Today

Linear Programming

Profit maximization.

Plant Carrots or Peas?

Profit maximization.

Plant Carrots or Peas?

2$ bushel of carrots.

Profit maximization.

Plant Carrots or Peas?

2$ bushel of carrots. 4$ for peas.

Profit maximization.

Plant Carrots or Peas?

2$ bushel of carrots. 4$ for peas.

Carrots take 3 unit of water/bushel.

Profit maximization.

Plant Carrots or Peas?

2$ bushel of carrots. 4$ for peas.

Carrots take 3 unit of water/bushel.
Peas take 2 units of water/bushel.

Profit maximization.

Plant Carrots or Peas?

2$ bushel of carrots. 4$ for peas.

Carrots take 3 unit of water/bushel.
Peas take 2 units of water/bushel.

100 units of water.

Profit maximization.

Plant Carrots or Peas?

2\$ bushel of carrots. 4\$ for peas.

Carrots take 3 unit of water/bushel.
Peas take 2 units of water/bushel.

100 units of water.

Peas get 3 sq. yards/bushel of sunny land.

Profit maximization.

Plant Carrots or Peas?

2$ bushel of carrots. 4$ for peas.

Carrots take 3 unit of water/bushel.
Peas take 2 units of water/bushel.

100 units of water.

Peas get 3 sq. yards/bushel of sunny land.
Carrots get 3 sq. yards/bushel of shady land.

Profit maximization.

Plant Carrots or Peas?

2$ bushel of carrots. 4$ for peas.

Carrots take 3 unit of water/bushel.
Peas take 2 units of water/bushel.

100 units of water.

Peas get 3 sq. yards/bushel of sunny land.
Carrots get 3 sq. yards/bushel of shady land.

Garden has 60 sq. yards of sunny land and 75 sq. yards of shady land.

Profit maximization.

Plant Carrots or Peas?

2$ bushel of carrots. 4$ for peas.

Carrots take 3 unit of water/bushel.
Peas take 2 units of water/bushel.

100 units of water.

Peas get 3 sq. yards/bushel of sunny land.
Carrots get 3 sq. yards/bushel of shady land.

Garden has 60 sq. yards of sunny land and 75 sq. yards of shady land.

Profit maximization.

Plant Carrots or Peas?

2$ bushel of carrots. 4$ for peas.

Carrots take 3 unit of water/bushel.
Peas take 2 units of water/bushel.

100 units of water.

Peas get 3 sq. yards/bushel of sunny land.
Carrots get 3 sq. yards/bushel of shady land.

Garden has 60 sq. yards of sunny land and 75 sq. yards of shady land.

To pea or not to pea, that is the question!

To pea or not to pea.

To pea or not to pea.

4$ for peas.

To pea or not to pea.

4$ for peas. 2$ bushel of carrots.

To pea or not to pea.

4$ for peas. 2$ bushel of carrots. x_1- to pea!

To pea or not to pea.

4$ for peas. 2$ bushel of carrots. x_1- to pea! x_2 carrots

To pea or not to pea.

4\$ for peas. 2\$ bushel of carrots. x_1- to pea! x_2 carrots

Money $4x_1 + 2x_2$

To pea or not to pea.

4$ for peas. 2$ bushel of carrots. x_1- to pea! x_2 carrots

Money $4x_1 + 2x_2$ maximize

To pea or not to pea.

4\$ for peas. 2\$ bushel of carrots. x_1- to pea! x_2 carrots
Money $4x_1 + 2x_2$ maximize $\max 4x_1 + 2x_2$.

To pea or not to pea.

4$ for peas. 2$ bushel of carrots. x_1- to pea! x_2 carrots

Money $4x_1 + 2x_2$ maximize $\max 4x_1 + 2x_2$.

Peas take 3 unit of water/bushel.

To pea or not to pea.

4\$ for peas. 2\$ bushel of carrots. x_1- to pea! x_2 carrots

Money $4x_1 + 2x_2$ maximize $\max 4x_1 + 2x_2$.

Peas take 3 unit of water/bushel.

Carrot take 2 units of water/bushel.

To pea or not to pea.

4\$ for peas. 2\$ bushel of carrots. x_1- to pea! x_2 carrots

Money $4x_1 + 2x_2$ maximize $\max 4x_1 + 2x_2$.

Peas take 3 unit of water/bushel.

Carrot take 2 units of water/bushel. 100 units of water.

To pea or not to pea.

4\$ for peas. 2\$ bushel of carrots. x_1- to pea! x_2 carrots
Money $4x_1 + 2x_2$ maximize $\max 4x_1 + 2x_2$.
Peas take 3 unit of water/bushel.
Carrot take 2 units of water/bushel. 100 units of water.

$3x_1 + 2x_2 \leq 100$

To pea or not to pea.

4\$ for peas. 2\$ bushel of carrots. x_1- to pea! x_2 carrots
Money $4x_1 + 2x_2$ maximize $\max 4x_1 + 2x_2$.
Peas take 3 unit of water/bushel.
Carrot take 2 units of water/bushel. 100 units of water.

$3x_1 + 2x_2 \leq 100$

Peas 3 yards/bushel of sunny land. Have 60 sq. yards.

To pea or not to pea.

4\$ for peas. 2\$ bushel of carrots. x_1- to pea! x_2 carrots
Money $4x_1 + 2x_2$ maximize $\max 4x_1 + 2x_2$.
Peas take 3 unit of water/bushel.
Carrot take 2 units of water/bushel. 100 units of water.

$3x_1 + 2x_2 \leq 100$

Peas 3 yards/bushel of sunny land. Have 60 sq. yards.
$3x_1 \leq 60$

To pea or not to pea.

4\$ for peas. 2\$ bushel of carrots. x_1- to pea! x_2 carrots
Money $4x_1 + 2x_2$ maximize $\max 4x_1 + 2x_2$.
Peas take 3 unit of water/bushel.
Carrot take 2 units of water/bushel. 100 units of water.

$3x_1 + 2x_2 \leq 100$

Peas 3 yards/bushel of sunny land. Have 60 sq. yards.
$3x_1 \leq 60$
Carrots get 3 yards/bushel of shady land. Have 75 sq. yards.

To pea or not to pea.

4\$ for peas. 2\$ bushel of carrots. x_1- to pea! x_2 carrots
Money $4x_1 + 2x_2$ maximize $\max 4x_1 + 2x_2$.
Peas take 3 unit of water/bushel.
Carrot take 2 units of water/bushel. 100 units of water.

$3x_1 + 2x_2 \leq 100$

Peas 3 yards/bushel of sunny land. Have 60 sq. yards.
$3x_1 \leq 60$
Carrots get 3 yards/bushel of shady land. Have 75 sq. yards.
$3x_2 \leq 75$

To pea or not to pea.

4\$ for peas. 2\$ bushel of carrots. x_1- to pea! x_2 carrots
Money $4x_1 + 2x_2$ maximize $\max 4x_1 + 2x_2$.
Peas take 3 unit of water/bushel.
Carrot take 2 units of water/bushel. 100 units of water.

$3x_1 + 2x_2 \leq 100$

Peas 3 yards/bushel of sunny land. Have 60 sq. yards.
$3x_1 \leq 60$
Carrots get 3 yards/bushel of shady land. Have 75 sq. yards.
$3x_2 \leq 75$
Can't make negative!

To pea or not to pea.

4\$ for peas. 2\$ bushel of carrots. x_1- to pea! x_2 carrots
Money $4x_1 + 2x_2$ maximize $\max 4x_1 + 2x_2$.
Peas take 3 unit of water/bushel.
Carrot take 2 units of water/bushel. 100 units of water.

$3x_1 + 2x_2 \leq 100$

Peas 3 yards/bushel of sunny land. Have 60 sq. yards.
$3x_1 \leq 60$
Carrots get 3 yards/bushel of shady land. Have 75 sq. yards.
$3x_2 \leq 75$
Can't make negative! $x_1, x_2 \geq 0$.

To pea or not to pea.

4\$ for peas. 2\$ bushel of carrots. x_1- to pea! x_2 carrots
Money $4x_1 + 2x_2$ maximize $\max 4x_1 + 2x_2$.
Peas take 3 unit of water/bushel.
Carrot take 2 units of water/bushel. 100 units of water.

$3x_1 + 2x_2 \leq 100$

Peas 3 yards/bushel of sunny land. Have 60 sq. yards.
$3x_1 \leq 60$
Carrots get 3 yards/bushel of shady land. Have 75 sq. yards.
$3x_2 \leq 75$
Can't make negative! $x_1, x_2 \geq 0$.

A linear program.

To pea or not to pea.

4\$ for peas. 2\$ bushel of carrots. x_1- to pea! x_2 carrots
Money $4x_1 + 2x_2$ maximize $\max 4x_1 + 2x_2$.
Peas take 3 unit of water/bushel.
Carrot take 2 units of water/bushel. 100 units of water.

$3x_1 + 2x_2 \leq 100$

Peas 3 yards/bushel of sunny land. Have 60 sq. yards.
$3x_1 \leq 60$
Carrots get 3 yards/bushel of shady land. Have 75 sq. yards.
$3x_2 \leq 75$
Can't make negative! $x_1, x_2 \geq 0$.

