MACROECONOMICS
73-240

Lecture 8

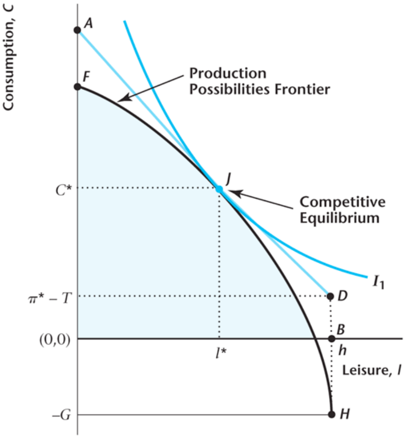
Shu Lin Wee

This version: September 25, 2019

Competitive Equilibrium Pt 2

Recap: Competitive equilibrium

A competitive equilibrium is achieved when the HH indifference curve is tangent to the budget line AND also tangent to the production possibility frontier!



Recap: Competitive Equilibrium

So eqm is achieved when we have:

- Tangency of HH indifference curve to PPF.

$$MRT_{l,C} = MP_N = w = MRS_{l,C}$$

- And consumption-leisure choice is feasible

$$C^* = zF(K, h - l^*) - G$$

- Also, the consumption allocation is affordable

$$C^* = w(h - l^*) + \pi - T$$

Recap: Mathematical Approach Explained

- Endogenous Objects to find: C, N^s, N^d, T, Y, w (6 objects!)

- Equilibrium Conditions:
  - HH optimality: 2 Conditions ($MRS_{l,c} = w$ and the Budget Constraint)
  - Firm optimality: 1 Condition ($MPN = w$)
  - Gov't Budget Constraint: 1 Condition ($G = T$)
  - Market Clearing: 2 conditions ($N^d = N^s$ and $C + G = Y$)

- Note: We must solve for 6 objects and we have 6 equilibrium conditions!

- Note π is endogenous, but we know $\pi = Y - wN^d$.

Recap: Roadmap

Hence to solve, we

1) Solve the HH's problem: solve for C, N^s in terms of π, w, T, h.

2) Solve the firm's problem: solve for N^d as a function of z, K, w

3) Use the fact that $G = T$ and substitute T for G

4) Labor market clears: $N^d = N^s = N^*$ at w^*. Solve for w^* by equating $N^s = N^d$.

5) Knowing $w^* \rightarrow$ know N^* from $N^d \rightarrow$ know π^* and Y^*

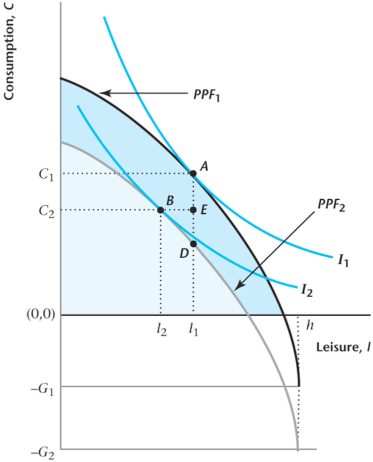
6) Knowing $Y^* \rightarrow$ know C^* from goods market clearing:

$$C = Y - G$$

Example: Increase in Government Spending

Government spending

Suppose government decides to spend more: $\Delta G > 0$



Government Spending

Suppose the government increases its expenditure : $\Delta G = G_2 - G_1 > 0$

1. Balanced budget if $G_2 > G_1$ then $T_2 > T_1$;
2. Reduces household's disposable income $C_2 < C_1$ and $l_2 < l_1$;
3. Increase in equilibrium hours worked: $N_2 > N_1$ implies $Y_2 > Y_1$.

Question 1: ΔC vs. ΔG?

Question 2: what has happened to the real wage?

Question 3: does GDP increase?

Question 4: does the household prefer the increase in G?

Government spending

Summarizing:

1) $G_1 \Rightarrow G_2$ with $G_2 > G_1$
2) $T_1 \Rightarrow T_2$ with $T_2 > T_1$
3) Negative income effect! Agent is poorer $C_2 < C_1$ and $l_2 < l_1$
4) $N_2 > N_1$ and $Y_2 > Y_1$
5) So output is higher because individuals decide to spend more time in labor.
6) Need to check if in equilibrium: C is lower.

If we measure welfare in terms of the utility of the household, is the household happier?

