Announcements

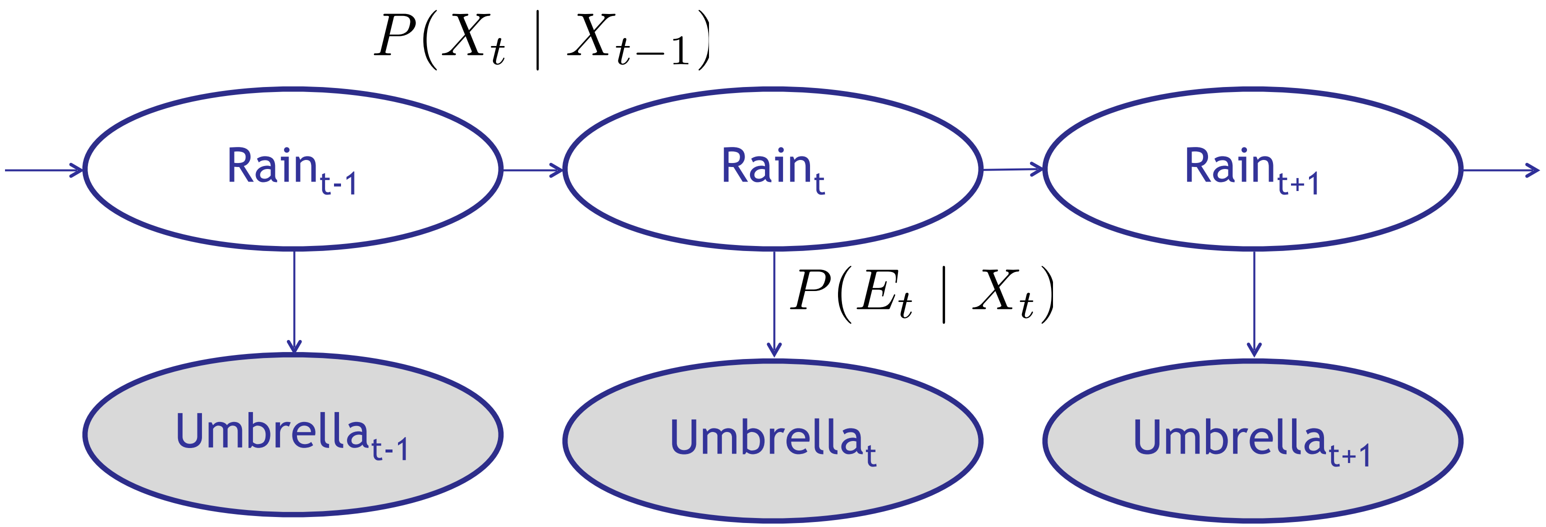
- Project 4: Due Friday Oct 28 5:00pm
- Homework 8: Due Monday Oct 31 11:59pm
- Midterm 2: Wednesday 11/9 7-9pm
  - Up to and including today's lecture.
- Upcoming Lectures – exact order TBD
  - Naïve Bayes
  - Perceptrons
  - Deep Learning
  - Advanced Topics

CS 188: Artificial Intelligence

Hidden Markov Models -- Filters

Instructors: Pieter Abbeel & Anca Dragan --- University of California, Berkeley

Example: Weather HMM

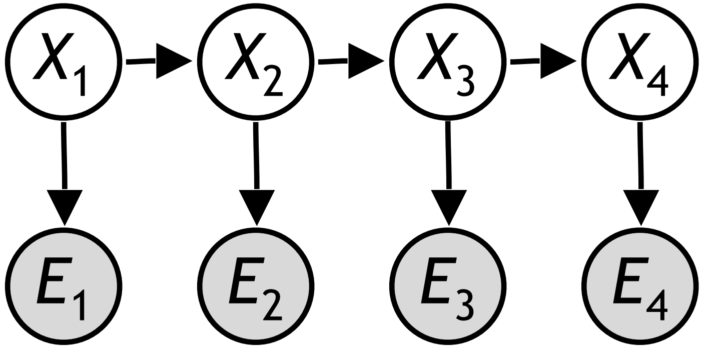


- An HMM is defined by:
  - Initial distribution: $P(X_1)$
  - Transitions: $P(X_t \mid X_{t-1})$
  - Emissions: $P(E_t \mid X_t)$

