Today

- Zero-Sum Games

Strategic Games.

N players.

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Each player has strategy set. $\{S_1, \ldots, S_N\}$.

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Vector valued payoff function: $u(s_1, \ldots, s_n)$ (e.g., $\in \Re^N$).

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Example:

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Example:

2 players

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2 players

Player 1: $\{$ **D**efect, **C**ooperate $\}$.
Player 2: $\{$ **D**efect, **C**ooperate $\}$.

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Player 1: { **D**efect, **C**ooperate }.
Player 2: { **D**efect, **C**ooperate }.

Payoff:

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Player 2: { **D**efect, **C**ooperate }.

Payoff:

	C	**D**
C	(3,3)	(0,5)
D	(5,0)	(1,1)

Famous because?

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What is the best thing for the players to do?

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Both cooperate. Payoff $(3, 3)$.

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What does player 2 do now?

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Stable now!

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What does player 2 do now?

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Nash Equilibrium: neither player has incentive to change strategy.

Two Person Zero Sum Games

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m strategies for player 1 n strategies for player 2

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"Player 1 pays a to player 2."

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m strategies for player 1 *n* strategies for player 2

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"Player 1 pays *a* to player 2."

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Payoffs by *m* by *n* matrix: *G*.

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Row player maximizes, column player minimizes.

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Roshambo: rock,paper, scissors.

	R	P	S
R	0	-1	1
P	1	0	-1
S	-1	1	0

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Any Nash Equilibrium?

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(R,R)?

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(R,R)? no.

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(R,R)? no. (P,R)?

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Any Nash Equilibrium?

(R,R)? no. (P,R)? no. (S,R)? no.

Mixed Strategies.

		R	P	S
R		0	-1	1
P		1	0	-1
S		-1	1	0

How do you play?

Mixed Strategies.

		R	P	S
R	$.3\overline{3}$	0	-1	1
P	$.3\overline{3}$	1	0	-1
S	$.3\overline{3}$	-1	1	0

How do you play?

Player 1: play each strategy with equal probability.

Mixed Strategies.

		R	P	S
		$.3\overline{3}$	$.3\overline{3}$	$.3\overline{3}$
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How do you play?

Player 1: play each strategy with equal probability.
Player 2: play each strategy with equal probability.

Mixed Strategies.

		R	P	S
		$.3\overline{3}$	$.3\overline{3}$	$.3\overline{3}$
R	$.3\overline{3}$	0	-1	1
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Player 1: play each strategy with equal probability.

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Player 1: play each strategy with equal probability.
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Definitions.

Mixed strategies: Each players plays distribution over strategies.

Mixed Strategies.

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		$.3\overline{3}$	$.3\overline{3}$	$.3\overline{3}$
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How do you play?

Player 1: play each strategy with equal probability.
Player 2: play each strategy with equal probability.

Definitions.

Mixed strategies: Each players plays distribution over strategies.

Pure strategies: Each player plays single strategy.

Payoffs: Equilibrium.

		R	P	S
		$.3\overline{3}$	$.3\overline{3}$	$.3\overline{3}$
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Payoffs?

[1] Remember zero sum games have one payoff.

Payoffs: Equilibrium.

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Payoffs? Can't just look it up in matrix!.

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Average Payoff.

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Average Payoff. Expected Payoff.

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Sample space: $\Omega = \{(i,j) : i,j \in [1,..,3]\}$

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Average Payoff. Expected Payoff.

Sample space: $\Omega = \{(i,j) : i,j \in [1,..,3]\}$
Random variable X (payoff).

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Average Payoff. Expected Payoff.

Sample space: $\Omega = \{(i,j) : i,j \in [1,..,3]\}$
Random variable X (payoff).

$$E[X] = \sum_{(i,j)} X(i,j)Pr[(i,j)].$$

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Payoffs: Equilibrium.

