CS188: Exam Practice Session 7

Q1. Probabilities

(a) Fill in the circles of **all** expressions that are equal to **1**,
given no independence assumptions:

- ○ $\sum_a P(A = a \mid B)$
- ○ $\sum_b P(A \mid B = b)$
- ○ $\sum_a \sum_b P(A = a, B = b)$
- ○ $\sum_a \sum_b P(A = a \mid B = b)$
- ○ $\sum_a \sum_b P(A = a)\ P(B = b)$
- ○ $\sum_a P(A = a)\ P(B = b)$
- ○ None of the above.

(b) Fill in the circles of **all** expressions that are equal to $\mathbf{P(A, B, C)}$,
given no independence assumptions:

- ○ $P(A \mid B, C)\ P(B \mid C)\ P(C)$
- ○ $P(C \mid A, B)\ P(A)\ P(B)$
- ○ $P(A, B \mid C)\ P(C)$
- ○ $P(C \mid A, B)\ P(A, B)$
- ○ $P(A \mid B)\ P(B \mid C)\ P(C)$
- ○ $P(A \mid B, C)\ P(B \mid A, C)\ P(C \mid A, B)$
- ○ None of the above.

(c) Fill in the circles of **all** expressions that are equal to $\mathbf{P(A \mid B, C)}$,
given no independence assumptions:

- ○ $\frac{P(A,B,C)}{\sum_a P(A=a,B,C)}$
- ○ $\frac{P(B,C|A)\ P(A)}{P(B,C)}$
- ○ $\frac{P(B|A,C)\ P(A|C)}{P(B|C)}$
- ○ $\frac{P(B|A,C)\ P(A|C)}{P(B,C)}$
- ○ $\frac{P(B|A,C)\ P(C|A,B)}{P(B,C)}$
- ○ $\frac{P(A,B|C)}{P(B|A,C)}$
- ○ None of the above.

(d) Fill in the circles of **all** expressions that are equal to $\mathbf{P(A \mid B)}$,
given that $\mathbf{A \perp\!\!\!\perp B \mid C}$:

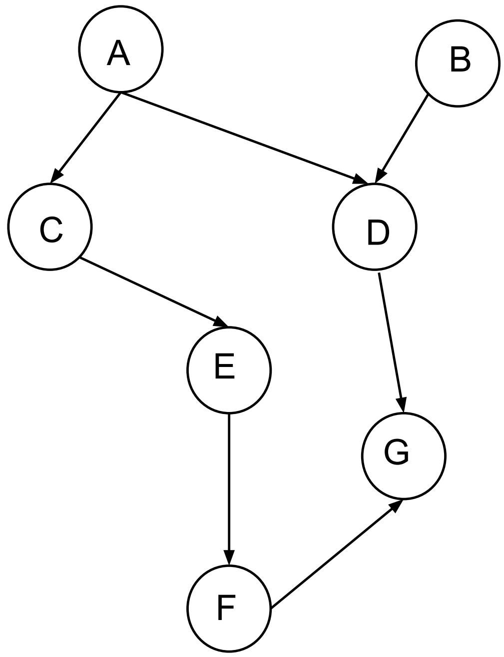
- ○ $\frac{P(A|C)\ P(B|C)}{P(B)}$
- ○ $\frac{P(A|C)\ P(B|C)}{P(B|C)}$
- ○ $\frac{\sum_c P(A|C=c)\ P(B|C=c)\ P(C=c)}{\sum_{c'} P(B|C=c')\ P(C=c')}$
- ○ $\frac{P(A|B,C)}{P(A|C)}$
- ○ $\frac{\sum_c P(B|A,C=c)\ P(A,C=c)}{P(B)}$
- ○ $\frac{\sum_c P(A,C=c)\ P(B|C=c)}{\sum_{c'} P(A,B,C=c')}$
- ○ None of the above.

(e) Fill in the circles of **all** expressions that are equal to $\mathbf{P(A, B, C)}$,
given that $\mathbf{A \perp\!\!\!\perp B \mid C}$ and $\mathbf{A \perp\!\!\!\perp C}$:

- ○ $P(A)\ P(B)\ P(C)$
- ○ $P(A)\ P(B, C)$
- ○ $P(A \mid B)\ P(B \mid C)\ P(C)$
- ○ $P(A \mid B, C)\ P(B \mid A, C)\ P(C \mid A, B)$
- ○ $P(A \mid C)\ P(B \mid C)\ P(C)$
- ○ $P(A \mid C)\ P(B \mid C)$
- ○ None of the above.

