

INTERMEDIATE MICROECONOMICS

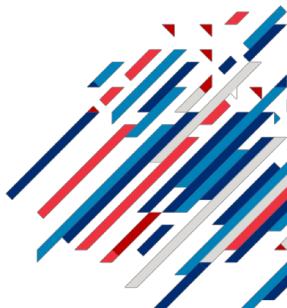
CONSUMER DEMAND

SPRING 2019, PROFESSOR ANH NGUYEN

1. Demand Functions

Demand Function

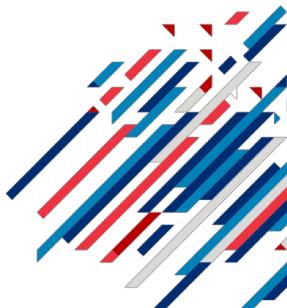
- Derive a demand curve from individual consumers' choices.
- Study how consumers react to changes in prices and income.
- Key concepts:
 - Demand Curve and Engel Curve
 - Price Elasticity and Income Elasticity
 - Normal Goods and Inferior Goods
- Reading: pp. 245-251, 258-265, 633-641, 652-655



Optimal Choice and Demand Function



- We used calculus to maximize consumer utility subject to a budget constraint.
 - This amounts to solving for the consumer's system of demand functions for the goods.

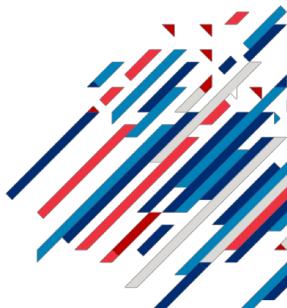


Optimal Choice and Demand Function

- Example: $x_1 = \text{pizza}$ and $x_2 = \text{burritos}$
 - **Demand functions** express these quantities in terms of the prices of both goods and income

$$x_1 = D_1(p_1, p_2, Y)$$

$$x_2 = D_2(p_1, p_2, Y)$$



Optimal Choice and Demand Function

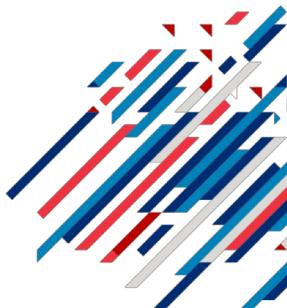


- Example: $x_1 = \text{pizza}$ and $x_2 = \text{burritos}$
 - Given a specific utility function, we can find closed-form solutions for the demand functions.
 - Cobb-Douglas utility function:

$$u(x_1, x_2) = x_1^a x_2^{1-a}$$

- We learned that the demand functions that result from this constrained optimization problem are:

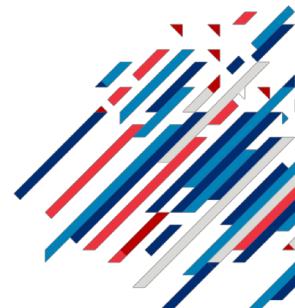
$$x_1 = \frac{aY}{p_1}, x_2 = \frac{(1 - a)Y}{p_2}$$



Demand Functions



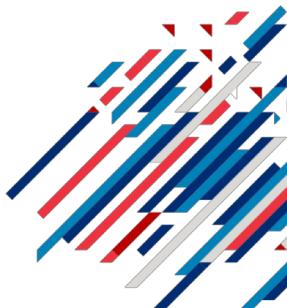
Utility Function	$U(q_1, q_2)$	Solution	Demand Functions	
			q_1	q_2
Perfect complements	$\min(q_1, q_2)$	interior	$Y/(p_1 + p_2)$	$Y/(p_1 + p_2)$
CES, $\rho \neq 0, \rho < 1, \sigma = 1/(\rho - 1)$	$(q_1^\rho + q_2^\rho)^{\frac{1}{\rho}}$	interior	$q_1 = \frac{Yp_1^\sigma}{p_1^{\sigma+1} + p_2^{\sigma+1}}$	$q_2 = \frac{Yp_2^\sigma}{p_1^{\sigma+1} + p_2^{\sigma+1}}$
Cobb-Douglas	$q_1^a q_2^{1-a}$	interior	aY/p_1	$(1 - a)Y/p_2$
Perfect substitutes, $p_1 = p_2 = p$ $p_1 < p_2$	$q_1 + q_2$	interior		$q_1 + q_2 = Y/p$
		corner	Y/p_1	0
		corner	0	Y/p_2
Quasilinear, $Y > a^2 p_2 / [4p_1]$	$aq_1^{0.5} + q_2$	interior	$\left(\frac{a}{2} \frac{p_2}{p_1}\right)^2$	$\frac{Y}{p_2} - \frac{a^2}{4} \frac{p_2}{p_1}$
		corner	Y/p_1	0



Demand Functions and Comparative Statics



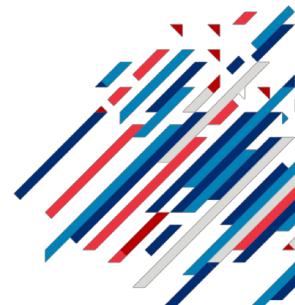
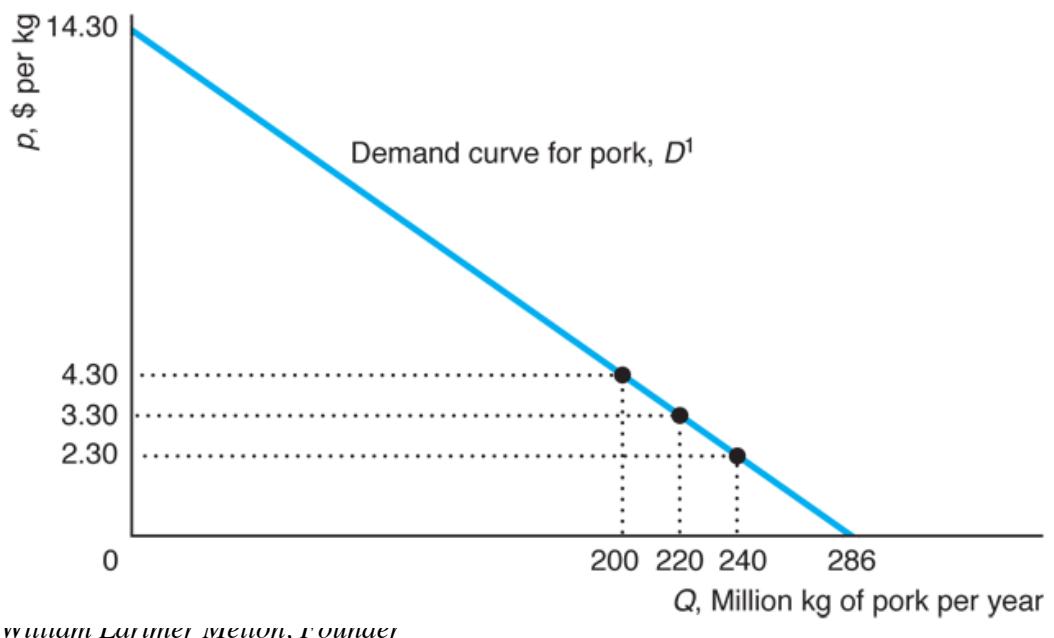
- We will look at how demand changes as we vary one independent variable at a time: *Comparative statics*
 - Price of each good: Demand curves
 - Income: Engel curves



Demand Curve



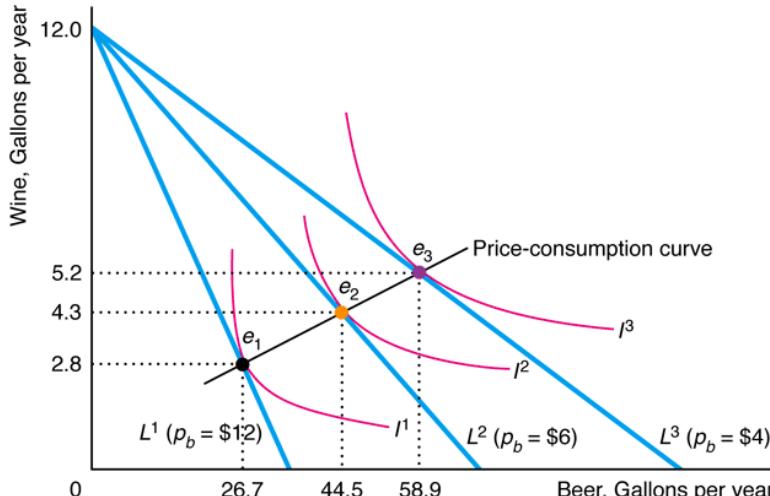
- If we hold people's tastes, their incomes, and the prices of other goods constant, a change in the price of a good will cause a movement along the demand curve.



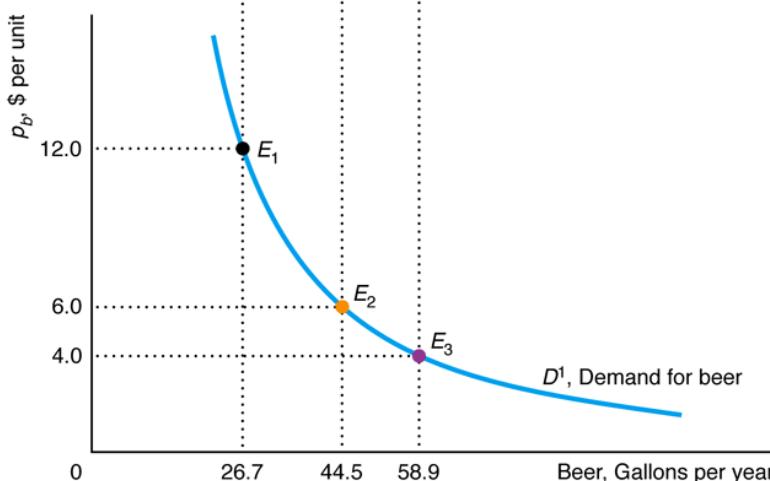
Individual Demand Curve



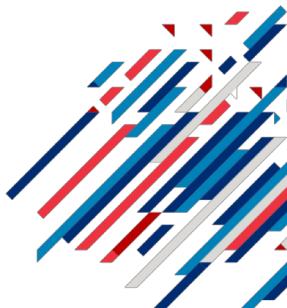
(a) Indifference Curves and Budget Constraints



(b) Demand Curve



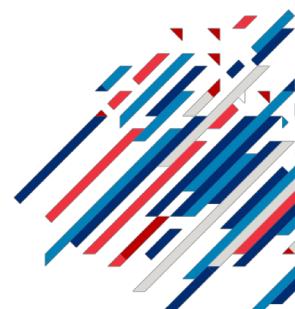
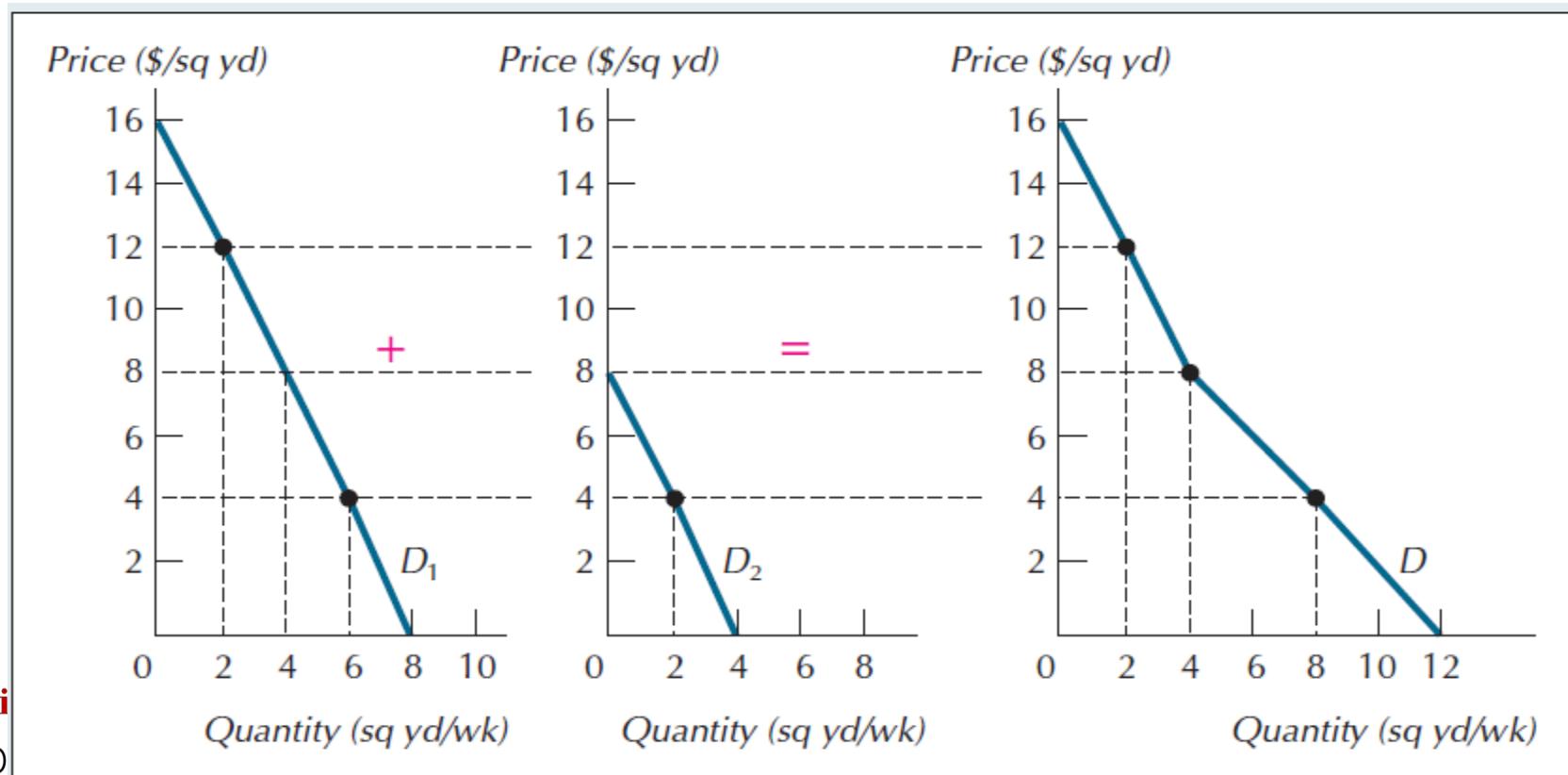
- Allowing the price of the good on the x-axis to fall, the budget constraint rotates out and shows how the optimal quantity of the x-axis good purchased increases.
- This traces out points along the demand curve.



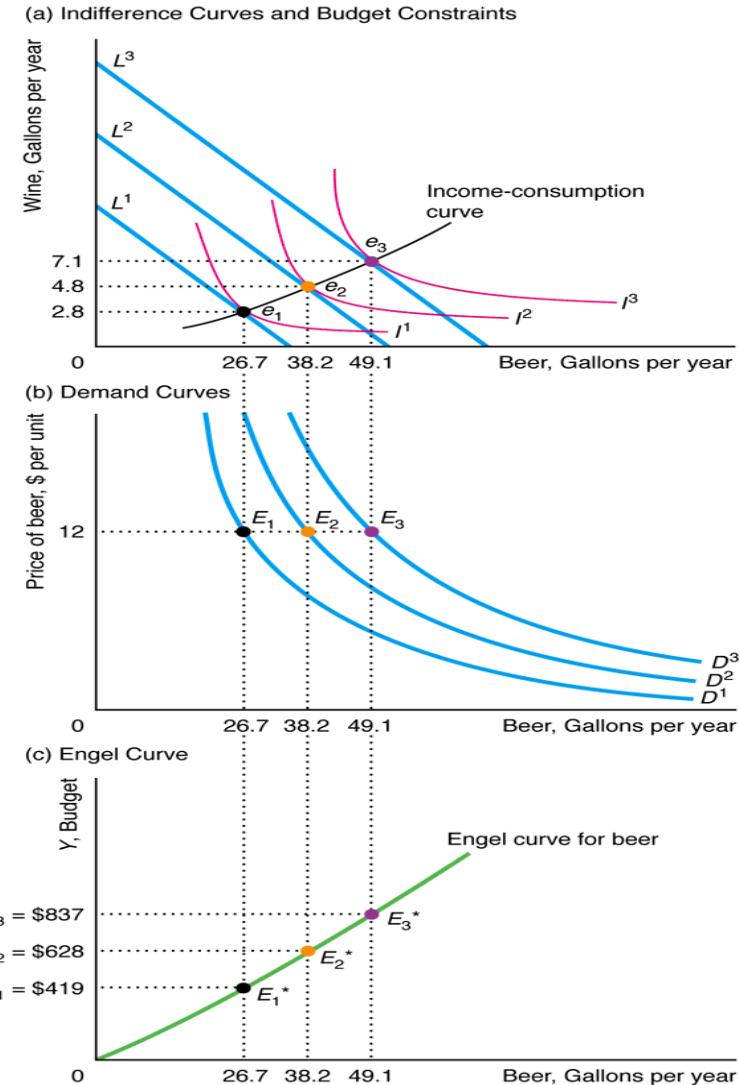
Aggregate Demand Curve



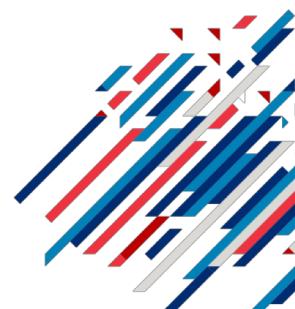
- The aggregate demand curve is derived by adding all individual demand curves.



Engel Curve



- An increase in an individual's income, holding tastes and prices constant, causes
 - New optimal choices
 - Demand curve shifts
- **Engel curve** shows the relationship between income and quantity demanded



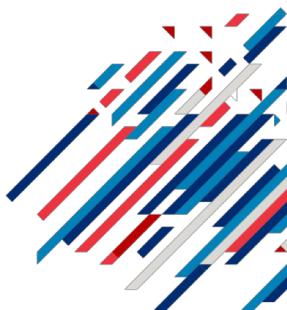
Price Elasticity



- *Price elasticity of demand:* Percentage change in the quantity demanded in response to a given percentage change in *price*

$$\varepsilon = \frac{\% \text{ change in demand}}{\% \text{ change in price}} = \frac{dx_1}{dp_1} \frac{p_1}{x_1}$$

- Elasticity is a unit-less measure, i.e. percentage differences



Price Elasticity and Expenditures



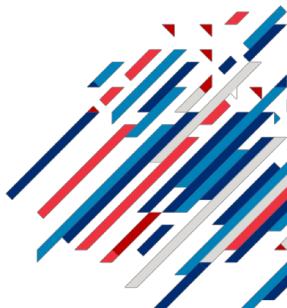
- How does a consumer's expenditure on good 1 change as its price increases?

- Expenditure on good 1:

$$E_1(p_1) = p_1 x_1(p_1)$$

- Take the derivative of E_1 with respect to p_1

$$\frac{dE_1}{dp_1} = x_1 + p_1 \frac{dx_1}{dp_1} = x_1(1 + \varepsilon)$$



Price Elasticity and Expenditures



- How does a consumer's expenditure on good 1 change as its price increases?

$$\frac{dE_1}{dp_1} = x_1 + p_1 \frac{dx_1}{dp_1} = x_1(1 + \varepsilon)$$

After the price increase, the consumer's spending on the good

- Increases if $|\varepsilon| < 1$ (**Inelastic demand**)
- Stays constant if $\varepsilon = 1$
- Decreases if $|\varepsilon| > 1$ (**Elastic demand**)

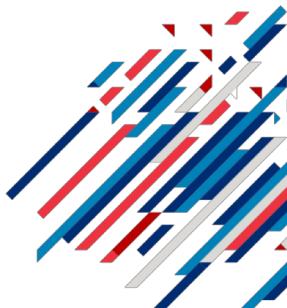


Price Elasticity: Example

- Consider a linear demand curve:

$$x_1(p_1) = 100 - 2p_1$$

- Solve for the price elasticity
- When is the demand price-elastic?
- When is the demand price-inelastic?

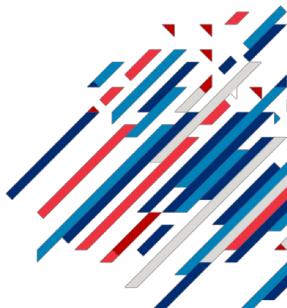


Income Elasticity



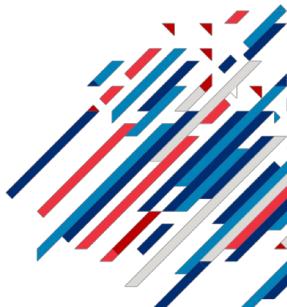
- *Income elasticity of demand:* Percentage change in the quantity demanded in response to a given percentage change in *income*

$$\xi = \frac{\% \text{ change in demand}}{\% \text{ change in income}} = \frac{dx_1}{dY} \frac{Y}{x_1}$$

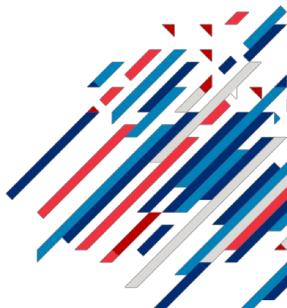
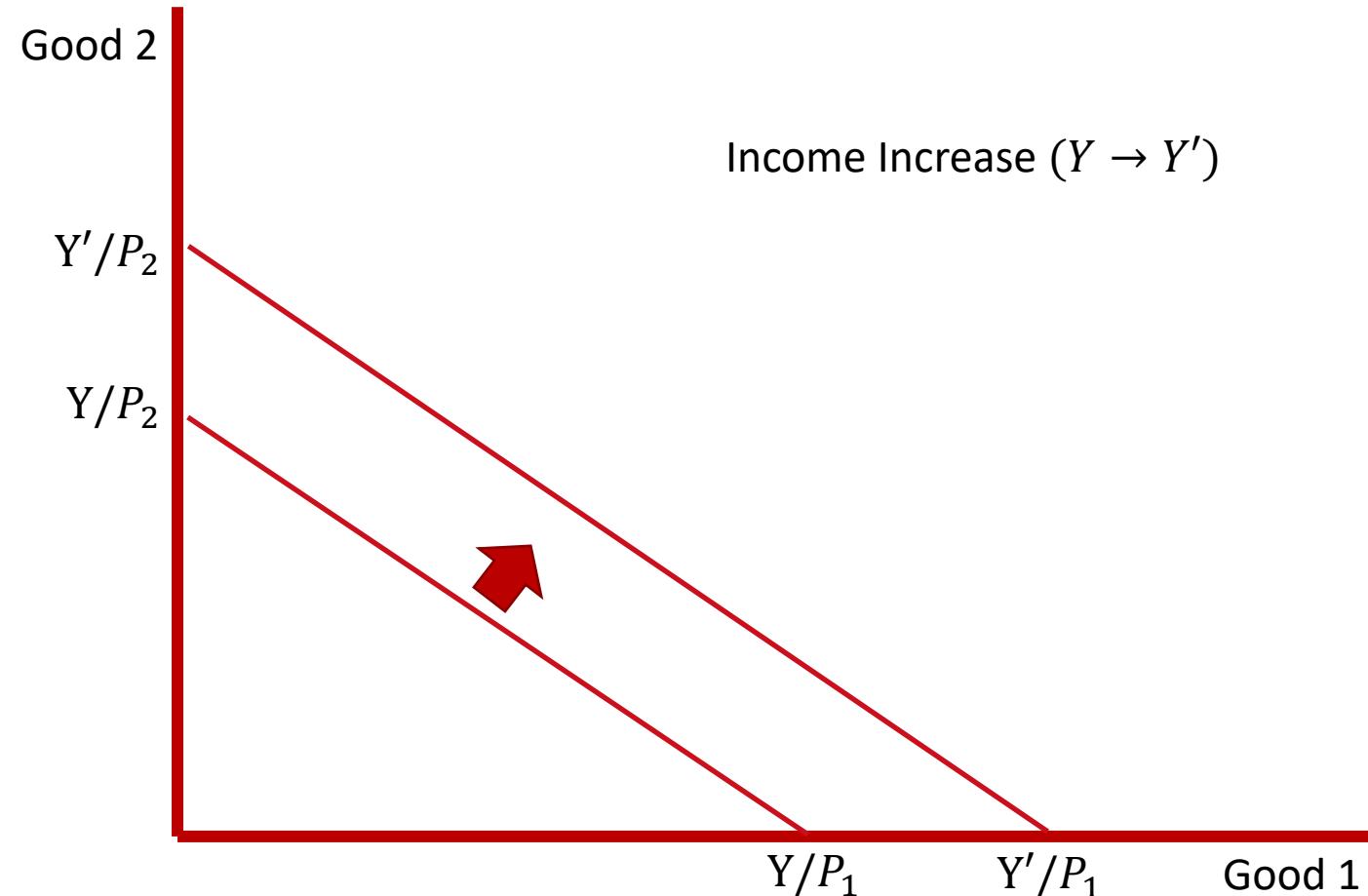


Normal Goods and Inferior Goods

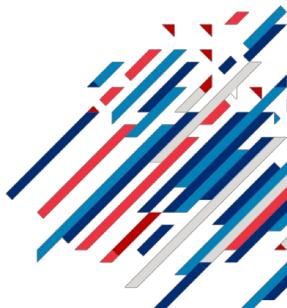
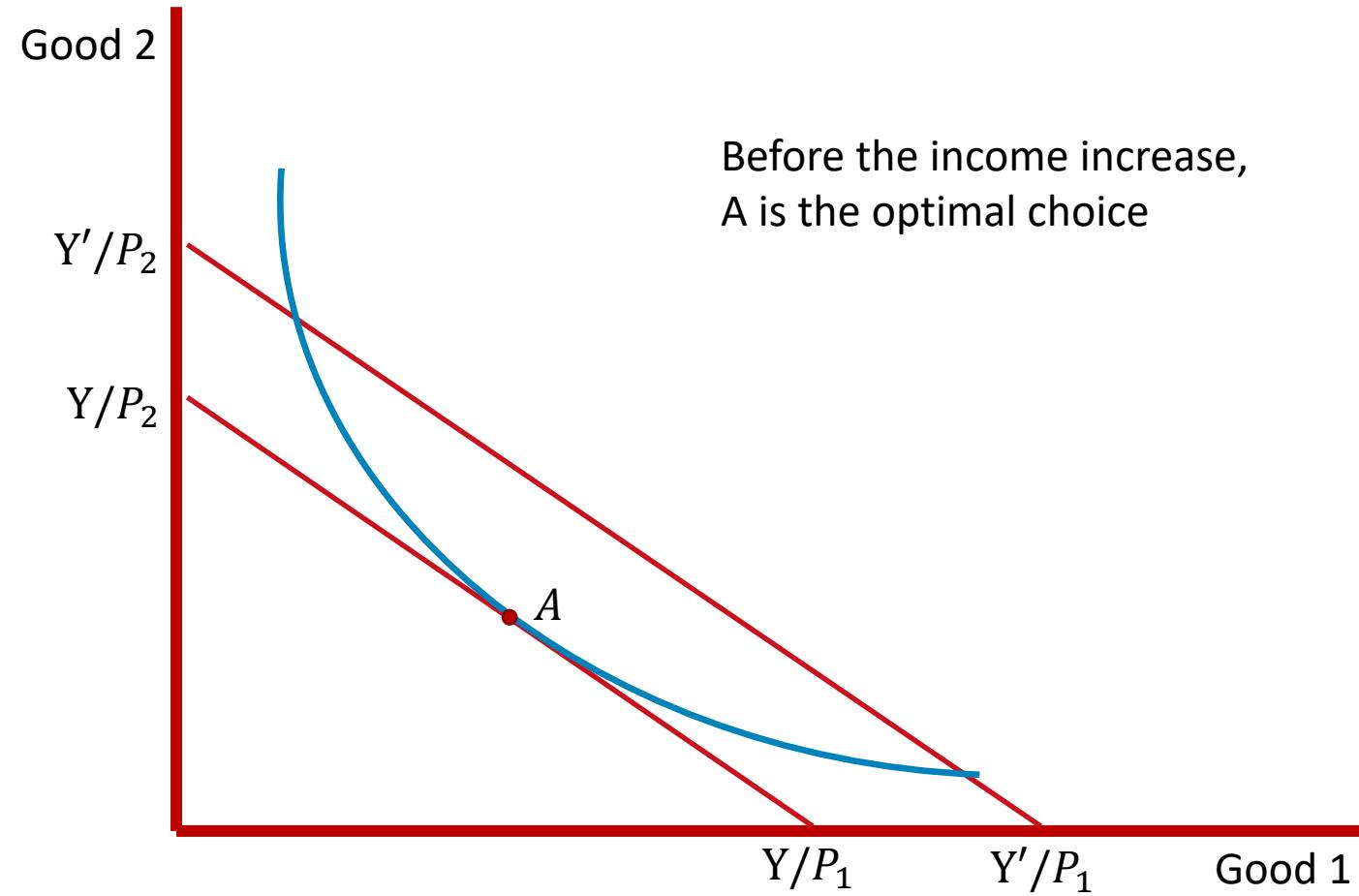
- **Normal goods**, those goods that we buy more of when our income increases, have a *positive* income elasticity.
 - *Luxury goods* are normal goods with an income elasticity greater than 1.
 - *Necessity goods* are normal goods with an income elasticity between 0 and 1.
- **Inferior goods**, those goods that we buy less of when our income increases, have a *negative* income elasticity.



