Answers to Exercises

Chapter 1

1.2.1 Because B and B^c are disjoint events whose union is the whole sample space, the law of total probability (Section 1.2.1) applies to give the desired formula.

1.2.3 **(b)** $f(x) = \begin{cases} 0 & \text{for } x \le 0; \\ 3x^2 & \text{for } 0 < x < 1; \\ 0 & \text{for } x \ge 1. \end{cases}$

(c) $E[X] = \frac{3}{4}$.

(d) $\Pr\left\{\frac{1}{4} \le X \le \frac{3}{4}\right\} = \frac{26}{64}$.

1.2.4 **(b)** $E[Z] = \frac{9}{8}$.

(c) $\mathrm{Var}[Z] = \frac{55}{64}$.

1.2.7 **(a)** $F_X(x) = \begin{cases} 0 & \text{for } x < 0; \\ x^R & \text{for } 0 \le x \le 1; \\ 1 & \text{for } 1 < x. \end{cases}$

(b) $E[X] = R/(1+R)$.

(c) $\mathrm{Var}[X] = R/\left[(R+2)(R+1)^2\right]$.

1.2.8 $f(v) = A(1-v)^{A-1} \quad \text{for } 0 \le v \le 1;$
$E[V] = 1/(A+1);$
$\mathrm{Var}[V] = A/\left[(A+2)(A+1)^2\right]$.

1.2.9 $F_X(x) = \begin{cases} 0 & \text{for } x < 0; \\ \frac{1}{2}x^2 & \text{for } 0 \le x \le 1; \\ 1 - \frac{1}{2}(2-x)^2 & \text{for } 1 < x \le 2; \\ 1 & \text{for } x > 2. \end{cases}$

$E[X] = 1$; $\mathrm{Var}[X] = \frac{1}{6}$.

1.3.1 $\Pr\{X = 3\} = \frac{10}{32}$.

1.3.2 $\Pr\{0 \text{ defective}\} = 0.3151$.
$\Pr\{0 \text{ or } 1 \text{ defective}\} = 0.9139$.

1.3.3 $\Pr\{N = 10\} = 0.0315$.

1.3.4 $\Pr\{X = 2\} = 2\mathrm{e}^{-2} = 0.2707$.
$\Pr\{X \le 2\} = 5\mathrm{e}^{-2} = 0.6767$.

1.3.5 $\Pr\{X \ge 8\} = 0.1334$.

1.3.6 **(a)** Mean $= \frac{n+1}{2}$; Variance $= \frac{n^2-1}{12}$.

(b) $\Pr\{Z=m\} = \begin{cases} \frac{m+1}{n^2} & \text{for } m=0,\ldots,n; \\ \frac{2n+1-m}{n^2} & \text{for } m=n+1,\ldots,2n. \end{cases}$

(c) $\Pr\{U=k\} = \frac{1+2(n-k)}{(n+1)^2} \quad \text{for } k=0,\ldots,n.$

1.4.1 $\Pr\{X>1.5\} = e^{-3} = 0.0498.$
$\Pr\{X=1.5\}=0.$

1.4.2 Median $= \frac{1}{\lambda}\log 2$; Mean $= \frac{1}{\lambda}$.

1.4.3 Exponential distribution with parameter $\lambda/2.54$.

1.4.4 Mean $=0$; Variance $=1$.

1.4.5 $\alpha^* = \frac{\sigma_Y^2-\rho\sigma_X\sigma_Y}{\sigma_X^2+\sigma_Y^2-2\rho\sigma_X\sigma_Y} \quad \text{for } \rho \neq \pm 1.$

1.4.6 **(a)** $f_Y(y) = e^{-y} \quad \text{for } y \geq 0.$

(b) $f_W(w) = \frac{1}{n}\left(\frac{1}{w}\right)^{(n-1)/n} \quad \text{for } 0<w<1.$

1.4.7 R has the gamma density $f_R(r) = \lambda^2 r e^{-\lambda r} \quad \text{for } r>0.$

1.5.1 $\Pr\{X \geq 1\} = 0.6835938$
$\Pr\{X \geq 2\} = 0.2617188$
$\Pr\{X \geq 3\} = 0.0507812$
$\Pr\{X \geq 4\} = 0.0039062.$

1.5.2 Mean $= \frac{5}{7}$.

1.5.3 $E[X] = \frac{1}{\lambda}$.

