

Quiz 3 Solution

True/False - No explanation needed. (1pt for correct, 0pt - no answer, -1pt - incorrect)

1. The number of solutions to the problem of distributing indistinguishable objects to distinguishable boxes is larger than that of distributing distinguishable objects to distinguishable boxes, given the same input. True/False
False. Say, one solution to the former can generate at least $k!$ solutions to the latter, where k is the number of objects in one box.
2. The solutions to the problem of distributing indistinguishable objects to distinguishable boxes involve Stirling numbers. True/False
False. The problem of distributing distinguishable objects to indistinguishable boxes may involve Stirling numbers.

Problems - Need justification. No justification means **zero**!

1. (5pts) How many non-negative integer solutions to the inequality $x_1 + x_2 + x_3 + x_4 \leq 20$, where $x_1 \geq 4$?

Change variable $X_1 = x_1 - 4$, the inequality becomes $X_1 + x_2 + x_3 + x_4 \leq 16$, where all variables are non-negative.

Add a dummy variable $x_5 = 16 - (X_1 + x_2 + x_3 + x_4) \geq 0$, the inequality becomes the equation $X_1 + x_2 + x_3 + x_4 + x_5 = 16$, where all variables are non-negative.

This is a typical problem of distributing 16 indistinguishable balls into 5 distinguishable boxes. The answer is $C(16 + 5 - 1, 5 - 1) = C(20, 4)$

2. (5pts) How many ways are there to deal 6 cards from a deck of 52 cards to 4 players if the players are indistinguishable?

Number of ways to pick a set of 6 cards out of 52 cards: $C(52, 6)$.

Number of ways to deal 6 distinguishable cards to 4 indistinguishable people: $S(6,1) + S(6,2) + S(6,3) + S(6,4)$.

The final answer: $C(52, 6) * (S(6,1) + S(6,2) + S(6,3) + S(6,4))$