Onset of Collective Behavior of Sedimenting Particles in the Knudsen Regime

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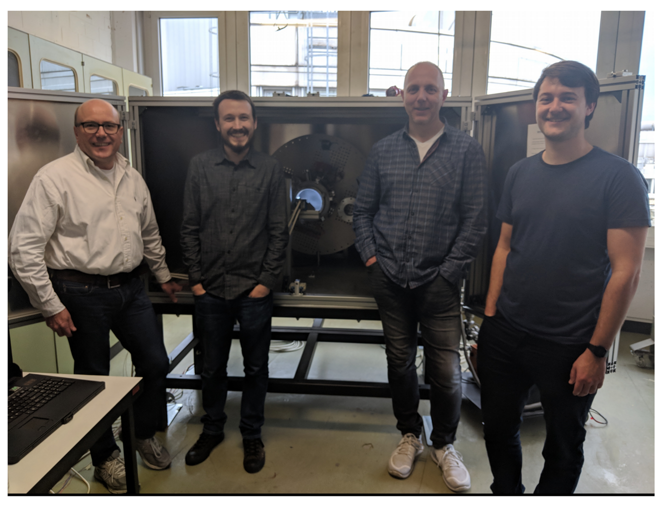
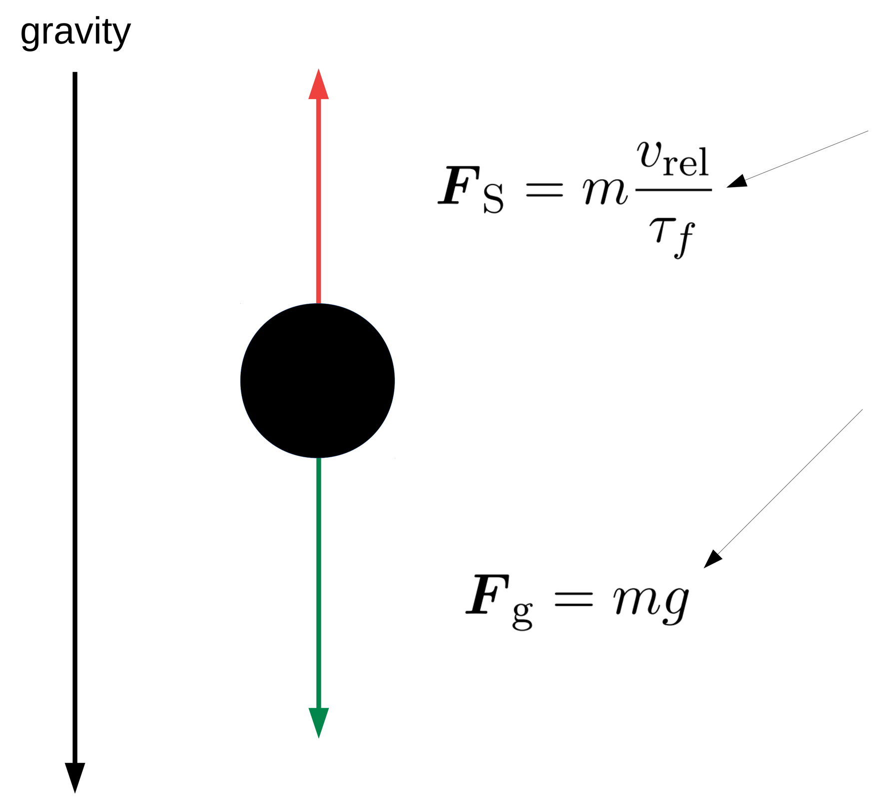
Based on work presented in Schneider et al 2019 (accepted for publication in ApJ) and ongoing work by Hubert Klahr and Vincent Carpenter.

Introduction

Understanding the motion of small solid bodies through fluids is crucial for many fields in science and engineering; for planet formation, the behavior of groups of particles is of particular interest. Niclas Schneider and Gerhard Wurm have conducted experiments that demonstrate conditions that cause a transition between individual motion and collective motion of particles. Hubert Klahr and I are endeavoring to understand their results, using simple analytical arguments and numerical hydrodynamics simulations.