A model with a simpler production function

- Technology: $Y = zN^d$
  In This Example production is LINEAR in N

- Preferences: $U(C, l) = \log(C) + \log(l)$
  (with $h = 1$ so that $l = 1 - N^s$)

- Marginal Product of labor: $MPN = z$

- Marginal Rate of Substitution: $MRS_{l,C} = \frac{C}{1-N^s}$

- Recall, for Comp. Eq. we solve for C, l, N^d, Y, w (and $T = G$)

Firm's Problem

- Firm's optimal decision:

$$\max_{N^d} zN^d - wN^d$$

- Suppose $w < z$, then firm would choose $N^d = \infty$

- Suppose $w > z$, then firm would choose $N^d = 0$
  Neither can be consistent with equilibrium!

- Only candidate equilibrium wage: $w = z$
  Useful Result: If production is linear and there are no taxes on firms, then in any equilibrium, $w = z$.

- If $w = z$, firm happy to choose any N^d. How do we determine it? What are profits?

Household's Problem

- Household's decision is standard:

$$\max_{C,l} U(C,l) = \log c + \log l$$

s.t.

$$C = w(h-l) + \pi - T$$

where $h = 1$ and $N^s = h - l$

- Derive FOC and solve for N^s

$$w = \frac{C}{1-N^s} \text{ and } C = wN^s - T$$

- Solving for labor supply:

$$N^s(w) = \frac{1}{2} + \frac{T}{2w}$$

and Consumption:

$$C = w(1-N^s)$$

Carnegie Mellon Tepper SCHOOL OF BUSINESS

Household's Problem

- In equilibrium, $w = z$ and $G = T$, we thus have

$$N^s = \frac{1}{2} + \frac{G}{2z} \text{ and } C = z(1 - N^s) = \frac{z}{2} - \frac{G}{2}$$

Equilibrium Outcomes

- Let's check goods market clearing:

$$Y = zN^d = zN^s = \frac{z}{2} + \frac{G}{2}$$

and

$$C + G = \frac{z}{2} - \frac{G}{2} + G \implies C = \frac{z}{2} - \frac{G}{2}$$

- Increase G_1 to $G_2 > G_1$

$$\frac{dY}{dG} = \frac{1}{2}$$

- Increasing G by \$1 increases Y by \$0.50 cents!

How Happy is the Household?

- Household utility in equilibrium. Recall:

$$C^* = wl^* \text{ and } l^* = 1 - N^{s*} = \frac{1}{2} - \frac{G}{2z}$$

$$\begin{aligned} U(C^*, l^*) &= \log(C^*) + \log(l^*) \\ &= \log(wl^*) + \log(l^*) \\ &= \log(w) + 2\log(l^*) \\ &= \log(w) + 2\log\left(\frac{1}{2} - \frac{G}{2z}\right) \end{aligned}$$

- In this example, higher G makes household worse off!

Carnegie Mellon Tepper School of Business

How Happy is the Household?

- This model had the prediction that more government spending makes the household worse off in terms of utility
- Is this always true? What assumption did we start with?

Testing the prediction of the model: ΔG

- Question: Can exogenous govt spending shocks be a driver of business cycle fluctuations?

Testing the prediction of the model: ΔG

For example 1: using data we can

- Look at the relation between ΔG and ΔC (we expect a negative correlation)
- Look at the relation between ΔG and ΔN (we expect a positive correlation)

For G and C we use NIPA table 1.1.6:
http://www.bea.gov/

For N we use FRED:
http://research.stlouisfed.org/fred2/series/CE16OV?cid=12

Model Predictions and Data

Relation between government spending and fluctuations:

- Model: Increase in G implies

$$Y\uparrow, N\uparrow, C\downarrow$$

- With concave production function, $N\uparrow \implies w\downarrow$

- Data (in the Short-Run):
  - Employment is pro-cyclical (similar to model)
  - Consumption and real wage are procyclical (different from model)

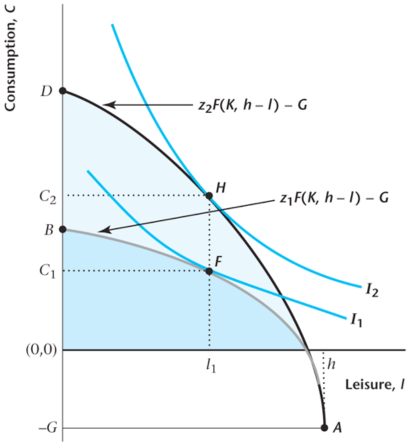
- Question: Is the change in G in the data always exogenous?