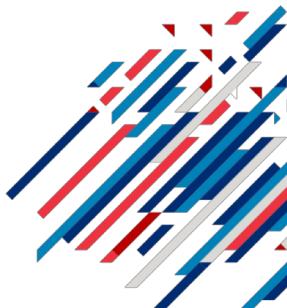
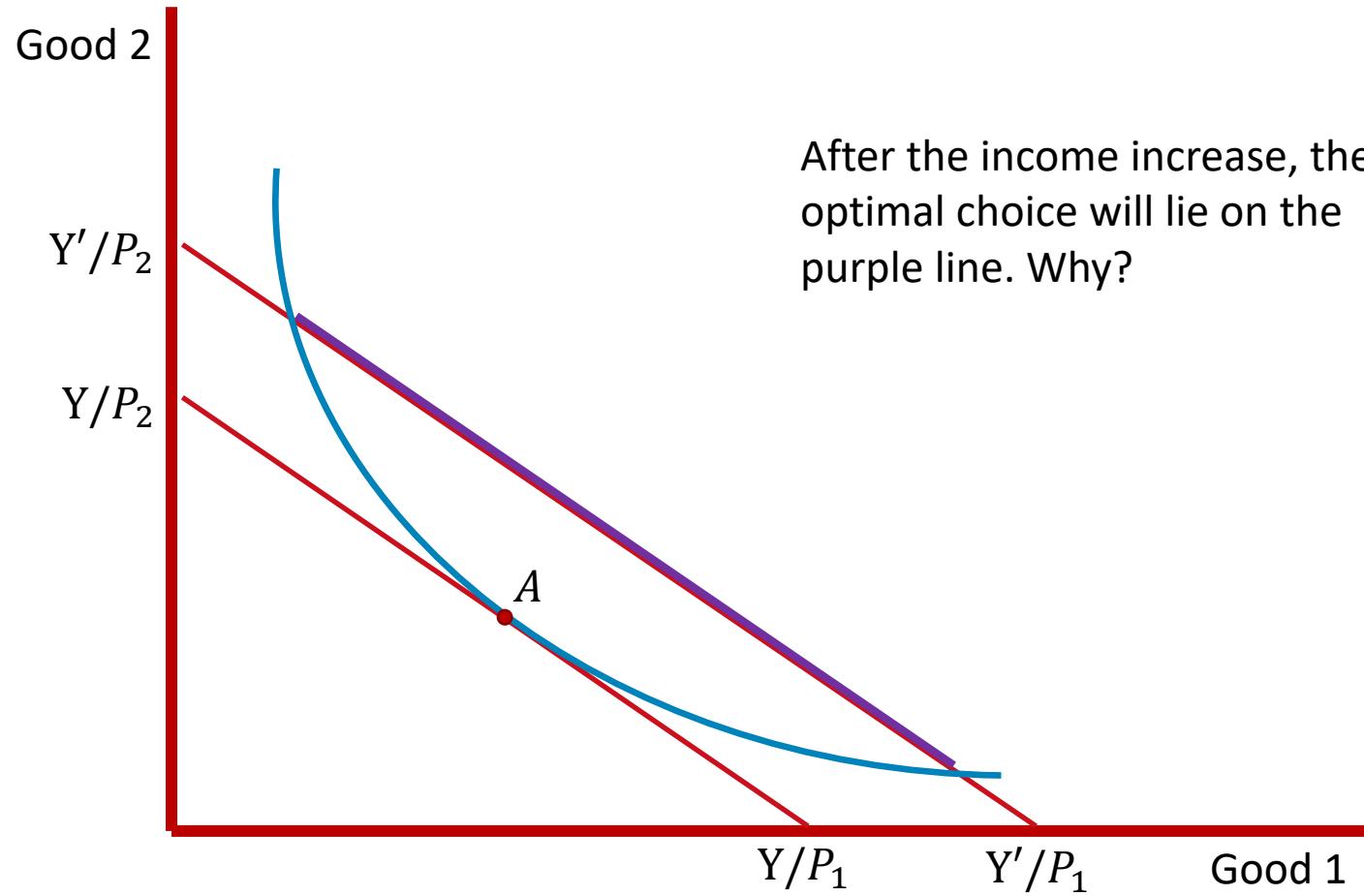
Normal Goods and Inferior Goods



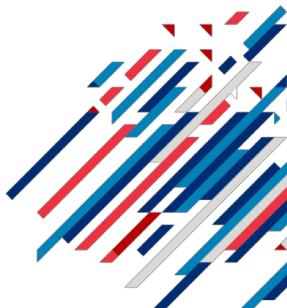
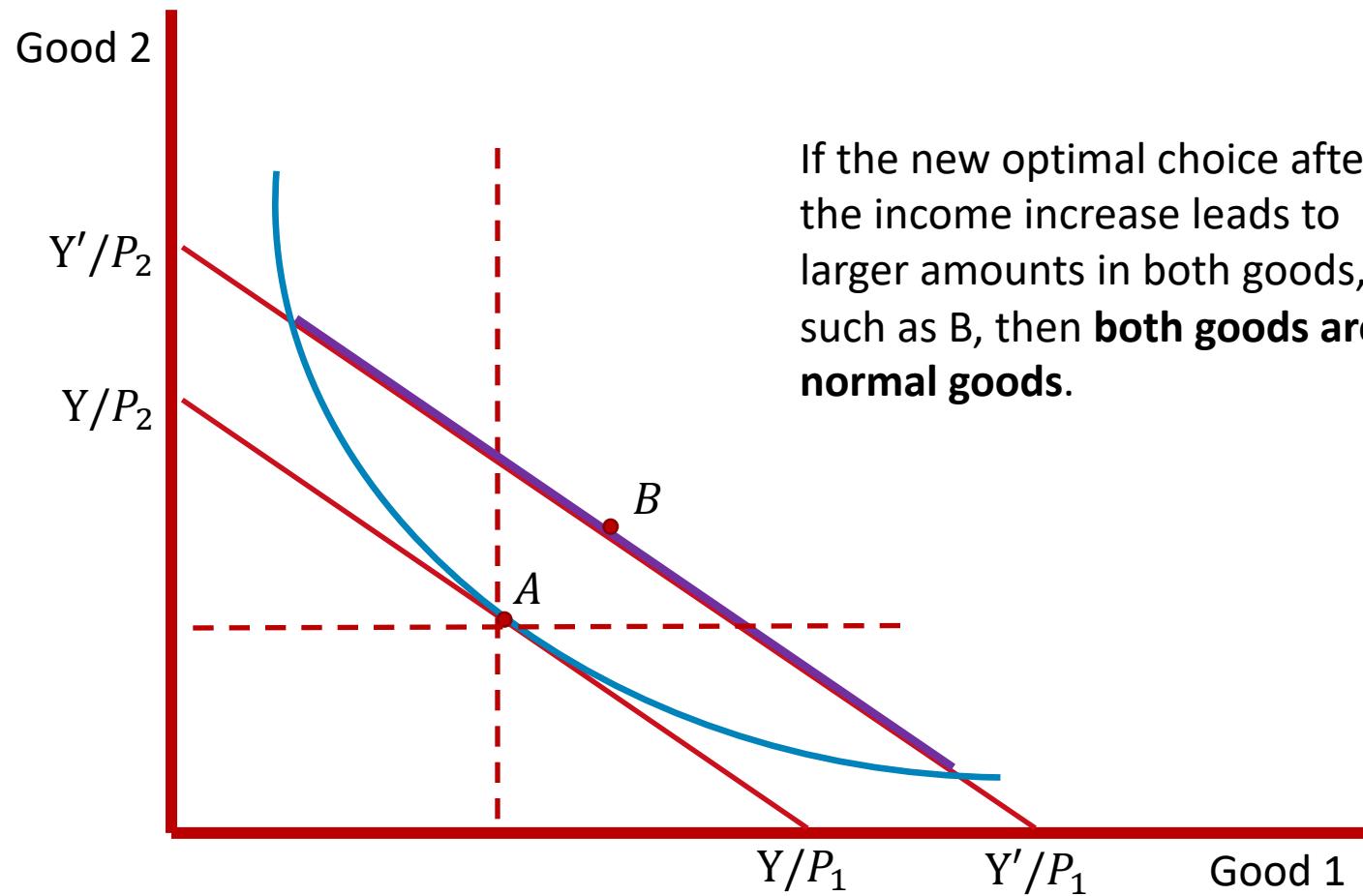
Normal Goods and Inferior Goods



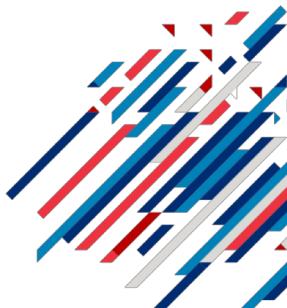
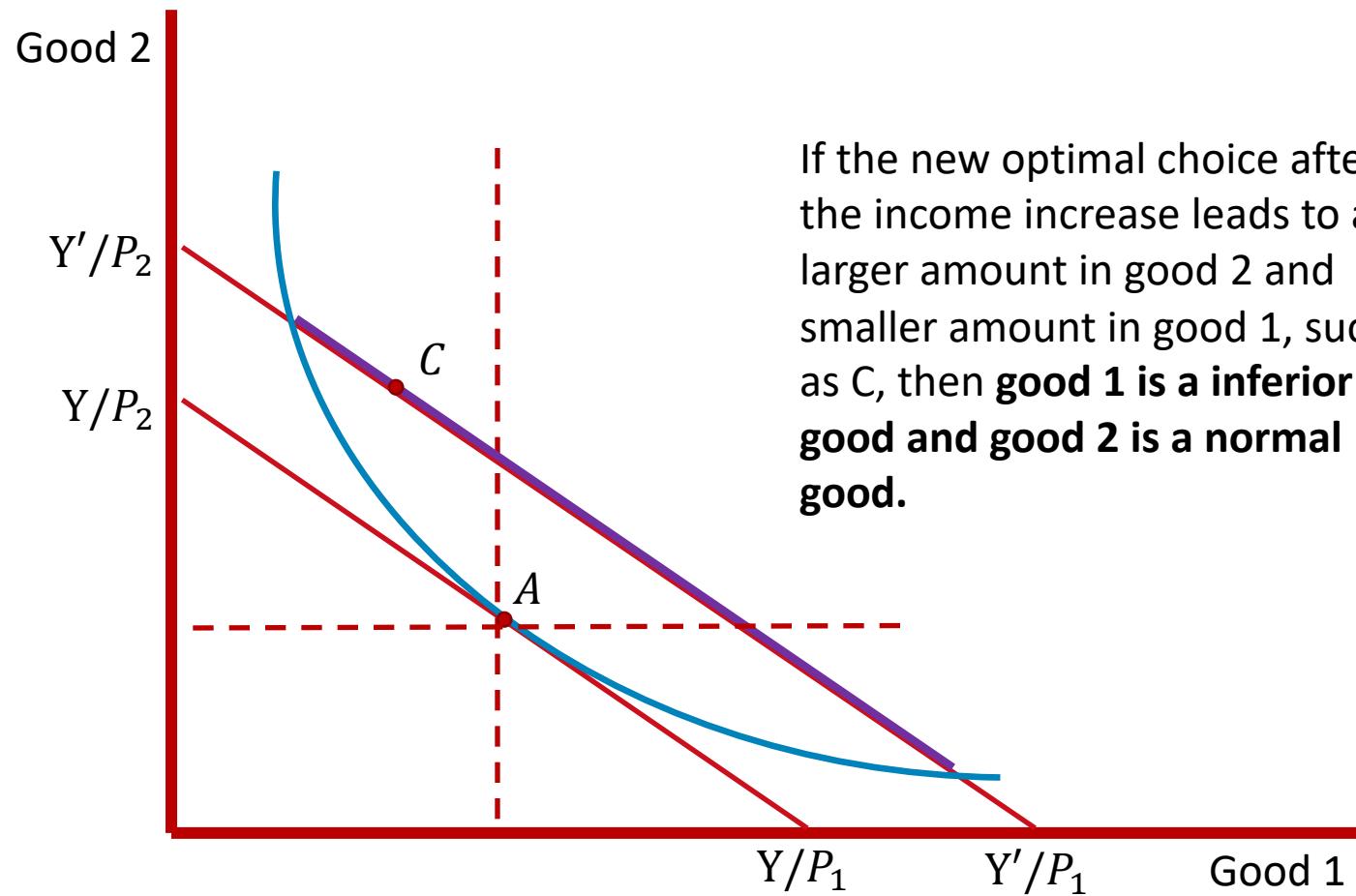
Normal Goods and Inferior Goods



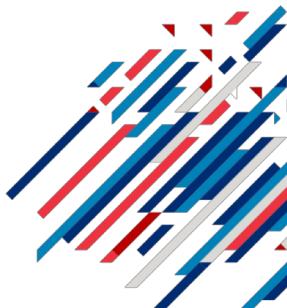
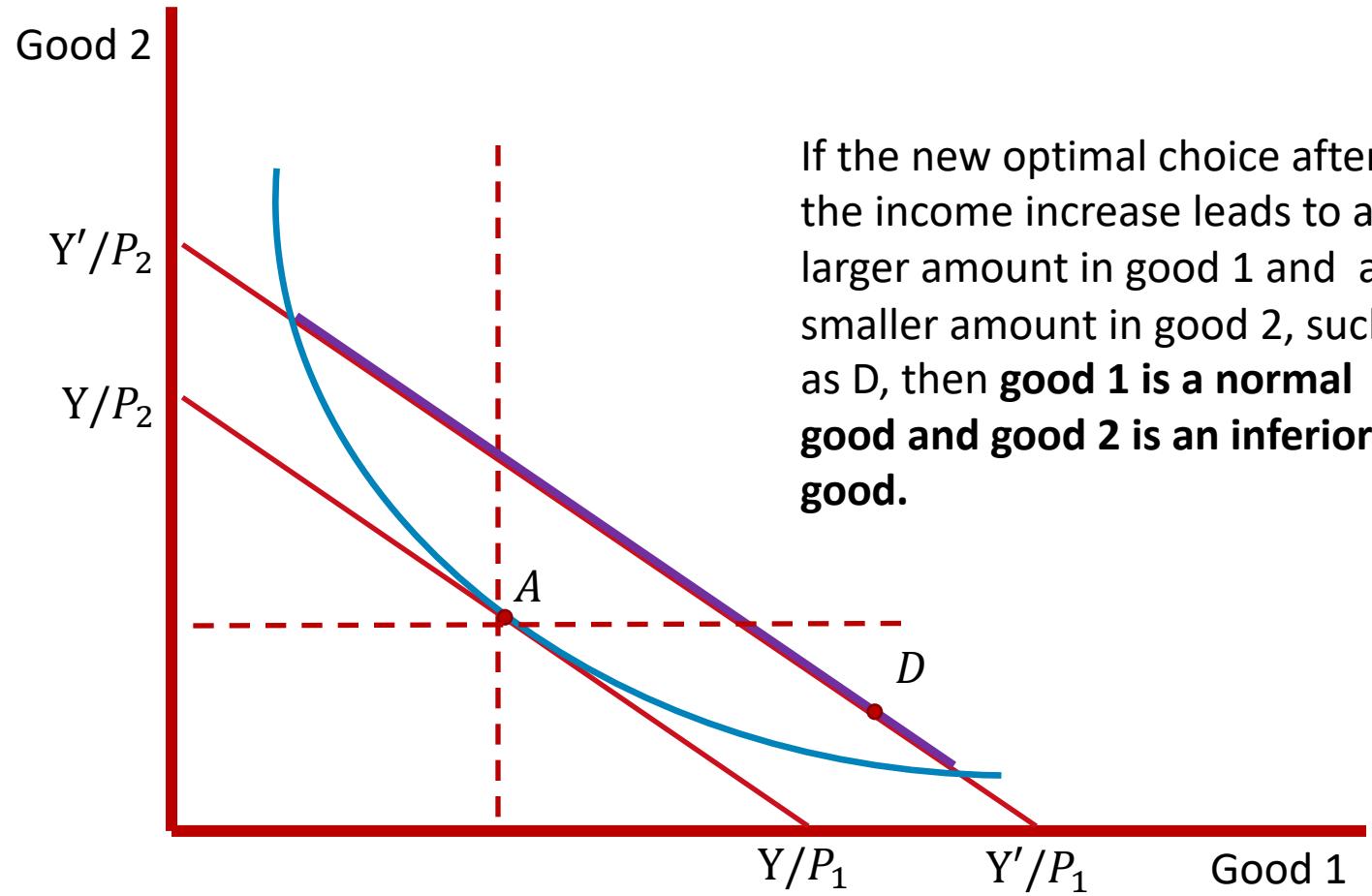
Normal Goods and Inferior Goods



Normal Goods and Inferior Goods

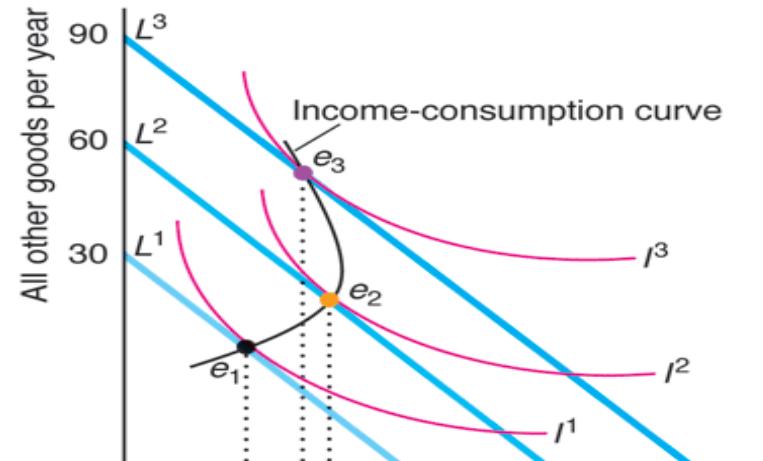


Normal Goods and Inferior Goods

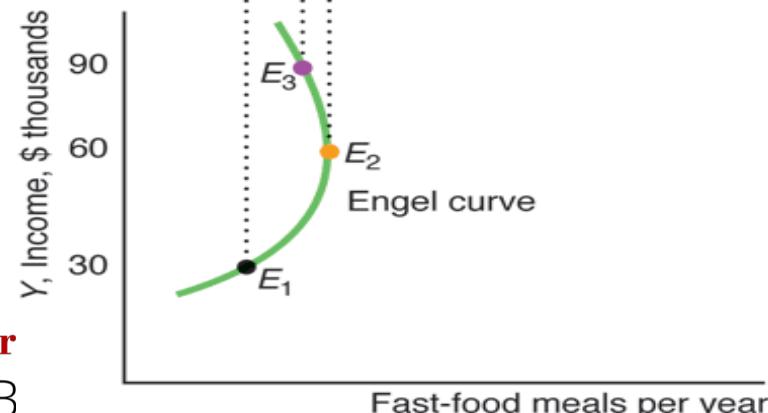


Normal Goods and Inferior Goods

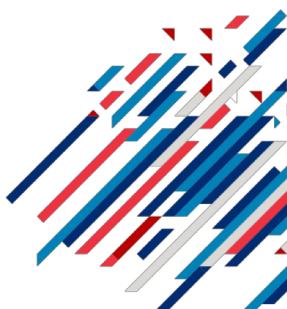
(a) Indifference Curves and Budget Constraints



(b) Engel Curve



- The shape of Engel curves can change in ways that indicate goods can be *both normal and inferior*, depending on an individual's income level

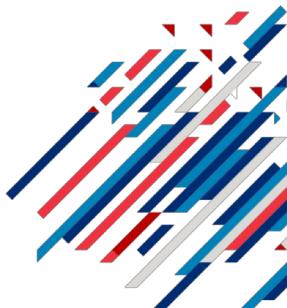


2. Substitution and Income Effects

Substitution and Income Effects



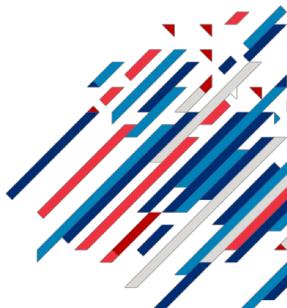
- We decompose the effects of the change in the price of a good on the optimal quantity of the good into two effects
 1. Substitution effect
 2. Income effect
- We study when the demand is increasing in price (Giffen goods)
- Reading: pp. 180-204



Effects of a Price Increase



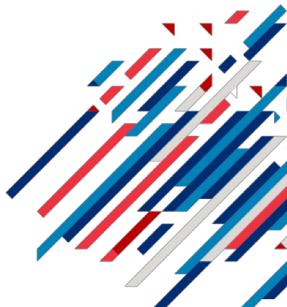
- Holding tastes, other prices, and income constant, an increase in the price of a good has two effects on an individual's demand:
 1. **Substitution effect:** the change in quantity demanded when the good's price increases, *holding other prices and consumer utility constant*.
 2. **Income effect:** the change in quantity demanded when income changes, holding prices constant.



