Stat 150, Fall 2018, HW #8

This homework is not for marks or to be handed in.

1. Pinsky and Karlin [PK], Problems (*not* exercises):

   7.1.3 (see p349 for the definition of γ_t)

   7.3.1

   7.3.5

   7.4.1

   7.4.4

   7.4.5

2. Let $(N(t), t \geq 0)$ be a renewal process. Let $M(t) = \mathbb{E}N(t)$ denote the expected number of events observed by time t, and $F(x) = \mathbb{P}(X \leq x)$ the common CDF for the interarrival times $(X_i, i \geq 1)$ between events.

   (a) Show that M and F satisfy the *renewal equation*

   $$M(t) = F(t) + \int_0^t M(t-x)dF(x).$$

   (b) Suppose that $X_i \sim \text{Unif}(0,1)$. Show that $M(t) = e^t - 1$ for $t \in [0,1]$. *Hint:* Let $H(x) = M(t) + 1$. Use the renewal equation to solve for $H(x)$.

   (c) Let $U_1, U_2, \ldots$ be IID $\text{Unif}(0,1)$, and put $N = \min\{k \geq 1 : \sum_{i=1}^k U_i > 1\}$. Show that $\mathbb{E}N = e$.