R_{t-1}	R_t	$P(R_t \mid R_{t-1})$
+r	+r	0.7
+r	-r	0.3
-r	+r	0.3
-r	-r	0.7

R_t	U_t	$P(U_t \mid R_t)$
+r	+u	0.9
+r	-u	0.1
-r	+u	0.2
-r	-u	0.8

Online Belief Update

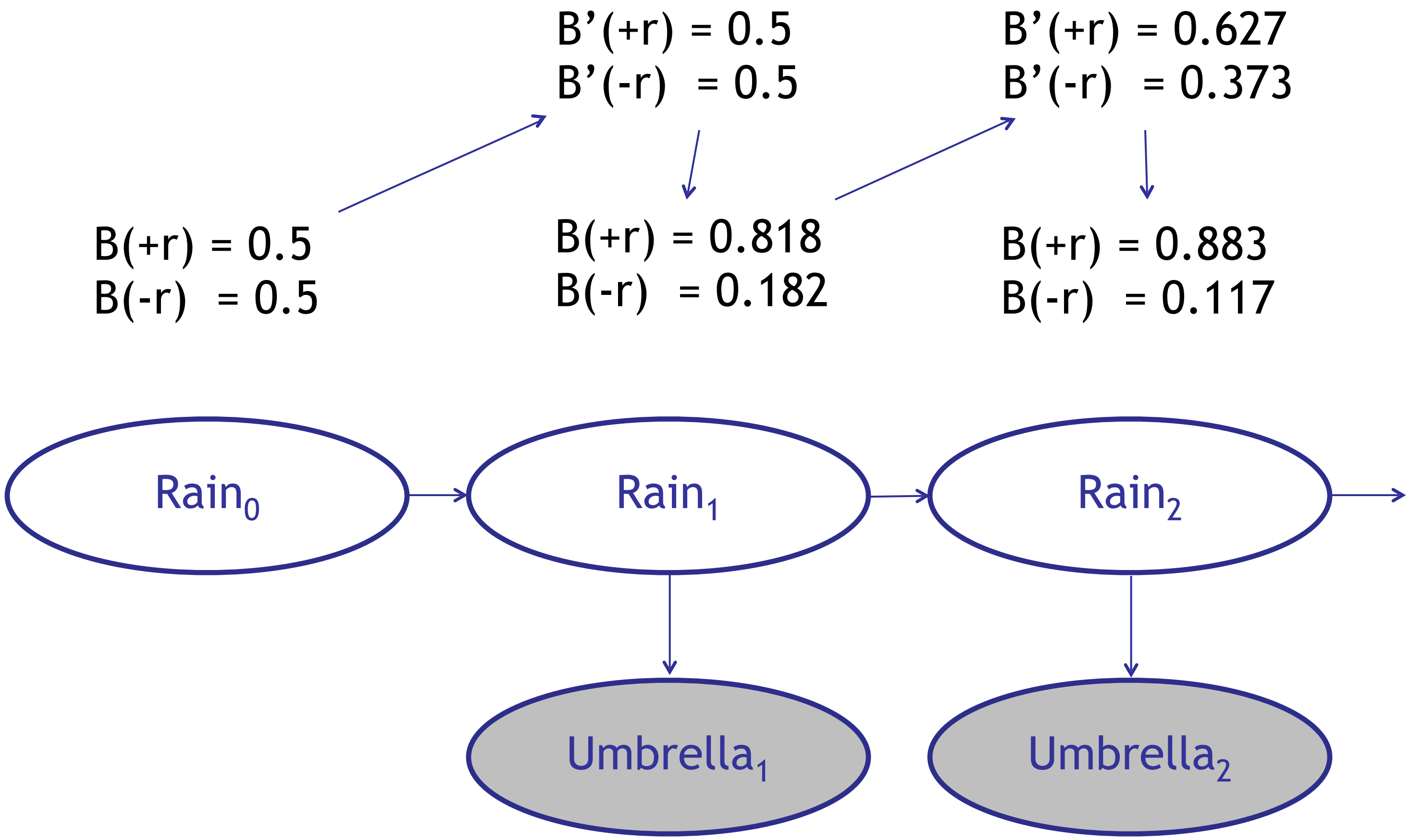


$$B_3(X) = P(X_3|e_{1:3})$$

$$B'_4(X) = P(X_4|e_{1:3}) = \sum_{x_3} P(X_4|x_3)B_3(X)$$

$$B_4(X) = P(X_4|e_{1:4}) \propto_{X_4} P(e_4|X_4)B'_4(X)$$

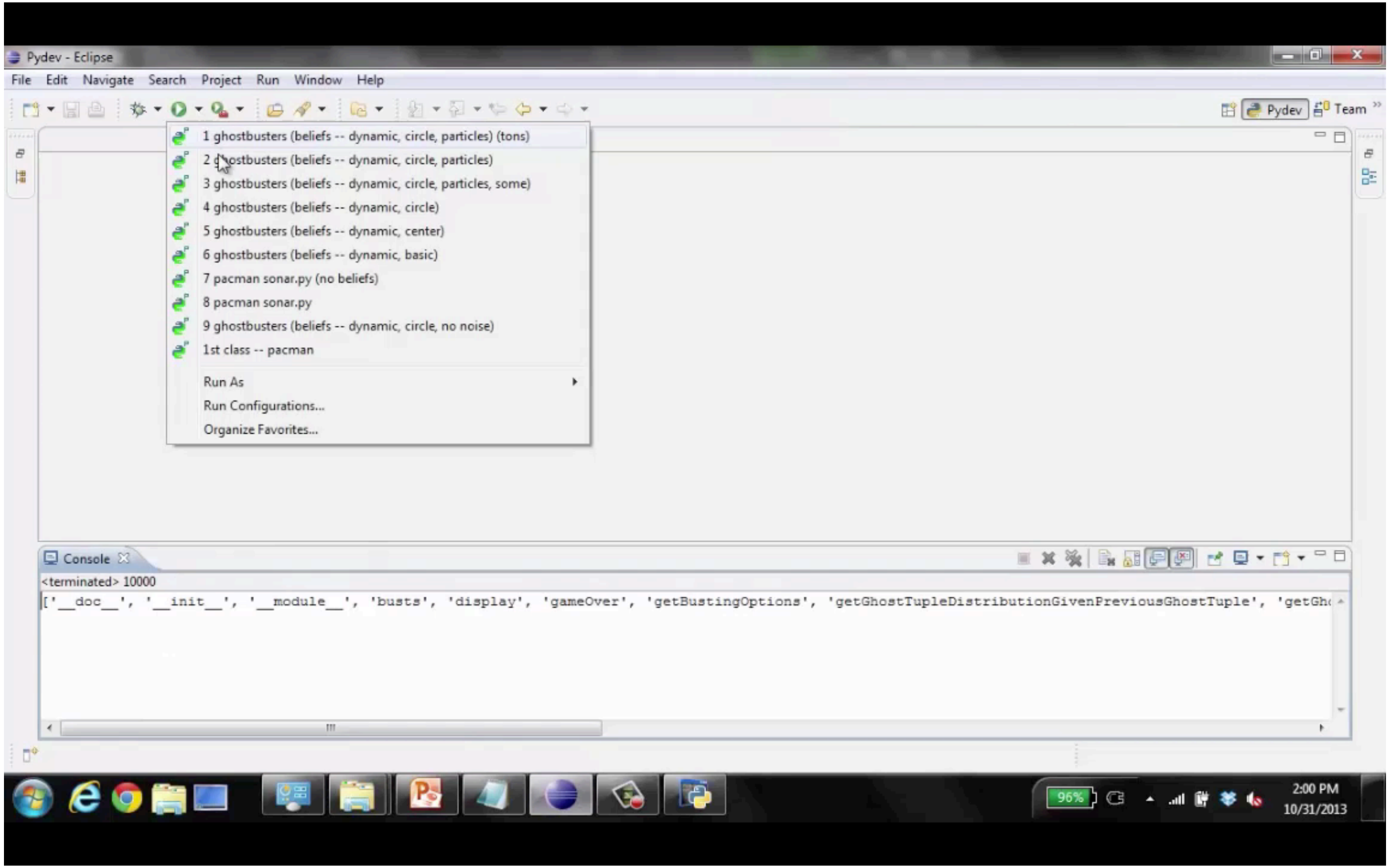
Example: Weather HMM



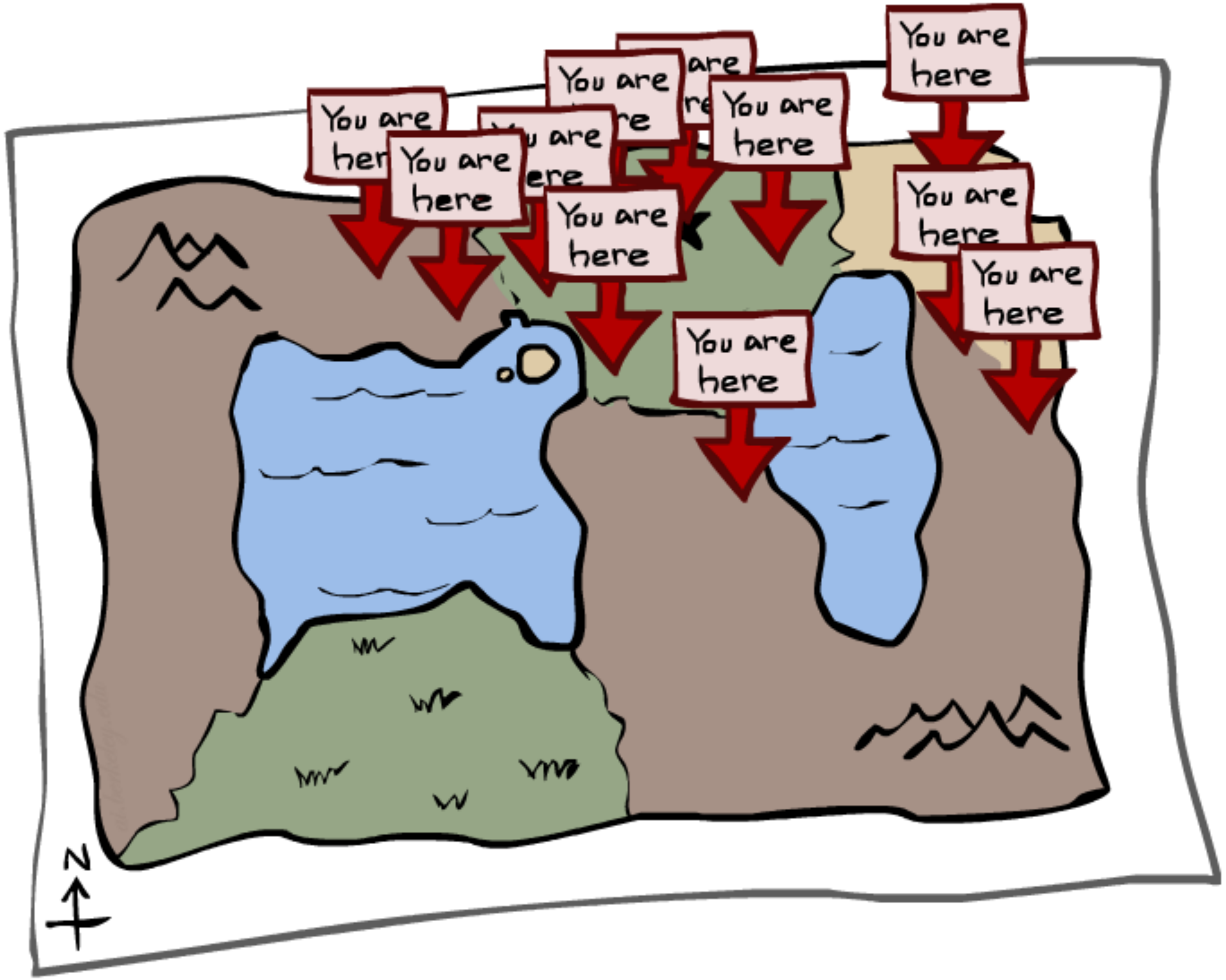
R_t	R_{t+1}	$P(R_{t+1} \mid$
+r	*+r*	*0.7*
+r	-r	0.3
-r	*+r*	*0.3*
-r	-r	0.7

R_t	U_t	$P(U_t \mid R_t)$
+r	*+u*	*0.9*
+r	-u	0.1
-r	*+u*	*0.2*
-r	-u	0.8

Video of Demo Pacman – Sonar (with beliefs)

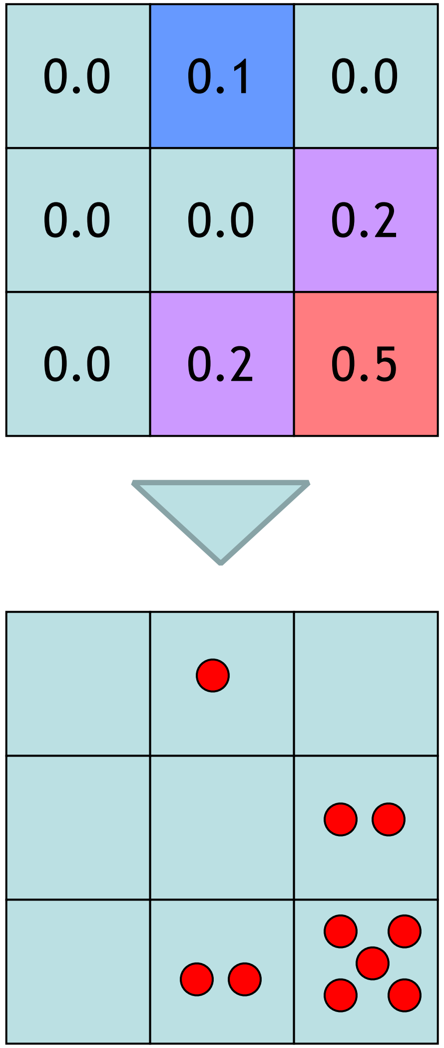


Particle Filtering

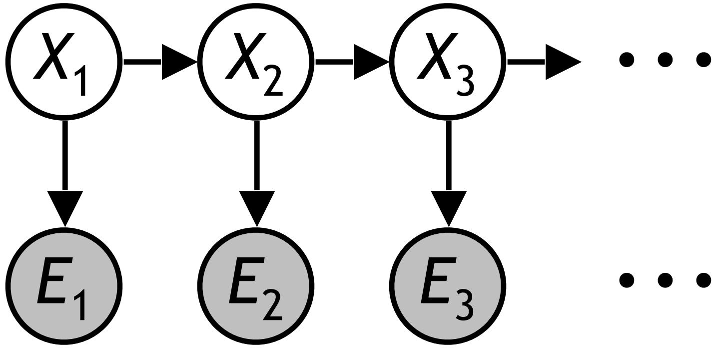


Particle Filtering

- Filtering: approximate solution
- Sometimes |X| is too big to use exact inference
  - |X| may be too big to even store B(X)
  - E.g. X is continuous
- Solution: approximate inference
  - Track samples of X, not all values
  - Samples are called particles
  - Time per step is linear in the number of samples
  - But: number needed may be large
  - In memory: list of particles, not states
- This is how robot localization works in practice
- Particle is just new name for sample



When is Exact Inference Too Big?



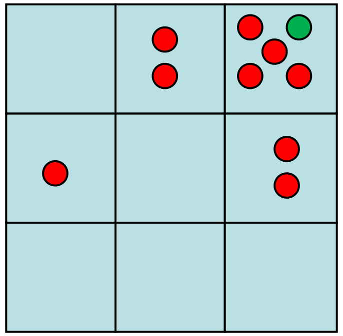
- For an HMM, when would you want to avoid *exact* inference?
  - The domain of X is big, but the domain of E is small?
  - The domain of E is big, but the domain of X is small?
  - Both domains are big?

Representation: Particles

- Our representation of P(X) is now a list of N particles (samples)
  - Generally, N << |X|
  - Storing map from X to counts would defeat the point

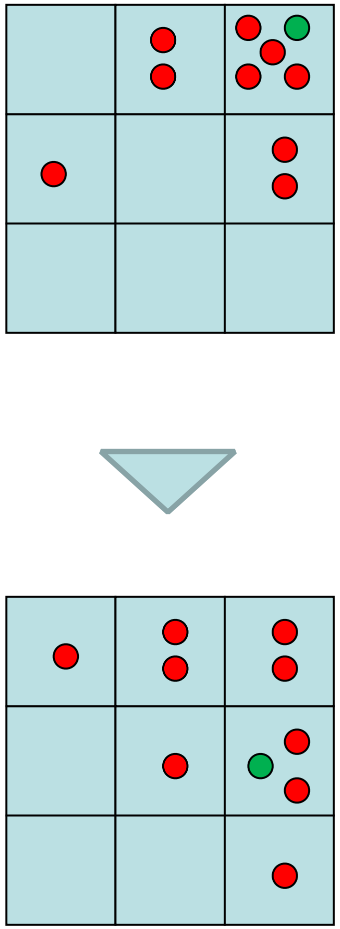
- P(x) approximated by number of particles with value x
  - So, many x may have P(x) = 0!
  - More particles, more accuracy

- For now, all particles have a weight of 1



Particles:
(3,3)
(2,3)
(3,3)
(3,2)
(3,3)
(3,2)
(1,2)
(3,3)
(3,3)
(2,3)