A linear program.

$$\begin{aligned}
\max 4x_1 + 2x_2 \\
3x_1 \leq 60 \\
3x_2 \leq 75 \\
3x_1 + 2x_2 \leq 100 \\
x_1, x_2 \geq 0
\end{aligned}$$

$$
\begin{aligned}
\max 4x_1 + 2x_2 \\
3x_1 \leq 60 \\
3x_2 \leq 75 \\
3x_1 + 2x_2 \leq 100 \\
x_1, x_2 \geq 0
\end{aligned}
$$

Optimal point?

$$\begin{aligned} \max 4x_1 + 2x_2 \\ 3x_1 \leq 60 \\ 3x_2 \leq 75 \\ 3x_1 + 2x_2 \leq 100 \\ x_1, x_2 \geq 0 \end{aligned}$$

Optimal point?

Try every point

$$
\begin{aligned}
\max 4x_1 + 2x_2 \\
3x_1 \leq 60 \\
3x_2 \leq 75 \\
3x_1 + 2x_2 \leq 100 \\
x_1, x_2 \geq 0
\end{aligned}
$$

Optimal point?

Try every point if we only had time!

$$\begin{aligned}\max 4x_1 + 2x_2 \\ 3x_1 \leq 60 \\ 3x_2 \leq 75 \\ 3x_1 + 2x_2 \leq 100 \\ x_1, x_2 \geq 0\end{aligned}$$

Optimal point?

Try every point if we only had time!

How many points?

$$
\begin{aligned}
\max 4x_1 + 2x_2 \\
3x_1 \leq 60 \\
3x_2 \leq 75 \\
3x_1 + 2x_2 \leq 100 \\
x_1, x_2 \geq 0
\end{aligned}
$$

Optimal point?

Try every point if we only had time!

How many points?

Real numbers?

$$\begin{aligned} \max 4x_1 + 2x_2 \\ 3x_1 \leq 60 \\ 3x_2 \leq 75 \\ 3x_1 + 2x_2 \leq 100 \\ x_1, x_2 \geq 0 \end{aligned}$$

Optimal point?

Try every point if we only had time!

How many points?

Real numbers?

Infinite.

$$
\begin{aligned}
\max 4x_1 + 2x_2 \\
3x_1 \leq 60 \\
3x_2 \leq 75 \\
3x_1 + 2x_2 \leq 100 \\
x_1, x_2 \geq 0
\end{aligned}
$$

Optimal point?

Try every point if we only had time!

How many points?

Real numbers?

Infinite. Uncountably infinite!

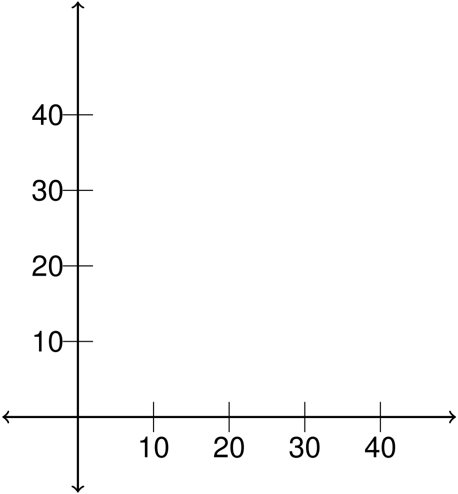
A linear program.

A linear program.

$$\begin{aligned} \max 4x_1 + 2x_2 \\ 3x_1 \leq 60 \\ 3x_2 \leq 75 \\ 3x_1 + 2x_2 \leq 100 \\ x_1, x_2 \geq 0 \end{aligned}$$

A linear program.

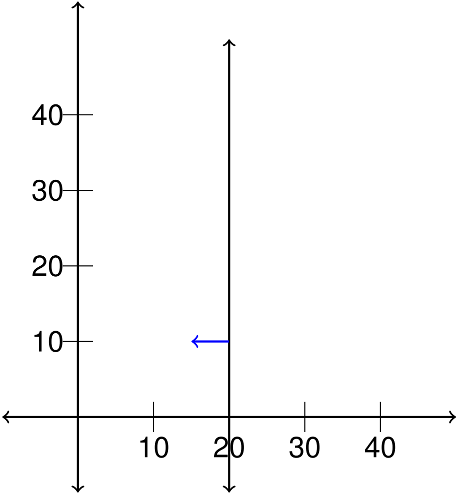
$$\begin{aligned} \max 4x_1 + 2x_2 \\ 3x_1 \leq 60 \\ 3x_2 \leq 75 \\ 3x_1 + 2x_2 \leq 100 \\ x_1, x_2 \geq 0 \end{aligned}$$



Optimal point?

A linear program.

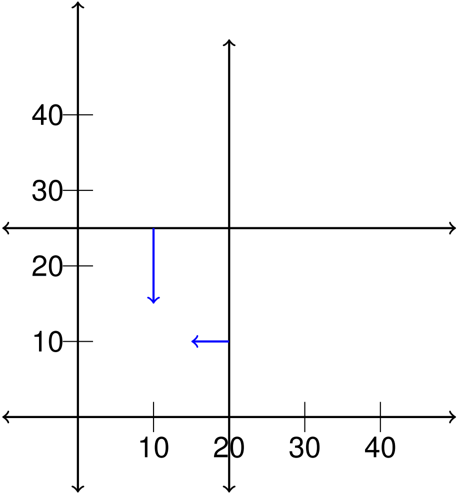
$$\max 4x_1 + 2x_2$$
$$3x_1 \leq 60$$
$$3x_2 \leq 75$$
$$3x_1 + 2x_2 \leq 100$$
$$x_1, x_2 \geq 0$$



Optimal point?

A linear program.

$$\begin{aligned} \max 4x_1 + 2x_2 \\ 3x_1 \leq 60 \\ 3x_2 \leq 75 \\ 3x_1 + 2x_2 \leq 100 \\ x_1, x_2 \geq 0 \end{aligned}$$



Optimal point?

A linear program.

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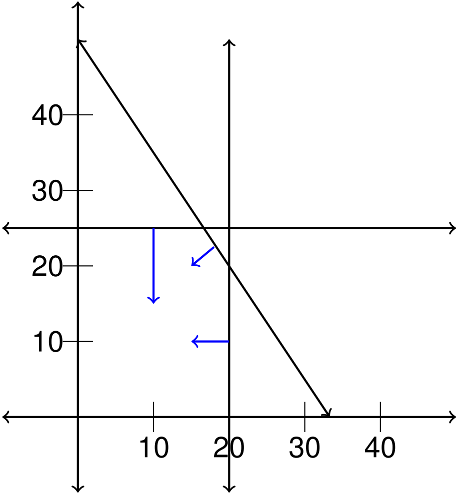
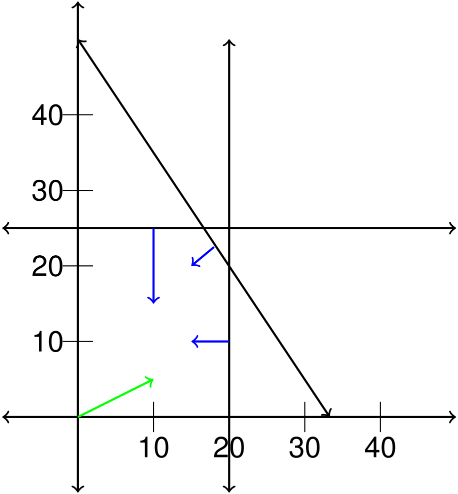


Optimal point?

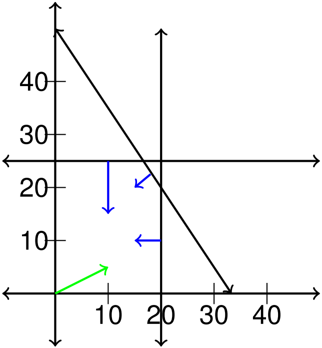
A linear program.

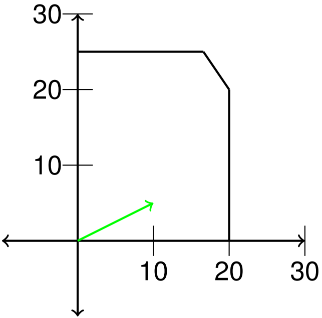
$$\begin{aligned} \max 4x_1 + 2x_2 \\ 3x_1 \leq 60 \\ 3x_2 \leq 75 \\ 3x_1 + 2x_2 \leq 100 \\ x_1, x_2 \geq 0 \end{aligned}$$



Optimal point?

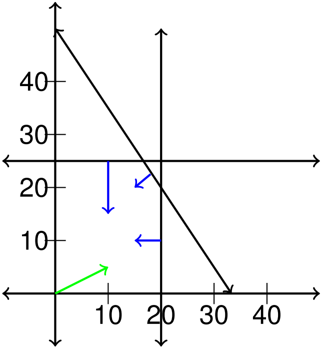
Feasible Region.

