Stochastic Volatility model on British Pound Exchange

Bayesian Statistics Course Project

Member:

- Vincent
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- LIN HAOJIAN
- LIN HUI

OUTLINE

01 Brief Review & Layer Partition

02 Realizationealization

03 Volatility and VAR Analysis

04 Model Comparison

05 Sensitivity Analysis

00

Brief Review & Layer Partition

Brief Review

- US Dollar → British Pound
- The observe data is mean-corrected return defined by:

$$y_t = 100 \times \left\{ \log(r_t/r_{t-1}) - \frac{1}{n}\sum_{i=1}^{n} \log(r_i/r_{i-1}) \right\}.$$

- (Stochastic Volatility Model)
- $$\begin{aligned} &y_t \sim N\left(0, \exp(\lambda + \sigma b_t)\right), \text{ for } t = 1, \ldots, n, \\ &b_1 \sim N(0, 1/(1-\phi^2)), \\ &b_{t+1} \sim N(\phi b_t, 1), \text{ for } t = 2, \ldots, n. \end{aligned}$$

$$\phi = \frac{\exp(\psi)}{\exp(\psi)+1} \quad \sigma = \exp(\alpha)$$

$$\alpha \sim N(0, \sigma_\alpha^2),\ \lambda \sim N(0, \sigma_\lambda^2),\ \psi \sim N(0, \sigma_\psi^2) \qquad \sigma_\alpha^2 = \sigma_\lambda^2 = \sigma_\psi^2 = 100.$$

Partition the Layer

(α, λ, ψ)

$$\phi = \frac{\exp(\psi)}{\exp(\psi)+1} \quad \sigma = \exp(\alpha)$$

$\downarrow$

Brief Review & Layer Partition

$(b_1, b_2, \ldots, b_n)$

$\downarrow$

$(y_1, y_2, \ldots, y_n)$

01

MCMC Realization

Data preparation

Source: Garch dataset (British pound/US dollar exchange rates)

Preprocess:

- log-return
- center & scale

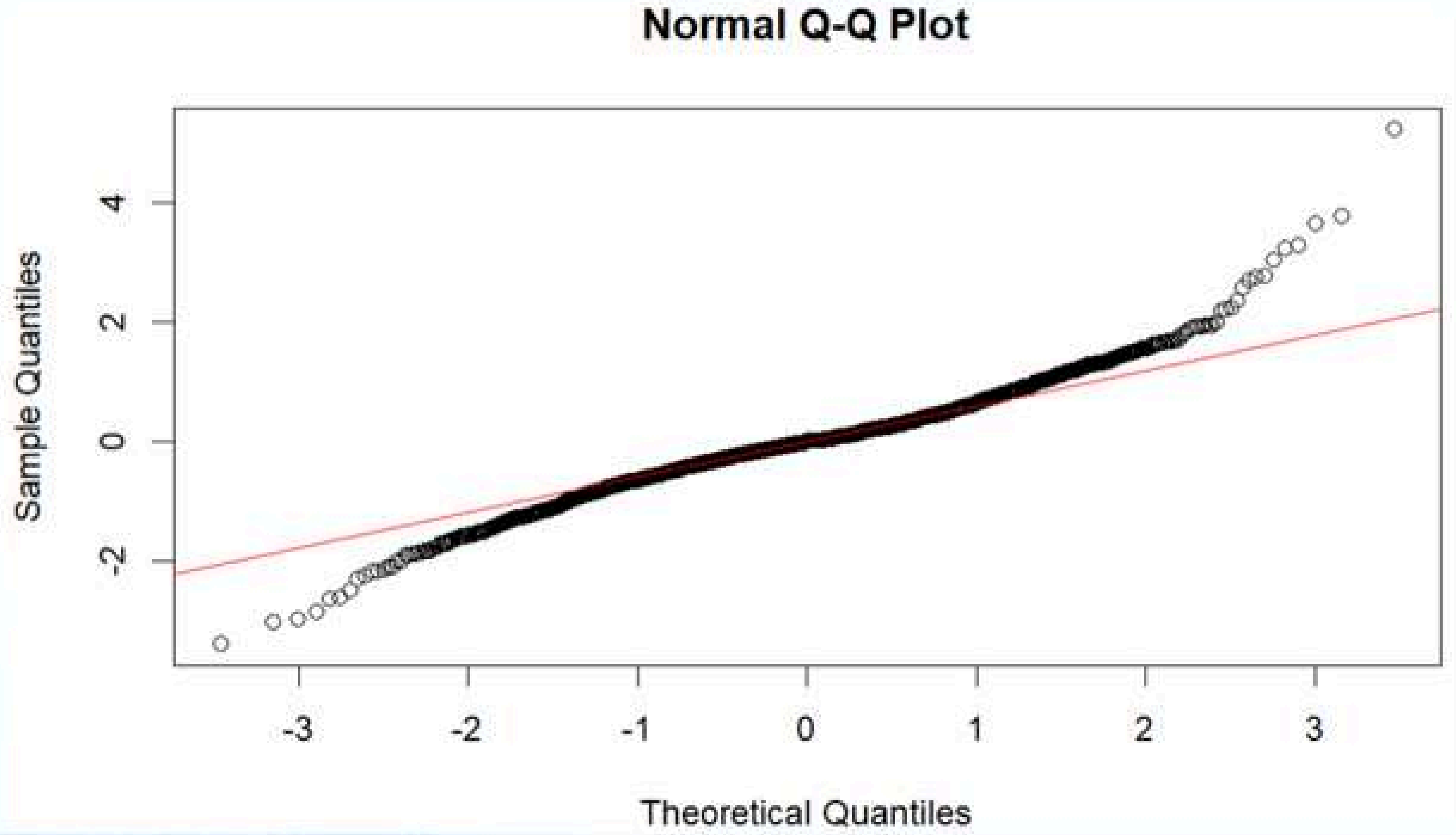
```
n <- length(exchange_rates)

log_returns <- diff(log(exchange_rates))

#yt setting
y <- 100 * (log_returns - mean(log_returns))
n <- length(y)
```

Data preparation

Stationarity Check (ADF Test)

```
        Augmented Dickey-Fuller Test

data:  y
Dickey-Fuller = -12.679, Lag order = 12, p-value = 0.01
alternative hypothesis: stationary
```

Diagnostic Tests




```
        Jarque Bera Test

data:  y
X-squared = 753.79, df = 2, p-value < 2.2e-16
```

```
        Box-Ljung test

data:  y^2
X-squared = 451.71, df = 20, p-value < 2.2e-16
```

MCMC & HMC

- Using Stan(NUTS)

```
model {
  // Priors
  alpha ~ normal(0, 10);  // Weakly informative prior for sigma = exp(alpha)
  lambda ~ normal(0, 10);
  psi ~ normal(0, 10);    // Prior for phi via logit transformation
  z_b ~ normal(0, 1);     // White noise for non-centered parameterization

  // Observation equation (corrected standard deviation)
  for (t in 1:n) {
    y[t] ~ normal(0, exp(0.5 * (lambda + sigma * b[t])));  // Correct parameterization
  }
}
```

- Sampling setup

Stan Configuration:

Chains: 4

Iterations: 2,000 (Warmup: 1,000)

Target acceptance rate: $\delta=0.95$

```
fit <- sampling(
  model,
  data = stan_data,
  seed = 456,
  iter = 2000,
  warmup = 1000,
  chains = 4,
  cores = 4,
  control = list(adapt_delta = 0.95)
)
```

