

Relational Database Design(2)



Friday , March 6, 2015

REVIEW

■ $R = (\text{ID}, \text{Name}, \text{City})$

- If ID is a key

■ What are the FDs?

ID	Name	City
1	IBM	New York
2	MS	LA
3	Apple	Texas
4	MS	LA

REVIEW

■ $R = (\text{ID}, \text{Name}, \text{City})$

- $\text{ID} \rightarrow \text{Name}$

Which rows violate the FD?

(It takes at least 2 rows to violate FD)

ID	Name	City
1	IBM	New York
1	MS	LA
3	Apple	Texas
4	MS	Seattle

REVIEW

■ $R = (\text{ID}, \text{Name}, \text{City})$

- $\text{ID} \rightarrow \text{Name}$

Which rows violate the FD?

Row 1 and 2

$\text{ID} \rightarrow \text{Name}$

means

Each value of ID can have only a single name,
but no restriction on name.

ID	Name	City
1	IBM	New York
1	MS	LA
3	Apple	Texas
4	MS	Seattle

REVIEW

■ R= (ID, Name, City)

- ID → Name,
- ID → City,
- Name → City

Which rows violate each FD?

(It take at least 2 rows to violate FD)

ID	Name	City
1	IBM	New York
2	IBM	LA
3	Apple	Texas
4	MS	Seattle
4	MS	New York

REVIEW

■ $R = (\text{ID}, \text{Name}, \text{City})$

$\text{ID} \rightarrow \text{Name}$,

-

$\text{ID} \rightarrow \text{City}$,

r4 and r5

$\text{Name} \rightarrow \text{City}$

r1 and r2, r4 and r5

ID	Name	City
1	IBM	New York
2	IBM	LA
3	Apple	Texas
4	MS	Seattle
4	MS	New York

Transitive Dependencies

- If $ID \rightarrow Name$ and $Name \rightarrow City$
- then we have a new $ID \rightarrow City$ (transitive rule)

ID	Name	City
1	IBM	New York
2	IBM	New York
3	Apple	Texas
4	MS	Seattle
5	Apple	Texas

The diagram illustrates a transitive dependency in a database table. A red circle is drawn around the 'Name' column header. Two red arrows point from the circled 'Name' entries in rows 3 and 5 to the circled 'City' entry in row 3, indicating that if IBM is associated with New York and Apple is associated with Texas, then Apple must also be associated with Texas.

REVIEW

■ $R = (\text{ID}, \text{Name}, \text{City})$

- $\text{ID} \rightarrow \text{Name}$,
- $\text{ID} \rightarrow \text{City}$
- $\text{Name} \rightarrow \text{City}$

■ What is a key?

■ Is this a 2NF?

■ Is this a 3NF?

ID	Name	City
1	IBM	New York
2	IBM	New York
3	Apple	Texas
4	MS	Seattle
5	Apple	Texas

Boyce-Codd Normal Form

1. *Domains of all attributes of R are atomic*
2. **Every determinant must be a superkey.**

2NF: all **nonkey attributes** are dependent on **the whole key.**

3NF: no transitive dependency.



Example

$R = \{ID, Name, city\}$

ID and Name are both keys.

$ID \rightarrow Name, City$

$Name \rightarrow ID, City$

■ Is it in 3NF?

$ID \rightarrow Name$ and $Name \rightarrow City$

■ Any redundant data can exist?

No because Name \rightarrow ID

ID	Name	City
1	IBM	New York
1	IBM	New York
2	MS	New York
3	Apple	Texas
4	HP	Seattle

Many-Many Relationship



Create an associative table !

SUPPLIER_PART



supplier_no	supplier_name	part_no	quantity
1	IBM	101	20
1	IBM	202	100
2	HP	101	50
2	HP	202	89
2	HP	303	70

What are the keys ?

Candidate Keys

- Assume that supplier names are always unique to each supplier.
- Thus we have two candidate keys:
 - ❖ **(supplier_no, part_no)** and **(supplier_name, part_no)**

supplier_no	supplier_name	part_no	quantity
1	IBM	101	20
1	IBM	202	100
2	HP	101	50
2	HP	202	89
2	HP	303	70

Functional Dependencies:

- Thus we have the following dependencies:
 - $(\text{supplier_no}, \text{part_no}) \rightarrow \text{quantity}$ ✓
 - $(\text{supplier_name}, \text{part_no}) \rightarrow \text{quantity}$ ✓
 - $\text{supplier_name} \rightarrow \text{supplier_no}$ *unique name
 - $\text{supplier_no} \rightarrow \text{supplier_name}$
- No transitive dependency (so it is in 3NF)
- $\text{supplier_name} \rightarrow \text{supplier_no} \rightarrow \text{supplier_name}$

Reflexive

Functional Dependencies:

