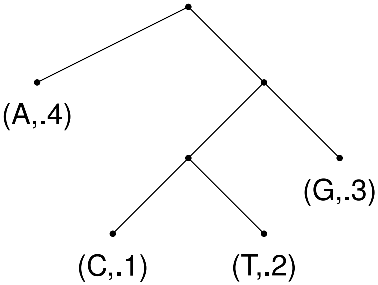

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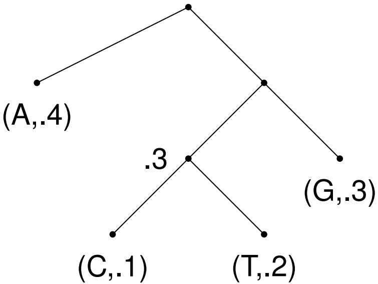

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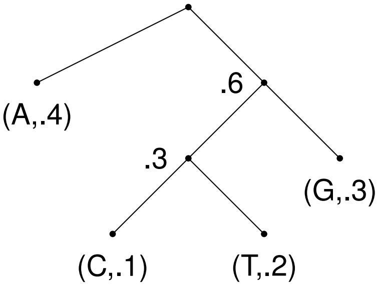

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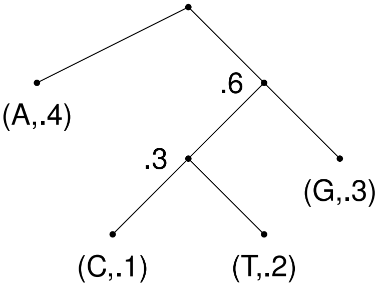

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to make low freq internal symbol.

Let's see method!

$(A, .4), (C, .1), (T, .2), (G, .3)$

$(A, .4), (\{C, T\}, .3), (G, .3)$

$(A, .4), (\{\{C, T\}, G\}, .6)$

$(\{A, \{\{C, T\}, G\}\}, 1)$

Algorithm.

Cost2: Sum over all nodes, except root, of their “frequency”.

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(A,.4)

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(C,.1) (T,.2)

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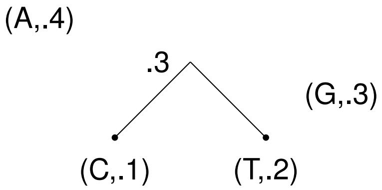

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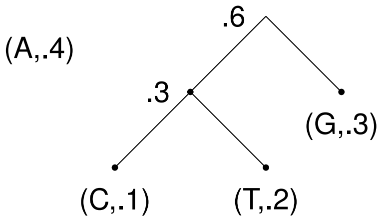

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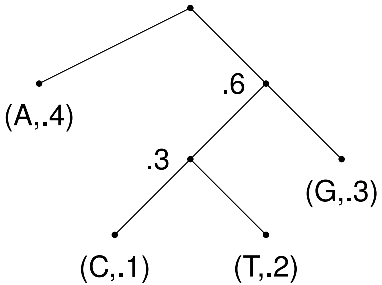

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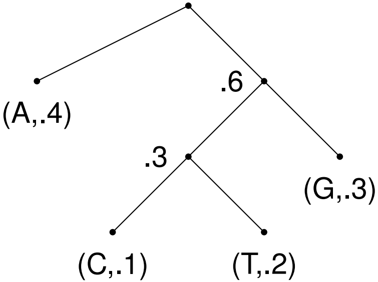

Implementation: priority queue to get lowest frequency trees.

Correctness..

Recall MST: added a “could have” edge in tree every time.

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“must have” if no ties.

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Huffman: An optimal tree

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Consider optimal tree.

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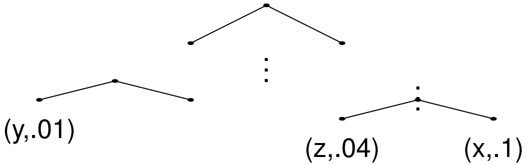

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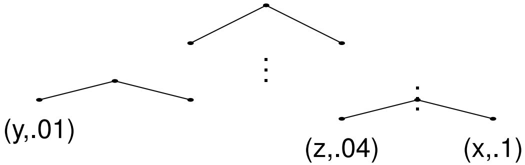

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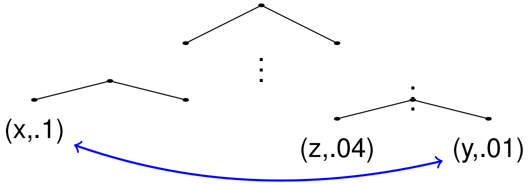

Cost gets better with this switch.

