Stat 150, Fall 2018, HW #4

Due Thurs Oct 18 at the start of class 9:30 AM in Evans 10.
Late assignments will not be accepted.

1. Pinsky and Karlin [PK], Problems (*not* exercises):

   5.3.5

   5.3.6. *Hint:* Relate $\int_0^T N(t)dt$ to the amount of time the first $Q-1$ customers wait to be processed.

   5.4.2

   5.4.4. *Hint:* Let $Z_t = \min\{W_1+Z_1, \ldots, W_{X(t)}+Z_{X(t)}\}$. Find $\mathbb{P}(Z_t > z)$, and then let $t \to \infty$ to obtain $\mathbb{P}(Z > z)$.

2. Durrett [D], Exercises:

   2.38

   2.53

3. Let $(X_t : t \geq 0)$ be a Poisson process with rate $\lambda > 0$. Let W_n be the time of the nth event. Find:

   (a) $\mathbb{E}(X_5)$
   (b) $\mathbb{E}(W_3)$
   (c) $\mathbb{P}(X_5 < 3)$
   (d) $\mathbb{P}(W_3 > 5)$
   (e) $\mathbb{P}(W_3 > 5 | X_2 = 1)$.

4. Robins and blackbirds make short, independent visits to a bird feeder. The number of robins seen by time t is a Poisson process $(R_t : t \geq 0)$ with rate $\lambda > 0$. The number of blackbirds seen by time t is a Poisson process $(B_t : t \geq 0)$ with rate $\mu > 0$.

   (a) Argue that $(T_t = R_t + B_t : t \geq 0)$ is a Poisson process, and give its rate.
   (b) Find the probability that the first bird to arrive is a robin.
   (c) Given that n birds have arrived by time t, identify the conditional distribution of the number of robins that have arrived by time t.