Tutorial 10
STAT 150

In every question, let $W(t)$ be a standard Brownian Motion.

Exercise 1 :

Find :

(a) $\text{Cov}(W_1^2, W_2)$.

(b) $\mathbb{P}[1 \leq W_1 \leq 2 | W_2 = 1]$.

(c) The conditionnal distribution of $W(2)$ given that $W(1) = 2$ and $W(3) = 4$.

(d) $\mathbb{E}[\int_0^2 W_t dt | W_1 = 1]$.

Exercise 2 :

Suppose $0 < a < b$.

(a) Conditional on $W_a = x$ find the probability that the Brownian Motion has no zeros in (a, b).

(b) Find the unconditional probability that the Brownian Motion has no zeros in (a, b).

(c) Let $T = \max\{s \in (0, 1) : W_s = 0\}$. Find the cumulative distribution function of T.

Exercise 3 :

Show that the following processes are martingales :

(a) $B_t^2 - t$.

(b) $\exp(\lambda B_t - \frac{\lambda^2 t}{2})$ for every $\lambda \in \mathbb{R}$.

(c) $W(t)t - \int_0^t W(s)ds$.