

**True/False** - No explanation needed. (1pt for correct, 0pt - no answer, -1pt - incorrect)

1. A random variable has the expected value 0, maximum value 10 and variance 20. Is it possible? True/False  
True/Yes. The variance has different dimension from the RV (Var has dimension of  $X^2$ ), so it can be larger than maximum value.
2. When we scale up the random variable  $X$  by 2 times, i.e. into  $2X$ , the expected value and the variance are also scaled up by 2 times, i.e.  $E[X]$  becomes  $2E[X]$  and  $Var(X)$  becomes  $2Var(X)$ , respectively. True/False  
False.  $E$  scales up by 2 times,  $Var$  scales up by  $2^2 = 4$  times.

**Problems** - Need justification. No justification means **zero**!

1. (10pts) Let  $X_1$ ,  $X_2$ , and  $X_3$  denote the numbers that come up on three rolls of a fair four-sided die. Let  $X = X_2$ ,  $Y = X_1 + X_3$ 
  - a) Find the expectation and variance of  $Y$ .
  - b) Find  $Cov(X, Y)$ .

a)  $E(X_1) = E(X_3) = \frac{1}{4} * 1 + \frac{1}{4} * 2 + \frac{1}{4} * 3 + \frac{1}{4} * 4 = \frac{5}{2}$ , where  $1/4$  is the probability of each number in one roll.

$$E(Y) = E(X_1) + E(X_3) = 5$$

$$Var(X_1) = Var(X_3) = E(X_1^2) - E^2(X_1)$$
$$E(X_1^2) = \frac{1}{4} * 1^2 + \frac{1}{4} * 2^2 + \frac{1}{4} * 3^2 + \frac{1}{4} * 4^2 = \frac{15}{2}$$

$$\text{So, } Var(X_1) = Var(X_3) = \frac{15}{2} - \frac{5^2}{2^2} = \frac{5}{4}$$

$$Var(Y) = Var(X_1) + Var(X_3) = \frac{5}{2} \text{ since } X_1 \text{ and } X_3 \text{ are independent}$$

b)  $Cov(X, Y) = Cov(X_2, X_1 + X_3) = Cov(X_2, X_1) + Cov(X_2, X_3) = 0 + 0 = 0$  since  $X_1$  and  $X_2$  are independent, so are  $X_1$  and  $X_3$ .