Horn SAT

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$$\implies A,\ A \implies B,\ \overline{B}.$$

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Or of negative literals.

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Taint Example:

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Is this satisfiable?

Horn Sat: another view.

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x_1 \wedge x_2 &\implies x_4 \\
x_3 &\implies x_2 \\
x_1 &\implies x_3 \\
x_5 \wedge x_1 &\implies x_3 \\
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\end{aligned}$$

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Problem: Find consistent assignment with fewest “True” variables.

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Example:
x_1 must be true

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Problem: Find consistent assignment with fewest "True" variables.

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Example:
x_1 must be true so x_3 must be true

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Problem: Find consistent assignment with fewest "True" variables.

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Example:
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Example:
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Solution: $\{x_1, x_2, x_3, x_4\}$ are True

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Same as horn sat!

Why same as HornSAT?

Horn SAT had negative clauses.

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Horn has negative clauses.

Negative clauses only problem for true variables.

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So if a negative clause is false, it must be.

Efficient implementation

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For each clause: keep count of true antecedents:

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When all antecedents true, than make consequent true.

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Connect variable to clauses with var as antecendent.

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For each clause: keep count of true antecedents:

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Data Structure:

Connect variable to clauses with var as antecendent.

When variable is set to true see if connected clauses are invoked.

Any SAT formula?

$$x_1 \implies x_2 \vee x_3.$$

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More about this...

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More about this... later in the course.

Set Cover.

Input:

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Items: City Blocks.
Sets: Possible cellphone tower location.

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Each cell phone tower location covers some subset of blocks.

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Items: Customers.

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Sets: Walmart locations covers subset of customers.

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Greedy Algorithm

Choose set S_i that has largest number of elements.

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Remove elements in S_i from all sets.

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Repeat.

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Choose set S_i that has largest number of elements.

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- Repeat.

Number of sets is number of iterations.

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Property: Set cover of size k

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Property: Set cover of size k (best solution)

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Number of sets is number of iterations.
How many iterations?

Property: Set cover of size k (best solution)
$\implies$ there exists a set that contains $\frac{1}{k}$ of remaining elements.

Greedy Algorithm

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Property: Set cover of size k (best solution)
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Analysis:
n_t elements remain at time t (after using t sets.)

Greedy Algorithm

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Property: Set cover of size k (best solution)
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Analysis:
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In iteration t, cover $\frac{1}{k}n_t$ remaining elements.

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When do we stop?

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Recall: $n_t \leq (1 - \frac{1}{k})^t n_0$

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(A) $t = \log n$

(B) $t = k$

(C) $t = k \ln n$.

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Plug in $t = k \ln n + 1$

Bound iterations.

When do we stop?

When $n_t < 1$?

Recall: $n_t \leq (1 - \frac{1}{k})^t n_0$

For what t must $n_t < 1$?

(A) $t = \log n$

(B) $t = k$

(C) $t = k \ln n$.

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Plug in $t = k \ln n + 1$ and clearly $n_t < 1$.

Bound iterations.

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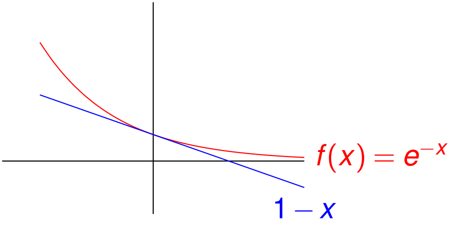

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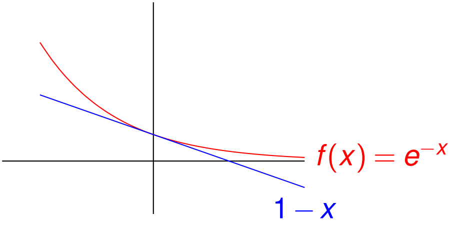

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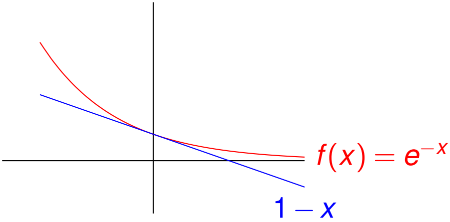

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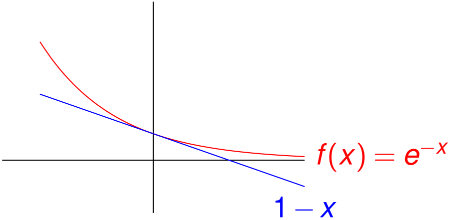

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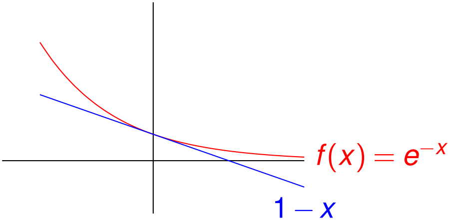

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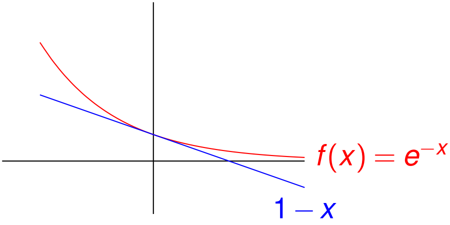

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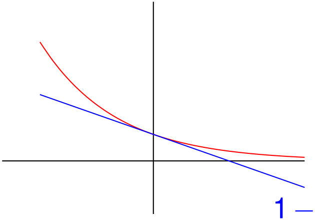

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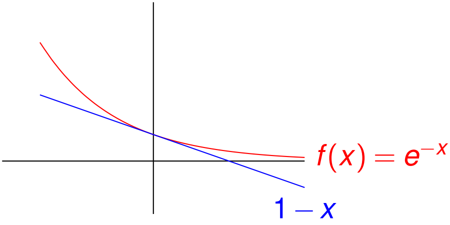

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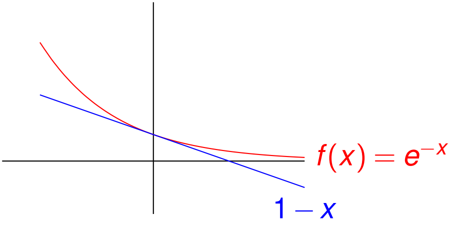

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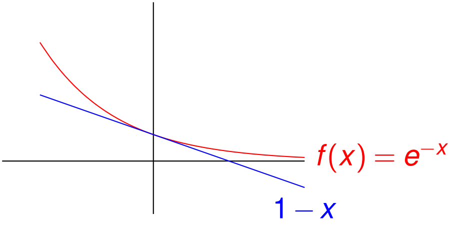

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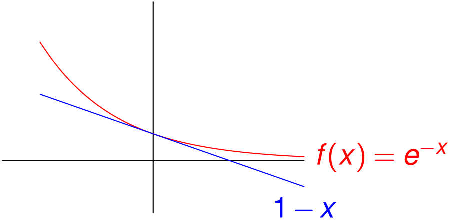

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Within $\ln n$ factor (almost) of the best possible!

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We did not find optimal solution!

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More later in the course.

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What to do?

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$L(i)$ is length of longest increasing subsequence ending at position i.

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$$L(j) = \max_{j<i \,\wedge\, a[j]<a[i]} \{L(j) + 1\}$$

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"DAG of problems to solve."

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Recursive instead of iterative?

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Same as "iterative".