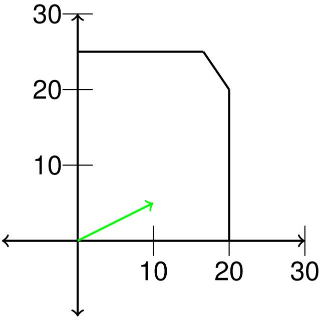




Convex.

Feasible Region.

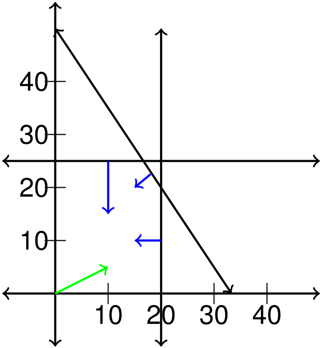


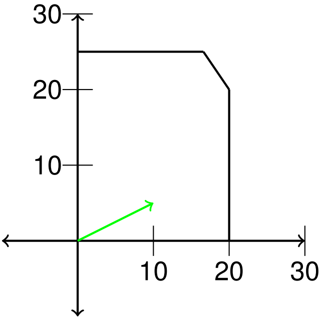


Convex.

Any two points in region connected by a line in region.

Feasible Region.



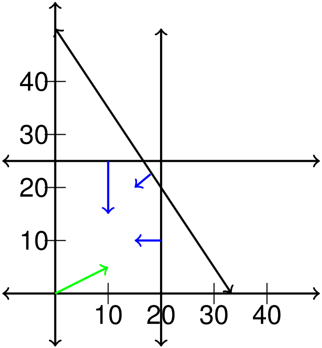


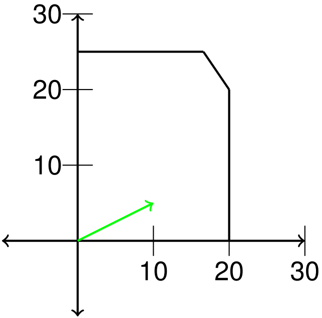
Convex.

Any two points in region connected by a line in region.

Algebraically:

Feasible Region.





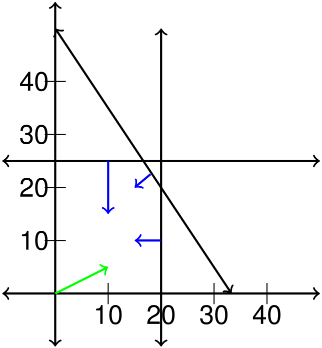
Convex.

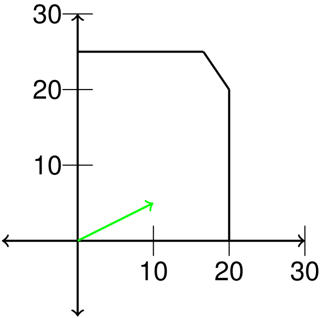
Any two points in region connected by a line in region.

Algebraically:

If x and x' satisfy an constraint, so does $x'' = \alpha x + (1 - \alpha)x'$

Feasible Region.





Convex.

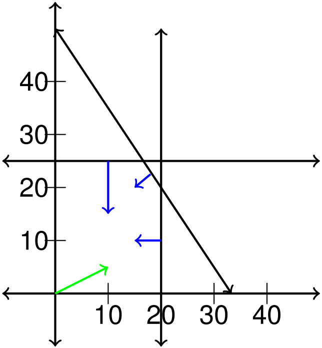
Any two points in region connected by a line in region.

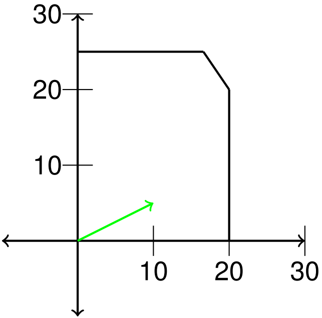
Algebraically:

If x and x' satisfy an constraint, so does $x'' = \alpha x + (1-\alpha)x'$

E.g. $3x \leq 60$ and $3x' \leq 60$

Feasible Region.





Convex.

Any two points in region connected by a line in region.

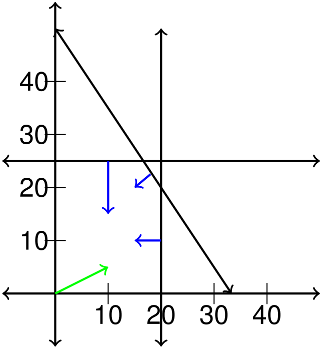
Algebraically:

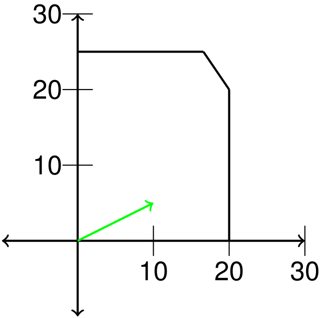
If x and x' satisfy an constraint, so does $x'' = \alpha x + (1-\alpha)x'$

E.g. $3x \leq 60$ and $3x' \leq 60$

$\rightarrow 3\alpha x \leq \alpha(60)$

Feasible Region.





Convex.

  Any two points in region connected by a line in region.

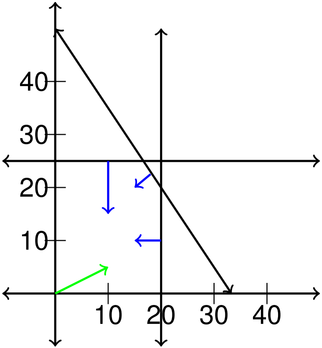
Algebraically:

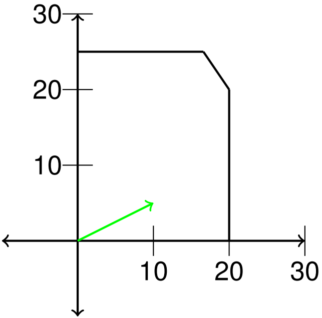
If x and x' satisfy an constraint, so does $x'' = \alpha x + (1-\alpha)x'$

E.g. $3x \leq 60$ and $3x' \leq 60$

  $\rightarrow 3\alpha x \leq \alpha(60)$ and $3(1-\alpha)x' \leq (1-\alpha)60$

Feasible Region.





Convex.

Any two points in region connected by a line in region.

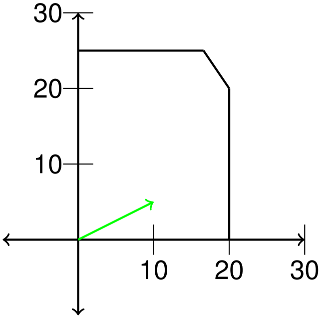
Algebraically:

If x and x' satisfy an constraint, so does $x'' = \alpha x + (1-\alpha)x'$

E.g. $3x \leq 60$ and $3x' \leq 60$

$\rightarrow 3\alpha x \leq \alpha(60)$ and $3(1-\alpha)x' \leq (1-\alpha)60$

$\rightarrow 3(\alpha(x) + (1-\alpha)x') \leq (\alpha + (1-\alpha))60 = 60$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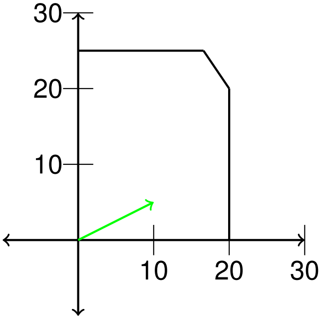
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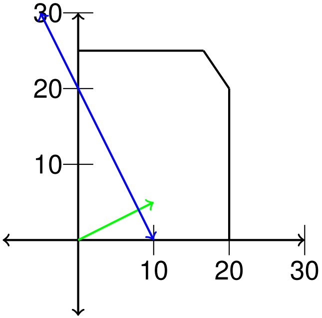
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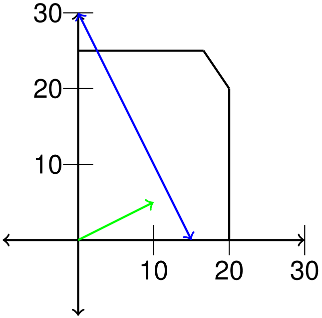
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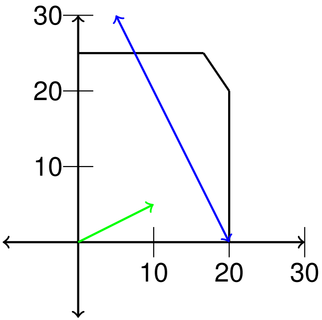
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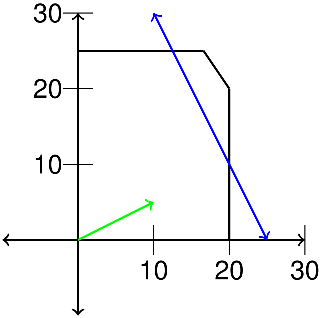
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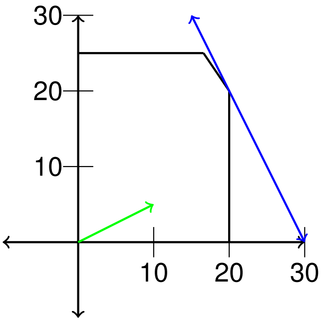
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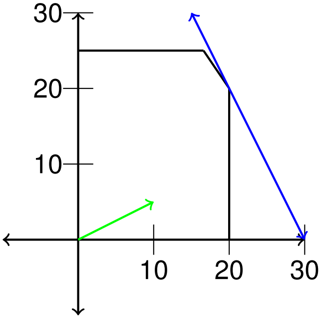
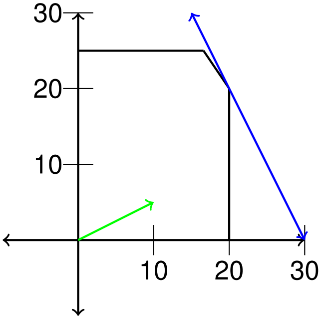
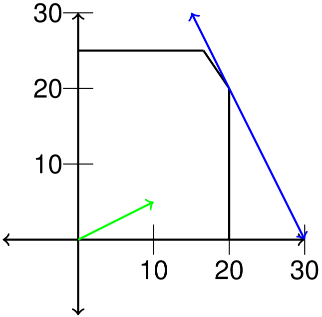