Sedimentation of Particles in Stokes' Drag Regime

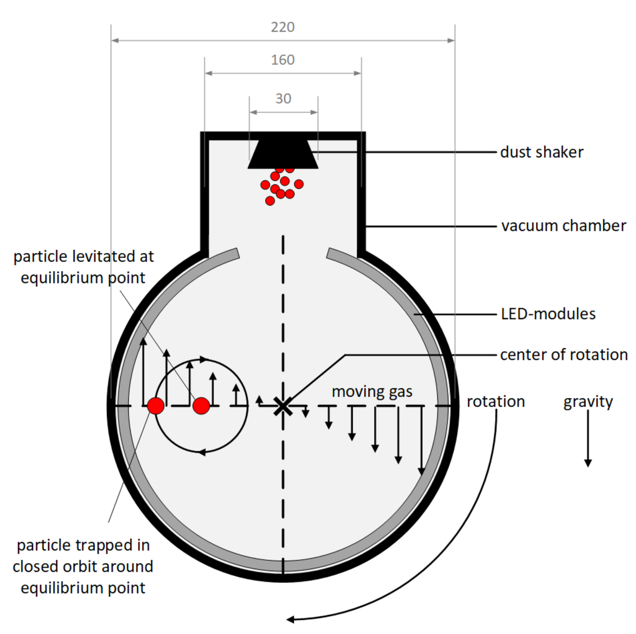


When particle Reynolds numbers are low, the drag force they experience is proportional to their velocity relative to the fluid

If a force, such as gravity, acts on a particle in a constant direction and with constant magnitude, the relative velocity will increase until the drag force is equal to the gravitational force, with the particle traveling at a constant terminal velocity:

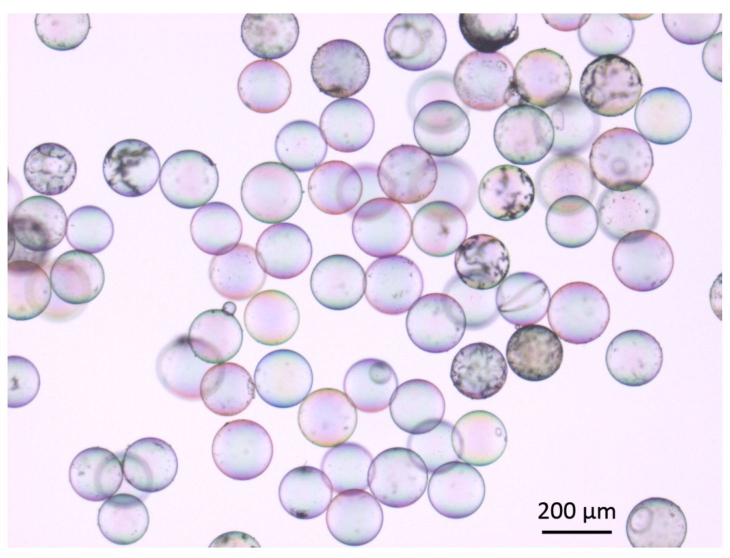
$$m\frac{v_{\rm rel,t}}{\tau_f} = mg \implies v_{\rm rel,t} = \tau_f g$$