1.5.4 **(a)** $E[X_A] = \frac{1}{2}; E[X_B] = \frac{1}{3};$
(b) $E[\min\{X_A, X_B\}] = \frac{1}{5};$
(c) $\Pr\{X_A < X_B\} = \frac{2}{5};$
(d) $E[X_B - X_A | X_A < X_B] = \frac{1}{3}.$

1.5.5 **(a)** $\Pr\{\text{Naomi is last}\} = \frac{1}{2};$
(b) $\Pr\{\text{Naomi is last}\} = \frac{282}{2500} = 0.1128;$
(c) $c = 2+\sqrt{3}.$

Chapter 2

2.1.1 $\Pr\{N=3, X=2\} = \frac{1}{16};$
$\Pr\{X=5\} = \frac{1}{48};$
$E[X] = \frac{7}{4}.$

2.1.2 $\Pr\{\text{two nickel heads}|N=4\} = \frac{3}{7}.$

2.1.3 $\Pr\{X \geq 1 | X \geq 1\} = 0.122184.$
$\Pr\{X > 1 | \text{Ace of spades}\} = 0.433513.$

2.1.4 $\Pr\{X=2\} = 0.2204.$

2.1.5 $E[X|X \text{ is odd}] = \lambda\left(\frac{e^{\lambda}+e^{-\lambda}}{e^{\lambda}-e^{-\lambda}}\right).$

2.1.6 $\Pr\{U=u, Z=z\} = \rho^2(1-\rho)^z, \quad 0 \leq u \leq z;$
$\Pr\{U=u|Z=n\} = \frac{1}{n+1}, \quad 0 \leq u \leq n.$

2.2.1 $\Pr\{\text{Game ends in a } 4\} = \frac{1}{4}$.

2.2.3 $\Pr\{\text{Win}\} = 0.468984$.

2.3.1

k	$\Pr\{Z = k\}$	
		$E[Z] = \frac{7}{4}$;
0	0.16406	$\mathrm{Var}[Z] = 1.604167$.
1	0.31250	
2	0.25781	
3	0.16667	
4	0.07552	
5	0.02083	
6	0.00260	

2.3.2 $E[Z] = \frac{3}{2}$; $\mathrm{Var}[Z] = \frac{9}{8}$;
$\Pr\{Z = 2\} = 0.29663$.

2.3.3 $E[Z] = \mu^2$; $\mathrm{Var}[Z] = \mu(1+\mu)\sigma^2$.

2.3.4 $\Pr\{X = 2\} = 0.2204$;
$E[X] = 2.92024$.

2.3.5 $E[Z] = 6$; $\mathrm{Var}[Z] = 26$.

2.4.1 $\Pr\{X = 2\} = \frac{1}{4}$.

2.4.2 $\Pr\{\text{System operates}\} = \frac{1}{2}$.

2.4.3 $\Pr\left\{U > \frac{1}{2}\right\} = 1 - \frac{1}{2}(1 + \log 2) = 0.1534$.

2.4.4 $f_Z(z) = \frac{1}{(1+z)^2}$ for $0 < z < \infty$.

2.4.5 $f_{U,V}(u, v) = e^{-(u+v)}$ for $u > 0, v > 0$.

2.5.1

x	$\frac{1}{2}$	1	2
$\Pr\{X > x\}$	0.61	0.37	0.14
$\frac{1}{x}E[X]$	2	1	$\frac{1}{2}$.

2.5.2 $\Pr\{X \geq 1\} = E[X] = p$.

Chapter 3

3.1.1 0.

3.1.2 0.12, 0.12.

3.1.3 0.03.

3.1.4 0.02, 0.02.

3.1.5 0.025, 0.0075.

3.2.1 **(a)** $\mathbf{P}^2 = \left\| \begin{matrix} 0.47 & 0.13 & 0.40 \\ 0.42 & 0.14 & 0.44 \\ 0.26 & 0.17 & 0.57 \end{matrix} \right\|$.

(b) 0.13.
(c) 0.16.

3.2.2

n	0	1	2	3	4
$\Pr\{X_n = 0 \mid X_0 = 0\}$	1	0	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{8}$

3.2.3 0.264, 0.254.

3.2.4 0.35.

3.2.5 0.27, 0.27.

3.2.6 0.42, 0.416.

3.3.1

$$\mathbf{P} = \begin{array}{c} \\ -1 \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccccc} -1 & 0 & 1 & 2 & 3 \end{array} \\ \left\|\begin{array}{ccccc} 0 & 0 & 0.3 & 0.3 & 0.4 \\ 0 & 0 & 0.3 & 0.3 & 0.4 \\ 0.3 & 0.3 & 0.4 & 0 & 0 \\ 0 & 0.3 & 0.3 & 0.4 & 0 \\ 0 & 0 & 0.3 & 0.3 & 0.4 \end{array}\right\| \end{array}$$

3.3.2 $P_{ii} = \left(\frac{i}{N}\right)p + \left(\frac{N-i}{N}\right)q$;
$P_{i,i+1} = \left(\frac{N-i}{N}\right)p$;
$P_{i,i-1} = \left(\frac{i}{N}\right)q$.

