Worksheet 20

Bounding Probabilities

Simple intuition:

1. Draw the normal pdf. Highlight the portion of the pdf capturing $\{|X - \mu| \geq k\sigma\}$ for $k = 0.5, 1, 2, 5$, roughly.
2. If X and Y are two different random variables, is it possible for Chebyshev to yield the exact same bound for them?
3. What are some reasons Chebyshev may be lossy? What are some reasons it may be sharp?

Calculations:

1. Suppose X is now Poisson with parameter λ. What are μ and σ for this distribution?
   (a) Compute $\mathbb{P}[|X - \mu| > 2 \cdot \sigma]$.
   (b) Approximate $\mathbb{P}[|X - \mu| > 2 \cdot \sigma]$ using Chebyshev.
   (c) Approximate $\mathbb{P}[|X - \mu| \leq 0.5 \cdot \sigma]$ using Chebsyhev.
2. Suppose that X has Laplace distribution with mean 0, i.e. its pdf is

$$f(x) = \frac{1}{2}e^{-|x|}.$$

   Note that the variance of this distribution is 2.
   (a) Compute $\mathbb{P}[|X| > 4]$.
   (b) Compute $\mathbb{P}[|X| \geq 4]$.
   (c) Use Chebyshev to approximate $\mathbb{P}[|X| > 4]$.

Source: Rosen's *Discrete Mathematics and its Applications*.