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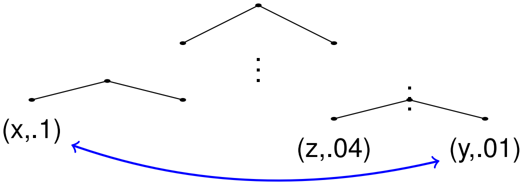
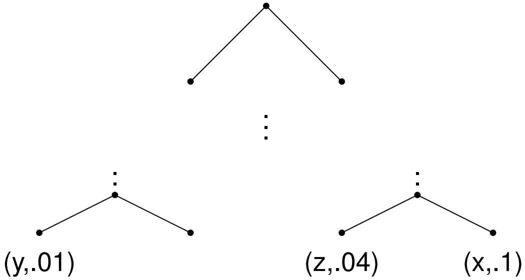


Cost gets better with this switch. Lowest frequency made siblings.

What about same depth?



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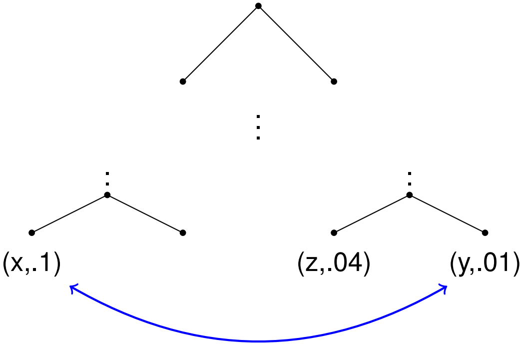
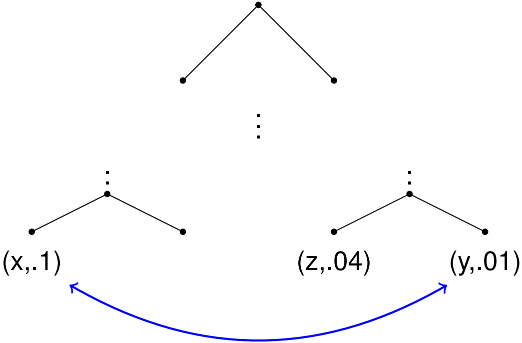


Cost stays the same,

What about same depth?



Cost stays the same, but can make lowest into sibling.

Huffman codes wrap-up

Questions ??

Huffman codes wrap-up

Questions ??

Can the shape of a Huffman tree be a

Huffman codes wrap-up

Questions ??

Can the shape of a Huffman tree be a

1. Caterpillar? Yes / No ?

Huffman codes wrap-up

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Huffman codes wrap-up

Questions ??

Can the shape of a Huffman tree be a

1. Caterpillar? Yes / No ?
2. Full balanced binary tree on k levels? Yes/No?
3. Any shape of a full binary tree? Yes/No ?

Taint Analysis

Taint Analysis

$a = http.read\_response();$

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$\vdots$

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$a = http.read\_response();$

$\vdots$

$b = a + c;$

$\vdots$

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a is input from web.
"a is tainted."

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A - "a is tainted"

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A - "a is tainted"

B - "b is tainted"

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$\implies A$

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$\implies A \text{ , } A \implies B$

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$\implies A \, , A \implies B \, , \overline{B}.$

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$$\implies A\,,\, A \implies B\,,\, \overline{B}.$$

Satisfiable?

Not in this case.

Horn SAT

Implications:

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And of positive literals imply **one** positive literal.

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Taint Example:

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And of positive literals imply **one** positive literal.

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Taint Example:

$$\implies A, A \implies B, \overline{B}.$$

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$$x \wedge y \rightarrow z$$

Negative clauses:

Or of negative literals.

$$\overline{u} \vee \overline{v}$$

Taint Example:

$$\implies A, A \implies B, \overline{B}.$$

Is this satisfiable?

