

**Homework 5**  
**Statistics 200B**  
**Due March 7, 2019**

1. Let  $X_1, \dots, X_n \stackrel{iid}{\sim} \text{Poisson}(\lambda)$ .

(a) Let  $\lambda_0 > 0$ . Find the size  $\alpha$  Wald test for

$$H_0 : \lambda = \lambda_0 \text{ versus } H_1 : \lambda \neq \lambda_0.$$

(b) (Computer experiment) Let  $\lambda_0 = 1$ ,  $n = 20$ , and  $\alpha = 0.05$ . Simulate  $X_1, \dots, X_n \sim \text{Poisson}(\lambda_0)$  and perform the Wald test. Repeat many times and count how often you reject the null hypothesis. How close is the type I error rate to 0.05?

2. Let  $X \sim \text{Binomial}(n, p)$ . Construct the likelihood ratio test for

$$H_0 : p = p_0 \text{ versus } H_1 : p \neq p_0.$$

Compare to the Wald test.

3. Suppose  $X_1, \dots, X_n$  are iid with PDF

$$f(x; \theta) = \begin{cases} e^{-(x-\theta)} & x \geq \theta \\ 0 & x < \theta \end{cases}$$

Consider testing  $H_0 : \theta \leq \theta_0$  versus  $H_1 : \theta > \theta_0$ . Find the likelihood ratio test statistic  $T(X) = \frac{\sup_{\theta \in \Theta} \mathcal{L}_n(\theta)}{\sup_{\theta \in \Theta_0} \mathcal{L}_n(\theta)}$ . Hint: Consider separately the cases  $\min\{X_1, \dots, X_n\} \leq \theta_0$  and  $\min\{X_1, \dots, X_n\} > \theta_0$ .

4. Suppose  $X_1, \dots, X_n \stackrel{iid}{\sim} N(\mu, \sigma^2)$ . For each of the following cases, derive the power function of the size 0.05 LRT, and plot it in R for  $n = 1, 4, 16, 64$ .

(a)  $H_0 : \mu \leq 0$  versus  $H_1 : \mu > 0$

(b)  $H_0 : \mu = 0$  versus  $H_1 : \mu \neq 0$

5. There is a theory that people can postpone their death until after an important event. To test the theory, Philips and King (1988) collected data on deaths around the Jewish holiday of Passover. Of 1919 daths, 922 died the week before the holiday and 997 died the week after. Think of this as a binomial and test the null hypothesis that  $\theta = 1/2$ . Report and interpret the p-value.

6. In 1861, 10 essays appeared in the *New Orleans Daily Crescent*. They were signed “Quintus Curtius Snodgrass”, and some people suspected they were actually written by Mark Twain. To investigate this, we will consider the proportion of three letter words found in each author’s work.

From eight Twain essays we have:

.225 .262 .217 .240 .230 .229 .235 .217

From 10 Snodgrass essays we have:

.209 .205 .196 .210 .202 .207 .224 .223 .220 .201

- (a) Perform a Wald test for equality of the means. Use the nonparametric plug-in estimator. Report the p-value and a 95% confidence interval for the difference in means.
- (b) Can you see any problems with the implicit assumptions we’ve made in this analysis?