INTERMEDIATE MICROECONOMICS

CONSUMER CHOICE: APPLICATIONS

SPRING 2019, PROFESSOR ANH NGUYEN

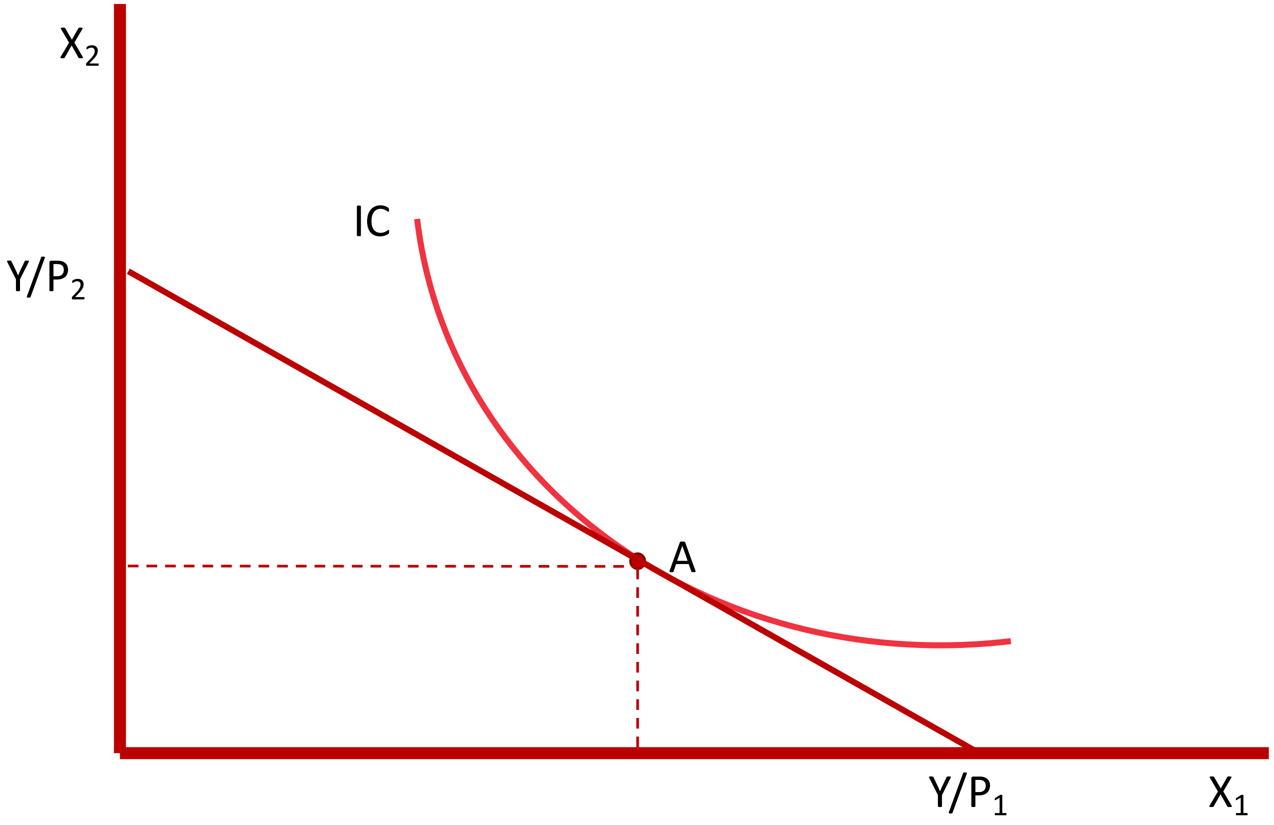
1. Cost of Living Adjustments

Cost of Living Adjustments

- Cost of living adjustments are used to ensure that an individual's utility stays the same when prices change.
- Examples: Pension payments adjusted for inflation

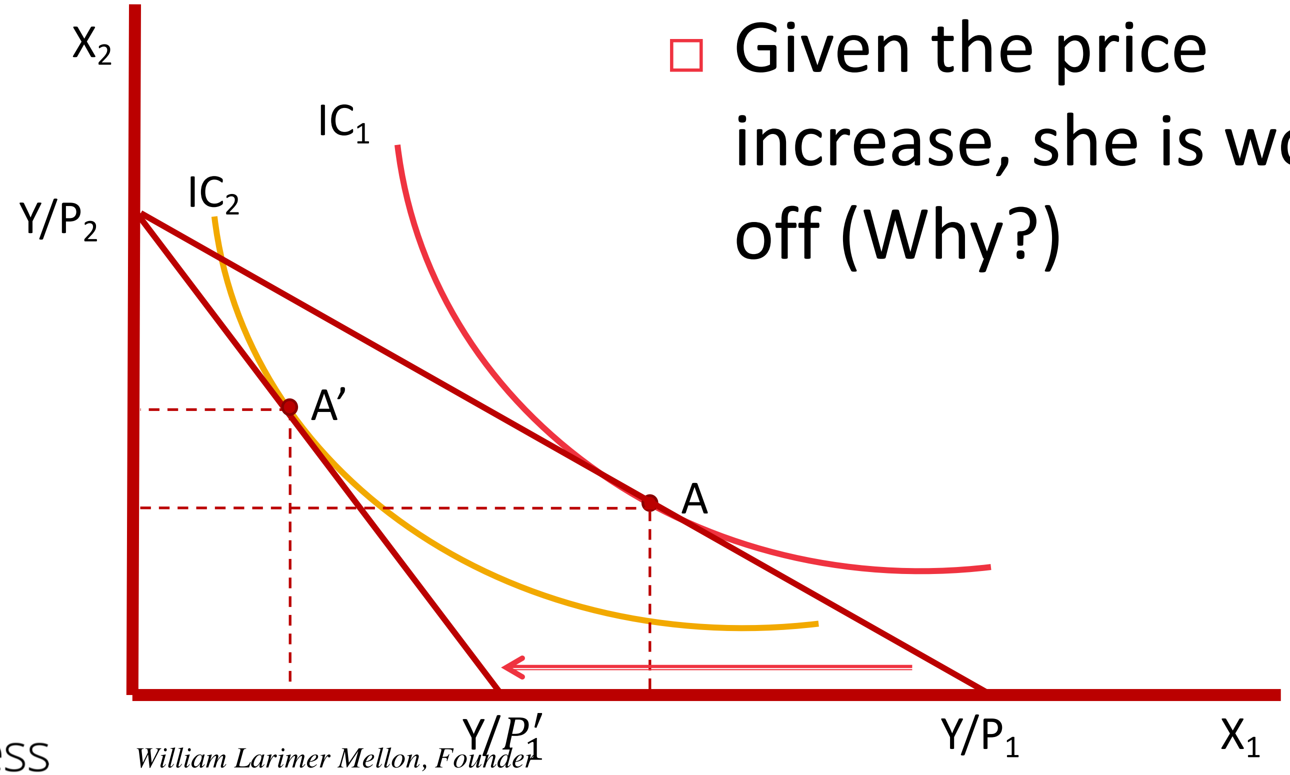
Cost of Living Adjustments

- Evelyn's optimal consumption: A



Cost of Living Adjustments

- Suppose the price of good 1 increases.
- Evelyn's new optimal consumption: A'



- Given the price increase, she is worse off (Why?)

Cost of Living Adjustments

- Question: How much should be given to Evelyn to compensate for the price change?
  - Objective: For her to get ***the same utility as before*** the price change.