3.3.3

$$\mathbf{P} = \begin{array}{c} \\ -1 \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccccc} -1 & 0 & 1 & 2 & 3 \end{array} \\ \left\|\begin{array}{ccccc} 0 & 0 & 0.1 & 0.4 & 0.5 \\ 0 & 0 & 0.1 & 0.4 & 0.5 \\ 0.1 & 0.4 & 0.5 & 0 & 0 \\ 0 & 0.1 & 0.4 & 0.5 & 0 \\ 0 & 0 & 0.1 & 0.4 & 0.5 \end{array}\right\| \end{array}$$

3.3.4

$$\begin{array}{c} \\ -2 \\ -1 \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccccc} -2 & -1 & 0 & 1 & 2 & 3 \end{array} \\ \left\|\begin{array}{cccccc} 0 & 0 & 0.2 & 0.3 & 0.4 & 0.1 \\ 0 & 0 & 0.2 & 0.3 & 0.4 & 0.1 \\ 0 & 0 & 0.2 & 0.3 & 0.4 & 0.1 \\ 0.2 & 0.3 & 0.4 & 0.1 & 0 & 0 \\ 0 & 0.2 & 0.3 & 0.4 & 0.1 & 0 \\ 0 & 0 & 0.2 & 0.3 & 0.4 & 0.1 \end{array}\right\| \end{array}$$

3.3.5

$$\mathbf{P} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \left\|\begin{array}{ccc} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{array}\right\| \end{array}$$

3.4.1 $v_{03} = 10$.

3.4.2 **(a)** $u_{10} = \frac{1}{4}$;
(b) $v_1 = \frac{5}{2}$.

3.4.3 **(a)** $u_{10} = \frac{40}{105}$;
(b) $v_1 = \frac{10}{3}$.

3.4.4 $v_0 = 6$.

3.4.5 $u_{\mathrm{H,TT}} = \frac{1}{3}$.

3.4.6 **(a)** $u_{10} = \frac{6}{23}$;
(b) $v_1 = \frac{50}{23}$.

3.4.7 $w_{11} = \frac{20}{11}$; $w_{12} = \frac{25}{11}$
$v_1 = \frac{45}{11}$.

3.4.8 $w_{11} = 1.290$; $w_{12} = 0.323$
$v_1 = 1.613$.

3.4.9 $u_{10} = \frac{9}{22} = 0.40909\ldots$;

$$P_{10}^{(2)} = 0.17;$$
$$P_{10}^{(4)} = 0.2658;$$
$$P_{10}^{(8)} = 0.35762\ldots;$$
$$P_{10}^{(16)} = 0.40245\ldots.$$

3.5.1 0.71273.

3.5.2 **(a)** 0.8044; 0.99999928….
(b) 0.3578; 0.00288….

3.5.3 $\mathbf{P}^2 = \begin{Vmatrix} 0.58 & 0.42 \\ 0.49 & 0.51 \end{Vmatrix}$.

$$\mathbf{P}^3 = \begin{Vmatrix} 0.526 & 0.474 \\ 0.553 & 0.447 \end{Vmatrix}.$$

$$\mathbf{P}^4 = \begin{Vmatrix} 0.5422 & 0.4578 \\ 0.5341 & 0.4659 \end{Vmatrix}.$$

$$\mathbf{P}^5 = \begin{Vmatrix} 0.53734 & 0.46266 \\ 0.53977 & 0.46023 \end{Vmatrix}.$$

3.5.4

$$\mathbf{P} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{Vmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{Vmatrix} \end{array}.$$

3.5.5 $P_{\mathrm{GG}}^{(8)} = 0.820022583$.

3.5.6 2.73.

3.5.7 $u_{10} = 0.3797468$.

3.5.8 $p_0 = \alpha, r_0 = 1 - \alpha$;
$p_i = \alpha(1-\beta), q_i = \beta(1-\alpha)$,
$r_j = \alpha\beta + (1-\alpha)(1-\beta)$, for $i \geq 1$.

3.5.9 $p_0 = 1, q_0 = 0$,
$p_i = p, q_i = q, r_i = 0$ for $i \geq 1$.

3.6.1 **(a)** $u_{35} = \frac{3}{5}$;

(b) $u_{35} = \left[1 - \left(\frac{q}{p}\right)^3\right] \Big/ \left[1 - \left(\frac{q}{p}\right)^5\right]$.

3.6.2 $u_{10} = 0.65$.

3.6.3 $v = 2152.777\ldots$.

3.6.4 $v_1 = 2.1518987$.