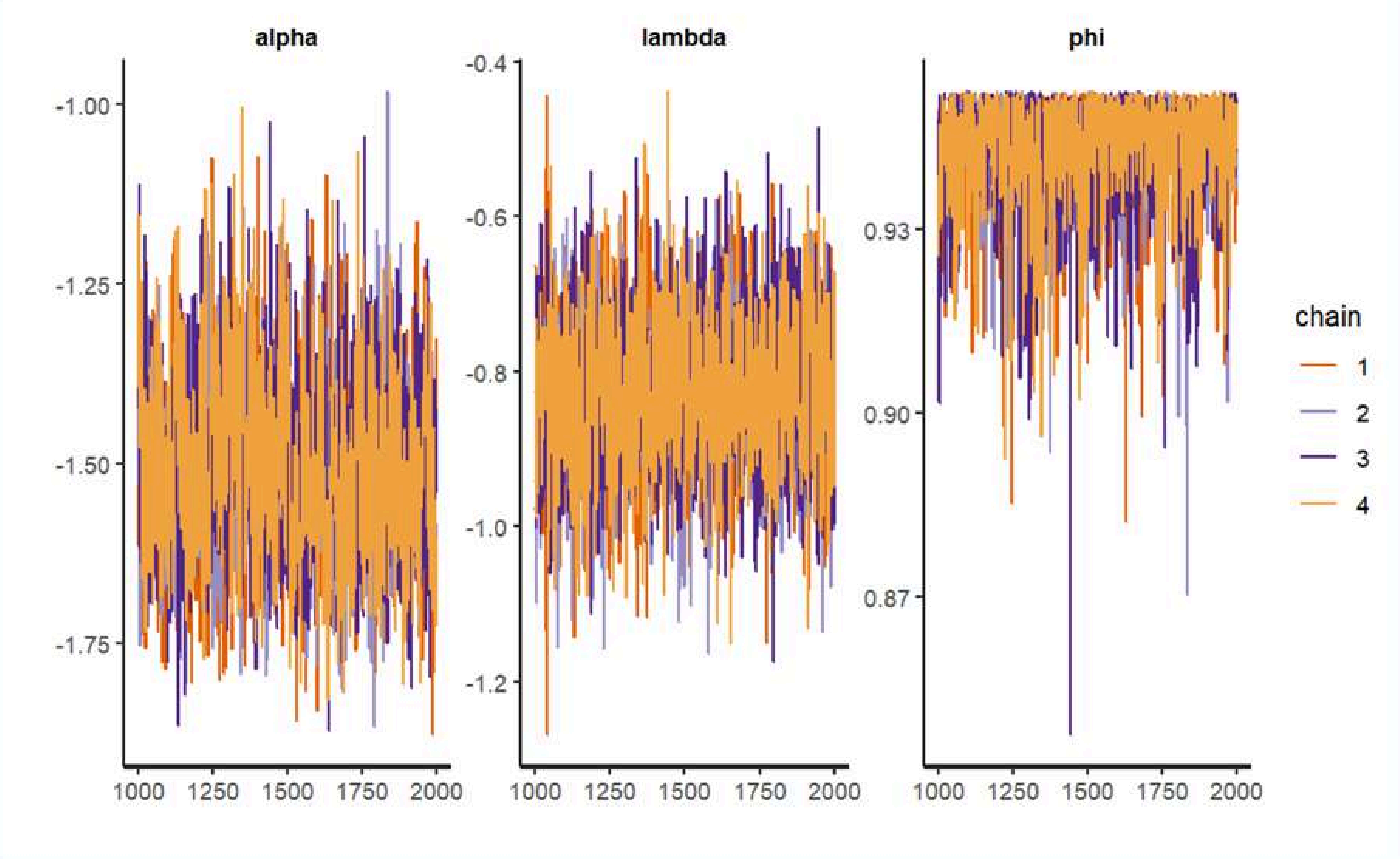
Convergence diagonostics

- Rhat statictics
- Trace plot
- ESS

```
Inference for Stan model: anon_model.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

        mean se_mean   sd  2.5%   25%   50%   75% 97.5% n_eff Rhat
alpha  -1.50       0 0.13 -1.73 -1.59 -1.51 -1.42 -1.23  1407    1
lambda -0.83       0 0.10 -1.02 -0.89 -0.82 -0.76 -0.63  7243    1
psi     2.82       0 0.16  2.41  2.74  2.87  2.94  2.99  2341    1
phi     0.94       0 0.01  0.92  0.94  0.95  0.95  0.95  2369    1
sigma   0.22       0 0.03  0.18  0.20  0.22  0.24  0.29  1412    1
```

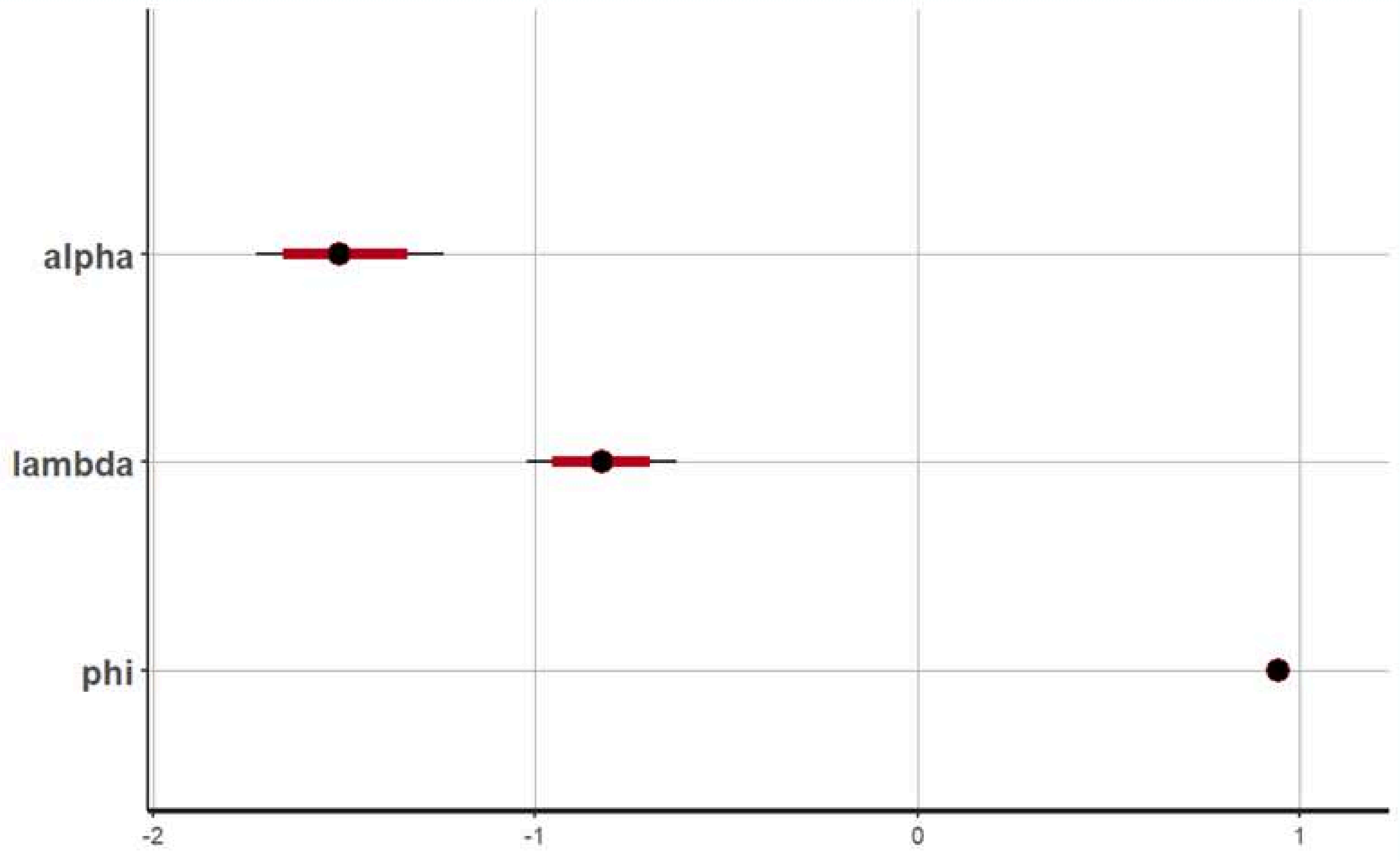
Convergence diagonostics

- Rhat statictics
- Trace plot
- ESS



Convergence diagonostics

- Rhat statictics
- Trace plot
- ESS




```
> cat(mean_ess)
8007.561
```

```
> cat(ess_per_sec)
267.9727 222.7478 282.1551 194.4716 290.7506 291.1735 290.0765 270.8918
```

02

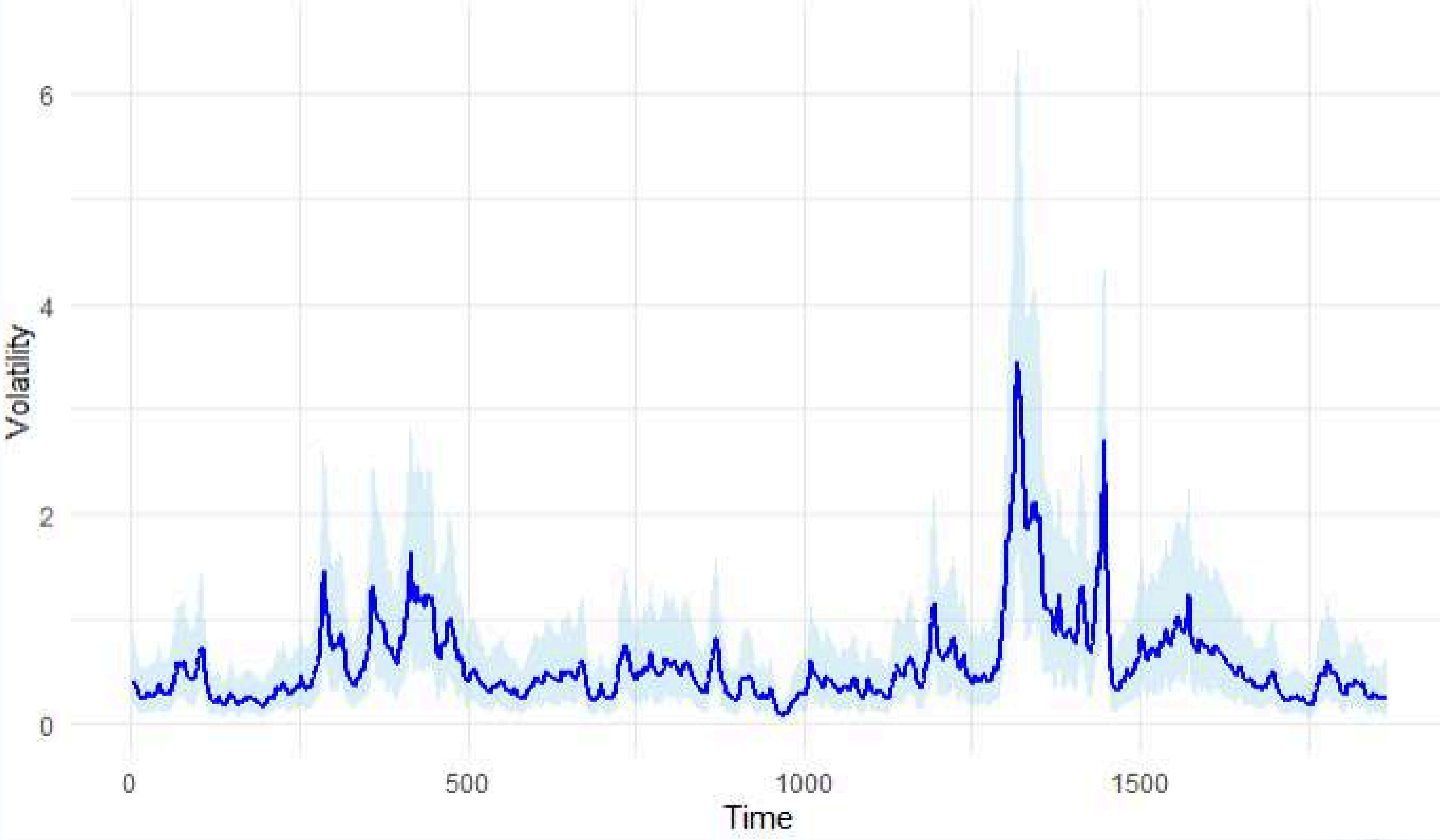
Volatility and VAR Analysis

林晖 12211713

Posterior Volatility Estimation

Blue line: Represents the posterior mean estimate of volatility at each time point.
Light blue shaded area: Indicates the 95% posterior credible interval, which reflects the uncertainty in the volatility estimates.