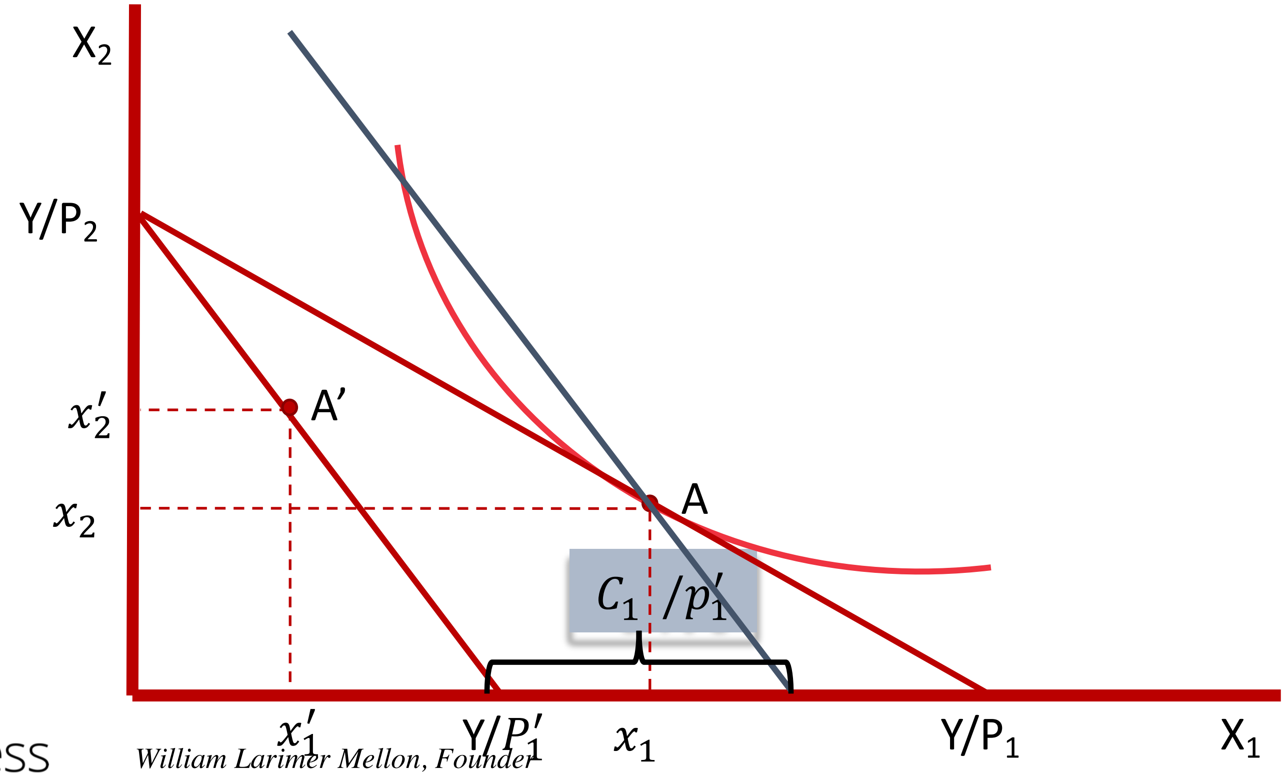
Cost of Living Adjustments

- Case 1: Compensate her so that she can buy the original consumption bundle, $A = (x_1 \ , x_2)$
  - She currently receives $Y = p_1 \ x_1 \ + p_2 x_2$
  - To buy A, she needs to have $Y' = \boldsymbol{p'_1} x_1 \ + p_2 x_2$
  - Therefore, compensation must be:

$$Y' - Y = x_1 \ (p'_1 - p_1)$$

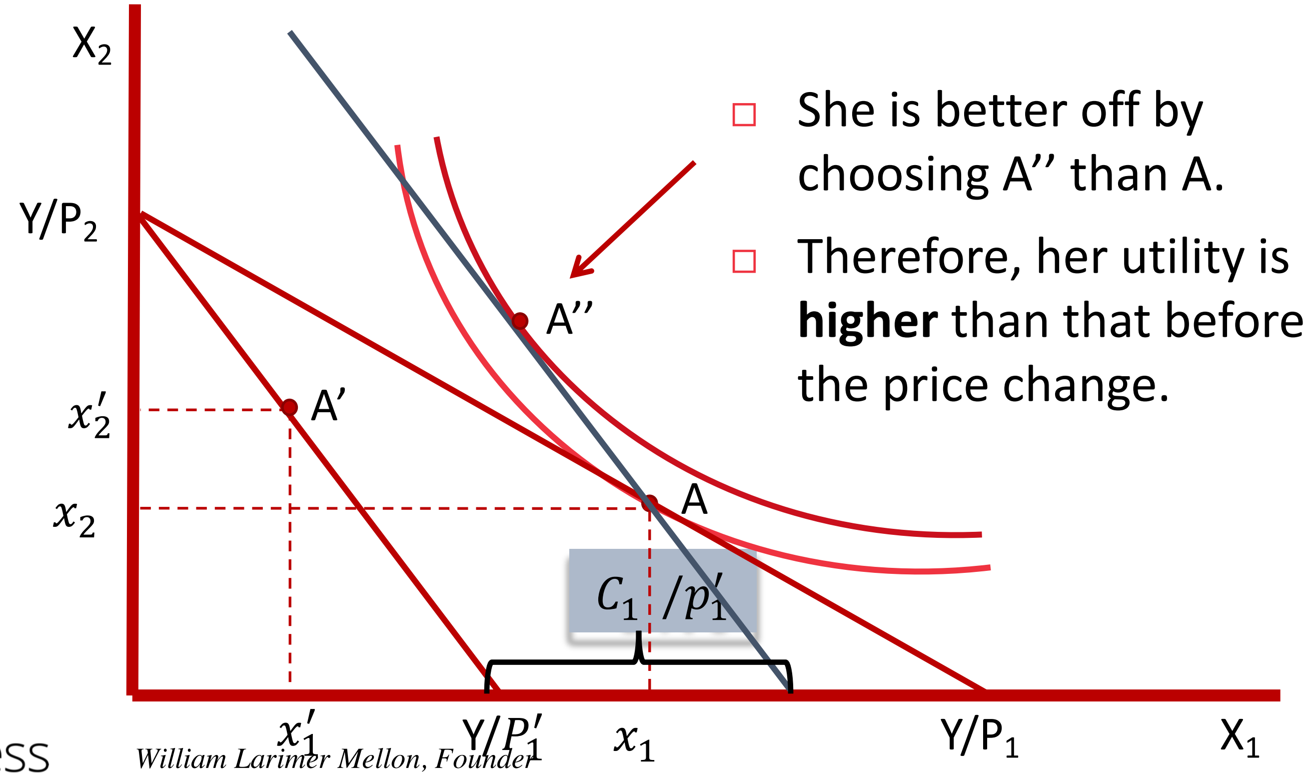
Cost of Living Adjustments

- Case 1: Compensate her so that she can buy A
  - Compensation: $C_1 = x_1\,(p_1' - p_1)$



Cost of Living Adjustments

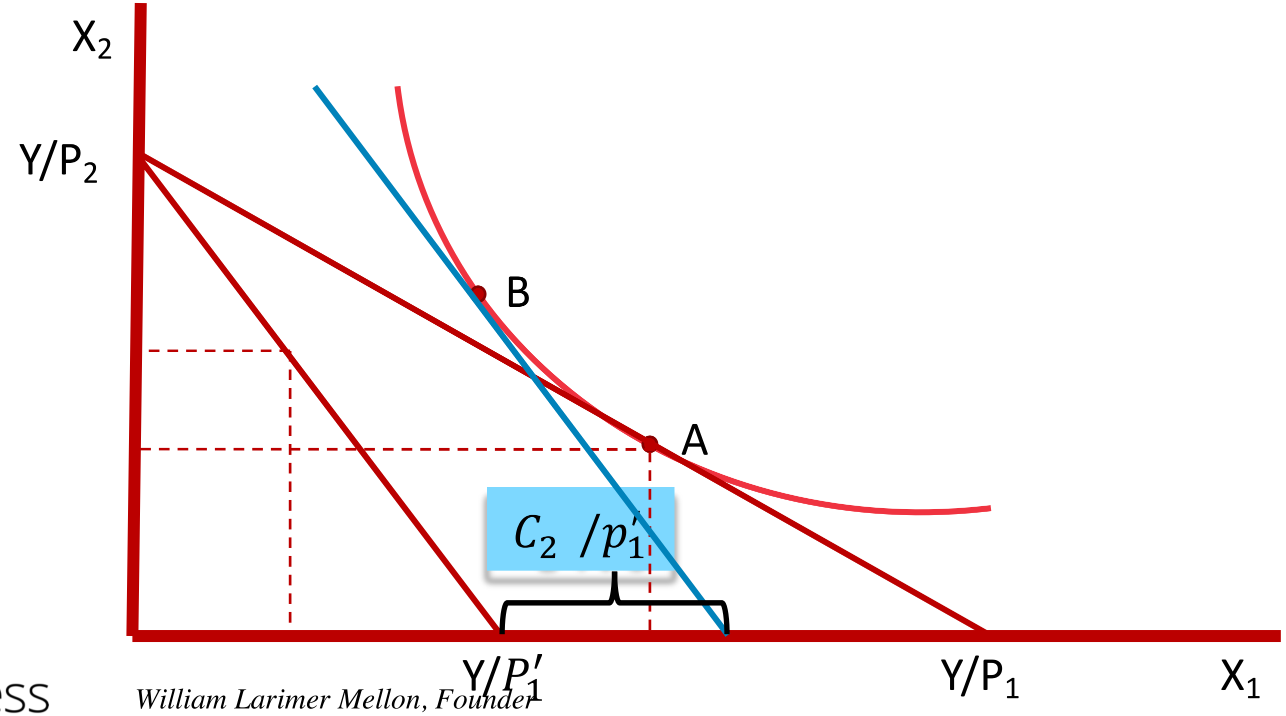
- Case 1: Compensate her so that she can buy A
  - Given the new payment C_1 , her choice won't be A



- She is better off by choosing A'' than A.
- Therefore, her utility is **higher** than that before the price change.

