Tutorial 11
STAT 150

Exercise 1 :

A birth and death chain has an absorbing state at 0, so $P_{0,0} = 1$ and transition probabilities for $i \geq 1$ given by, $P_{i,i-1} = q_i > 0$,$P_{i,i+1} = p_i > 0$ and $P_{i,i} = 1 - p_i - q_i = r_i$.

(a) Find a non-constant P-harmonic function for the Markov chain. (i.e h s.t $Ph = h$)

(b) Starting from position x find the probability of reaching state n before state 0.

(c) Find the probability of ever reaching state 0 starting from x.

Exercise 2 :

Prove that $\displaystyle\int_0^1 B_t dt \sim \mathcal{N}(0, \frac{1}{3})$

Hint : For every continuous function f we have : $\displaystyle\int_0^1 f(t)dt = \lim_{n\to+\infty} \frac{1}{n}\sum_{k=1}^{n} f(\frac{k}{n})$

Exercise 3 :

Let $N(t)$ be a renewal process with increments X_i having a CDF F; Let W_i the time of the i-th arrival (i.e $W_i = X_1 + .. + X_i$), let F_n be the CDF of W_n which is the n-th convolution of F.

Now for every $t \geq 0$, let $\delta_t = t - W_{N(t)}$ be the current life and $\gamma_t = W_{N(t)+1} - t$ be the residual lifetime.

(a) Find the joint distribution of δ_t and γ_t in function of the convolutions F_n's.

(b) Compute it explicitly for the case when $N(t)$ is a Poisson process (i.e when the increments X_i are exponential with parameter λ). We have $dF_k(z) = \frac{\lambda^k z^{k-1}}{(k-1)!} e^{-\lambda z} dz$