- Not BCNF , determinant is not a superkey.
 - $\text{supplier_name} \rightarrow \text{supplier_no}$ ❌
 - $\text{supplier_no} \rightarrow \text{supplier_name}$ ❌

Duplicate Data Exist

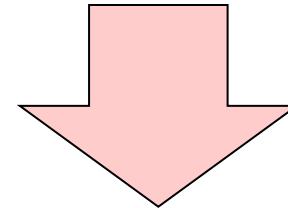
supplier_no	supplier_name	part_no	quantity
1	IBM	101	20
1	IBM	202	100
2	HP	101	50
2	HP	202	89
2	HP	303	70

Decomposition (into BCNF):

SUPPLIER_PART

(**supplier_no**, **supplier_name**, **part_no**, **quantity**)

- Decompose the *schema* into:



SUPPLIER_ID (**supplier_no**, **supplier_name**)

SUPPLIER_PARTS (**supplier_no**, **part_no**, **quantity**)

Decompose 3NF into BCNF

SUPPLIER_PART (supplier_no, supplier_name, part_no, quantity)

split

SUPPLIER_ID

<u>supplier_no</u>	<u>supplier_name</u>
1	IBM
2	HP

SUPPLIER_PARTS

<u>supplier_no</u>	<u>part_no</u>	<u>quantity</u>
1	1	20
1	2	100
2	1	50
2	2	89
2	3	70

Goal — Devise a Theory for the Following

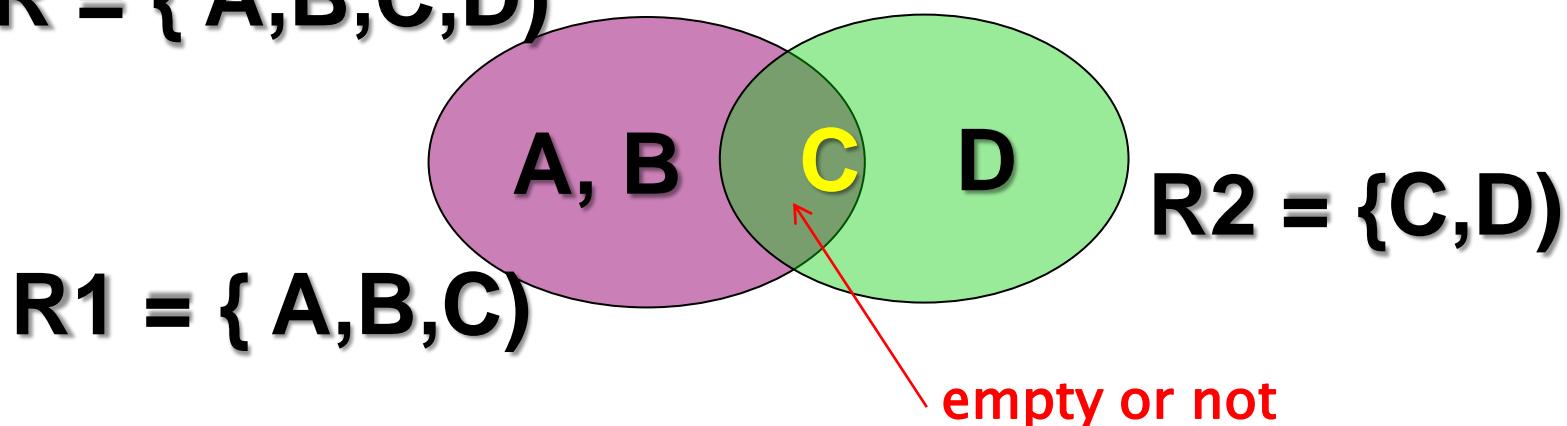
- Decide whether a particular relation R is in “good” form.
- In the case that a relation R is not in “good” form, decompose it into a set of relations $\{R_1, R_2, \dots, R_n\}$ such that
 - each relation is in a good form
 - the decomposition is a **lossless-join** decomposition
- Our theory is based on:
 - *functional dependencies*
 - *multi-valued dependencies*

Decomposition

- All attributes of an original schema (R) must appear in the decomposition (R_1, R_2):

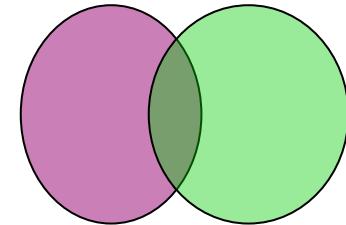
$$R = R_1 \cup R_2$$

$R = \{ A, B, C, D \}$



Lossless-join Decomposition

$$\blacksquare R = R_1 \cup R_2$$



- **Lossless-join** decomposition.
 - For all possible relations r on schema R

$$r = \Pi_{R1}(r) \bowtie \Pi_{R2}(r)$$

Join give the same result as r

Π = column projection
 \bowtie = cross product

Functional Dependencies



Revisited

Functional Dependencies (Cont.)

