

## Quiz 9

**True/False** - No explanation needed. (1pt for correct, 0pt - no answer, -1pt - incorrect)

1. The CDF of a continuous RV is always continuous in the range of that RV. True/False  
True. It may not be smooth, but it is continuous over the RV range.
2. The maximum value of a PDF can exceed 1. True/False  
True. Say,  $Uniform(10, 10.1)$

**Problems** - Need justification. No justification means **zero**!

1. (10pts) Given a function  $f(x) = c(4 - x^2)$  for  $-2 \leq x \leq 2$  and  $f(x) = 0$  otherwise.
  - a) Find  $c$  so that  $f(x)$  is a PDF of a RV  $X$ .
  - b) Find CDF of  $X$ .

$$\text{a) } 1 = \int_{-2}^2 f(x)dx = \int_{-2}^2 c(4 - x^2)dx = c \left( 4x - \frac{x^3}{3} \right) \Big|_{-2}^2 = \frac{32c}{3} \Rightarrow c = \frac{3}{32}$$

$$\text{b) } \int_{-\infty}^x f(s)ds = \int_{-2}^x \frac{3}{32}(4 - s^2)ds = \frac{3}{32} \left( 4s - \frac{s^3}{3} \right) \Big|_{-2}^x = -\frac{x^3}{32} + \frac{3x}{8} + \frac{1}{2}$$

Thus, CDF of  $X$  is  $F(x) = -\frac{x^3}{32} + \frac{3x}{8} + \frac{1}{2}$  for  $-2 \leq x \leq 2$ ,  $F(x) = 0$  for  $x < -2$  and  $F(x) = 1$  for  $x > 2$