Homework 1
Statistics 200B
Due Jan. 31, 2019

1. Let $X \sim N(0,1)$ and let $Y = e^X$. Find the PDF for Y.

2. Let $X \sim Poisson(\lambda)$ and $Y \sim Poisson(\mu)$ and assume that X and Y are independent. Show that the distribution of X given that $X + Y = n$ is $Binomial(n, \pi)$ where $\pi = \lambda/(\lambda + \mu)$.

3. Let X have PDF $f_X(x) = \begin{cases} 1/4 & 0 < x < 1 \\ 3/8 & 3 < x < 5 \\ 0 & \text{otherwise} \end{cases}$

   (a) Find the CDF of X.

   (b) Let $Y = 1/X$. Find the probability density function $f_Y(y)$ for Y. Hint: Consider three cases: $1/5 \leq y \leq 1/3$, $1/3 \leq y \leq 1$, and $y > 1$.

4. Let X and Y have joint density

$$f_{X,Y}(x,y) = \begin{cases} c(x+y) & 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find c.

   (b) Find $f_{Y|X}(y|x)$.

   (c) Find $P(Y > 1/2 | X = 1)$.

   (d) Find $P(Y > 1/2 | X < 1/2)$

5. Let X be a continuous random variable with CDF F. Suppose that $P(X > 0) = 1$ and that $E[X]$ exists. Show that $E[X] = \int_0^\infty P(X > x)dx$. Hint: Consider integrating by parts. The following fact is helpful: if $E[X]$ exists then $\lim_{x\to\infty} x[1 - F(x)] = 0$.

6. Let $X \sim Exponential(\beta)$. (See page 29 of the textbook for the definition.) Find $P(|X - \mu_X| \geq k\sigma_X)$ for $k > 1$, where μ_X and σ_X denote the mean and standard deviation of the distribution, both equal to β in this case. Calculate

an upper bound for this probability using Chebyshev's inequality. Make a plot (a rough sketch is ok) comparing the exact probability to the bound, both as a function of k.

7. Suppose that $P(X = 1) = P(X = -1) = 1/2$. Define

$$X_n = \begin{cases} X & \text{with probability } 1 - 1/n \\ e^n & \text{with probability } 1/n \end{cases}$$

   Show why X_n does or does not converge to X

   (a) in probability.

   (b) in distribution.

   (c) in quadratic mean.

8. Given a sequence of random variables such that X_n converges to μ (μ is a constant) in probability, give one example where:

   (a) $E(X_n)$ does not converge to μ.

   (b) $E(X_n - \mu)^2$ does not converge to 0.