Particle Filtering: Elapse Time



Particle Filtering: Elapse Time

- Each particle is moved by sampling its next position from the transition model

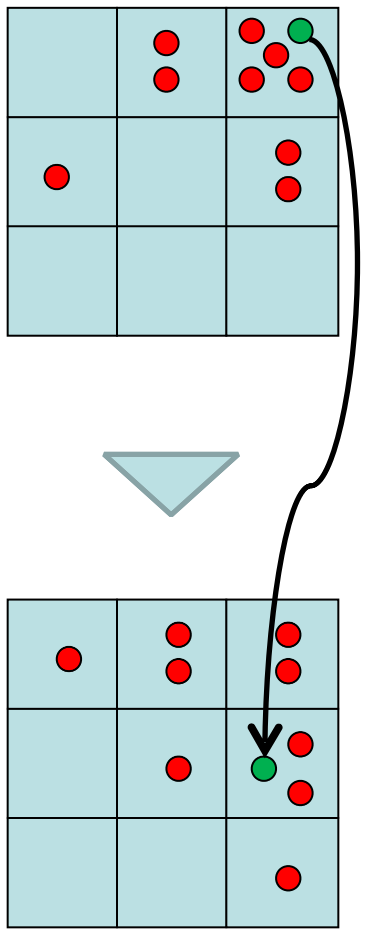
$$x' = \text{sample}(P(X'|x))$$

  - This is like prior sampling – samples' frequencies reflect the transition probabilities
  - Here, most samples move clockwise, but some move in another direction or stay in place

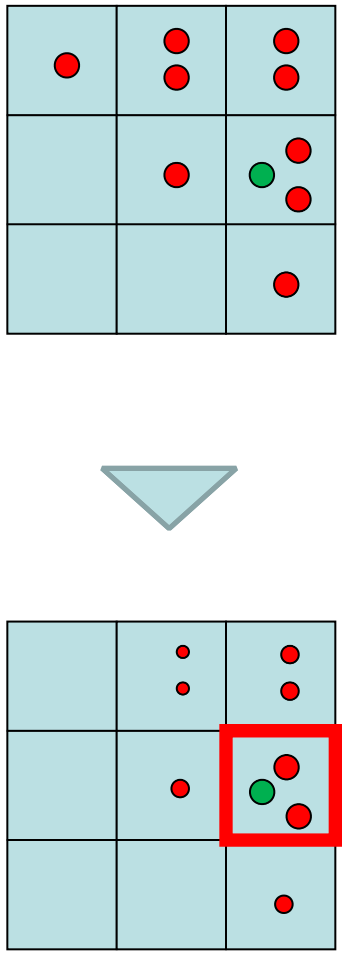
- This captures the passage of time
  - If enough samples, close to exact values before and after (consistent)

Particles:
(3,3)
(2,3)
(3,3)
(3,2)
(3,3)
(3,2)
(1,2)
(3,3)
(3,3)
(2,3)

Particles:
(3,2)
(2,3)
(3,2)
(3,1)
(3,3)
(3,2)
(1,3)
(2,3)
(3,2)
(2,2)



Particle Filtering: Observe



Particle Filtering: Observe

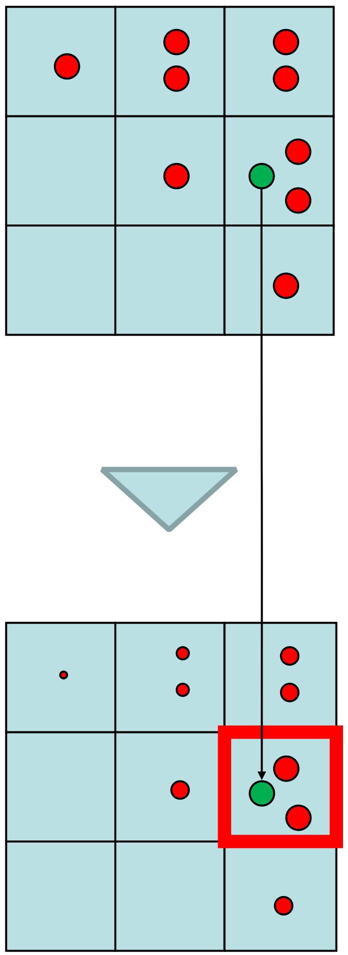
- Slightly trickier:
  - Don't sample observation, fix it
  - Similar to likelihood weighting, downweight samples based on the evidence

$$w(x) = P(e|x)$$

$$B(X) \propto P(e|X)B'(X)$$

  - As before, the probabilities don't sum to one, since all have been downweighted (in fact they now sum to (N times) an approximation of P(e))

Particles:
(3,2)
(2,3)
(3,2)
(3,1)
(3,3)
(3,2)
(1,3)
(2,3)
(3,2)
(2,2)



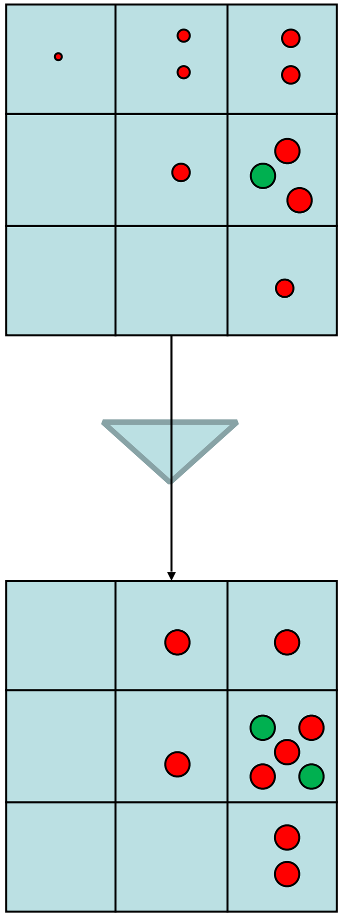
Particles:
(3,2) w=.9
(2,3) w=.2
(3,2) w=.9
(3,1) w=.4
(3,3) w=.4
(3,2) w=.9
(1,3) w=.1
(2,3) w=.2
(3,2) w=.9
(2,2) w=.4

Particle Filtering: Resample

- Rather than tracking weighted samples, we resample
- N times, we choose from our weighted sample distribution (i.e. draw with replacement)
- This is equivalent to renormalizing the distribution
- Now the update is complete for this time step, continue with the next one

Particles:
(3,2) w=.9
(2,3) w=.2
(3,2) w=.9
(3,1) w=.4
(3,3) w=.4
(3,2) w=.9
(1,3) w=.1
(2,3) w=.2
(3,2) w=.9
(2,2) w=.4

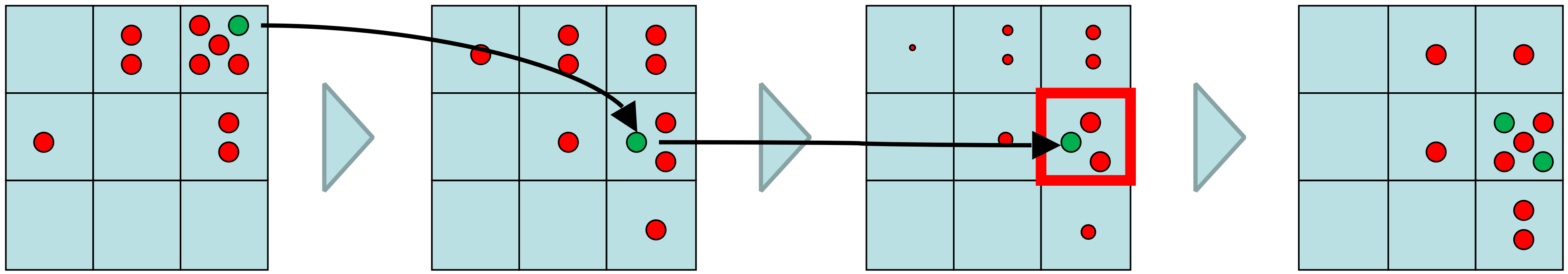
(New) Particles:
(3,2)
(2,2)
(3,2)
(2,3)
(3,3)
(3,2)
(1,3)
(2,3)
(3,2)
(3,2)