- K is a **superkey** for relation schema R if and only if $K \rightarrow R$
- K is a candidate key for R iff
 - $K \rightarrow R$, and
 - no $Y \subset K, Y \rightarrow R$

minimal key

Example

$R = (sid, name, grade)$

$sid, name \rightarrow grade$ and

$sid \rightarrow name, grade$

Hence $\{sid, name\}$ is a super key.

but

$\{sid, name\}$ is not a candidate key;

$\{sid\} \subset \{sid, name\}$, $sid \rightarrow R$

Functional Dependencies (Cont.)

- Functional dependencies allow us to express constraints that cannot be expressed using superkeys.

Use of Functional Dependencies

- Test relations legal under F .
 - whether r satisfies F .
- Specify constraints on the set of legal relations
 - F holds on R if all legal relations on R satisfy F .

r1(R), r2(R) , r3(R) or
tables r1, r2, r3 have the
same schemas

Functional Dependencies (Cont.)

■ Trivial functional dependency

E.g.

- ❖ **customer-name, loan-number → customer-name**
- ❖ **customer-name → customer-name**

○ In general, $Y \rightarrow X$ is trivial if $X \subseteq Y$

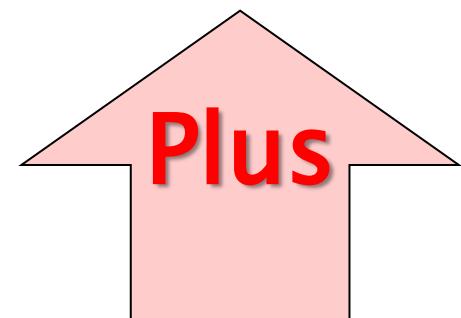
Closure of a Set of Functional Dependencies

- Given a set F of functional dependencies, there are certain other functional dependencies that are *logically implied by F .*
 - E.g. If $A \rightarrow B$ and $B \rightarrow C$, then we can infer that $A \rightarrow C$
- The set of all functional dependencies logically implied by F is the **$\text{closure of } F (F^+)$**

Example

$$F = \{A \rightarrow B, B \rightarrow C\}$$

$$F^+ = \{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$$



Finding all of F^+

■ Armstrong's Axioms:

- if $X \subseteq Y$, then $Y \rightarrow X$
(reflexivity)
- if $Y \rightarrow X$, then $ZY \rightarrow ZX$
(augmentation)
- if $Y \rightarrow X$, and $X \rightarrow Z$, then $Y \rightarrow Z$
(transitivity)

Example

- if $X \subseteq Y$, then $Y \rightarrow X$ (reflexivity)

trivial rule: ID, Name \rightarrow ID ;always true

ID \rightarrow ID

- if $Y \rightarrow X$, then $ZY \rightarrow ZX$ (augmentation)

If ID \rightarrow GPA then

ID, Name \rightarrow GPA, Name

Example (transitivity rule)

- If $Y \rightarrow X$, and $X \rightarrow Z$, then $Y \rightarrow Z$

IF $ID \rightarrow Name$, $Name \rightarrow City$

THEN $ID \rightarrow City$

Homework

- Consider 1NF, 2NF, 3NF, BCNF
- Compute F+ according to the Armstrong axioms

Example

- $R = (A, B, C, G, H, I)$
 $F = \{ A \rightarrow B, A \rightarrow C, CG \rightarrow H, CG \rightarrow I, B \rightarrow H \}$
- some members of F^+
 - $A \rightarrow H$
 - We have $A \rightarrow B, A \rightarrow C, CG \rightarrow H, B \rightarrow H$
 - Can we derive from $\underbrace{A \rightarrow C, CG \rightarrow H}_{\text{cannot derive}}$ contain only single A on left hand side
 - We have $A \rightarrow B$ and $B \rightarrow H$
 - we can derive $A \rightarrow H$ (transitivity)

Example

- $R = (A, B, C, G, H, I)$
 $F = \{ A \rightarrow B, A \rightarrow C, CG \rightarrow H, CG \rightarrow I, B \rightarrow H \}$
- $AG \rightarrow I$
 - We can eliminate $A \rightarrow B$, $B \rightarrow H$, $CG \rightarrow H$
 - No **H** on LHS of any FD
 - Now consider $A \rightarrow C$, $CG \rightarrow I$,
 1. $AG \rightarrow CG$ (by augmenting $A \rightarrow C$ with G)
 2. $CG \rightarrow I$ (from F)
 3. $AG \rightarrow I$ (1-2 and transitivity)

Union Rule

$$F = \{ ID \rightarrow \text{name} , ID \rightarrow \text{dept} \}$$

■ *ID → Name, Dept*

❖ from *ID → Name and ID → Dept* : “union rule”

can be inferred from FD definition,

1. *ID → IDName* by augmenting *ID → Name* with *ID*

2. *ID Name → Dept Name* by augmenting

ID → Dept with *Name*,

3. *ID → Dept Name* 1-2 transitivity

Exercise

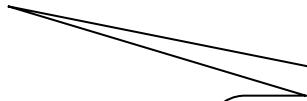
$$F = \{ A \rightarrow B, A \rightarrow C, CG \rightarrow H, CG \rightarrow I, B \rightarrow H \}$$