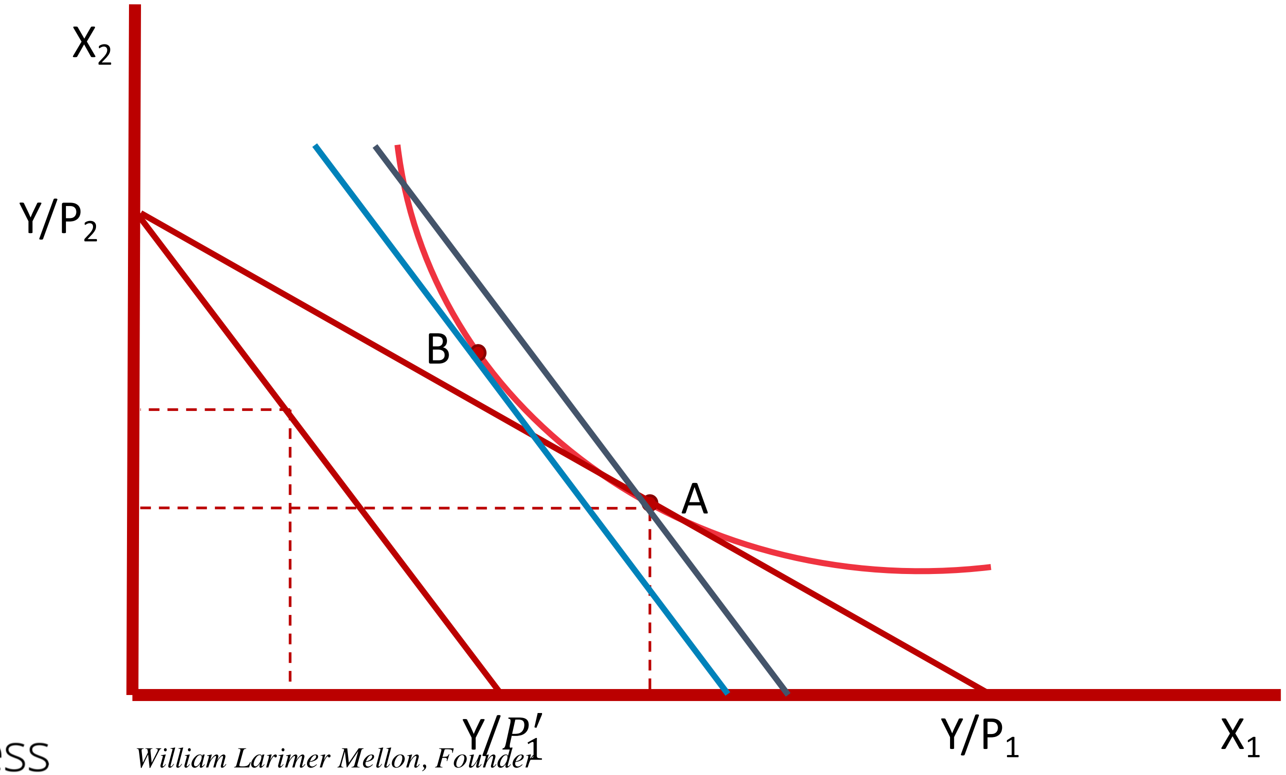
Cost of Living Adjustments

- Case 2: Compensate her so that she can buy B, where she is on the original IC at the new prices
  - Compensation: C_2



Cost of Living Adjustments

- C_2 is cheaper than C_1. Why?



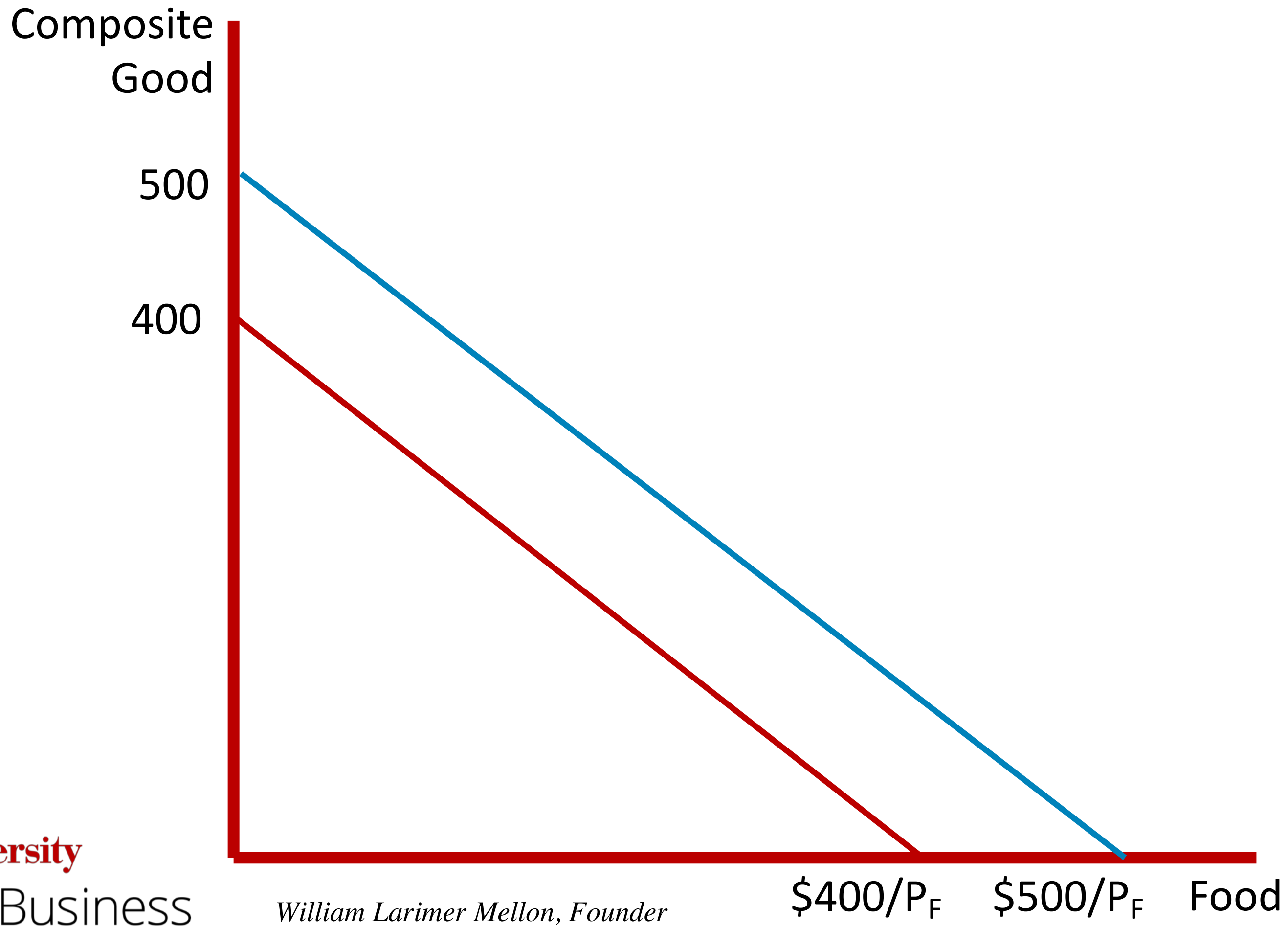
2. Cash vs. Food Stamps

Cash vs. Food Stamps

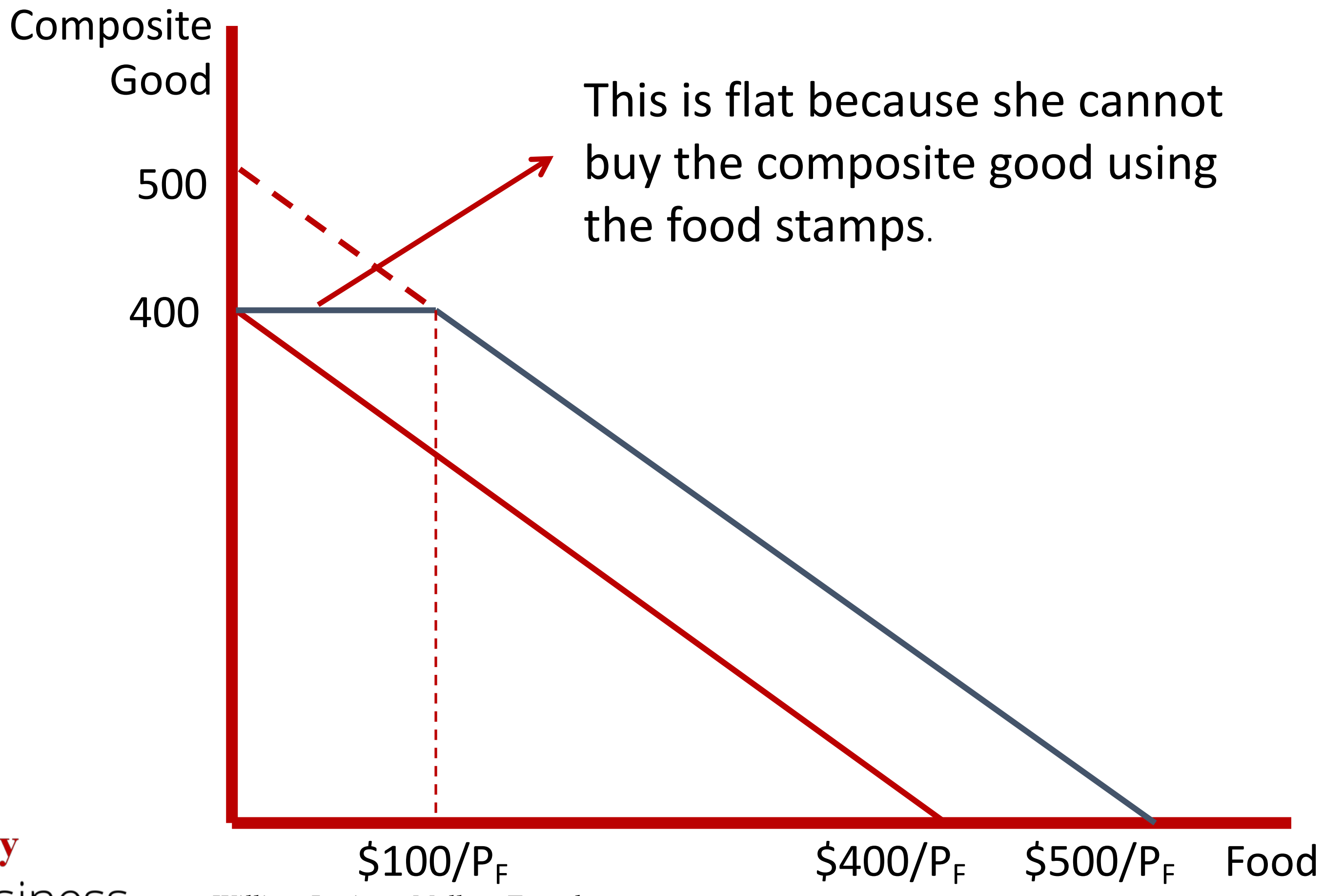
- Evelyn has income $400
- Two goods: a composite good and food
  - Price of a composite good: $1
  - Price of a unit of food: $\$p_F$
- She is eligible for $100 in food stamps