Recap: Particle Filtering

- Particles: track samples of states rather than an explicit distribution

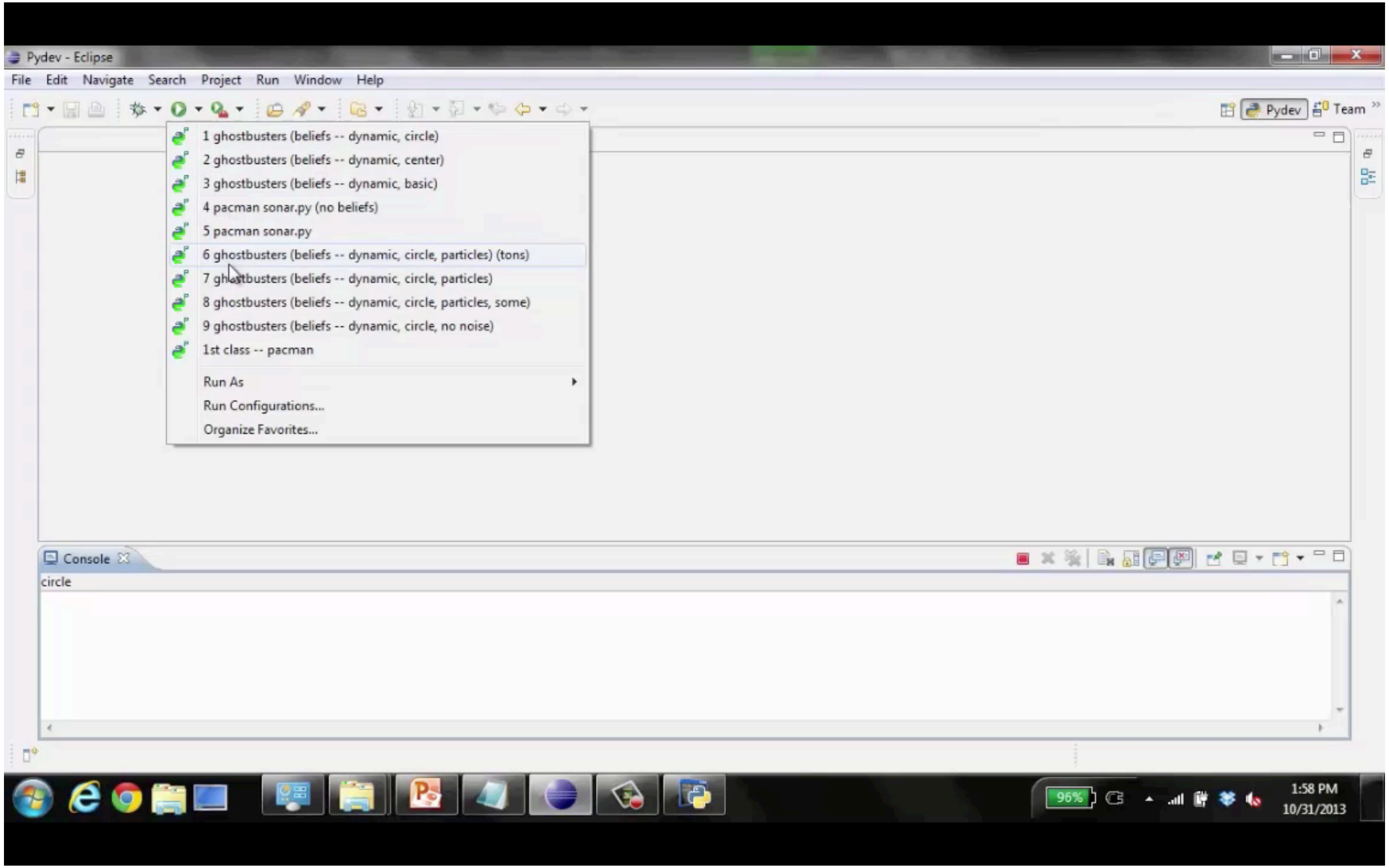
Elapse Weight Resample



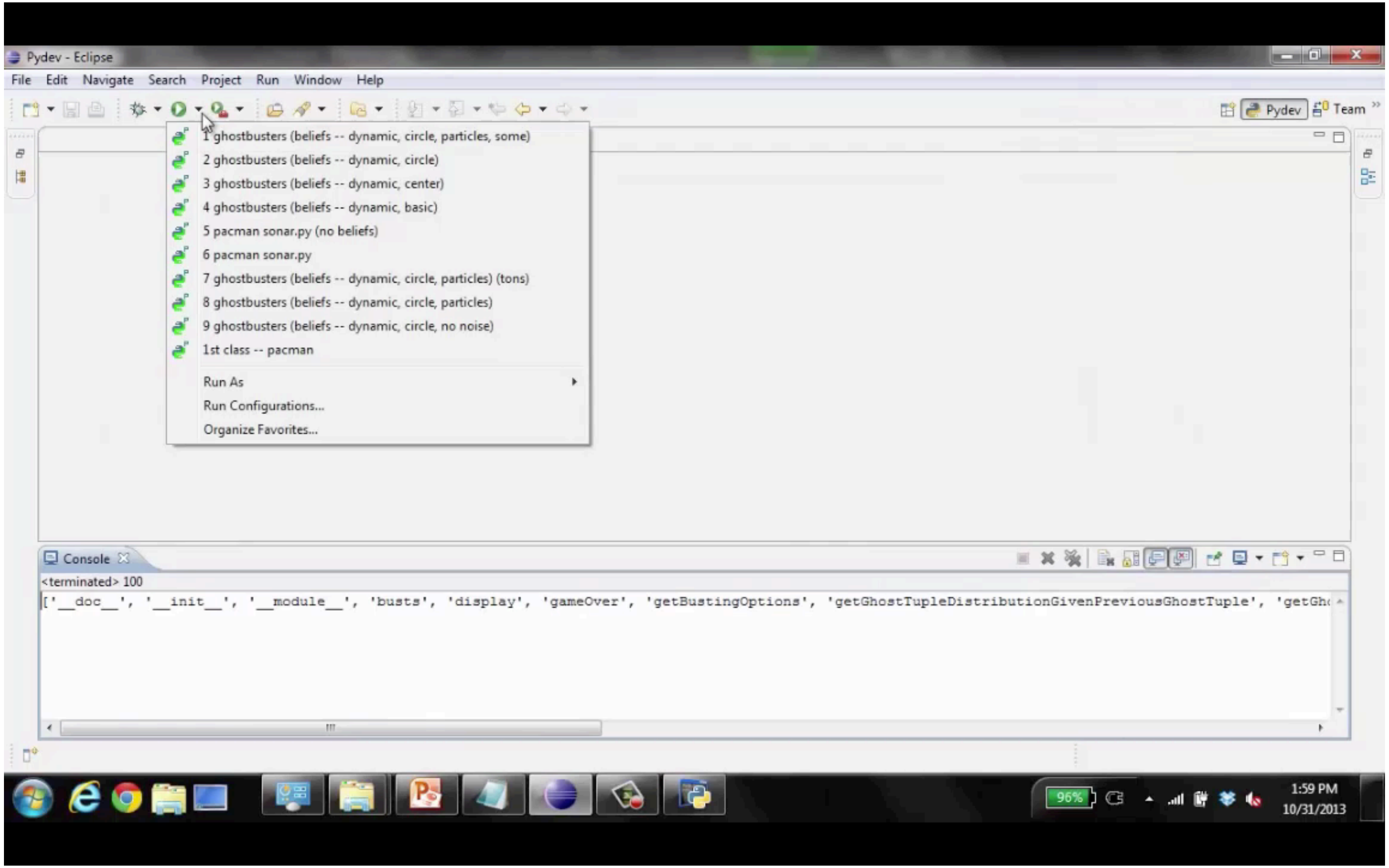
Particles:		Particles:		Particles:	(New) Particles:
(3,3)		(3,2)		(3,2) w=.9	(3,2)
(2,3)		(2,3)		(2,3) w=.2	(2,2)
(3,3)	Transition	(3,2)	Emissions	(3,2) w=.9	(3,2)
(3,2)	$P(X_t \mid X_{t-1})$	(3,1)	$P(E_t \mid X_t)$	(3,1) w=.4	(2,3)
(3,3)		(3,3)		(3,3) w=.4	(3,3)
(3,2)		(3,2)		(3,2) w=.9	(3,2)
(1,2)		(1,3)		(1,3) w=.1	(1,3)
(3,3)		(2,3)		(2,3) w=.2	(2,3)
(3,3)		(3,2)		(3,2) w=.9	(3,2)
(2,3)		(2,2)		(2,2) w=.4	(3,2)

[Demos: ghostbusters particle filtering

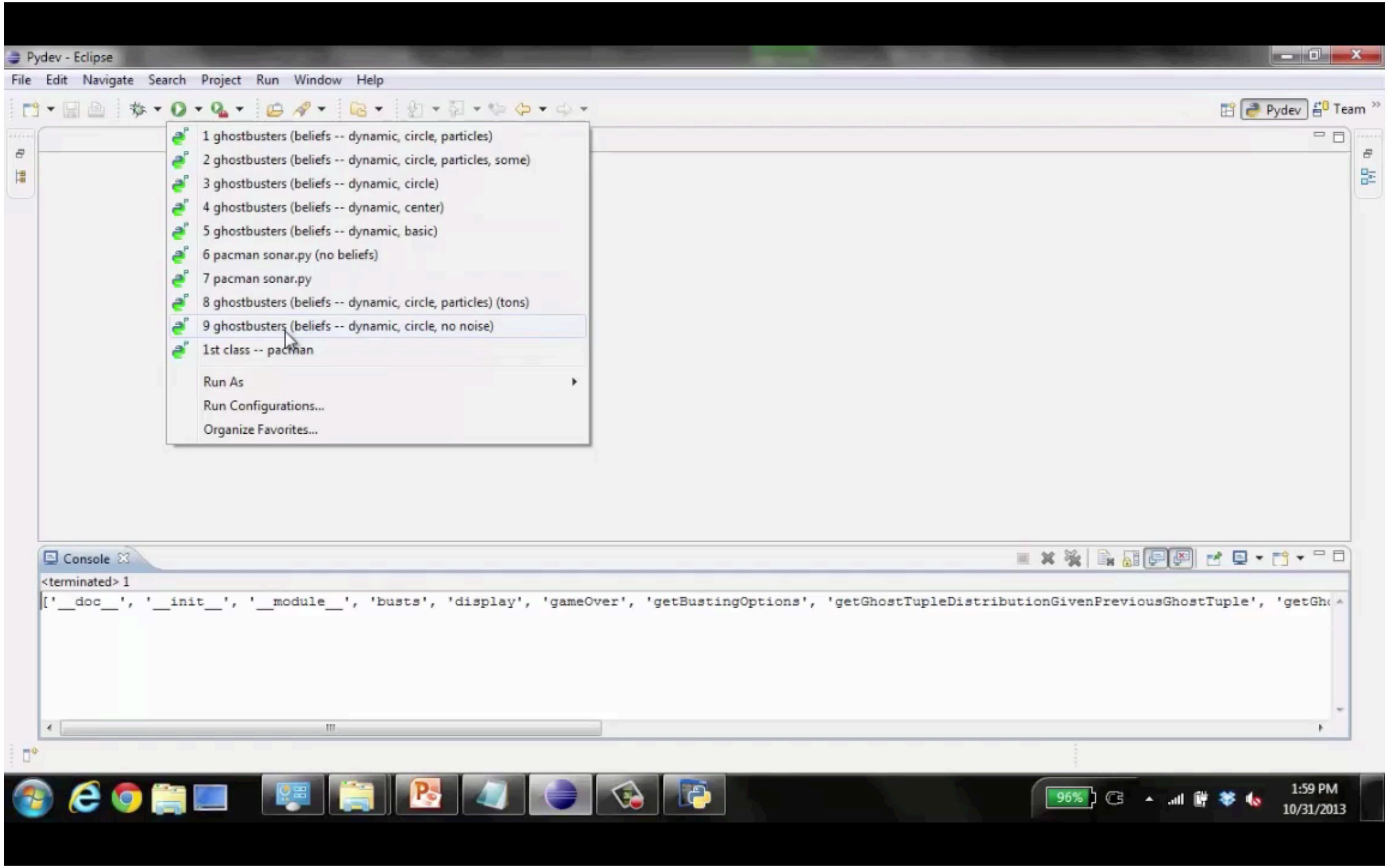
Video of Demo – Moderate Number of Particles



