MATH 10B with Prof. Stankova | Student:
Section #209 ; time: 3:30-5pm | SID:
GSI: Ninh DO | Tue 2/12/19

Quiz 3 Solution

True/False - No explanation needed. (1pt for correct, 0pt - no answer, -1pt - incorrect)

1. The number of solutions to the problem of distributing indistinguishable objects to indistinguishable boxes is larger than that of distributing indistinguishable objects to distinguishable boxes, given the same input. True/False
   False. Say, one solution to the former can generate at least $k!$ solutions to the latter, where k is the number of boxes.

2. The solutions to the problem of distributing distinguishable objects to indistinguishable boxes involve Stirling numbers. True/False
   True or False. Some students indicate the corner cases that do not involse Stirling numbers.

Problems - Need justification. No justification means **zero**!

1. (5pts) How many non-negative integer solutions to the inequality $x_1 + x_2 + x_3 + x_4 \leq 24$, where $x_1 \geq 4$?

   Change variable $X_1 = x_1 - 4$, the inequality becomes $X_1 + x_2 + x_3 + x_4 \leq 20$, where all variables are non-negative.
   Add a dummy variable $x_5 = 20 - (X_1 + x_2 + x_3 + x_4) \geq 0$, the in equality becomes the equation $X_1 + x_2 + x_3 + x_4 + x_5 = 20$, where all variables are non-negative.
   This is a typical problem of distributing 16 indistinguishable balls into 5 distinguishable boxes. The answer is C(20 + 5 - 1, 5 - 1) = C(24, 4)

2. (5pts) How many ways are there to to deal 10 cards from a deck of 52 cards to 2 players if the players are indistinguishable?

   Number of ways to pick a set of 10 cards out of 52 cards: C(52, 10).
   Number of ways to deal 10 distinguishable cards to 2 indistinguishable people: S(10,1) + S(10,2).
   The final answer: C(52, 10) * (S(10,1) + S(10,2))