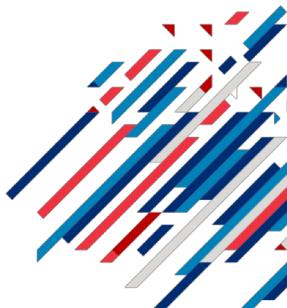
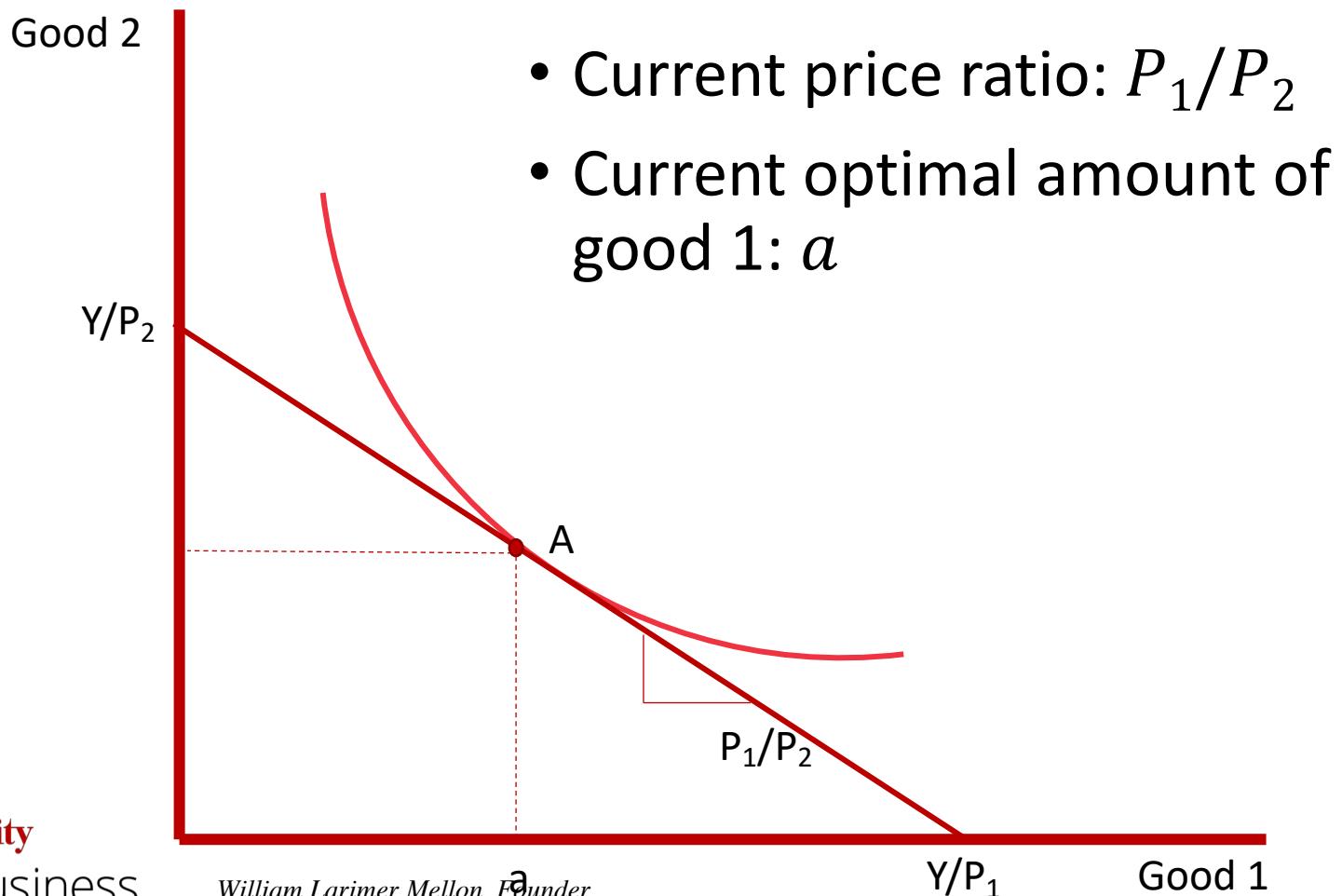
Effects of a Price Increase



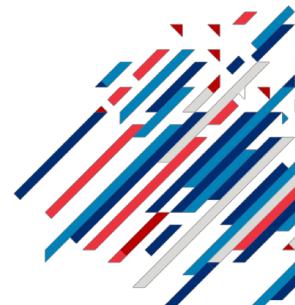
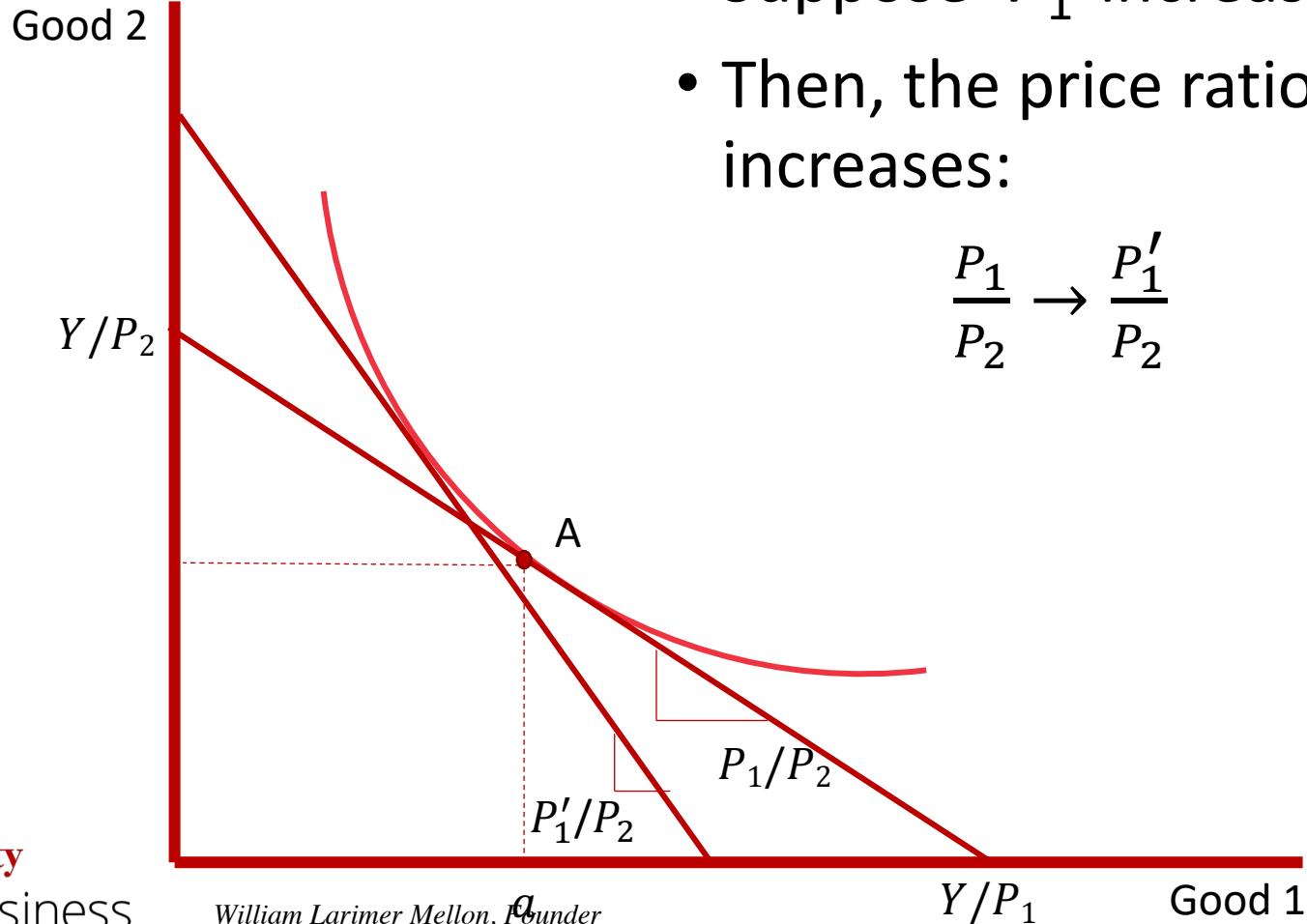
- When the price of a good increases, *the total change in quantity demanded is the sum of the substitution and income effects.*



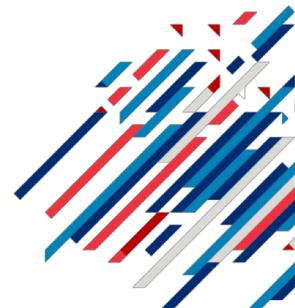
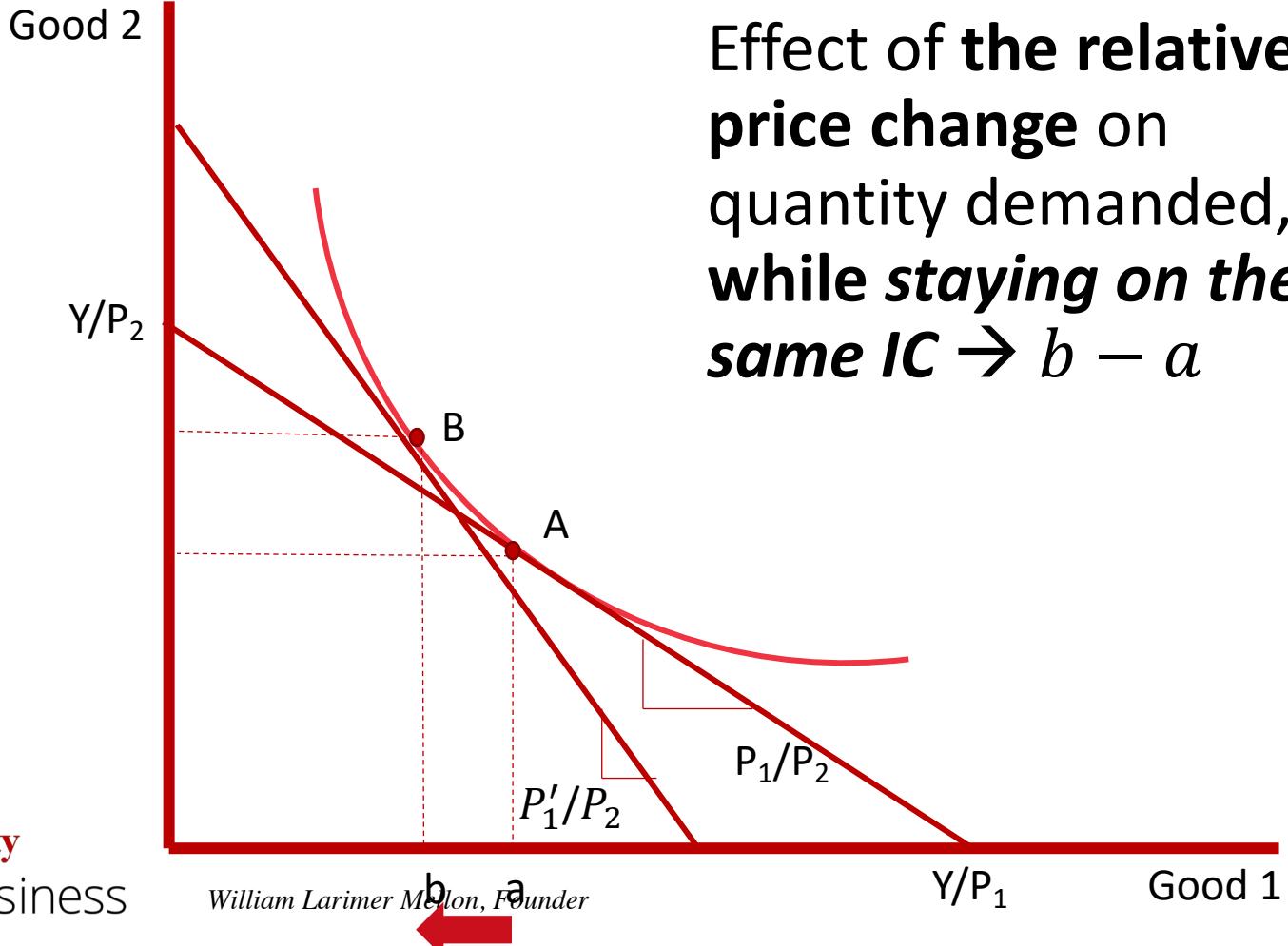
Substitution Effect



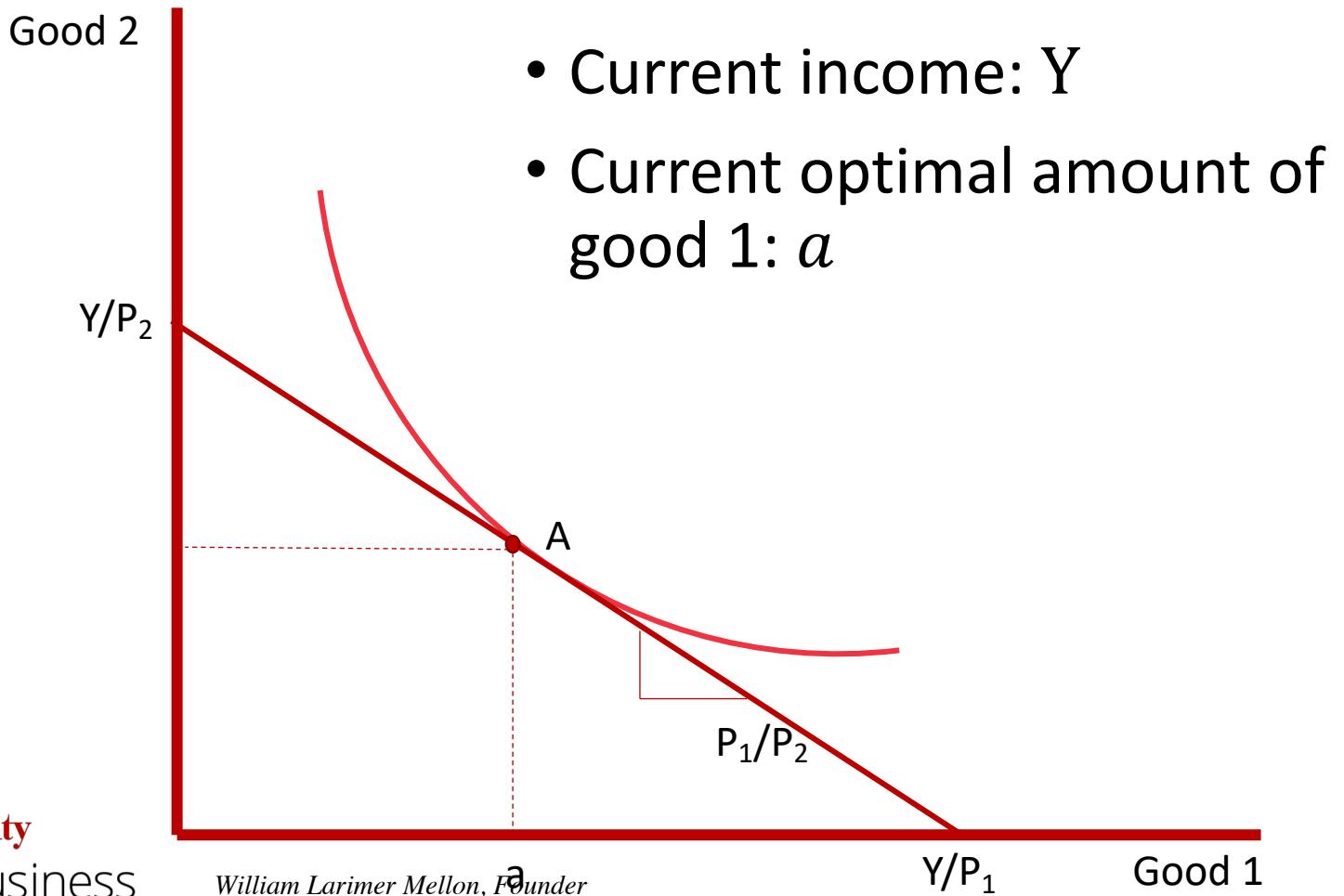
Substitution Effect



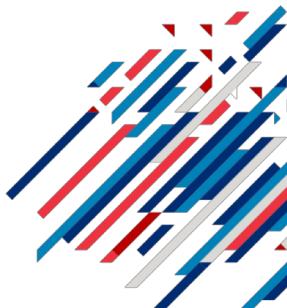
Substitution Effect



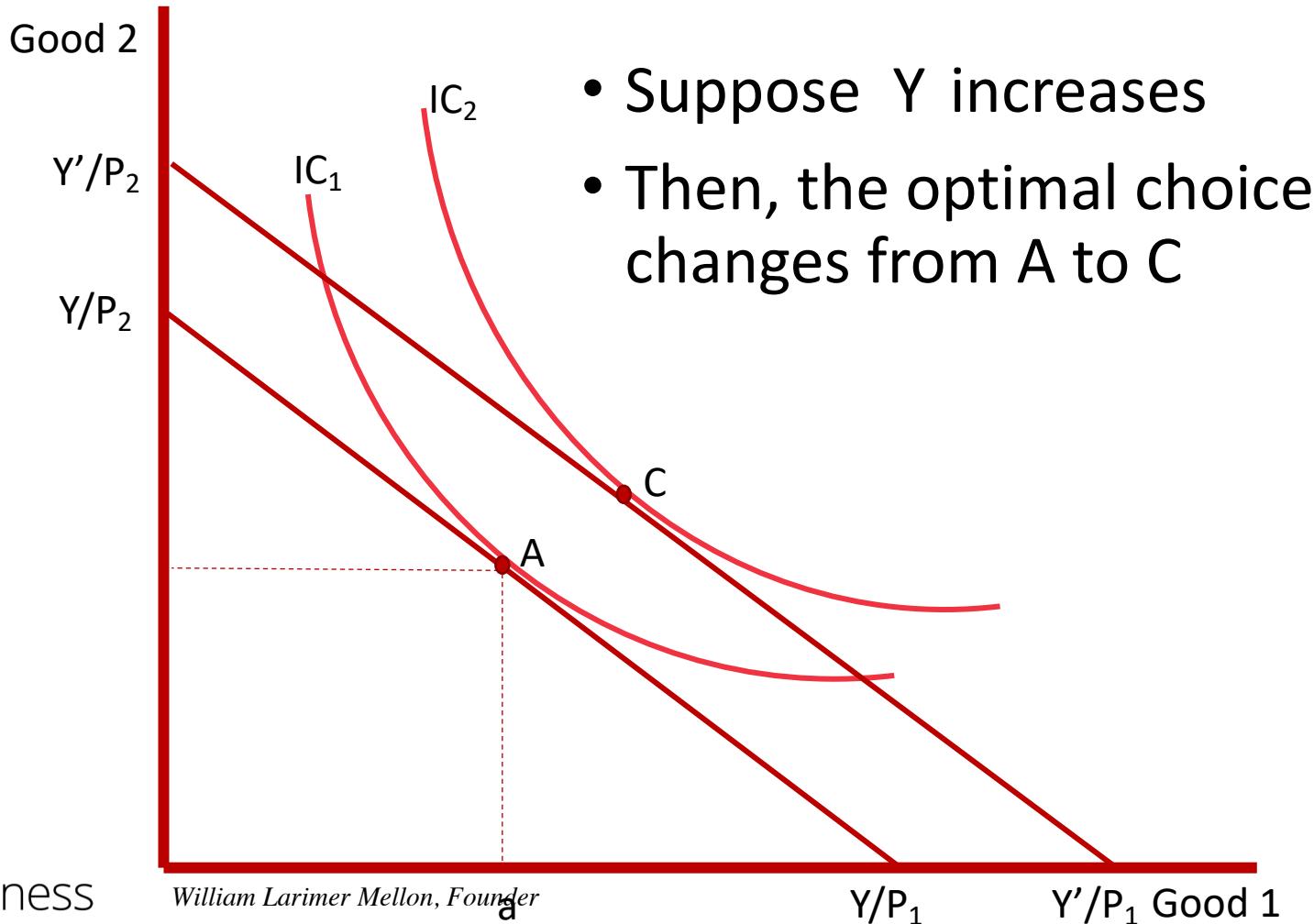
Income Effect



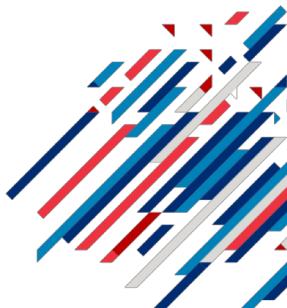
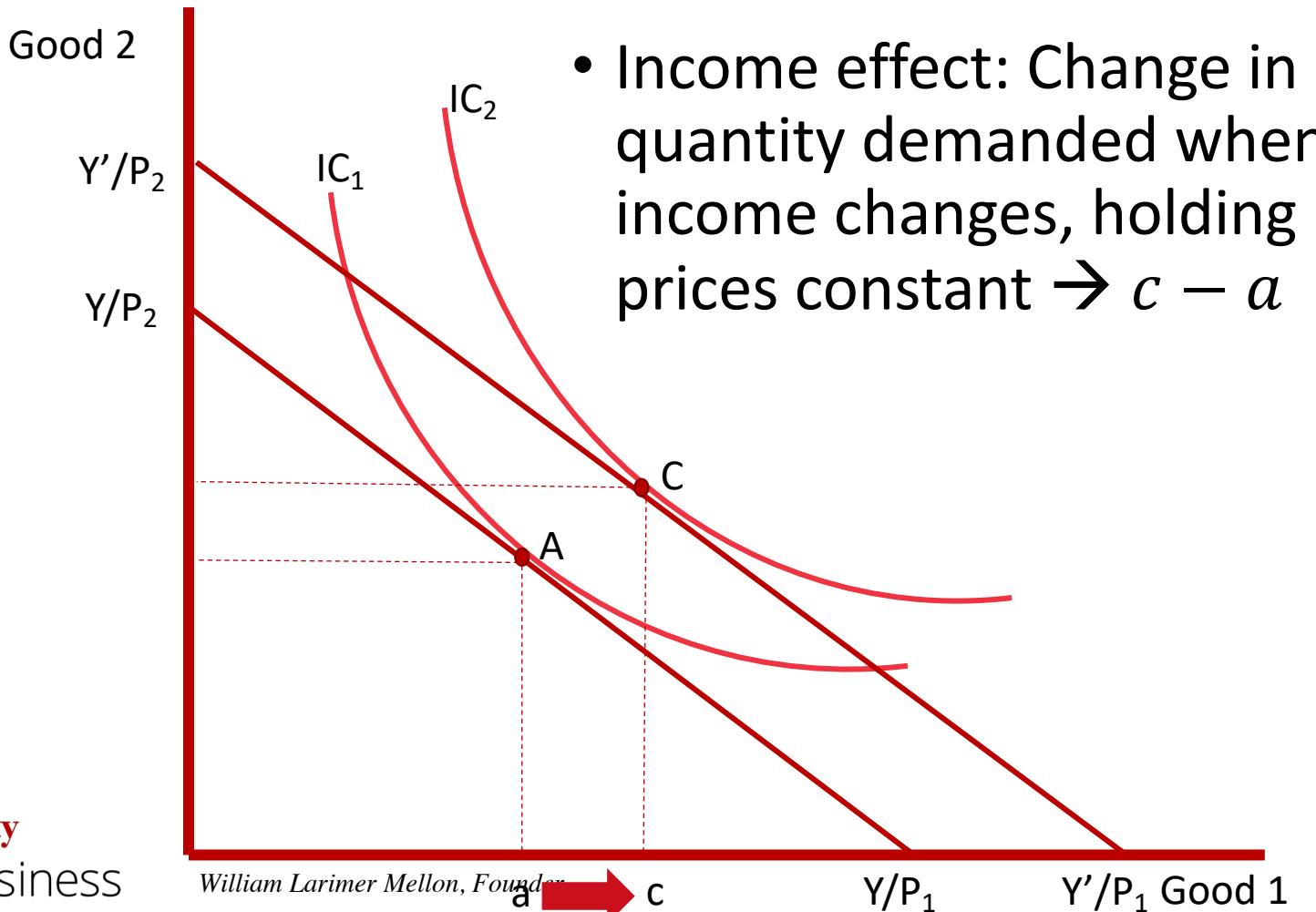
- Current income: Y
- Current optimal amount of good 1: a



Income Effect



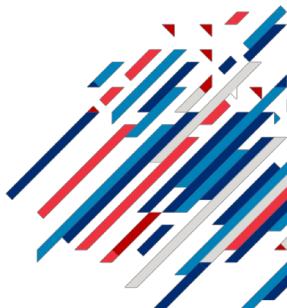
Income Effect



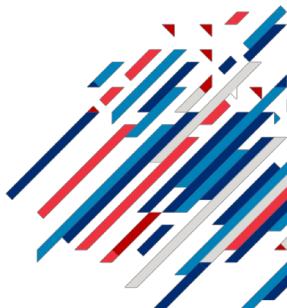
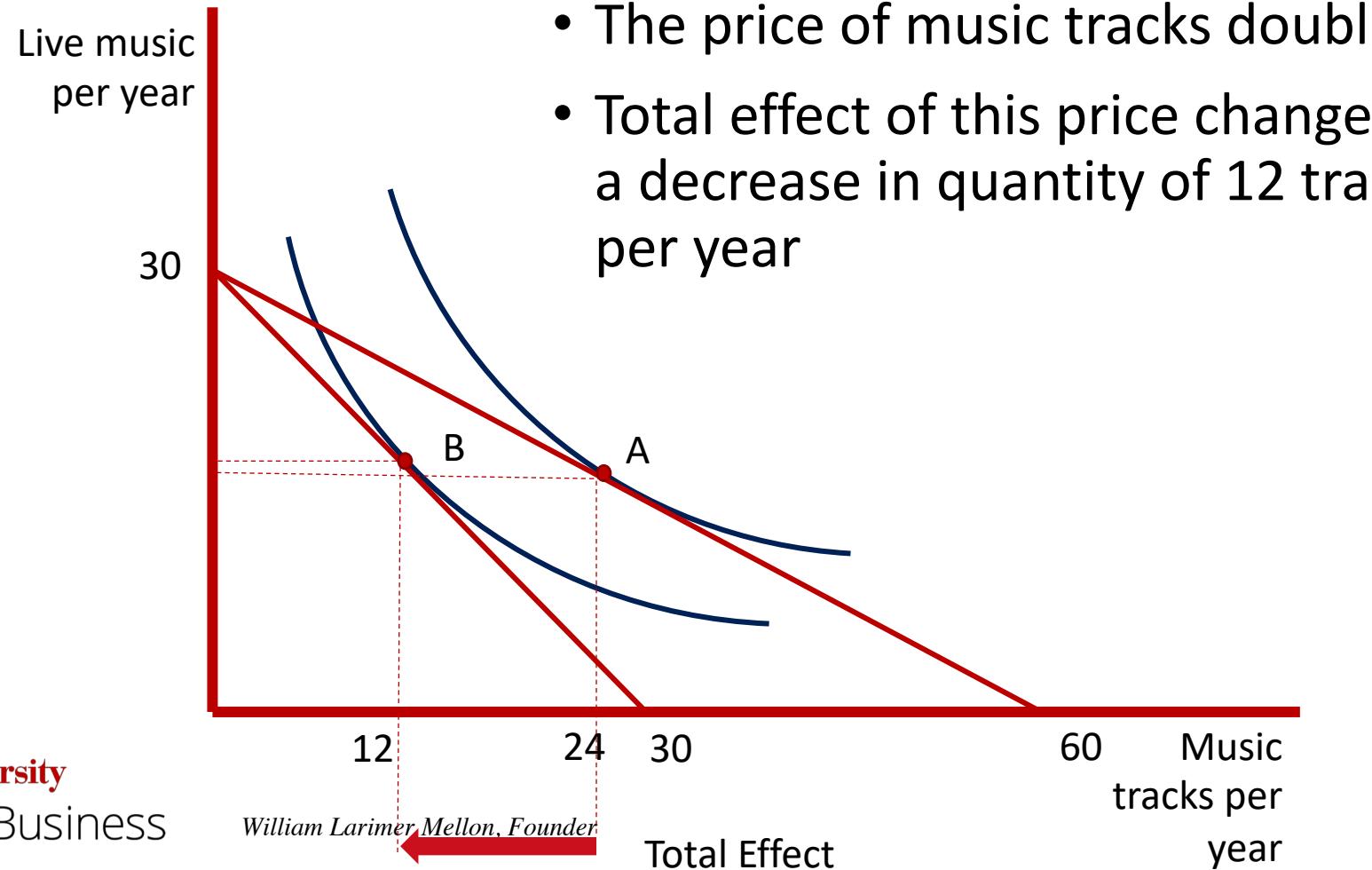
Substitution and Income Effects