- Question: Does she *always* prefer $100 in cash instead of food stamps?

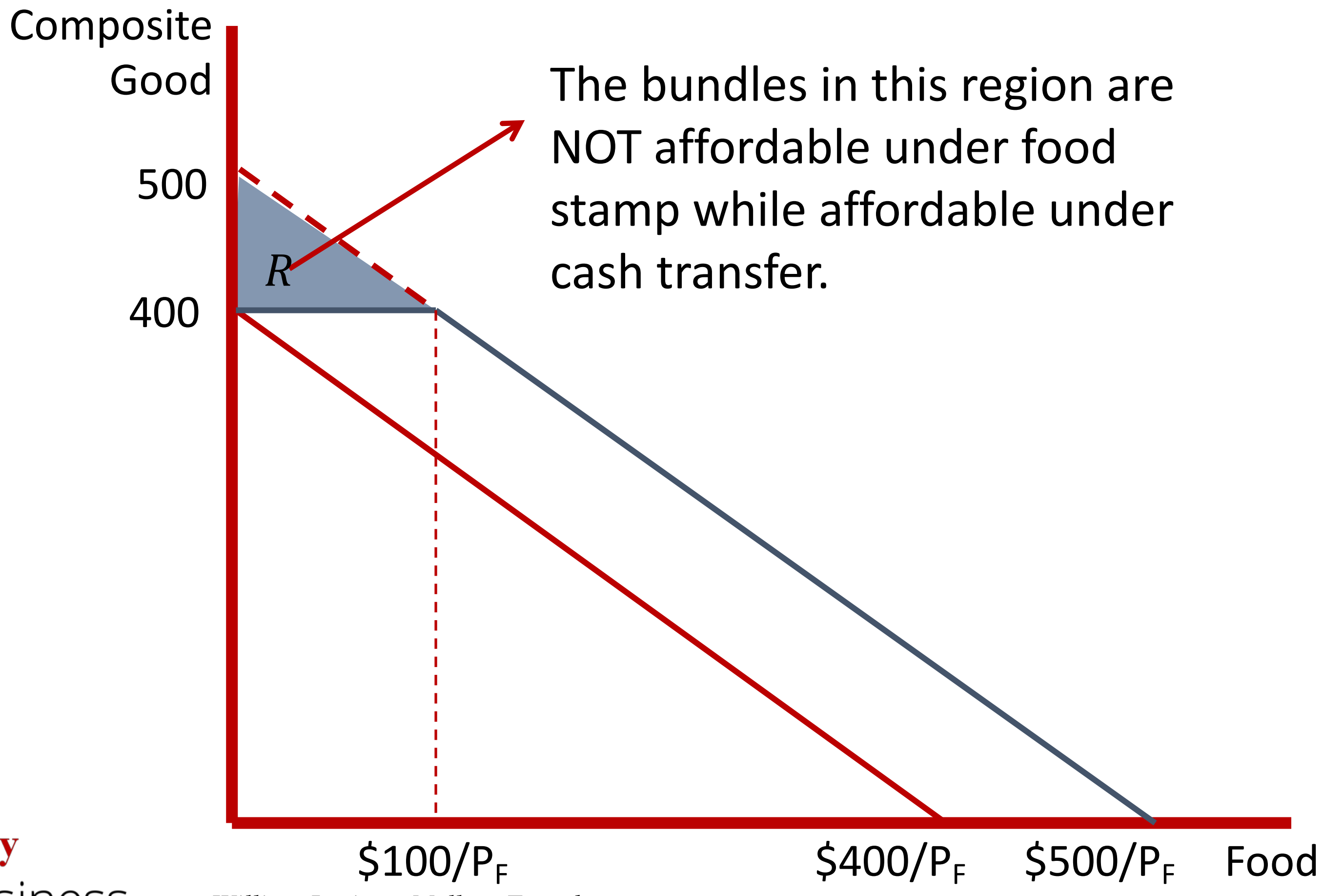
Budget Constraint with Cash Subsidy



Budget Constraint with Food Stamps



Budget Constraint with Food Stamps

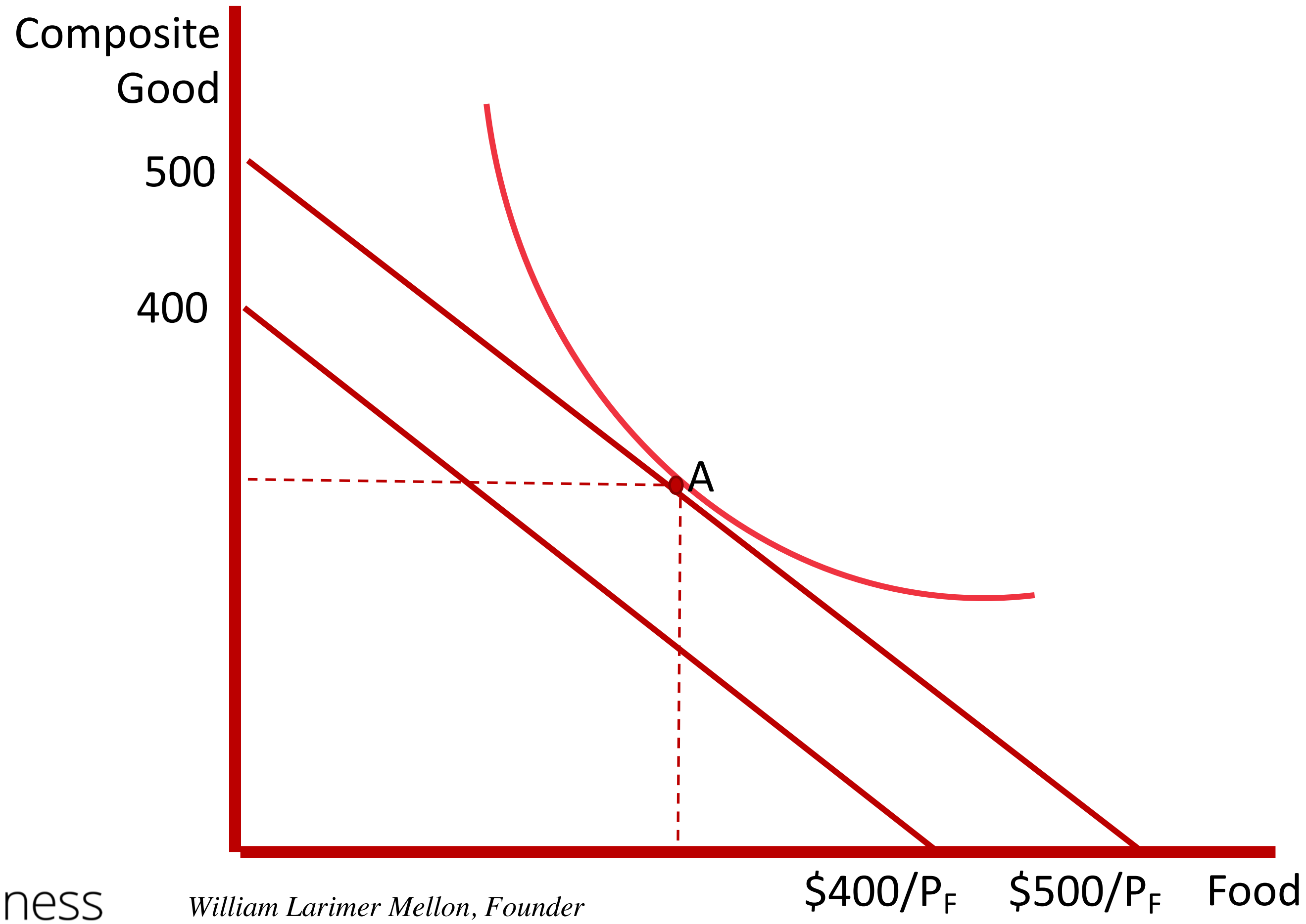


The bundles in this region are NOT affordable under food stamp while affordable under cash transfer.