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$$E[X] = \sum_{(i,j)} X(i,j) Pr[(i,j)].$$

Each player chooses independently:
$Pr[(i,j)] = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

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$$E[X] = 0.$$

---

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Sample space: $\Omega = \{(i,j) : i,j \in [1,..,3]\}$
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Each player chooses independently:
$Pr[(i,j)] = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

$$E[X] = 0.^{1}$$

[1] Remember zero sum games have one payoff.

Equilibrium

		R	P	S
		$.3\overline{3}$	$.3\overline{3}$	$.3\overline{3}$
R	$.3\overline{3}$	0	1	-1
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S	$.3\overline{3}$	1	-1	0

Will Player 1 change strategy?

Equilibrium

		R	P	S
		$.3\overline{3}$	$.3\overline{3}$	$.3\overline{3}$
R	$.3\overline{3}$	0	1	-1
P	$.3\overline{3}$	-1	0	1
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Will Player 1 change strategy? Mixed strategies uncountable!

Equilibrium

		R	P	S
		$.3\overline{3}$	$.3\overline{3}$	$.3\overline{3}$
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Will Player 1 change strategy? Mixed strategies uncountable!

Expected payoffs for pure strategies for player 1.

Equilibrium

		R	P	S
		$.3\overline{3}$	$.3\overline{3}$	$.3\overline{3}$
R	$.3\overline{3}$	0	1	-1
P	$.3\overline{3}$	-1	0	1
S	$.3\overline{3}$	1	-1	0

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Expected payoffs for pure strategies for player 1.

Expected payoff of Rock?

Equilibrium

		R	P	S
		$.3\overline{3}$	$.3\overline{3}$	$.3\overline{3}$
R	$.3\overline{3}$	0	1	-1
P	$.3\overline{3}$	-1	0	1
S	$.3\overline{3}$	1	-1	0

Will Player 1 change strategy? Mixed strategies uncountable!

Expected payoffs for pure strategies for player 1.

Expected payoff of Rock? $\frac{1}{3} \times 0 + \frac{1}{3} \times 1 + \frac{1}{3} \times -1 = 0$.

Equilibrium

		R	P	S
		$.3\overline{3}$	$.3\overline{3}$	$.3\overline{3}$
R	$.3\overline{3}$	0	1	-1
P	$.3\overline{3}$	-1	0	1
S	$.3\overline{3}$	1	-1	0

Will Player 1 change strategy? Mixed strategies uncountable!

Expected payoffs for pure strategies for player 1.

Expected payoff of Rock? $\frac{1}{3} \times 0 + \frac{1}{3} \times 1 + \frac{1}{3} \times -1 = 0$.
Expected payoff of Paper?

Equilibrium

		R	P	S
		$.3\overline{3}$	$.3\overline{3}$	$.3\overline{3}$
R	$.3\overline{3}$	0	1	-1
P	$.3\overline{3}$	-1	0	1
S	$.3\overline{3}$	1	-1	0

Will Player 1 change strategy? Mixed strategies uncountable!

Expected payoffs for pure strategies for player 1.

Expected payoff of Rock? $\frac{1}{3} \times 0 + \frac{1}{3} \times 1 + \frac{1}{3} \times -1 = 0$.
Expected payoff of Paper? $\frac{1}{3} \times -1 + \frac{1}{3} \times 0 + \frac{1}{3} \times 1 = 0$.

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		$.3\overline{3}$	$.3\overline{3}$	$.3\overline{3}$
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Expected payoffs for pure strategies for player 1.

Expected payoff of Rock? $\frac{1}{3} \times 0 + \frac{1}{3} \times 1 + \frac{1}{3} \times -1 = 0$.
Expected payoff of Paper? $\frac{1}{3} \times -1 + \frac{1}{3} \times 0 + \frac{1}{3} \times 1 = 0$.
Expected payoff of Scissors?

