CS170 Discussion Section 12

Dominating Sets

In an undirected graph $G = (V, E)$, we say $D \subseteq V$ is a *dominating set* if every $v \in V$ is either in D or adjacent to at least one member of D. In the DOMINATING SET problem, the input is a graph and a budget b, and the aim is to find a dominating set in the graph of size at most b, if one exists. Show that DOMINATING SET is NP-complete.

Independent Set Approximation

Given an undirected graph $G = (V, E)$ in which each node has degree $\leq d$, show how to efficiently find an independent set whose size is at least $1/(d + 1)$ times that of the largest independent set.

3-SAT

Consider the optimization version of 3-SAT where the objective is to find a variable assignment that satisfies as many clauses as possible.

1. Consider the 3-SAT instance $\overline{x_1} \vee x_2 \vee x_3$. Suppose we set, for $i = 1, 2, 3$, x_i to be 0 or 1 with probability 1/2 independently. What is the probability the instance is satisfied?
2. Give a randomized algorithm with a 7/8-approximation in expectation, i.e. when the input contains n clauses, the expected number of satisfied clauses is $7n/8$.
3. Give a deterministic 7/8-approximation algorithm; the number of satisfied clauses should be at least $7n/8$.

Feedback Edge Set

Given a directed graph, return the largest subset of edges that corresponds to a DAG. In other words, remove edges from the original graph until we are left with a DAG, while maximizing the number of edges in the resulting DAG. Find a $\frac{1}{2}$ approximation algorithm; propose an algorithm that keeps at least half of the number of edges that would have been kept in the optimal solution.