Experiment Setup

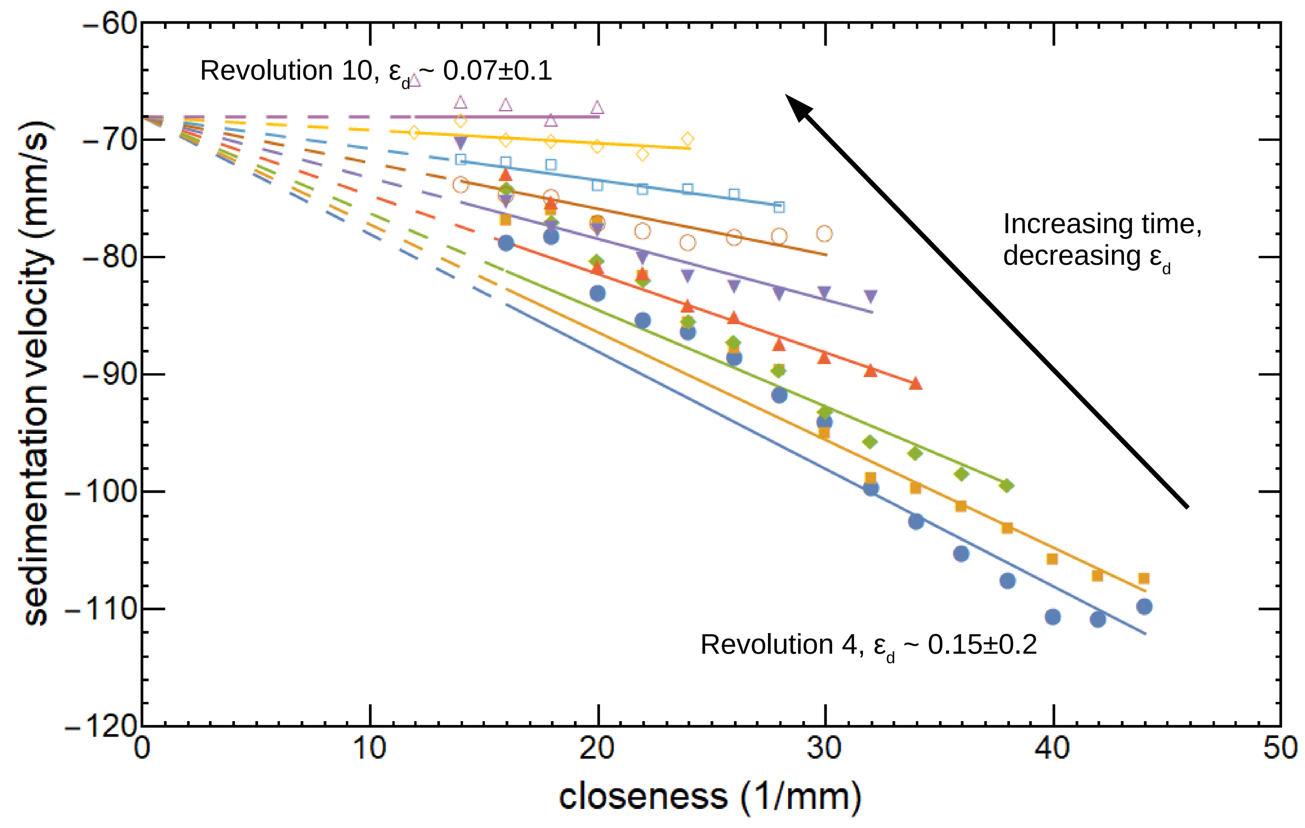


The experiment consists of a sealed cylindrical chamber of air, at room temperature and evacuated to a pressure of around a kilopascal, which is rotated about its axis by a motor, causing the gas within to assume a solid-body rotation profile (see diagram to the left). Hollow glass beads (pictured below) are dropped into the chamber, and a high speed camera captures about 100 images per second of the positions of the beads, from which their velocities can be reconstructed. Important physical parameters of the experiment are collected in the table at bottom left.

Parameter	Value
particle size	165 μm $\pm$ 15 μm
particle density	60 kg/m^3
initial particle number	650
gas pressure	950 Pa $\pm$ 100 Pa
initial dust-to-gas	0.15$\pm$0.02
final dust-to-gas	0.07 $\pm$0.01
max. local dust-to-gas	1.4 $\pm$0.1
initial volume filling factor	3 x10^{-5}
friction time	7 ms
Knudsen number	0.08
rotation frequency f	0.336 Hz
Stokes number	0.014
$Re_{\rm Setup}$ with d = 22 cm	18.5