In areas with obvious peaks and large shadow areas, market fluctuations are severe, and the uncertainty of model predictions is also high. On the contrary, relatively calm, and more confident.

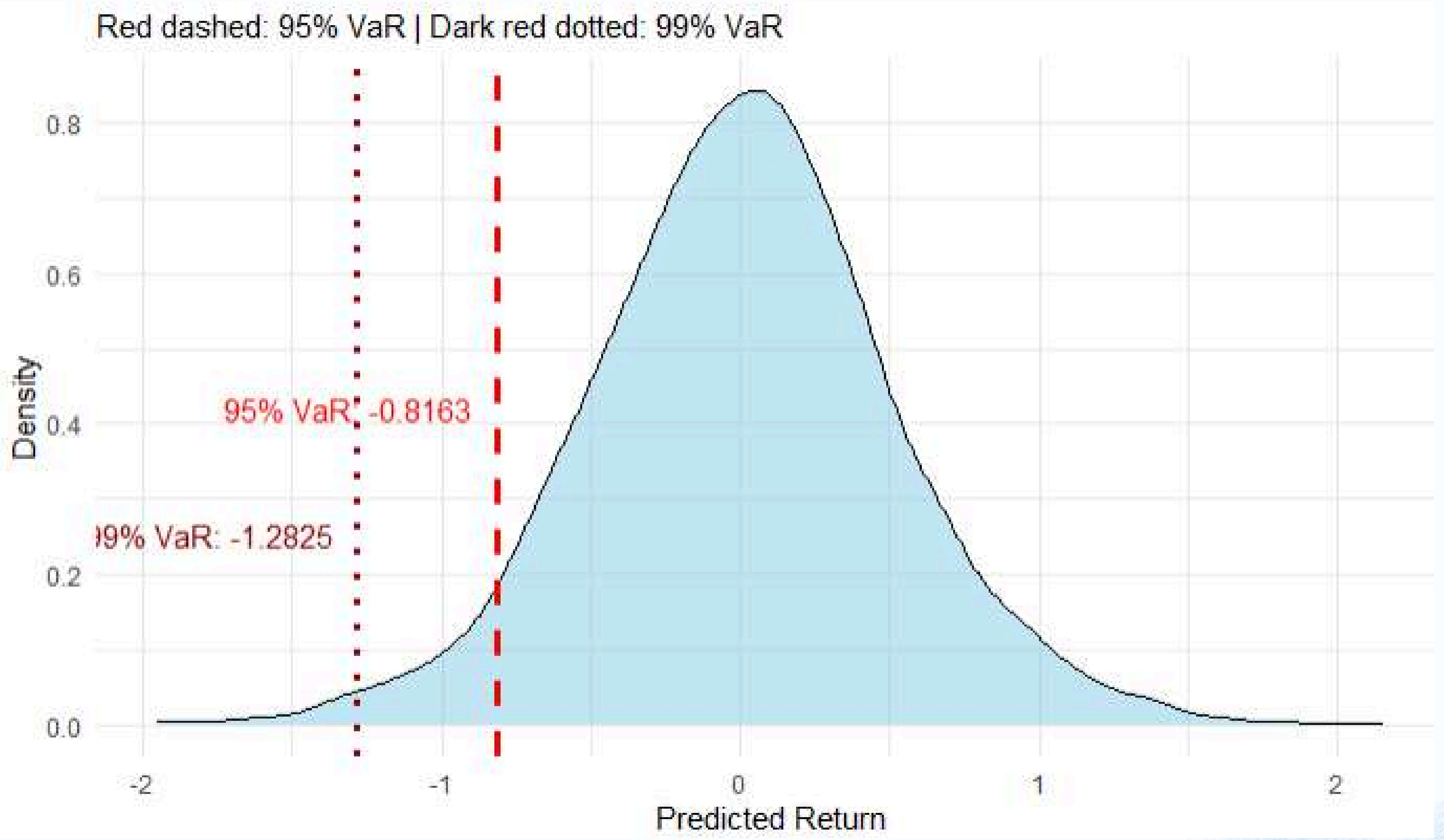


One-Day VaR Estimation

The graph shows the posterior predictive return distribution (blue density curve).

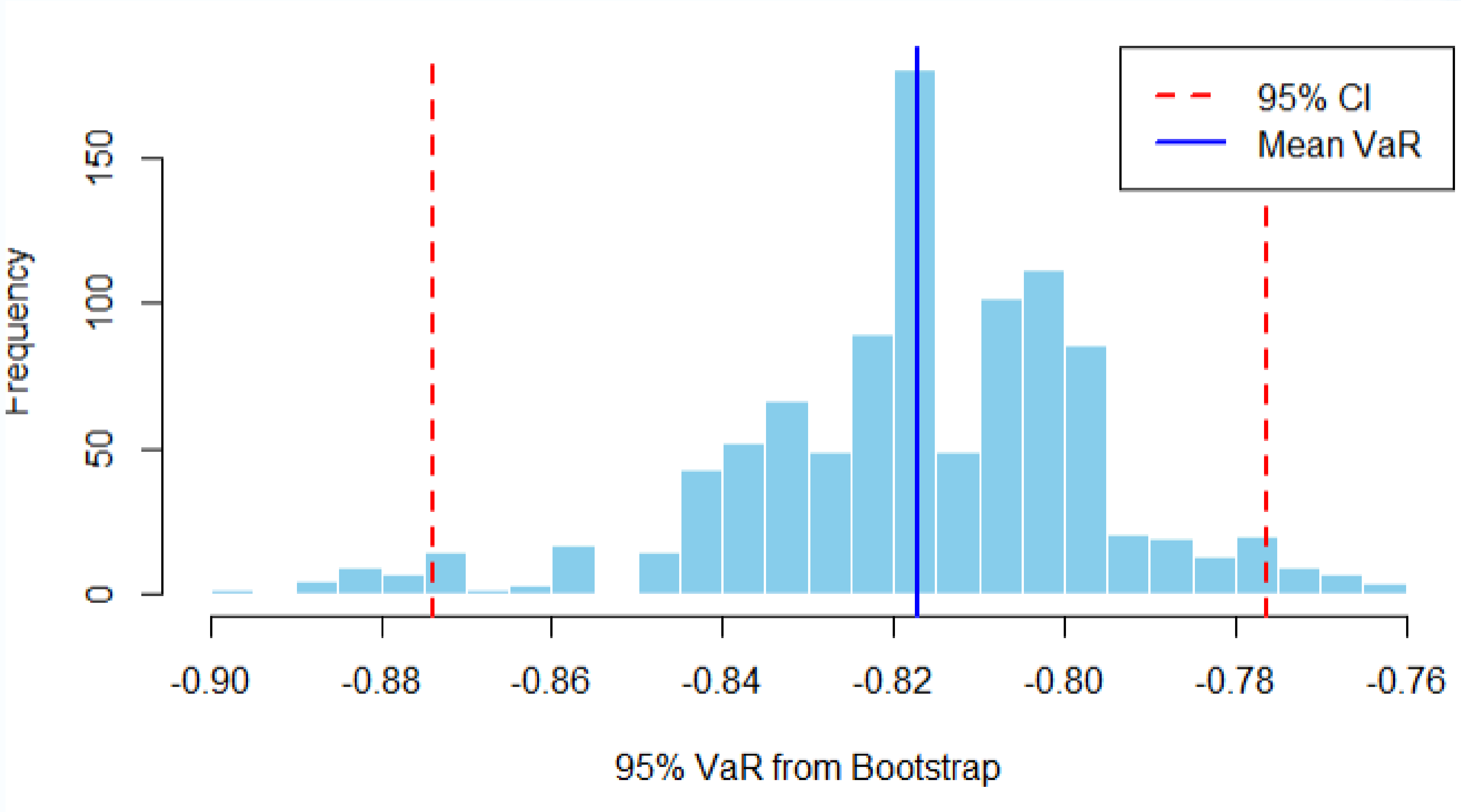
This means that, under the given return distribution, the maximum one-period loss is not expected to exceed **0.8163** at 95% confidence or **1.2825** at 99% confidence.

If actual losses exceed the VaR threshold, this may indicate that the risk model underestimates tail risk.



VaR Uncertainty Analysis (Bootstrap)

The bootstrap distribution of 95% VaR estimates appears approximately symmetric but with slight skewness.
The mean VaR estimate is around -0.82, consistent with the main model result.
The 95% confidence interval ranges approximately between [-0.88, -0.78], indicating the uncertainty in the VaR estimate.
The relatively narrow confidence interval suggests that the VaR estimate from the model is stable.
Overall, despite some sampling uncertainty, the risk estimate is robust.



