MA329 Statistical linear models 22-23

Assignment 2 (Due date: Oct 18, 11pm. For late submission, each day costs 10 percent)

1. (10 marks) If $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{A}+\boldsymbol{PBQ}$ are nonsingular, prove that

$$(\boldsymbol{A}+\boldsymbol{PBQ})^{-1} = \boldsymbol{A}^{-1} - \boldsymbol{A}^{-1}\boldsymbol{PB}(\boldsymbol{B}+\boldsymbol{BQA}^{-1}\boldsymbol{PB})^{-1}\boldsymbol{BQA}^{-1}$$

2. (15 marks) $\boldsymbol{A} = \begin{pmatrix} 4 & 2 & 2 \\ 2 & 2 & 0 \\ 3 & 0 & 3 \end{pmatrix}$

   (a) Find a symmetric generalized inverse for $\boldsymbol{A}$;

   (b) Find a nonsymmetric generalized inverse for $\boldsymbol{A}$.

3. (15 marks) Prove that $\mathbf{X}(\mathbf{X}'\mathbf{X})^{-}$ is a generalized inverse of $\mathbf{X}'$ for any generalized inverse of $\mathbf{X}'\mathbf{X}$.

Assignment 2.

1. Based on the results for matrix partition for

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \stackrel{d}{=} \begin{pmatrix} -B^{-1} & Q \\ P & A \end{pmatrix}$$

we have

$$(A_{22} - A_{21} A_{11}^{-1} A_{12})^{-1}$$

$$= A_{22}^{-1} + A_{22}^{-1} A_{21} (A_{11} - A_{12} A_{22}^{-1} A_{21})^{-1} A_{12} A_{22}^{-1}$$

Let $A_{22} = A$, $A_{21} = P$, $A_{11} = -B^{-1}$, $A_{12} = Q$

$$= A^{-1} + A^{-1} P (-B^{-1} - Q A^{-1} P)^{-1} Q A^{-1}$$

$$= A^{-1} - A^{-1} P B \underbrace{B^{-1} (B^{-1} + Q A^{-1} P)^{-1} B^{-1}} B Q A^{-1}$$

$$= A^{-1} - A^{-1} P B (B + B Q A^{-1} P B)^{-1} B Q A^{-1}$$

2. (a) $$\underset{\sim}{A} = \left(\begin{array}{c|cc} 4 & 2 & 2 \\ \hline 2 & 2 & 0 \\ 3 & 0 & 3 \end{array}\right)$$

$$\underset{\sim 1}{A^-} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$$

(b) $$\underset{\sim}{A} = \left(\begin{array}{cc|c} 4 & 2 & 2 \\ \hline 2 & 2 & 0 \\ 3 & 0 & 3 \end{array}\right)$$

$$\underset{\sim 2}{A^-} = \left(\begin{array}{c|cc} 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{2} & -\frac{1}{3} \\ \hline 0 & 0 & 0 \end{array}\right)$$

3. Since

$$(X'X)(X'X)^-(X'X) = X'X$$

Let $\underset{m\times n}{X} = \underset{m\times r}{B}\ \underset{r\times n}{C}$

$r = \text{rank}(X)$ i.e. B and C are full rank

and $\underset{r\times n}{C}\ \underset{n\times r}{R} = I_r$ i.e. R is a right inverse of R

or $X' = C'B'$

Then we have

$$C'B'BC(X'X)^-C'B'BC = C'B'BC$$

$$\Rightarrow R'C'B'BC(X'X)^-C'B'BCR = R'C'B'BCR$$

$$\Rightarrow B'BC(X'X)^-C'B'B = B'B$$

$B'B$ is non-singular

$$\Rightarrow B'BC(X'X)^-C' = I_r$$

$$\Rightarrow C'B'BC(X'X)^-C'B' = C'B'$$

$$\Rightarrow X'\left[X(X'X)^-\right]X' = X'$$

$\Rightarrow$ $X(X'X)^-$ is a g-inverse of X'.