Problem #6

Prove that $E[X+Y] = E[X] + E[Y]$

$Var[X+Y] = Var[X] + Var[Y] + 2\,Cov(X,Y)$

Solution:

$$E[X] = \frac{\sum X_i}{n} \qquad E[Y] = \frac{\sum Y_i}{n}$$

$$E[X+Y) = \frac{\sum (X_i + Y_i)}{n}$$

$$\Rightarrow \quad E[X] + E[Y] = \frac{\sum X_i}{n} + \frac{\sum Y_i}{n} = \frac{\sum (X_i + Y_i)}{n} = E[X+Y]$$

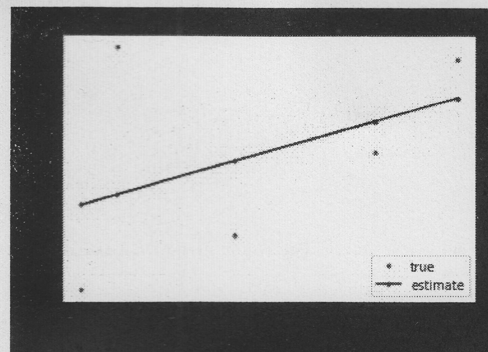
$$Var[X+Y] = E[(X+Y-\mu_{x+y})^2]$$

$$= E[(X+Y)^2] - E[X+Y]^2$$

$$= E[X^2] + E[Y^2] + 2E[XY]$$

$$+ E[X]^2 + E[Y]^2 + 2E[X]E[Y]$$

$$= Var[X] + Var[Y] + 2\,Cov(X,Y)$$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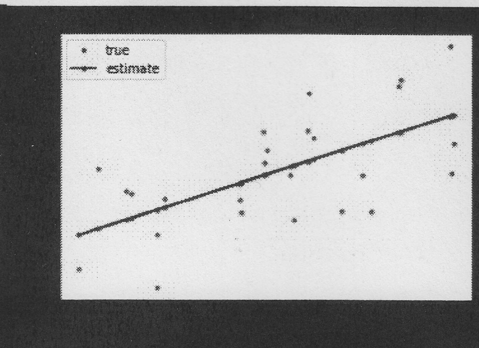
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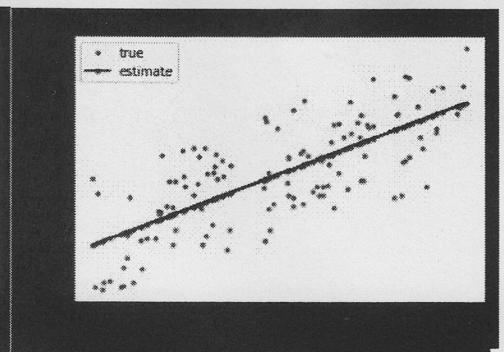
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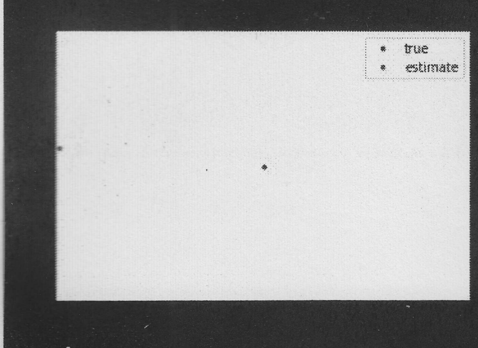
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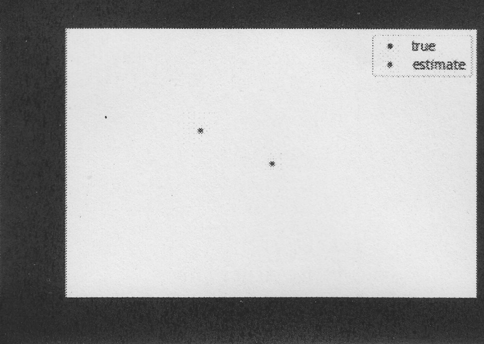
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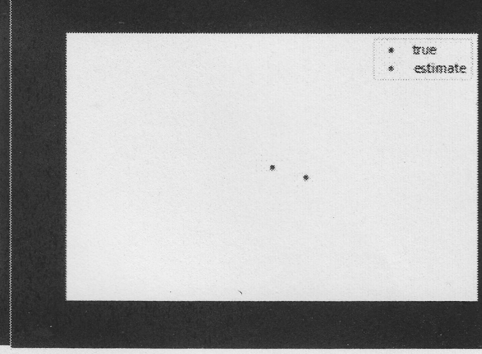
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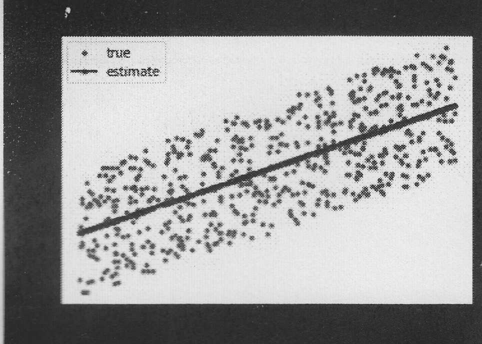
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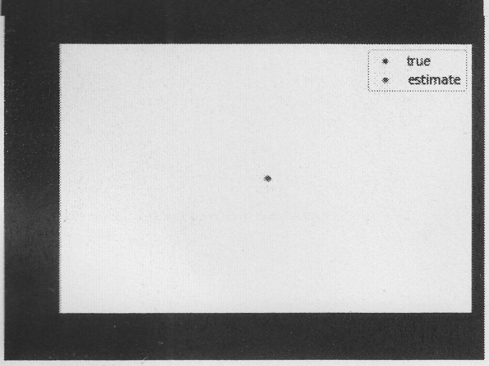
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prob 2e