Carnegie Mellon Tepper SCHOOL OF BUSINESS

Example: Changes in TFP

Changes in TFP

Suppose there is an increase in TFP: $\Delta z > 0$



Changes in TFP

Summarizing:

1. $z_1 \Rightarrow z_2$ with $z_2 > z_1$
2. Wage increases $w_2 > w_1$
3. Consumption increases $C_1 \Rightarrow C_2$
4. Hours worked? depends on Income and Substitution effects

An example with $G = 0$

- Technology: $Y = zK^{\alpha} N^{d,1-\alpha}$
- Preferences: $U(C, l) = \log(C) + \log(l)$
  (with $h = 1$ so that $l = 1 - N^s$)
- Marginal Product of labor: $MPN = (1-\alpha) z \left(\frac{K}{N^d}\right)^{\alpha}$
- Marginal Rate of Substitution: $MRS_{l,C} = \frac{C}{1-N^s}$
- Recall, for Comp. Eq. we solve for C, l, N^d, Y, w (and $T = G$)

Characterizing a competitive equilibrium

- Labor market clears, $N^s = N^d$ at equilibrium wage w^*

$$MRS_{l,C} = \frac{C}{1-N} = w^* = (1-\alpha)z\left(\frac{K}{N}\right)^{\alpha} = MPN$$

- Desired allocations must be feasible

$$C = zK^{\alpha}N^{1-\alpha}$$

Solve for N^*:

$$N^* = \frac{1-\alpha}{2-\alpha}$$

and C^*

$$C^* = zK^{\alpha}\left(\frac{1-\alpha}{2-\alpha}\right)^{1-\alpha}$$

- In this example, what happened to income and substitution effects?

Carnegie Mellon Tepper School of Business

Testing the prediction of the model: Δz

- Can productivity shocks be a driver of business cycle fluctuations?

Testing the prediction of the model: Δz

For example 2: using data we can

- Look at the relation between Δz and ΔC
  (we expect a positive correlation)
- Look at the relation between Δz and Δw
  (we expect a positive correlation)

For C we use NIPA table 1.1.6:
http://www.bea.gov/

For w we use FRED:
https://research.stlouisfed.org/fred2/series/CES0500000003

Carnegie Mellon Tepper School of Business

Model Predictions and Data

Relation between producitivity and fluctuations:

- Model: Increase in z implies

$$Y \uparrow, N \text{ ambiguous}, C \uparrow, w \uparrow$$

- Data:
  - Long-Run:
    - Output, consumption, real wages have risen, hours about the same (consistent if income and subst. effects cancel over the long-run)
  - Short-Run:
    - Consumption and real wages are pro-cyclical (consistent)
    - Employment is pro-cyclical (consistent if substitution effect dominates in the short run)

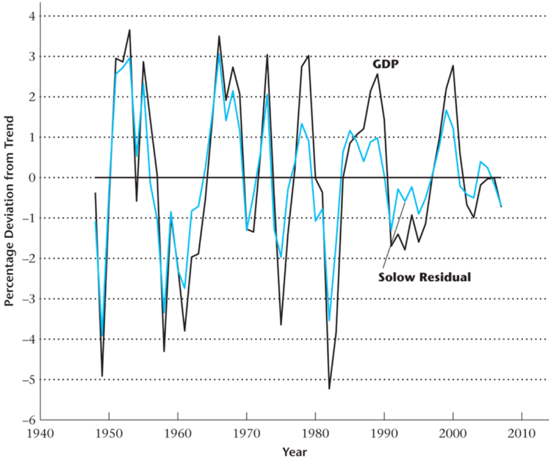
Measuring Short Run fluctuations in z

- Exactly as in Development accounting! Called the Solow Residual.
- In equilibrium, $Y = zK^{.36}N^{.64}$
- Measure z_t using

$$\ln z_t = lnY_t - 0.36 \ln K_t - 0.64 \ln N_t$$

- Data:
    - $Y_t \equiv$ Real GDP
    - $K_t \equiv$ sum of undepreciated capital (add up capital expenditures over time from NIPA)
    - $N_t \equiv$ total employment from Bureau of Labor Statistics

Solow Residual in the U.S.



- Primary motivation for business cycle theory

CONGRATULATIONS!

You just studied a model of the free-market economy!

For what good is a model of the economy?

- How is the model useful?
  - We can use the model to ask questions about how key aggregate variables: Y,C,N and w would change:
    - Forecasting: if $z \downarrow$ by 10%, what happens to Y,C,N, w?
    - Policy-making: if country A were to receive foreign aid via an injection of K, how would Y,C,N, w change?
    - Welfare considerations: is the economy performing the 'best' that it can? (to be discussed next!)

Carnegie Mellon Tepper SCHOOL OF BUSINESS

Roadmap

- Next Class... $\Rightarrow$ Pareto Optimality