- **Derive $CG \rightarrow HI$ using Axiom rules**
 - from $CG \rightarrow H$ and $CG \rightarrow I$: “union rule” can be inferred from FD definition, or

Exercise

$$F = \{ A \rightarrow B, A \rightarrow C, CG \rightarrow H, CG \rightarrow I, B \rightarrow H \}$$

- **Derive $CG \rightarrow HI$ by**
- **Consider $CG \rightarrow H, CG \rightarrow I,$**
 - **$CG \rightarrow CGI$ by augmenting $CG \rightarrow I$ with CG**
 - **$CGI \rightarrow HI$ by augmenting $CG \rightarrow H$ with $I,$**
 - **$CG \rightarrow HI$ transitivity**



**It is
 $\{C,G\} \rightarrow \{H,I\}$**

Procedure for Computing F^+

■ To compute the closure of a set of FD F :

■ $F^+ = F$

repeat

for each FD f in F^+

 apply reflexivity and augmentation rules on f

 add the resulting FD to F^+

for each pair of FD f_1 and f_2 in F^+

 if f_1 and f_2 can be combined using transitivity

 then add the resulting FD to F^+

until F^+ does not change any further

There exists
an easier
method

Closure of Functional Dependencies (Cont.)

- We can further simplify manual computation of F^+ by using the following additional rules.
 - If $Y \rightarrow X$ and $Y \rightarrow Z$ holds, then $Y \rightarrow XZ$ holds (union)
 - If $Y \rightarrow XZ$ holds, then $Y \rightarrow X$ and $Y \rightarrow Z$ holds (decomposition)
 - If $Y \rightarrow X$ and $ZX \rightarrow W$ holds, then $ZY \rightarrow W$ holds (pseudotransitivity) replace x with y

The above rules can be inferred from Armstrong's axioms.

Closure of Attribute Sets

Given a set of attributes G ,

Define the *closure* of G under $F(G^+)$ as
the set of attributes that are
functionally determined by G under F :

$$G \rightarrow X \text{ is in } F^+ \Leftrightarrow X \subseteq G^+$$

G can determine X , if X is a subset of G^+

Closure of Attribute Sets

- Algorithm to compute G^+ , the closure of G under F

```
result := G;  
while (changes to result) do  
for each  $X \rightarrow Y$  in  $F$  do  
begin  
    if  $X \subseteq result$  then  $result := result \cup Y$   
end
```

$G^+ = \text{Result}$

Example of Attribute Set Closure

$R = (A, B, C, G, H, I)$

$F = \{ A \rightarrow B, A \rightarrow C, CG \rightarrow H, CG \rightarrow I, B \rightarrow H \}$

$(AG)^+$

1. $\text{result} = AG$ start with AG
2. $\text{result} = ABCG$ add BC ($A \rightarrow C, A \rightarrow B; A \subseteq \text{result}$)
3. $\text{result} = ABCGH$ add H ($CG \rightarrow H; CG \subseteq \text{result}$)
4. $\text{result} = ABCGHI$ add I ($CG \rightarrow I; CG \subseteq \text{result}$)

$$(AG)^+ = ABCGHI$$

$R = (A, B, C, G, H, I)$

$F = \{ A \rightarrow B, A \rightarrow C, CG \rightarrow H, CG \rightarrow I, B \rightarrow H \}$

Is AG a candidate key?

1. Is AG a super key?

1. Does $AG \rightarrow R$? == Is $(AG)^+ \supseteq R$ yes

2. Is any subset of AG a superkey?

1. Does $A \rightarrow R$? == Is $(A)^+ \supseteq R$ no

2. Does $G \rightarrow R$? == Is $(G)^+ \supseteq R$ no

So AG is a candidate key

Uses of Attribute Closure

■ Testing for a superkey:

- ❖ compute Y^+ , and check if $Y^+ \supseteq R$.

■ Testing functional dependencies

- ❖ To check if $Y \rightarrow X$ holds (or, is in F^+),
 - just check if $X \subseteq Y^+$.
- ❖ a simple and cheap test, and very useful

Ex. $(AG)^+ = ABCGHI$
 $AG \rightarrow B, AG \rightarrow H, \dots$

Uses of Attribute Closure

■ Computing closure of F

- ❖ For each $A \subseteq R$, we find the closure A^+ , and
- ❖ For each $S \subseteq A^+$ ($A^+ = \{S, \dots\}$)
 - we output a functional dependency

$$A \rightarrow S.$$

Computing closure of F

■ $R = (A, B, C, G, H)$

■ $F = \{ A \rightarrow B,$
 $B \rightarrow C,$
 $CG \rightarrow H \}$

For each $X \subseteq R$, we find the closure X^+ ,

$$A^+ = ? \{A, B, C\}$$

$$B^+ = ? \{ B, C \}$$

$$C^+ = ? \{ C \}$$

$$G^+ = ? \{ G \}$$

$$H^+ = ? \{ H \}$$

No need to
compute
 AB^+, BC^+, AC^+
(trivial)

$$CG^+ = ? \{C, G, H\}$$

$$AG^+ = ? \{A, B, C, G, H\} = R$$

$$BG^+ = ? \{B, C, G, H\}$$

For each $S \subseteq X^+$, output $X \rightarrow S$.