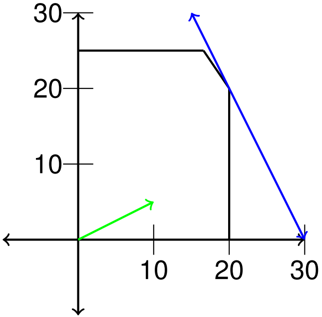
Optimal at pointy part of feasible region!



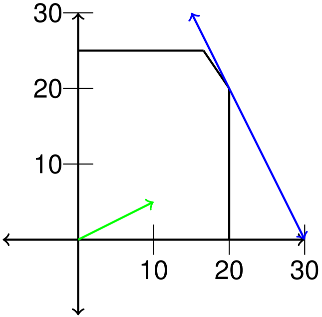
Optimal at pointy part of feasible region!
Vertex of region.



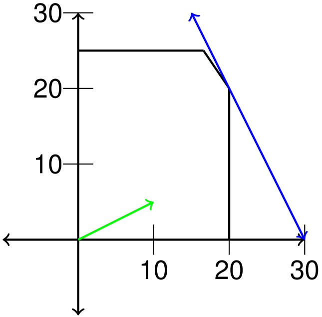
Optimal at pointy part of feasible region!
Vertex of region.
Intersection of two of the constraints! Which are lines.



Optimal at pointy part of feasible region!
Vertex of region.
Intersection of two of the constraints! Which are lines.
Try every vertex!



Optimal at pointy part of feasible region!
Vertex of region.
Intersection of two of the constraints! Which are lines.
Try every vertex! Choose best among the ones in the region.



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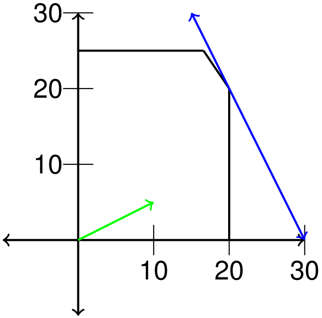
$O(m^2)$ if m constraints and 2 variables.



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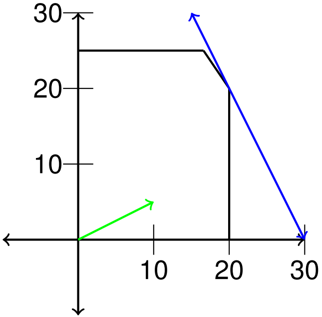
For n variables, m constraints, how many?



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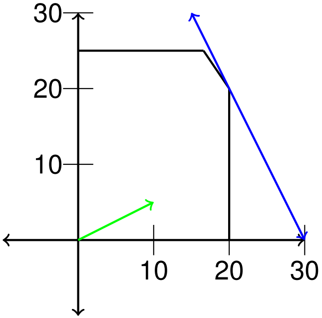
For n variables, m constraints, how many?
nm? $\binom{m}{n}$? $n+m$?



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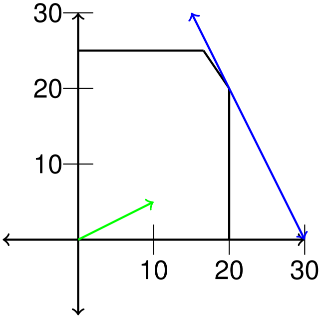

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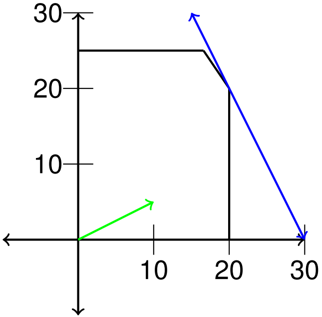
Finite!!!!!!



Optimal at pointy part of feasible region!
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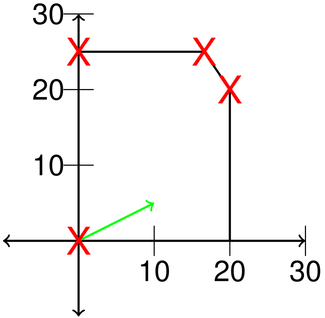
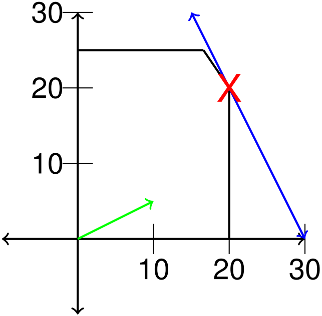
$O(m^2)$ if m constraints and 2 variables.
For n variables, m constraints, how many?
nm? $\binom{m}{n}$? $n+m$?
$\binom{m}{n}$
Finite!!!!!! But exponential in the number of variables.



Simplex: Start at vertex.

$$\max 4x_1 + 2x_2$$
$$3x_1 \leq 60$$
$$3x_2 \leq 75$$
$$3x_1 + 2x_2 \leq 100$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$



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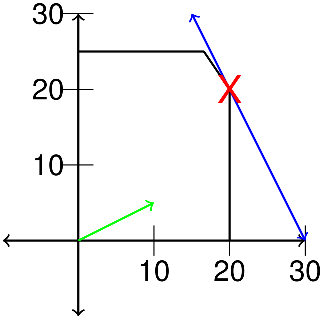

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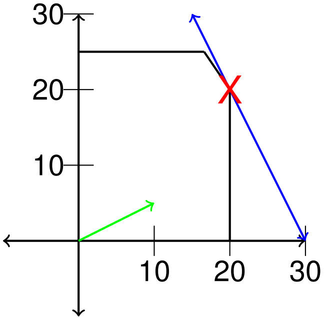
Simplex: Start at vertex. Move to better neighboring vertex.
Until you stop.



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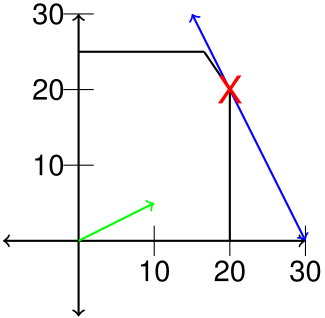

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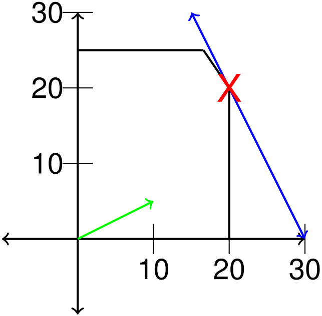
Until you stop. This example.
(0,0) objective 0.



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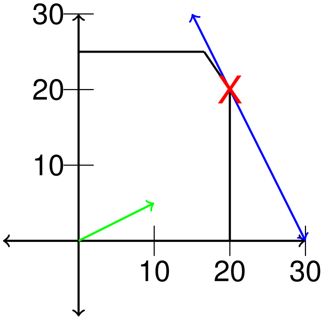
Simplex: Start at vertex. Move to better neighboring vertex.
Until you stop. This example.
(0,0) objective 0. $\rightarrow (0,25)$ objective 50.



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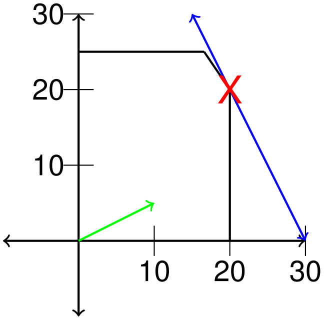
Simplex: Start at vertex. Move to better neighboring vertex.
Until you stop. This example.
(0,0) objective 0. $\rightarrow (0, 25)$ objective 50.
$\rightarrow (16\frac{2}{3}, 25)$ objective $116\frac{2}{3}$



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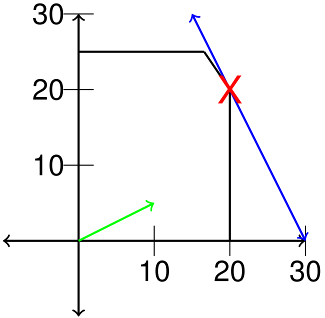
Simplex: Start at vertex. Move to better neighboring vertex.
Until you stop. This example.
(0,0) objective 0. $\rightarrow (0, 25)$ objective 50.
$\rightarrow (16\frac{2}{3}, 25)$ objective $116\frac{2}{3}$ $\rightarrow (20, 20)$ objective 120.



