

True/False - No explanation needed. (1pt for correct, 0pt - no answer, -1pt - incorrect)

1. The sample std is a biased estimate of the population std. True/False
False. It is unbiased estimate.
2. If the sample size is very large, the std of the sample mean will approach the population std.
True/False
False. It will approach 0.

Problems - Need justification. No justification means **zero**!

1. (10pts) Anna rolls a fair 4-side dice 100 times. She takes the average.
(a) Find the 87% CI of the average.

Hint: use $\bar{\sigma}^2 = \frac{\sigma^2}{n-1}$

$X_1, \dots, X_{100} \sim \text{Uniform}(1, 4)$ with $\mu = \frac{1+4}{2} = 2.5$ and $\sigma^2 = \frac{4^2-1}{12} = 1.25$

$\bar{X} = \frac{X_1 + \dots + X_{100}}{100}$ with $\bar{\mu} = \mu = 2.5$ and $\bar{\sigma}^2 = \frac{\sigma^2}{100-1} = 0.0126 \Rightarrow \bar{\sigma} = 0.112$

z-score for 87% CI is 1.514 (look at area = 0.435)

Thus, the 87% CI is: $\bar{\mu} \pm z_{score} * \bar{\sigma} = 2.5 \pm 1.514 * 0.112$

- (b) Find the probability that this average is less than 2.

$$P(\bar{X} < 2) = P\left(\frac{\bar{X} - \bar{\mu}}{\bar{\sigma}} < \frac{2 - 2.5}{0.112}\right) \approx P(Z < -4.4) \approx 0.5 - 0.5 = 0$$