

NP-Completeness

1. 4D Matching

Recall the 3-DIMENSIONAL MATCHING problem, asking you to match n girls, n boys, and n pets given a list of compatible triples. We know that it is **NP**-complete. In the 4-DIMENSIONAL MATCHING problem you are given compatible *quadruples* of n boys, n girls, n pets, and n homes, and again you want to create n harmonious households accommodating them all. Prove 4-DIMENSIONAL MATCHING is **NP**-complete.

2. Strongly Connected Subgraph

You are given a strongly connected directed graph $G = (V, E)$ and a budget B , and you are asked to find a subgraph (V, E') of G with $E' \subseteq E$ such that (1) (V, E') is strongly connected, and (2) $|E'| \leq B$. Prove that this problem is **NP**-complete.

3. $\frac{1}{3}$ Independent Set

In the $\frac{1}{3}$ -INDEPENDENT SET problem you are given a graph (V, E) and you are asked to find an independent set of the graph of size exactly $\frac{|V|}{3}$. In other words, the target size g is not part of the input, but it is always $\frac{|V|}{3}$. Prove this special case of INDEPENDENT SET is **NP**-complete.

4. Non-Partisan Traveling Senator Problem

The NON-PARTISAN TRAVELING SENATOR PROBLEM (NPTSP) is a variant of TSP. Each vertex now has one of two colorings (red, blue), and a path cannot include more than three consecutive vertices with the same color. Formulate a reduction from TSP to NPTSP.