



Economics Department

Business Economics Homework
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1 Markets and pollution externalities (6pts)

A firm with a marginal cost of 2 produces a good whose inverse demand is $P(q) = 10 - q$. Production generates a pollution externality that costs society 3 per unit of good produced. Consumers of the product are assumed to bear the pollution cost.

(i) If the firm behaves in a perfectly competitive way, what is the equilibrium price and what are the profit and surplus?

(ii) What would be the socially optimal quantity (that is, considering consumers, the firm and the pollution damage)? Compare with (i).

(iii) What is the allocation (price and quantity) if the government sets a unit tax of 3 (paid by the firm) per unit sold of the product?

(iv) Suppose now that the firm behaves as a monopoly. What price and quantity is it going to choose? Compare with (ii).

2 Franchise Fee and Efficiency (7pts)

Consider an upstream monopoly firm M producing a good with a cost function $C(Q) = \frac{1}{2}Q^2$. This good is sold at a price \bar{p} to two downstream retailers, R1 and R2, who sell the good to final consumers at a price p . This price p is given by the inverse demand function $p = 1 - q_1 - q_2$ where q_1 and q_2 are the quantities sold respectively by R1 and R2, with $Q = q_1 + q_2$.

1. For given \bar{p} (and assuming $\bar{p} < 1$), characterize the Cournot outcome on the downstream market, i.e. the equilibrium quantities and the profit of each retailer.

2. Write the profit of the monopoly M as a function of \bar{p} and then show that the price maximizing the profit of firm M is $\bar{p} = \frac{5}{8}$.
3. Derive then the monopoly M profit and the price p on the final market.

We now assume that the monopoly can impose a franchise fee to the retailers. More precisely, to buy the good, each retailer must pay a fixed amount F (the franchise fee) and a price \bar{p} per unit.

4. For given \bar{p} , what is the maximal level F the monopoly can choose?
5. Assuming that the monopoly chooses this maximal level (for the two retailers), determine the monopoly profit maximizing price \bar{p} and the level of the franchise fee.
6. Compute then the profit of the monopoly, as well as the retail price p on the final market.
7. Focusing on the consumers surplus, should one promote such a system of franchise fee?

3 Competition with limited attention (7pts)

Two firms (F1 and F2) compete in prices and sell differentiated goods. Their marginal cost of production is supposed to be zero. We model this competition by assuming that the consumers are uniformly distributed on the unit line $[0, 1]$ and firms are located at both ends of this line. When a consumer located at point x buys a good at a price p_i from a firm i located at distance d , we assume that his utility is given by $\bar{s} - p_i - dt$ with $t > 0$ and differentiation parameter.

We first assume that all consumers are aware of the existence of both firms.

1. For given prices p_1 and p_2 , compute the market share of each firm.
2. Write down the profit function of each firm and derive their best-response function.
3. Characterize the equilibrium prices and profits for both firms.

We now assume that the firms must advertise their products to let consumers know about them. More precisely, to inform a share λ_i of consumers about the existence of product i , firm i incurs costs of $A(\lambda_i) = \alpha\lambda_i^2/2$. Moreover, we assume that if a consumer is reached by two ads, he will be able to understand both with probability β but only one (chosen randomly) with probability $1 - \beta$. When an ad is not understood, it is never considered by consumers. Finally, we assume (to guarantee an interior solution) that $t < 2\alpha\beta$.

4. For given values of $\lambda_1, \lambda_2, p_1, p_2$ and α , what is the demand addressed to F1?
5. Given the actions of F2, derive the first-order conditions with respect to p_1 and λ_1 maximizing the profit of F1.
6. Focusing on the symmetric equilibrium (with $\lambda_1 = \lambda_2 = \lambda$ and $p_1 = p_2 = p$), compute the equilibrium price and advertising intensity.
7. How are these values influenced by a change in β ? Explain.