3.7.1 $\mathbf{W} = \begin{Vmatrix} \frac{20}{11} & \frac{25}{11} \\ \frac{10}{11} & \frac{40}{11} \end{Vmatrix}$.

(a) $u_{10} = \frac{9}{22}$;

(b) $w_{11} = \frac{20}{11}$; $w_{12} = \frac{25}{11}$.

3.7.2 $\mathbf{W} = \begin{Vmatrix} \frac{100}{79} & \frac{70}{79} \\ \frac{30}{79} & \frac{100}{79} \end{Vmatrix}$.

(a) $u_{\mathrm{I}0} = \frac{30}{79}$;

(b) $w_{11} = \frac{100}{79}$; $w_{12} = \frac{70}{79}$.

3.8.1 $M(n) = 1$, $V(n) = n$.

3.8.2 $\mu = b + 2c$; $\sigma^2 = b + 4c - (b + 2c)^2$.

3.8.3

n	1	2	3	4	5
u_n	0.5	0.625	0.695	0.742	0.775

3.8.4 $M(n) = \lambda^n$, $V(n) = \lambda^n\left(\frac{1-\lambda^n}{1-\lambda}\right)$, $\lambda \neq 1$.

3.9.1

n	1	2	3	4	5
u_n	0.333	0.480	0.564	0.619	0.658

$u_\infty = 0.82387$.

3.9.2 $\varphi(s) = p_0 + p_2 s^2$.

3.9.3 $\varphi(s) = p + q s^N$.

3.9.4 $\frac{\varphi(s) - \varphi(0)}{1 - \varphi(0)}$.

Chapter 4

4.1.1 $\pi_0 = \frac{10}{21}, \pi_1 = \frac{5}{21}, \pi_2 = \frac{6}{21}$.

4.1.2 $\pi_0 = \frac{31}{66}, \pi_1 = \frac{16}{66}, \pi_2 = \frac{19}{66}$.

4.1.3 $\pi_1 = \frac{3}{13}$.

4.1.4 2.94697.

4.1.5 $\pi_0 = \frac{10}{29}, \pi_1 = \frac{5}{29}, \pi_2 = \frac{5}{29}, \pi_3 = \frac{9}{29}$.

4.1.6 $\pi_0 = \frac{5}{14}, \pi_1 = \frac{6}{14}, \pi_2 = \frac{3}{14}$.

4.1.7 $\pi_0 = \frac{140}{441}, \pi_1 = \frac{40}{441}, \pi_2 = \frac{135}{441}, \pi_3 = \frac{126}{441}$.

4.1.8 $\pi_u = \frac{4}{17}$.

4.1.9 $\pi_0 = \frac{2}{7}, \pi_1 = \frac{3}{7}, \pi_2 = \frac{2}{7}$.

4.1.10 $\pi_{\text{late}} = \frac{17}{40}$.

4.2.1 $\pi_s = \frac{8}{9}$.

4.2.2 One facility: $\Pr\{\text{Idle}\} = \frac{q^2}{1+p^2}$;

Two facilities: $\Pr\{\text{Idle}\} = \frac{1}{1+p+p^2}$.

4.2.3 **(a)**

p	0	0.02	0.04	0.06	0.08	0.10
AFI	0.10	0.11	0.12	0.13	0.14	0.16
AOQ	0	0.018	0.036	0.054	0.072	0.090

(b)

p	0	0.02	0.04	0.06	0.08	0.10
AFI	0.20	0.23	0.27	0.32	0.37	0.42
AOQ	0	0.016	0.032	0.048	0.064	0.080

4.2.4

p	0.05	0.10	0.15	0.20	0.25
R_1	0.998	0.990	0.978	0.962	0.941
R_2	0.998	0.991	0.981	0.968	0.952

4.2.5 $\pi_A = \frac{1}{5}$.

4.2.6 $\pi_0 = \frac{1}{3}$.

4.2.7 **(a)** 0.6831;

(b) $\pi_1 = \pi_2 = \frac{10}{21}, \pi_3 = \frac{1}{21}$;

(c) $\pi_3 = \frac{1}{21}$.

4.2.8 $\pi_3 = \frac{8}{51}$.

4.3.1 $\left\{n \geq 1; \mathrm{P}_{00}^{(n)} > 0\right\} = \{5, 8, 10, 13, 15, 16, 18, 20, 21, 23, 24, 25, 26, 28, \ldots\}$

$d(0) = 1, P_{5,7}^{(37)} = 0, P_{i,j}^{(38)} > 0 \quad$ for all i, j.

4.3.2 Transient states: $\{0, 1, 3\}$.
Recurrent states: $\{2, 4, 5\}$.

4.3.3 **(a)** $\{0, 2\}, \{1, 3\}, \{4, 5\}$;
(b) $\{0\}, \{5\}, \{1, 2\}, \{3, 4\}$.

