

Quiz 5 Solution

True/False - No explanation needed. (1pt for correct, 0pt - no answer, -1pt - incorrect)

1. The probability of the event A or the event B occurring is upper bounded by the maximum of the probability of the event A and the probability of the event B, i.e.

$$P(A \cup B) \leq \max(P(A), P(B))$$

True/False

False. Draw a Venn's diagram, LHS is the union of A and B, RHS is the bigger of A and B circles. So LHS is always larger than or equal to RHS.

2. The probability of the event A occurring is always less than or equal to the probability of the event A occurring given that the event B occurs, i.e. $P(A) \leq P(A|B)$. True/False

False. In the case of A and B mutually exclusive, the latter is 0. So the former cannot be always less than or equal to the latter.

Problems - Need justification. No justification means **zero**!

1. (10pts) A family has 5 kids in a row. The probability of having a boy is 0.4. Calculate:
(a) The probability of 3 boys and 2 girls.
(b) The probability of 2 boys given at least 1 girl.

Hint: You do not need to simplify your answers.

(a) The probability of having 3 boys and 2 girls is $0.4^3 * 0.6^2$. There are $C(5, 3)$ ways to combine 3 boys and 2 girls in a row. So the final answer is: $C(5, 3) * 0.4^3 * 0.6^2$

(b) Let A be the event of having at least 1 girl, B be the event of having 2 boys.

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Denominator: $P(A) = 1 - 0.4^5$

Nominator is the event of having 2 boys and at least 1 girl. This is equivalent to having 2 boys and 3 girls. $P(A \cap B) = C(5, 2) * 0.4^2 * 0.6^3$

The final answer is:

$$P(B|A) = \frac{C(5, 2) * 0.4^2 * 0.6^3}{1 - 0.4^5}$$