Equilibrium

		R	P	S
		$.3\overline{3}$	$.3\overline{3}$	$.3\overline{3}$
R	$.3\overline{3}$	0	1	-1
P	$.3\overline{3}$	-1	0	1
S	$.3\overline{3}$	1	-1	0

Will Player 1 change strategy? Mixed strategies uncountable!

Expected payoffs for pure strategies for player 1.

Expected payoff of Rock? $\frac{1}{3} \times 0 + \frac{1}{3} \times 1 + \frac{1}{3} \times -1 = 0$.
Expected payoff of Paper? $\frac{1}{3} \times -1 + \frac{1}{3} \times 0 + \frac{1}{3} \times 1 = 0$.
Expected payoff of Scissors? $\frac{1}{3} \times 1 + \frac{1}{3} \times 0 + \frac{1}{3} \times -1 = 0$.

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Will Player 1 change strategy? Mixed strategies uncountable!

Expected payoffs for pure strategies for player 1.

Expected payoff of Rock? $\frac{1}{3} \times 0 + \frac{1}{3} \times 1 + \frac{1}{3} \times -1 = 0$.
Expected payoff of Paper? $\frac{1}{3} \times -1 + \frac{1}{3} \times 0 + \frac{1}{3} \times 1 = 0$.
Expected payoff of Scissors? $\frac{1}{3} \times 1 + \frac{1}{3} \times 0 + \frac{1}{3} \times -1 = 0$.

No better pure strategy.

Equilibrium

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		$.3\overline{3}$	$.3\overline{3}$	$.3\overline{3}$
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Will Player 1 change strategy? Mixed strategies uncountable!

Expected payoffs for pure strategies for player 1.

Expected payoff of Rock? $\frac{1}{3} \times 0 + \frac{1}{3} \times 1 + \frac{1}{3} \times -1 = 0.$
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Expected payoff of Scissors? $\frac{1}{3} \times 1 + \frac{1}{3} \times 0 + \frac{1}{3} \times -1 = 0.$

No better pure strategy. $\implies$ No better mixed strategy!

Equilibrium

		R	P	S
		$.3\overline{3}$	$.3\overline{3}$	$.3\overline{3}$
R	$.3\overline{3}$	0	1	-1
P	$.3\overline{3}$	-1	0	1
S	$.3\overline{3}$	1	-1	0

Will Player 1 change strategy? Mixed strategies uncountable!

Expected payoffs for pure strategies for player 1.

Expected payoff of Rock? $\frac{1}{3} \times 0 + \frac{1}{3} \times 1 + \frac{1}{3} \times -1 = 0$.
Expected payoff of Paper? $\frac{1}{3} \times -1 + \frac{1}{3} \times 0 + \frac{1}{3} \times 1 = 0$.
Expected payoff of Scissors? $\frac{1}{3} \times 1 + \frac{1}{3} \times 0 + \frac{1}{3} \times -1 = 0$.

No better pure strategy. $\implies$ No better mixed strategy!

Mixed strat. payoff is weighted av. of payoffs of pure strats.

Equilibrium

		R	P	S
		$.3\overline{3}$	$.3\overline{3}$	$.3\overline{3}$
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$E[X]=\sum_{(i,j)}(Pr[i]\times Pr[j])X(i,j)$

Equilibrium

		R	P	S
		$.3\overline{3}$	$.3\overline{3}$	$.3\overline{3}$
R	$.3\overline{3}$	0	1	-1
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$E[X] = \sum_{(i,j)} (Pr[i] \times Pr[j]) X(i,j) = \sum_i Pr[i] (\sum_j Pr[j] \times X(i,j))$

Equilibrium

		R	P	S
		$.3\overline{3}$	$.3\overline{3}$	$.3\overline{3}$
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Equilibrium

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		$.3\overline{3}$	$.3\overline{3}$	$.3\overline{3}$
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Mixed strategy can't be better than the best pure strategy.

