In [70]:
```
import numpy as np
import matplotlib.pyplot as plt


# Question 2b

x0 = np.array([0, 0]).reshape(2, 1)
b = np.array([4.5, 6]).reshape(2, 1)
f = lambda x: np.sqrt( np.dot( (x - b).T, (x - b) ) )
delta_f = lambda x: (x - b) / np.sqrt( np.dot( (x - b).T, (x - b) ) )

for _ in range(10):
    x0 = x0 - delta_f(x0)
    print(x0.flatten(), f(x0).flatten())
```

```
[ 0.6  0.8] [ 6.5]
[ 1.2  1.6] [ 5.5]
[ 1.8  2.4] [ 4.5]
[ 2.4  3.2] [ 3.5]
[ 3.  4.] [ 2.5]
[ 3.6  4.8] [ 1.5]
[ 4.2  5.6] [ 0.5]
[ 4.8  6.4] [ 0.5]
[ 4.2  5.6] [ 0.5]
[ 4.8  6.4] [ 0.5]
```

In [73]:
```
# Question 2c

x0 = np.array([0, 0]).reshape(2, 1)
b = np.array([3, 4]).reshape(2, 1)
f = lambda x: np.sqrt( np.dot( (x - b).T, (x - b) ) )
delta_f = lambda x: (x - b) / np.sqrt( np.dot( (x - b).T, (x - b) ) )

for i in range(10):
    x0 = x0 - (5 / 6) ** i * delta_f(x0)
    print(x0.flatten(), f(x0).flatten())
```

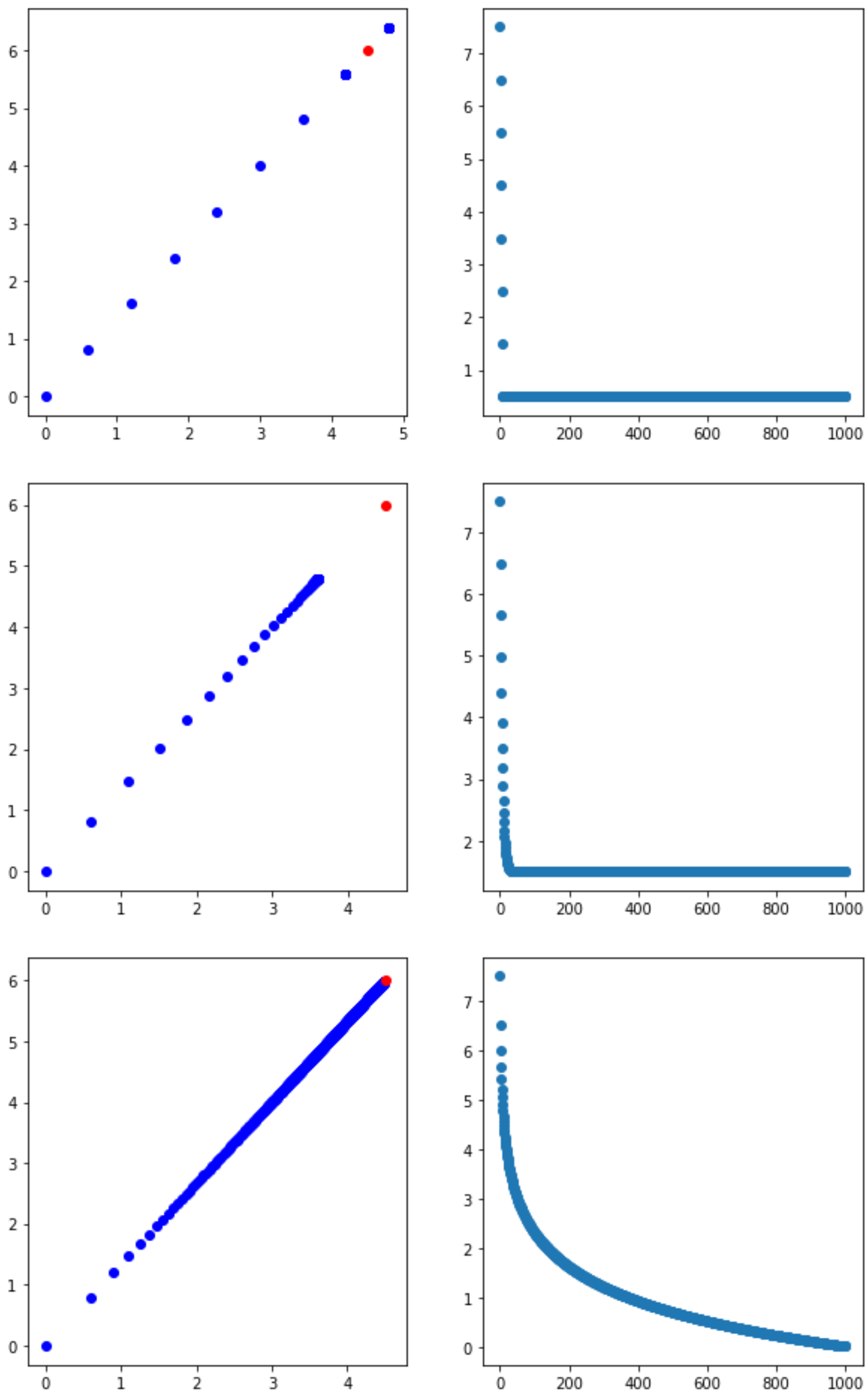
```
[ 0.6  0.8] [ 4.]
[ 1.1         1.46666667] [ 3.16666667]
[ 1.51666667  2.02222222] [ 2.47222222]
[ 1.86388889  2.48518519] [ 1.89351852]
[ 2.15324074  2.87098765] [ 1.41126543]
[ 2.39436728  3.19248971] [ 1.00938786]
[ 2.59530607  3.46040809] [ 0.67448988]
[ 2.76275506  3.68367341] [ 0.39540824]
[ 2.90229588  3.86972784] [ 0.1628402]
[ 3.0185799  4.0247732] [ 0.0309665]
```