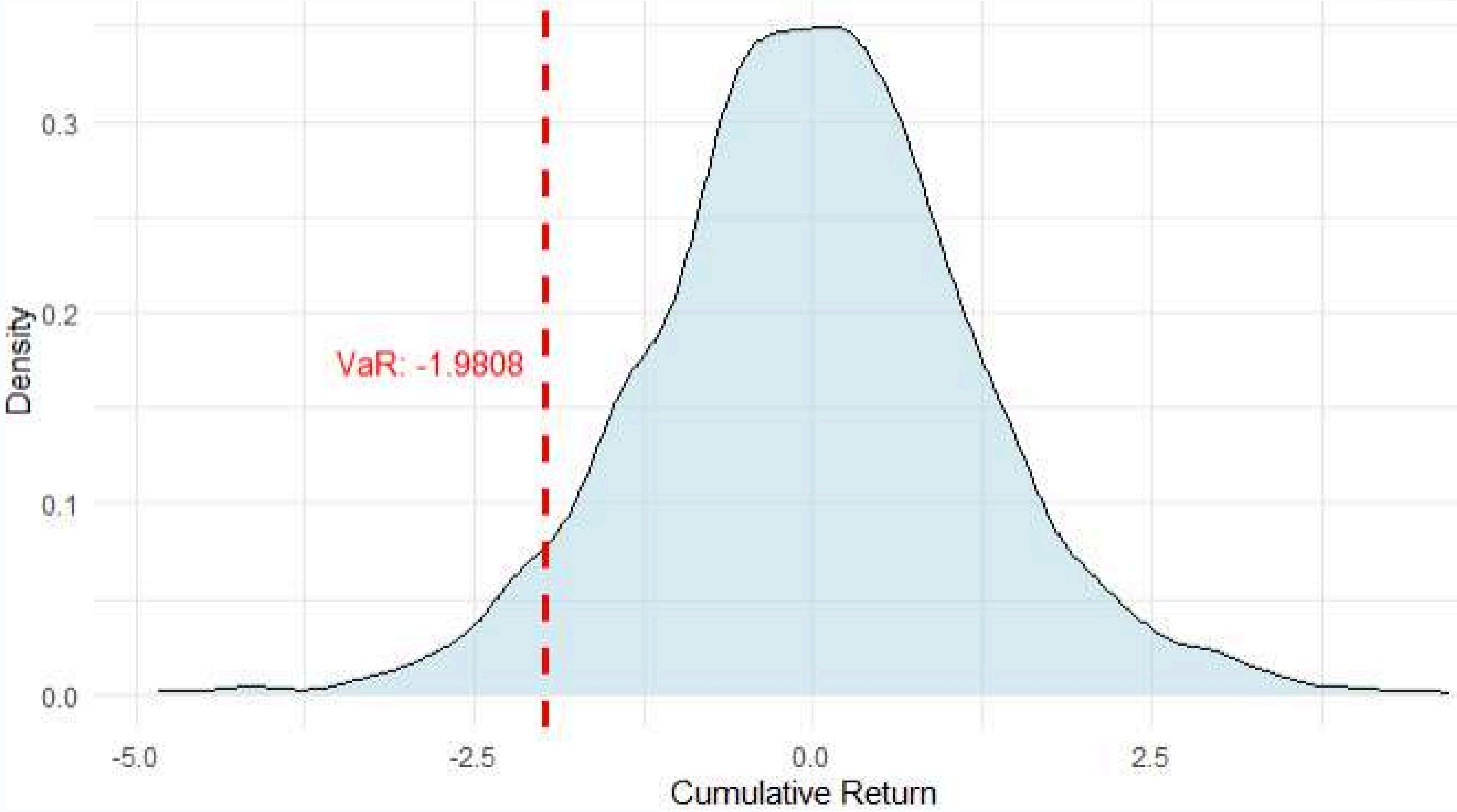
5-Day Cumulative VaR Analysis

The figure illustrates the posterior distribution of 5-day cumulative returns and the corresponding VaR risk measure. The estimated 5-day 95% VaR is **-1.9808**, indicating a 5% probability that returns will fall below -1.9808 over the next 5 days.

The distribution is slightly right-skewed, with the peak slightly below 0, suggesting mostly positive returns but with some negative tail risk.

The VaR confidence bound lies in the left tail of the distribution, signifying substantial but quantifiable risk.

Overall, the model effectively captures the multi-day VaR risk, providing quantitative insights for portfolio risk management.



03

Model Comparison

Name SID

What is leverage?

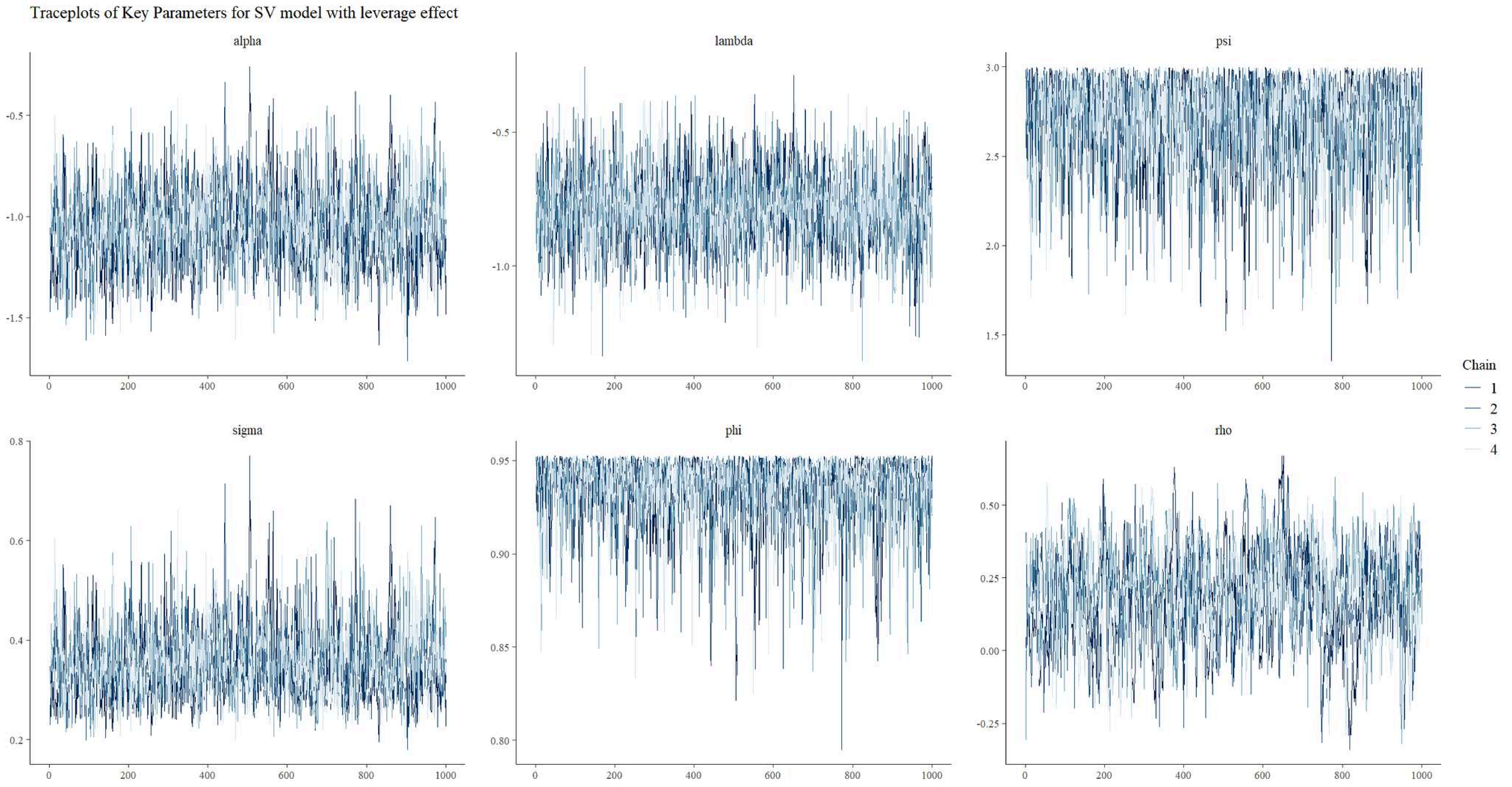
where the correlation matrix of (ε_t, η_t) is

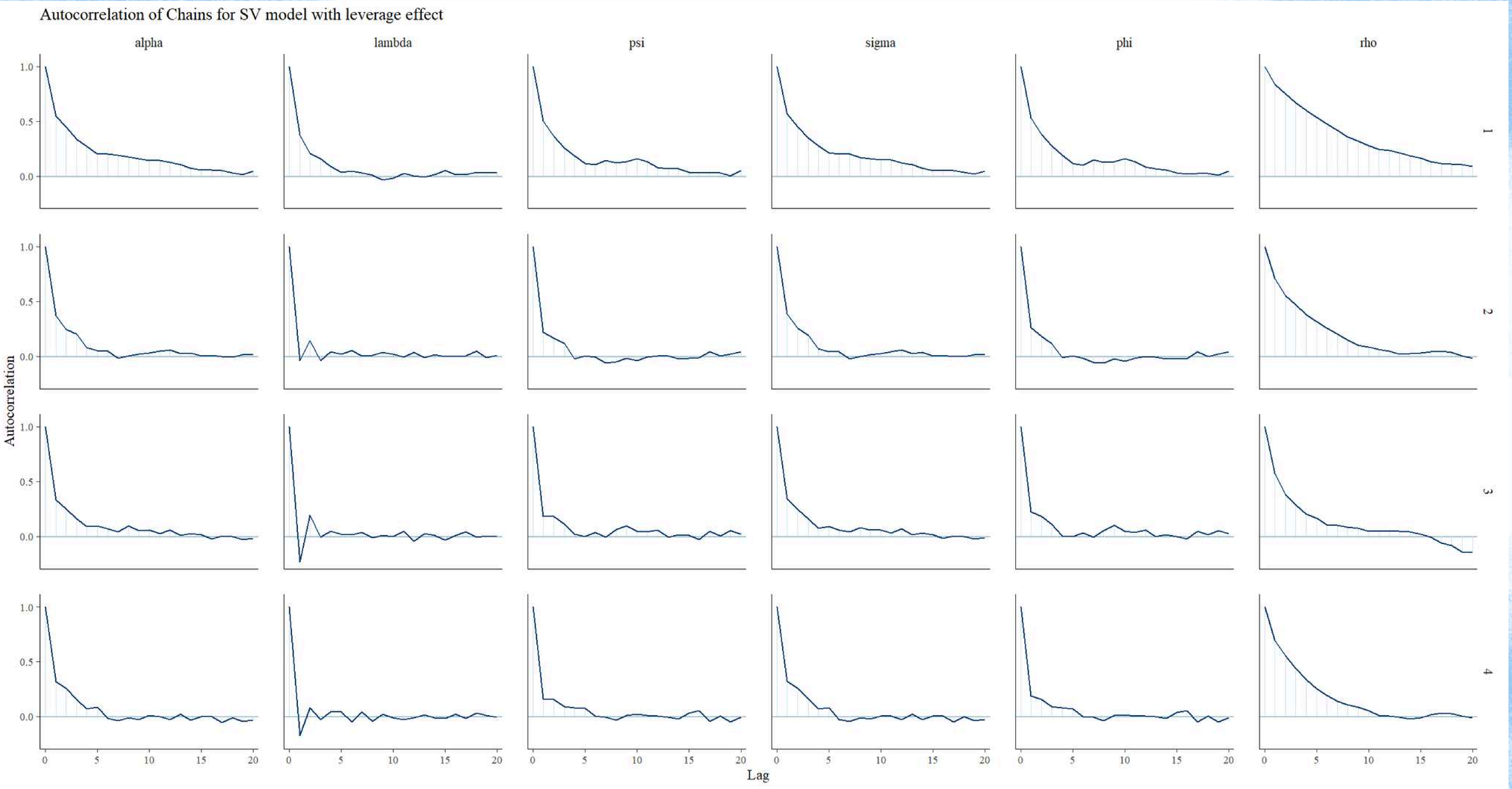
$$\boldsymbol{\Sigma}^{\rho} = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}. \tag{4}$$

