Systematic risk in an equally weighted portfolio

Systematic risk is defined as the risk that cannot be diversified by adding stocks in the portfolio. With n stocks, the portfolio variance is

$$\sigma_p^2 = \sum_{1 \le i \le n} \frac{\sigma_i^2}{n^2} + \sum_{1 \le i \le n} \sum_{1 \le j \le n, i \ne j} \frac{\sigma_i \sigma_j \rho_{ij}}{n^2}$$

Denote the average variance by

$$\bar{\sigma} = \frac{1}{n} \sum_{1 \le i \le n} \sigma_i^2$$

And the average covariance by

$$\overline{Cov} = \frac{1}{n(n-1)} \sum_{1 \le i \le n} \sum_{1 \le j \le n, i \ne j} \sigma_i \sigma_j \rho_{ij}$$

Then the portfolio variance can be expressed as

$$\sigma_p^2 = \frac{1}{n} \bar{\sigma} + \frac{n-1}{n} \overline{Cov}$$

When $n \to \infty$, $\sigma_p^2 \to \overline{Cov}$. The quantity $\overline{Cov}$ is the systematic risk component. If we decompose the risk in smaller portfolios into a systematic and an idiosyncratic part, the systematic part will not be diversified away while the idiosyncratic part will go to zero as the number of (not perfectly correlated) stocks increases.