Video of Demo – One Particle

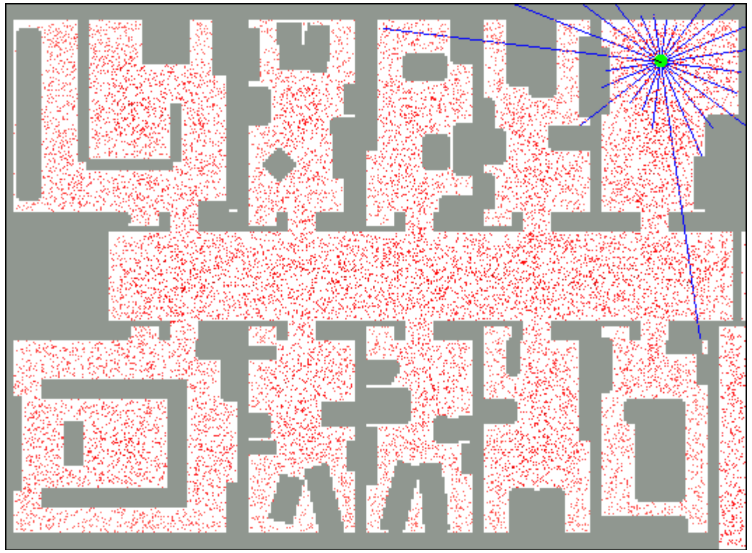


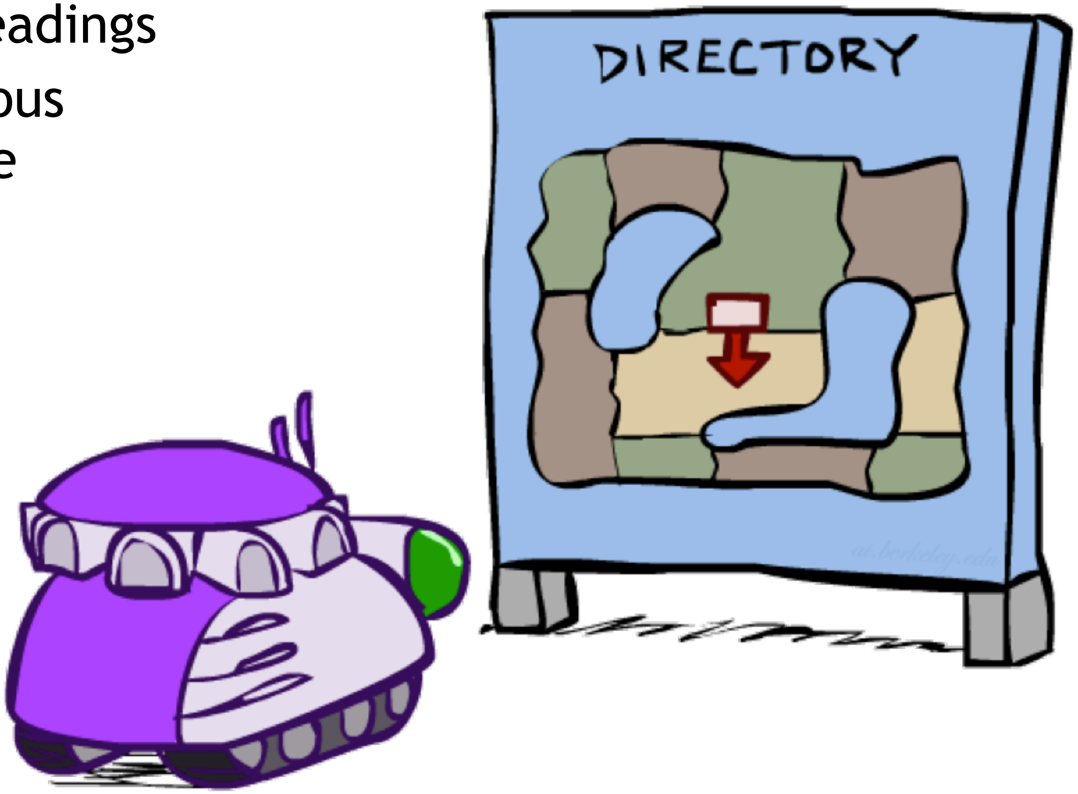
Video of Demo – Huge Number of Particles



Robot Localization

- In robot localization:
  - We know the map, but not the robot’s position
  - Observations may be vectors of range finder readings
  - State space and readings are typically continuous (works basically like a very fine grid) and so we cannot store B(X)
  - Particle filtering is a main technique



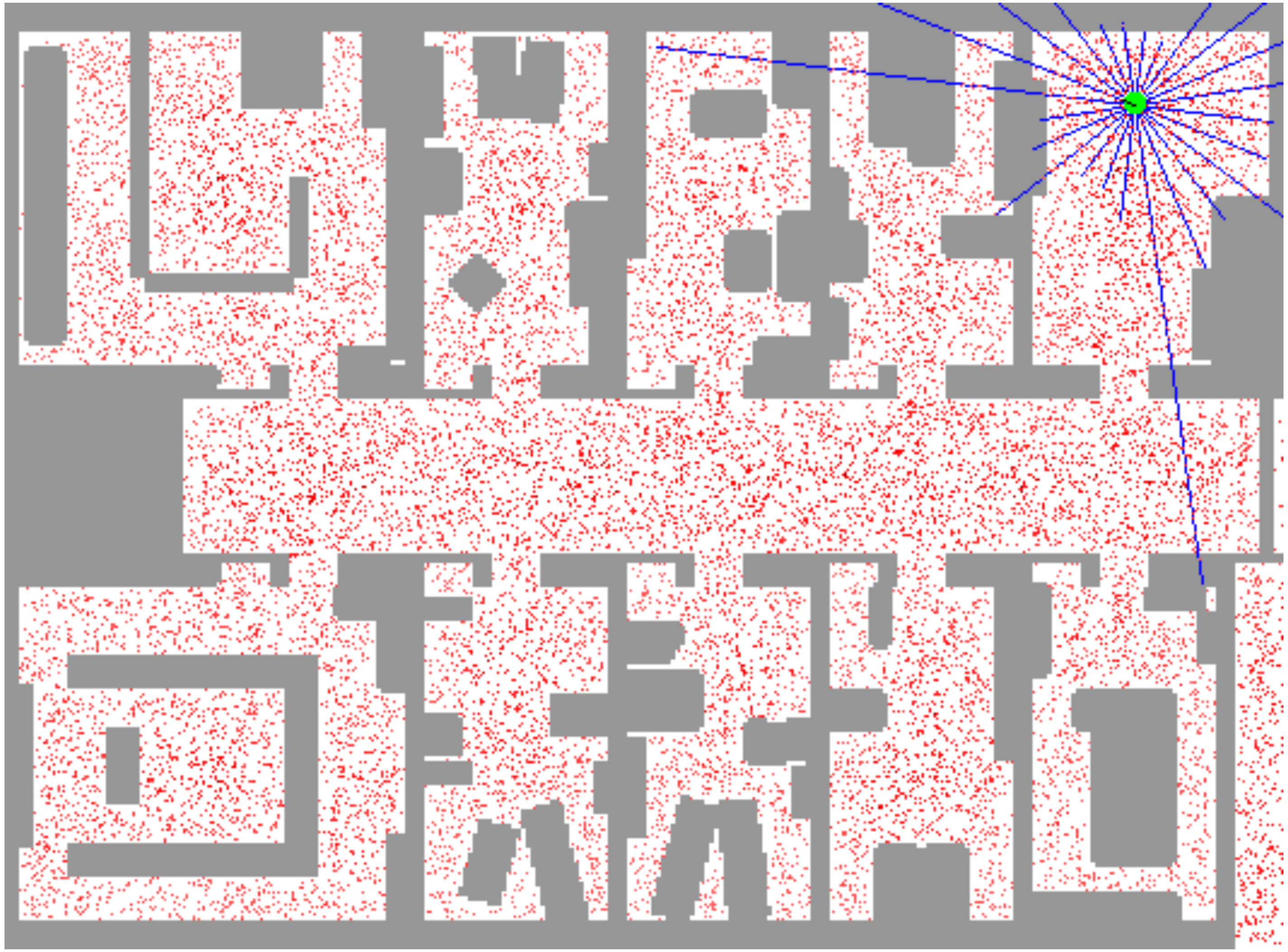


Particle Filter Localization (Sonar)

[Dieter Fox, et al.]

[Video: global-sonar-uw-annotated.avi]

Particle Filter Localization (Laser)

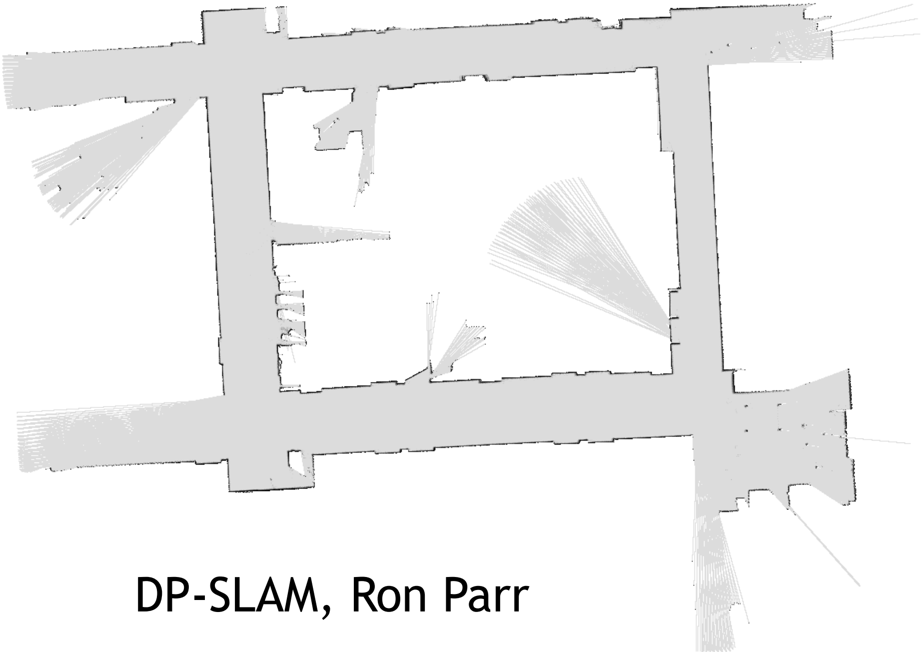


[Dieter Fox, et al.]

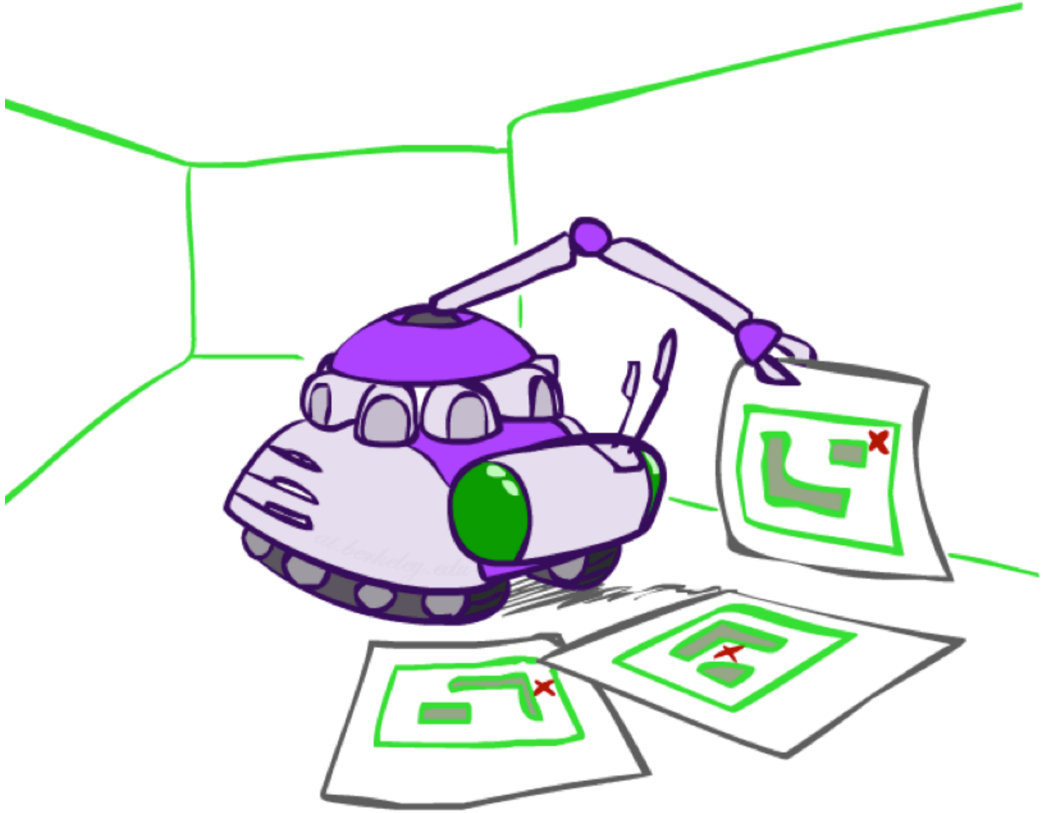
[Video: global-floor.gif]

