Homework 4
Statistics 200B
Due Feb. 28, 2019

1. Verify the statements made in class about the Fisher information matrix $I_n(\mu, \sigma)$ and its inverse when $X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$.

2. Let $X_1, \ldots, X_n \overset{iid}{\sim} Gamma(\alpha, \beta)$. (The definition of Gamma distribution can be found at http://en.wikipedia.org/wiki/Gamma_distribution.)

   (a) Find the MLE of β assuming α is known.

   (b) Find the Fisher information and construct an approximate 95% normal-based confidence interval for β.

   (c) When both α and β are unknown, there is no closed-form expression for the MLE. The file `berkeleyprecip.csv` on bCourse contains total monthly precipitation data for Berkeley, CA, going back to 1919. In `R`, calculate the total winter precipitation for each year (removing missing values) using

   ```
   precip <- read.csv("berkeleyprecip.csv", header = TRUE)
   precip[precip==-99999] <- NA # Missing values
   winter.precip <- precip$DEC + precip$JAN + precip$FEB
   winter.precip <- winter.precip[!is.na(winter.precip)]
   ```

   Numerically find the MLEs for α and β under the model that the values for each year are *iid* with distribution $Gamma(\alpha, \beta)$. Approximate the observed Fisher information matrix and use it to construct 95% normal-based confidence intervals for α and β. Hint: Look at the steps in `betaexample.R`. Turn in your MLEs and confidence intervals along with a comment about the numerical optimization: what evidence do you have about whether the algorithm found a global optimum?

3. Let $X_1, \ldots, X_n \overset{iid}{\sim} Unif(0, \theta)$. Show that the MLE is consistent. Hint: Let $Y = \max\{X_1, \ldots, X_n\}$. For any c, $P(Y < c) = P(X_1 < c, X_2 < c, \ldots, X_n < c) = P(X_1 < c)P(X_2 < c) \cdots P(X_n < c)$.

4. Let $X_1, \ldots, X_n \overset{iid}{\sim} N(\theta, 1)$. Define $Y_i = I\{X_i > 0\}$. Let $\psi = P(Y_1 = 1)$.

(a) Find the MLE of ψ.

(b) Find an approximate 95% confidence interval for ψ.

(c) Define $\tilde{\psi} = \frac{1}{n}\sum_{i=1}^{n} Y_i$. Show that $\tilde{\psi}$ is a consistent estimator of ψ.

(d) Compute the asymptotic relative efficiency of $\tilde{\psi}$ to $\hat{\psi}$. Hint: Use the delta method to get the standard error of the MLE. Then compute the standard error (i.e., the standard deviation) of $\tilde{\psi}$.

(e) Suppose that the data are not really normal. Show that $\hat{\psi}$ is not consistent. What, if anything, does $\hat{\psi}$ converge to?