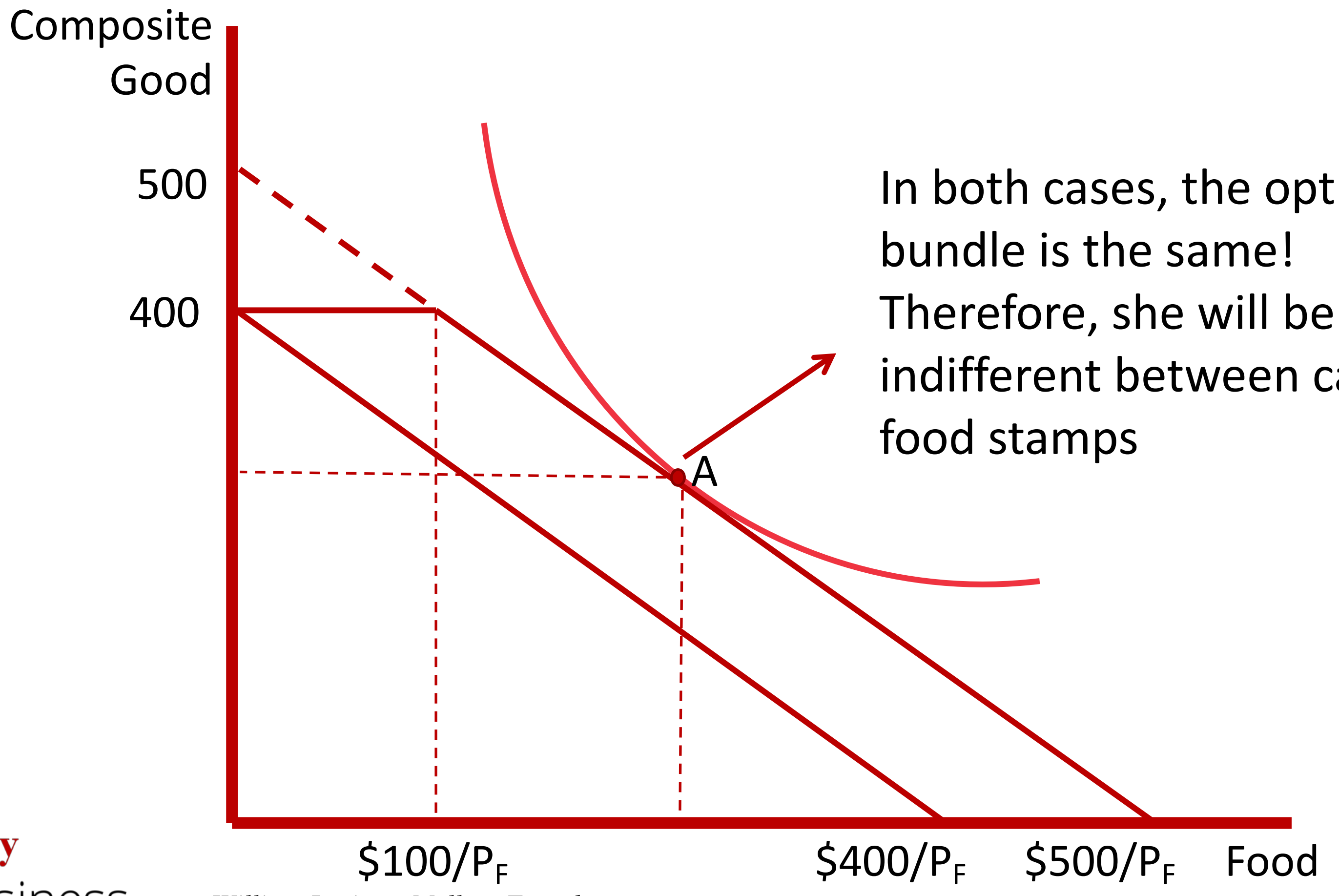
Cash vs. Food Stamps

- **Question**: Does she *always* prefer $100 in cash instead of food stamps?
- **Answer**: She prefers cash to food stamps ***only if*** her optimal bundle under cash is in region R. Otherwise, she is ***indifferent*** between cash and food stamps.

Indifferent btw Cash and Food Stamps

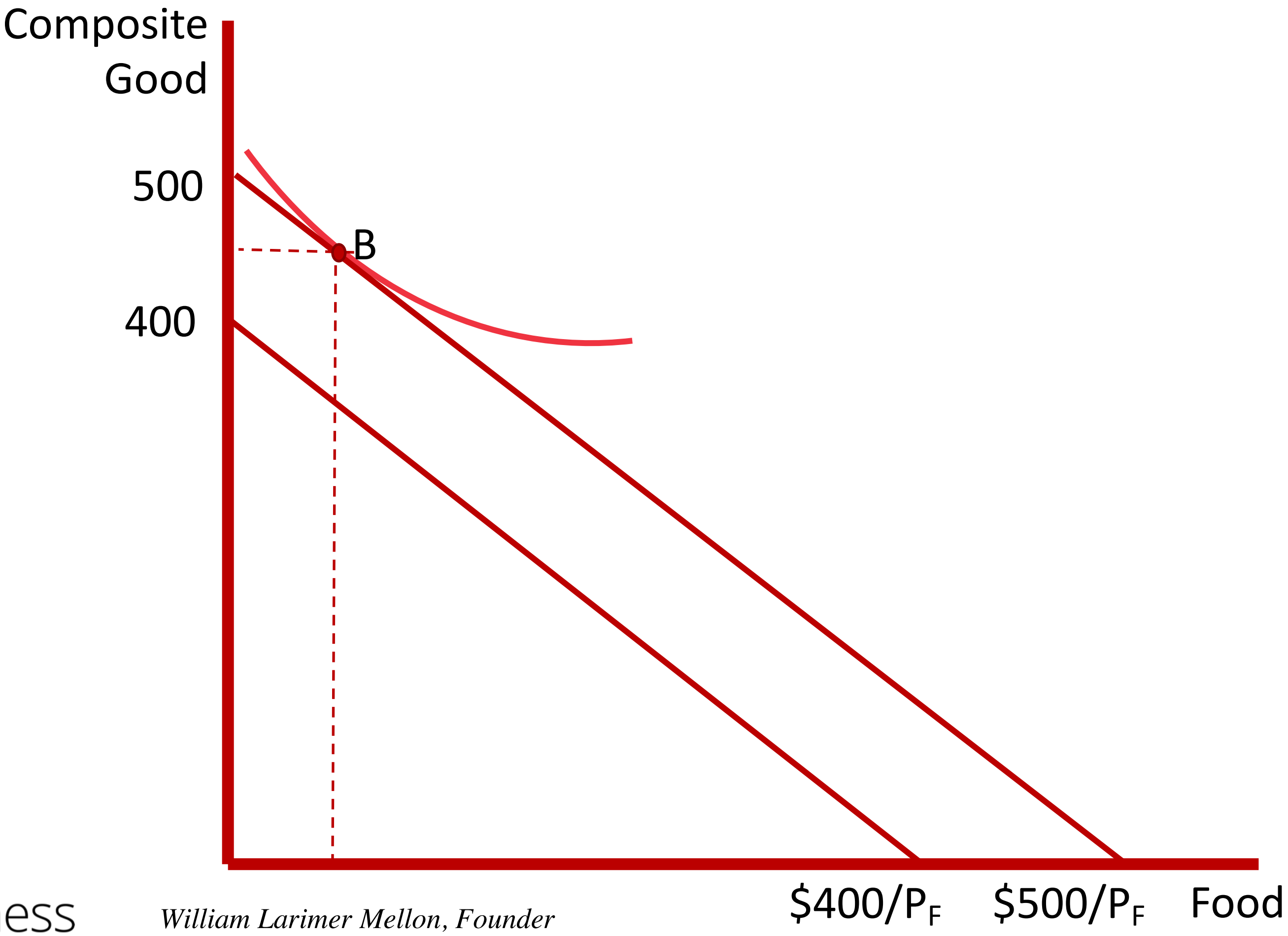


Indifferent btw Cash and Food Stamps



In both cases, the optimal bundle is the same! Therefore, she will be indifferent between cash and food stamps