Robot Mapping

- SLAM: Simultaneous Localization And Mapping
  - We do not know the map or our location
  - State consists of position AND map!
  - Main techniques: Kalman filtering (Gaussian HMMs) and particle methods

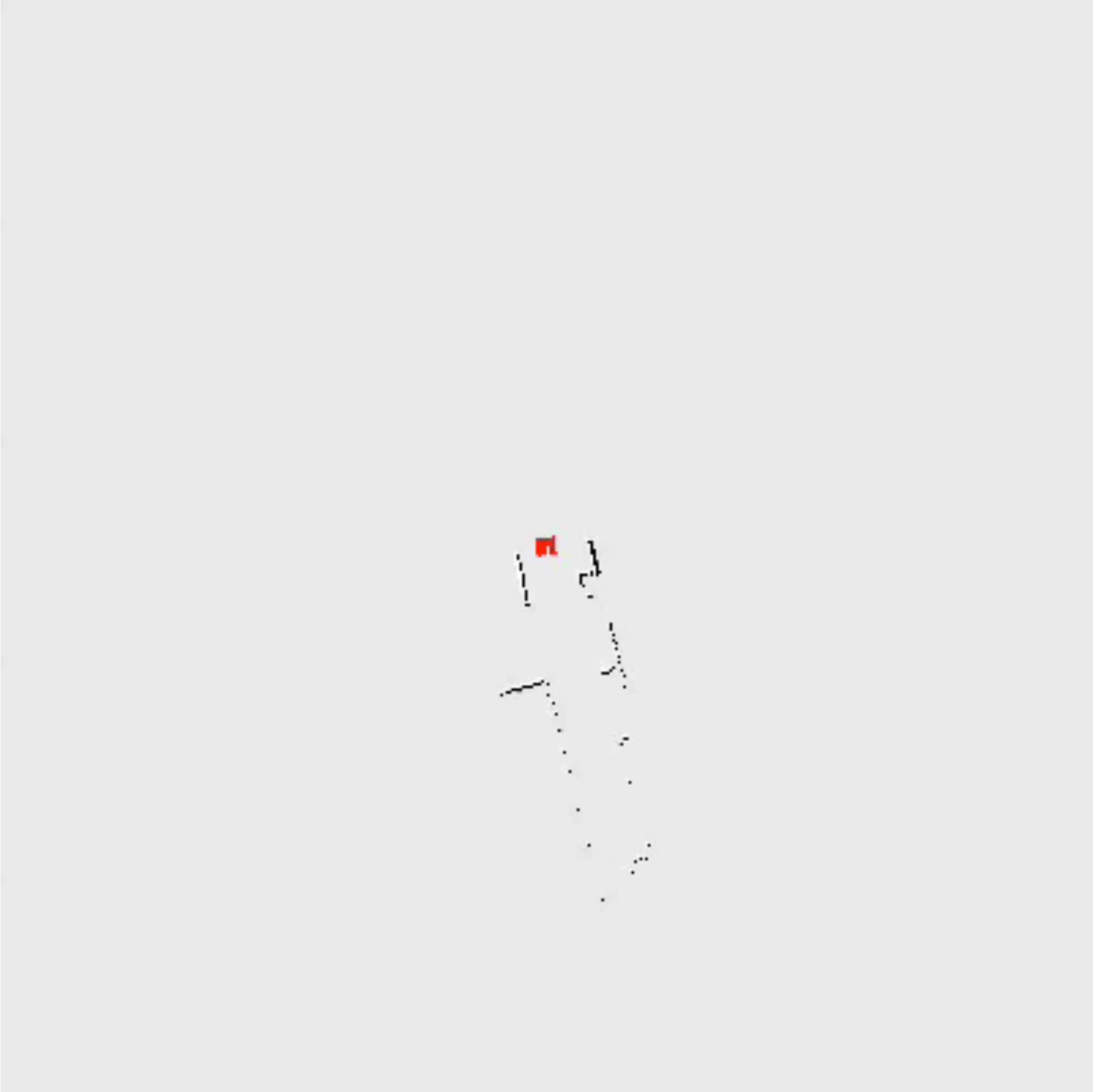


DP-SLAM, Ron Parr



[Demo: PARTICLES-SLAM-mapping1-new.a

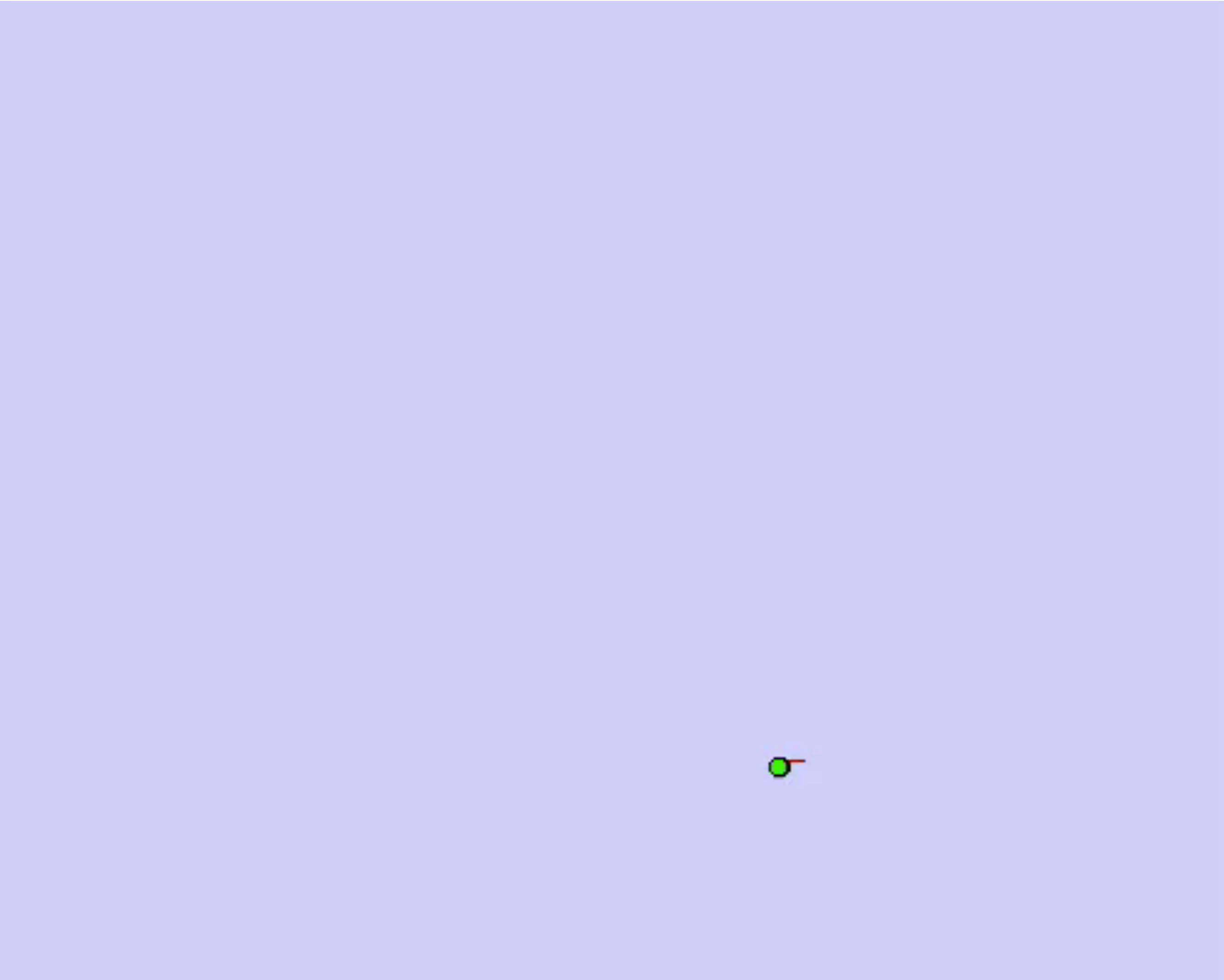
Particle Filter SLAM – Video 1



[Sebastian Thrun, et al.]

