

True/False - No explanation needed. (1pt for correct, 0pt - no answer, -1pt - incorrect)

1. The MLE always maximizes the probability the data occur. True/False
True. This is the purpose of MLE.
2. Formulating the data likelihood, taking its logarithm (log) and setting the derivative of the log likelihood to 0 is always a good method to find the MLE. True/False
False. Sometimes we don't have to take log nor even taking the derivative but still being able to find MLE.

Problems - Need justification. No justification means **zero**!

1. (10pts) Over all years, the MATH 10B midterm mean is 50 and the std is 10. This year, the instructor takes a sample of 36 students whose midterm mean is 60. He believes that students of this year are smarter than the average over years. Take the significance level $\alpha = 0.05$.

(a) Perform the hypothesis test and draw a conclusion, i.e. formulate hypotheses, state one-sided/two-sided test, calculate p-value, draw conclusion.

Hint: use $\bar{\sigma} = \frac{\sigma}{\sqrt{n}}$

$H_0 : \mu = \mu_0 = 50$ vs $H_1 : \mu > \mu_0 = 50$, one-sided test

$$z\text{-score} = \frac{60 - 50}{10/\sqrt{36}} = 6$$

$p\text{-val} \approx 0.5 - 0.5 = 0 < \alpha = 0.05$, so rejecting the null H_0 . Conclusion: there is strong evidence supporting that this year students are smarter than the previous year ones.

(b) Calculate critical value and find the rejection region.

Critical value: $50 + 1.96 * 10/\sqrt{36} \approx 53.3$, rejection region is $[53.3, \infty)$