

True/False - No explanation needed. (1pt for correct, 0pt - no answer, -1pt - incorrect)

1. If the the variance exists, the mean exists. True/False
True. This is the reverse version of the statement "if the mean does not exist, the variance does not either".
2. If a RV X is scaled up by 2 times, i.e. $2X$, its standard deviation is scaled up by 4 times.
True/False
False. Variance scaled up by 4 times, but std by 2.

Problems - Need justification. No justification means **zero**!

1. (10pts) Given a RV X having the PDF $f(x) = c(4 - x^2)$ for $-1 \leq x \leq 2$ and $f(x) = 0$ otherwise. Find the variance of X (in term of c).

$$\mu = \int_{-\infty}^{\infty} xf(x)dx = \int_{-1}^2 xc(4 - x^2)dx = c \left(2x^2 - \frac{x^4}{4} \right) \Big|_{-1}^2 = \frac{9c}{4}$$

$$\begin{aligned} \sigma^2 &= \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx = \int_{-\infty}^{\infty} x^2 f(x)dx - \mu^2 = \int_{-1}^2 x^2 c(4-x^2)dx - \mu^2 = c \left(\frac{4x^3}{3} - \frac{x^5}{5} \right) \Big|_{-1}^2 - \\ &\left(\frac{9c}{4} \right)^2 = \frac{27c}{3} - \frac{81c^2}{16} \end{aligned}$$