[Demo: PARTICLES-SLAM-mapping1-new.a

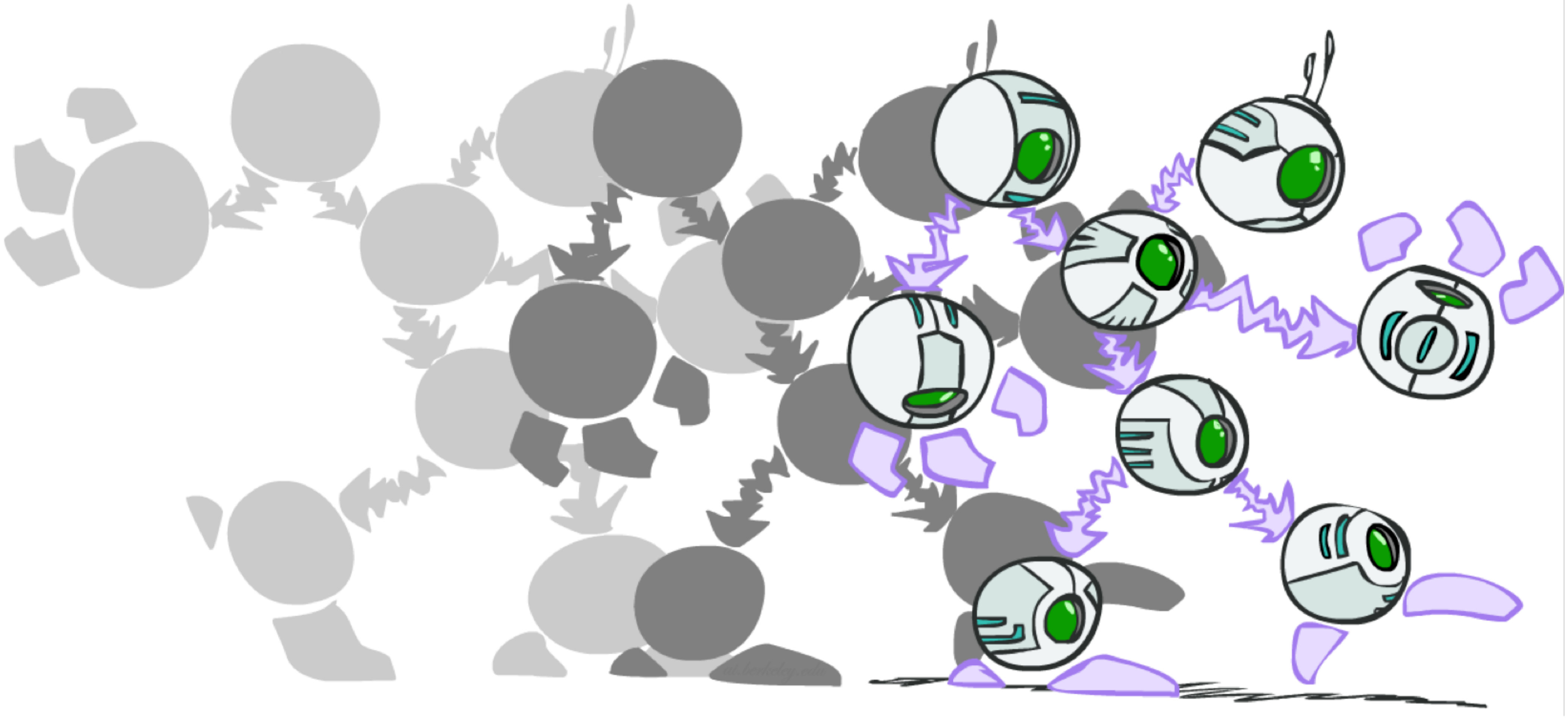
Particle Filter SLAM – Video 2



[Dirk Haehnel, et al.]

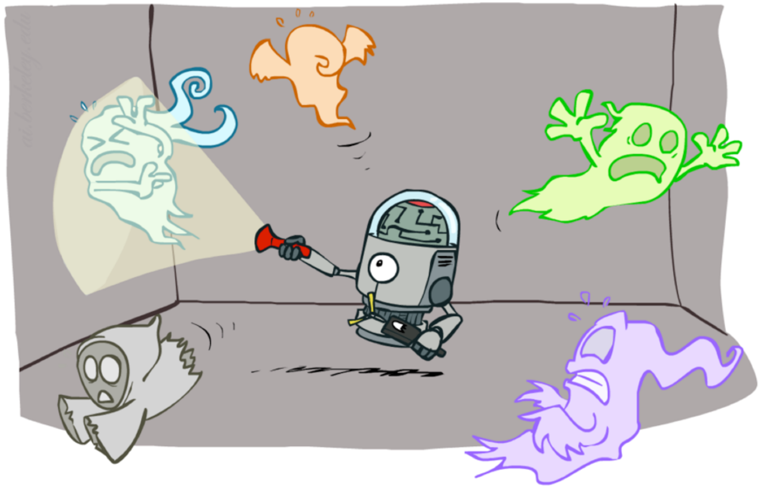
[Demo: PARTICLES-SLAM-fastslam.a

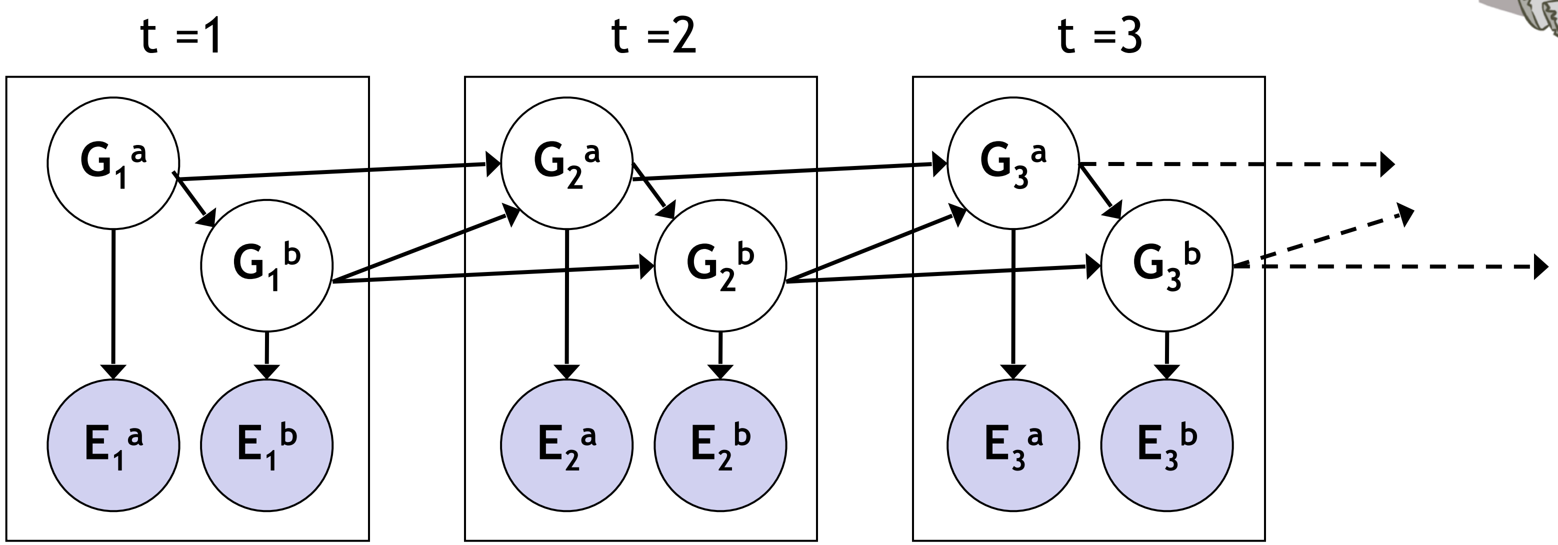
Dynamic Bayes Nets



Dynamic Bayes Nets (DBNs)

- We want to track multiple variables over time, using multiple sources of evidence
- Idea: Repeat a fixed Bayes net structure at each time
- Variables from time *t* can condition on those from *t-1*

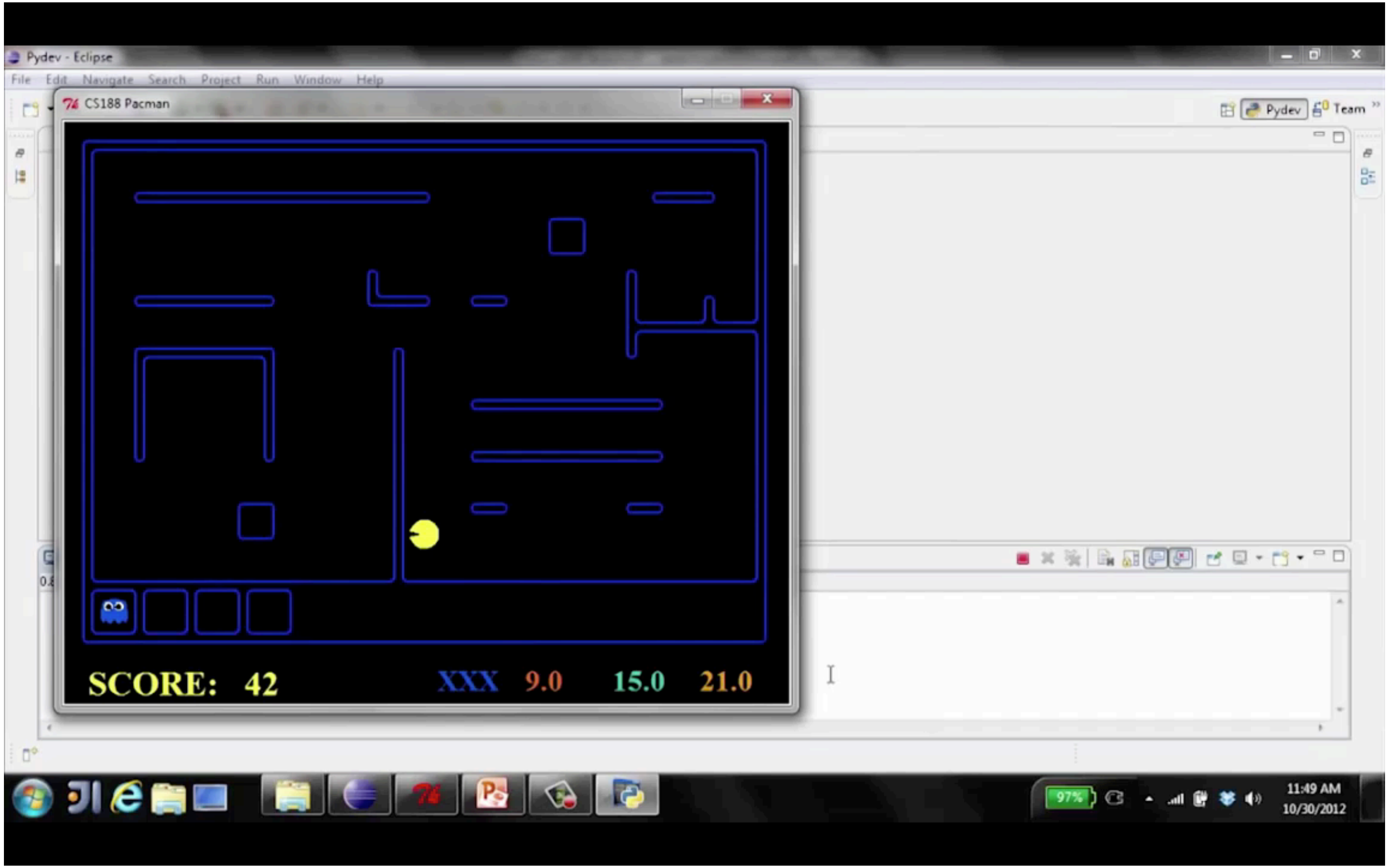




- Dynamic Bayes nets are a generalization of HMMs

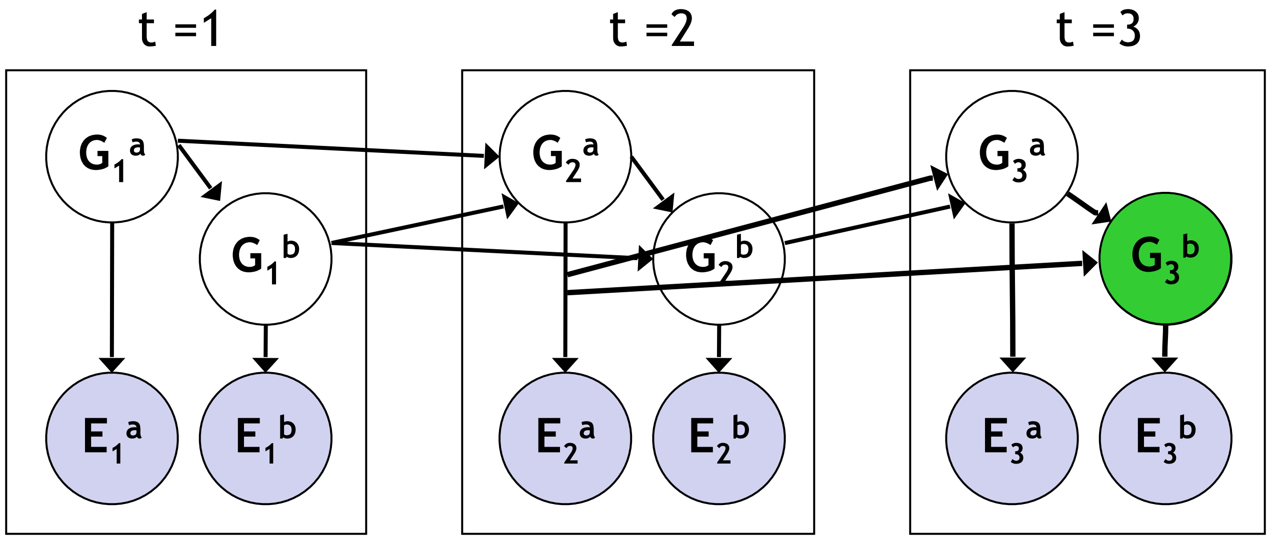
[Demo: pacman sonar ghost DBN model (L15D6)]