Horn Sat: another view.

$$\begin{aligned} x_1 \wedge x_2 &\implies x_4 \\ x_3 &\implies x_2 \\ x_1 &\implies x_3 \\ x_5 \wedge x_1 &\implies x_3 \\ x_2 \wedge x_6 &\implies x_5 \\ &\implies x_1 \end{aligned}$$

Horn Sat: another view.

$$\begin{aligned}
x_1 \wedge x_2 &\implies x_4 \\
x_3 &\implies x_2 \\
x_1 &\implies x_3 \\
x_5 \wedge x_1 &\implies x_3 \\
x_2 \wedge x_6 &\implies x_5 \\
&\implies x_1
\end{aligned}$$

Problem: Find consistent assignment with fewest "True" variables.

Horn Sat: another view.

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Problem: Find consistent assignment with fewest "True" variables.

Greedy algorithm:

Horn Sat: another view.

$$
\begin{aligned}
x_1 \wedge x_2 &\implies x_4 \\
x_3 &\implies x_2 \\
x_1 &\implies x_3 \\
x_5 \wedge x_1 &\implies x_3 \\
x_2 \wedge x_6 &\implies x_5 \\
&\implies x_1
\end{aligned}
$$

Problem: Find consistent assignment with fewest “True” variables.

Greedy algorithm: Only set variables to true if you have to.

Horn Sat: another view.

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\begin{aligned}
x_1 \wedge x_2 &\implies x_4 \\
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Problem: Find consistent assignment with fewest “True” variables.

Greedy algorithm: Only set variables to true if you have to.

Example:
x_1 must be true

Horn Sat: another view.

$$\begin{aligned}
x_1 \wedge x_2 &\implies x_4 \\
x_3 &\implies x_2 \\
x_1 &\implies x_3 \\
x_5 \wedge x_1 &\implies x_3 \\
x_2 \wedge x_6 &\implies x_5 \\
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\end{aligned}$$

Problem: Find consistent assignment with fewest “True” variables.

Greedy algorithm: Only set variables to true if you have to.

Example:
x_1 must be true so x_3 must be true

Horn Sat: another view.

$$\begin{aligned} x_1 \wedge x_2 &\implies x_4 \\ x_3 &\implies x_2 \\ x_1 &\implies x_3 \\ x_5 \wedge x_1 &\implies x_3 \\ x_2 \wedge x_6 &\implies x_5 \\ &\implies x_1 \end{aligned}$$

Problem: Find consistent assignment with fewest "True" variables.

Greedy algorithm: Only set variables to true if you have to.

Example:
x_1 must be true so x_3 must be true
so x_2 must be true

Horn Sat: another view.

$$
\begin{aligned}
x_1 \wedge x_2 &\implies x_4 \\
x_3 &\implies x_2 \\
x_1 &\implies x_3 \\
x_5 \wedge x_1 &\implies x_3 \\
x_2 \wedge x_6 &\implies x_5 \\
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x_1 must be true so x_3 must be true
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Solution:

Horn Sat: another view.

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Example:
x_1 must be true so x_3 must be true
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Solution: $\{x_1, x_2, x_3, x_4\}$ are True

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Could also set x_5 to true, or both x_5 and x_6 to true...

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Could also set x_5 to true, or both x_5 and x_6 to true...but don't!

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Same as horn sat!

Why same as HornSAT?

Horn SAT had negative clauses.

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The $k+1$ set variable set to true

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Negative clauses only problem for true variables.

Why same as HornSAT?

Horn SAT had negative clauses.

No negative clauses for above algorithm.

Algorithm: Set a variable true ..if you have to!

Property: any variable set to true must be true in *any* satisfying assignment.

By induction. First k set to true... must be!
The $k+1$ set variable set to true
is set to true to satisfy a clause
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Horn has negative clauses.

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So if a negative clause is false, it must be.

Efficient implementation

$$\begin{aligned}
x_1 \wedge x_2 &\implies x_4 \\
x_3 &\implies x_2 \\
x_1 &\implies x_3 \\
x_5 \wedge x_1 &\implies x_3 \\
x_2 \wedge x_6 &\implies x_5 \\
&\implies x_1
\end{aligned}$$

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For each clause: keep count of true antecedents:

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For each clause: keep count of true antecedents:

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Connect variable to clauses with var as antecendent.

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For each clause: keep count of true antecedents:
When all antecedents true, than make consequent true.

Data Structure:
Connect variable to clauses with var as antecendent.
When variable is set to true see if connected clauses are invoked.