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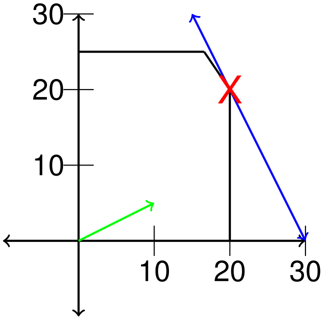
Duality:



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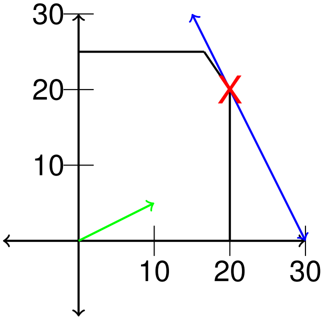
Duality:
Add blue equations to get objective function?



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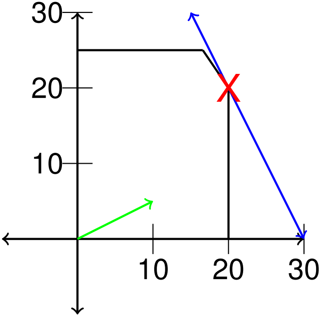
Duality:
Add blue equations to get objective function?
$1/3$ times first plus third.



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Duality:
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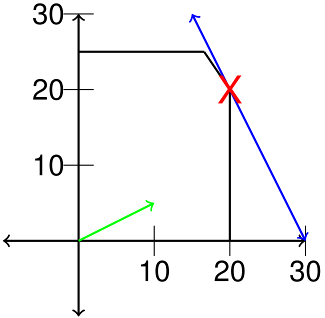
$1/3$ times first plus third.
Get $4x_1 + 2x_2 \le 120$.



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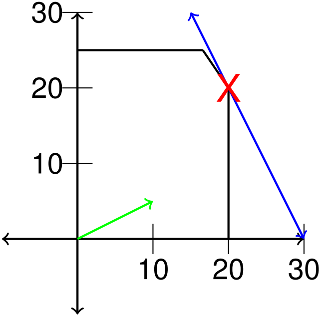
Duality:
Add blue equations to get objective function?
$1/3$ times first plus third.
Get $4x_1 + 2x_2 \leq 120$. Every solution must satisfy this inequality!



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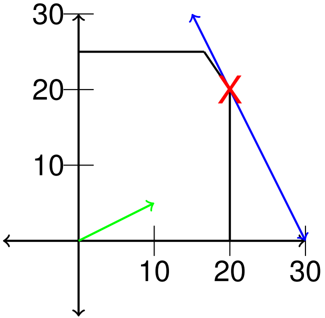
Objective value: 120.



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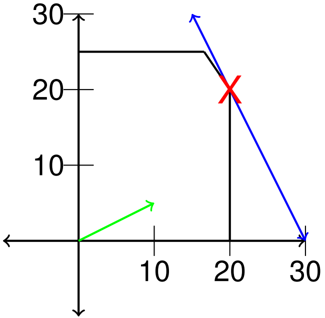
Can we do better?



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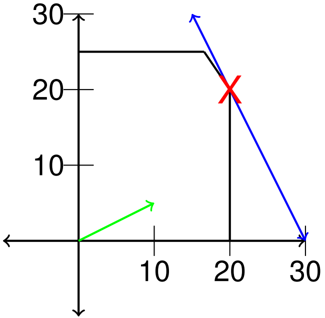
Can we do better? Yes?



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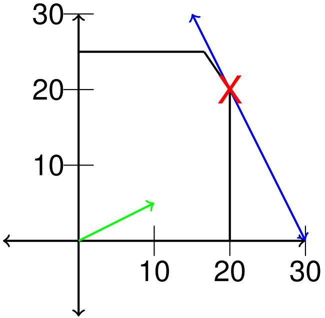
Can we do better? Yes? No?



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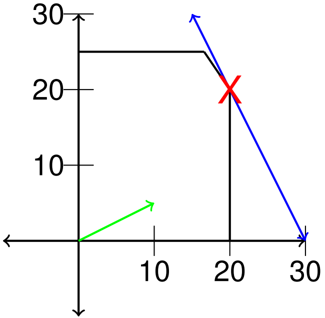

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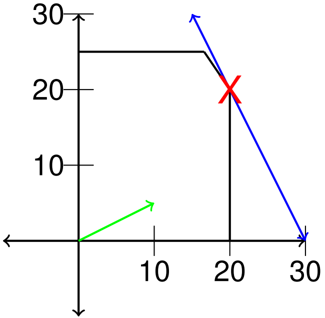

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Add blue equations to get objective function?
$1/3$ times first plus third.
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Objective value: 120.
Can we do better? Yes? No? Maybe? No!
Dual problem: add equations to get best upper bound.

More variables.

More vegetables.

More variables.

More vegetables. How about some Kale!

More variables.

More vegetables. How about some Kale!
3$ per bushel.

More variables.

More vegetables. How about some Kale!
3$ per bushel.
2 sq. yards/bushel per sunny or 3 sq. yards/bushel for shady land.

More variables.

More vegetables. How about some Kale!
3$ per bushel.
2 sq. yards/bushel per sunny or 3 sq. yards/bushel for shady land.
2 units of water.

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x_3 - sunny kale

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More variables.

More vegetables. How about some Kale!
3$ per bushel.
2 sq. yards/bushel per sunny or 3 sq. yards/bushel for shady land.
2 units of water.
x_3 - sunny kale x_4 - shady kale.

$$\begin{aligned} \max 4x_1 + 2x_2 + 3x_3 + 3x_4 \\ 3x_1 + 2x_3 \leq 60 \\ 3x_2 + 3x_4 \leq 75 \\ 3x_1 + 2x_2 + 2x_3 + 2x_4 \leq 100 \\ x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

Carpet production planning.

Demands: $d_1, d_2, \ldots, d_{12}$, range: $440 - 920$

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30 employees. 20 carpets/month. 2000/month.

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Overtime: \$180 extra per carpet. Also at most 30% for one employee.

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Variables.

w_i - number of workers in month i

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Variables.

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o_i - overtime carpets in month i

Carpet production planning.

Demands: $d_1, d_2, \ldots, d_{12}$, range: $440 - 920$

30 employees. 20 carpets/month. 2000/month.

Overtime: \$180 extra per carpet. Also at most 30% for one employee.

Hiring/firing: 320/400.

Storage: 8/carpet and no storage at the end of year.

Variables.

w_i - number of workers in month i ; $w_0 = 30$

x_i - carpets made in month i

o_i - overtime carpets in month i

h_i, f_i - hired/fired in month i

Carpet production planning.

Demands: $d_1, d_2, \ldots, d_{12}$, range: $440 - 920$
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h_i, f_i - hired/fired in month i
s_i - number stored at end of month i

Carpet production planning.

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s_i - number stored at end of month i ; $s_{12} = 0$

Carpet production planning.

Demands: $d_1, d_2, \ldots, d_{12}$, range: $440 - 920$

30 employees. 20 carpets/month. 2000/month.

Overtime: \$180 extra per carpet. Also at most 30% for one employee.

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s_i - number stored at end of month i ; $s_{12} = 0$

Carpet production planning.

Demands: $d_1, d_2, \ldots, d_{12}$, range: $440 - 920$
30 employees. 20 carpets/month. 2000/month.
Overtime: \$180 extra per carpet. Also at most 30% for one employee.
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Variables.

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x_i - carpets made in month i
o_i - overtime carpets in month i
h_i, f_i - hired/fired in month i
s_i - number stored at end of month i ; $s_{12} = 0$

Nonnegative:

Carpet production planning.

Demands: $d_1, d_2, \ldots, d_{12}$, range: $440 - 920$
30 employees. 20 carpets/month. 2000/month.
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Hiring/firing: 320/400.
Storage: 8/carpet and no storage at the end of year.

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Nonnegative: $w_i, x_i, o_i, h_i, f_i, s_i \geq 0$

Carpet production planning.

Demands: $d_1, d_2, \ldots, d_{12}$, range: $440 - 920$
30 employees. 20 carpets/month. 2000/month.
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Nonnegative: $w_i, x_i, o_i, h_i, f_i, s_i \geq 0$
Production:

Carpet production planning.

Demands: $d_1, d_2, \ldots, d_{12}$, range: $440 - 920$
30 employees. 20 carpets/month. 2000/month.
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Hiring/firing: 320/400.
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x_i - carpets made in month i
o_i - overtime carpets in month i
h_i, f_i - hired/fired in month i
s_i - number stored at end of month i ; $s_{12} = 0$

Nonnegative: $w_i, x_i, o_i, h_i, f_i, s_i \geq 0$
Production: $x_i = 20w_i + o_i$

Carpet production planning.