In [56]:
```
# Question 2d

x0 = np.array([0, 0]).reshape(2, 1)
b = np.array([4.5, 6]).reshape(2, 1)
f = lambda x: np.sqrt( np.dot( (x - b).T, (x - b) ) )
delta_f = lambda x: (x - b) / np.sqrt( np.dot( (x - b).T, (x - b) ) )

for i in range(10):
    x0 = x0 - 1 / (i + 1) * delta_f(x0)
    print(x0.flatten(), f(x0).flatten())
```

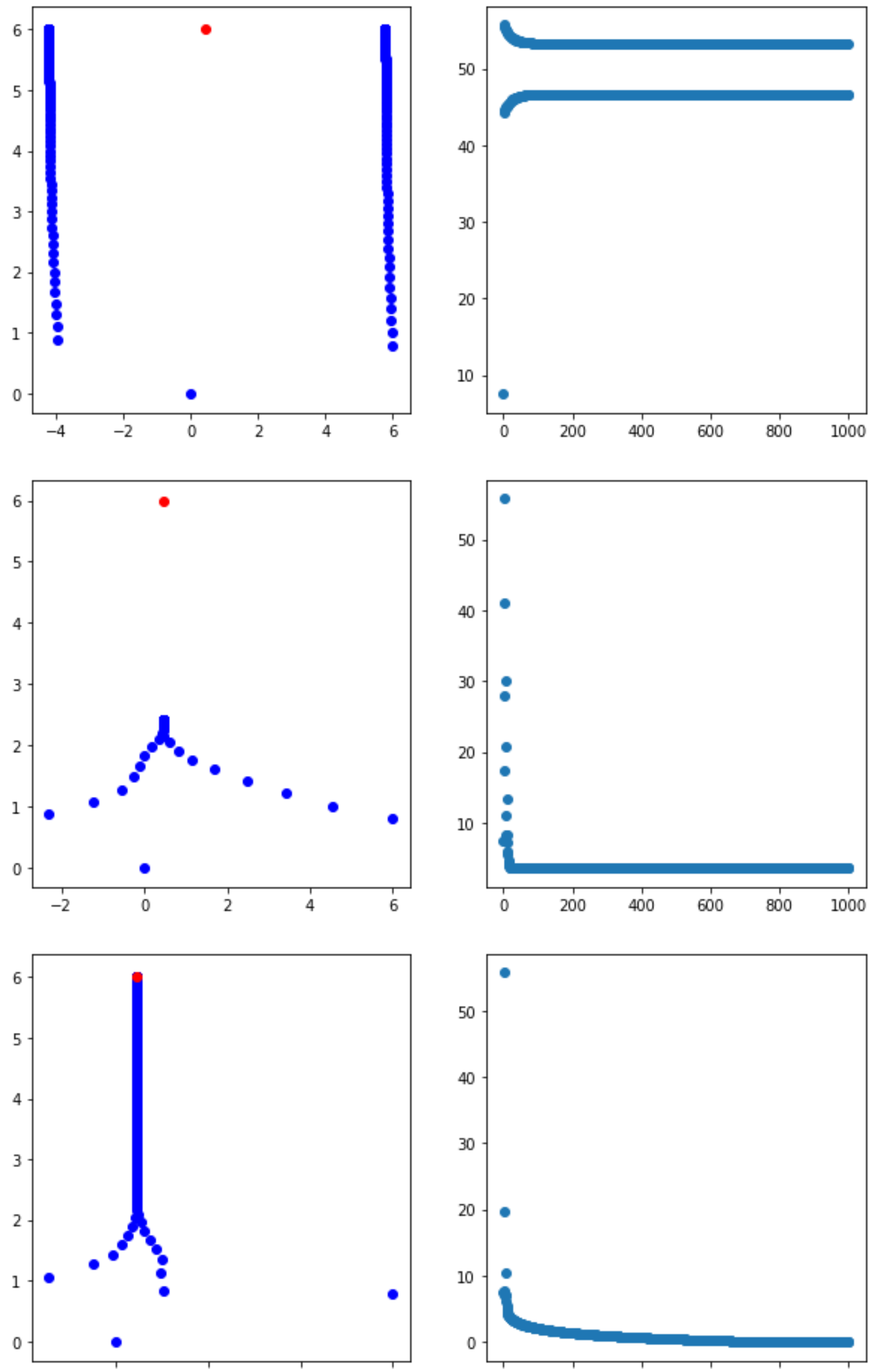
```
[ 0.6  0.8] [ 6.5]
[ 0.9  1.2] [ 6.]
[ 1.1         1.46666667] [ 5.66666667]
[ 1.25        1.66666667] [ 5.41666667]
[ 1.37        1.82666667] [ 5.21666667]
[ 1.47  1.96] [ 5.05]
[ 1.55571429  2.07428571] [ 4.90714286]
[ 1.63071429  2.17428571] [ 4.78214286]
[ 1.69738095  2.2631746 ] [ 4.67103175]
[ 1.75738095  2.3431746 ] [ 4.57103175]
```

In [65]:
```
def main(A, step_size):
    ##########################################################################
#########
    # TODO(student): Input Variables
    A  # do not change this until the last part
    b = np.array([4.5, 6])  # b in the equation ||Ax-b||
    initial_position = np.array([0, 0])  # position at iteration 0
    total_step_count = 1000  # number of GD steps to take
    step_size  # step size at iteration i
    ##########################################################################
#########

    # computes desired number of steps of gradient descent
    positions = compute_updates(A, b, initial_position, total_step_count, s
tep_size)

    # print out the values of the x_i
