MATH 10B with Prof. Stankova
Section #209 ; time: 3:30-5pm
GSI: Ninh DO

Quizz 2

Student:
SID:
Tue 2/5/19

True/False - No explanation needed. (1pt for correct, 0pt - no answer, -1pt - incorrect)

1. $C(n,r) = C(n, n-r)$. True/False
   True

2. A number of ways to arange n objects is $\frac{n!}{(n-r)!}$, where r is the number of the distinct objects. True/False
   False. There is no such theorem.

Problems - Need justification. No justification means **zero**!

1. (5pts) Show that in a group of 31 people, all of whom are female, male or non-binary, at least one of the following must be true:

   (a) At least 5 are female

   (b) At least 24 are male

   (c) At least 4 are non-binary

   Hint: $31 - 4 - 3 = 24$

   PHP: at least $\lceil 31/3 \rceil = 11$ are either female, male, non-binary. Assuming that they are non-binary, (c) is true. Otherwise, there are at most 3 non-binary and at least $31 - 3 = 28$ female or make. PHP: at least $\lceil 28/2 \rceil = 14$ female or male. Assuming that they are female, (a) is true. Otherwise, there are at most 4 female out of at least 28, and at least $28 - 4 = 24$ male, i.e. (b) is true.

   Another approach: using contradiction. Assuming none of the statement is true, i.e. at most 4 are female AND at most 23 are male AND at most 3 are non-binary. This implies there are at most $4 + 23 + 3 = 30$ people. But there are 31 people, the extra one must fall into one of three categories to satisfy at least one statement.

2. (5pts) How many ways are there to arrange 8 men and 12 women standing in a line so that no two men stand next to each other?

   Number of ways to arrange 12 women: 12!
   Number of ways to arrange 8 men: 8!
   Put men in the spaces between women. 12 women make 13 spaces. Number of ways to select 8 spaces for 8 men: $C(13, 8)$
   Final answer: $12! \cdot 8! \cdot C(13, 8)$