■ $F^+ = \{ A \rightarrow B, A \rightarrow C,$
 $CG \rightarrow H, B \rightarrow C,$
 $AG \rightarrow H, AG \rightarrow B,$
 $AG \rightarrow C, BG \rightarrow H,$
 $BG \rightarrow C \}$

Benefit of F⁺

Normalization

Goals of Normalization

- Decide whether a particular relation R is in “good” form.
- In the case that a relation R is not in “good” form, decompose it into a set of relations $\{R_1, R_2, \dots, R_n\}$ such that
 - each relation is in good form
 - the decomposition is a lossless-join decomposition
- Our theory is based on:
 - functional dependencies
 - multivalued dependencies

Decomposition

- All attributes of an original schema (R) must appear in the decomposition (R_1 , R_2):

$$R = R_1 \cup R_2$$

- Lossless-join decomposition.
For all possible relations r on schema R

$$r = \Pi_{R1}(r) \bowtie \Pi_{R2}(r)$$

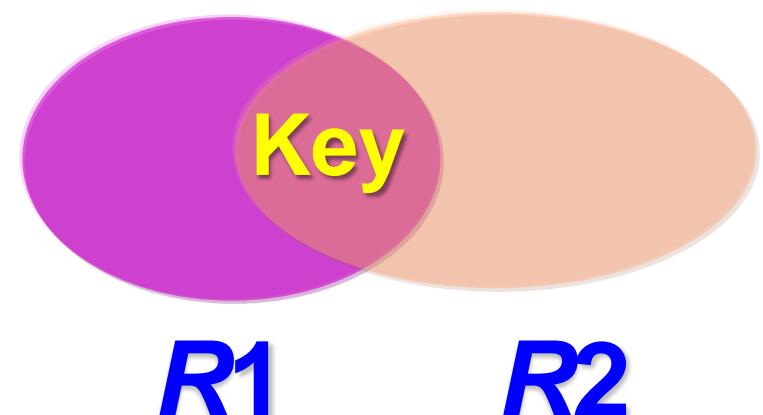
Result of join must
 $= r$

Lossless-join Decomposition

- A decomposition of R into R_1 and R_2 is lossless join if and only if at least one of the following dependencies is in F^+ :

- $(R_1 \cap R_2) \rightarrow R_1$ or
- $(R_1 \cap R_2) \rightarrow R_2$

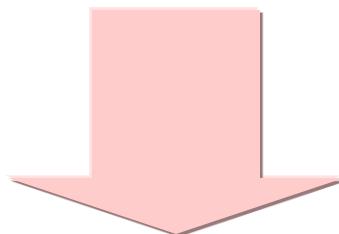
Key for joining
 R_1, R_2



Decomposition

Employee

<u>Name</u>	Address	Province	Phone1	Phone2
John	1	01	-	-
John	2	01	021111111	0222222222
Mary	3	02	-	0255555555
Sam	4	02	-	-



Decomposition

Employee(R1)

<u>Name</u>	Address	Province
John	1	01
John	2	01
Mary	3	02
Sam	4	02

R1 = { Name, Address, Province }

R2 = { Name, Phone }

$R1 \cap R2 = \{ \text{Name} \}$

$\text{Name} \rightarrow R_1$, $\text{Name} \rightarrow R_2$

Telephone(R2)