4.3.4 $\{0\}, d = 1$;
$\{1\}, d = 0$;
$\{2, 3, 4, 5\}, d = 1$.

4.4.1 $\pi_k = p^k / \left(1 + p + p^2 + p^3 + p^4\right) \quad$ for $\quad k = 0, \ldots, 4$.

4.4.2 **(a)** $\pi_0 = \frac{1449}{9999}$.

(b) $m_{10} = \frac{8550}{1449}$.

4.4.3 $\pi_0 = \pi_1 = 0.2, \pi_2 = \pi_3 = 0.3$.

4.5.1 $\lim P_{00}^{(n)} = \lim P_{10}^{(n)} = 0.4$;

$\lim P_{20}^{(n)} = \lim P_{30}^{(n)} = 0$;

$\lim P_{40}^{(n)} = 0.4$.

4.5.2 **(a)** $\frac{3}{11}$, **(e)** $\frac{3}{11}$,
(b) 0, **(f)** X,
(c) $\frac{2}{33}$, **(g)** $\frac{1}{3}$,
(d) $\frac{2}{9}$, **(h)** $\frac{4}{27}$.

Chapter 5

5.1.1 **(a)** e^{-2};
(b) e^{-2}.

5.1.2 $(p_k/p_{k-1}) = \lambda/k, \quad k = 0, 1, \ldots.$

5.1.3 $\Pr\{X = k|N = n\} = \binom{n}{k} p^k(1-p)^{n-k}, \quad p = \frac{\alpha}{\alpha+\beta}.$

5.1.4 **(a)** $\frac{(\lambda t)^k e^{-\lambda t}}{k!}, \quad k = 0, 1, \ldots;$
(b) $\Pr\{X(t) = n+k|X(s) = n\} = \frac{[\lambda(t-s)]^k e^{-\lambda(t-s)}}{k!},$
$E[X(t)X(s)] = \lambda^2 ts + \lambda s.$

5.1.5 $\Pr\{X = k\} = (1-p)p^k$ for $k = 0, 1, \ldots$ where $p = 1/(1+\theta)$.

5.1.6 **(a)** e^{-12};
(b) Exponential, parameter $\lambda = 3$.

5.1.7 **(a)** $2e^{-2}$;
(b) $\frac{64}{3}e^{-6}$;
(c) $\binom{6}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^4$;
(d) $\frac{32}{3}e^{-4}$.

5.1.8 **(a)** $5e^{-2}$;
(b) $4e^{-4}$;
(c) $\frac{1-3e^{-2}}{1-e^{-2}}$.

5.1.9 **(a)** 4;
(b) 6;
(c) 10.

5.2.1

k	0	1	2
(a)	0.290	0.370	0.225
(b)	0.296	0.366	0.221
(c)	0.301	0.361	0.217

5.2.2 Law of rare events, e.g., (a) Many potential customers who could enter store, small probability for each to actually enter.

5.2.3 The number of distinct pairs is large; the probability of any particular pair being in sample is small.

5.2.4 $\Pr\{\text{Three pages error free}\} \approx e^{-12}$.

5.3.1 e^{-6}.

5.3.2 **(a)** $e^{-6} - e^{-10}$;
(b) $4e^{-4}$.

5.3.3 $\frac{1}{4}$.

5.3.4 $\binom{5}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^3 = \frac{80}{243}$.

5.3.5 $\binom{n}{m}\left(\frac{t}{T}\right)^m\left(1-\frac{t}{T}\right)^{n-m}, \quad m = 0, 1, \ldots, n.$

5.3.6 $F(t) = \left(1 - \mathrm{e}^{-\lambda t}\right)^n$.

5.3.7 $t + \frac{2}{\lambda}$.

5.3.8 $\binom{12}{5}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^7$.

5.3.9 $\Pr\{W_r \le t\} = 1 - \sum_{k=0}^{r-1} \frac{(\lambda t)^k \mathrm{e}^{-\lambda t}}{k!}$.

5.4.1 $\frac{1}{n+1}$.

5.4.2 $\frac{1}{4}$.

5.4.3 $\frac{5}{2}$.

5.4.4 See equation (5.23).

5.4.5 $\left[1 - \frac{1-\mathrm{e}^{-\alpha}}{\alpha}\right]^5$.

5.5.1 0.9380.

5.5.2 0.05216.

5.5.3 0.1548.

5.6.1 0.0205.

5.6.2 Mean $= \frac{\lambda t}{\theta}$, Variance $= \frac{2\lambda t}{\theta^2}$.

5.6.3 $\frac{\mathrm{e}^{-\lambda G(z)t} - \mathrm{e}^{-\lambda t}}{1 - \mathrm{e}^{-\lambda t}}$.