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		R	P	S
		$.3\overline{3}$	$.3\overline{3}$	$.3\overline{3}$
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Equilibrium

		R	P	S
		$.3\overline{3}$	$.3\overline{3}$	$.3\overline{3}$
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Mixed strategy can't be better than the best pure strategy.

Player 1 has no incentive to change! Same for player 2.

Equilibrium

		R	P	S
		$.3\overline{3}$	$.3\overline{3}$	$.3\overline{3}$
R	$.3\overline{3}$	0	1	-1
P	$.3\overline{3}$	-1	0	1
S	$.3\overline{3}$	1	-1	0

Will Player 1 change strategy? Mixed strategies uncountable!

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Mixed strategy can't be better than the best pure strategy.

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Equilibrium!

Another example plus notation.

Rock, Paper, Scissors, prEempt.

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PreEmpt ties preEmpt, beats everything else.

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Payoffs.

Another example plus notation.

Rock, Paper, Scissors, prEempt.

PreEmpt ties preEmpt, beats everything else.

Payoffs.

	R	P	S	E
R	0	-1	1	-1
P	1	0	-1	-1
S	-1	1	0	-1
E	1	1	1	0

Equilibrium?

Another example plus notation.

Rock, Paper, Scissors, prEempt.

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Payoffs.

	R	P	S	E
R	0	-1	1	-1
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Equilibrium? **(E,E)**.

Another example plus notation.

Rock, Paper, Scissors, prEempt.

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Payoffs.

	R	P	S	E
R	0	-1	1	-1
P	1	0	-1	-1
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Equilibrium? **(E,E)**. Pure strategy equilibrium.

Another example plus notation.

Rock, Paper, Scissors, prEempt.

PreEmpt ties preEmpt, beats everything else.

Payoffs.

	R	P	S	E
R	0	-1	1	-1
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E	1	1	1	0

Equilibrium? **(E,E)**. Pure strategy equilibrium.

Notation:

Another example plus notation.

Rock, Paper, Scissors, prEempt.

PreEmpt ties preEmpt, beats everything else.

Payoffs.

	R	P	S	E
R	0	-1	1	-1
P	1	0	-1	-1
S	-1	1	0	-1
E	1	1	1	0

Equilibrium? **(E,E)**. Pure strategy equilibrium.

Notation: Rock is 1, Paper is 2, Scissors is 3, prEmpt is 4.

Another example plus notation.

Rock, Paper, Scissors, prEempt.
PreEmpt ties preEmpt, beats everything else.
Payoffs.

	R	P	S	E
R	0	-1	1	-1
P	1	0	-1	-1
S	-1	1	0	-1
E	1	1	1	0

Equilibrium? **(E,E)**. Pure strategy equilibrium.
Notation: Rock is 1, Paper is 2, Scissors is 3, prEmpt is 4.
Payoff Matrix.

$$G = \begin{bmatrix} 0 & -1 & 1 & -1 \\ 1 & 0 & -1 & -1 \\ -1 & 1 & 0 & -1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Playing the boss...

Row has extra strategy:Cheat.

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Note: column knows row cheats.

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Rock is strategy 1, Paper is 2, Scissors is 3, and Cheat is 4 (for row.)

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Note: column knows row cheats. Why play?
Row is column's boss.

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Ties with rock and scissors, beats paper. (Scissors, or no rock!)
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Note: column knows row cheats. Why play?
Row is column's boss.

Equilibrium: play the boss...

$$G = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equilibrium:

Equilibrium: play the boss...

$$G = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equilibrium:
Row: $(0, \frac{1}{3}, \frac{1}{6}, \frac{1}{2})$.

Equilibrium: play the boss...

$$G = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equilibrium:
Row: $(0, \frac{1}{3}, \frac{1}{6}, \frac{1}{2})$. Column: $(\frac{1}{3}, \frac{1}{2}, \frac{1}{6})$.

Equilibrium: play the boss...

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Payoff?