<u>Name</u>	Phone
John	0211111111
Mary	0255555555
John	0222222222

Normalization Using Functional Dependencies

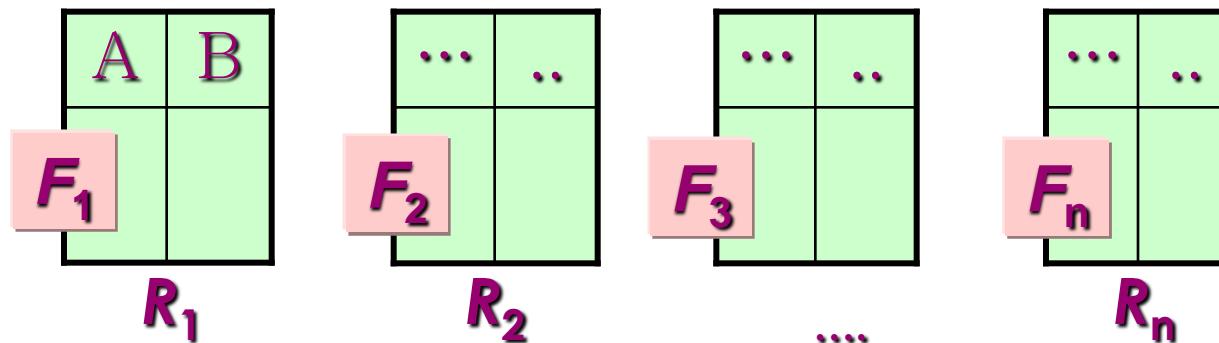
- When we decompose a relation schema R with a set of functional dependencies F into R_1, R_2, \dots, R_n we want
 - Lossless-join decomposition: Otherwise information loss.
 - No redundancy: R_i should be in either Boyce-Codd Normal Form or Third Normal Form.
 - Dependency preservation: the decomposition should be dependency preserving

Dependency preservation

- Before

A	B		R
$F = \{A \rightarrow B, \dots\}$				

- After decompose R into R_1, R_2, \dots, R_n



- dependency preserving

$$(F_1 \cup F_2 \cup \dots \cup F_n)^+ = F^+$$

Example

- $R = (A, B, C)$
 $F = \{A \rightarrow B, B \rightarrow C\}$
 - ❖ Can be decomposed in two different ways

R	A	B	C

$F = \{A \rightarrow B, B \rightarrow C\}$

■ First Method

A	B

R_1

B	C

R_2

$$F_2 = \{B \rightarrow C\}$$

$$F_1 = \{A \rightarrow B\}$$

■ Lossless-join decomposition:

$$R_1 \cap R_2 = \{B\} \text{ and } B \rightarrow BC$$

■ Dependency preserving $(F_1 \cup F_2)^+ = F^+$

Example

- $R = (A, B, C)$
 $F = \{A \rightarrow B, B \rightarrow C\}$
 - ❖ Can be decomposed in two different ways

R	A	B	C

$$F = \{A \rightarrow B, B \rightarrow C\}$$

$$F^+ = \{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$$

■ Second Method

A	B

A	C

R_1

R_2

$$F_1 = \{A \rightarrow B\}$$

■ Lossless-join decomposition:

$$R_1 \cap R_2 = \{A\} \text{ and } A \rightarrow B$$

■ No dependency preserving

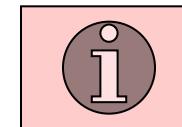
$$(F_1 \cup F_2)^+ \neq F^+$$

(cannot check $B \rightarrow C$ without computing $R_1 \times R_2$)

Boyce-Codd Normal Form

A relation schema R is in BCNF with respect to a set F of FD if for all FDs in F^+ of the form $Y \rightarrow X$, where $Y \subseteq R$ and $X \subseteq R$,
at least one of the following holds:

- $Y \rightarrow X$ is trivial (i.e., $X \subseteq Y$) หรือ
- Y is a superkey for R
all determinants are keys.

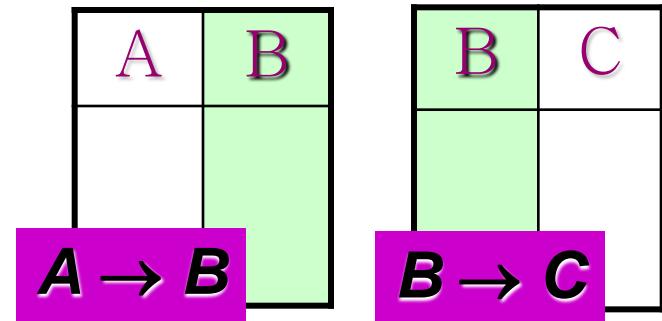


BCNF

Example

- $R = (A, B, C)$
 $F = \{A \rightarrow B, B \rightarrow C\}$
Key = {A}
- R is not in BCNF (B is not a key but $B \rightarrow C$)
- Decomposition $R_1 = (A, B)$, $R_2 = (B, C)$