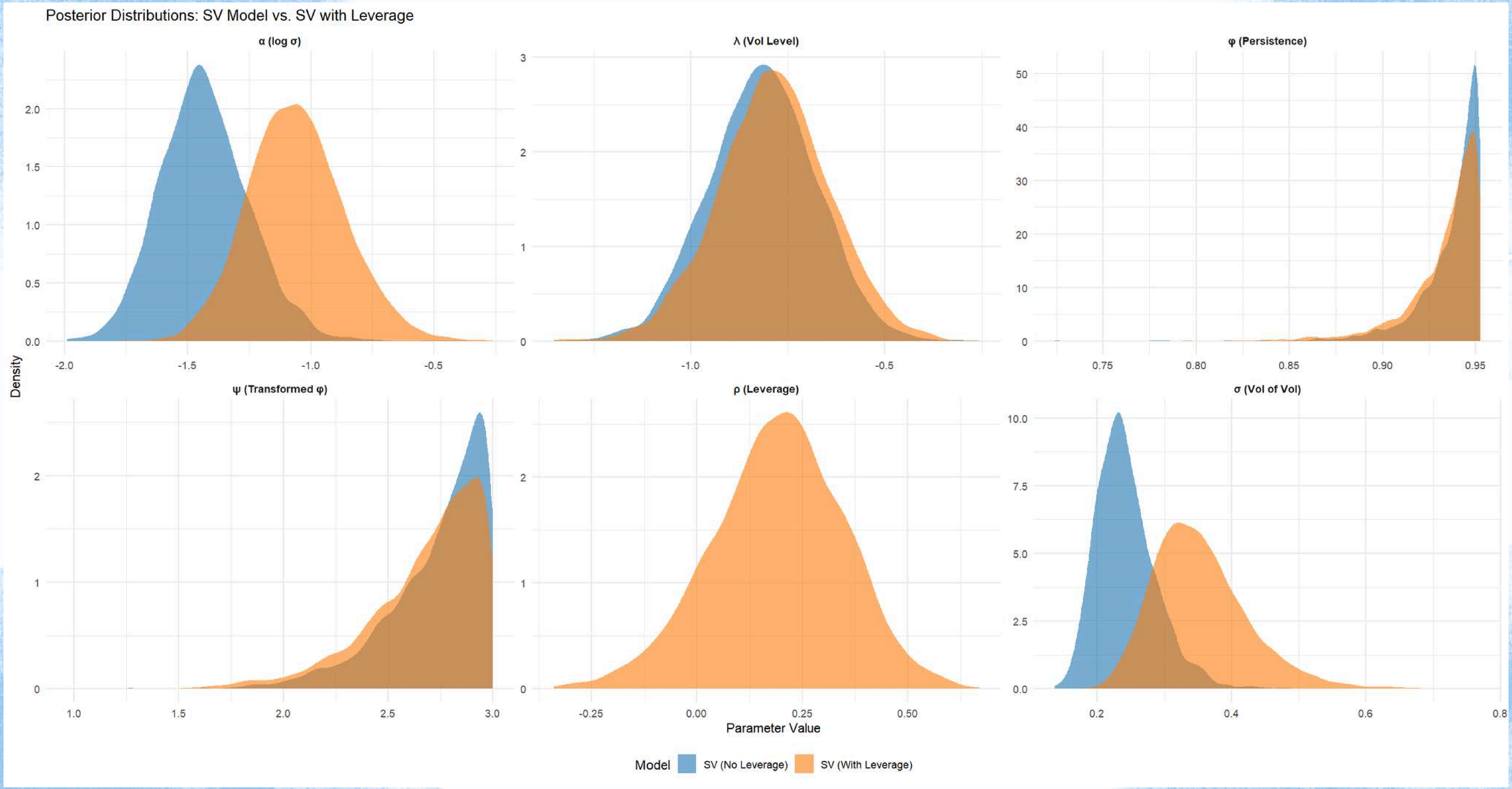
The vector $\boldsymbol{\zeta} = (\mu, \varphi, \sigma, \rho)^{\top}$ collects the SV parameters. The new parameter compared to Equation 1 is a correlation term ρ which relates the residuals of the observations to the innovations of the variance process. Equation 1 is therefore a special case of Equation 3 with a pre-fixed $\rho = 0$.

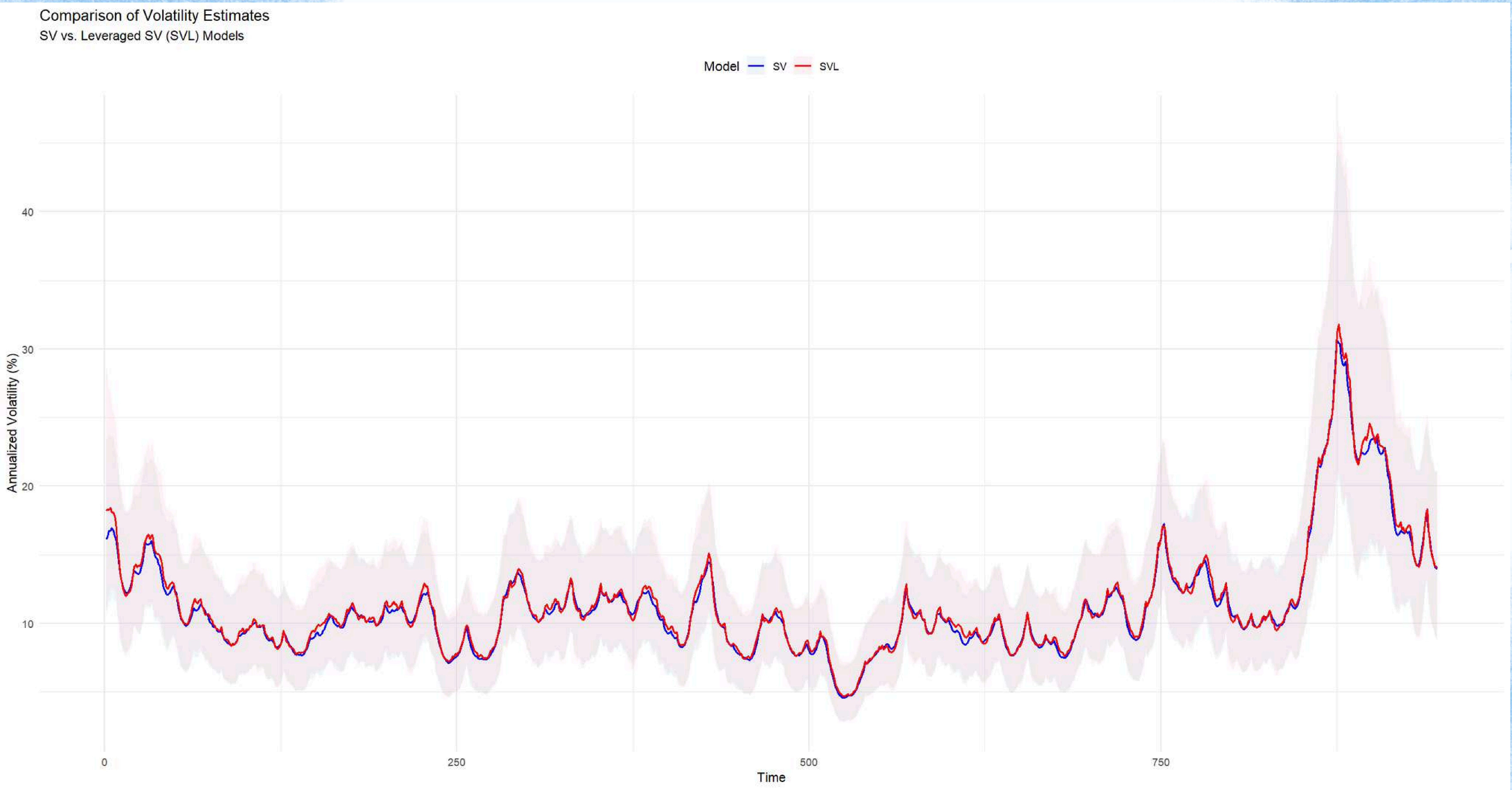
```
  rho ~ uniform(-1, 1);          // Prior for leverage parameter

  // Observation equation (corrected standard deviation)
  for (t in 1:n) {
    y[t] ~ normal(0, exp(0.5 * (lambda + sigma * b[t])));
    if (t < n) {
      b[t+1] ~ normal(phi * b[t] + rho * y[t] / exp(0.5 * (lambda + sigma * b[t])),
                      sqrt(1 - rho^2));
    }
  }
```

```
> print(fit.svl, pars = c("alpha", "lambda", "psi", "sigma", "phi", "rho"))
Inference for Stan model: anon_model.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

        mean se_mean   sd  2.5%   25%   50%   75% 97.5% n_eff Rhat
alpha  -1.06    0.01 0.19 -1.42 -1.19 -1.06 -0.93 -0.65   883 1.00
lambda -0.79    0.00 0.14 -1.07 -0.88 -0.79 -0.69 -0.50  2564 1.00
psi     2.68    0.01 0.26  2.02  2.54  2.75  2.88  2.99  1268 1.00
sigma   0.35    0.00 0.07  0.24  0.30  0.35  0.39  0.52   871 1.00
phi     0.93    0.00 0.02  0.88  0.93  0.94  0.95  0.95  1233 1.00
rho     0.19    0.01 0.16 -0.13  0.09  0.20  0.30  0.48   432 1.01

Samples were drawn using NUTS(diag_e) at Thu Jun  5 12:44:21 2025.
For each parameter, n_eff is a crude measure of effective sample size,
and Rhat is the potential scale reduction factor on split chains (at
convergence, Rhat=1).
```