#     print(positions)
#     print(np.dot(np.linalg.inv(A), b))

    fig = plt.figure(figsize=(10, 5))
    # plot the values of the x_i
    ax1 = fig.add_subplot(121)
    ax1.scatter(positions[:, 0], positions[:, 1], c='blue')
    ax1.scatter(np.dot(np.linalg.inv(A), b)[0],
                np.dot(np.linalg.inv(A), b)[1], c='red')
    ax1.plot()

    # plot the values of f
    f = []
    for x in positions:
        f.append( np.linalg.norm(A.dot(x) - b) )
    ax2 = fig.add_subplot(122)
    ax2.plot(f, "o")
    plt.show()


def compute_gradient(A, b, x):
    """Computes the gradient of ||Ax-b|| with respect to x."""
    return np.dot(A.T, (np.dot(A, x) - b)) / np.linalg.norm(np.dot(A, x) -
b)


def compute_update(A, b, x, step_count, step_size):
    """Computes the new point after the update at x."""
    return x - step_size(step_count) * compute_gradient(A, b, x)


def compute_updates(A, b, p, total_step_count, step_size):
    """Computes several updates towards the minimum of ||Ax-b|| from p.

    Params:
        b: in the equation ||Ax-b||
        p: initialization point
        total_step_count: number of iterations to calculate
        step_size: function for determining the step size at step i
    """
    positions = [np.array(p)]
    for k in range(total_step_count):
        positions.append(compute_update(A, b, positions[-1], k, step_size))
    return np.array(positions)
```