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Payoff? Remember: weighted average of pure strategies.

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Payoff? Remember: weighted average of pure strategies.

Row Player.

Equilibrium: play the boss...

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Payoff? Remember: weighted average of pure strategies.

Row Player.

Strategy 1: $\frac{1}{3} \times 0 + \frac{1}{2} \times 1 + \frac{1}{6} \times -1$

Equilibrium: play the boss...

$$G = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equilibrium:
Row: $(0, \frac{1}{3}, \frac{1}{6}, \frac{1}{2})$. Column: $(\frac{1}{3}, \frac{1}{2}, \frac{1}{6})$.

Payoff? Remember: weighted average of pure strategies.

Row Player.

Strategy 1: $\frac{1}{3} \times 0 + \frac{1}{2} \times 1 + \frac{1}{6} \times -1 = \frac{1}{3}$

Equilibrium: play the boss...

$$G = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equilibrium:
Row: $(0, \frac{1}{3}, \frac{1}{6}, \frac{1}{2})$. Column: $(\frac{1}{3}, \frac{1}{2}, \frac{1}{6})$.

Payoff? Remember: weighted average of pure strategies.

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Strategy 1: $\frac{1}{3} \times 0 + \frac{1}{2} \times 1 + \frac{1}{6} \times -1 = \frac{1}{3}$
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Strategy 3: $\frac{1}{3} \times 1 + \frac{1}{2} \times -1 + \frac{1}{6} \times 0 = -\frac{1}{6}$
Strategy 4: $\frac{1}{3} \times 0 + \frac{1}{2} \times 0 + \frac{1}{6} \times -1$

Equilibrium: play the boss...

$$G = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equilibrium:
Row: $(0, \frac{1}{3}, \frac{1}{6}, \frac{1}{2})$. Column: $(\frac{1}{3}, \frac{1}{2}, \frac{1}{6})$.

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Equilibrium:
Row: $(0, \frac{1}{3}, \frac{1}{6}, \frac{1}{2})$. Column: $(\frac{1}{3}, \frac{1}{2}, \frac{1}{6})$.

Payoff? Remember: weighted average of pure strategies.

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Strategy 1: $\frac{1}{3} \times 0 + \frac{1}{2} \times 1 + \frac{1}{6} \times -1 = \frac{1}{3}$
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Row: $(0, \frac{1}{3}, \frac{1}{6}, \frac{1}{2})$. Column: $(\frac{1}{3}, \frac{1}{2}, \frac{1}{6})$.

Payoff? Remember: weighted average of pure strategies.

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Payoff is $0 \times \frac{1}{3} + \frac{1}{3} \times (-\frac{1}{6}) + \frac{1}{6} \times (-\frac{1}{6}) + \frac{1}{2} \times (-\frac{1}{6})$

Equilibrium: play the boss...

$$G = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equilibrium:
Row: $(0, \frac{1}{3}, \frac{1}{6}, \frac{1}{2})$. Column: $(\frac{1}{3}, \frac{1}{2}, \frac{1}{6})$.

Payoff? Remember: weighted average of pure strategies.

Row Player.

Strategy 1: $\frac{1}{3} \times 0 + \frac{1}{2} \times 1 + \frac{1}{6} \times -1 = \frac{1}{3}$
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Equilibrium: play the boss...

$$G = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equilibrium:
Row: $(0, \frac{1}{3}, \frac{1}{6}, \frac{1}{2})$. Column: $(\frac{1}{3}, \frac{1}{2}, \frac{1}{6})$.

Payoff? Remember: weighted average of pure strategies.

Row Player.

