

True/False - No explanation needed. (1pt for correct, 0pt - no answer, -1pt - incorrect)

1. The sample mean (or sample average) is not a RV and its value is roughly $\bar{\mu} = \mu$, where μ is the population mean. True/False
False. The sample mean is a RV as it is a sum of a bunch of RVs.
2. If the sample size is very large, the variance of the sample mean will approach the population variance. True/False
False. The variance of the sample mean will approach 0.

Problems - Need justification. No justification means **zero**!

1. (10pts) John rolls a fair 6-side dice 100 times. He takes the average.
(a) Find the 87% CI of the average.

Hint: use $\bar{\sigma}^2 = \frac{\sigma^2}{n-1}$

$X_1, \dots, X_{100} \sim Uniform(1, 6)$ with $\mu = \frac{1+6}{2} = 3.5$ and $\sigma^2 = \frac{6^2-1}{12} \approx 2.92$

$\bar{X} = \frac{X_1 + \dots + X_{100}}{100}$ with $\bar{\mu} = \mu = 3.5$ and $\bar{\sigma}^2 = \frac{\sigma^2}{100-1} = 0.0295 \Rightarrow \bar{\sigma} = 0.172$

z-score for 87% CI is 1.514 (look at area = 0.435)

Thus, the 87% CI is: $\bar{\mu} \pm z_{score} * \bar{\sigma} = 3.5 \pm 1.514 * 0.172$

- (b) Find the probability that this average is less than 4.

$$P(\bar{X} < 4) = P\left(\frac{\bar{X} - \bar{\mu}}{\bar{\sigma}} < \frac{4 - 3.5}{0.172}\right) = P(Z < 2.91) \approx 0.5 + 0.4982 = 0.9982$$