MATH 10B with Prof. Stankova
Section #208 ; time: 2-3:30pm
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Student:
SID:
Tue 2/19/19

Quiz 4 Solution

True/False - No explanation needed. (1pt for correct, 0pt - no answer, -1pt - incorrect)

1. The bubble sort algorithm is efficient if the list is already sorted in increasing order. True/False
   False. The buble sort algorithm still goes through the whole procedure, i.e. comparing all adjacent pairs, when running.

2. Suppose there are n men and n women that have the strictly opposite preferences, i.e. all men prefer $w_1 > w_2 > ... > w_n$, all women prefer $m_n > m_{n-1} > ... > m_1$. If we run the stable matching algorithm to couple them, the result is unstable, i.e. there is at least one couple who both can find a better match. True/False
   False. The stable matching algorithm always gives the stable result.

Problems - Need justification. No justification means **zero**!

1. (10pts) Prove, for all positive integer n:

$$\frac{1}{2*5}+\frac{1}{5*8}+\frac{1}{8*11}+...+\frac{1}{(3n-1)*(3n+2)}=\frac{n}{6n+4}$$

Step 1: Check with n = 1

$$\frac{1}{2*5}=\frac{1}{6*1+4}=\frac{1}{10}$$

Step 2: Assume the statement is correct up to $n = k$, i.e.

$$\frac{1}{2*5}+\frac{1}{5*8}+\frac{1}{8*11}+...+\frac{1}{(3k-1)*(3k+2)}=\frac{k}{6k+4}$$

Step 3: Prove the statement is correct with $n = k + 1$, i.e.

$$\frac{1}{2*5}+\frac{1}{5*8}+\frac{1}{8*11}+...+\frac{1}{(3k-1)*(3k+2)}+\frac{1}{(3(k+1)-1)*(3(k+1)+2)}=\frac{k+1}{6(k+1)+4}$$

$$LHS=\frac{k}{6k+4}+\frac{1}{(3k+2)*(3k+5)}=\frac{k}{2*(3k+2)}+\frac{1}{(3k+2)*(3k+5)}$$

$$=\frac{1}{3k+2}\left[\frac{k}{2}+\frac{1}{3k+5}\right]=\frac{1}{3k+2}\frac{3k^2+5k+2}{2(3k+5)}=\frac{1}{3k+2}\frac{(3k+2)(k+1)}{2(3k+5)}$$

$$\frac{k+1}{2(3k+5)}=\frac{k+1}{6k+10}=RHS$$

Conclusion: The statement is correct.