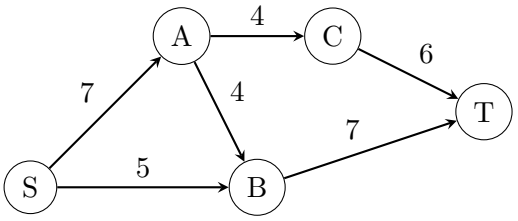
CS170 Discussion Section 9: *3/22*

Network Flows

Consider the following graph with edge capacities as shown:



1. Draw the residual graph after pushing 4 units of flow through $S \to A \to B \to T$.

2. Starting from that residual graph, push four units of flow through $S \to B \to A \to C \to T$, and draw the new residual graph.

3. Compute a maximum flow of the above graph. Draw the residual graph of the maximum flow. Find a minimum cut.

Verifying a max-flow

Suppose someone presents you with a solution to a max-flow problem on some network. Give a *linear* time algorithm to determine whether the solution does indeed give a maximum flow.

Repairing a Flow

In a particular network $G = (V, E)$ whose edges have integer capacities c_e, we have already found the maximum flow f from node s to node t. However, we now find out that one of the capacity values we used was wrong: for edge (u, v) we used c_{uv} whereas it should have been $c_{uv} - 1$. This is unfortunate because the flow f uses that particular edge at full capacity: $f_{uv} = c_{uv}$. We could redo the flow computation from scratch, but there's a faster way. Show how a new optimal flow can be computed in $O(|V| + |E|)$ time.

Secret Santa

Imagine you are throwing a party and you want to play Secret Santa. Thus you would like to assign to every person at the party another partier to whom they must anonymously give a gift. However, there are some restrictions on who can give gifts to who. For instance, nobody should be assigned to give a gift to themselves or to their spouse. Since you are the host, you know all of these restrictions. Give an efficient algorithm that determines if you can play Secret Santa.