Q2. Bayes Nets: Independence

Consider a Bayes Net with the following graph:



Which of the following are guaranteed to be true without making any additional conditional independence assumptions, other than those implied by the graph? (Mark all true statements)

○ $P(A \mid C, E) = P(A \mid C)$

○ $P(A, E \mid G) = P(A \mid G) * P(E \mid G)$

○ $P(A \mid B = b) = P(A)$

○ $P(A \mid B, G) = P(A \mid G)$

○ $P(E, G \mid D) = P(E \mid D) * P(G \mid D)$

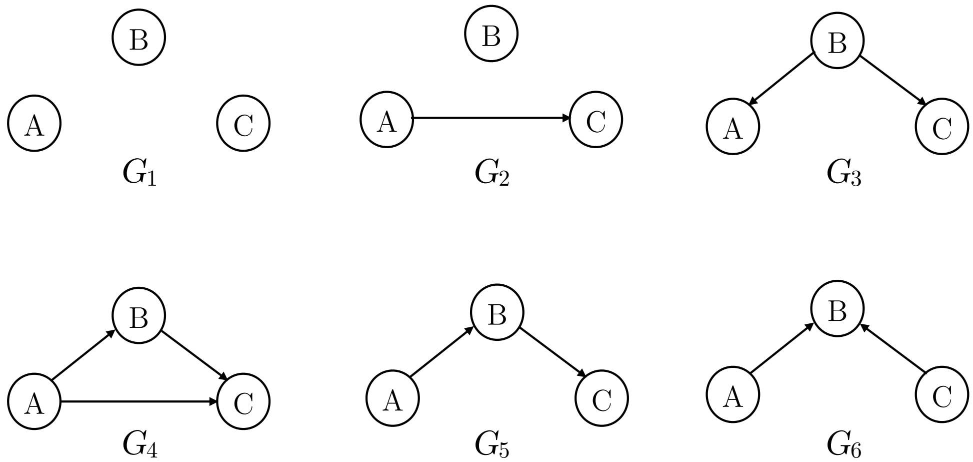
○ $P(A, B \mid F) = P(A \mid F) * P(B \mid F)$

Q3. Bayes Nets: Representation

(a) Graph structure: Representational Power

Recall that any directed acyclic graph G has an associated family of probability distributions, which consists of all probability distributions that can be represented by a Bayes' net with structure G.

For the following questions, consider the following six directed acyclic graphs:



(i) Assume all we know about the joint distribution $P(A, B, C)$ is that it can be represented by the product $P(A|B, C)P(B|C)P(C)$. Mark each graph for which the associated family of probability distributions is guaranteed to include $P(A, B, C)$.

○ G_1 ○ G_2 ○ G_3

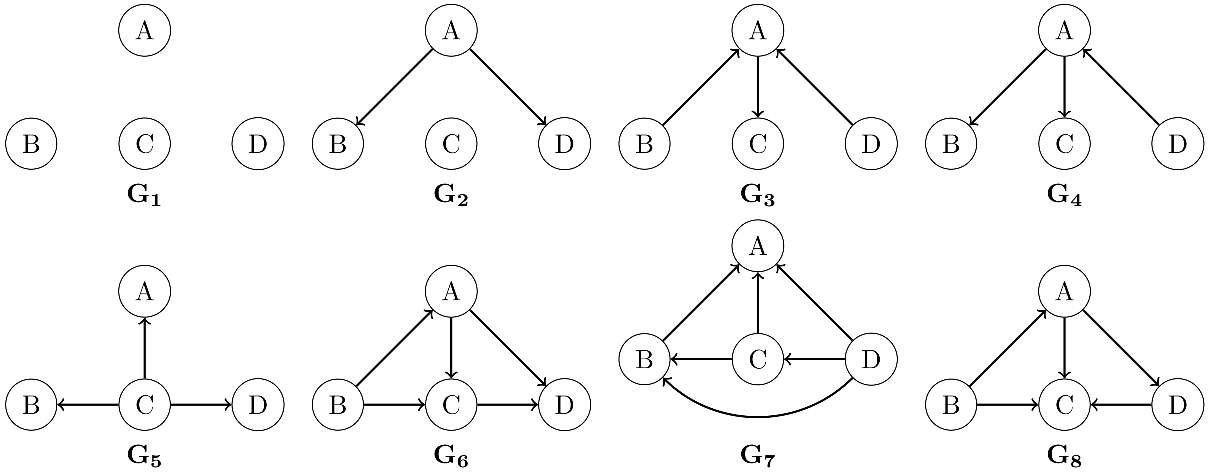
○ G_4 ○ G_5 ○ G_6

(ii) Now assume all we know about the joint distribution $P(A, B, C)$ is that it can be represented by the product $P(C|B)P(B|A)P(A)$. Mark each graph for which the associated family of probability distributions is guaranteed to include $P(A, B, C)$.