Demands: $d_1, d_2, \ldots, d_{12}$, range: $440 - 920$
30 employees. 20 carpets/month. 2000/month.
Overtime: \$180 extra per carpet. Also at most 30% for one employee.
Hiring/firing: 320/400.
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x_i - carpets made in month i
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s_i - number stored at end of month i ; $s_{12} = 0$

Nonnegative: $w_i, x_i, o_i, h_i, f_i, s_i \geq 0$
Production: $x_i = 20w_i + o_i$
Employment:

Carpet production planning.

Demands: $d_1, d_2, \ldots, d_{12}$, range: $440 - 920$
30 employees. 20 carpets/month. 2000/month.
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Hiring/firing: 320/400.
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s_i - number stored at end of month i ; $s_{12} = 0$

Nonnegative: $w_i, x_i, o_i, h_i, f_i, s_i \geq 0$
Production: $x_i = 20w_i + o_i$
Employment: $w_i = w_{i-1} + h_i - f_i$

Carpet production planning.

Demands: $d_1, d_2, \ldots, d_{12}$, range: $440 - 920$
30 employees. 20 carpets/month. 2000/month.
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Production: $x_i = 20w_i + o_i$
Employment: $w_i = w_{i-1} + h_i - f_i$
Inventory:

Carpet production planning.

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30 employees. 20 carpets/month. 2000/month.
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Regulations:

Carpet production planning.

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Inventory: $s_i = s_{i-1} + x_i - d_i$
Regulations: $o_i \leq 6w_i$

Carpet production planning.

Demands: $d_1, d_2, \ldots, d_{12}$, range: $440 - 920$
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Overtime: \$180 extra per carpet. Also at most 30% for one employee.
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Regulations: $o_i \leq 6w_i$
Objective:

Carpet production planning.

Demands: $d_1, d_2, \ldots, d_{12}$, range: $440 - 920$
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Objective:

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Regulations: $o_i \leq 6 w_i$
Objective:

$$\min \quad 2000 \sum_i w_i$$

Carpet production planning.

Demands: $d_1, d_2, \ldots, d_{12}$, range: $440 - 920$
30 employees. 20 carpets/month. 2000/month.
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Production: $x_i = 20w_i + o_i$
Employment: $w_i = w_{i-1} + h_i - f_i$
Inventory: $s_i = s_{i-1} + x_i - d_i$
Regulations: $o_i \leq 6w_i$
Objective:

$$\min \quad 2000\sum_i w_i + 320\sum_i h_i$$

Carpet production planning.

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Inventory: $s_i = s_{i-1} + x_i - d_i$
Regulations: $o_i \leq 6w_i$
Objective:

$$\min \quad 2000\sum_i w_i + 320\sum_i h_i + 400\sum f_i$$

Carpet production planning.

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Inventory: $s_i = s_{i-1} + x_i - d_i$
Regulations: $o_i \leq 6w_i$
Objective:

$$\min \quad 2000\sum_i w_i + 320\sum_i h_i + 400\sum f_i + 8\sum s_i$$

Carpet production planning.

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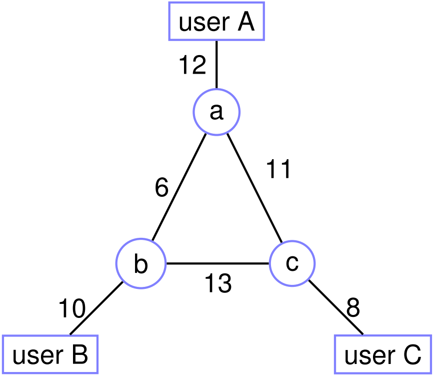
Employment: $w_i = w_{i-1} + h_i - f_i$
Inventory: $s_i = s_{i-1} + x_i - d_i$
Regulations: $o_i \leq 6w_i$
Objective:

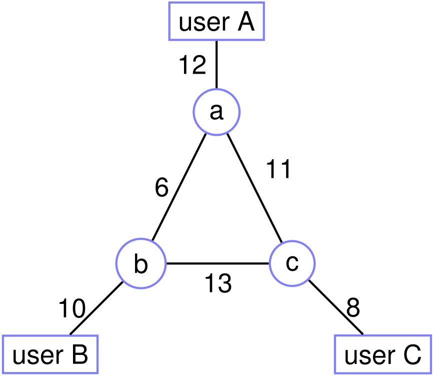
$$\min \quad 2000 \sum_i w_i + 320 \sum_i h_i + 400 \sum f_i + 8 \sum s_i + 180 \sum_i o_i.$$

Bandwidth.



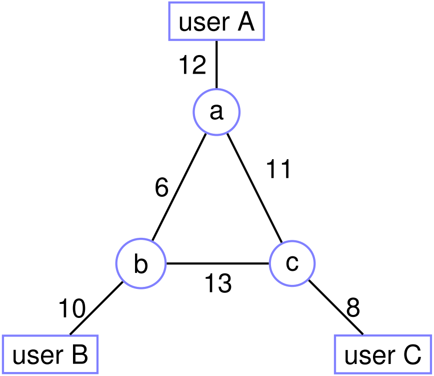
$A - B$ pays 3\$ per unit,
$A - C$ pays 2\$ per unit,
$B - C$ pays 4\$ per unit.

Bandwidth.



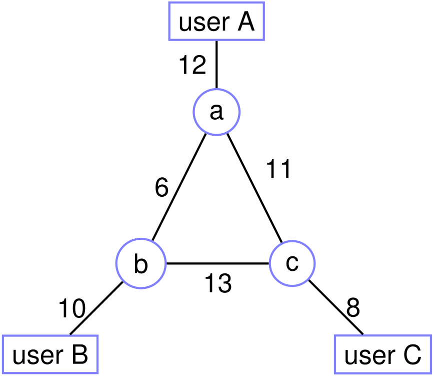
$A - B$ pays 3$ per unit,
$A - C$ pays 2$ per unit,
$B - C$ pays 4$ per unit.
Every pair gets 2 units.

Bandwidth.



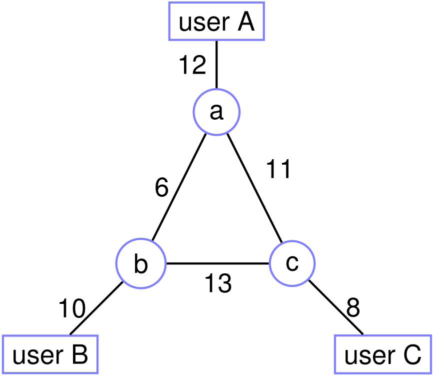
$A - B$ pays 3\$ per unit,
$A - C$ pays 2\$ per unit,
$B - C$ pays 4\$ per unit.
Every pair gets 2 units.
X_{AB} - flow along $A - a - b - B$.

Bandwidth.



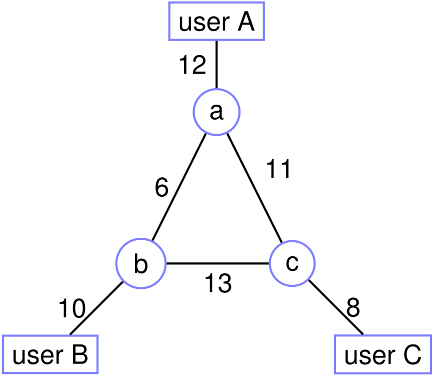
$A-B$ pays 3\$ per unit,
$A-C$ pays 2\$ per unit,
$B-C$ pays 4\$ per unit.
Every pair gets 2 units.
X_{AB} - flow along $A-a-b-B$.
X'_{AB} is flow along path $A-a-c-b-B$

Bandwidth.



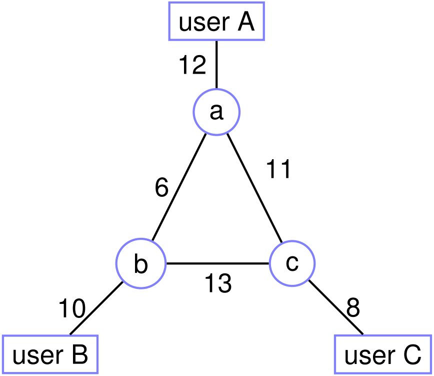
$A - B$ pays 3\$ per unit,
$A - C$ pays 2\$ per unit,
$B - C$ pays 4\$ per unit.
Every pair gets 2 units.
X_{AB} - flow along $A - a - b - B$.
X'_{AB} is flow along path $A - a - c - b - B$
Capacity constraint on edge (a, b):

Bandwidth.



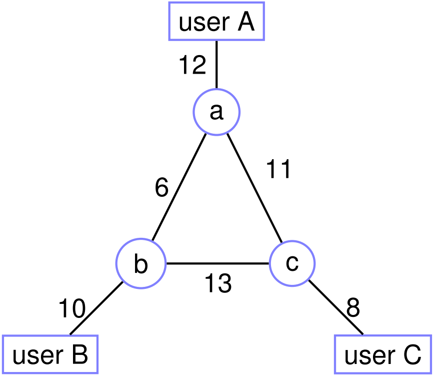
$A - B$ pays 3$ per unit,
$A - C$ pays 2$ per unit,
$B - C$ pays 4$ per unit.
Every pair gets 2 units.
X_{AB} - flow along $A - a - b - B$.
X'_{AB} is flow along path $A - a - c - b - B$
Capacity constraint on edge (a,b):
$X_{AB} + X'_{BC} + X'_{AC}$

Bandwidth.