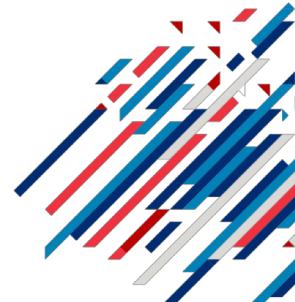
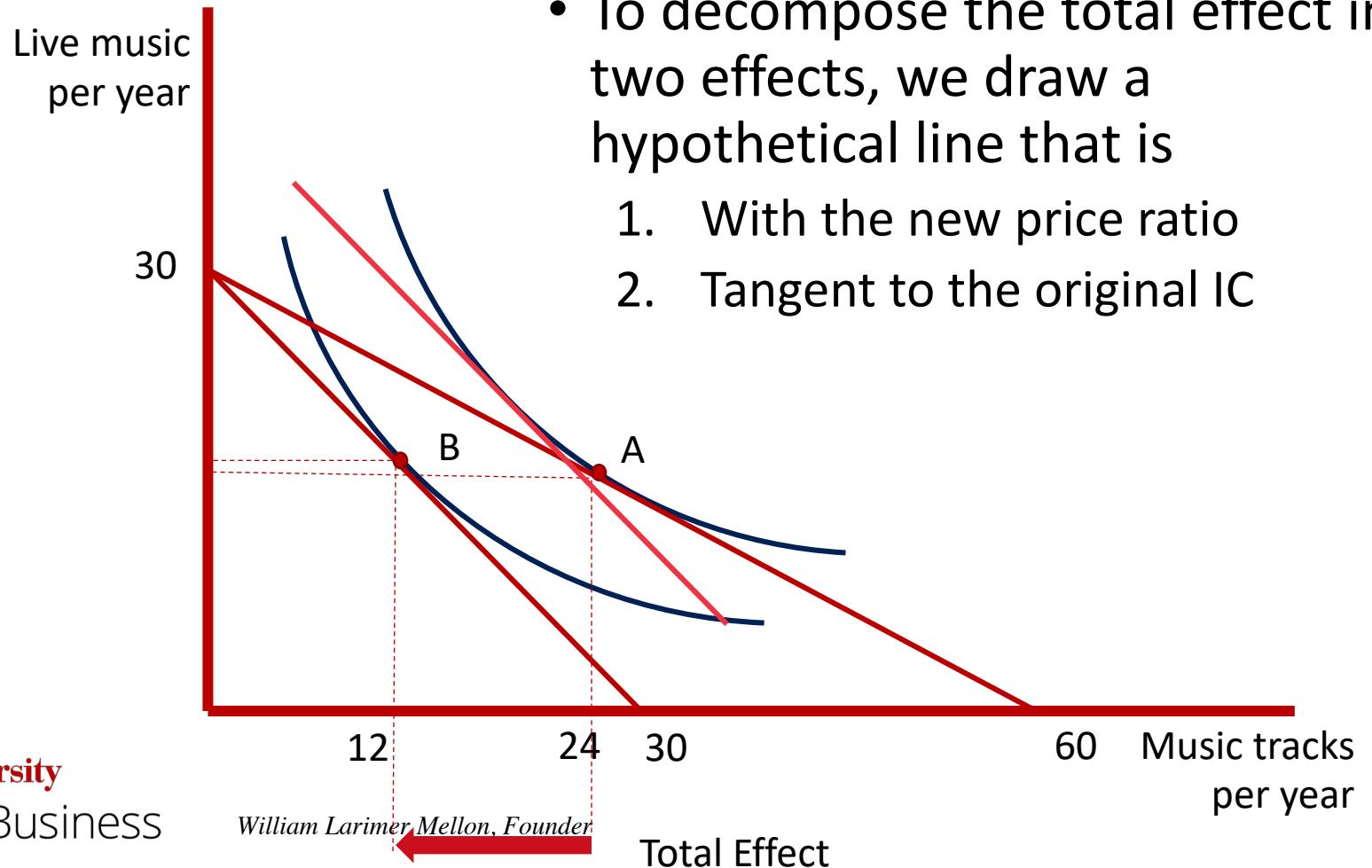
- Consider an **increase** in the price of a good.
- The direction of the substitution effect is **always negative**.
 - When price increases, individuals consume less of it because they are substituting away from it.
- The direction of the income effect **depends upon whether the good is normal or inferior**.
 - When price increases and the good is normal, the income effect is negative.
 - When price increases and the good is inferior, the income effect is positive.



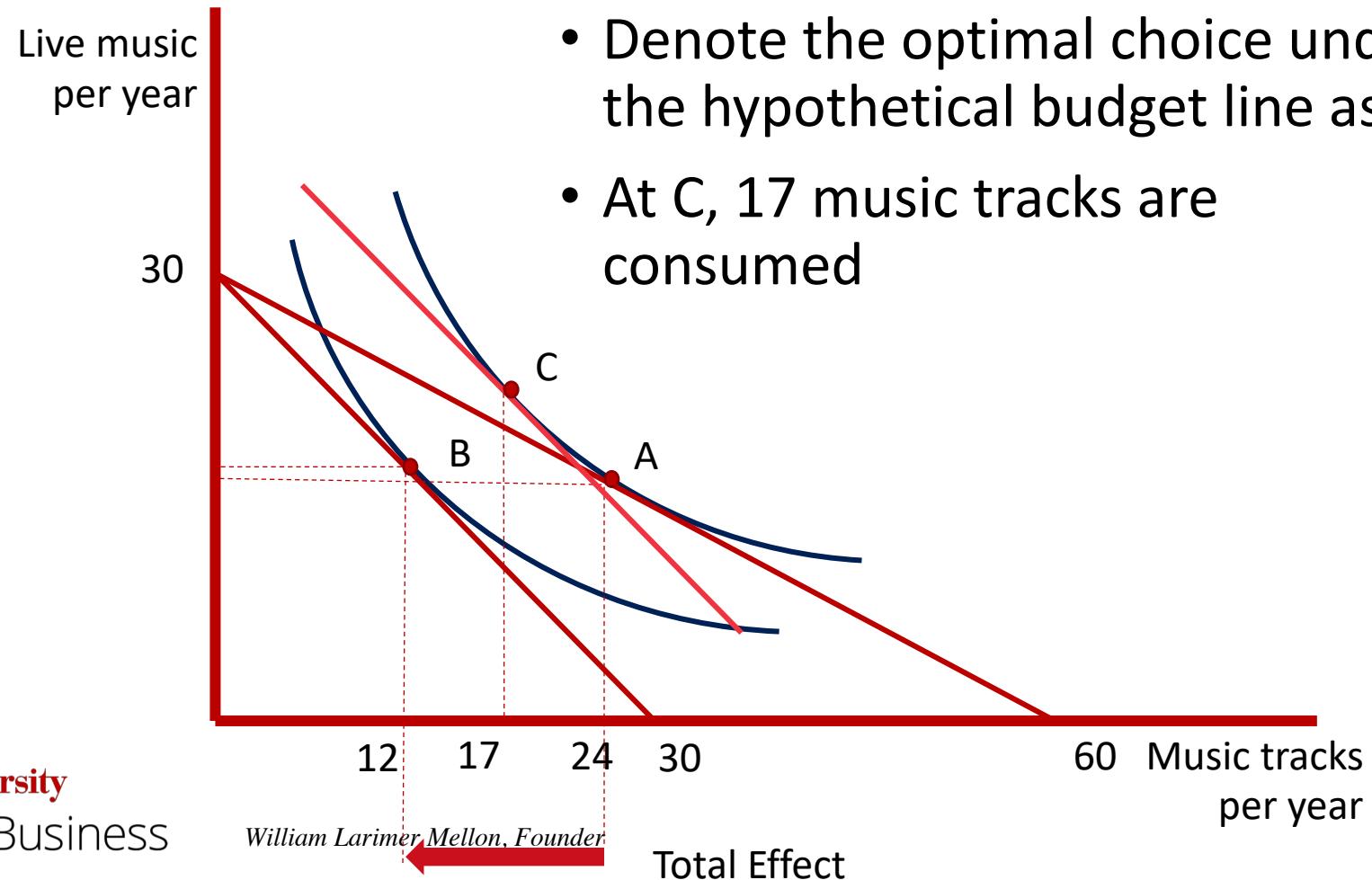
Substitution and Income Effects: Normal Good



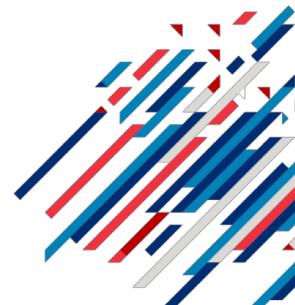
Substitution and Income Effects: Normal Good



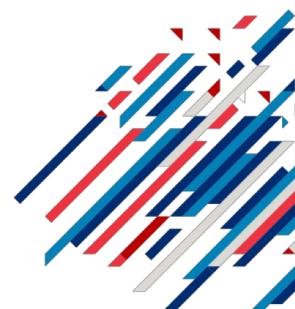
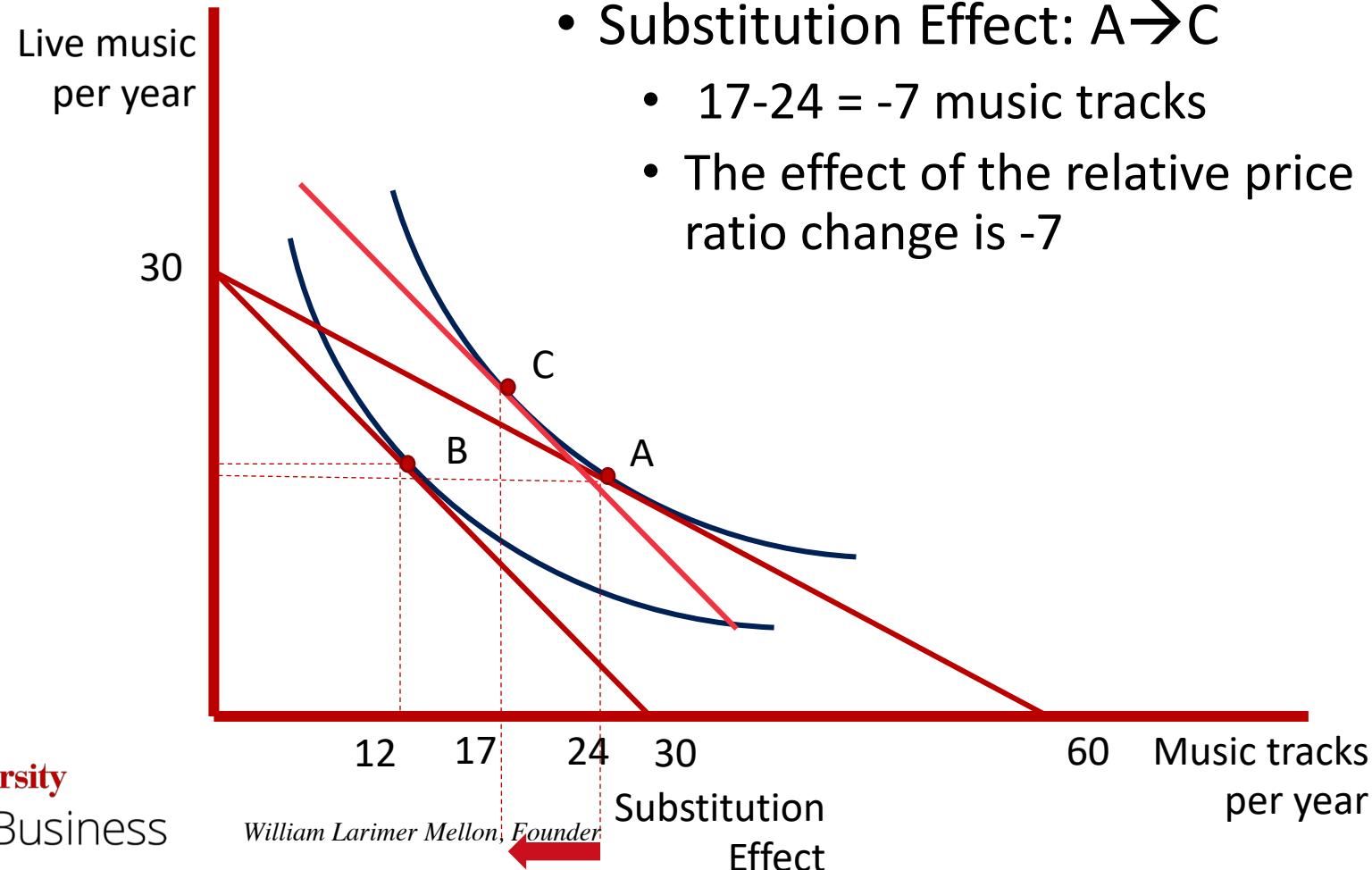
Substitution and Income Effects: Normal Good



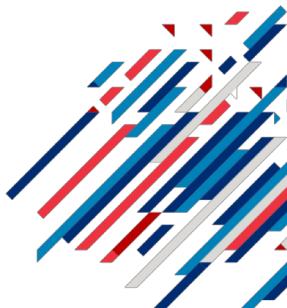
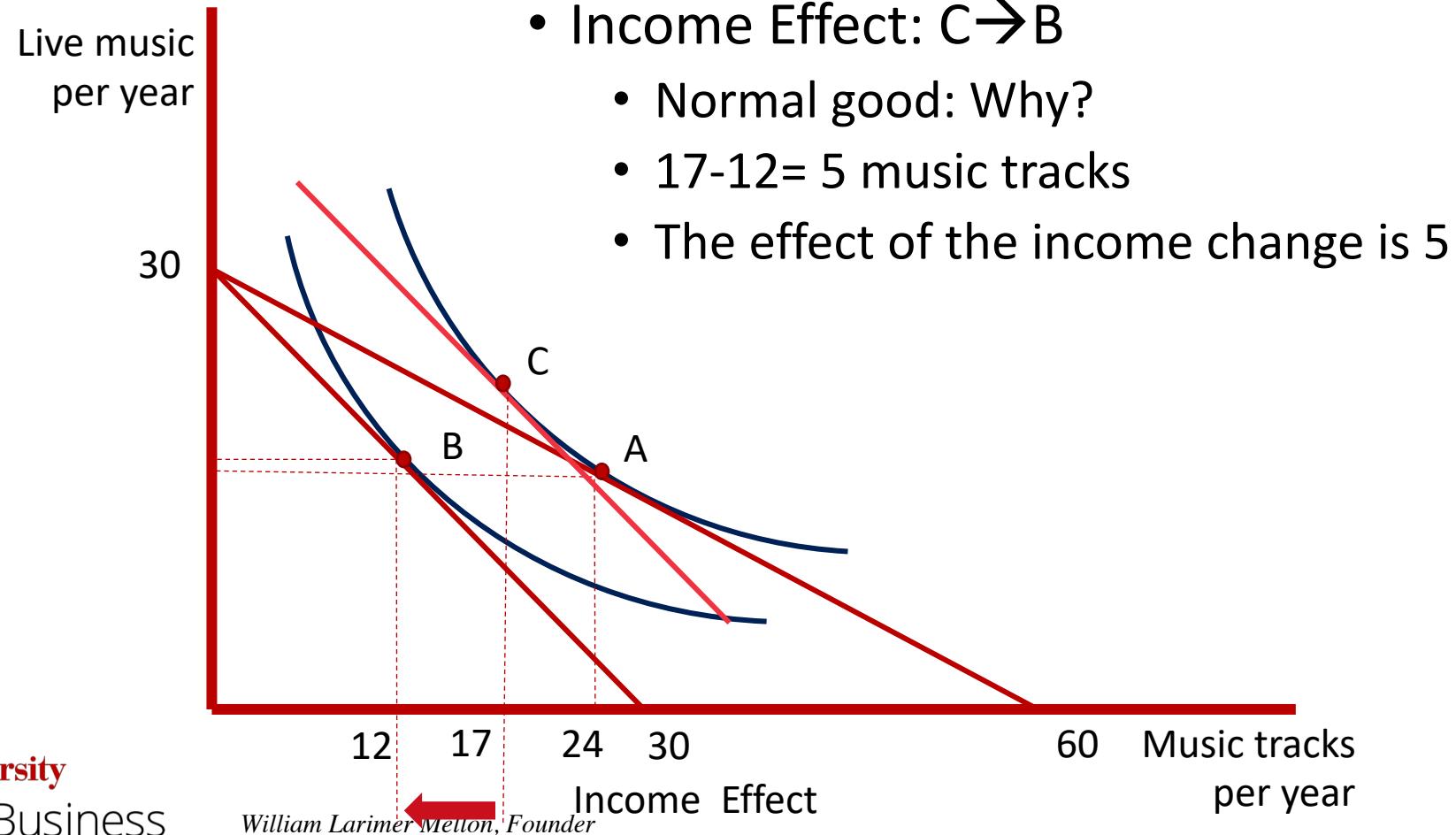
- Denote the optimal choice under the hypothetical budget line as C
- At C, 17 music tracks are consumed



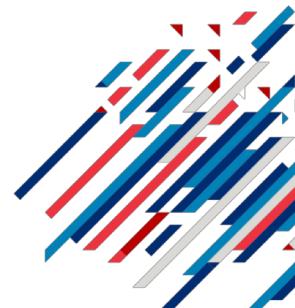
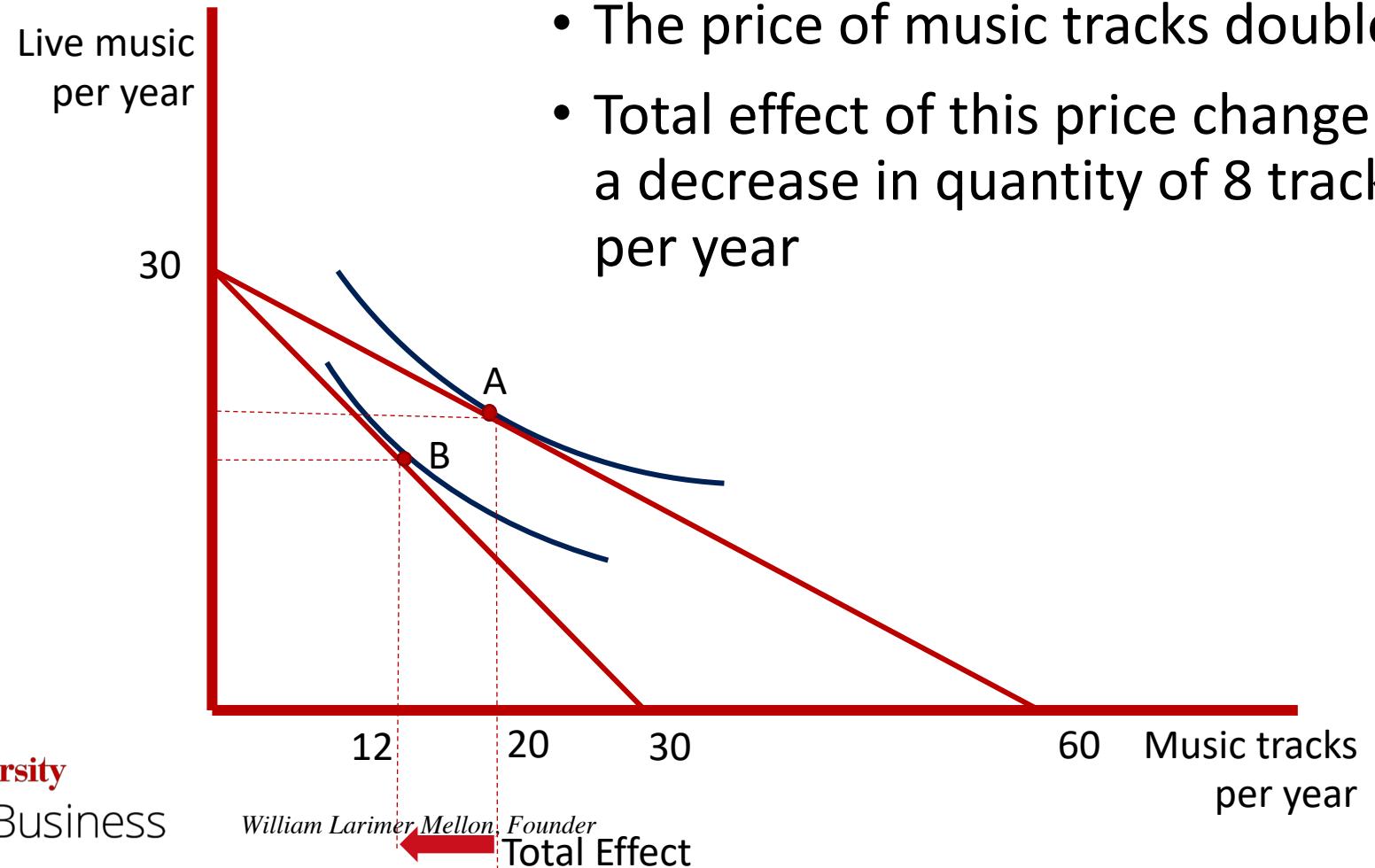
Substitution and Income Effects: Normal Good



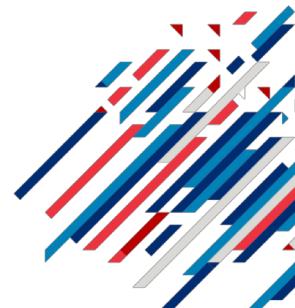
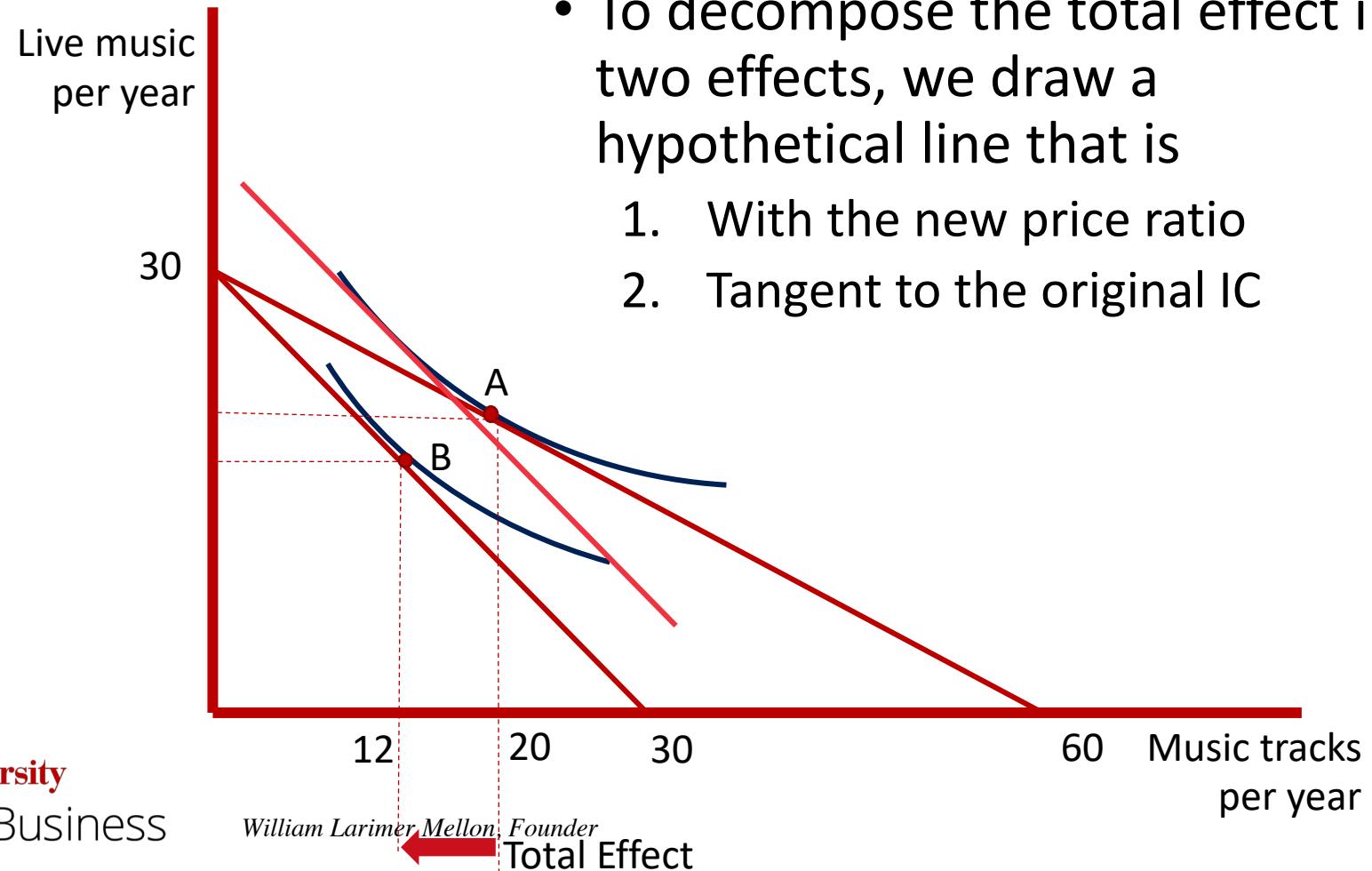
Substitution and Income Effects: Normal Good