Strategy 1: $\frac{1}{3} \times 0 + \frac{1}{2} \times 1 + \frac{1}{6} \times -1 = \frac{1}{3}$
Strategy 2: $\frac{1}{3} \times -1 + \frac{1}{2} \times 0 + \frac{1}{6} \times 1 = -\frac{1}{6}$
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Strategy 4: $\frac{1}{3} \times 0 + \frac{1}{2} \times 0 + \frac{1}{6} \times -1 = -\frac{1}{6}$

Payoff is $0 \times \frac{1}{3} + \frac{1}{3} \times (-\frac{1}{6}) + \frac{1}{6} \times (-\frac{1}{6}) + \frac{1}{2} \times (-\frac{1}{6}) = -\frac{1}{6}$

Column player: every column payoff is $-\frac{1}{6}$.

Equilibrium: play the boss...

$$G = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equilibrium:
Row: $(0, \frac{1}{3}, \frac{1}{6}, \frac{1}{2})$. Column: $(\frac{1}{3}, \frac{1}{2}, \frac{1}{6})$.

Payoff? Remember: weighted average of pure strategies.

Row Player.

Strategy 1: $\frac{1}{3} \times 0 + \frac{1}{2} \times 1 + \frac{1}{6} \times -1 = \frac{1}{3}$
Strategy 2: $\frac{1}{3} \times -1 + \frac{1}{2} \times 0 + \frac{1}{6} \times 1 = -\frac{1}{6}$
Strategy 3: $\frac{1}{3} \times 1 + \frac{1}{2} \times -1 + \frac{1}{6} \times 0 = -\frac{1}{6}$
Strategy 4: $\frac{1}{3} \times 0 + \frac{1}{2} \times 0 + \frac{1}{6} \times -1 = -\frac{1}{6}$

Payoff is $0 \times \frac{1}{3} + \frac{1}{3} \times (-\frac{1}{6}) + \frac{1}{6} \times (-\frac{1}{6}) + \frac{1}{2} \times (-\frac{1}{6}) = -\frac{1}{6}$

Column player: every column payoff is $-\frac{1}{6}$.

Both only play optimal strategies!

Equilibrium: play the boss...

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Both only play optimal strategies!

Why not play just one? Changes payoff for other guy!

Two person zero sum games.

$m \times n$ payoff matrix G.

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Row mixed strategy: $x = (x_1, \ldots, x_m)$.

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Payoff for strategy pair (x, y):

$$p(x, y) = x^t G y$$

That is,

$$\sum_i x_i \left(\sum_j G[i,j] y_j \right) = \sum_j \left(\sum_i x_i G[i,j] \right) y_j.$$

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Recall row maximizes, column minimizes.

Best Response/Defense.

Row goes first:

Best Response/Defense.

Row goes first:
Find x, where best column is not too low..

$$R = \min_y \max_x (x^t G y).$$

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Note: y can be $(0,0,\ldots,1,\ldots 0)$.

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Example: Roshambo.

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Example: Roshambo. Value of R?

		R	P	S
x_1	R	0	-1	1
x_2	P	1	0	-1
x_3	S	-1	1	0

Pick x_1,x_2,x_3 to maximize

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Pick x_1,x_2,x_3 to maximize

$$\min\{x_2 - x_3, -x_1 + x_3, x_1 - x_2\}$$

Best Defense: row

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Linear program:

Best Defense: row

Pick x_1,x_2,x_3 to maximize

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Linear program: $z = \min\{x_2 - x_3, -x_1 + x_3, x_1 - x_2\}$

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Linear program: $z = \min\{x_2 - x_3, -x_1 + x_3, x_1 - x_2\}$

$$\begin{aligned} \max z \\ z \le x_2 - x_3 \\ z \le -x_1 + x_3 \\ z \le x_1 - x_2 \\ x_1 + x_2 = 1 \end{aligned}$$

or in standard form...

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Pick x_1,x_2,x_3 to maximize

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$$\begin{aligned} &\max z \\ x_2 - x_3 - z &\geq 0 \\ -x_1 + x_3 - z &\geq 0 \\ x_1 - x_2 - z &\geq 0 \\ x_1 + x_2 &= 1 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Computing best defense: column.