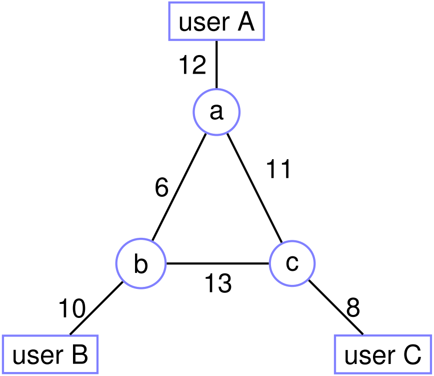
$A-B$ pays 3\$ per unit,
$A-C$ pays 2\$ per unit,
$B-C$ pays 4\$ per unit.
Every pair gets 2 units.
X_{AB} - flow along $A-a-b-B$.
X'_{AB} is flow along path $A-a-c-b-B$
Capacity constraint on edge (a,b):
$X_{AB}+X'_{BC}+X'_{AC}\leq 6$
Bandwidth constraint:

Bandwidth.



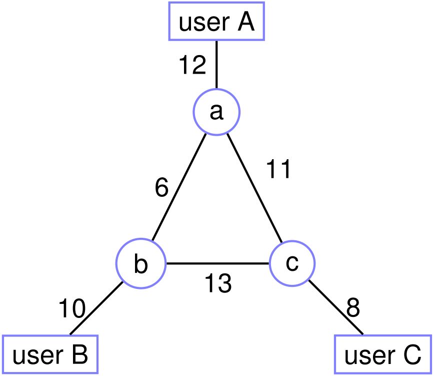
$A - B$ pays 3\$ per unit,
$A - C$ pays 2\$ per unit,
$B - C$ pays 4\$ per unit.
Every pair gets 2 units.
X_{AB} - flow along $A - a - b - B$.
X'_{AB} is flow along path $A - a - c - b - B$
Capacity constraint on edge (a, b):
$X_{AB} + X'_{BC} + X'_{AC} \leq 6$
Bandwidth constraint:
$X_{AB} + X'_{AB}$

Bandwidth.



$A - B$ pays 3$ per unit,
$A - C$ pays 2$ per unit,
$B - C$ pays 4$ per unit.
Every pair gets 2 units.
X_{AB} - flow along $A - a - b - B$.
X'_{AB} is flow along path $A - a - c - b - B$
Capacity constraint on edge (a, b):
$X_{AB} + X'_{BC} + X'_{AC} \leq 6$
Bandwidth constraint:
$X_{AB} + X'_{AB} \geq 2$

Bandwidth.



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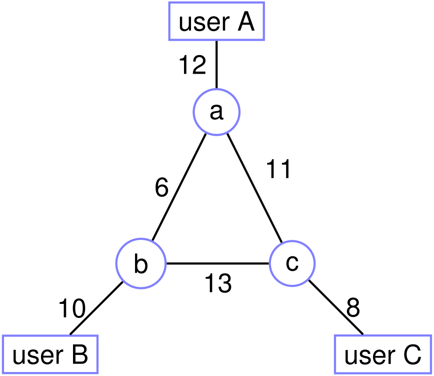
Every pair gets 2 units.
X_{AB} - flow along $A - a - b - B$.
X'_{AB} is flow along path $A - a - c - b - B$
Capacity constraint on edge (a, b):
$X_{AB} + X'_{BC} + X'_{AC} \leq 6$
Bandwidth constraint:
$X_{AB} + X'_{AB} \geq 2$

Bandwidth.



How many edge constraints?

$A - B$ pays 3\$ per unit,
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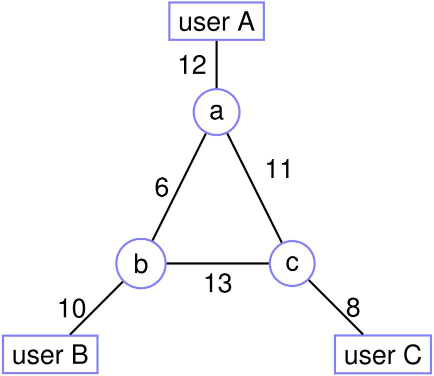
Every pair gets 2 units.
X_{AB} - flow along $A - a - b - B$.
X'_{AB} is flow along path $A - a - c - b - B$
Capacity constraint on edge (a, b):
$X_{AB} + X'_{BC} + X'_{AC} \leq 6$
Bandwidth constraint:
$X_{AB} + X'_{AB} \geq 2$

Bandwidth.



$A - B$ pays 3$ per unit,
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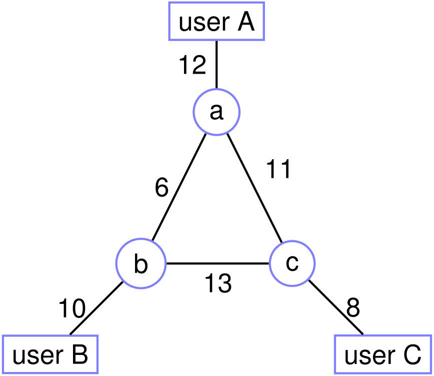
$B - C$ pays 4$ per unit.
Every pair gets 2 units.
X_{AB} - flow along $A - a - b - B$.
X'_{AB} is flow along path $A - a - c - b - B$
Capacity constraint on edge (a, b):
$X_{AB} + X'_{BC} + X'_{AC} \leq 6$
Bandwidth constraint:
$X_{AB} + X'_{AB} \geq 2$

How many edge constraints? 6.

Bandwidth.



$A - B$ pays 3\$ per unit,
$A - C$ pays 2\$ per unit,
$B - C$ pays 4\$ per unit.
Every pair gets 2 units.
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How many bandwidth constraints?

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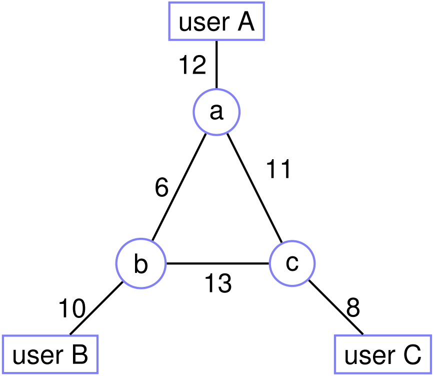

How many edge constraints? 6.
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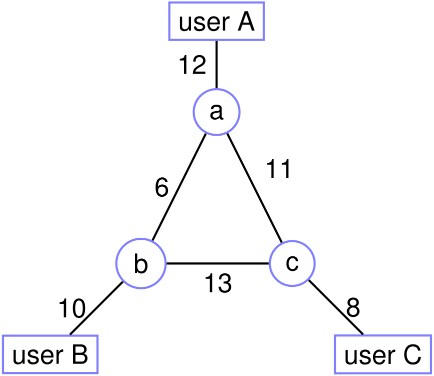

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Objective function?

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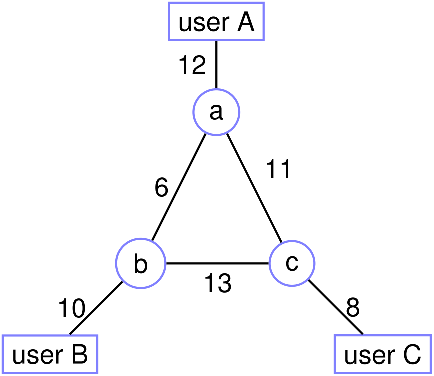

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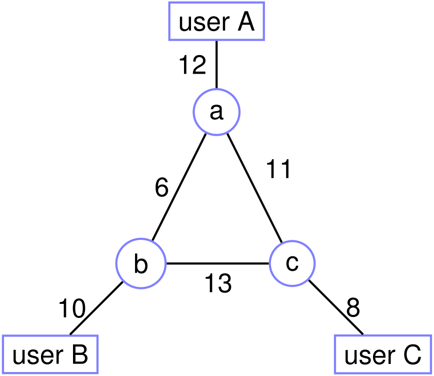

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How many edge constraints? 6.
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Objective function?

$$3(X_{AB}+X'_{AB})+4(X_{BC}+X'_{BC})+2(X_{AC}+X'_{AC})$$

A linear program.

A linear program.

$$\begin{aligned} \max 4x_1 + 2x_2 \\ 2x_1 \leq 60 \\ 3x_2 \leq 75 \\ 3x_1 + 2x_2 \leq 100 \\ x_1, x_2 \geq 0 \end{aligned}$$

A linear program.

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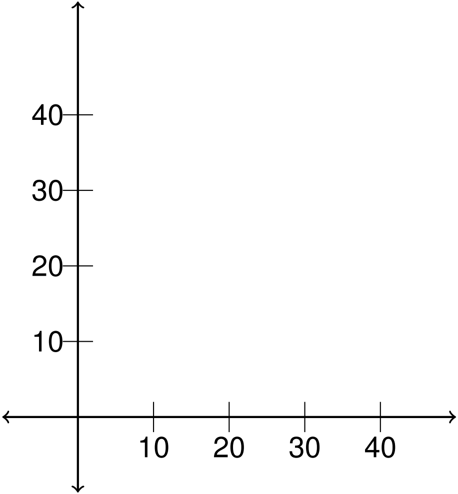

Optimal point?