Video of Demo Pacman Sonar Ghost DBN Model



Exact Inference in DBNs

- Variable elimination applies to dynamic Bayes nets
- Procedure: "unroll" the network for T time steps, then eliminate variables until $P(X_T|e_{1:T})$ is computed



- Online belief updates: Eliminate all variables from the previous time step; store factors for current time only

DBN Particle Filters

- A particle is a complete sample for a time step
- **Initialize**: Generate prior samples for the t=1 Bayes net
  - Example particle: $\mathbf{G_1^a} = (3,3)$ $\mathbf{G_1^b} = (5,3)$
- **Elapse time**: Sample a successor for each particle
  - Example successor: $\mathbf{G_2^a} = (2,3)$ $\mathbf{G_2^b} = (6,3)$
- **Observe**: Weight each *entire* sample by the likelihood of the evidence conditioned on the sample
  - Likelihood: $P(\mathbf{E_1^a} \mid \mathbf{G_1^a}) * P(\mathbf{E_1^b} \mid \mathbf{G_1^b})$
- **Resample:** Select prior samples (tuples of values) in proportion to their likelihood

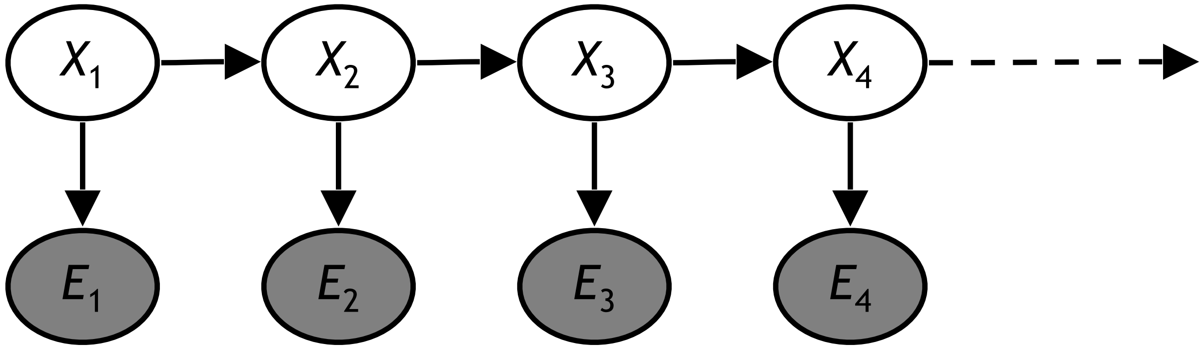
Most Likely Explanation

HMMs: MLE Queries

- HMMs defined by
  - States X
  - Observations E
  - Initial distribution:
  - Transitions:
  - Emissions:

- New query: most likely explanation:

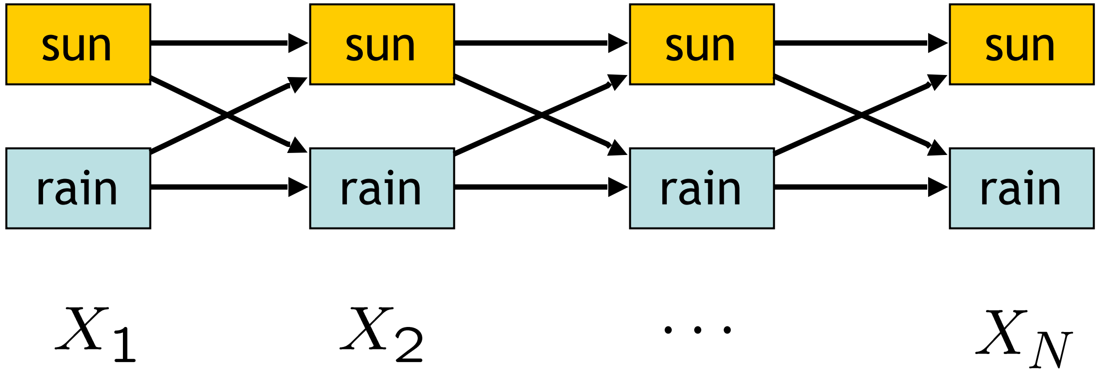
$$P(X_1)$$
$$P(X|X_{-1})$$
$$P(E|X)$$



$$\arg\max_{x_{1:t}} P(x_{1:t}|e_{1:t})$$

State Trellis

- State trellis: graph of states and transitions over time



- Each arc represents some transition $x_{t-1} \rightarrow x_t$
- Each arc has weight $P(x_t|x_{t-1})P(e_t|x_t)$
- Each path is a sequence of states
- The product of weights on a path is that sequence's probability along with the evidence
- Forward algorithm computes sums of paths, Viterbi computes best paths

State Trellis

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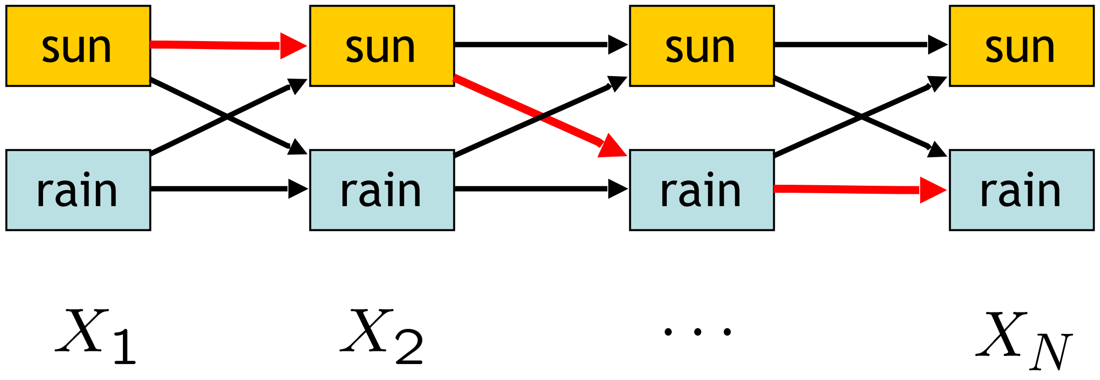

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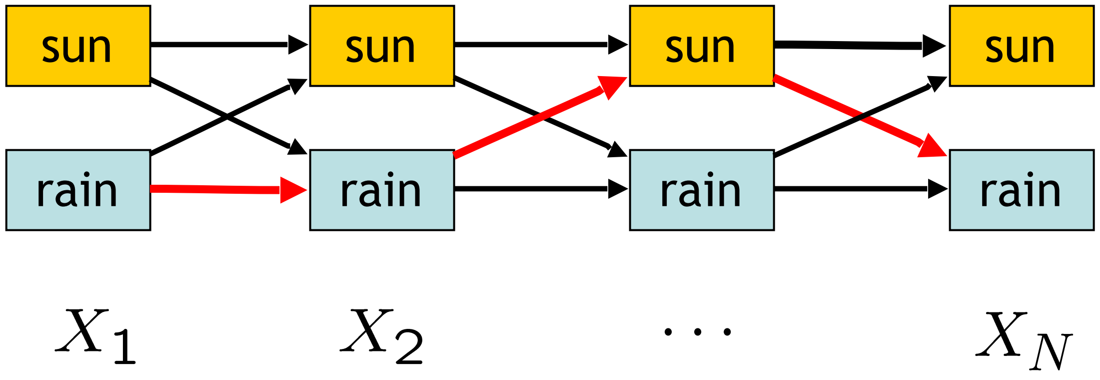

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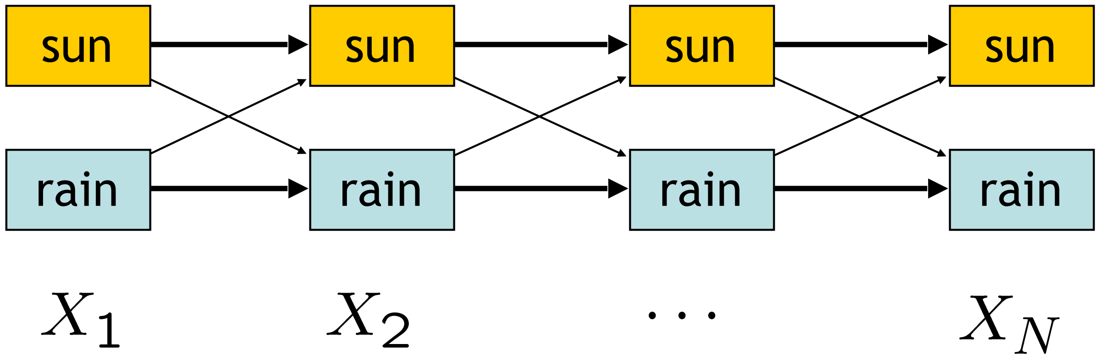
Forward / Viterbi Algorithms



Forward Algorithm (Sum)

$$f_t[x_t] = P(x_t, e_{1:t})$$

$$= P(e_t|x_t) \sum_{x_{t-1}} P(x_t|x_{t-1}) f_{t-1}[x_{t-1}]$$

Viterbi Algorithm (Max)

$$m_t[x_t] = \max_{x_{1:t-1}} P(x_{1:t-1}, x_t, e_{1:t})$$

$$= P(e_t|x_t) \max_{x_{t-1}} P(x_t|x_{t-1}) m_{t-1}[x_{t-1}]$$