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Column goes first:
Find y, where best row is not high.

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Again: x of form $(0, 0, \ldots, 1, \ldots 0)$.

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Again: x of form $(0,0,\ldots,1,\ldots 0)$.

Example: Roshambo. Value of C?

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Find y, where best row is not high.

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Again: x of form $(0, 0, \ldots, 1, \ldots 0)$.

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Example: Roshambo. Value of C? 0.

	y_1 R	y_2 P	y_3 S
R	0	-1	1
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	y_1 R	y_2 P	y_3 S
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Find y to minimize $\max\{y_2 - y_3, -y_1 + y_3, y_1 - y_2\}$.

Column Best Defense: LP.

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Column Best Defense: LP.

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$$
\begin{aligned}
&\min w \\
y_2 - y_3 &\geq w \\
-y_1 + y_3 &\geq w \\
y_1 - y_2 &\geq w \\
y_1 + y_2 &= 1
\end{aligned}
$$

Column Best Defense: LP.

Find y to minimize $\max\{y_2 - y_3, -y_1 + y_3, y_1 - y_2\}$.

$$\begin{aligned} \min w \\ y_2 - y_3 \geq w \\ -y_1 + y_3 \geq w \\ y_1 - y_2 \geq w \\ y_1 + y_2 = 1 \end{aligned}$$

..in standard form..

$$\begin{aligned} \min w \\ y_2 - y_3 - w \geq 0 \\ -y_1 + y_3 - w \geq 0 \\ y_1 - y_2 - w \geq 0 \\ y_1 + y_2 = 1 \\ y_1, y_2, y_3 \geq 0 \end{aligned}$$

Both.

$$
\begin{aligned}
&\max z \\
&x_2 - x_3 - z \geq 0 \\
&-x_1 + x_3 - z \geq 0 \\
&x_1 - x_2 - z \geq 0 \\
&x_1 + x_2 = 1 \\
&x_1, x_2 \geq 0
\end{aligned}
\qquad\qquad
\begin{aligned}
\min w& \\
y_2 - y_3 - w \geq 0& \\
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y_1 + y_2 = 1& \\
y_1, y_2, y_3 \geq 0&
\end{aligned}
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Note: “column” defense lp is upper bound on “row” defense lp.

Both.

$$
\begin{array}{ll}
\max z & \min w \\
x_2 - x_3 - z \geq 0 & y_2 - y_3 - w \geq 0 \\
-x_1 + x_3 - z \geq 0 & -y_1 + y_3 - w \geq 0 \\
x_1 - x_2 - z \geq 0 & y_1 - y_2 - w \geq 0 \\
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x_1, x_2 \geq 0 & y_1, y_2, y_3 \geq 0
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Note: “column” defense lp is upper bound on “row” defense lp.

Linear programming dual?

Both.

$$\begin{aligned} &\max z \\ &x_2 - x_3 - z \geq 0 \\ &-x_1 + x_3 - z \geq 0 \\ &x_1 - x_2 - z \geq 0 \\ &x_1 + x_2 = 1 \\ &x_1, x_2 \geq 0 \end{aligned} \qquad \begin{aligned} \min w \\ y_2 - y_3 - w \geq 0 \\ -y_1 + y_3 - w \geq 0 \\ y_1 - y_2 - w \geq 0 \\ y_1 + y_2 = 1 \\ y_1, y_2, y_3 \geq 0 \end{aligned}$$

Note: “column” defense lp is upper bound on “row” defense lp.

Linear programming dual? ...Yes!!!

