Tutorial 1
STAT 150

Exercise 1. Let X have a Poisson distribution with parameter $\lambda > 0$. Suppose λ itself is random, following an exponential density with parameter θ.

(a) What is the marginal distribution of X?
(b) Determine the conditional density for λ given $X = k$.

Exercise 2.
Let (ξ_i) be i.i.d such that $\mathbb{P}(\xi_i = 1) = \mathbb{P}(\xi_i = -1) = \frac{1}{2}$, and let (X_t) be the random walk defined by :

$$X_t = \sum_{i=1}^{t} \xi_i$$

Let $K > 0$ be fixed integer. Find an explicit formula for :

1)

$$p(x) = \mathbb{P}(\text{the walk hits K before } 0 | X_0 = x)$$

2)

$$s(x) = \mathbb{E}(\text{number of steps before reaching either 0 or K} | X_0 = x)$$

Exercise 3.
Let T be uniform on $[0, 1]$, and let U be uniform on $[0, T]$ given T. What is $\mathbb{P}(U > \frac{1}{2})$?

Exercise 4.
Max runs a dolphin-watch business. Every day, he is unable to run the trip due to bad weather with probability p, independently of all the other days. Let Y be the number of days he works on between two bad-weather days. Let X be the number of customers who go on Max's trip in this period of Y days. Conditional on Y, the distribution of X is :

$$(X|Y) \sim \text{Poisson}(\mu Y)$$

(a) Name the distribution of Y , and state $\mathbb{E}[Y]$ and $\text{Var}(Y)$.
(b) Find $\mathbb{E}[X]$ and $\text{Var}(X)$.