CS170 Discussion Section 4: *2/15*

1 Coraysoda

The land of Coraysoda has a network of settlements, S, and roads, R. There is always at least one path from any settlement to any other settlement. Most roads have a specified tax cost T_r that you must pay to traverse it. All roads are bidirectional. It is helpful to consider this as a graph $G = (V, E)$, where $V = S$ and $E = R$.

- The king and queen of Coraysoda have built massive golden statues of themselves in two different settlements. To encourage people to travel by it, any paths that visit either of the two statues will have half of their total trip cost reimbursed (visiting both statues does not reimburse more). Given a starting settlement A, and a destination settlement B, find the cheapest route there.

- A few settlements are protesting the Coraysoda government by blocking off all the roads coming in and out of the settlement. You want to determine which settlements have been "disrupted". We've defined a settlement is disrupted if it now can only reach less than half of the settlements it used to be able to reach. Can you use the result of the previous part? Your runtime should be $O(S + R)$.

- Finally, all the riots are over, and the road network is back to normal. But we want to upgrade the road system. A road can be upgraded into a cutting edge paved road, for a cost of $2T_r$. A paved road is free to traverse, still bidirectional, and allows traversal via car. While spending the least amount possible, which roads should you upgrade to connect every settlement via car? You should make it so that starting from any settlement, it is possible to reach any other settlement without getting out of the car.

2 Special Case Shortest Path

Consider a connected graph G whose edge weights are always 1 or 2. Design a shortest path algorithm to compute the shortest path from the starting vertex S to the destination vertex T. Your runtime should be $O(|E| + |V|)$.

3 New road

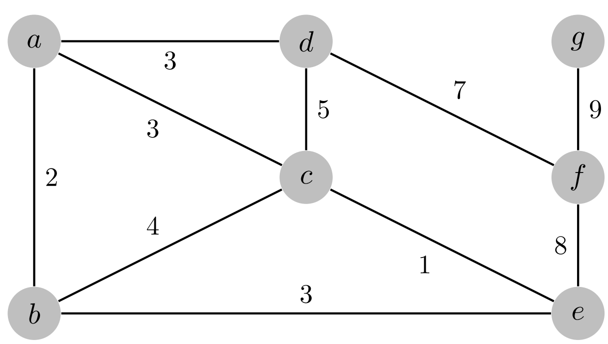
Consider a directed positive-weighted graph $G = (V, E)$, a number k, and a candidate list of new positive-weighted directed edges E' that we can add. Unfortunately, we can only add add k edges to our network, or in other words, we need to choose a subset $S \subseteq E'$, with $|S| \leq k$. In order to estimate how good these edges are, we consider the improvement of the shortest path from two predetermined nodes s, t from $G = (V, E)$ and $G' = (V, E \cup S)$ (that is, we consider how much shorter the path will become after adding all the new edges).

1. Show how to compute the best improvement efficiently when $k = 1$.

2. Show how to do this efficiently for every other $k > 1$ (optional, explain how to get the best set S)

4 Kruskal

Use Kruskal's greedy algorithm to find the minimum spanning tree of the following weighted graph.



5 Midterm Prep

Good luck next Tuesday! There is a review session 7:00-9:00 tonight in Hearst Annex A1. Slides will be posted on Piazza afterwards. If there is extra time at the end of discussion, you could ask the TA to review certain topics for the midterm.