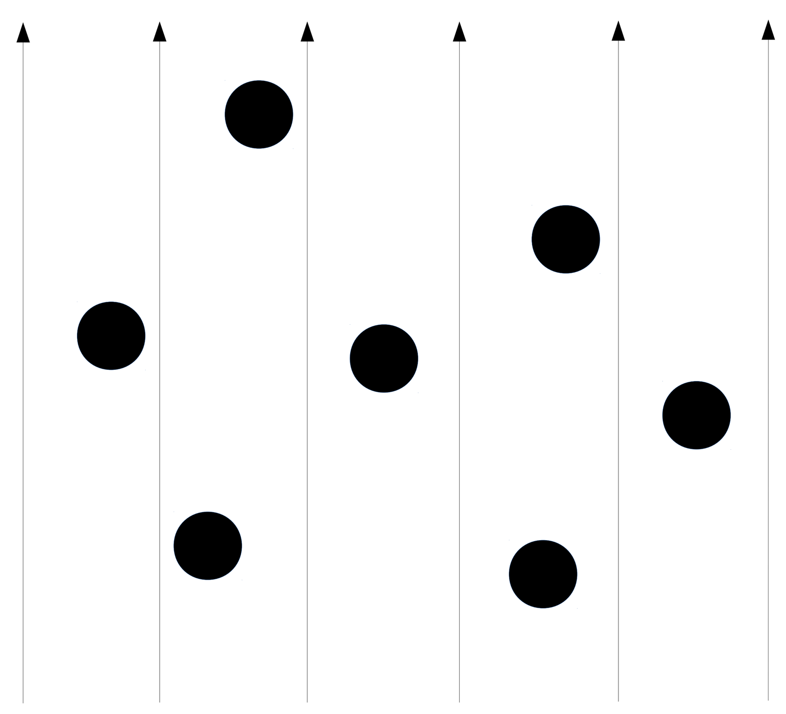
Experiment Results



While the experiment has obtained many fascinating restults, there is only one with which we are concerned at the moment: at sufficiently high dust to gas ratios, closely packed groups of particles sediment with a faster terminal velocity than the single-particle terminal velocity of ~71 mm/s. Their appears to be a roughly linear relationship between the excess velocity and the closeness, which measures how tightly packed the particles are.

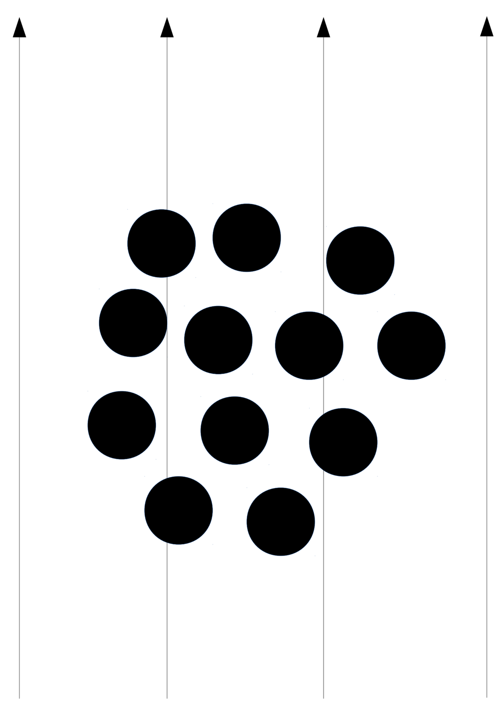
Particle Back-Reaction

The experiment finds that, while single particles indeed exhibit the terminal velocity predicted from particle and gas properties, groups of particles do not. We can get a rough idea of why this might be by considering the effect the particles have on the gas as it moves past them.



The gas exerts a drag force on the particles, so they must exert the same force back on the gas; we call this the back-reaction. When the particles are loosely packed, they don't have as big of an influence on the gas, and it is able to flow normally through a group of them.