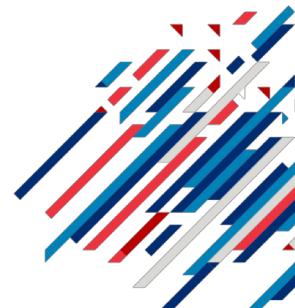
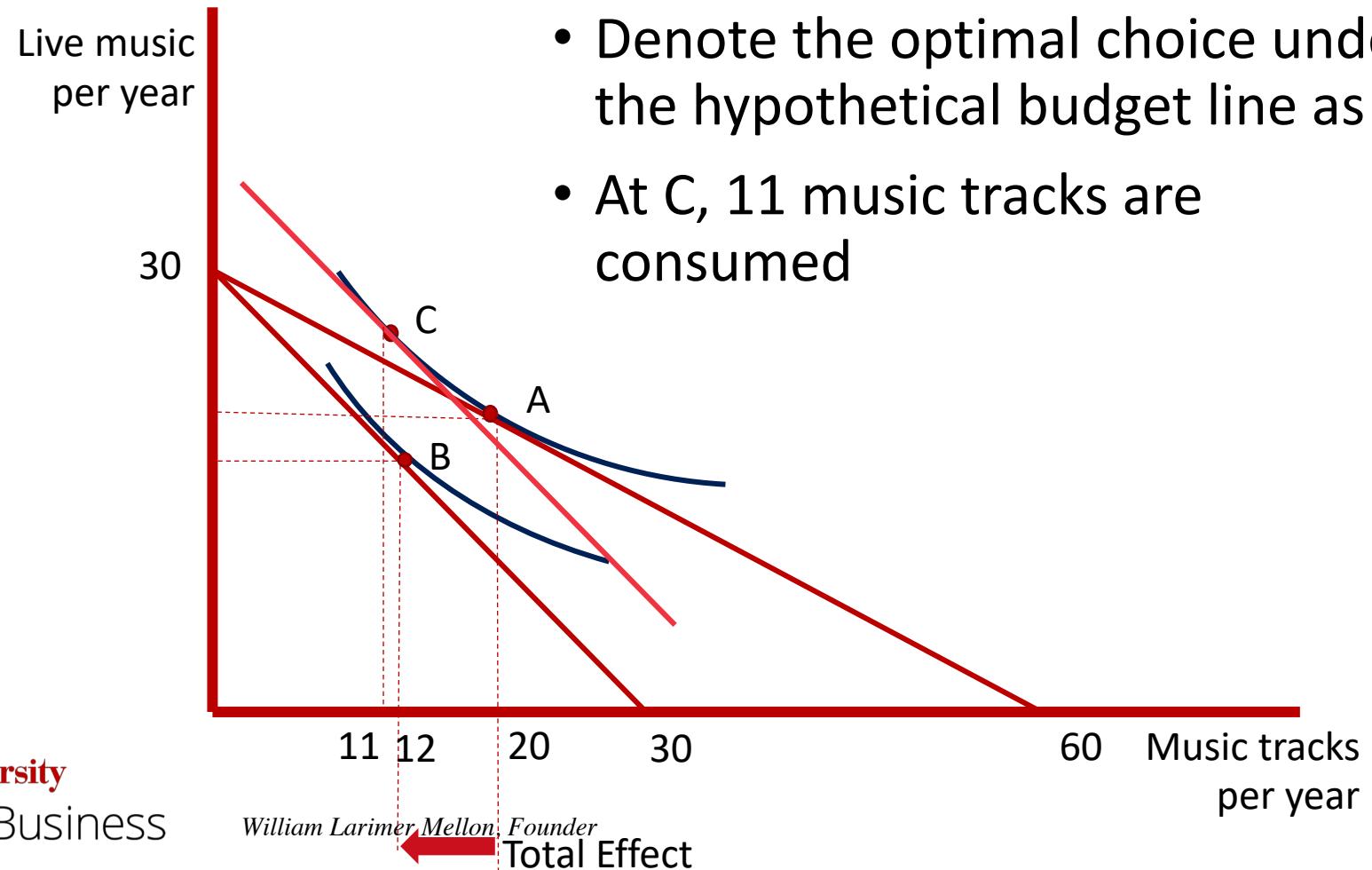
Substitution and Income Effects: Inferior Good



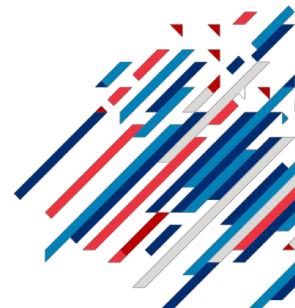
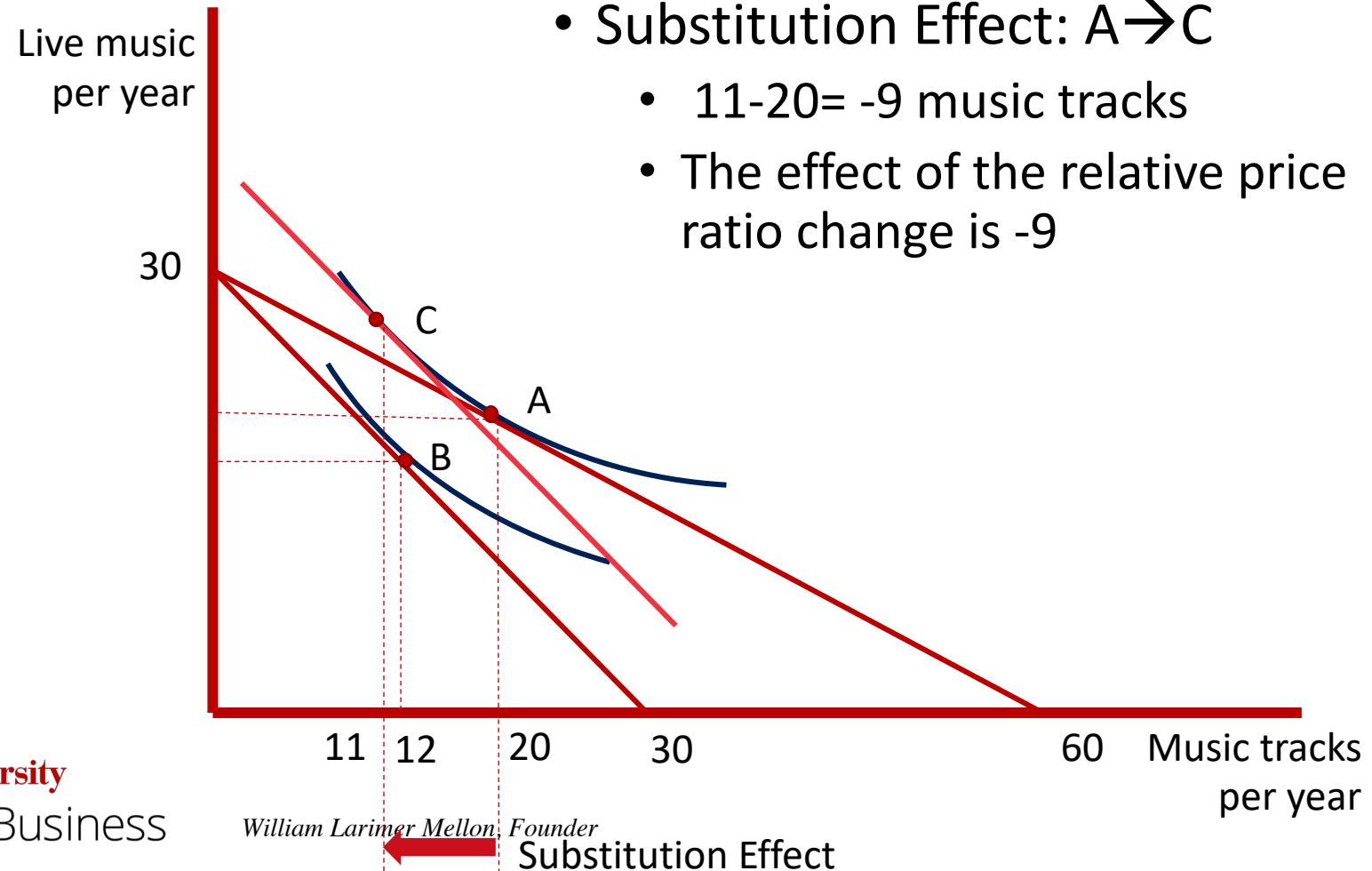
Substitution and Income Effects: Inferior Good



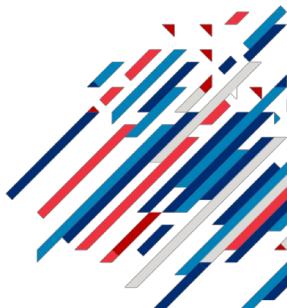
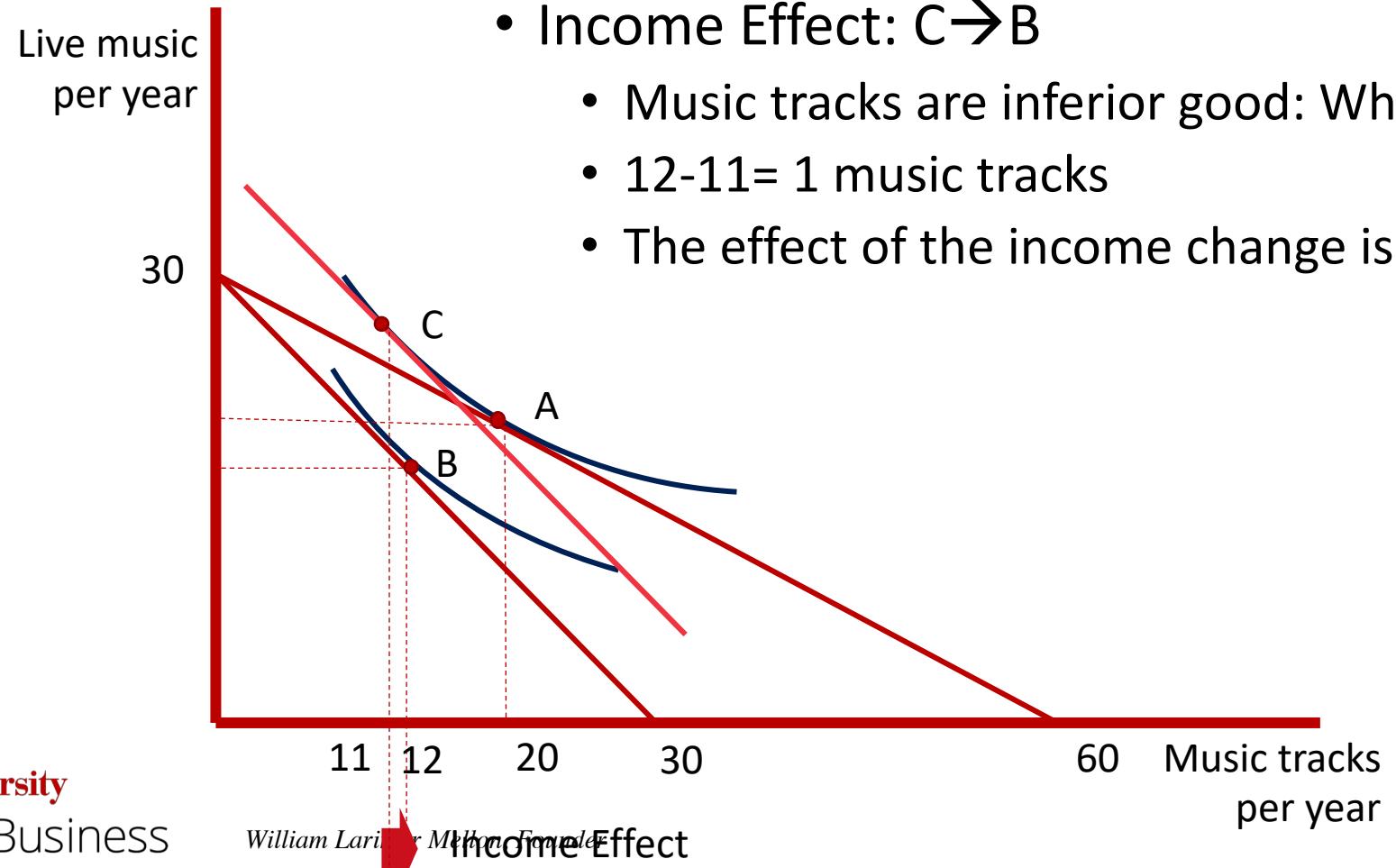
Substitution and Income Effects: Inferior Good



Substitution and Income Effects: Inferior Good



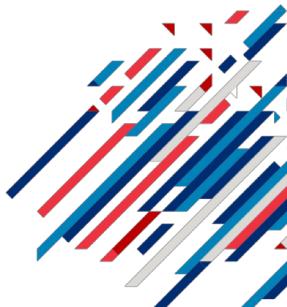
Substitution and Income Effects: Inferior Good



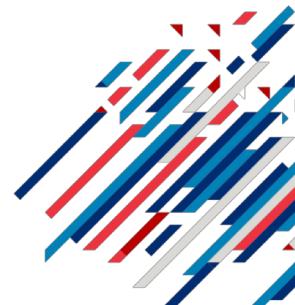
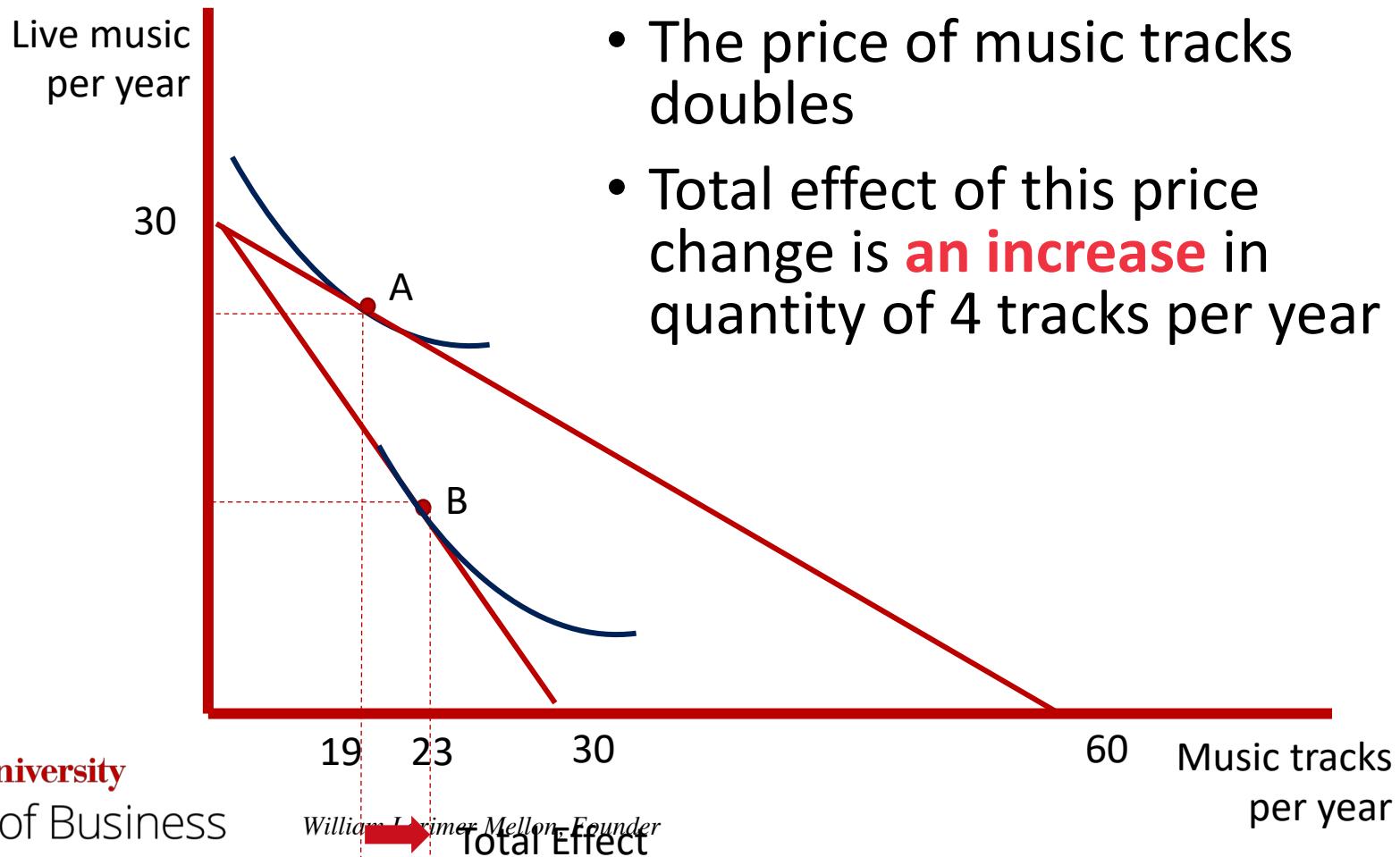
Inferior Goods and Giffen Goods



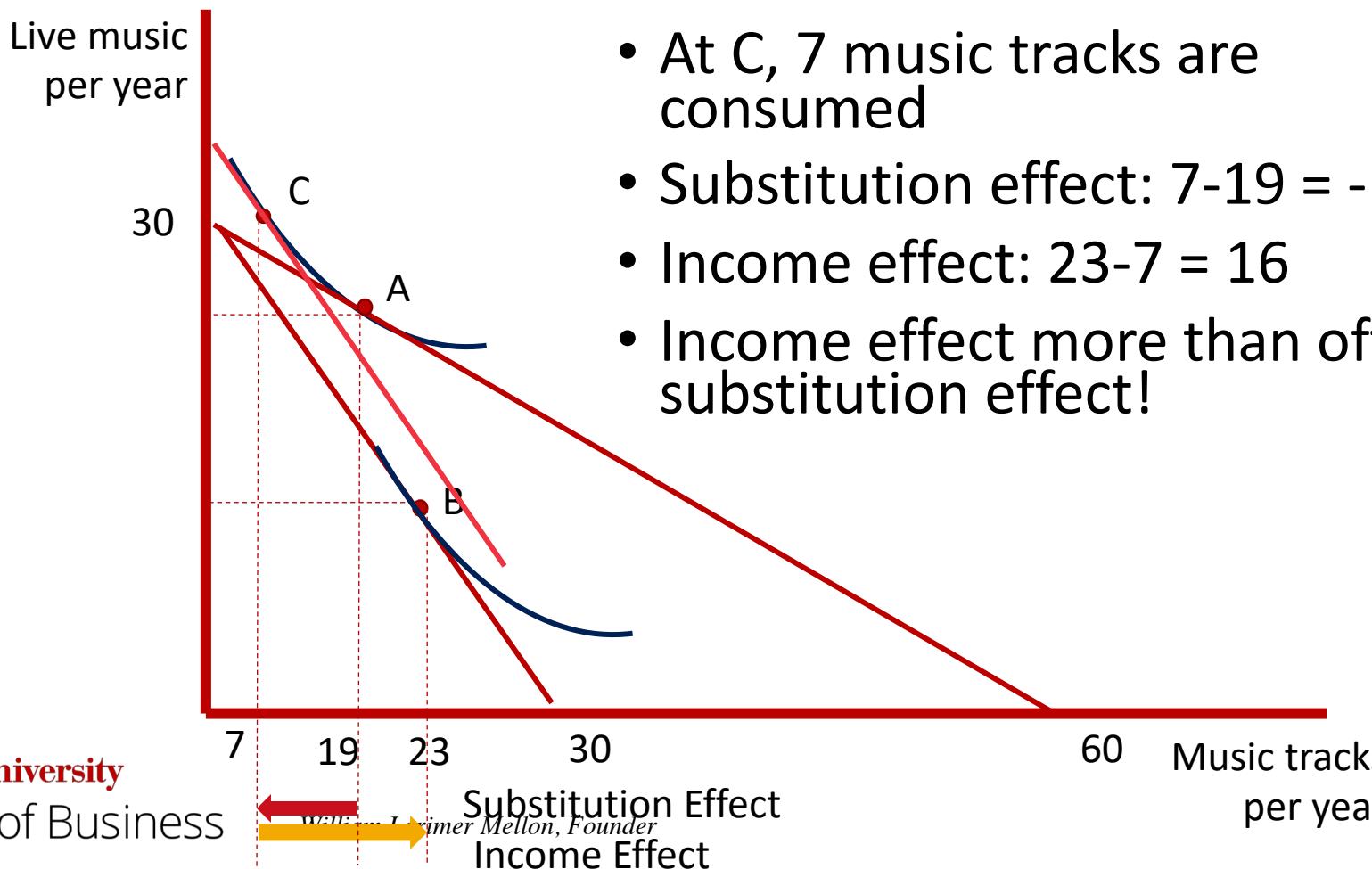
- For an inferior good, the income effect goes in the opposite direction from the substitution effect
- If the income effect more than offsets the substitution effect, then the demand curve has an upward slope
 - In such case, the good is called a **Giffen good**
 - For a Giffen good, a decrease in its price causes the quantity demanded to fall



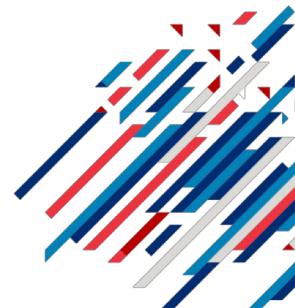
Giffen Goods



Giffen Goods



- At C, 7 music tracks are consumed
- Substitution effect: $7-19 = -12$
- Income effect: $23-7 = 16$
- Income effect more than offsets substitution effect!

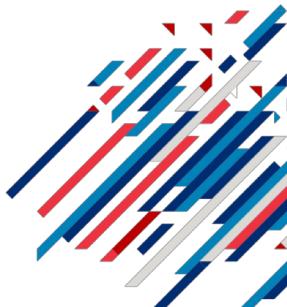


3. Labor Supply

Labor Supply



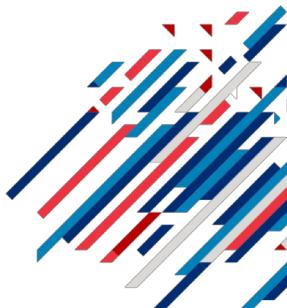
- Using the consumer choice framework, we study the decisions involving consumption and leisure, which are behind the supply of labor
- We study the effects of various policies on labor supply
 - Unemployment benefits
 - Income taxes: Flat vs. Progressive
- Reading: pp. 49-51, 59-60, 251-253, 265-268



Labor Supply: Road Map



1. Represent the budget constraint where the X-axis is for the hours of leisure, and the Y-axis is the amount of consumption
2. Solve for the optimal labor supply
3. Study how the wage rate change affects the labor supply using substitution and income effects
 - As the wage rate increases, some people may work LESS
4. Study the effects of various policies on labor supply
 - Unemployment benefits
 - Income taxes: Flat vs. Progressive



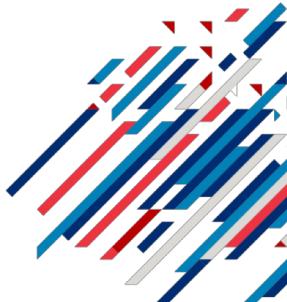
Budget Constraint



- Standard budget constraint: A consumer with fixed income Y , who is choosing quantities of goods 1 and 2, with prices p_1 and p_2 :

$$p_1x_1 + p_2x_2 \leq Y$$

- Our consumer earns money by working: Her income is wl , where w is the wage rate, in dollars per hour of work, and l is the number of hours of work.



Budget Constraint (Cont'd)

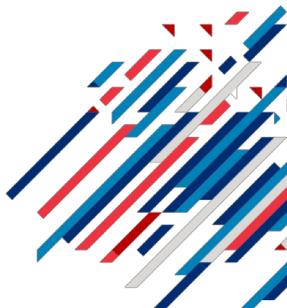
- Our budget constraint: “What you spend is less than or equal to what you have”

$$pc \leq wl,$$

where p is the price of consumption.

