

Quizz 2

True/False - No explanation needed. (1pt for correct, 0pt - no answer, -1pt - incorrect)

1. $P(n, r) = P(n, n - r)$. True/False
False. True for combination but not for permutation.
2. A number of ways to arrange n objects is $\frac{n!}{(n-r)!}$, where r is the number of the non-distinct objects. True/False
False. There is no such theorem.

Problems - Need justification. No justification means **zero**!

1. (5pts) Show that in a group of 16 people, all of whom are female, male or non-binary, at least one of the following must be true:
 - (a) At least 5 are female
 - (b) At least 4 are male
 - (c) At least 9 are non-binary

Hint: $16 - 4 - 3 = 9$

PHP: at least $\lceil 16/3 \rceil = 6$ are either female, male, non-binary. Assuming that they are male, (b) is true. Otherwise, there are at most 3 male and at least $16 - 3 = 13$ female or non-binary. PHP: at least $\lceil 13/2 \rceil = 7$ female or non-binary. Assuming that they are female, (a) is true. Otherwise, there are at most 4 female out of at least 13, and at least $13 - 4 = 9$ non-binary, i.e. (c) is true.

Another approach: using contradiction. Assuming none of the statement is true, i.e. at most 4 are female AND at most 3 are male AND at most 8 are non-binary. This implies there are at most $4 + 3 + 8 = 15$ people. But there are 16 people, the extra one must fall into one of three categories to satisfy at least one statement.

2. (5pts) How many ways are there to arrange 4 men and 7 women standing in a line so that no two men stand next to each other?

Number of ways to arrange 7 women: $7!$

Number of ways to arrange 4 men: $4!$

Put men in the spaces between women. 7 women make 8 spaces. Number of ways to select 4 spaces for 4 men: $C(8, 4)$

Final answer: $7! \cdot 4! \cdot C(8, 4)$