

True/False - No explanation needed. (1pt for correct, 0pt - no answer, -1pt - incorrect)

1. The probability of the event A and the event B occurring is lower bounded by the minimum of the probability of the event A and the probability of the event B, i.e.

$$P(A \cap B) \geq \min(P(A), P(B))$$

True/False

False. Draw a Venn's diagram, LHS is the intersection of A and B that can be 0 if A and B are mutually exclusive, RHS is the smaller of A and B circles that can be positive. So LHS cannot be larger than or equal to RHS.

2. The probability of the event A occurring is always less than or equal to the probability of the event A occurring given that the event B occurs, i.e. $P(A) \leq P(A|B)$. True/False
False. In the case of A and B mutually exclusive, the latter is 0. So the former cannot be always less than or equal to the latter.

Problems - Need justification. No justification means **zero**!

1. (10pts) A family has 5 kids in a row. The probability of having a boy is 0.6. Calculate:
(a) The probability of 3 girls and 2 boys.
(b) The probability of 2 girls given at least 1 boy.

Hint: You do not need to simplify your answers.

- (a) The probability of having 3 girls and 2 boys is $0.4^3 * 0.6^2$. There are $C(5, 3)$ ways to combine 3 girls and 2 boys in a row. So the final answer is: $C(5, 3) * 0.4^3 * 0.6^2$
(b) Let A be the event of having at least 1 boy, B be the event of having 2 girls.

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Denominator: $P(A) = 1 - 0.4^5$

Nominator is the event of having 2 girls and at least 1 boy. This is equivalent to having 2 girls and 3 boys. $P(A \cap B) = C(5, 2) * 0.4^2 * 0.6^3$

The final answer is:

$$P(B|A) = \frac{C(5, 2) * 0.4^2 * 0.6^3}{1 - 0.4^5}$$