Practice Midterm I
Statistics 200B

1. Suppose that $Y_i = \beta x_i + \epsilon_i$, where $\epsilon_1, \ldots, \epsilon_n$ are independent with $E(\epsilon_i) = 0$ and $V(\epsilon_i) = \sigma^2$. Let $\hat{\beta}_n = \sum_{i=1}^n x_i Y_i / \sum_{i=1}^n x_i^2$.

   (a) Prove that $\hat{\beta}_n$ is unbiased as long as one $x_i \neq 0$.

   (b) Prove that $\hat{\beta}_n$ is a consistent estimator subject to a condition on the x_i's. Be explicit about what the condition is.

2. Let $X_1, \ldots, X_n$ be *iid* observations from a distribution F. Let

$$\theta = T(F) = \frac{E_F(X)}{\sqrt{V_F(X)}}$$

   Derive the plug-in estimator $\hat{\theta}_n$. (Show how it is obtained by substituting the ECDF $\hat{F}_n$ for F.)

3. Let $X_1, \ldots, X_n \overset{iid}{\sim} Unif(-\theta, \theta)$, where $\theta > 0$.

   (a) Find the likelihood function for θ.

   (b) Find the MLE $\hat{\theta}_n$.

   (c) Find the exact distribution (expressed in terms of CDF or PDF – your choice) for $\hat{\theta}_n$.

4. One observation, X, is taken from a $N(0, \sigma^2)$ population.

   (a) Find an unbiased estimator of σ^2.

   (b) Find the MLE of σ.

5. Let $X_1, \ldots, X_n \overset{iid}{\sim} N(\theta, 1)$. Define

$$Y_i = \begin{cases} 1 & X_i > 0 \\ -1 & X_i \leq 0 \end{cases}$$

   Let $\psi = E(Y_i)$.

   (a) Find the MLE $\hat{\psi}_n$ of ψ.

(b) Find an approximate 95% confidence interval for ψ.

(c) Define $\tilde{\psi}_n = \frac{1}{n}\sum_{i=1}^{n} Y_i$. Compute the asymptotic relative efficiency of $\tilde{\psi}_n$ to $\hat{\psi}_n$.