Today - Special Topic: Cryptography

- Commitments
- Zero-Knowledge Proofs

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- Best Algorithm: $e^{(3^{2/3} - o(1))(\log p)^{\frac{1}{3}} (\log\log p)^{\frac{2}{3}}}$

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- Using large enough primes primes the discrete log problem is believed to be hard!

Commitment Schemes

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  - **Binding**: C can not find $(0, s_0)$ and $(1, s_1)$ such that R outputs 1 on both.

Commitment Protocol

$Commiter(b; s_C)$

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 & & \textit{Store } Y
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Output 1 if $g^b h^{s_C} \stackrel{?}{=} Y$

Else output 0

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- Y contains no information about b.
- If $g^b h^s = Y$ then $g^{1-b} h^{s'} = Y$ where $s' = \frac{2b-1}{x} + s \mod p-1$.[1]

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How would you prove that a NP problem is true?

- A *NP problem* I is true if there exists a solution S such that $\mathscr{C}(I, S) = \mathit{true}$, where $\mathscr{C}$ is the checking algorithms.

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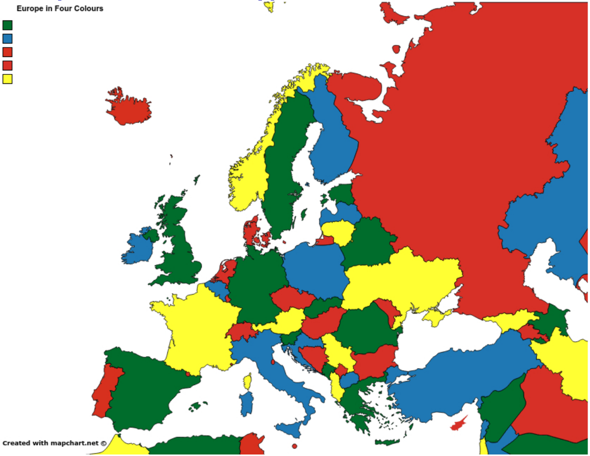
- A *NP problem* I is true if there exists a solution S such that $\mathscr{C}(I, S) = \mathit{true}$, where $\mathscr{C}$ is the checking algorithms.
- You can send the solution S to your friend.
- However, this leaks the solution to your friend.

Graph 3-coloring



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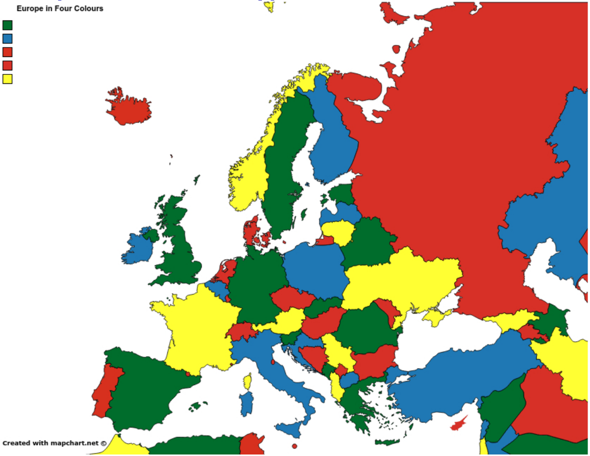

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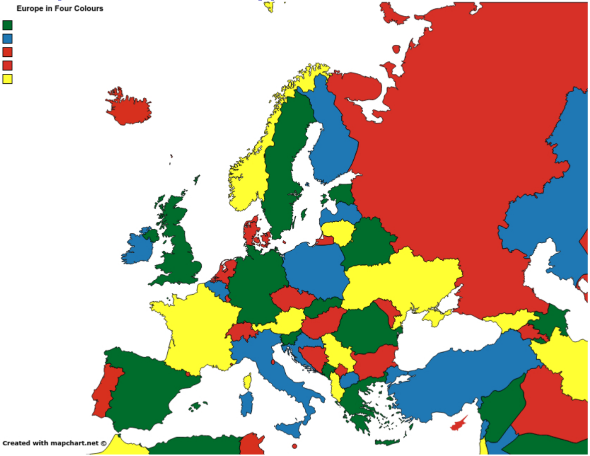

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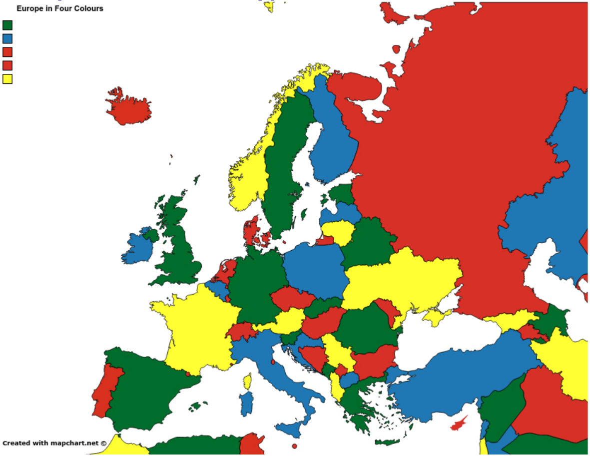

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- This problem is NP-complete.

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  - Zero-Knowledge: No cheating $\mathscr{V}^*$ learns anything about P's coloring function c.

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$$\xrightarrow{\text{open } c_u, c_v}$$

Zero-Knowledge Protocol

$\mathscr{P}(G, c; r)$

π be a random function

$\{R, B, G\} \to \{R, B, G\}$

$\mathscr{V}(G, s)$

$$\xrightarrow{\forall v \in V, c_v = com(\pi(c(v)))}$$

$$e \xleftarrow{\$} E$$

$$\xleftarrow{e = (u, v)}$$

$$\xrightarrow{\text{open } c_u, c_v}$$

Output 1
if diff
Else 0

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- Must use fresh randomness (namely π) in each.

Zero-Knowledge

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- What does a cheating verifier $\mathscr{V}^*$ learn in one execution?
- Nothing! :)

- CS194 on Cryptography: Next Semester