- The two goods in our framework are consumption(c) and leisure (L).
- Hours of labor plus hours of leisure must equal the total number of hours (T)

$$l + L = T.$$



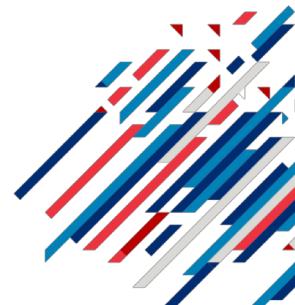
Budget Constraint (Cont'd)

- We have two expressions:

$$\begin{aligned} pc &\leq wl \\ l &= T - L \end{aligned}$$

- Substituting l in the budget constraint:

$$\begin{aligned} pc &\leq w(T - L) \\ \rightarrow pc + wL &\leq wT \end{aligned}$$

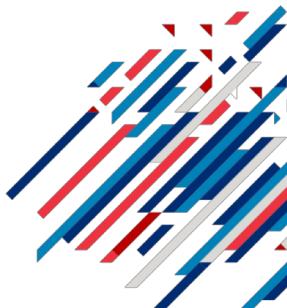


Budget Constraint (Cont'd)

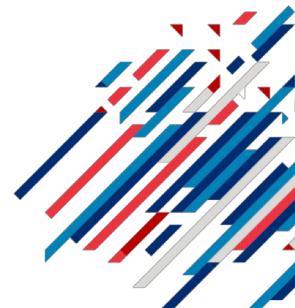
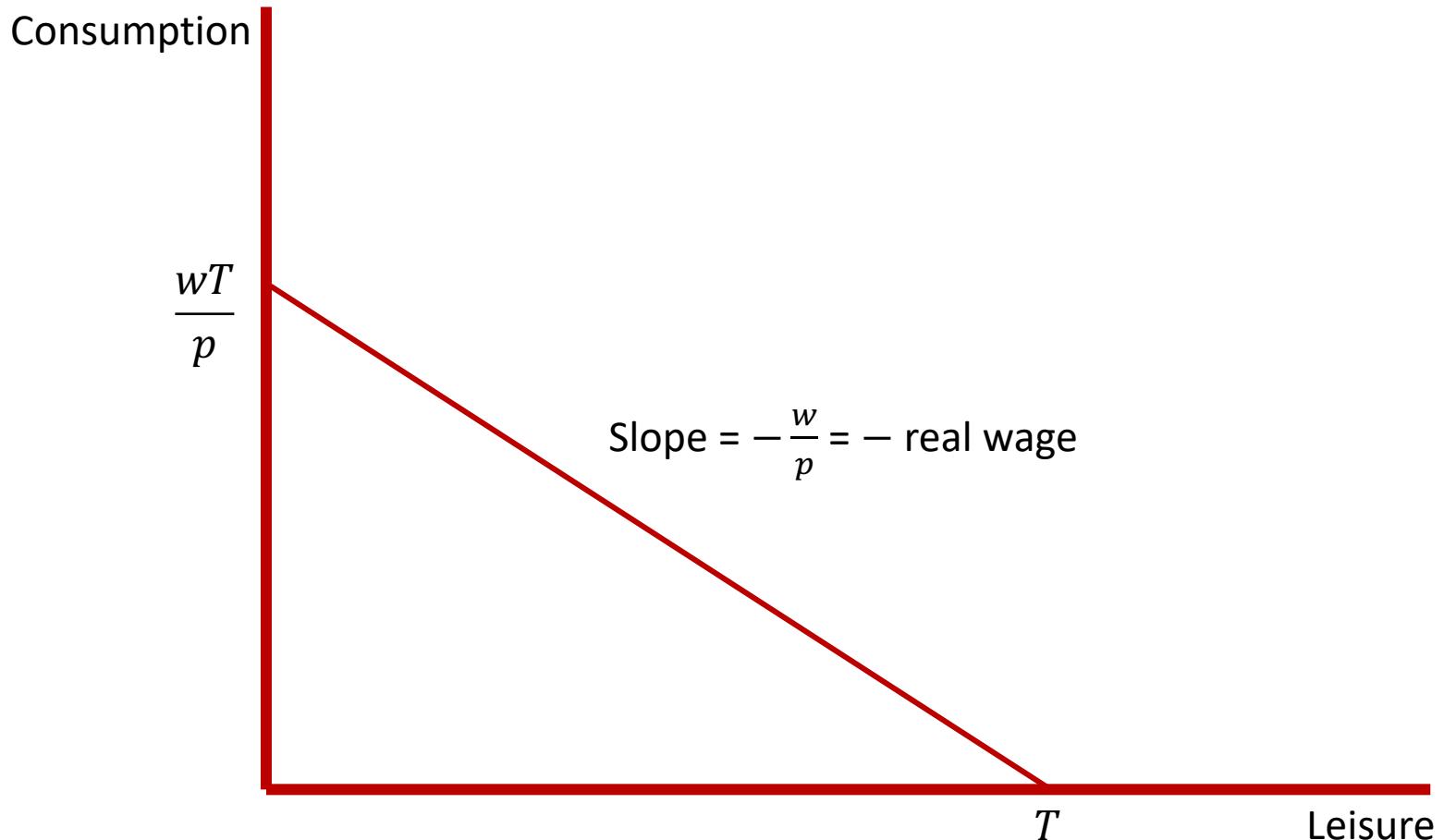
- Budget constraint:

$$pc + wL \leq wT$$

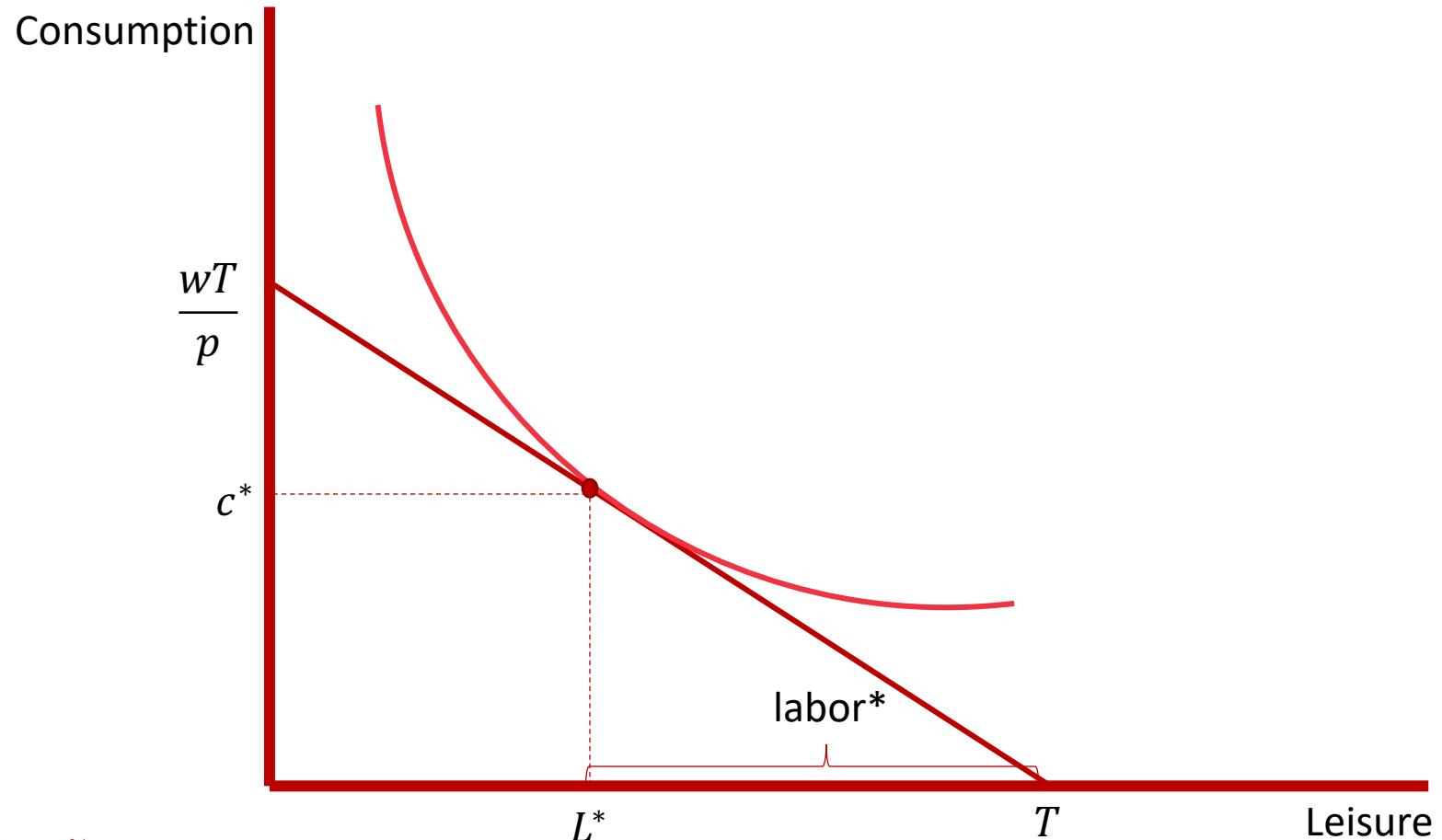
- Two goods: Consumption (c) and leisure (L)
- Price of consumption: p
- Price of leisure: the wage rate (w)
- Income: What the consumer would get by working all the time available, wT



Budget Constraint (Cont'd)



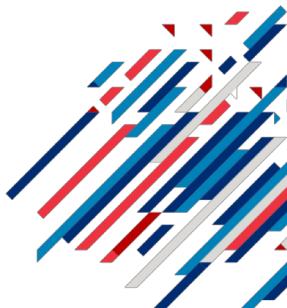
Optimal Choice



Carnegie Mellon University

Tepper School of Business

William Larimer Mellon, Founder



Labor Supply



- Demand functions:

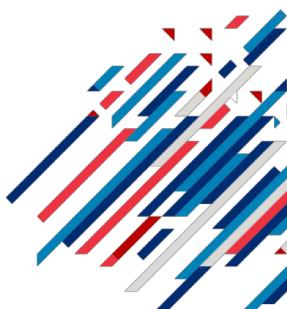
$$c^* = c(w, p), L^* = L(w, p)$$

- Labor supply function:

$$l^* = l(w, p) = T - L(w, p)$$

- If p is fixed and only w is allowed to vary, then the labor supply is a function of the wage rate only.

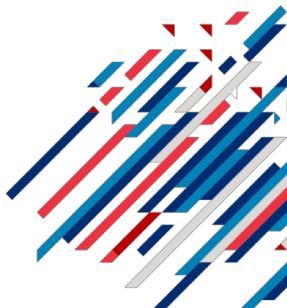
- Labor supply curve: $l(w)$



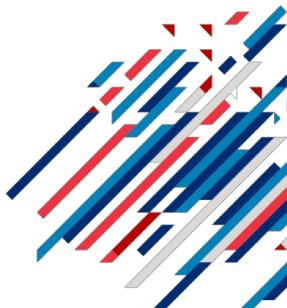
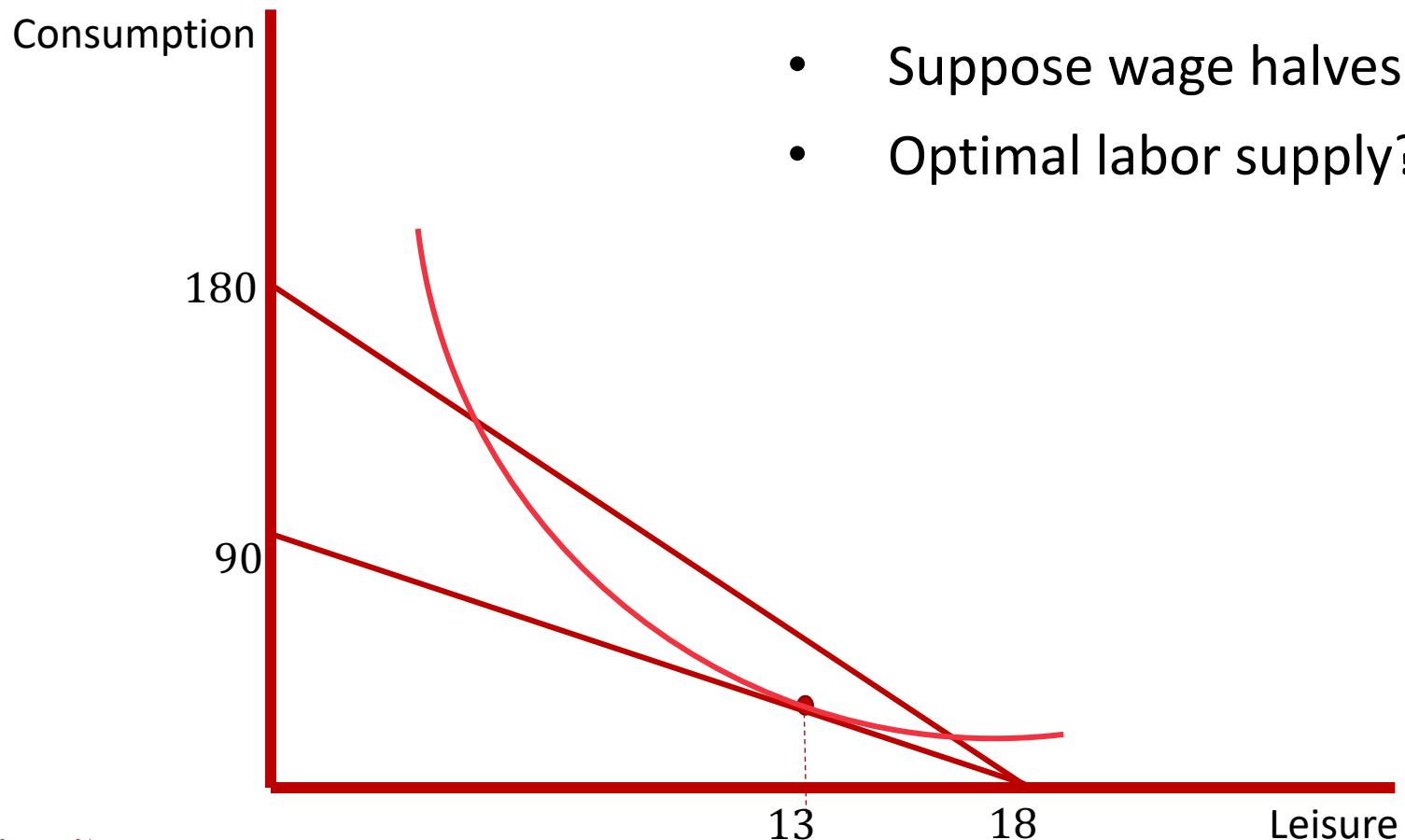
Derivation of Labor Supply Curve



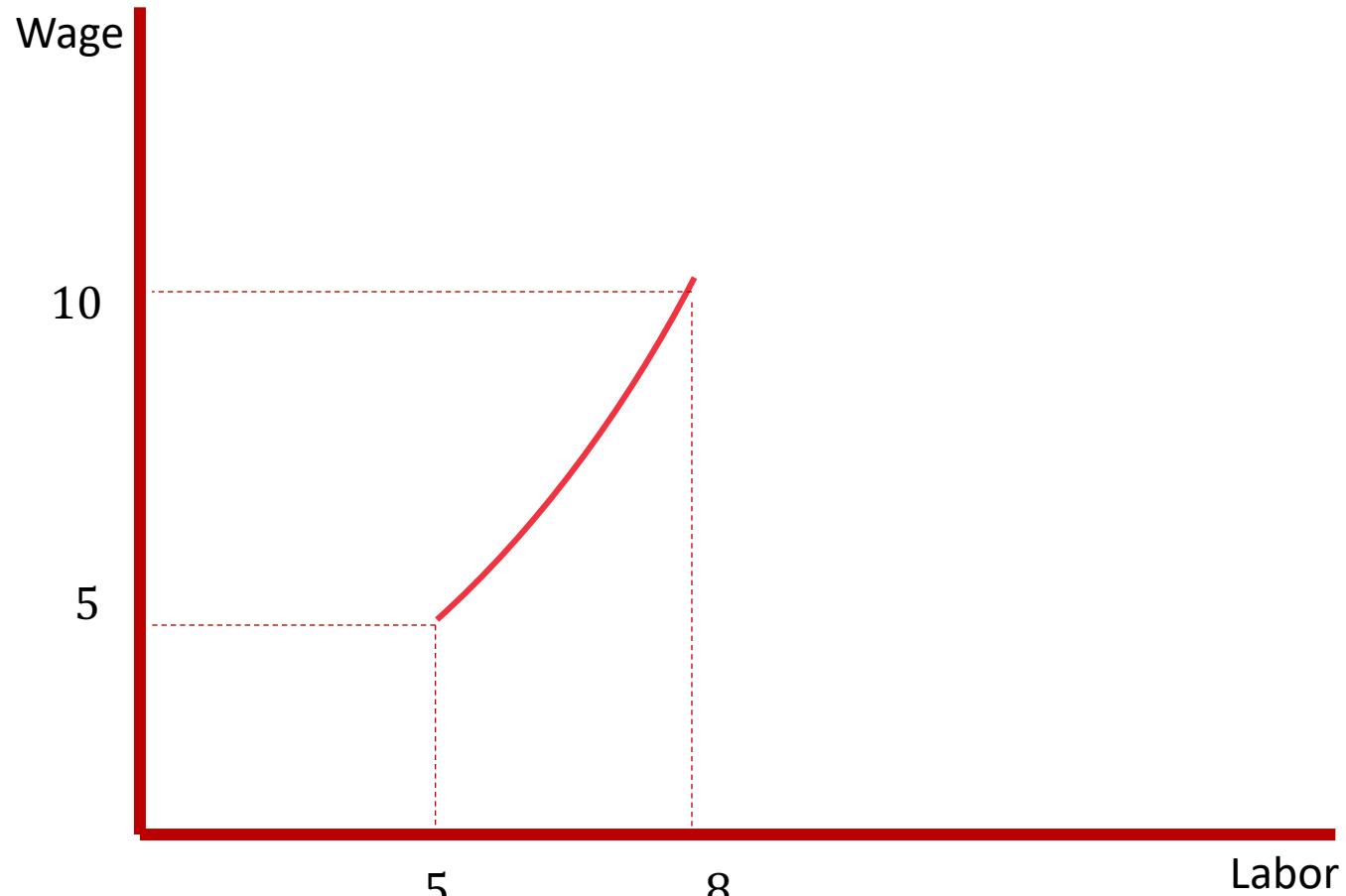
- Price of consumption :\$1
- Total time = 18 hours
- Wage rate?
- Optimal labor supply?



Derivation of Labor Supply Curve



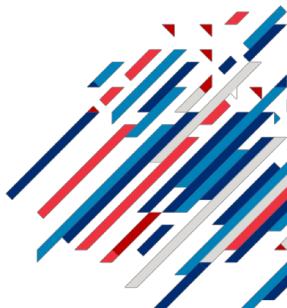
Derivation of Labor Supply Curve



Carnegie Mellon University

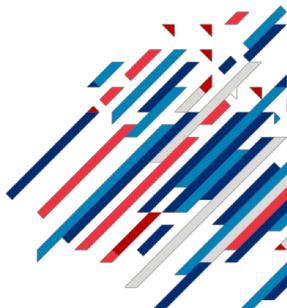
Tepper School of Business

William Larimer Mellon, Founder



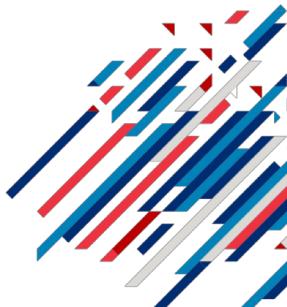
Labor Supply Curve

- Labor supply curve: $l(w)$
- **Question:** Does the labor supply curve upward sloping or downward sloping? In other words, as the wage rate increases, does the labor supply increase or decrease?

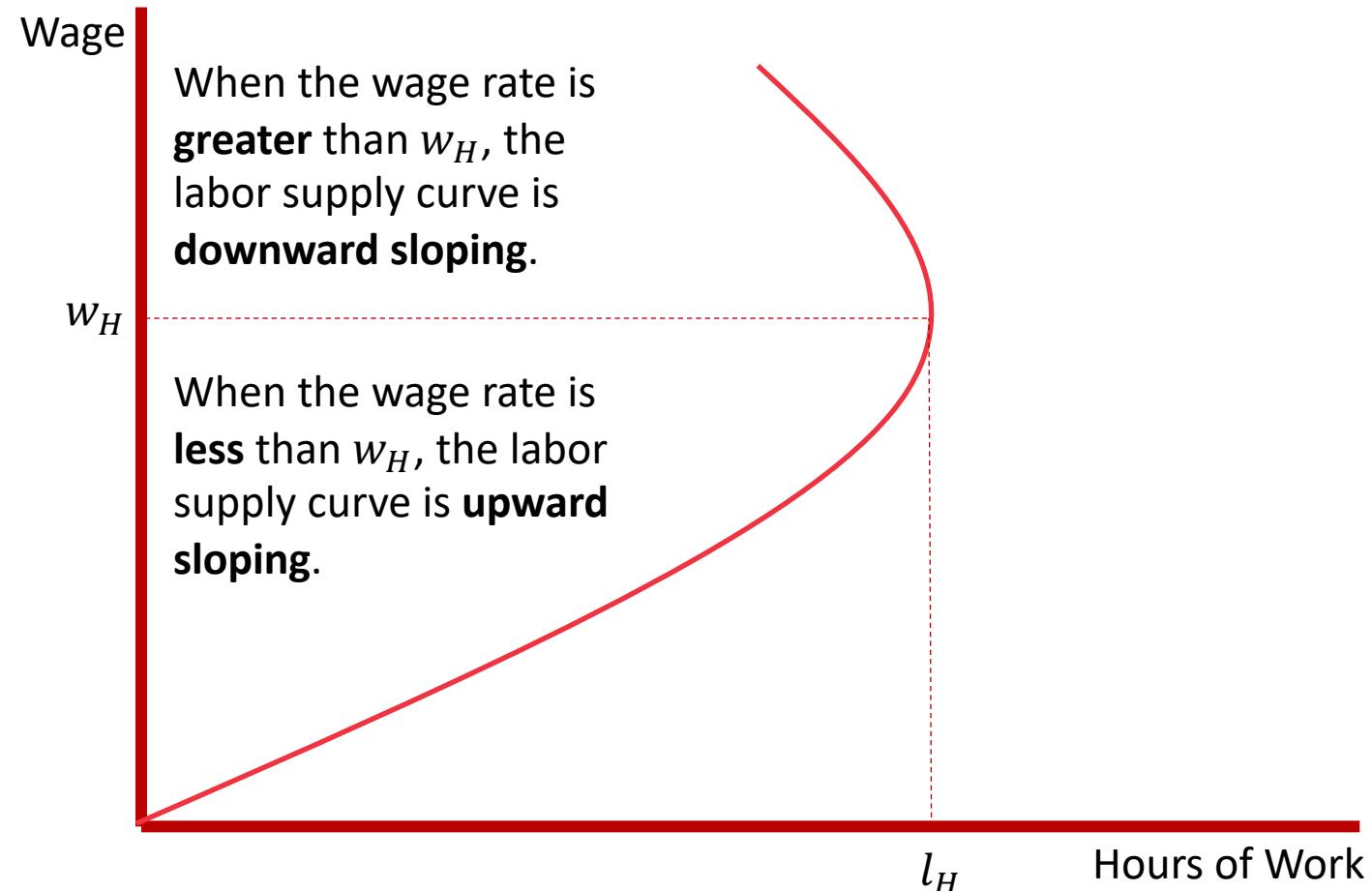


Labor Supply Curve

- **Answer:** It depends on the relative sizes of substitution effect and income effect
 - Substitution effect: Higher wage rate → Higher relative price of leisure → Less leisure → More work
 - Income effect: Higher wage rate → Higher income per hour → More leisure (leisure is a normal good) → Less work