Both.

$$\max z$$
$$x_2 - x_3 - z \geq 0$$
$$-x_1 + x_3 - z \geq 0$$
$$x_1 - x_2 - z \geq 0$$
$$x_1 + x_2 = 1$$
$$x_1, x_2 \geq 0$$

$$\min w$$
$$y_2 - y_3 - w \geq 0$$
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Linear programming dual? ...Yes!!!

$$\implies z = w!$$

Both.

$$\max z \qquad\qquad \min w$$

$$x_2 - x_3 - z \geq 0 \qquad\qquad y_2 - y_3 - w \geq 0$$

$$-x_1 + x_3 - z \geq 0 \qquad\qquad -y_1 + y_3 - w \geq 0$$

$$x_1 - x_2 - z \geq 0 \qquad\qquad y_1 - y_2 - w \geq 0$$

$$x_1 + x_2 = 1 \qquad\qquad y_1 + y_2 = 1$$

$$x_1, x_2 \geq 0 \qquad\qquad y_1, y_2, y_3 \geq 0$$

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$\max_x \min_y x^t G y = \min_y \max_x x^t G y.$

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\begin{array}{ll}
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Wow

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$\max_x \min_y x^t Gy = \min_y \max_x x^t Gy.$

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x_1 + x_2 = 1 & y_1 + y_2 = 1 \\
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$\max_x \min_y x^t G y = \min_y \max_x x^t G y.$

Wow! ! ! ! ! ! ! ! (von Neumann's minimax theorem.)

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Column goes first, row gets to respond.

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Test!

Von Neumann's Minimax theorem.

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Test!

Play rock first?

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Test!

Play rock first?

Mixed strategy.

Von Neumann's Minimax theorem.

$$\max_x \min_y x^t G y = \min_y \max_x x^t G y.$$

Column goes first, row gets to respond.

same as

Row goes first, column gets to respond.

Test!

Play rock first?

Mixed strategy. Play rock, paper,scissors uniformly.

Search Problems

Shortest paths, minimum msts, maximum matchings, maximum increasing subsequences, maximum flows.

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Efficient algorithms:

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Try everything: $n!$ matchings!

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Try everything: n! matchings! n^{n-2} spanning trees...

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Are there always *efficient* algorithms for optimization problems?

Perhaps not.

Satisfiability or **SAT**.

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An Instance.

$$\phi = (x \vee y \vee z)(x \vee \overline{y})(y \vee \overline{z})(z \vee \overline{x}) \vee (\overline{x} \vee \overline{y} \vee \overline{z})$$

Perhaps not.

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$$\phi = (x \vee y \vee z)(x \vee \overline{y})(y \vee \overline{z})(z \vee \overline{x}) \vee (\overline{x} \vee \overline{y} \vee \overline{z})$$

Is there any way to satisfy the formula above?

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Is there any way to satisfy the formula above?

Search problem: Given instance I find a solution S.

S is short

Perhaps not.

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Is there any way to satisfy the formula above?

Search problem: Given instance *I* find a solution *S*.

S is short and easy to check.

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Is there any way to satisfy the formula above?

Search problem: Given instance I find a solution S.

S is short and easy to check.

A *search problem* has efficient checking algorithm $\mathscr{C}$:
S is solution for S if and only if $\mathscr{C}(I, S) = \textit{true}$.

Perhaps not.

Satisfiability or **SAT**.

An Instance.

$$\phi = (x \vee y \vee z)(x \vee \overline{y})(y \vee \overline{z})(z \vee \overline{x}) \vee (\overline{x} \vee \overline{y} \vee \overline{z})$$

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A *search problem* has efficient checking algorithm $\mathscr{C}$:
S is solution for S if and only if $\mathscr{C}(I, S) = true$.

For SAT, what is S?

Perhaps not.

Satisfiability or **SAT**.

An Instance.

$$\phi = (x \vee y \vee z)(x \vee \overline{y})(y \vee \overline{z})(z \vee \overline{x}) \vee (\overline{x} \vee \overline{y} \vee \overline{z})$$

Is there any way to satisfy the formula above?

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Greedy and strongly connected components.

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Efficient algorithm for 3-SAT?

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Efficient algorithm for 3-SAT? Don't think so!

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Efficient algorithm for 3-SAT? Don't think so! More later.