Cash is Preferred to Food Stamps



Cash is Preferred to Food Stamps

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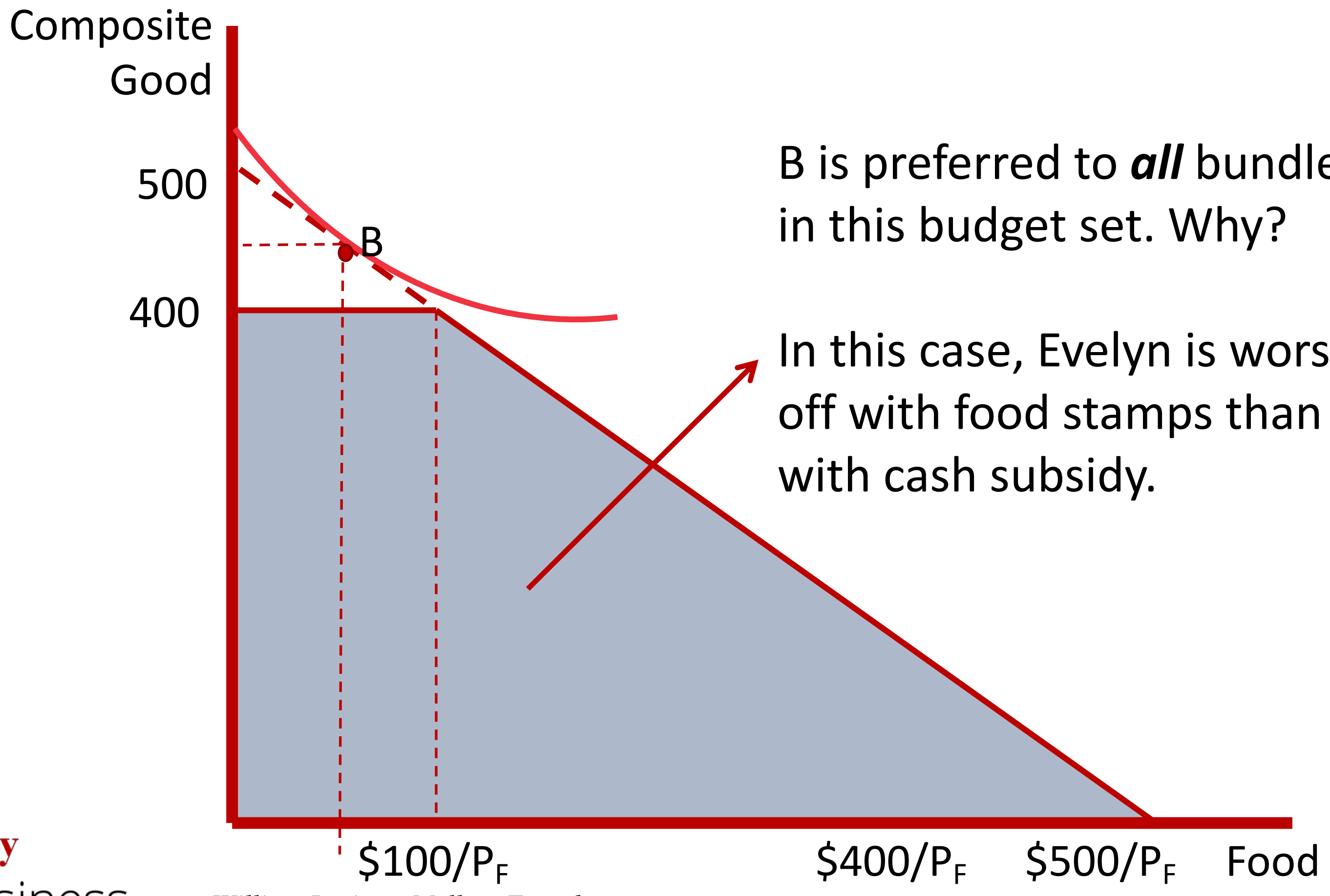


B is preferred to ***all*** bundles in this budget set. Why?

In this case, Evelyn is worse off with food stamps than with cash subsidy.

3. Quantity Discount

Quantity Discount

- A pizza chain recently offered the following special promotion:

  "Buy one pizza at full price and get your next

  three pizzas for just $5 each!"

  - The full price of a pizza: $10
  - The price of all other goods: $1 per unit
  - Evelyn's daily income: $40

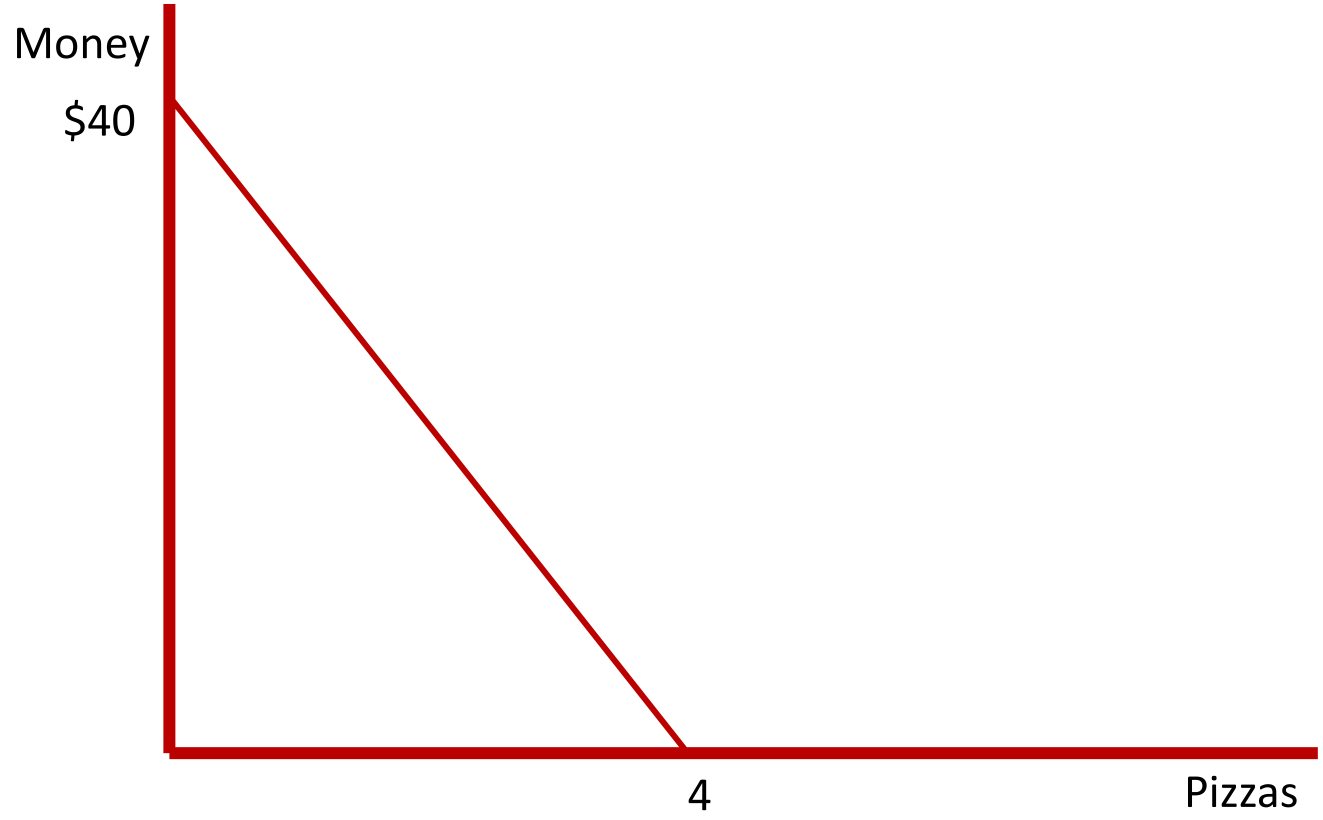
Quantity Discount

- Draw budget constraints for pizza and all other goods before and during the special promotion

- Discuss the following statement:
  - “All consumers will buy more pizzas during the promotion than before/after the promotion.”

