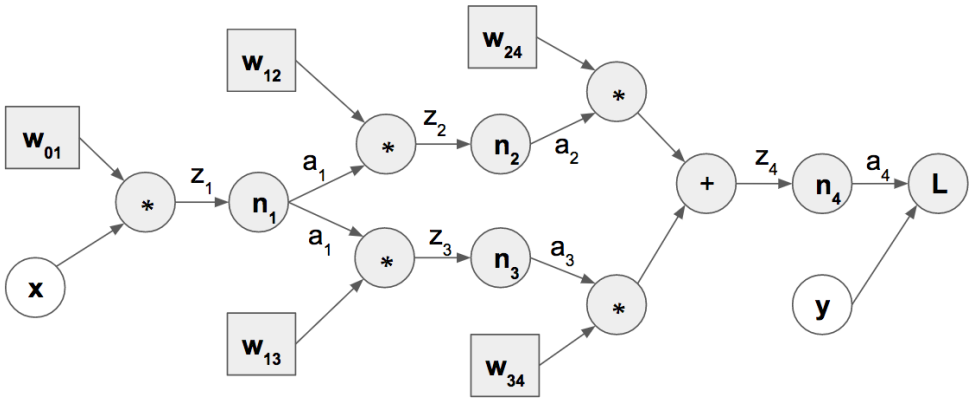
CS188: Exam Practice Session 11 Solutions

Q1. Deep Learning

(a) Consider the neural network below. We refer to nodes that apply the activation function as neurons numbered from top-down, left-right. Draw the computational graph of the following neural network, labeling all the bolded variables listed below. (Please use the same conventions as from project 6)

- $\mathbf{x}$, a scalar, is the input. $\mathbf{a_4}$, also a scalar, is the output.
- There are three hidden layers of size 1, 2, and 1.
- All neurons $\mathbf{n_i}$ have the same activation function $g(x) = e^x$.
  (numbered from top-down, left-right)
- Each $\mathbf{a_i}$ value is the final output for neuron i in the network.
- Each $\mathbf{z_i}$ value is the pre-activation value for neuron i in the network (i.e. after the dot product, before the nonlinear activation).
- Let $\mathbf{w_{ij}}$ be the weight used between neuron n_i and n_j.
  (You can consider x to be n_0 for the purposes of labeling weights.)
- $\mathbf{L}(y, a_4)$ is the loss function. It is $(y - a_4)^2$, where $\mathbf{y}$ is the training label.



(b) Use the Chain Rule and the equations given to calculate $\delta L/\delta z_2$. You may use x and y, along with all of the w, a, and z values.

$$\frac{\delta L}{\delta z_2} = \frac{\delta L}{\delta a_4}\frac{\delta a_4}{\delta z_4}\frac{\delta z_4}{\delta a_2}\frac{\delta a_2}{\delta z_2} = -2(y - a_4) * e^{z_4} * w_{24} * e^{z_2}$$

(c) Use the Chain Rule and the equations given to calculate $\delta L/\delta z_3$. You may use x and y, along with all of the w, a, and z values.

$$\frac{\delta L}{\delta z_3} = \frac{\delta L}{\delta a_4}\frac{\delta a_4}{\delta z_4}\frac{\delta z_4}{\delta a_3}\frac{\delta a_3}{\delta z_3} = -2(y - a_4) * e^{z_4} * w_{34} * e^{z_3}$$

(d) Use the Chain Rule and the equations given to calculate $\delta L/\delta w_{01}$. You may use x and y, along with all of the w, a, and z values. You may symbolically use previous parts.

$$\frac{\delta L}{\delta w_{01}} =$$

$$\frac{\delta L}{\delta a_1}\frac{\delta a_1}{\delta z_1}\frac{\delta z_1}{\delta w_{01}} =$$

$$\left(\frac{\delta L}{\delta z_2}\frac{\delta z_2}{\delta a_1} + \frac{\delta L}{\delta z_3}\frac{\delta z_3}{\delta a_1}\right)\frac{\delta a_1}{\delta z_1}\frac{\delta z_1}{\delta w_{01}} = \left(\frac{\delta L}{\delta z_2}\, w_{12} + \frac{\delta L}{\delta z_3}\, w_{13}\right) * e^{z_1} * x$$

Q2. Gradients

Given that

$$p(y=1 \mid f(x); w) = \frac{e^{w^\top f(x)}}{e^{w^\top f(x)} + e^{-w^\top f(x)}}$$
$$p(y=0 \mid f(x); w) = \frac{e^{-w^\top f(x)}}{e^{w^\top f(x)} + e^{-w^\top f(x)}}$$
$$g(x; w) = \begin{cases} w^\top x & w^\top x > 0 \\ \alpha(e^{w^\top x} - 1) & w^\top x \leq 0 \end{cases}$$

Take the following gradients:

(a) $\frac{\partial}{\partial w} g(x; w)$

Let n be the dimension of the vectors x and w. Let's consider the two cases in the conditional:

1. $w^\top x \leq 0$:

$$\begin{aligned} \frac{\partial}{\partial w_i} w^\top x &= \frac{\partial}{\partial w_i} \sum_{k=1}^{n} w_k x_k \\ &= \frac{\partial}{\partial w_i} (w_i x_i + \sum_{k \neq i} w_k x_k) \\ &= x_i + 0 \end{aligned}$$

So $\frac{\partial}{\partial w} = [\frac{\partial}{\partial w_1}, \ldots, \frac{\partial}{\partial w_n}] = [x_1, \ldots, x_n] = x$

2. $w^\top x > 0$:

$$\begin{aligned} \frac{\partial}{\partial w_i} \alpha(e^{w^\top x} - 1) &= \alpha e^{w^\top x} \frac{\partial}{\partial w_i}(w^\top x) && \text{by the chain rule} \\ &= \alpha e^{w^\top x} x_i && \text{using the result from above} \\ \frac{\partial}{\partial w} \alpha(e^{w^\top x} - 1) &= \alpha e^{w^\top x} x \end{aligned}$$

So

$$\frac{\partial}{\partial w} g(x; w) = \begin{cases} x & w^\top x > 0 \\ \alpha e^{w^\top x} x & w^\top x \leq 0 \end{cases}$$

(b) $\frac{\partial}{\partial w} \sum_{i=1}^{m} \log p(y = y^{(y)} \mid f(x^{(i)}); w)$

Let's first consider differentiating the log probability for the case where $y = 1$:

$$\begin{aligned} \frac{\partial}{\partial w} \log p(y=1 \mid f(x); w) &= \frac{\partial}{\partial w} \log \frac{e^{w^\top f(x)}}{e^{w^\top f(x)} + e^{-w^\top f(x)}} \\ &= \frac{\partial}{\partial w} \log \frac{e^{2w^\top f(x)}}{e^{2w^\top f(x)} + e^0} && \text{multiplying top and bottom by } e^{w^\top f(x)} \\ &= \frac{\partial}{\partial w} (2w^\top f(x) - \log(e^{2w^\top f(x)} + 1)) && \text{properties of log} \\ &= 2f(x) - \frac{1}{e^{2w^\top f(x)} + 1} \frac{\partial}{\partial w} (e^{2w^\top f(x)} + 1) && \text{using the chain rule on the log term} \\ &= 2f(x) - \frac{2f(x) e^{2w^\top f(x)}}{e^{2w^\top f(x)} + 1} && \text{similar derivation to second case of part (a)} \\ &= \frac{2f(x)}{e^{2w^\top f(x)} + 1} && \text{combine fractions and simplify} \end{aligned}$$

Now for the case where $y = 0$, we either do a similar derivation or observe that we can substitute $-f(x)$ for $f(x)$ in the definition of $p(y = 1 \mid \ldots)$ to get $p(y = 0 \mid \ldots)$. Since $f(x)$ functions as a constant when we are computing the partial derivative with respect to w, we can substitute $-f(x)$ for $f(x)$ in the equation for the gradient as well. So the gradient will be

$$\frac{\partial}{\partial w} \log p(y = 0 \mid f(x); w) = \frac{-2f(x)}{e^{-2w^\top f(x)} + 1}$$

Now we need to put these cases together to get a partial derivative for

$$p(y = y^{(i)} | f(x^{(i)}); w)$$

using the value of $y^{(i)}$. We can use the exponentiation trick for this:

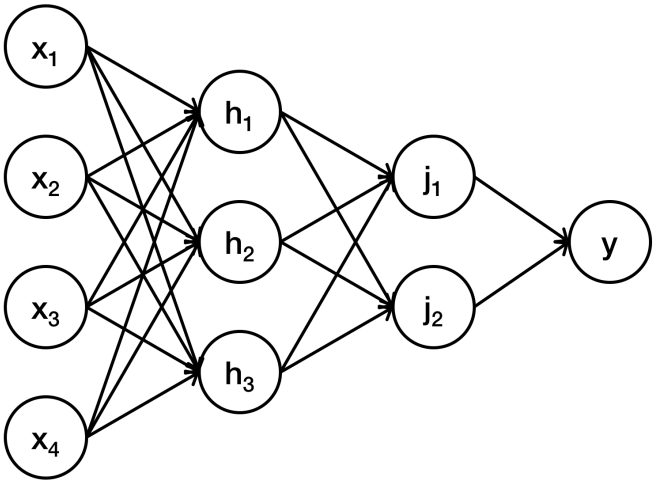
$$\frac{\partial}{\partial w} \log p(y = y^{(i)} \mid f(x^{(i)}); w) = \left(\frac{2f(x^{(i)})}{e^{2w^\top f(x^{(i)})} + 1}\right)^{y^{(i)}} \left(\frac{-2f(x^{(i)})}{e^{-2w^\top f(x^{(i)})} + 1}\right)^{1-y^{(i)}}$$

