

CS170 Discussion Section 11 : 4/12

Definitions

1. **P**: the set of all problems that can be solved (not just verified) in polynomial time.
2. **NP**: the set of all problems that can be verified in polynomial time.
3. **NP-complete**: a problem to which all other problems in NP reduce. In other words, if we could efficiently solve any **NP**-complete problem, we could efficiently solve any problem in NP.

Reduction Basics

Assume A reduces to B in polynomial time. In each part you will be given a fact about one of the problems. Determine what, if anything, this allows you to determine about the other problem. (*You can answer each part in one sentence.*)

1. A is in **P**.
2. B is in **P**.
3. A is **NP**-hard.
4. B is **NP**-hard.

Stingy SAT

Stingy SAT is the following problem: given a set of clauses (each a disjunction of literals), and an integer k , find a satisfying assignment in which at most k variables are True, if such an assignment exists. Prove that Stingy SAT is NP-Complete.

More Reductions

1. Give a reduction from Vertex Cover to Set Cover. In Vertex Cover your input is a graph G and a budget b , and you want to find b vertices that touch every edge. In Set Cover, we are given a set E and several subsets of it S_1, \dots, S_m , along with a budget b . We are asked to select b of these subsets so that their union is E .
2. Give a reduction from 3D Matching to Set Cover.

Reliable Network

Reliable Network is the following problem: We are given two $n \times n$ matrices, a distance matrix d_{ij} , and a connectivity requirement matrix r_{ij} , as well as a budget b ; we must find a graph $G = (\{1, \dots, n\}, E)$ such that the total cost of all edges is b or less and between any two distinct vertices i and j there are r_{ij} vertex-disjoint paths. Show that Reliable Network is NP-Complete.