5.6.4 **(a)** $\frac{1}{9}$;
(b) $\frac{11}{27}$.

5.6.5 $\Pr\{M(t) = k\} = \frac{\Lambda(t)^k \mathrm{e}^{-\Lambda(t)}}{k!}, \quad \text{where} \quad \Lambda(t) = \lambda \int_0^t [1 - G(u)]\mathrm{d}u.$

Chapter 6

6.1.1 $P_0(t) = \mathrm{e}^{-t}$;

$P_1(t) = \frac{1}{2}\mathrm{e}^{-t} - \frac{1}{2}\mathrm{e}^{-3t}$;

$P_2(t) = 3\left[\frac{1}{2}\mathrm{e}^{-t} + \frac{1}{2}\mathrm{e}^{-3t} - \frac{1}{2}\mathrm{e}^{-2t}\right]$

$P_3(t) = 6\left[\frac{1}{8}\mathrm{e}^{-t} + \frac{1}{4}\mathrm{e}^{-3t} - \frac{1}{3}\mathrm{e}^{-2t} - \frac{1}{24}\mathrm{e}^{-5t}\right]$.

6.1.2 **(a)** $\frac{11}{6}$;
(b) $\frac{25}{6}$;
(c) $\frac{49}{36}$.

6.1.3 $X(t)$ is a Markov process (memoryless property of exponential distribution) for which the sojourn time in state k is exponentially distributed with parameter λk.

6.1.4 **(a)** $\Pr\{X=0\}=(1-\alpha h)^n=1-n\alpha h+o(h)$;
(b) $\Pr\{X=1\}=n\alpha h(1-\alpha h)^n=n\alpha h+o(h)$;
(c) $\Pr\{X\geq 2\}=1-\Pr\{X=0\}-\Pr\{X-1\}=o(h)$.

6.1.5 $E[X(t)]=\frac{1}{p}$, $\operatorname{Var}[X(t)]=\frac{1-p}{p^2}$, where $p=\mathrm{e}^{-\beta t}$.

6.1.6 **(a)** $P_1(t)=\mathrm{e}^{-5t}$;
(b) $P_2(t)=5\left[\frac{1}{2}\mathrm{e}^{-3t}-\frac{1}{2}\mathrm{e}^{-5t}\right]$
(c) $P_3(t)=15\left[\frac{1}{20}\mathrm{e}^{-3t}-\frac{1}{16}\mathrm{e}^{-5t}+\frac{1}{80}\mathrm{e}^{-13t}\right]$.

6.2.1 $P_3(t)=\mathrm{e}^{-5t}$;
$P_2(t)=5\left[\frac{1}{3}\mathrm{e}^{-2t}-\frac{1}{3}\mathrm{e}^{-5t}\right]$;
$P_1(t)=10\left[\frac{1}{6}\mathrm{e}^{-5t}+\frac{1}{3}\mathrm{e}^{-2t}-\frac{1}{2}\mathrm{e}^{-3t}\right]$;
$P_0(t)=1-P_1(t)-P_2(t)-P_3(t)$.

6.2.2 **(a)** $\frac{31}{30}$;
(b) $\frac{29}{15}$;
(c) $\frac{361}{900}$.

6.2.3 $P_3(t)=\mathrm{e}^{-t}$;
$P_2(t)=\left[\mathrm{e}^{-t}-\mathrm{e}^{-2t}\right]$;
$P_1(t)=2\left[\frac{1}{2}\mathrm{e}^{-t}-\mathrm{e}^{-2t}+\frac{1}{2}\mathrm{e}^{-3t}\right]$;
$P_0(t)=1-P_1(t)-P_2(t)-P_3(t)$.

6.2.4 $P_2(t)=10\mathrm{e}^{-4t}\left(1-\mathrm{e}^{-2t}\right)^3$.

6.3.1 $\lambda_n=\lambda,\mu_n=n\mu$ for $n=0,1,\ldots$.

6.3.2 Assume that $\Pr\{\text{Particular patient exits in } [t,t+h)|k \text{ patients}\}=\frac{1}{m_k}h+o(h)$.

6.4.2 $\pi_k=\binom{N}{k}p^k(1-p)^{N-k}$, where $p=\frac{\alpha}{\alpha+\beta}$.

6.4.3 $\pi_k=\frac{\lambda^k\mathrm{e}^{-\lambda}}{k!}$, where $\lambda=\frac{\alpha}{\beta}$.