Finally, combining these into the sum:

$$\frac{\partial}{\partial w} \sum_{i=1}^{m} \log p(y = y^{(i)} \mid f(x^{(i)}); w) = \sum_{i=1}^{m} \left(\frac{2f(x^{(i)})}{e^{2w^\top f(x^{(i)})} + 1}\right)^{y^{(i)}} \left(\frac{-2f(x^{(i)})}{e^{-2w^\top f(x^{(i)})} + 1}\right)^{1-y^{(i)}}$$

Q3. Neural Network Data Sufficiency

The next few problems use the below neural network as a reference. Neurons h_{1-3} and j_{1-2} all use ReLU activation functions. Neuron y uses the identity activation function: $f(x) = x$. In the questions below, let $w_{a,b}$ denote the weight that connects neurons a and b. Also, let o_a denote the value that neuron a outputs to its next layer.



Given this network, in the following few problems, you have to decide whether the data given are sufficient for answering the question.

(a) Given the above neural network, what is the value of o_y?

Data item 1: the values of all weights in the network and the values o_{h_1}, o_{h_2}, o_{h_3}
Data item 2: the values of all weights in the network and the values o_{j_1}, o_{j_2}

- ○ Data item (1) alone is sufficient, but data item (2) alone is not sufficient to answer the question.
- ○ Data item (2) alone is sufficient, but data item (1) alone is not sufficient to answer the question.
- ○ Both statements taken together are sufficient, but neither data item alone is sufficient.
- ● Each data item alone is sufficient to answer the question.
- ○ Statements (1) and (2) together are not sufficient, and additional data is needed to answer the question.

(b) Given the above neural network, what is the value of o_{h_1}?

Data item 1: the neuron input values, i.e., o_{x_1} through o_{x_4}
Data item 2: the values o_{j_1}, o_{j_2}

- ○ Data item (1) alone is sufficient, but data item (2) alone is not sufficient to answer the question.
- ○ Data item (2) alone is sufficient, but data item (1) alone is not sufficient to answer the question.
- ○ Both statements taken together are sufficient, but neither data item alone is sufficient.
- ○ Each data item alone is sufficient to answer the question.
- ● Statements (1) and (2) together are not sufficient, and additional data is needed to answer the question.

(c) Given the above neural network, what is the value of o_{j_1}?

Data item 1: the values of all weights connecting neurons h_1, h_2, h_3 to j_1, j_2
Data item 2: the values o_{h_1}, o_{h_2}, o_{h_3}

- ○ Data item (1) alone is sufficient, but data item (2) alone is not sufficient to answer the question.
- ○ Data item (2) alone is sufficient, but data item (1) alone is not sufficient to answer the question.
- ● Both statements taken together are sufficient, but neither data item alone is sufficient.
- ○ Each data item alone is sufficient to answer the question.
- ○ Statements (1) and (2) together are not sufficient, and additional data is needed to answer the question.

(d) Given the above neural network, what is the value of $\partial o_y / \partial w_{j_2,y}$?

Data item 1: the value of o_{j_2}
Data item 2: all weights in the network and the neuron input values, i.e., o_{x_1} through o_{x_4}

- ○ Data item (1) alone is sufficient, but data item (2) alone is not sufficient to answer the question.
- ○ Data item (2) alone is sufficient, but data item (1) alone is not sufficient to answer the question.
- ○ Both statements taken together are sufficient, but neither data item alone is sufficient.
- ● Each data item alone is sufficient to answer the question.
- ○ Statements (1) and (2) together are not sufficient, and additional data is needed to answer the question.

(e) Given the above neural network, what is the value of $\partial o_y / \partial w_{h_2,j_2}$?

Data item 1: the value of $w_{j_2,y}$
Data item 2: the value of $\partial o_{j_2} / \partial w_{h_2,j_2}$

- ○ Data item (1) alone is sufficient, but data item (2) alone is not sufficient to answer the question.
- ○ Data item (2) alone is sufficient, but data item (1) alone is not sufficient to answer the question.
- ● Both statements taken together are sufficient, but neither data item alone is sufficient.
- ○ Each data item alone is sufficient to answer the question.
- ○ Statements (1) and (2) together are not sufficient, and additional data is needed to answer the question.

(f) Given the above neural network, what is the value of $\partial o_y / \partial w_{x_1,h_3}$?

Data item 1: the value of all weights in the network and the neuron input values, i.e., o_{x_1} through o_{x_4}
Data item 2: the value of w_{x_1,h_3}

- ● Data item (1) alone is sufficient, but data item (2) alone is not sufficient to answer the question.
- ○ Data item (2) alone is sufficient, but data item (1) alone is not sufficient to answer the question.
- ○ Both statements taken together are sufficient, but neither data item alone is sufficient.
- ○ Each data item alone is sufficient to answer the question.
- ○ Statements (1) and (2) together are not sufficient, and additional data is needed to answer the question.