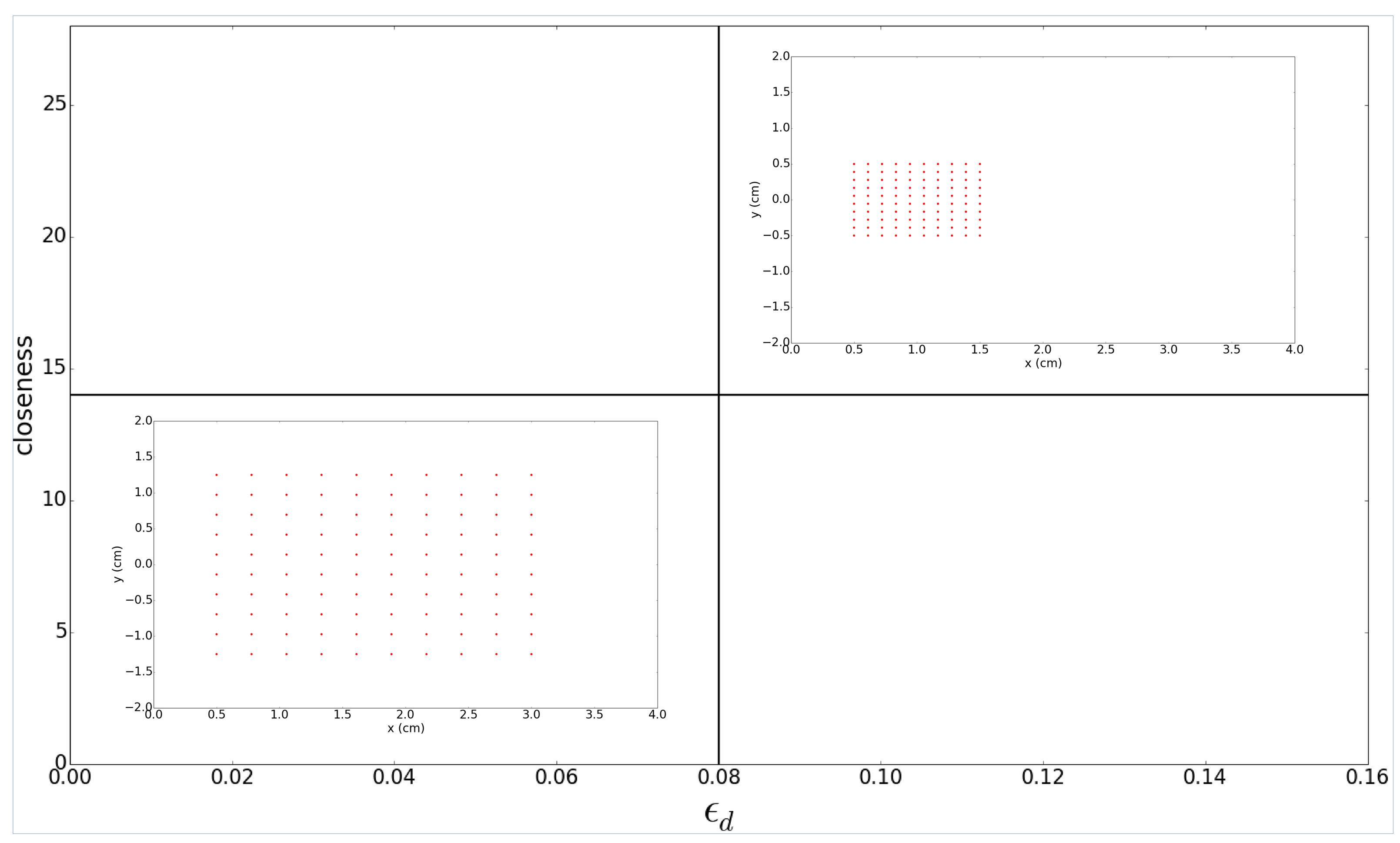
When the particles are packed more tightly, however, are able to more drastically alter the fluid flow through them; this in turn may allow for the violation of the drag condition used to determine the terminal velocity of a single particle.



We therefore intend to examine the effects of particle back-reaction on the gas flow through groups of particles in order to explain the observed results.

Ongoing Numerical Simulations

It is only possible to learn so much analytically, and to further our understanding of what is happening here we must conduct numerical simulations. We do this using the Pencil Code hydrodynamics simulation framework, which allows us to simulate the hydrodynamics of the gas with high precision, and to include Lagrangian particles that feel drag and exert a back-reaction force on the gas.



The experiment measured an effect which depends on dust to gas ratio and closeness, we wish to reproduce this relationship in our simulations. As a first step, we are designing and testing a simple, two dimensional simulation, with small numbers of particles, that should nonetheless be able to capture the relevant behavior. We will run it with two different spatial distributions of particles, one which gives the conditions for collective motion as predicted by the experiment, and one which does not. Once the simulations are running, if we find that we are able to reproduce the experiment in this most basic sense, we can use different particle distributions to populate the phase space more richly, and determine how well we can quantitatively reproduce the observed sedimentation velocity vs closeness relationship.