CSCE 643 Multi-View Geometry CV Homework I

Yukun Zeng
Department of Computer Science and Engineering
Texas A&M University
College Station, TX 77840
Email: yzeng@tamu.edu

I. Four point rectification

First I'd like to briefly summarize the goal and methodology for this rectification problem. As we know, the pictures taken by a camera is actually projections of Euclidean prototype of real-world scenarios. Assuming we have a point in the Euclidean space whose homogenized coordinate is (x, y, z), and its counterpart in projective space, or in the picture, is (x', y', z). According to the definition of projective transformation, the planar projective transformation can be represented by a non-singular 3×3 matrix:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{1}$$

More briefly as $\mathbf{x}' = \mathbf{H}\mathbf{x}$, this can be further transformed into (note that by default we assume $h_{33} = 1$):

$$\begin{aligned} x' &= h_{11}x + h_{12}y + h_{13}z \\ y' &= h_{21}x + h_{22}y + h_{23}z \\ z' &= h_{31}x + h_{32}y + z \end{aligned} \tag{2}$$

However, this is up scale, and if we go down scale and set $z' = 1, z = 1$, it turns out that it can be transformed through equation 3–5:

$$\begin{aligned} x' &= \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + 1} \\ y' &= \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + 1} \end{aligned} \tag{3}$$

$$\begin{aligned} x'(h_{31}x + h_{32}y + 1) &= h_{11}x + h_{12}y + h_{13} \\ y'(h_{31}x + h_{32}y + 1) &= h_{21}x + h_{22}y + h_{23} \end{aligned} \tag{4}$$

$$\begin{aligned} x' &= h_{11}x + h_{12}y + h_{13} - h_{31}x'x - h_{32}xy \\ y' &= h_{21}x + h_{22}y + h_{23} - h_{31}xy' - h_{32}yy' \end{aligned} \tag{5}$$

which could also be represented by matrix forms:

$$\begin{bmatrix} x & y & 1 & 0 & 0 & 0 & -xx' & -yx' & -x' \\ 0 & 0 & 0 & x & y & 1 & -xy' & -yy' & -y' \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = 0 \tag{6}$$

Now, say we have four points whose coordinate in Euclidean space is $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$, and their corresponding coordinates in projective space is $(x'_1, y'_1), (x'_2, y'_2), (x'_3, y'_3), (x'_4, y'_4)$, let:

$$\begin{aligned} p_i &= \begin{pmatrix} x_i & y_i & 1 & 0 & 0 & 0 & -x_i x'_i & -y_i x'_i & -x'_i \end{pmatrix} \\ p'_i &= \begin{pmatrix} 0 & 0 & 0 & x_i & y_i & 1 & -x_i y'_i & -y_i y'_i & -y'_i \end{pmatrix} \end{aligned} \tag{7}$$

we can then easily scale equation 6 for our current point set:

$$\begin{bmatrix} p_1 \\ p'_1 \\ p_2 \\ p'_2 \\ p_3 \\ p'_3 \\ p_4 \\ p'_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ 1 \end{bmatrix} = 0 \tag{8}$$

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