6.4.4 **(a)** $\pi_0=1\Big/\left(1+\frac{\lambda}{\mu}+\frac{1}{2}\left(\frac{\lambda}{\mu}\right)^2\right)$;
(b) $\pi_0=1\Big/\left(1+\frac{\lambda}{\mu}+\left(\frac{\lambda}{\mu}\right)^2\right)$.

6.4.5 $\pi_k=(k+1)(1-\theta)^2\theta^k$.

6.4.6 $\pi_k=\frac{\theta^k\mathrm{e}^{-\theta}}{k!}$, where $\theta=\frac{\lambda}{\mu}$.

6.5.1 Use $\log\frac{1}{1-x}=1+x+\frac{1}{2}x^2+\cdots|x|<1$.

6.5.2 Use $\frac{\mathrm{K}}{\mathrm{K}-i}\cong 1$ for $\mathrm{K}\gg i$.

6.6.1 $1-\left(\frac{\beta_A}{\alpha_A+\beta_A}\right)\left(\frac{\beta_B}{\alpha_B+\beta_B}\right)$.

6.6.2 $P_{00}(t) = \left\{\frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu}\mathrm{e}^{-(\lambda+\mu)t}\right\}^2.$

6.7.1 $f(t) = \frac{2-\sqrt{2}}{4}\mathrm{e}^{-(2+\sqrt{2})t} + \frac{2+\sqrt{2}}{4}\mathrm{e}^{-(2-\sqrt{2})t}.$

6.7.2 $f_0'(t) = -\frac{\sqrt{2}}{4}\mathrm{e}^{-(2-\sqrt{2})t} + \frac{\sqrt{2}}{4}\mathrm{e}^{-(2+\sqrt{2})t};$

$f_1'(t) = -\frac{2-\sqrt{2}}{4}\mathrm{e}^{-(2-\sqrt{2})t} - \frac{2+\sqrt{2}}{4}\mathrm{e}^{-(2+\sqrt{2})t}.$

Chapter 7

7.1.1 The age δ_t of the item in service at time t cannot be greater than t.

7.1.2 $F_2(t) = 1 - \mathrm{e}^{-\lambda t} - \lambda t\mathrm{e}^{-\lambda t}.$

7.1.3 **(a)** True;
(b) False;
(c) False.

7.1.4 **(a)** W_k has a Poisson distribution with parameter λk.
(b) $\Pr\{N(t) = k\} = \sum_{n=0}^{t} \frac{\mathrm{e}^{-\lambda k}(\lambda k)^n}{n!} - \sum_{n=0}^{t} \frac{\mathrm{e}^{-\lambda(k+1)}(\lambda k+\lambda)^n}{n!}.$

7.2.1 $\frac{1}{a} + \frac{1}{b}.$

7.2.2 The system starts from an identical condition at those instants when first both components are OFF.

7.2.3

n	$M(n)$	$u(n)$
1	0.4	0.4
2	0.66	0.26
3	1.104	0.444
4	1.6276	0.5236
5	2.0394	0.41184
6	2.4417	0.4023
7	2.8897	0.44798
8	3.3374	0.44769
9	3.7643	0.42693
10	4.1947	0.4304

7.3.1 $\mathrm{e}^{-\lambda s}; t + \frac{1}{\lambda}.$

7.3.2 The inter-recording time is a random sum with a geometric number of exponential terms and is exponential with parameter λp. (See the example following Chapter 2, (2.34)).

7.3.3 $\Pr\{N(t) = n, W_{N(t)+1} > t+s\}$

$= \left\{\frac{(\lambda t)^n \mathrm{e}^{-\lambda t}}{n!}\right\}\mathrm{e}^{-\lambda s}.$

7.4.1 $M(t) \approx \frac{3}{2}t - \frac{7}{16}.$

7.4.2 $\frac{3}{8}.$

7.4.3 $T^* = \sqrt{\frac{\sqrt{273}-15}{8}}.$

7.4.4 $c(T)$ is a decreasing function of T.

7.4.5 $h(x) = 2(1-x) \quad \text{for} \quad 0 \le x \le 1.$

7.4.6 **(a)** $\frac{\alpha}{\alpha+\beta}$;

(b) $13\frac{\alpha}{\alpha+\beta}$;

(c) $\frac{7}{\alpha+\beta}$.

7.5.1 $\frac{\mu\lambda}{1+\mu\lambda}$.

7.5.2 $\frac{2}{9}$.

7.5.3 $T^* = \infty$.

7.6.1–7.6.2

n	v_n	u_n
0	0.6667	1.33333
1	1.1111	0.88888
2	0.96296	1.03704
3	1.01235	0.98765
4	0.99588	1.00412
5	1.00137	0.99863
6	0.99954	1.00046
7	1.00015	0.99985
8	0.99995	1.00005
9	1.00002	0.99998
10	0.99999	1.00001

7.6.3 $E[X] = 1$; $\Sigma b_k = 1$.

