MA329 Statistical linear models 23-24

Assignment 3 (Due date: Nov 3, 11pm. For late submission, each day costs 10 percent)

1. (10 marks) Consider a nonsingular $n \times n$ matrix $\boldsymbol{A}$ whose elements are functions of the scalar x. Also consider the full-rank $p \times n$ matrix $\boldsymbol{B}$. Let $\boldsymbol{H} = \boldsymbol{B}'(\boldsymbol{B}\boldsymbol{A}\boldsymbol{B}')^{-1}\boldsymbol{B}$. Show that
$$\frac{\partial \boldsymbol{H}}{\partial x} = -\boldsymbol{H}\frac{\partial \boldsymbol{A}}{\partial x}\boldsymbol{H}$$

2. (20 marks) Let $\boldsymbol{X} \sim N_3(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where $\mathbf{X}' = (X_1, X_2, X_3)$, $\boldsymbol{\mu}' = (3, -2, 0)$ and
$$\boldsymbol{\Sigma} = \begin{pmatrix} 5 & 0 & -3 \\ 0 & 9 & 0 \\ -3 & 0 & 2 \end{pmatrix}$$
   (a) Are X_2 and $2X_1 - X_3$ independent? Explain.

   (b) Find the distribution of $\begin{pmatrix} 2X_1 - 5X_3 \\ X_1 + X_2 \end{pmatrix}$.

   (c) Find the conditional distribution of X_3, given that $X_1 = 1$ and $X_2 = -2$.

3. (10 marks) Let $\mathbf{Y} = (Y_1,\ \ Y_2,\ \ Y_3)'$. $E(\mathbf{Y}) = (2,\ \ 3,\ \ 4)'$ and the covariance matrix of $\mathbf{Y}$ is
$$\boldsymbol{\Sigma} = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & 3 \end{pmatrix}.$$
   Let $U = \sum_{i=1}^{3}(Y_i - \overline{Y})^2$. Find the expected value of U.

4. (10 marks) Let $\mathbf{Y} = (Y_1, ..., Y_n)'$. $E(\mathbf{Y}) = \mu\mathbf{1}$ and the covariance matrix of $\mathbf{Y}$ is $\sigma^2\mathbf{I}$. Let
$$U = \sum_{i<j}(Y_i - Y_j)^2.$$
   Find the expected value of U, and find a constant k such that kU is an unbiased estimator of σ^2.