Traceplots of Key Parameters for SV model with leverage effect
alpha
lambda
psi
sigma
phi
rho
Chain
1
2
3
4
-0.5
-1.0
-1.5
1.5
2.0
2.5
3.0
0.2
0.4
0.6
0.8
0.80
0.85
0.90
0.95
-0.25
0.00
0.25
0.50
0
200
400
600
800
1000


Autocorrelation of Chains for SV model with leverage effect
alpha
lambda
psi
sigma
phi
rho
Autocorrelation
Lag
1
2
3
4


Posterior Distributions: SV Model vs. SV with Leverage
α (log σ)
λ (Vol Level)
φ (Persistence)
ψ (Transformed φ)
ρ (Leverage)
σ (Vol of Vol)
Density
Parameter Value
Model
SV (No Leverage)
SV (With Leverage)



Comparison of Volatility Estimates
SV vs. Leveraged SV (SVL) Models
Model
SV
SVL
Annualized Volatility (%)
40
30
20
10
0
250
500
750
Time

SV WAIC: 1973.6
SVL WAIC: 1970.1
Difference: 3.4

	RMSE	MAE
SV	1.131207	0.9667018
SVL	1.140530	0.9765608

04

Sensitivity Analysis

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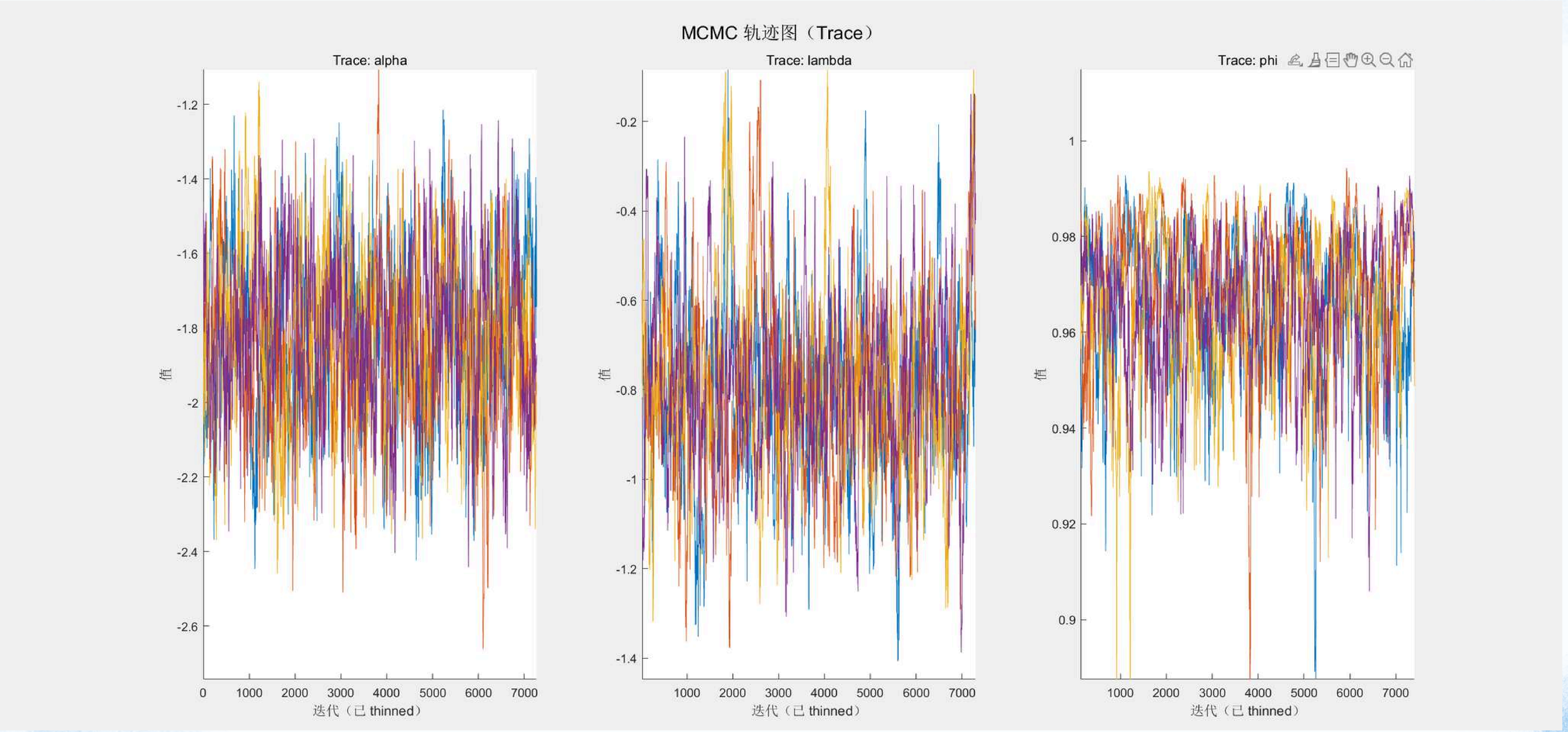
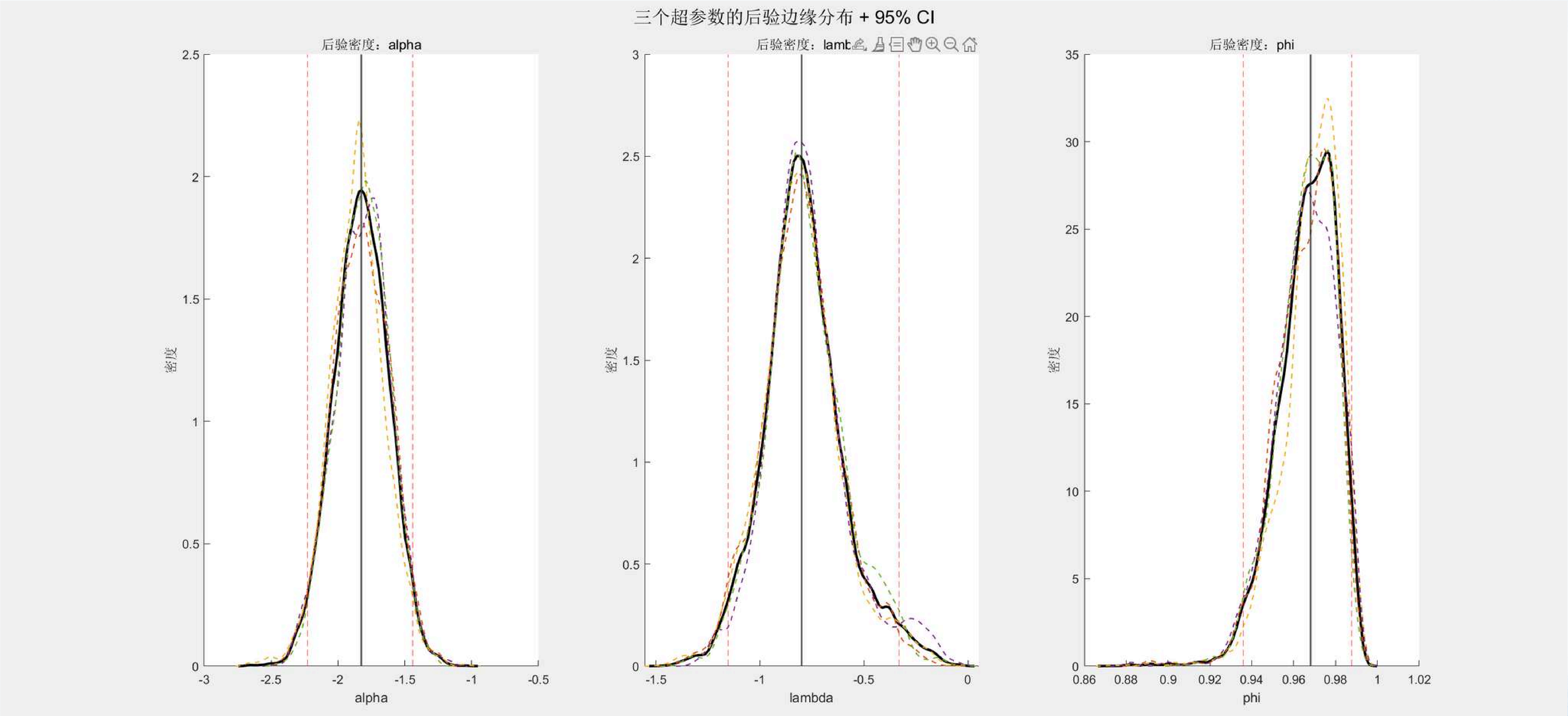
Result Using MH-within Gibbs