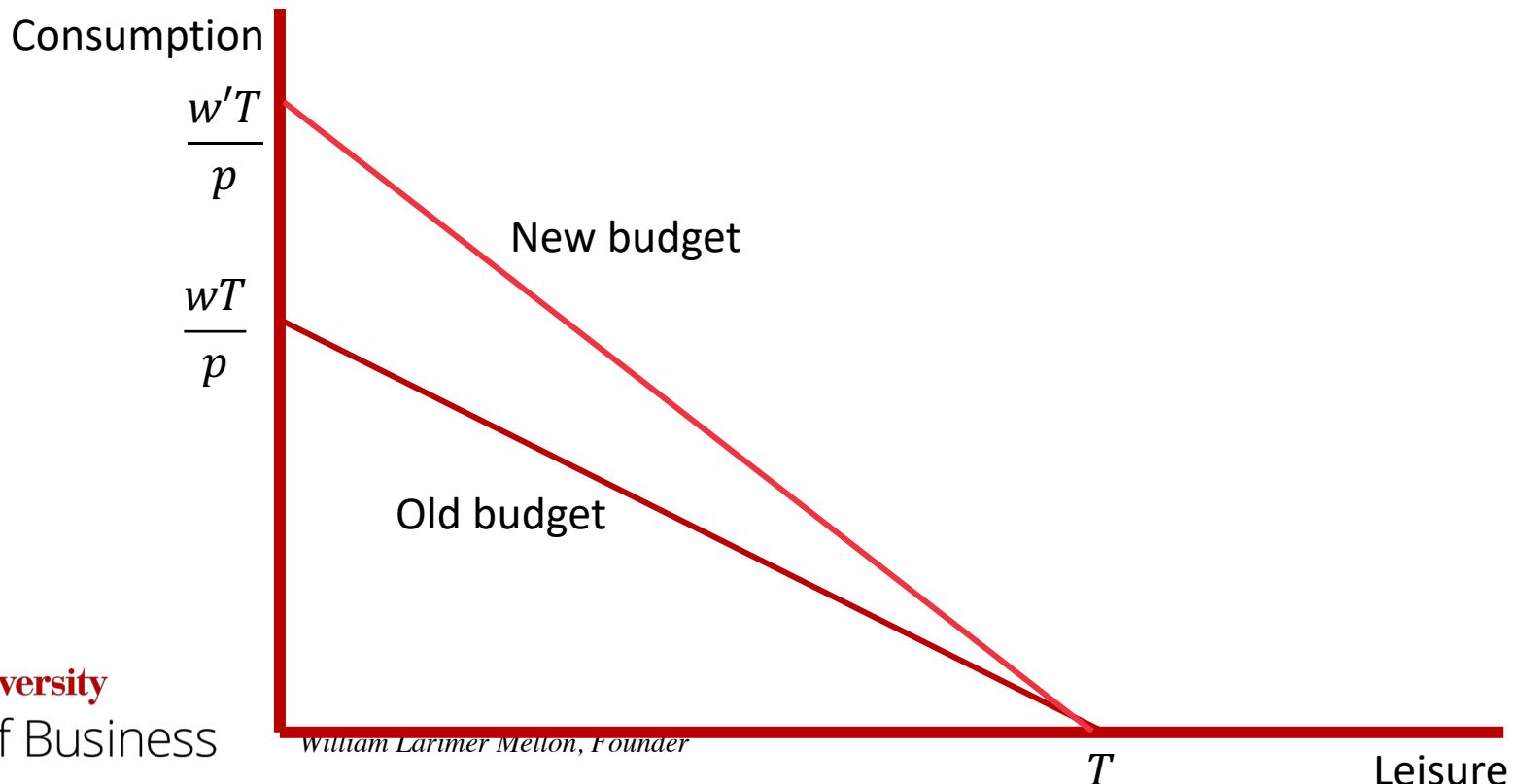
An Example of Labor Supply Curve



Wage Rate Increase



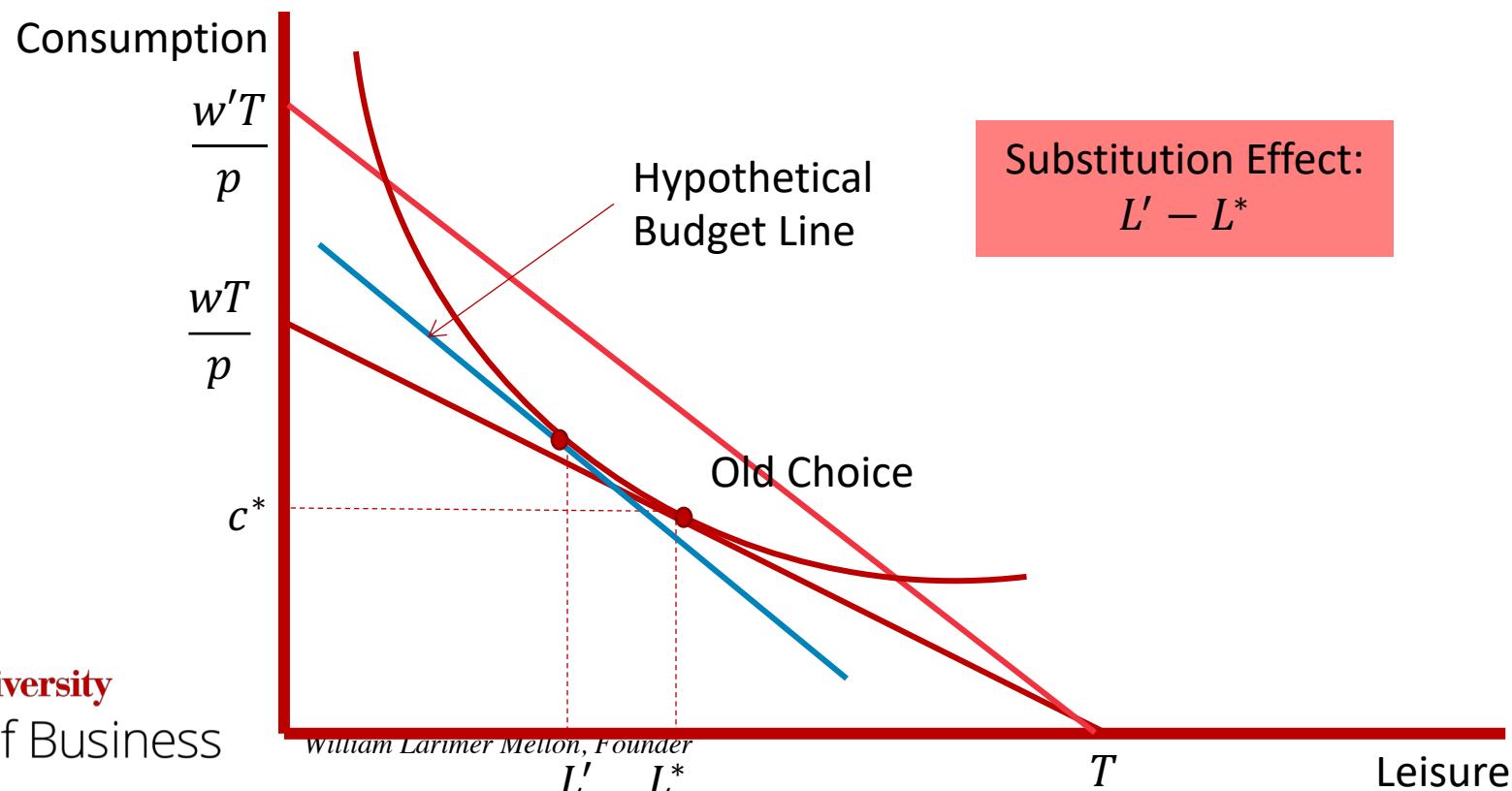
- As the wage rate increases from w to w' :
 1. Slope change: Relative price of leisure increases
 2. Location change (outward): Relative income increases



Substitution Effect



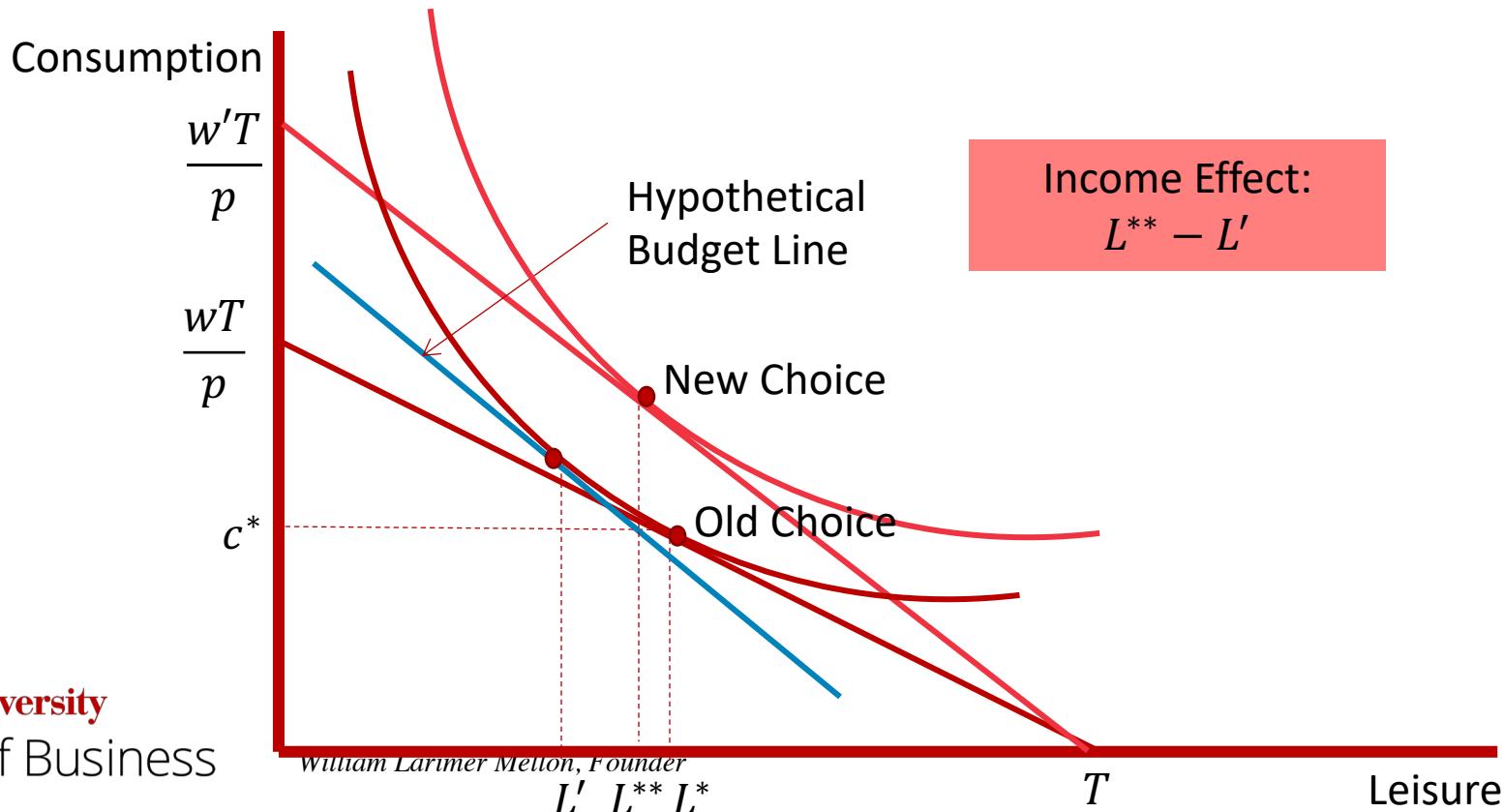
- As the wage rate increases from w to w' :
 1. Relative price of leisure increases → Substitution effect
 2. Relative income increases



Income Effect



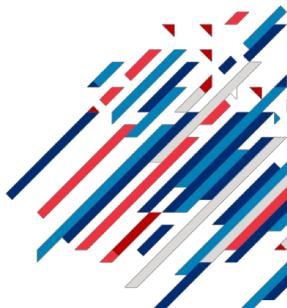
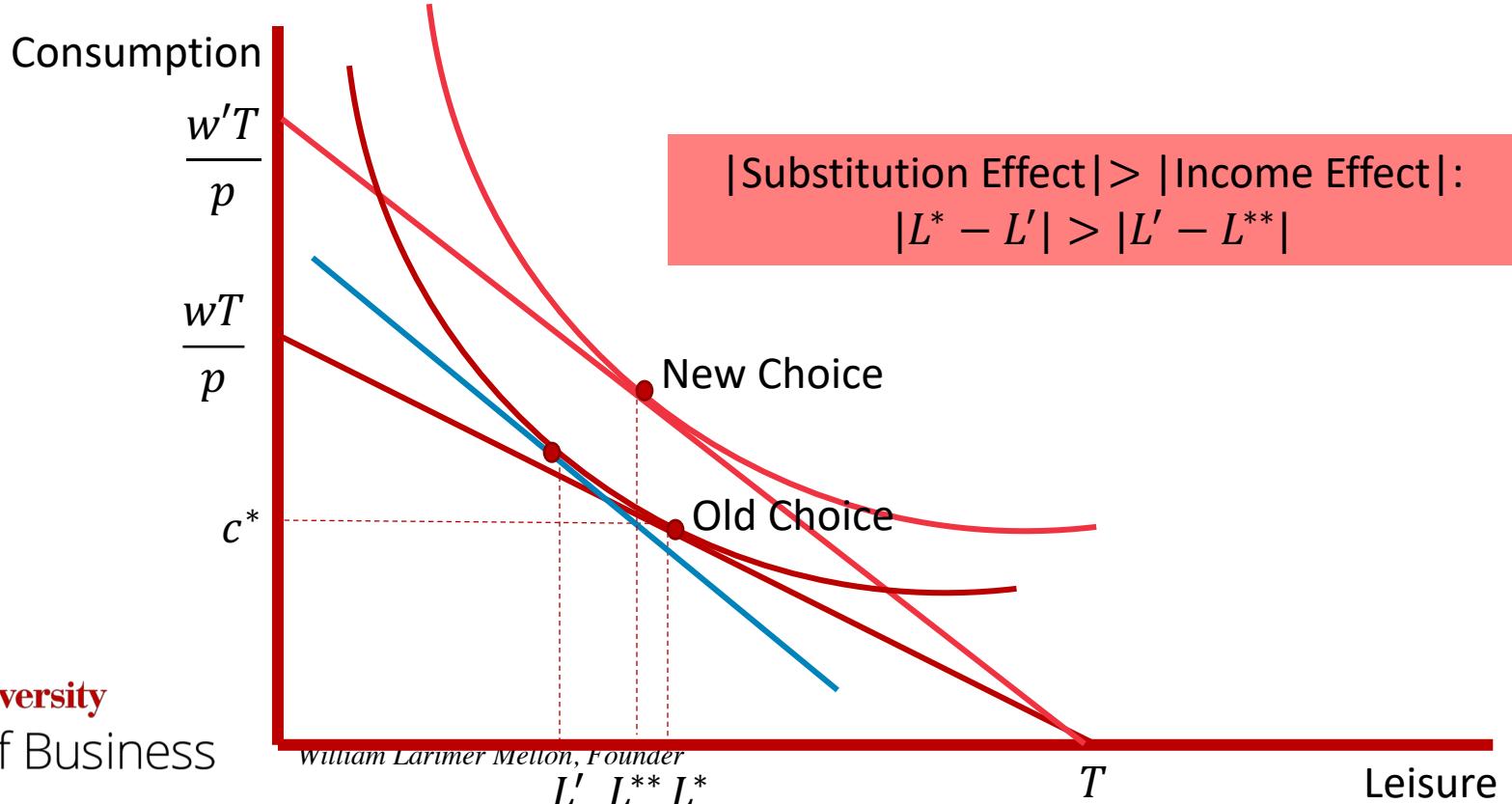
- As the wage rate increases from w to w' :
 - Relative price of leisure increases
 - Relative income increases → Income effect



Labor Supply Curve



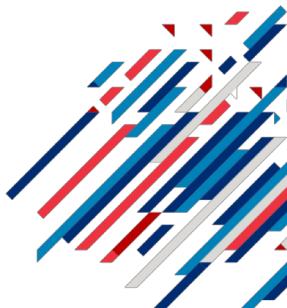
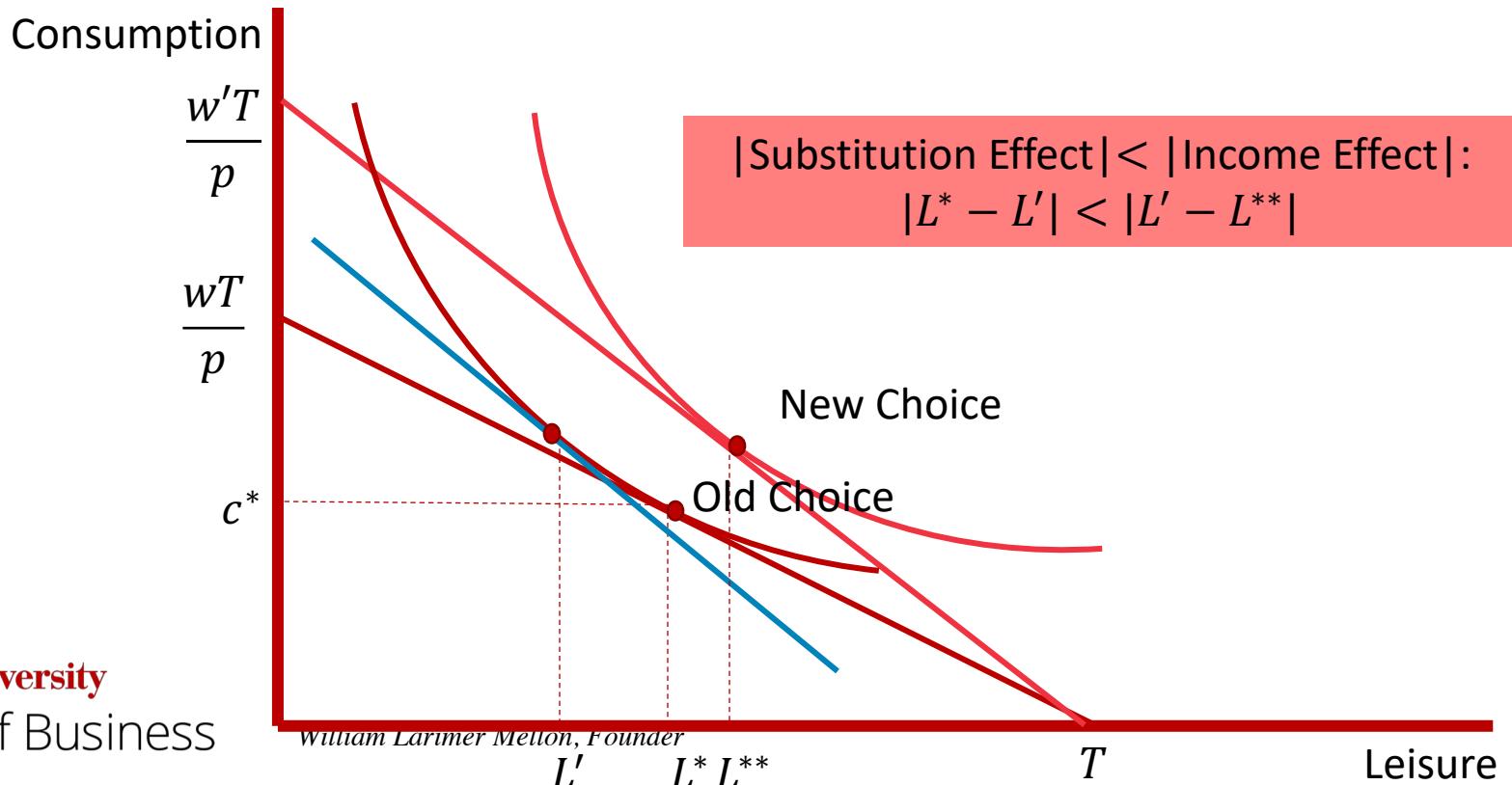
- Here, the labor supply is **upward sloping** ($l^* < l^{**}$)
- Substitution effect and income effect are opposite
- $|\text{Substitution effect}| > |\text{Income effect}|$



Labor Supply Curve

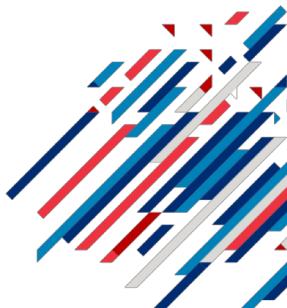


- Here, the labor supply is **downward sloping** ($l^* > l^{**}$)
- Substitution effect and income effect are opposite
- $| \text{Substitution effect} | < | \text{Income effect} |$



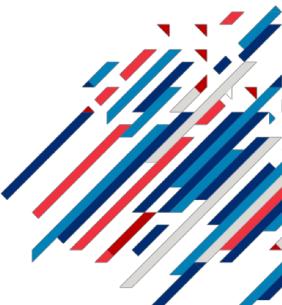
Labor Policy 1: Unemployment Benefit

- Suppose the government considers providing unemployment benefit U to people who do not work.
- Questions:
 1. Represent the budget constraint under this policy
 2. With the unemployment benefit, will the labor supply increase?



Labor Policy 2: Labor Income Tax

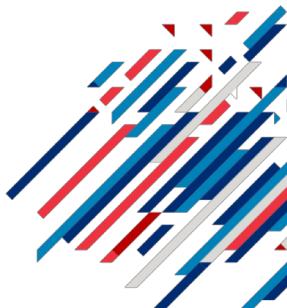
- There are two types of labor income tax.
 1. Flat tax: The tax rate is the same for everyone
 2. Progressive tax: The tax rate depends on before tax (e.g. no tax for low income earners; and tax the rest)
- Questions:
 1. Represent the budget constraint under this policy
 2. Compare the two tax regimes by consumer welfare and tax revenues



Labor Policy 1: Unemployment Benefit

- Suppose the government provides unemployment benefit U to people who do not work.
- Budget constraint:

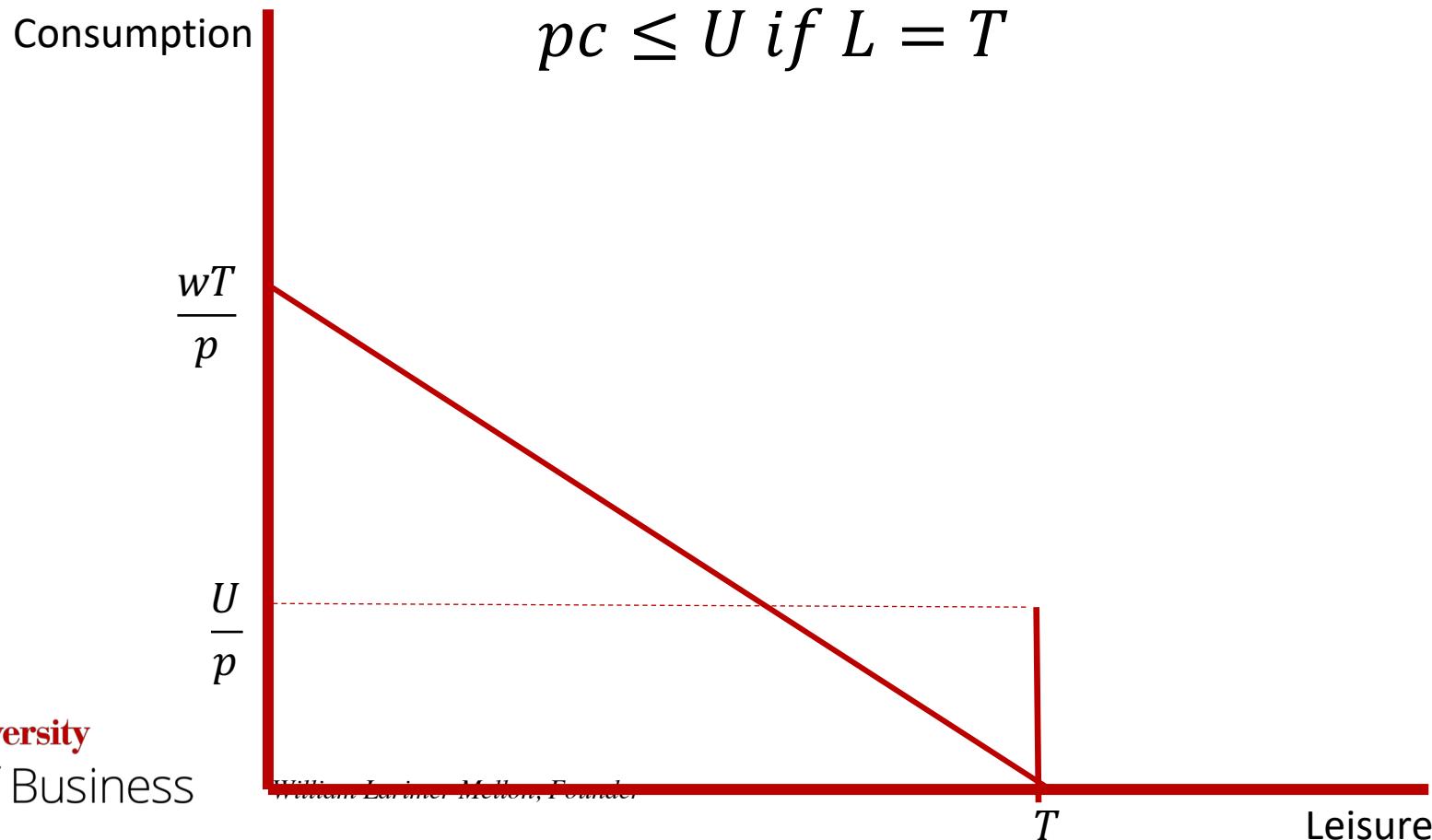
$$\begin{aligned} pc + wL &\leq wT \text{ if } L < T \\ pc &\leq U \text{ if } L = T \end{aligned}$$