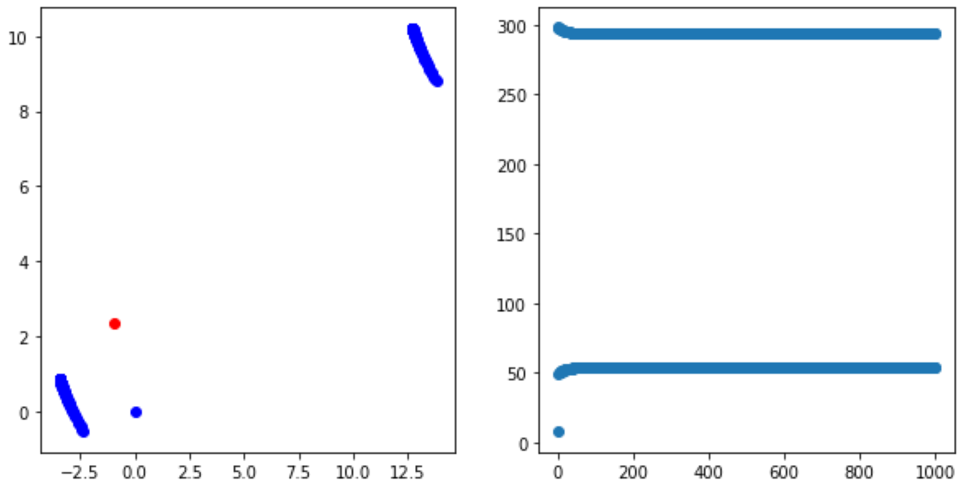
```
[[1 0]
 [0 1]]
```

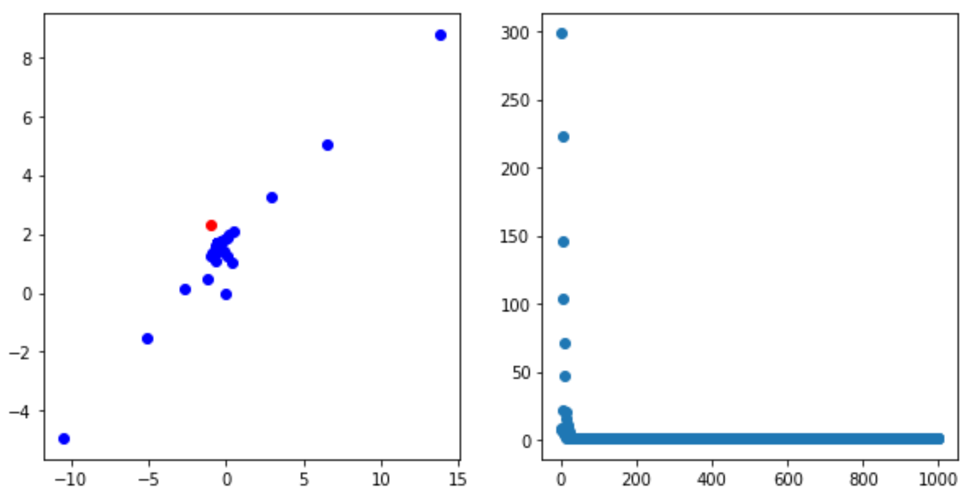

0 1 2 3 4 5 6
0 1 2 3 4 5
1 2 3 4 5 6 7
0 200 400 600 800 1000
0 1 2 3 4 5 6
0 1 2 3 4
2 3 4 5 6 7
0 200 400 600 800 1000
0 1 2 3 4 5 6
0 1 2 3 4
0 1 2 3 4 5 6 7
0 200 400 600 800 1000

