Stat 200A, Fall 2018
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WEEK 7 EXERCISES

You are expected to do all these problems, but for **Homework 7** please turn in **only Problems 2, 3, 5, and 6** on **Thursday October 11 at the start of lecture**.

1. Poisson MGF

Let X have Poisson (μ) distribution, and let Y independent of X have Poisson (λ) distribution.

a) Find the mgf of X.

b) Use the result of Part **a** to show that the distribution of $X+Y$ is Poisson.

2. Bilateral Exponential Moments

Let X have density

$$f_X(x) \quad = \quad \frac{1}{2}e^{-|x|}, \qquad -\infty < x < \infty$$

a) Sketch a graph of f_X.

b) Find the mgf of X (careful about where it is defined).

c) Use the mgf to find the even moments of X. The odd moments are of course all 0.

d) Check your answer to Part **c** by finding the even moments by direct integration.

3. Gamma Tail Bound

Let X have the gamma (r, λ) distribution. Show that

$$P\left(X \geq \frac{2r}{\lambda}\right) \quad \leq \quad \left(\frac{2}{e}\right)^r$$

4. Poisson Phone Calls

This is Pitman Exercise 4.2.5.

Suppose calls are arriving at a telephone exchange at an average rate of 2 per second, according to a Poisson arrival process. Find

a) the probability that the fourth call after time $t = 0$ arrives within 2 seconds of the third call;

b) the probability that the fourth call arrives by time $t = 5$ seconds;

c) the expected time at which the fourth call arrives

5. More Poisson Phone Calls

This is Pitman Exercise 4.Review.13.

Local calls are coming into a telephone exchange according to a Poisson process with rate λ_{loc} calls per minute. Indpendently of this, long distance calls are coming in at a rate of λ_{dis} calls per minute. Write down expressions for the probabilities of the following events:

a) exactly 5 local calls and 3 long distance calls arrive in a given minute

b) exactly 50 calls (counting both local and long distance) arrive in a given three-minute period

c) starting from a fixed time, the first ten calls to arrive are local

6. Poisson Particles

This is Pitman Exercise 4.Review.14.

Particles arrive at a Geiger counter according to a Poisson process with rate 3 per minute.

a) Find the chance that fewer than 4 particles arrive in the time interval 0 to 2 minutes.

b) Let T_n minutes denote the arrival time of the nth particle. Find

$$P(T_1 < 1, T_2 - T_1 < 1, T_3 - T_2 < 1)$$

c) Find the conditional distribution of the number of arrivals in 0 to 2 minutes given that there were 10 arrivals in 0 to 4 minutes. Recognize this as a named distribution, and state the parameters.

7. Poisson Toll Booth

This is a version of Pitman Exercise 4.Review.16

Cars arrive at a toll booth according to a Poisson process at rate of 3 arrivals per minute. Of the cars arriving at the booth, it is known that over the long run 60% are Japanese imports. What is the probability that the cars that arrive in a given 10-minute period consist of 10 that are Japanese imports and 5 that are not? State your assumptions clearly.