- R_1 and R_2 in BCNF
- Lossless-join decomposition
- Dependency preserving

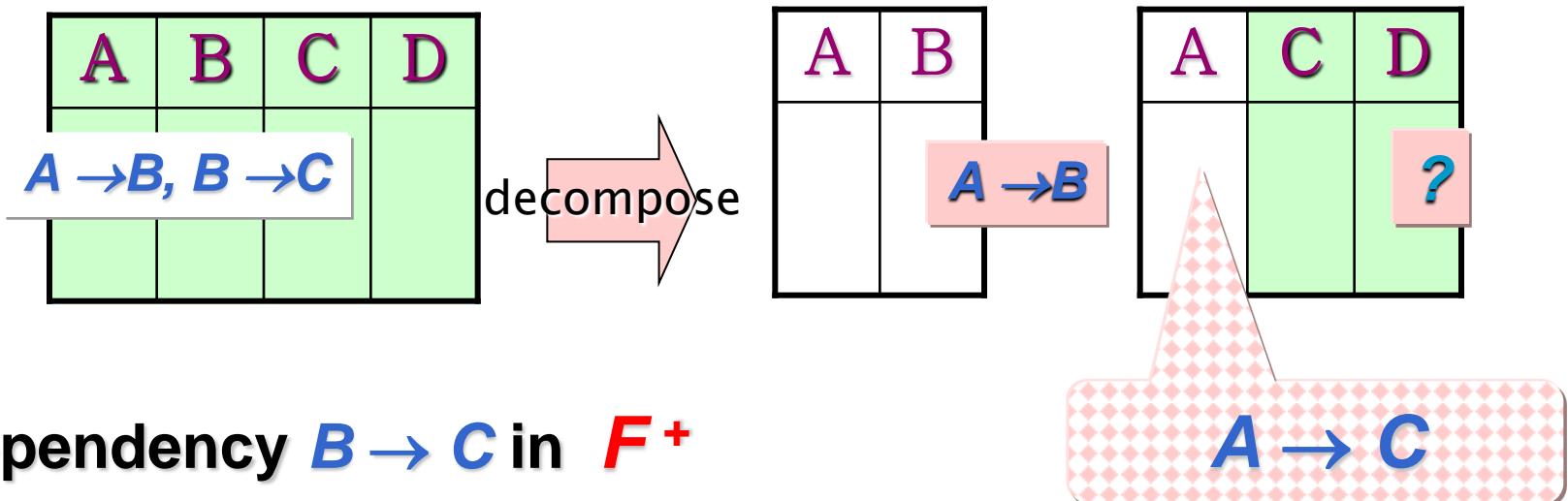


Testing for BCNF

- To check if a non-trivial dependency $X \rightarrow Y$ causes a violation of BCNF
 - 1. compute X^+ and
 - 2. verify that X^+ includes all attributes of R,
that is, X is a superkey of R .
- check **only** the dependencies in the given set F for violation of BCNF, rather than checking all dependencies in F^+ .

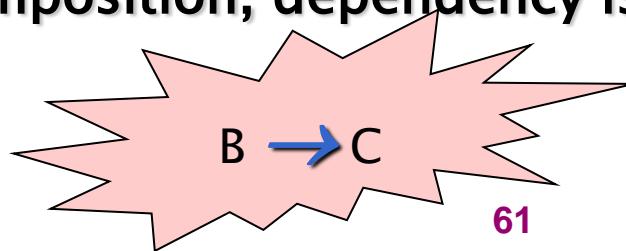
Testing for BCNF

- However, using only F is **incorrect** when testing a relation in a **decomposition** of R
 - E.g. Consider $R(A, B, C, D)$, with $F = \{ A \rightarrow B, B \rightarrow C \}$
 - Decompose R into $R_1(A, B)$ and $R_2(A, C, D)$



- dependency $B \rightarrow C$ in F^+

After decomposition, dependency is not preserved



BCNF Decomposition Algorithm

- $R = (A, B, C)$ $F = \{A \rightarrow B, B \rightarrow C\}$
Key = {A}

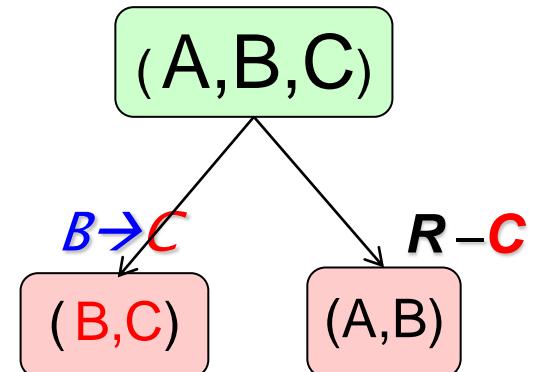
- **R is not in BCNF**
(B is not a key *but* $B \rightarrow C$)

Decomposition

- **Result = { R };** Compute $F_+ = \{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$
o **Repeat**

- Select $R_i = (ABC)$ **not in BCNF**
 - Select $B \rightarrow C$ in F_+
 - Remove R_i from the result
 - **Result = { $R_1 = (B, C)$, $R_2 = R_i - C = (A, B)$ }**

- o **Until done** (No R_i not in BCNF)



BCNF Decomposition Algorithm

result := { R };

Start with R not in BCNF

done := false;

F^+ on R

compute F^+ ;

*Check for FD that
 X is not a key*

while (not **done**) **do**

if (there is a schema R_i in **result** that is not in BCNF)
 then

begin

 let $X \rightarrow Y$ be a nontrivial FD on R_i ,

result := (**result** – R_i) $\cup \{X, Y\} \cup (R_i – Y)$;

end

else **done** := true;

1. Remove R_i
to decompose

2. Add R_1

Add a schema in BCNF

3. Add R_2 : the rest of
 R_i without Y , keep X
for lossless join

Note: each R_i is in BCNF, and decomposition is lossless-join.

