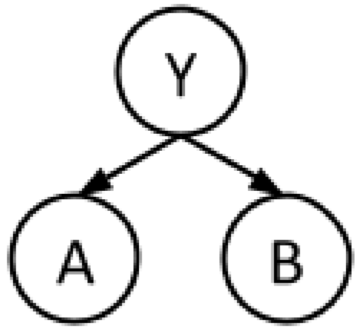
CS188 Fall 2017 Section 10: Machine Learning

1 Naive Bayes

In this question, we will train a Naive Bayes classifier to predict class labels Y as a function of input features A and B. Y, A, and B are all binary variables, with domains 0 and 1. We are given 10 training points from which we will estimate our distribution.

A	1	1	1	1	0	1	0	1	1	1
B	1	0	0	1	1	1	1	0	1	1
Y	1	1	0	0	0	1	1	0	0	0



1. What are the maximum likelihood estimates for the tables $P(Y)$, $P(A|Y)$, and $P(B|Y)$?

Y	$P(Y)$
0	3/5
1	2/5

A	Y	$P(A\|Y)$
0	0	1/6
1	0	5/6
0	1	1/4
1	1	3/4

B	Y	$P(B\|Y)$
0	0	1/3
1	0	2/3
0	1	1/4
1	1	3/4

2. Consider a new data point ($A = 1$, $B = 1$). What label would this classifier assign to this sample?

$$
\begin{aligned}
P(Y=0, A=1, B=1) &= P(Y=0)P(A=1|Y=0)P(B=1|Y=0) && (1)\\
&= (3/5)(5/6)(2/3) && (2)\\
&= 1/3 && (3)\\
P(Y=1, A=1, B=1) &= P(Y=1)P(A=1|Y=1)P(B=1|Y=1) && (4)\\
&= (2/5)(3/4)(3/4) && (5)\\
&= 9/40 && (6)\\
& && (7)
\end{aligned}
$$

Our classifier will predict label 0.

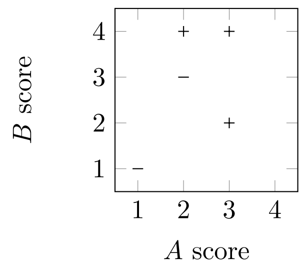
3. Let's use Laplace Smoothing to smooth out our distribution. Compute the new distribution for $P(A|Y)$ given Laplace Smoothing with $k = 2$.

A	Y	$P(A\|Y)$
0	0	3/10
1	0	7/10
0	1	3/8
1	1	5/8

2 Perceptron

You want to predict if movies will be profitable based on their screenplays. You hire two critics A and B to read a script you have and rate it on a scale of 1 to 4. The critics are not perfect; here are five data points including the critics' scores and the performance of the movie:

#	Movie Name	A	B	Profit?
1	Pellet Power	1	1	-
2	Ghosts!	3	2	+
3	Pac is Bac	2	4	+
4	Not a Pizza	3	4	+
5	Endless Maze	2	3	-



1. First, you would like to examine the linear separability of the data. Plot the data on the 2D plane above; label profitable movies with + and non-profitable movies with − and determine if the data are linearly separable. The data are linearly separable.

2. Now you decide to use a perceptron to classify your data. Suppose you directly use the scores given above as features, together with a bias feature. That is $f_0 = 1$, $f_1 =$ score given by A and $f_2 =$ score given by B.

   Run one pass through the data with the perceptron algorithm, filling out the table below. Go through the data points in order, e.g. using data point #1 at step 1.

step	Weights	Score	Correct?
1	[-1, 0, 0]	$-1 \cdot 1 + 0 \cdot 1 + 0 \cdot 1 = -1$	yes
2	[-1, 0, 0]	$-1 \cdot 1 + 0 \cdot 3 + 0 \cdot 2 = -1$	no
3	[0, 3, 2]	$0 \cdot 1 + 3 \cdot 2 + 2 \cdot 4 = 14$	yes
4	[0, 3, 2]	$0 \cdot 1 + 3 \cdot 3 + 2 \cdot 4 = 17$	yes
5	[0, 3, 2]	$0 \cdot 1 + 3 \cdot 2 + 2 \cdot 3 = 12$	no

   Final weights: [-1, 1, -1]

3. Have weights been learned that separate the data? With the current weights, points will be classified as positive if $-1 \cdot 1 + 1 \cdot A + -1 \cdot B \geq 0$, or $A - B \geq 1$. So we will have incorrect predictions for data points 3:

$$-1 \cdot 1 + 1 \cdot 2 + -1 \cdot 4 = -3 < 0$$

   and 4:

$$-1 \cdot 1 + 1 \cdot 3 + -1 \cdot 4 = -2 < 0$$

   Note that although point 2 has $w \cdot f = 0$, it will be classified as positive (since we classify as positive if $w \cdot f \geq 0$).

4. More generally, irrespective of the training data, you want to know if your features are powerful enough to allow you to handle a range of scenarios. Circle the scenarios for which a perceptron using the features above can indeed perfectly classify movies which are profitable according to the given rules:

   (a) Your reviewers are awesome: if the total of their scores is more than 8, then the movie will definitely be profitable, and otherwise it won't be. Can classify (consider weights $[-8, 1, 1]$)

   (b) Your reviewers are art critics. Your movie will be profitable if and only if each reviewer gives either a score of 2 or a score of 3. Cannot classify

   (c) Your reviewers have weird but different tastes. Your movie will be profitable if and only if both reviewers agree. Cannot classify

3 Maximum Likelihood

A Geometric distribution is a probability distribution of the number X of Bernoulli trials needed to get one success. It depends on a parameter p, which is the probability of success for each individual Bernoulli trial. Think of it as the number of times you must flip a coin before flipping heads. The probability is given as follows:

$$P(X = k) = p(1 - p)^{k-1} \tag{8}$$

p is the parameter we wish to estimate.

We observe the following samples from a Geometric distribution: $x_1 = 5$, $x_2 = 8$, $x_3 = 3$, $x_4 = 5$, $x_5 = 7$. What is the maximum likelihood estimate for p?

$$\begin{align}
L(p) &= P(X = x_1)P(X = x_2)P(X = x_3)P(X = x_4)P(X = x_5) \tag{9}\\
&= P(X = 5)P(X = 8)P(X = 3)P(X = 5)P(X = 7) \tag{10}\\
&= p^5(1 - p)^{23} \tag{11}\\
\log(L(p)) &= 5\log(p) + 23\log(1 - p) \tag{12}\\
& \tag{13}
\end{align}$$

We must maximize the log-likelihood of p, so we will take the derivative, and set it to 0.

$$\begin{align}
0 &= \frac{5}{p} - \frac{23}{1 - p} \tag{14}\\
p &= 5/28 \tag{15}
\end{align}$$