Stat 200A Fall 2018 Midterm Reference Sheet — A. Adhikari

name and parameters	values	mass fn. or pdf	cdf or survival fn.	expectation	variance	mgf $M(t)$
Uniform	$m \le k \le n$	$1/(n-m+1)$		$(m+n)/2$	$((n-m+1)^2-1)/12$	
Bernoulli (p)	0, 1	$p_1 = p, p_0 = q$		p	pq	$q + pe^t$
Binomial (n, p)	$0 \le k \le n$	$\binom{n}{k} p^k q^{n-k}$		np	npq	$(q+pe^t)^n$
Poisson (μ)	$k \ge 0$	$e^{-\mu}\mu^k/k!$		μ	μ	$\exp(\mu(e^t-1))$
Geometric (p)	$k \ge 1$	$q^{k-1}p$	$P(X > k) = q^k$	$1/p$	q/p^2	
"Negative binomial" (r, p)	$k \ge r$	$\binom{k-1}{r-1} p^{r-1} q^{k-r} p$		r/p	rq/p^2	
Geometric (p)	$k \ge 0$	$q^k p$	$P(X > k) = q^{k+1}$	q/p	q/p^2	
Negative binomial (r, p)	$k \ge 0$	$\binom{k+r-1}{r-1} p^{r-1} q^k p$		rq/p	rq/p^2	
Hypergeometric (N, G, n)	$0 \le g \le n$	$\binom{G}{g}\binom{B}{b}/\binom{N}{n}$		$n\frac{G}{N}$	$n\frac{G}{N} \cdot \frac{B}{N} \cdot \frac{N-n}{N-1}$	
Uniform	$x \in (a, b)$	$1/(b-a)$	$F(x) = (x-a)/(b-a)$	$(a+b)/2$	$(b-a)^2/12$	
Beta (r, s)	$x \in (0, 1)$	$\frac{\Gamma(r+s)}{\Gamma(r)\Gamma(s)} x^{r-1}(1-x)^{s-1}$		$r/(r+s)$	$rs/((r+s)^2(r+s+1))$	
Exponential (λ) = Gamma $(1, \lambda)$	$x \ge 0$	$\lambda e^{-\lambda x}$	$F(x) = 1 - e^{-\lambda x}$	$1/\lambda$	$1/\lambda^2$	
Gamma (r, λ)	$x \ge 0$	$\frac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x}$	by Pois. proc. for integer r	r/λ	r/λ^2	$(\lambda/(\lambda - t))^r$ for $t < \lambda$
Chi-square (n)	$x \ge 0$	same as gamma $(n/2, 1/2)$		n	$2n$	
Normal $(0, 1)$	$x \in$ R	$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$	cdf: $\Phi(x)$	0	1	$\exp(t^2/2)$
Normal (μ, σ^2)	$x \in$ R	$\frac{1}{\sigma}\phi((x-\mu)/\sigma)$	cdf: $\Phi((x-\mu)/\sigma)$	μ	σ^2	
Rayleigh	$x \ge 0$	$xe^{-\frac{1}{2}x^2}$	$F(x) = 1 - e^{-\frac{1}{2}x^2}$	$\sqrt{\pi/2}$	$(4-\pi)/2$	
Cauchy	$x \in$ R	$1/\pi(1+x^2)$	$F(x) = \frac{1}{2} + \frac{1}{\pi} arctan(x)$			

- The mode of the binomial (n, p) distribution is the integer part of $(n+1)p$. If $(n+1)p$ is an integer then the previous integer is also a mode.
- The mode of the Poisson (μ) distribution is the integer part of μ. If μ is an integer then the previous integer is also a mode.
- If Z is standard normal then $E(|Z|) = \sqrt{2/\pi}$
- For $r > 0$, the integral $\Gamma(r) = \int_0^\infty x^{r-1}e^{-x}dx$ satisfies $\Gamma(r+1) = r\Gamma(r)$. So $\Gamma(r) = (r-1)!$ if r is an integer. Also, $\Gamma(1/2) = \sqrt{\pi}$.
- Let X and Y have joint density $f_{X,Y}$ and let $g : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be smooth and invertible. Let $(V, W) = g(X, Y) = (g_1(X, Y), g_2(X, Y))$ and let $(X, Y) = g^{-1}(V, W) = (h_1(V, W), h_2(V, W))$. Then the joint density of V and W is

$$f_{V,W}(v, w) = \frac{f_{X,Y}(x, y)}{abs(det(J(x, y)))} \quad \text{at the point } (x, y) = (h_1(v, w), h_2(v, w)), \quad \text{where } J(x, y) = \begin{bmatrix} \frac{\partial g_1(x,y)}{\partial x} & \frac{\partial g_1(x,y)}{\partial y} \\ \frac{\partial g_2(x,y)}{\partial x} & \frac{\partial g_2(x,y)}{\partial y} \end{bmatrix}$$

Equivalently

$$f_{V,W}(v, w) = f_{X,Y}(h_1(v, w), h_2(v, w)) \cdot abs(det(K(v, w))), \quad \text{where } K(v, w) = \begin{bmatrix} \frac{\partial h_1(v,w)}{\partial v} & \frac{\partial h_1(v,w)}{\partial w} \\ \frac{\partial h_2(v,w)}{\partial v} & \frac{\partial h_2(v,w)}{\partial w} \end{bmatrix}$$

- $(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k}$ • $\sum_{k=0}^n r^k = (1 - r^{n+1})/(1-r)$ • If $|r| < 1$ then $\sum_{k=0}^\infty r^k = 1/(1-r)$
- $e^x = \sum_{n=0}^\infty x^n/n!$ • $\log(1+x) \sim x$ for small x • This implies $\left(1 + \frac{x}{n}\right)^n \to e^x$ as $n \to \infty$.