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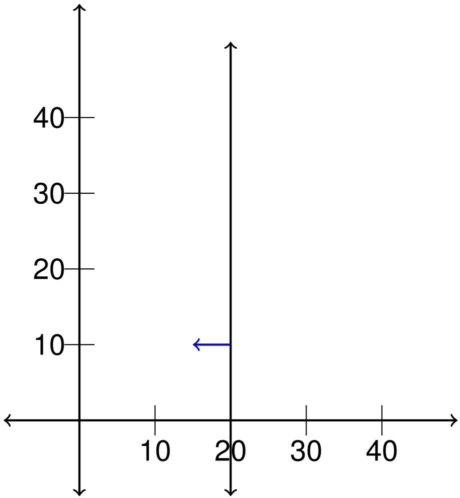

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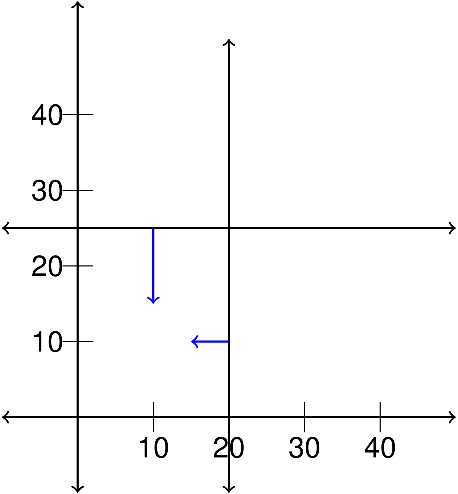

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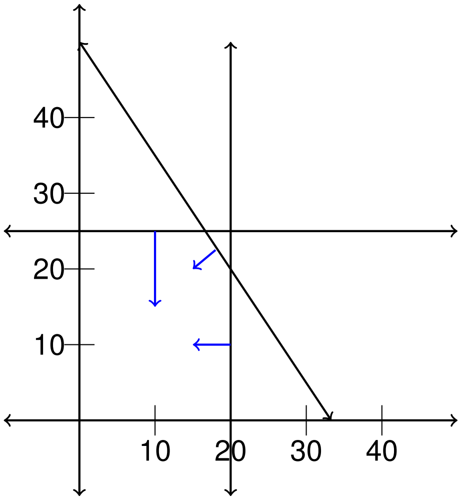

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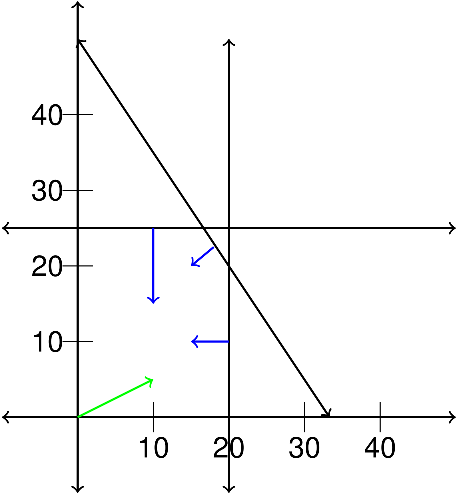

Optimal point?

Again with carpets!

Production:

Again with carpets!

Production: $x_i = 20w_i + o_i$
Employment:

Again with carpets!

Production: $x_i = 20w_i + o_i$
Employment: $w_i = w_{i-1} + h_i - f_i$
Inventory:

Again with carpets!

Production: $x_i = 20w_i + o_i$
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Regulations:

Again with carpets!

Production: $x_i = 20w_i + o_i$
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Again with carpets!

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Regulations: $o_i \leq 6w_i$

$$\min 2000\sum_i w_i + 320\sum_i h_i + 400\sum f_i + 8\sum s_i + 180\sum_i o_i.$$

Again with carpets!

Production: $x_i = 20w_i + o_i$
Employment: $w_i = w_{i-1} + h_i - f_i$
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$$\min 2000\sum_i w_i + 320\sum_i h_i + 400\sum f_i + 8\sum s_i + 180\sum_i o_i.$$

Different form!

Variants of linear programs.

1. Maximization or minimization.
2. Equations or inequalities.
3. Non-negative variables or unrestricted variables.

Reductions.

1. Maximization to minimization?

Reductions.

1. Maximization to minimization?
   Multiply objective function by -1.

Reductions.

1. Maximization to minimization?
   Multiply objective function by -1.
2. Less than inequalities into greater than?

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   Example: $4 \geq 3$

Reductions.

1. Maximization to minimization?
   Multiply objective function by -1.
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   Example: $4 \geq 3$ to $(-1)4 \leq (-1)3$.

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   Multiply objective function by -1.
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   Multiply both sides by (-1) again!
   Example: $4 \geq 3$ to $(-1)4 \leq (-1)3$.
3. Inequalities and equalities.
   (a) $\sum_i a_i x_i \leq b$ into equality?

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   Multiply objective function by -1.
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   Example: $4 \geq 3$ to $(-1)4 \leq (-1)3$.
3. Inequalities and equalities.

   (a) $\sum_i a_i x_i \leq b$ into equality?
   $\sum_i a_i x_i + s = b$

Reductions.

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   Multiply objective function by -1.
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   Multiply both sides by (-1) again!
   Example: $4 \geq 3$ to $(-1)4 \leq (-1)3$.
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4. Simulate unrestricted variable x with positive variable

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   - Introduce x_+, and x_-.

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   - Replace x by $(x_+ - x_-)$.

Reductions.

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4. Simulate unrestricted variable x with positive variables.
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   - Replace x by $(x_+ - x_-)$.

   $(x_+ - x_-)$ could be any real number!

Standard Form.

Standard form.

Standard Form.

Standard form.
Minimization,

Standard Form.

Standard form.
Minimization, positive variables,

Standard Form.

Standard form.
Minimization, positive variables, and “greater than” inequalities.

Standard Form.

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Peas and carrots.

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Minimization, positive variables, and “greater than” inequalities.

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$$\begin{aligned} \max 4x_1 + 2x_2 \\ 2x_1 \leq 60 \\ 3x_2 \leq 75 \\ 3x_1 + 2x_2 \leq 100 \\ x_1, x_2 \geq 0 \end{aligned}$$

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Matrix Form.

Recall Linear equations: $Ax = b$?

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Can do that here, too!

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\end{aligned}
$$

$$\min[-4, -2] \cdot [x_1, x_2]$$

$$\begin{pmatrix} -2 & 0 \\ 0 & -3 \\ -3 & -2 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \geq \begin{bmatrix} -60 \\ -75 \\ -100 \end{bmatrix}$$

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Inputs:
$m \times n$ matrix A;

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$$
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$$

Inputs:
$m \times n$ matrix A; m length vector b;

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Inputs:
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Inputs:

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Output: n length vector x.

$$\min cx$$

$$Ax \geq b$$

Linear Program Problem

Inputs:

Linear Program Problem

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Linear Program Problem

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Linear Program Problem

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$$\min cx$$
$$Ax \geq b$$

Oh yes, some complexities here.

Linear Program Problem

Inputs:
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Oh yes, some complexities here.

1. Program has constraints $x_1 \leq 1$ and $x_1 \geq 3$?

Linear Program Problem

Inputs:
$m \times n$ matrix A; m length vector b; n length vector c.
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   Has no feasible solution!

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   Has no feasible solution!
   Infeasible.

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Inputs:
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$$\min cx$$
$$Ax \geq b$$

Oh yes, some complexities here.

1. Program has constraints $x_1 \leq 1$ and $x_1 \geq 3$?
   Has no feasible solution!
   Infeasible.
2. Program $x_1 \geq 0$, $\max x_1$.

Linear Program Problem

Inputs:
$m \times n$ matrix A; m length vector b; n length vector c.
Output: n length vector x.

$$\min cx$$
$$Ax \geq b$$

Oh yes, some complexities here.

1. Program has constraints $x_1 \leq 1$ and $x_1 \geq 3$?
   Has no feasible solution!
   Infeasible.
2. Program $x_1 \geq 0$, $\max x_1$.
   Optimum?

Linear Program Problem

Inputs:
$m \times n$ matrix A; m length vector b; n length vector c.
Output: n length vector x.

$$\min cx$$
$$Ax \geq b$$

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   Optimum?
   100

Linear Program Problem

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$m \times n$ matrix A; m length vector b; n length vector c.
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$$\min cx$$
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Oh yes, some complexities here.

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   Infeasible.
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   Optimum?
   100 ,200

Linear Program Problem

Inputs:
$m \times n$ matrix A; m length vector b; n length vector c.
Output: n length vector x.

$$\min cx$$
$$Ax \geq b$$

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   Optimum?
   100 ,200 ,300

Linear Program Problem

Inputs:
$m \times n$ matrix A; m length vector b; n length vector c.
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   100 ,200 ,300 ... no limit!

Linear Program Problem

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2. Program $x_1 \geq 0$, $\max x_1$.
   Optimum?
   100 ,200 ,300 ... no limit!
   Unbounded.