MA329 Statistical linear models 22-23

Assignment 3 (Due date: Nov 2, 11pm. For late submission, each day costs 10 percent. The solution will be released at 6pm Nov 4 since midterm test is on Nov 7. This assignment will not be accepted once the solution is released.)

1. (10 marks) Consider a nonsingular $n \times n$ matrix $\boldsymbol{A}$ whose elements are functions of the scalar x. Also consider the full-rank $p \times n$ matrix $\boldsymbol{B}$. Let $\boldsymbol{H} = \boldsymbol{B}'(\boldsymbol{BAB}')^{-1}\boldsymbol{B}$. Show that
$$\frac{\partial \boldsymbol{H}}{\partial x} = -\boldsymbol{H}\frac{\partial \boldsymbol{A}}{\partial x}\boldsymbol{H}$$

2. (20 marks) Let $\boldsymbol{y} = (y_1, y_2, y_3)'$ be distributed as $N_3(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where
$$\boldsymbol{\mu} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \quad \boldsymbol{\Sigma} = \begin{pmatrix} 4 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}.$$
   (a) Find the distribution of $\begin{pmatrix} y_1 - y_2 + y_3 \\ 2y_1 + y_2 - y_3 \end{pmatrix}$;
   (b) The conditional distribution of (y_1, y_2) given y_3;
   (c) The partial correlation between y_1 and y_2 given y_3.

3. (10 marks) Let $\mathbf{Y} = (Y_1, \ \ Y_2, \ \ Y_3)'$. $E(\mathbf{Y}) = (2, \ \ 3, \ \ 4)'$ and the covariance matrix of $\mathbf{Y}$ is
$$\mathbf{\Sigma} = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & 3 \end{pmatrix}.$$
Let $U = \sum_{i=1}^{3}(Y_i - \overline{Y})^2$. Find the expected value of U.

4. (10 marks) Let $\mathbf{Y} = (Y_1, ..., Y_n)'$. $E(\mathbf{Y}) = \mu\mathbf{1}$ and the covariance matrix of $\mathbf{Y}$ is $\sigma^2\mathbf{I}$. Let
$$U = \sum_{i<j}(Y_i - Y_j)^2.$$
Find the expected value of U, and find a constant k such that kU is an unbiased estimator of σ^2.