

Basics of counting

1. How many different three-letter initials are there that begin with an A? contain A?
2. How many 6-element RNA sequences
 - (a) end with GU?
 - (b) contain only A or U?
3.
 - (a) How many different functions are there from a set with m elements to a set with n elements?
 - (b) How many different injective functions are there from a set with m elements to a set with n elements? You may assume $n \geq m$.
4. How many positive integers between 100 and 999 inclusive
 - (a) are divisible by 7?
 - (b) are divisible by 3 or 4?
 - (c) are divisible by 3 but not by 4?
5. How many strings of 5 decimal digits
 - (a) contain at least one 4?
 - (b) do not have the same digit?
 - (c) do not have two consecutive digits that are the same?
 - (d) either end in 4, or start with 6?
6. How many diagonals does a convex polygon with n sides have? (Recall that a polygon is convex if every line segment connecting two points in the interior or boundary of the polygon lies entirely within this set and that a diagonal of a polygon is a line segment connecting two vertices that are not adjacent.)
7. In how many ways can a photographer at a wedding arrange six people in a row, including the bride and groom, if
 - (a) the bride must be next to the groom?
 - (b) the bride is not next to the groom?
 - (c) the bride is positioned somewhere to the left of the groom?
8. ** Todd has ten apples and he hopes to finish them within ten days. If Todd will eat at least one apple a day, how many ways can he choose to achieve his goal? (This question uses a trick that we haven't seen yet. Don't worry if you can't solve it, but it is an interesting problem and worths thinking about!)

Source: Rosen's *Discrete Mathematics and its Applications*.