CS170 Discussion Section 11 : 4/12

Definitions

1. **P**: the set of all problems that can be solved (not just verified) in polynomial time.
2. **NP**: the set of all problems that can be verified in polynomial time.
3. **NP**-*complete*: a problem to which all other problems in NP reduce. In other words, if we could efficiently solve any **NP**-complete problem, we could efficiently solve any problem in NP.

Reduction Basics

Assume A reduces to B in polynomial time. In each part you will be given a fact about one of the problems. Determine what, if anything, this allows you to determine about the other problem. *(You can answer each part in one sentence.)*

1. A is in **P**.
2. B is in **P**.
3. A is **NP**-hard.
4. B is **NP**-hard.

Solution: If A reduces to B, we know B can be used to solve A, which means B is at least as hard as A. As a result, in case 2 we can say that A is in **P**, and in case 3 we can say that B is **NP**-hard. In cases 1 and 4, we cannot conclude anything interesting.

Stingy SAT

Stingy SAT is the following problem: given a set of clauses (each a disjunction of literals), and an integer k, find a satisfying assignment in which at most k variables are True, if such an assignment exists. Prove that Stingy SAT is NP-Complete.

Solution: It's a generalization of SAT; any instance of SAT on n variables is also an instance of Stingy SAT with $k = n$.

More Reductions

1. Give a reduction from Vertex Cover to Set Cover. In Vertex Cover your input is a graph G and a budget b, and you want to find b vertices that touch every edge. In Set Cover, we are given a set E and several subsets of it $S_1, ..., S_m$, along with a budget b. We are asked to select b of these subsets so that their union is E.

   Solution: Vertex cover is the special case in which E consists of the edges of the graph, and there is a subset S_i for each vertex, containing the edges adjacent to that vertex. The budget remains the same.

2. Give a reduction from 3D Matching to Set Cover.

   Solution: 3D Matching is a special case of Set Cover; E consists of n boys, n girls and n pets, and there is a subset for each valid triple (b, g, p). There is a valid 3D Matching iff the corresponding Set Cover has a budget of n.

Reliable Network

Reliable Network is the following problem: We are given two $n \times n$ matrices, a distance matrix d_{ij}, and a connectivity requirement matrix r_{ij}, as well as a budget b; we must find a graph $G = (\{1, ..., n\}, E)$ such that the total cost of all edges is b or less and between any two distinct vertices i and j there are r_{ij} vertex-disjoint paths. Show that Reliable Network is NP-Complete.

Solution: Reduction from Rudrata Cycle to Reliable Network. Given $G = (V, E)$, take $b = n$, $d_{ij} = 1\ \forall(i, j) \in E$ and $d_{ij} > 1$ otherwise; set $r_{ij} = 2\ \forall(i, j)$. Now we must find edges such that the sum of the weights of all the edges is n (so only n edges), and they are part of exactly one cycle (so there are two vertex-disjoint paths between each pair of vertices). This must contain all of the vertices since each pair satisfies this. This is exactly a Rudrata Cycle.