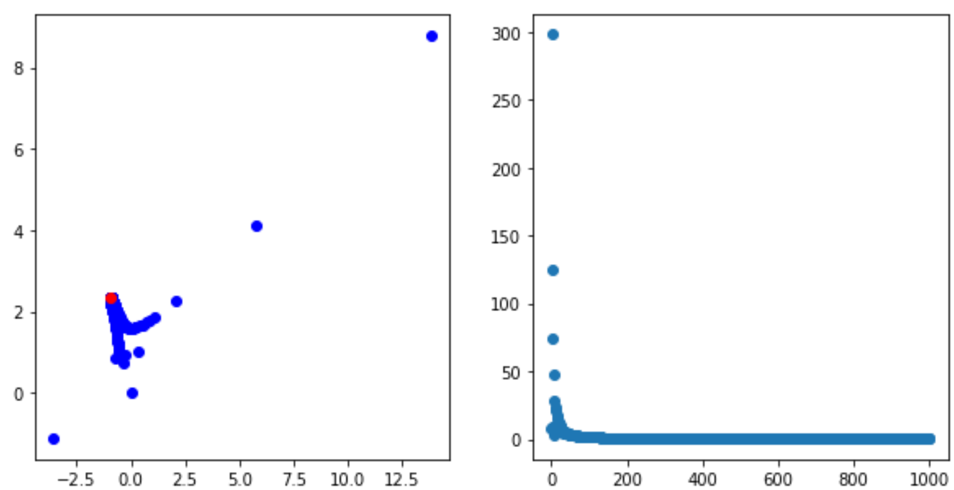
```
[[10  0]
 [ 0  1]]
```



```
[[15  8]
 [ 6  5]]
```







In [106]:
```
# Question 4

import scipy.spatial

def gradient(position, locations, distances):
    df_dx1_vector = 2 * (position[0] - locations[:, 0] - \
                 distances * (position[0] - locations[:, 0]) \
                 / np.sqrt( (locations[:, 0] - position[0]) ** 2 + (locati
ons[:, 1] - position[1]) ** 2 ))
    df_dx1 = np.sum(df_dx1_vector)

    df_dy1_vector = 2 * (position[1] - locations[:, 1] - \
                 distances * (position[1] - locations[:, 1]) \
                 / np.sqrt( (locations[:, 0] - position[0]) ** 2 + (locati
ons[:, 1] - position[1]) ** 2))
    df_dy1 = np.sum(df_dy1_vector)
    return np.array([df_dx1, df_dy1])


########################################################################
#########  Data Generating Functions ###################################
########################################################################
def generate_sensors(k = 7, d = 2):
        """
        Generate sensor locations.
        Input:
        k: The number of sensors.
        d: The spatial dimension.
        Output:
        sensor_loc: k * d numpy array.
        """
        sensor_loc = 100*np.random.randn(k,d)
        return sensor_loc

def generate_data(sensor_loc, k = 7, d = 2,
                                 n = 1, original_dist = True):
        """
        Generate the locations of n points.

        Input:
        sensor_loc: k * d numpy array. Location of sensor.
        k: The number of sensors.
        d: The spatial dimension.
        n: The number of points.
        original_dist: Whether the data are generated from the original
        distribution.

        Output:
        obj_loc: n * d numpy array. The location of the n objects.
        distance: n * k numpy array. The distance between object and
        the k sensors.
        """
        assert k, d == sensor_loc.shape

        obj_loc = 100*np.random.randn(n, d)
        if not original_dist:
           obj_loc += 1000

        distance = scipy.spatial.distance.cdist(obj_loc,

           sensor_loc,

           metric='euclidean')
```

```
Generated location:
[ 44.38632327  33.36743274]
Gradient Descent result:
[ 43.07188433  32.71217817]
```