Result Using Gibbs



Result Using Gibbs

```
alpha:  Mean = -1.8267,  95% CI = [-2.2276, -1.4420]
lambda: Mean = -0.7879,  95% CI = [-1.1515, -0.3311]
phi:    Mean = 0.9665,   95% CI = [0.9358, 0.9878]
```

```
alpha:  Rˆ = 1.0033
lambda: Rˆ = 1.0022
phi:    Rˆ = 1.0040
```

- analyze_sv_nuts_noncentered.m
- build_tree.m
- garch (1).csv
- leapfrog.m
- load_garch_data.m
- log_prior.m
- nuts_one_step.m
- nuts_sampler.m
- nuts_sampler_theta.m
- reconstruct_b.m
- run_sv_mwg_loguniform.m
- run_sv_nuts_noncentered.m
- set_priors.m
- sv_logpost_gibbs.m
- sv_logpost_grad_noncentered.m
- sv_logpost_grad_theta.m
- sv_logpost_zb.m
- sv_rwmh_samples.mat
- sv_rwmh_samples_fixed.mat
- sv_rwmh_samples_loguniform.mat

Different Prior

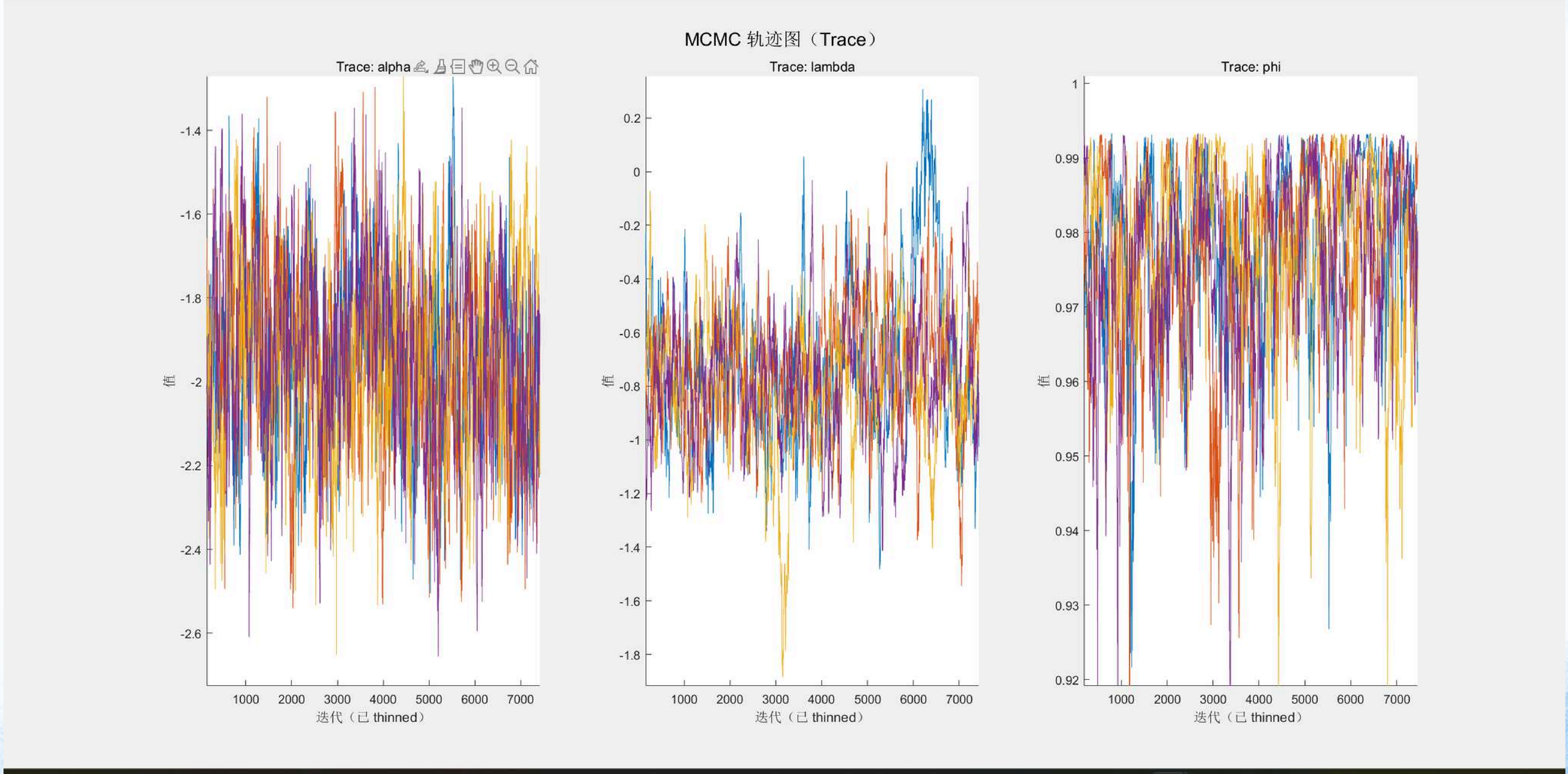
```matlab
function lp = log_prior(param, prior)
% 计算单个参数的对数先验概率密度
% 支持 normal, uniform, student

switch lower(prior.type)
    case 'normal'
        mu = prior.param1;
        sigma = prior.param2;
        lp = -0.5*log(2*pi*sigma^2) - 0.5*((param - mu)/sigma).^2;

    case 'uniform'
        a = prior.param1;
        b = prior.param2;
        if all(param >= a) && all(param <= b)
            lp = -log(b - a);
        else
            lp = -Inf;
        end

    case 'student'
        mu = prior.param1;
        sigma = prior.param2;
        df = prior.df;
        x_std = (param - mu) ./ sigma;
        lp = gammaln((df + 1)/2) - gammaln(df/2) - 0.5*log(pi*df) ...
             - log(sigma) - ((df + 1)/2)*log(1 + (x_std).^2 / df);

    otherwise
        error('Unsupported prior type: %s', prior.type);
end
end
```

Normal (Default)