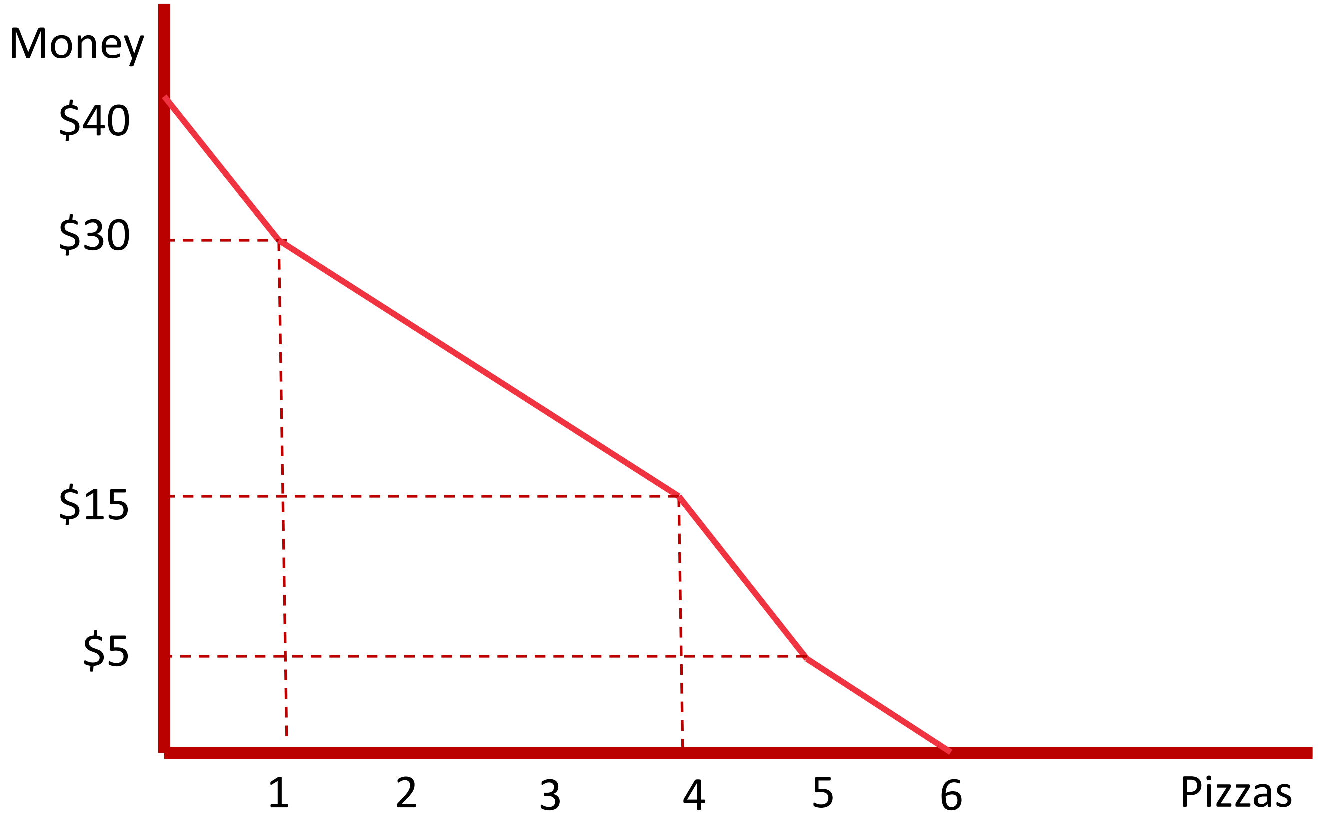
Budget Constraints

- Without the Promotion



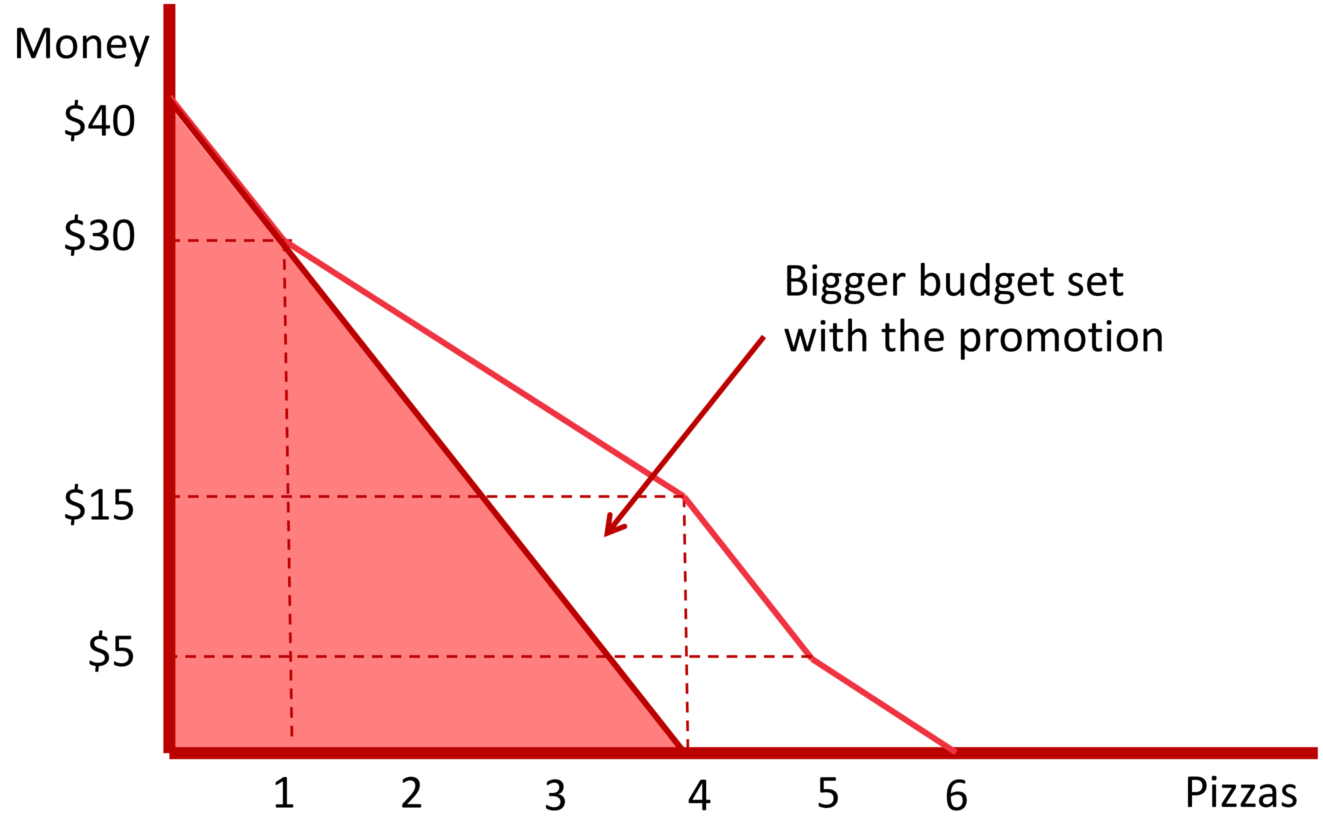
Budget Constraints

- With the Promotion



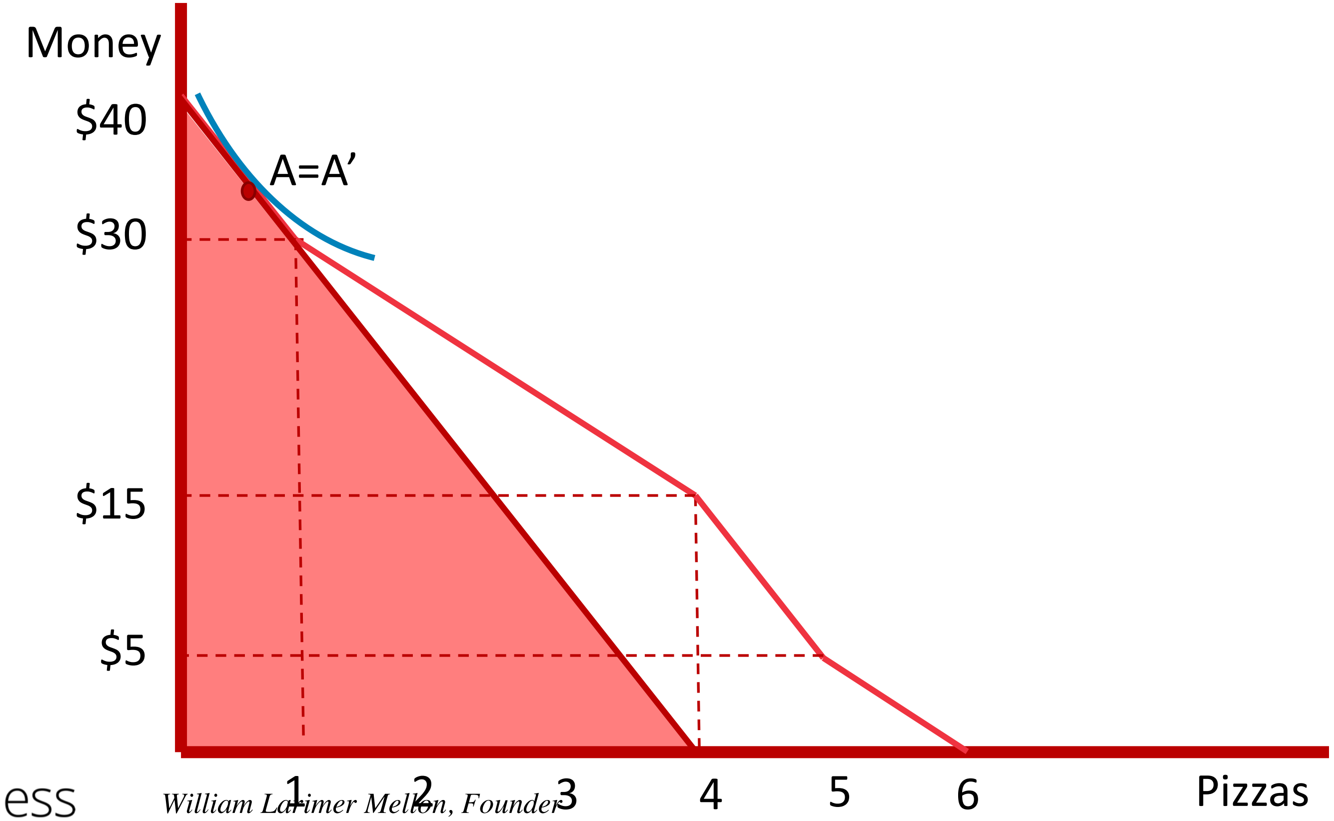
Budget Constraints

- With and without the Promotion



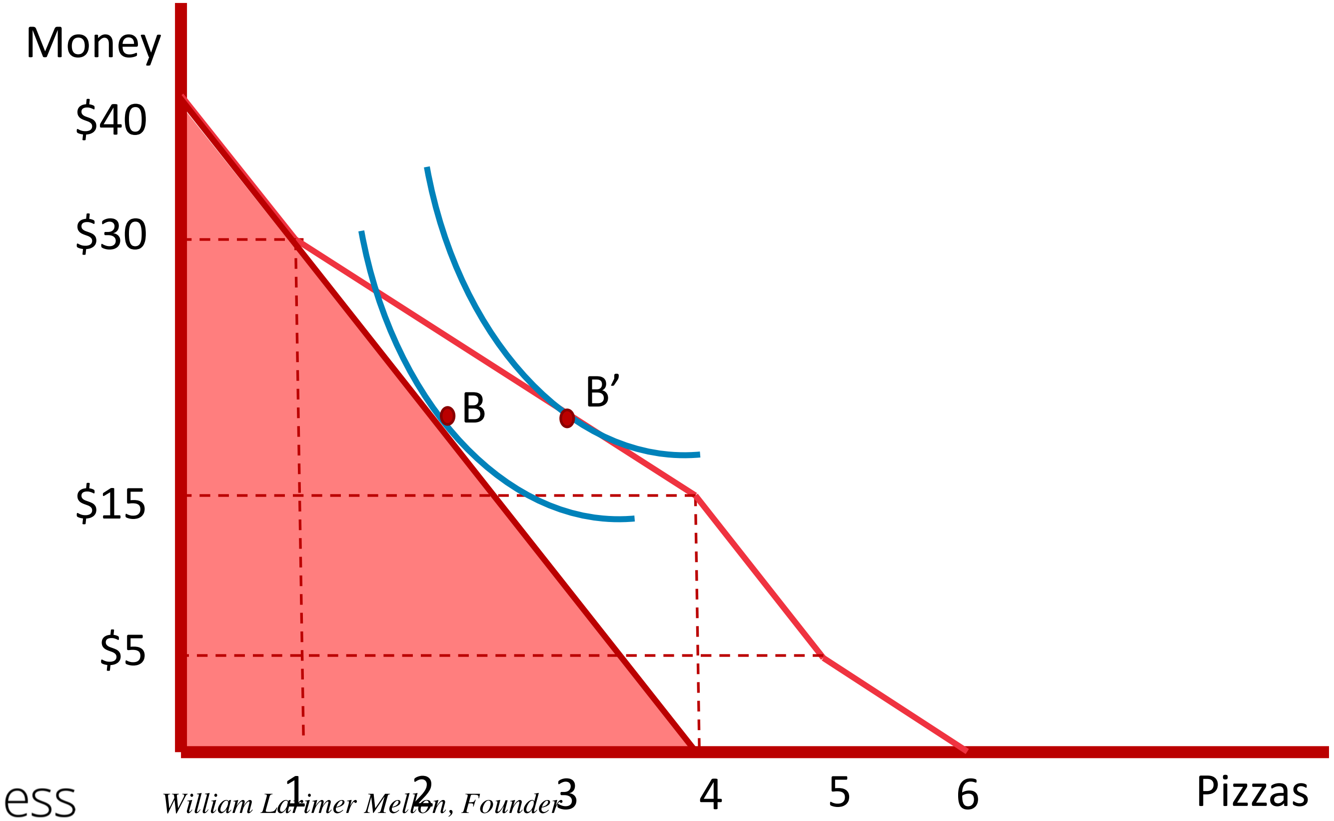
Optimal Choices

- Case 1: With and without promotion, the optimal choice does NOT change.



Optimal Choices

- Case 2: With promotion, more pizzas are consumed



Carnegie Mellon University
Tepper School of Business
William Larimer Mellon, Founder

4. Lagrangian Method

Consumer Choice with Calculus

- We represent consumers' problem by:

$$\max_{x_1, x_2} \quad u(x_1, x_2)$$

$$s.t. \, p_1 x_1 + p_2 x_2 = Y$$

- We have learned two methods to tackle this:
  - Brute-force method
  - Use-the-graphs method

Consumer Choice with Calculus

- We introduce another method: the ***Lagrangian method***
- You can choose to use any of the three methods to solve consumer's problem

Lagrangian Method

- The following problem is equivalent to the constrained optimization problem:

$$\max_{x_1,x_2,\lambda} \quad u(x_1,x_2) + \lambda(Y - p_1x_1 - p_2x_2)$$

  - This new objective function is called “Lagrangian”
  - If $\lambda > 0$, there is penalty in violating the budget constraint
  - Here we also choose the optimal level of λ
  - λ indicates *how binding the constraint is*

Lagrangian Method

- We take the first order conditions:

$$\max_{x_1,x_2,\lambda} \quad u(x_1, x_2) + \lambda(Y - p_1x_1 - p_2x_2)$$

- x_1: $\quad \frac{\partial u(x_1^*,x_2^*)}{\partial x_1} - \lambda p_1 = 0$
- x_2: $\quad \frac{\partial u(x_1^*,x_2^*)}{\partial x_2} - \lambda p_2 = 0$
- λ: $\quad Y - p_1x_1^* - p_2x_2^* = 0$

- We solve for 3 unknowns (x_1^*, x_2^*, λ) in the above 3 equations.

Lagrangian Method: FOC

- FOCs:
  - x_1: $$\frac{\partial u(x_1^*, x_2^*)}{\partial x_1} - \lambda p_1 = 0$$
  - x_2: $$\frac{\partial u(x_1^*, x_2^*)}{\partial x_2} - \lambda p_2 = 0$$
- Interpretation: At the optimum,
  - Marginal utility of a good is equal to its price times λ.
  - Marginal utility per dollar is equated:
$$\frac{MU_1}{p_1} = \frac{MU_2}{p_2} = \lambda$$
  - MRS is equal to the price ratio.

Lagrangian Method: λ

- What is λ?
  - Marginal benefit of income
  - Marginal cost of the budget constraint
- Recall the FOCs:
  - x_i: $$\frac{\partial u\left(x_i^*, x_j^*\right)}{\partial x_i} - \lambda p_i = 0.$$
  - Why isn't the marginal utility of a good just its price?
  - There's another cost other than its price: The cost of strengthening the constraint in the Lagrangian
  - FOC's still represent the idea that $MC = MB$!

5. Choice as a Function of Prices and Income

Choice as a Function of (p_1, p_2, Y)

- If Evelyn has a Cobb-Douglas utility function, what is her optimal consumption bundle (x_i^*, x_j^*) in terms of income, prices, and $0 < a < 1$?

$$u(x_1, x_2) = x_1^a x_2^{1-a}$$

Consumer Choice with Calculus

- *Brute force method*:
  - **Step 1 & 2**: Substitute $x_2 = \frac{Y - p_1 x_1}{p_2}$ and solve for x_1

$$\max_{x_1} {x_1}^a \left((Y - p_1 x_1)/p_2\right)^{1-a}$$

  - To solve for x_1, we take the ***first order condition***:

$$a{x_1}^{a-1}\left(\frac{Y - p_1 x_1}{p_2}\right)^{1-a} - (1-a){x_1}^a\left(\frac{p_1}{p_2}\right)\left(\frac{Y - p_1 x_1}{p_2}\right)^{-a} = 0.$$

  - Rearranging the equation:

$$x_1 = \frac{aY}{p_1}$$

Consumer Choice with Calculus

- *Brute force method*:
  - **Step 1**: $x_2 = (Y - p_1 x_1)/p_2$
  - **Step 2**: $x_1 = \frac{aY}{p_1}$
  - **Step 3:** Plug in the optimal x_1 to solve for x_2

$$x_2 = (Y - p_1 x_1)/p_2$$

    - $x_2 = \frac{(1-a)Y}{p_2}$

Consumer Choice with Calculus

- *Use-the-graphs method:*
  - **Step 1**: Using the given utility function, solve for MRS and obtain:

$$\frac{MU_1(x_1,x_2)}{MU_2(x_1,x_2)} = \frac{p_1}{p_2}$$

- $\frac{MU_1(x_1,x_2)}{MU_2(x_1,x_2)} = \frac{a x_1^{a-1} x_2^{1-a}}{(1-a) x_1^{a} x_2^{-a}} = \frac{a x_2}{(1-a) x_1}$
- Therefore, $\frac{a x_2}{(1-a) x_1} = \frac{p_1}{p_2}$

Consumer Choice with Calculus

- *Use-the-graphs method*
  - **Step 1**: $\frac{ax_2}{(1-a)x_1} = \frac{p_1}{p_2}$
  - **Step 2**: Write down the budget constraint:
    $$p_1 x_1 + p_2 x_2 = Y$$
  - **Step 3**: Solve for the two unknowns (x_1, x_2)
    - $x_1 = \frac{aY}{p_1}, \quad x_2 = \frac{(1-a)Y}{p_2}$

Choice under a Cobb-Douglas Utility

$$x_1 = \frac{aY}{p_1}, \qquad x_2 = \frac{(1-a)Y}{p_2}$$

- Optimal amount of a good is *decreasing* in its price.
- Optimal amount of a good is *increasing* in income.
- Optimal amount of good 1 is *increasing* in a.
- Optimal amount of good 2 is *decreasing* in a.
- Optimal amount of good 1 does *NOT depend* on the price of good 2.