Homework 3
Statistics 200B
Due Feb. 14, 2019

When not specified, please do not use R to get answers.

1. Consider again the cloud seeding data from Homework 2. Let θ be the difference in the median precipitation from the two groups. Find the plug-in estimate of θ. Using the bootstrap, estimate the standard error of the plug-in estimate and produce an approximate 95% Normal confidence interval for θ.

2. Let $X_1, \ldots, X_n$ be distinct observations (no ties). Let $X_1^*, \ldots, X_n^*$ denote a bootstrap sample (a sample from the empirical CDF), and let $\bar{X}_n^* = \frac{1}{n}\sum_{i=1}^n X_i^*$. Find: $E(\bar{X}_n^*|X_1, \ldots, X_n)$, $V(\bar{X}_n^*|X_1, \ldots, X_n)$, $E(\bar{X}_n^*)$, and $V(\bar{X}_n^*)$.

3. The file `bigcity.dat` on bCourse contains populations in thousands for $n = 49$ U.S. cities in 1920 (labeled u) and 1930 (labeled x). Demographers are interested in estimating $\theta = E_F[X]/E_F[U]$, where F represents the joint distribution of X and U. Calculate the plug-in estimate of θ and use the bootstrap to estimate the standard error and construct a 95% bootstrap pivotal interval. Hint: To sample once from $\hat{F}$, use something like

```
index <- sample(1:n, n, replace = TRUE)
u.star <- cities$u[index]
x.star <- cities$x[index]
```

4. Let $X_1, \ldots, X_n \overset{iid}{\sim} Unif(a, b)$, where a and b are unknown parameters and $a < b$.
   (a) Find the method of moments estimators for a and b.
   (b) Find the MLE $\hat{a}$ and $\hat{b}$.

5. Let $X_1, \ldots, X_n \overset{iid}{\sim} Poisson(\lambda)$. Find the MLE for λ and an estimated standard error.

6. Let $X_1, \ldots, X_n$ be iid with PDF $f(x; \theta) = 1/\theta$ for $0 \le x \le \theta$ and $\theta > 0$. Estimate θ using both the method of moments and maximum likelihood. Calculate the mean squared error for each estimator. Which one should be preferred and why?

7. Let $X_1, \ldots, X_n$ be *iid* with common distribution

$$P(X_i \leq x|\alpha, \beta) = \begin{cases} 0 & x < 0 \\ (x/\beta)^\alpha & 0 \leq x \leq \beta \\ 1 & x > \beta \end{cases}$$

   (a) Find the MLEs for α and β.

   (b) The length (in millimeters) of cuckoo's eggs found in hedge sparrow nests can be modeled with this distribution. For the data

   $$22.0, 23.9, 20.9, 23.8, 25.0, 24.0, 21.7, 23.8, 22.8, 23.1, 23.1, 23.5, 23.0, 23.0$$

   find the MLEs of α and β.

8. Consider the Normal linear regression model

$$Y_i \overset{indep}{\sim} N(\beta_0 + \beta_1 X_i, \sigma^2), \quad i = 1, \ldots, n$$

   Find the MLEs for β_0, β_1, and σ^2.