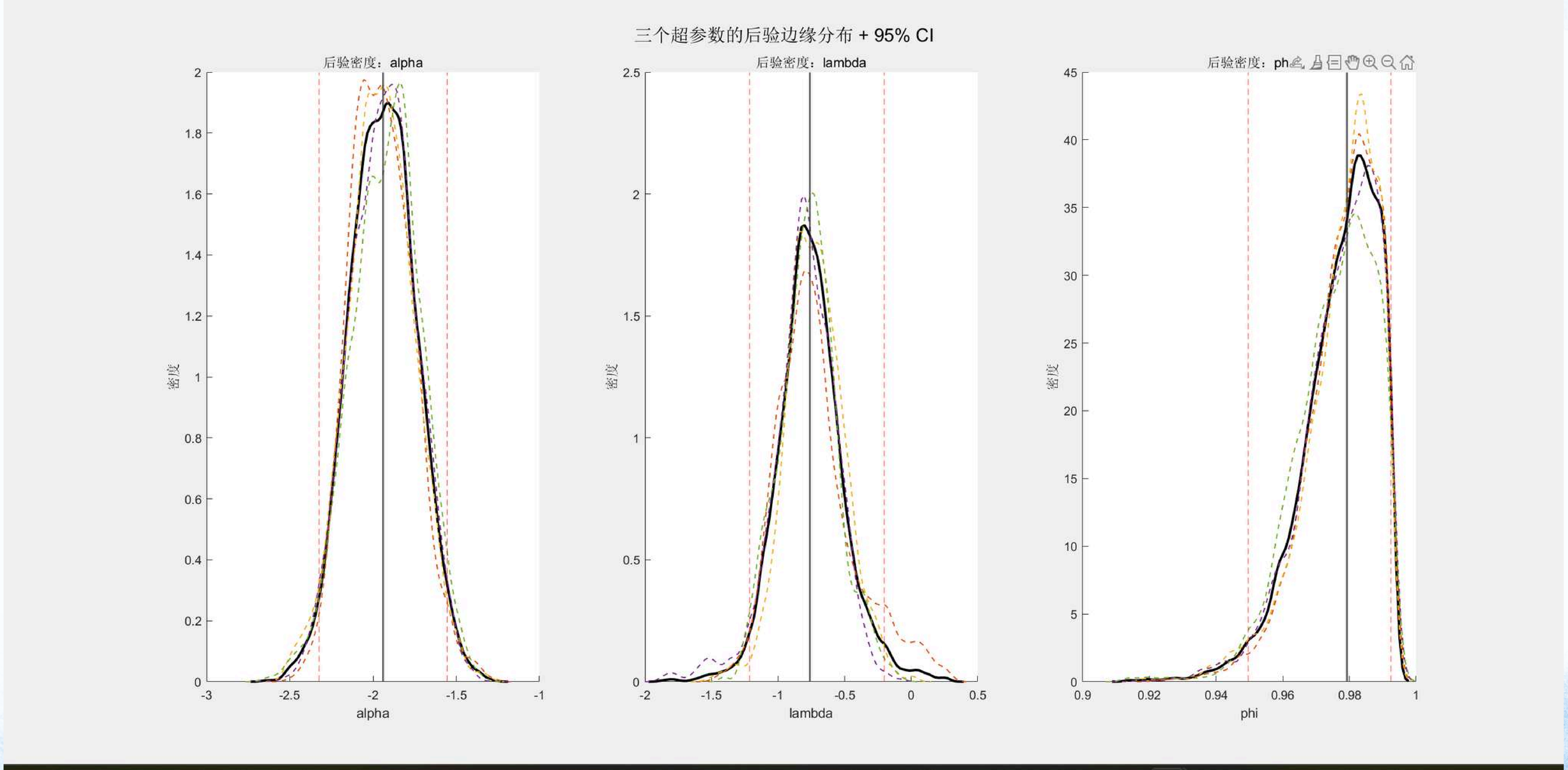
Uniform

Heavy-Tailed t

Different Prior : Uniform



Different Prior : Uniform

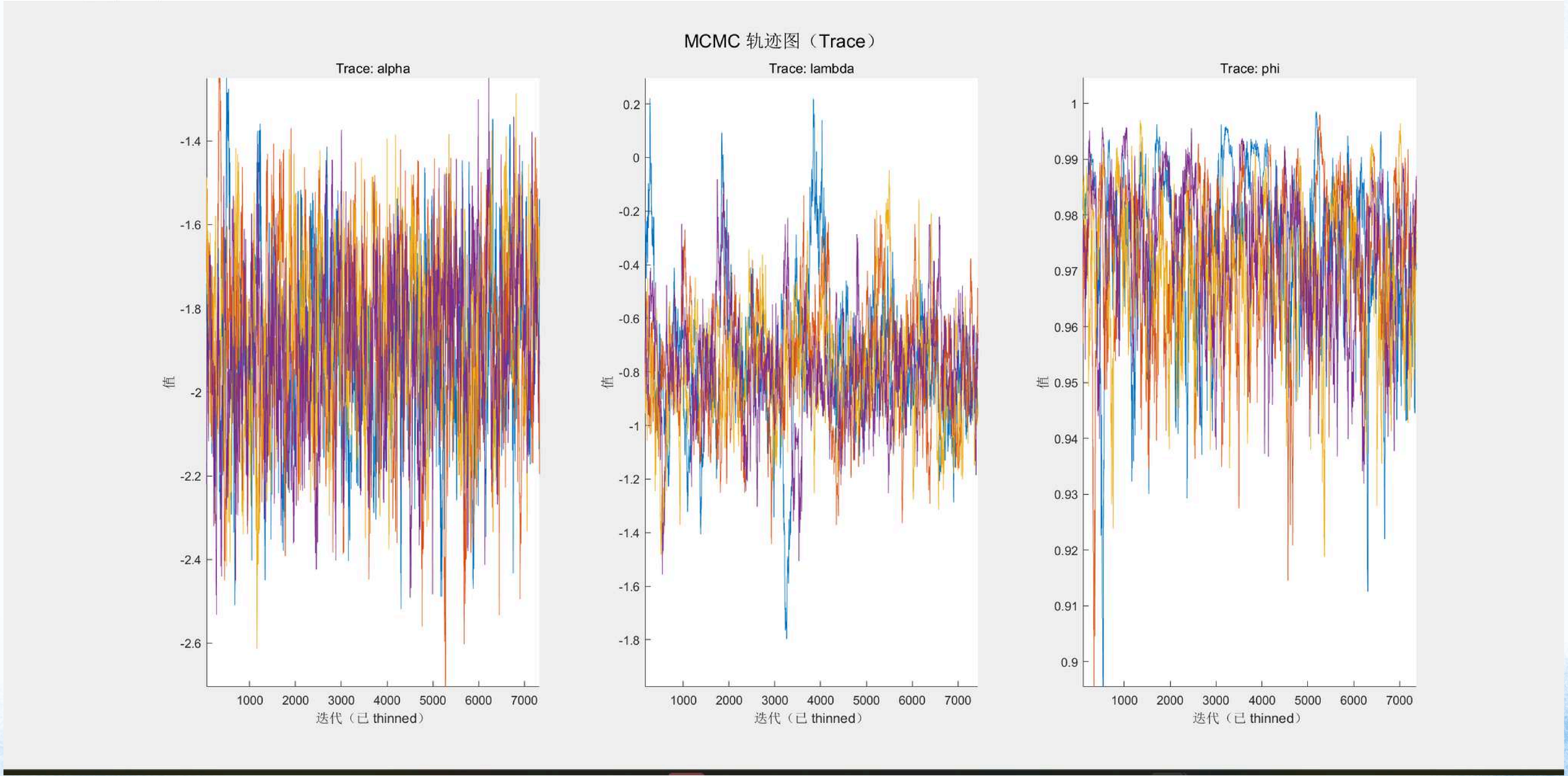


Different Prior : Uniform

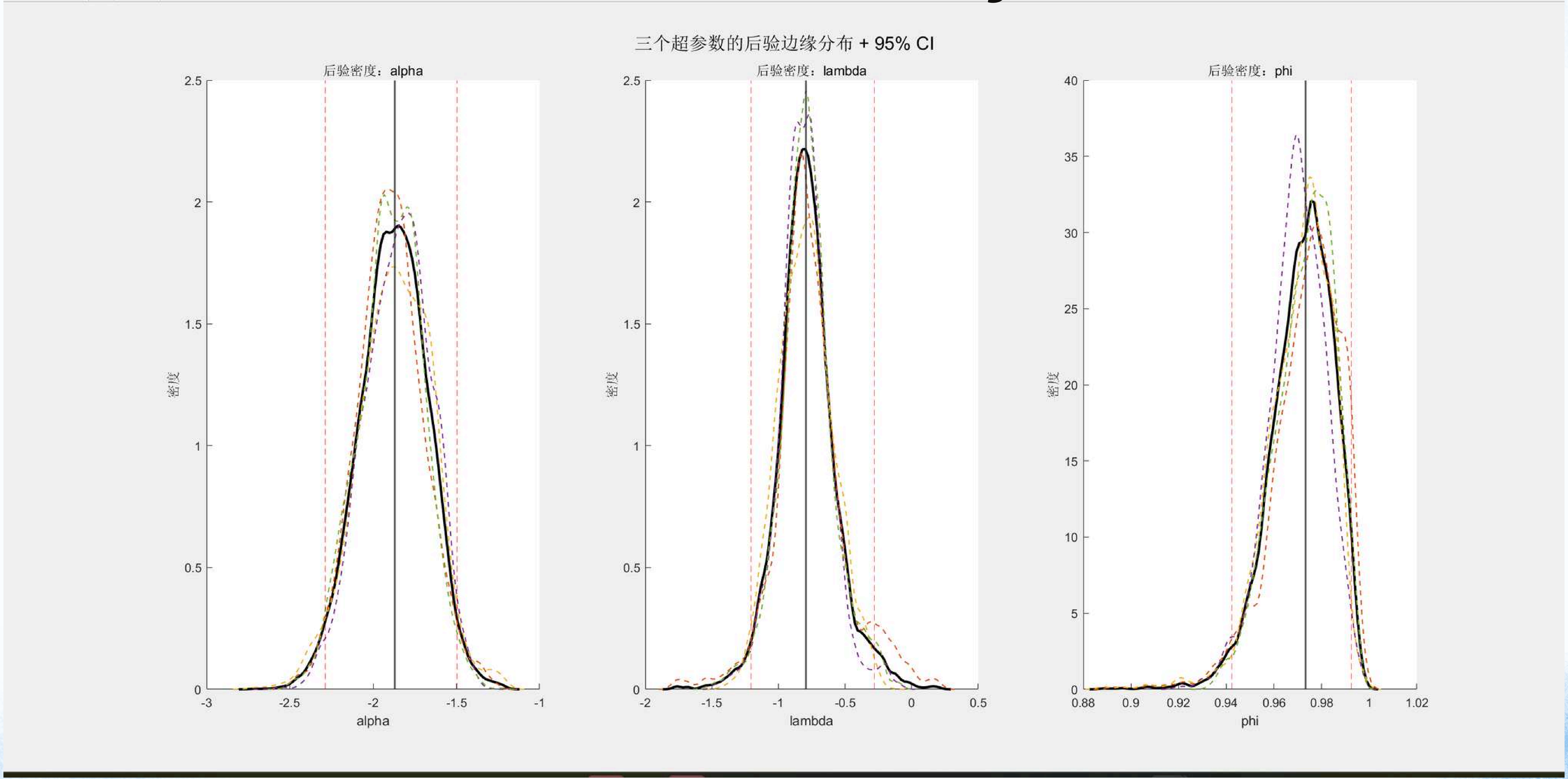
```
alpha:  Mean = -1.9396,  95% CI = [-2.3233, -1.5546]
lambda: Mean = -0.7530,  95% CI = [-1.2139, -0.2048]
phi:    Mean = 0.9772,   95% CI = [0.9496, 0.9923]
```

```
alpha:  R^ = 1.0020
lambda: R^ = 1.0094
phi:    R^ = 1.0028
```

Different Prior : Heavy-Tailed t



Different Prior : Heavy-Tailed t



Different Prior : Heavy-Tailed t

```
alpha:  Mean = -1.8763,  95% CI = [-2.2865, -1.4974]
lambda: Mean = -0.7854,  95% CI = [-1.2038, -0.2812]
phi:    Mean = 0.9718,   95% CI = [0.9424, 0.9924]
```

```
alpha:  R^ = 1.0039
lambda: R^ = 1.0026
phi:    R^ = 1.0082
```

AI Report

- For the Layer Partition, AI is NOT used.
- For the MCMC Realization AI is used at refining rstan code.
- For the Volatility and VAR Analysis, AI is used to do the analysis of pictures.
- For the Model Comparison, AI is used to assist the coding.
- For the Sensitivity Analysis, AI is used to assist the coding. The prior, likelihood and the is prerequired and sampling process is proposed. Then AI generate code according to the requirement. The logic checking and debugging is done by human.

Q&A

Thank you