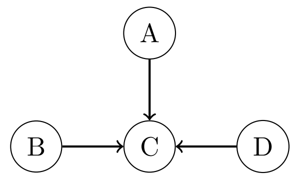
○ G_1 ○ G_2 ○ G_3

○ G_4 ○ G_5 ○ G_6

(b) For the following questions, consider the following eight directed acyclic graphs:



(i) Consider the Bayes' Net $\mathbf{B_1}$ below, and fill in **all the circles** (or select *None of the above*) corresponding to the Bayes' Nets $\mathbf{G_1}$ through $\mathbf{G_8}$ that are able to represent **at least one distribution** that $\mathbf{B_1}$ is able to represent.



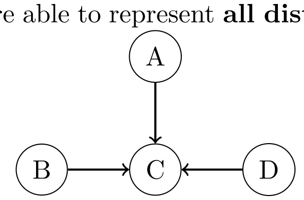
$\mathbf{B_1}$

○ $\mathbf{G_1}$ ○ $\mathbf{G_2}$ ○ $\mathbf{G_3}$ ○ $\mathbf{G_4}$

○ $\mathbf{G_5}$ ○ $\mathbf{G_6}$ ○ $\mathbf{G_7}$ ○ $\mathbf{G_8}$

○ None of the above.

(ii) Consider the Bayes' Net $\mathbf{B_1}$ below, and fill in **all the circles** (or select *None of the above*) corresponding to the Bayes' Nets $\mathbf{G_1}$ through $\mathbf{G_8}$ that are able to represent **all distributions** that $\mathbf{B_1}$ is able to represent.



$\mathbf{B_1}$

○ $\mathbf{G_1}$ ○ $\mathbf{G_2}$ ○ $\mathbf{G_3}$ ○ $\mathbf{G_4}$

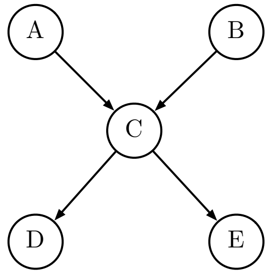
○ $\mathbf{G_5}$ ○ $\mathbf{G_6}$ ○ $\mathbf{G_7}$ ○ $\mathbf{G_8}$

○ None of the above.

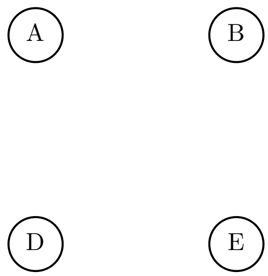
(c) Marginalization and Conditioning

Consider a Bayes' net over the random variables A, B, C, D, E with the structure shown below, with full joint distribution $P(A, B, C, D, E)$.

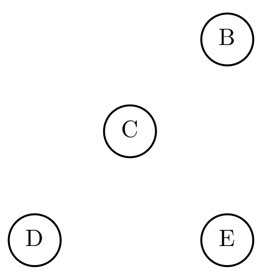
The following three questions describe different, unrelated situations (your answers to one question should not influence your answer to other questions).



(i) Consider the marginal distribution $P(A, B, D, E) = \sum_c P(A, B, c, D, E)$, where C was eliminated. On the diagram below, draw the minimal number of arrows that results in a Bayes' net structure that is able to represent this marginal distribution. If no arrows are needed write "No arrows needed."



(ii) Assume we are given an observation: $A = a$. On the diagram below, draw the minimal number of arrows that results in a Bayes' net structure that is able to represent the conditional distribution $P(B, C, D, E \mid A = a)$. If no arrows are needed write "No arrows needed."



(iii) Assume we are given two observations: $D = d, E = e$. On the diagram below, draw the minimal number of arrows that results in a Bayes' net structure that is able to represent the conditional distribution $P(A, B, C \mid D = d, E = e)$. If no arrows are needed write "No arrows needed."