Labor Policy 1: Unemployment Benefit

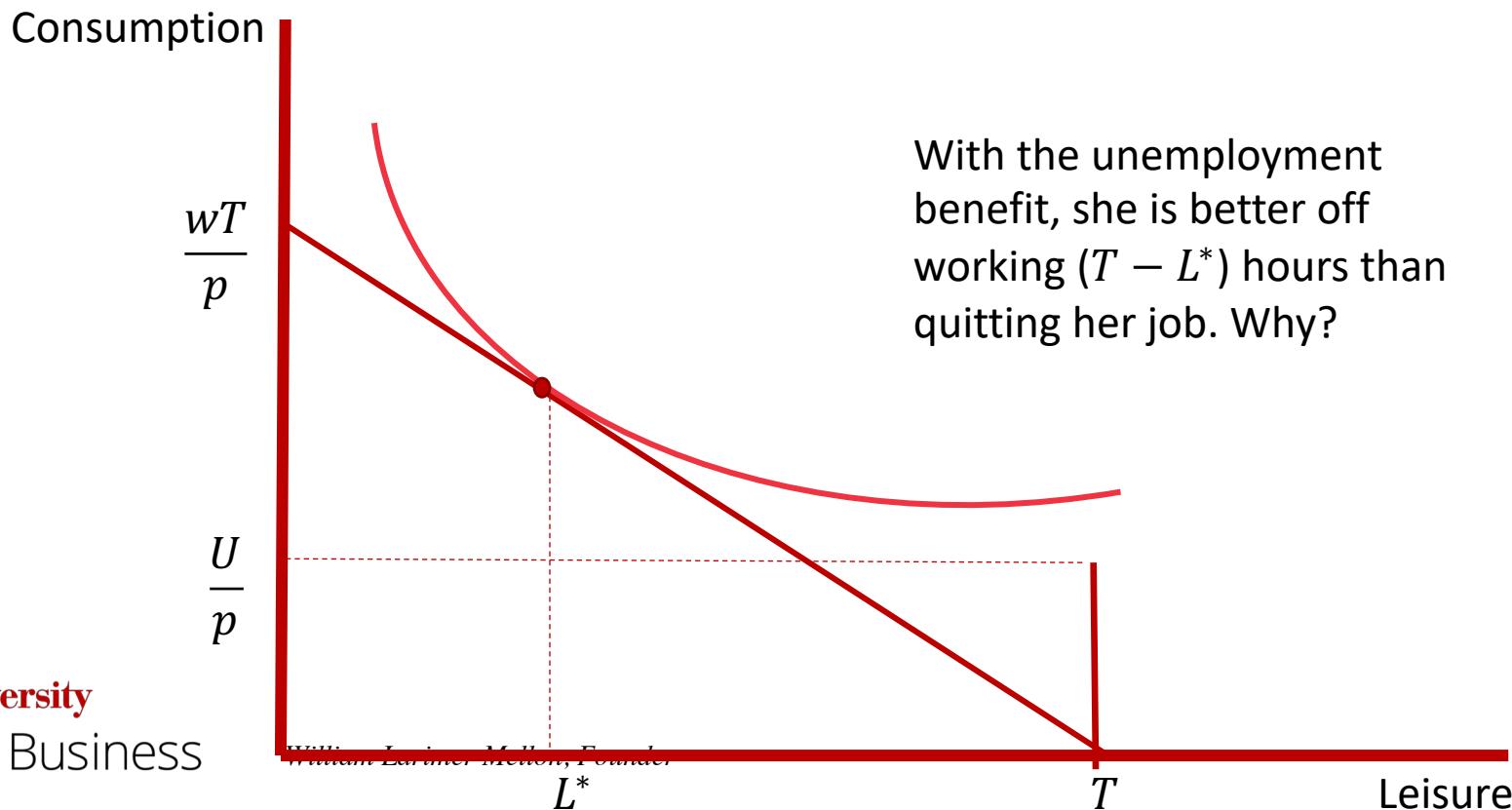
- Budget constraint:

$$pc + wL \leq wT \text{ if } L < T$$
$$pc \leq U \text{ if } L = T$$



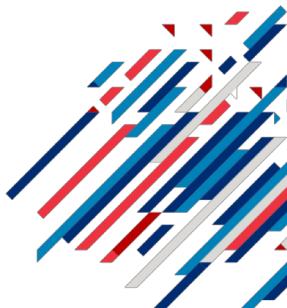
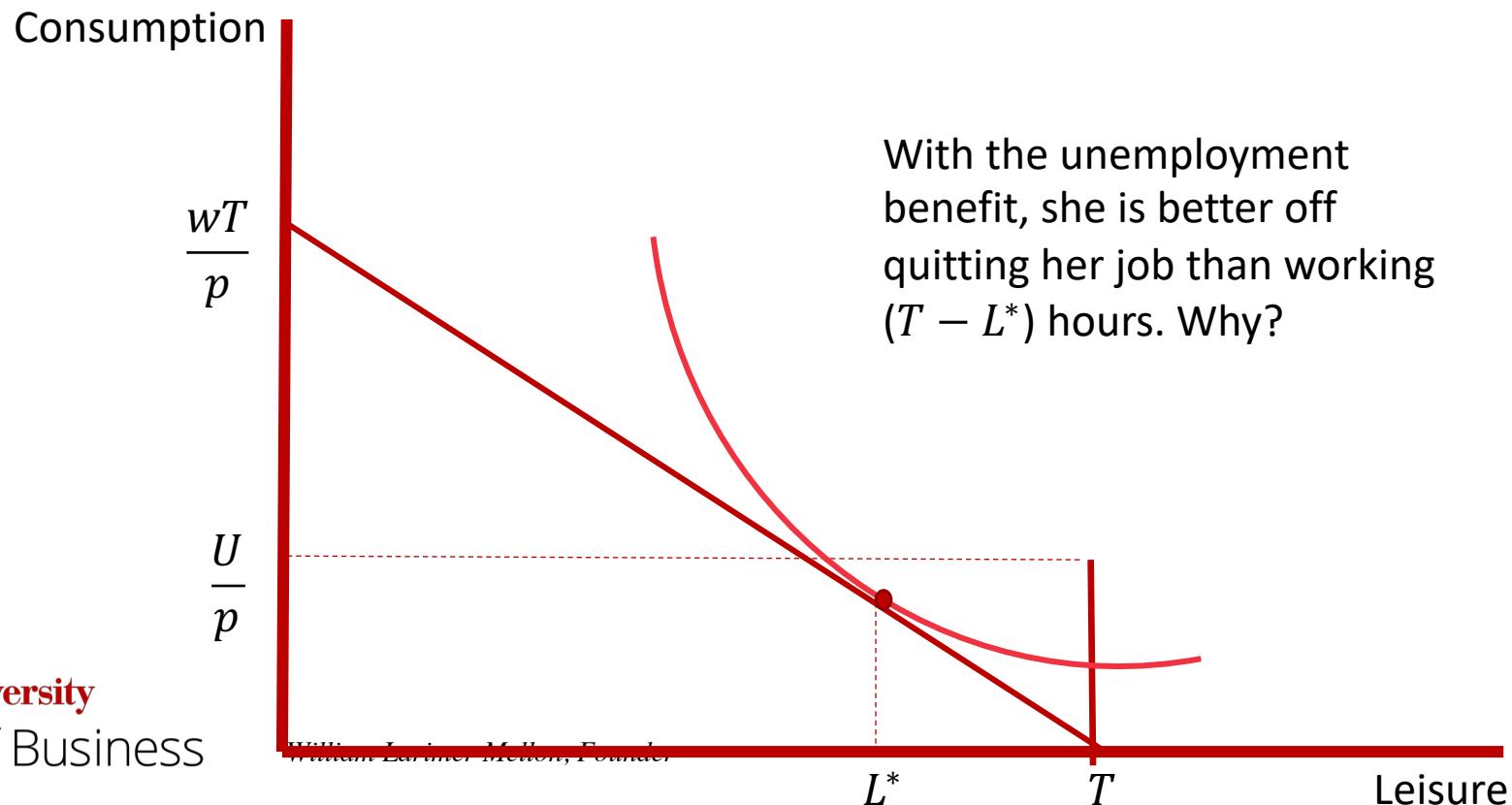
Labor Policy 1: Unemployment Benefit

- Evelyn: Unemployment benefit does not affect her labor supply



Labor Policy 1: Unemployment Benefit

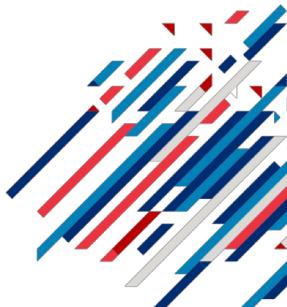
- Elliot: Unemployment benefit incentives her to quit her job



Labor Policy 2: Labor Income Tax

- Flat Tax
 - The tax rate t is the same for everyone
 - The after-tax wage rate is $w(1 - t)$
 - Budget constraint:

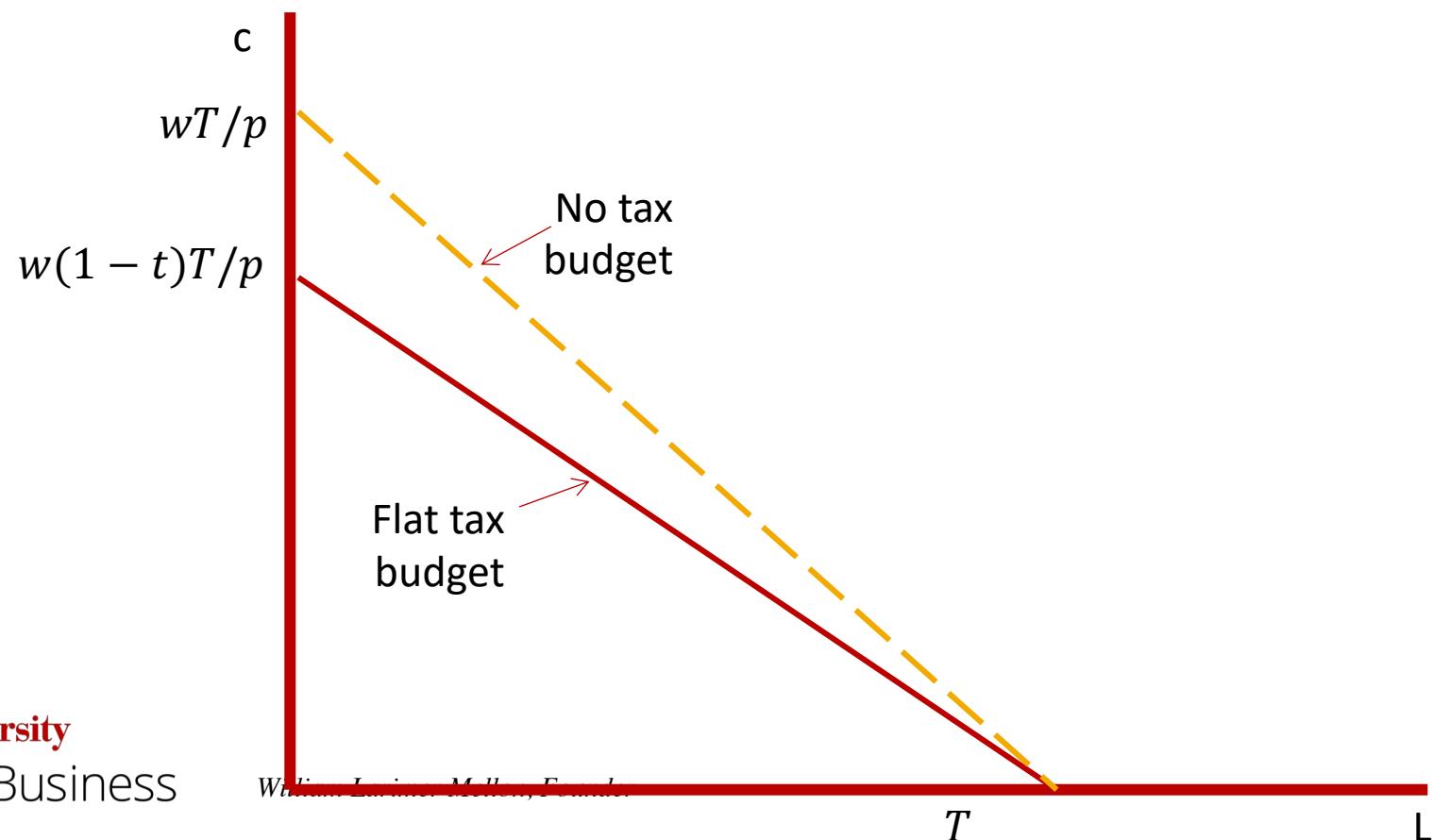
$$pc + w(1 - t)L \leq w(1 - t)T$$



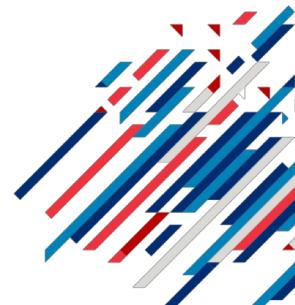
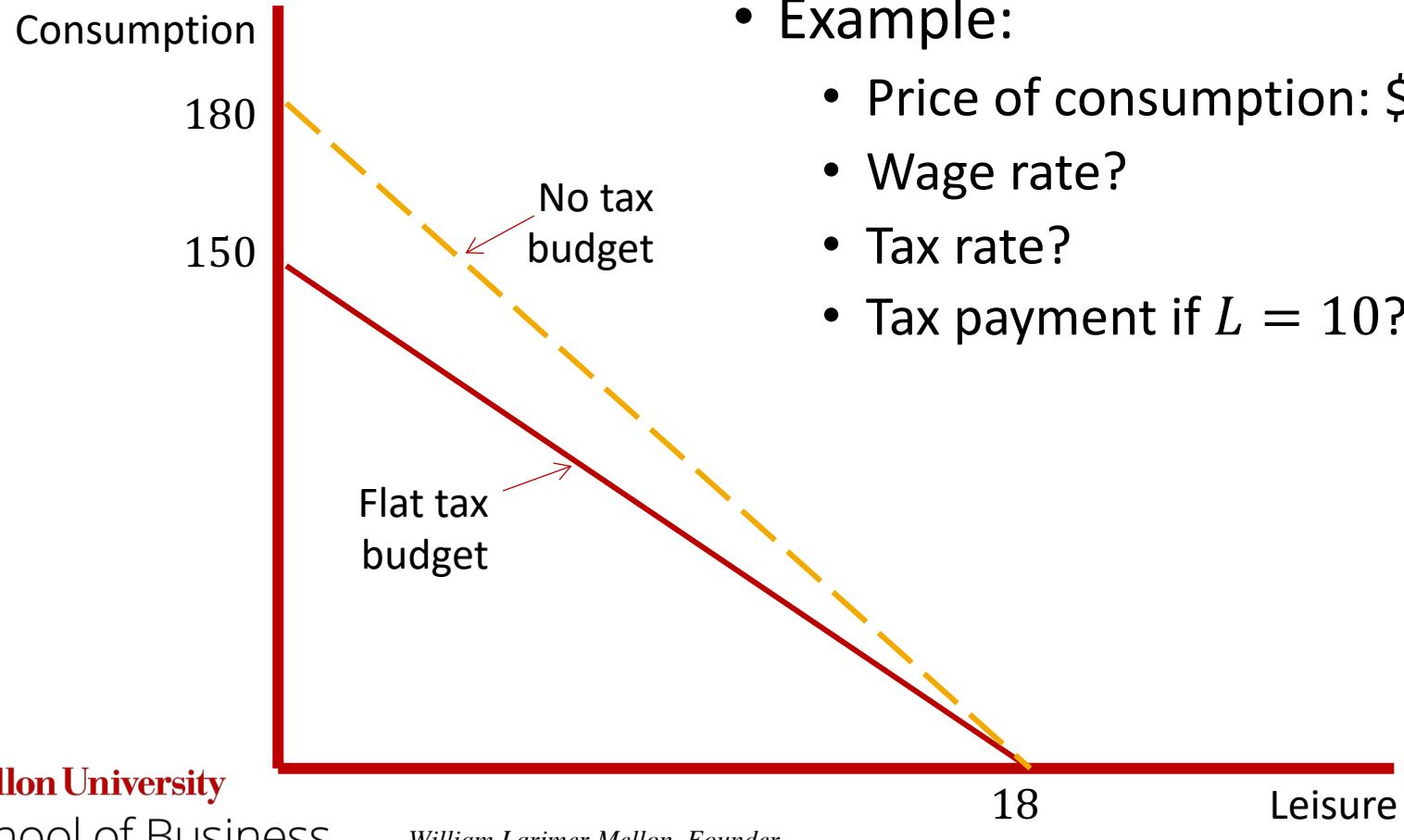
Labor Policy 2: Labor Income Tax

- Flat Tax Budget constraint:

$$pc + w(1 - t)L \leq w(1 - t)T$$



Labor Policy 2: Labor Income Tax



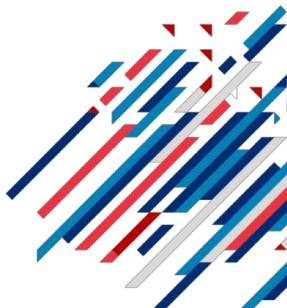
Labor Policy 2: Labor Income Tax

- Progressive Tax

- No tax for those who make less than $M = \frac{wT}{3}$
- Tax rate for those who make more than M is t'
 - If a consumer works less than $\frac{T}{3}$, she will not be taxed.
- Budget constraint:

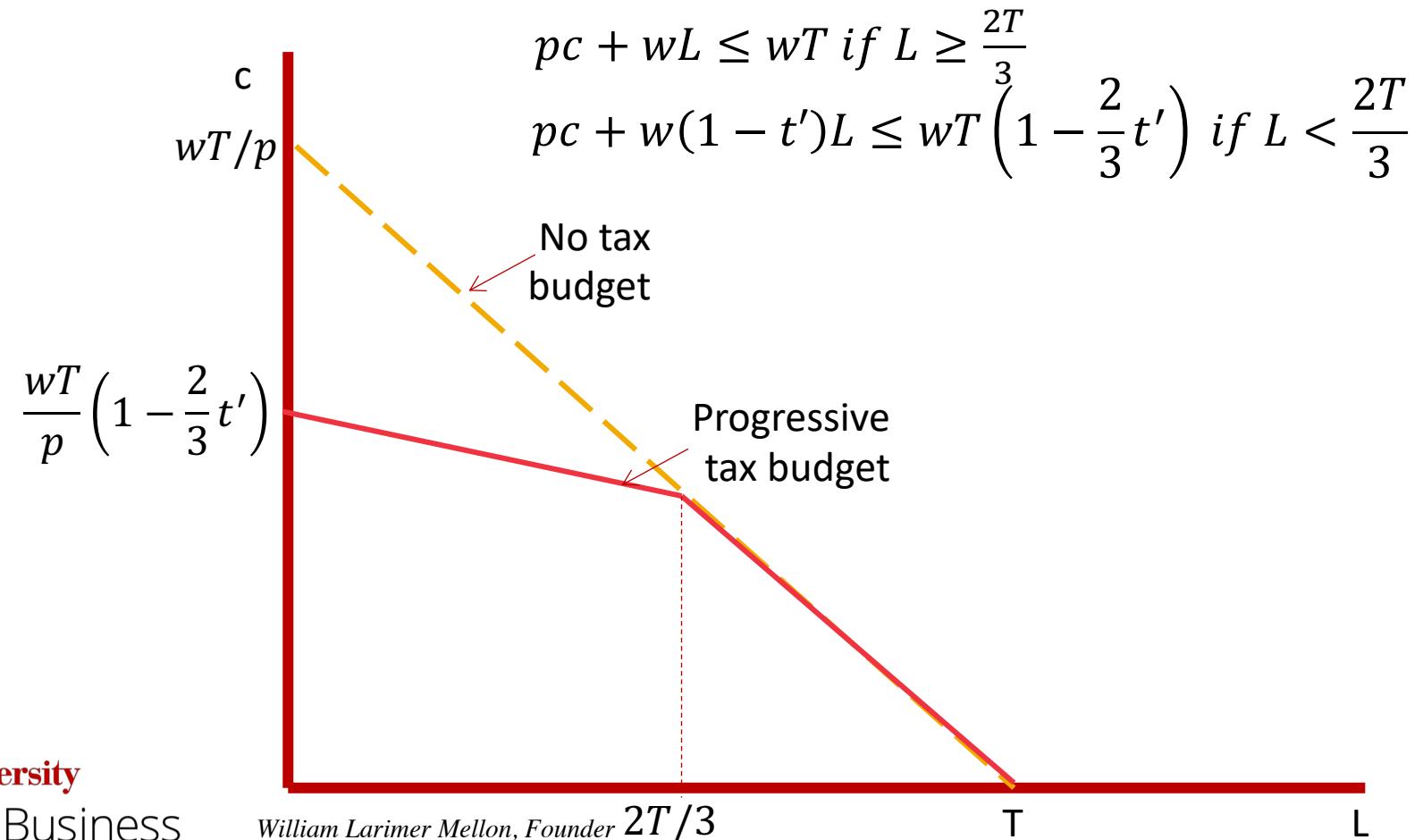
$$pc + wL \leq wT \text{ if } L \geq \frac{2T}{3}$$

$$pc + w(1 - t')L \leq wT \left(1 - \frac{2}{3}t'\right) \text{ if } L < \frac{2T}{3}$$

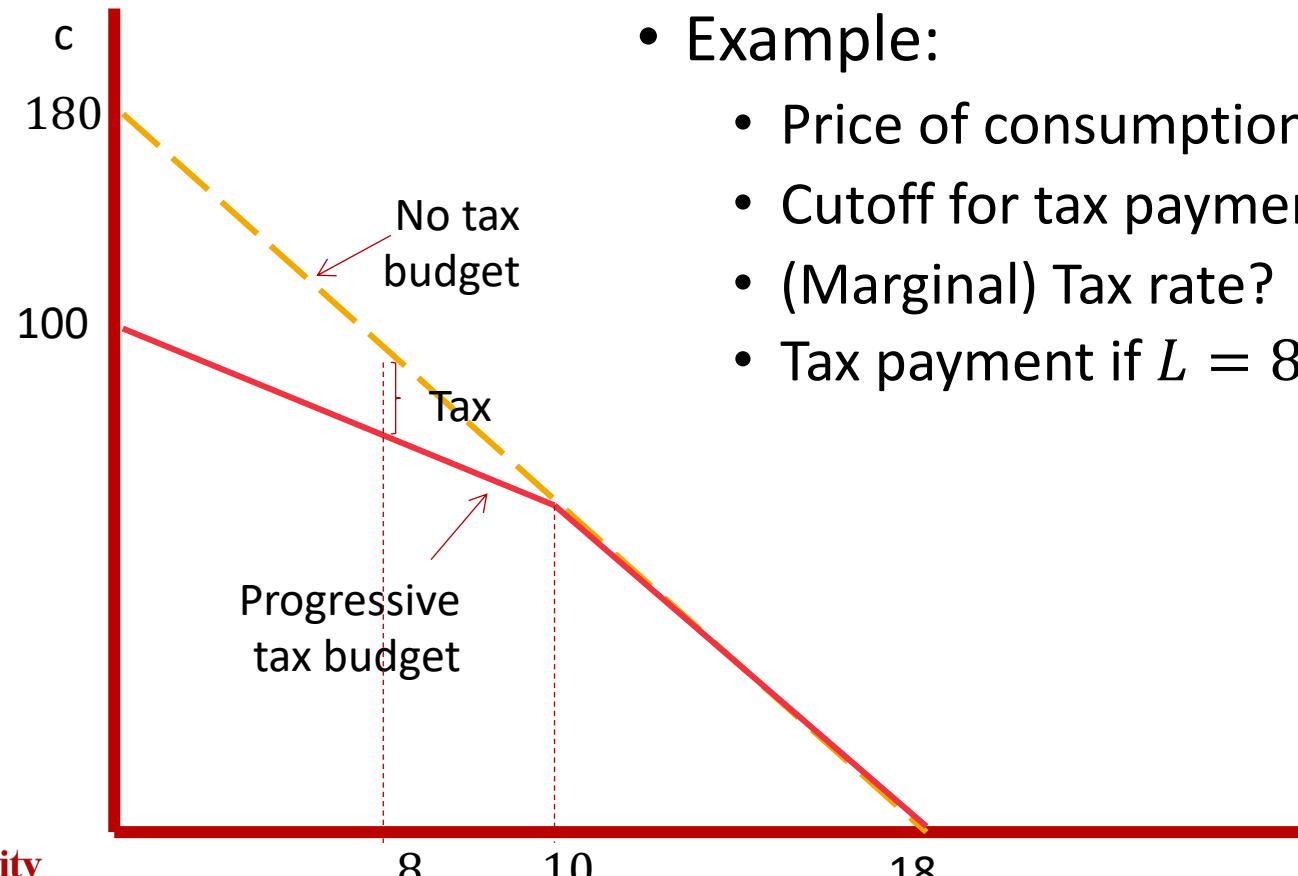


Labor Policy 2: Labor Income Tax

- Progressive Tax Budget constraint:

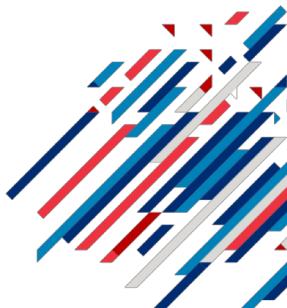


Labor Policy 2: Labor Income Tax



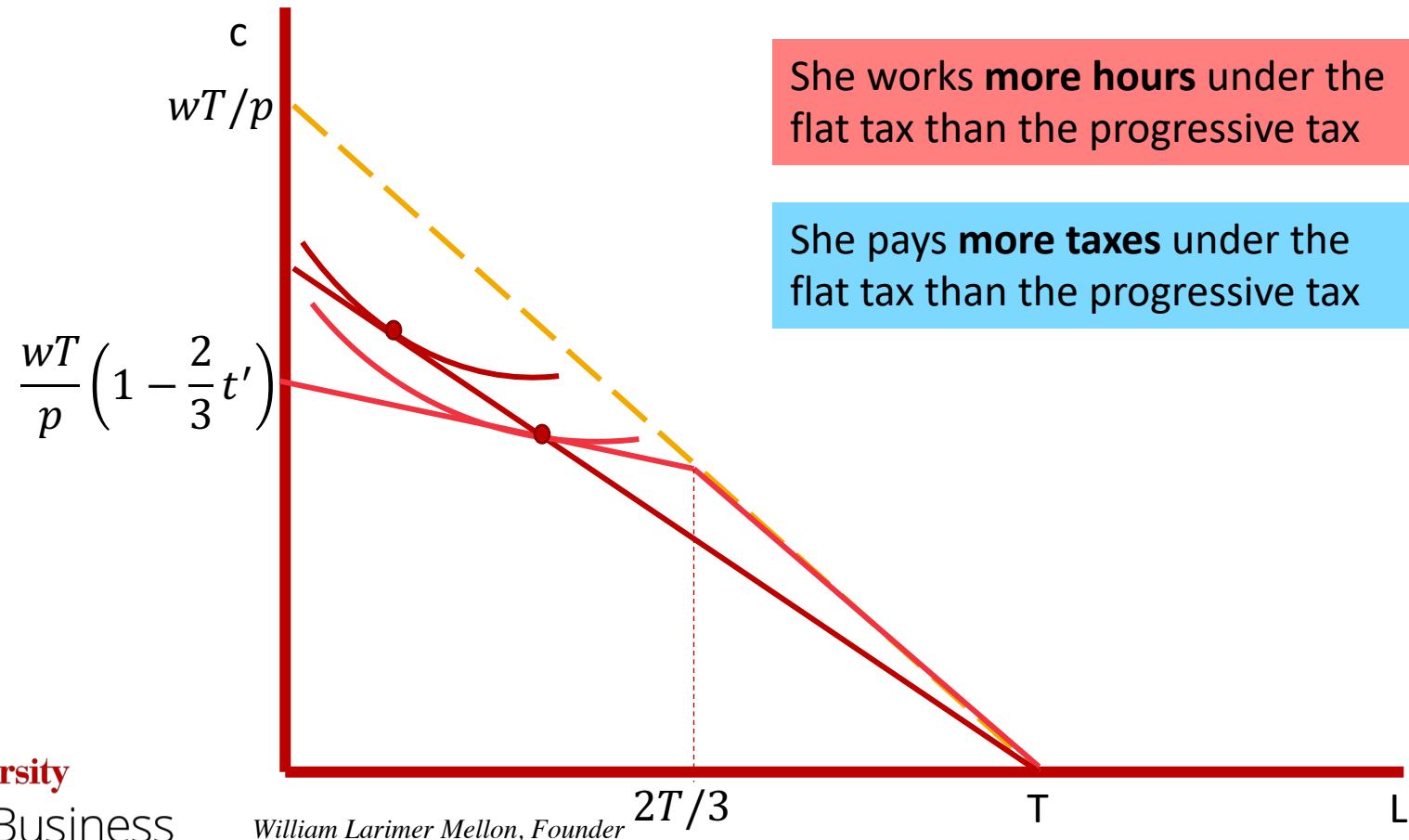
- Example:

- Price of consumption: \$1
- Cutoff for tax payment?
- (Marginal) Tax rate?
- Tax payment if $L = 8$?



Labor Policy 2: Labor Income Tax

- Evelyn prefers flat tax



Labor Policy 2: Labor Income Tax

- Elliot prefers progressive tax