Chapter 8

8.1.1 **(a)** 0.8413.

(b) 4.846.

8.1.2 $\text{Cov}[W(s), W(t)] = \min\{s, t\}$.

8.1.3 $\frac{\partial p}{\partial t} = \frac{1}{2}\varphi(z)t^{-\frac{3}{2}}\left[z^2 - 1\right]$;

$\frac{\partial p}{\partial x} = \frac{1}{t}z\varphi(z)$.

8.1.4 **(a)** 0.

(b) $3u^2 + 3uv + uw$.

8.1.5 **(a)** $e^{-|s-t|}$;

(b) $t(1-s) \quad \text{for} \quad 0 < t < s < 1$;

(c) $\min\{s, t\}$.

8.1.6 **(a)** Normal, $\mu = 0, \sigma^2 = 4u + v$;

(b) Normal, $\mu = 0, \sigma^2 = 9u + 4v + w$.

8.1.7 $\frac{s}{S}$.

8.2.1 **(a)** 0.6826.

(b) 4.935.

8.2.2 If $\tan\theta = \sqrt{s/t}$, then $\cos\theta = \sqrt{t/(s+t)}$.

8.2.3 0.90.

8.2.4 Reflection principle: $\Pr\{M_n \geq a, S_n < a\} =$
$\Pr\{M_n \geq a, S_n > a\} (= \Pr\{S_n > a\})$.
Also, $\Pr\{M_n \geq a, S_n = a\} = \Pr\{S_n = a\}$.

8.2.5 $\Pr\{\tau_0 < t\} = \Pr\{B(u) \neq 0 \quad \text{for all } t \leq u < a\} = 1 - \vartheta(t, a)$.

8.2.6 $\Pr\{\tau_1 < t\} = \Pr\{B(u) = 0 \quad \text{for some } u, b < u < t\}$.

8.3.1 $\Pr\{R(t) < y | R(0) = x\}$
$= \Pr\{-y < B(t) < y | B(0) = x\}$
$= \Pr\{B(t) < y | B(0) = x\} - \Pr\{B(t) < -y | B(0) = x\}$.

8.3.2 0.3174, 0.15735.

8.3.3 0.83995.

8.3.4 0.13534.

8.3.5 No, No.

8.4.1 0.05.

8.4.2 0.5125, 0.6225, 0.9933.

8.4.3 0.25, 24.5, 986.6.

8.4.4 $\tau = 43.3 \quad \text{versus} \quad E[T] = 21.2$

8.4.5 0.3325.

8.4.6 **(a)**
$$E\left[(\xi - a)^+\right] = \int_a^\infty x\varphi(x)\mathrm{d}x - a\int_a^\infty \varphi(x)\mathrm{d}x$$
$$= \varphi(a) - a[1 - \Phi(a)].$$

(b) $(X - b)^+ = \sigma\left(\xi - \frac{b-\mu}{\sigma}\right)^+$.

8.5.1 0.8643, 0.7389, 0.7357.

8.5.2 0.03144, 0.4602, 0.4920.

8.5.3 **(a)** $E[V_n] = (1 - \beta)^n v$
$\mathrm{Cov}[V_n, V_{n+k}] = (1 - \beta)^k$.
(b) $E[\Delta V | V_n = v] = -\beta v$
$\mathrm{Var}[\Delta V | V_n = v] = 1$.

Chapter 9

9.1.1 **(a)** Probability waiting planes exceed available air space.
(b) Mean number of cars in lot.

9.1.2 The standard deviation of the exponential distribution equals the mean.

9.1.3 $1\frac{1}{2}$ days.

9.2.1 $\frac{25}{36}$.

9.2.3 $L = 1 \quad \text{versus} \quad L = 5$.

9.3.1 $\nu = \frac{1}{\mu}, \tau^2 = \frac{1}{\mu^2}, L = \frac{\rho}{1-\rho}$.

9.3.2 $L = \rho + \frac{\rho^2(1+\alpha)}{2\alpha(1-\rho)}$.

9.3.3 $3\frac{1}{4}$.

9.3.4 $\frac{27}{36}$ versus 3.

9.3.5 $M(t) = \lambda \int_0^t [1 - G(y)]\mathrm{d}y \to \lambda\nu$.

9.4.1 8 per hour.

9.4.2 $\frac{5}{16}$.

9.4.3 8.55.

9.4.4 0.0647.

9.5.1 $\frac{1}{5}\left(\frac{4}{5}\right) = \frac{4}{25}$.

9.5.2 $\Pr\{Z \leq 20\} = 0.9520$.