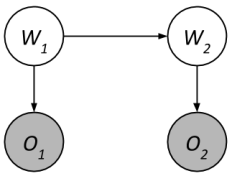
CS188 Fall 2017 Section 10: HMMs and Naive Bayes

1 HMMs

Consider the following Hidden Markov Model.



W_1	$P(W_1)$
0	0.3
1	0.7

W_t	W_{t+1}	$P(W_{t+1}\|W_t)$
0	0	0.4
0	1	0.6
1	0	0.8
1	1	0.2

W_t	O_t	$P(O_t\|W_t)$
0	A	0.9
0	B	0.1
1	A	0.5
1	B	0.5

Suppose that we observe $O_1 = A$ and $O_2 = B$.
Using the forward algorithm, compute the probability distribution $P(W_2|O_1 = A, O_2 = B)$ one step at a time.

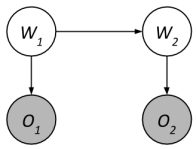
1. Compute $P(W_1, O_1 = A)$.

2. Using the previous calculation, compute $P(W_2, O_1 = A)$.

3. Using the previous calculation, compute $P(W_2, O_1 = A, O_2 = B)$.

4. Finally, compute $P(W_2|O_1 = A, O_2 = B)$.

Let's use Particle Filtering to estimate the distribution of $P(W_2|O_1 = A, O_2 = B)$. Here's the HMM again:



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We start with two particles representing our distribution for W_1.
$P_1 : W_1 = 0$
$P_2 : W_1 = 1$
Use the following random numbers to run particle filtering:

$$[0.22, 0.05, 0.33, 0.20, 0.84, 0.54, 0.79, 0.66, 0.14, 0.96]$$

1. **Observe**: Compute the weight of the two particles after evidence $O_1 = A$.

2. **Resample**: Using the random numbers, resample P_1 and P_2 based on the weights.

3. **Elapse Time**: Now let's compute the elapse time particle update. Sample P_1 and P_2 from applying the time update.

4. **Observe**: Compute the weight of the two particles after evidence $O_2 = B$.

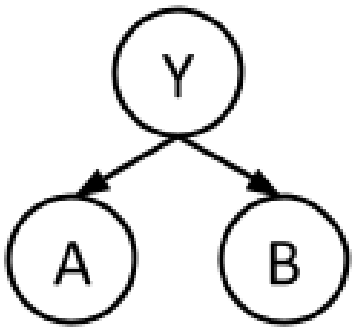
5. **Resample**: Using the random numbers, resample P_1 and P_2 based on the weights.

6. What is our estimated distribution for $P(W_2|O_1 = A, O_2 = B)$?

2 Naive Bayes

In this question, we will train a Naive Bayes classifier to predict class labels Y as a function of input features A and B. Y, A, and B are all binary variables, with domains 0 and 1. We are given 10 training points from which we will estimate our distribution.

A	1	1	1	1	0	1	0	1	1	1
B	1	0	0	1	1	1	1	0	1	1
Y	1	1	0	0	0	1	1	0	0	0



1. What are the maximum likelihood estimates for the tables $P(Y)$, $P(A|Y)$, and $P(B|Y)$?

Y	$P(Y)$
0	
1	

A	Y	$P(A\|Y)$
0	0	
1	0	
0	1	
1	1	

B	Y	$P(B\|Y)$
0	0	
1	0	
0	1	
1	1	

2. Consider a new data point ($A = 1$, $B = 1$). What label would this classifier assign to this sample?

3. Let's use Laplace Smoothing to smooth out our distribution. Compute the new distribution for $P(A|Y)$ given Laplace Smoothing with $k = 2$.

A	Y	$P(A\|Y)$
0	0	
1	0	
0	1	
1	1	