Example of BCNF Decomposition

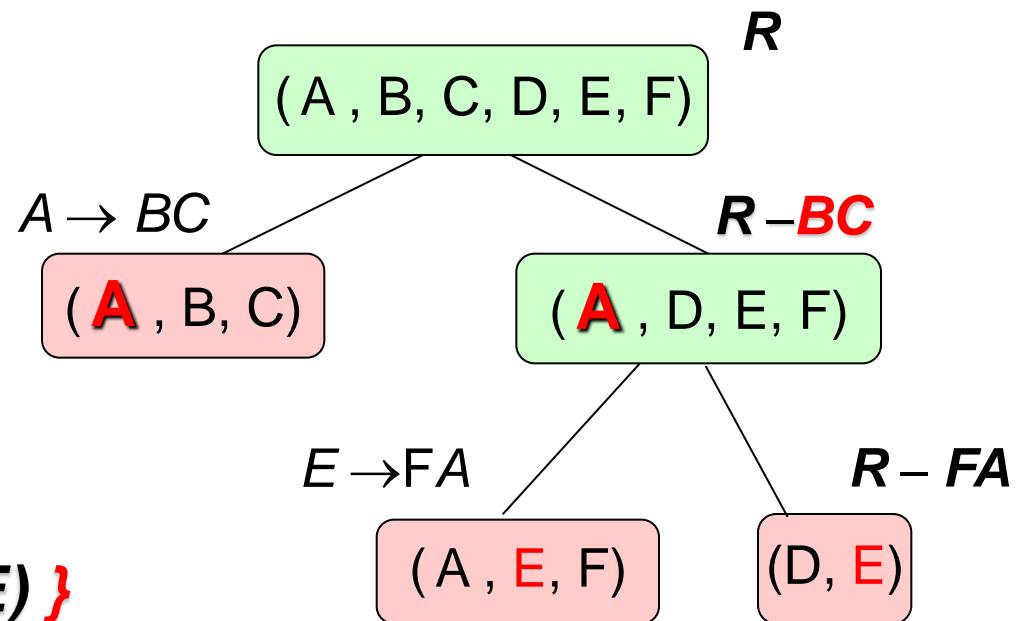
- $R = (A, B, C, D, E, F)$
 $F = \{A \rightarrow CB, E \rightarrow FA\}$ Not in BCNF because
Key = {E, D} A nor E is not a key
- Need Decomposition
- First compute F^+ by
- For each $A \subseteq R$, we find the closure A^+
 - $A^+ \rightarrow ACB$
 - $E^+ \rightarrow FACB$
 - $(ED)^+ \rightarrow FABCD$
- $F^+ = \{A \rightarrow CB, E \rightarrow ACBF, ED \rightarrow ABCDF\}$

Example of BCNF Decomposition

■ $R = (A, B, C, D, E, F)$

$$F^+ = \{A \rightarrow BC, E \rightarrow \underline{ABC}F, ED \rightarrow ABCDF\}$$

■ Decomposition



Final decomposition

$\{(A, B, C), (A, E, F), (D, E)\}$
 $(F_1 \cup F_2 \cup F_3) + \text{equals to } F^+$

All the leave nodes in the result

BCNF and Dependency Preservation

It is not always possible to get a BCNF decomposition that is dependency preserving

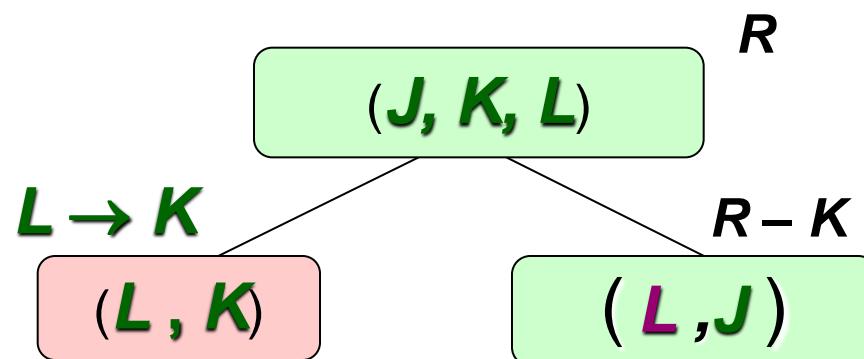
- $R = (J, K, L)$

$$F = \{JK \rightarrow L, L \rightarrow K\}$$

Two candidate keys = JK and JL

- R is not in BCNF ($L \rightarrow K$ but L is not a key)

- Any decomposition of R will fail to preserve $JK \rightarrow L$



Question ?

