MACROECONOMICS
73-240

Lecture 9

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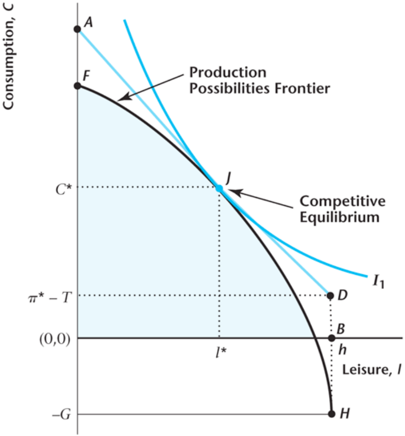
Recap

1) We have just finished up on competitive equilibria.

Summary

In deriving a competitive equilibrium, needed to solve for the endogenous variables

- But it was actually sufficient to solve for (C^*, l^*)



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Summary

- If I know l^*, I have to know $N^* = h - l^*$
- If I know C^*, I have to know $Y* = C^* + G$
  - Also if I know l^*, and hence N^*, I know $Y = zF(K, N^*)$
- Once I know l^* and hence N^*, I know $w^* = MPN$
- Once I know N^*, w^*, Y^*, I know π^*
- and we knew $T^* = G$ in equilibrium

Recap cont'd

1) We have just finished up on competitive equilibria.

2) Given our model of the aggregate economy, we saw that:
   - Exogenous government spending shocks do not drive business cycles: get the wrong co-movement!
   - Exogenous productivity shocks can drive business cycles: get the right co-movement!

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What are Productivity Shocks?

1) Difficult to measure technical change.

2) Possible Candidates:

- R& D expenditure
- Patents
- Productivity estimates:

$$\ln z = \ln Y - \alpha \ln K - (1-\alpha)\ln N$$

- Innovation counts, technology titles/publications

3) Measure should capture inventions as close as possible to the moment of commercialization

4) Be quantifiable and consistent

Plan for This Lecture

1) How good are competitive equilibrium outcomes?

   - Pareto optimum
   - Welfare Theorems

Privately Optimal vs. Socially Optimal

1) So far in a competitive equilibrium:
   - Households choose what is privately optimal for them given (w, π, T)
   - Firms choose what is privately optimal for them given (z, K, w)
   - Governments choose taxes to finance exogenous G
   - Markets coordinate these decisions

2) The competitive equilibrium is a model of the free market economy

3) But does the free market economy give us an outcome that is socially optimal?

Questions we will ask:

- Using the model we have built thus far, we want to ask:

  1) Does the competitive equilibrium (free market economy) give rise to an efficient allocation of time between work and leisure?

  2) Does the competitive equilibrium give rise to an efficient allocation of output between consumption and government spending?

Efficiency: Pareto Optimality

Definition

-KEY DEFINITION-

Pareto Optimality

An equilibrium is Pareto optimal if there is no rearrangement of production or consumption that makes the consumer better off.

- Pareto optimality represents efficiency, not fairness!

The Social Planner

- Suppose a benevolent dictator decides how much people work, rest, and consume

- Then, the dictator will try to make people as happy as possible given the technological constraints in the economy.

The pareto optimum is chosen by a social planner that:

1) Allocates factors to production, consumption and leisure to maximize the utility of the household

2) Only subject to feasibility

The Social Planner (SP)

What can the social planner (SP) control?

- z, K, G still exogenous. SP cannot control
- SP can control how much labor to use in production
- SP can control how much households should consume.
- Since he has direct control over C and N
  - which means the SP does not need markets to coordinate actions. (No prices!)
  - which means the SP does not need to collect taxes to finance G (can directly allocate part of output to G)

Pareto Optimality

Some notes on the Social Planner

- The Social Planner's problem is essentially an assignment problem
- The Social Planner wants to make households as happy as possible given what the economy can feasibly produce.
- The Social Planner does **not** need to care about the HH budget constraint or the Govt budget constraint, he only needs to care about the PPF when thinking about feasibility.
- This is because the Social Planner chooses how to direct all the resources in the economy towards either production or consumption.

The Planner Problem

The planner wants to make households as happy as possible:

$$\max_{C,l} U(C, l)$$

but is constrained but what the economy can produce:

$$C = zF(k, h - l) - G$$

Derive the optimality conditions for the social planner's problem! (Do they look familiar?)

The Planner Problem

The planner solves

$$\max_{C,l} U(C,l)$$

subject to

$$C = zF(k, h-l) - G$$

Optimality implies:

$$\begin{aligned} zF_N(k, h-l) &= \frac{U_L(C,l)}{U_C(C,l)} \\ C &= zF(k, h-l) - G \end{aligned}$$

In words, the marginal rate of transformation must equal the marginal rate of substitution between C and l and C, l must be feasible.
Pareto optimality requires $MRT = MRS$.
But those equations also characterize a competitive equilibrium, so...

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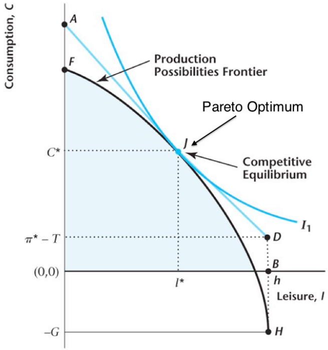
Pareto Optimality

First welfare theorem: a competitive equilibrium is Pareto optimal



- It means: equilibrium outcomes in competitive market are desirable
- The real wage is equal to minus the slope of the PPF and minus the slope of the indifference curve at J

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Pareto Optimality

Second welfare theorem: Any Pareto optimum can be achieved by the outcome of a competitive equilibrium with suitable transfers

- Second welfare theorem tells us that out of all the possible Pareto optimal outcomes, any desired Pareto efficient allocation can be achieved by a competitive equilibrium if we are able to redistribute endowments and then let markets take over

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Pareto Optimality: An Example

- Purpose of this example is to show the usefulness of the welfare theorems

- We will see that by solving the social planner's problem, we get the same allocations that we found by solving the equilibrium!

Example with no government

- Production function: $Y = 2K^{0.5}N^{0.5}$ with $K = 1$
- Utility function: $u(C, l) = \log(C) + \log(l)$ and $l = 1 - N^s$ (with $h = 1$)
- Marginal product of labor: $MP_N = N^{-0.5}$
- Marginal rate of substitution: $MRS_{l,C} = \frac{C}{l} = \frac{C}{1-N}$

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The Social Planner's Problem

- The planner solves

$$\max_{C,l} \log(C) + \log(l)$$

  subject to $C = 2N^{0.5}$

- Optimality implies

$$\frac{C}{1-N} = N^{-0.5} \text{ and } C = 2N^{0.5}$$

- Let's do a bit of algebra...
- Then $N = \frac{1}{3}$ and $C = \frac{2}{\sqrt{3}}$
- This is exactly what we found last time!

Competitive Equilibrium and Pareto Optimality

- From the Social Planner's problem, we see that Pareto Optimality requires:

$$MRT = MRS$$

- From the CE, we have:

$$MRT = w = MRS$$

- Notice that if the 1st equality is violated, or the 2nd equality is violated, CE $\neq$ PO.
- When do we fail to have Pareto optimality?
  1) Externalities (pollution)
  2) Missing markets (financing frictions, moral hazard)
  3) Non competitive firms
  4) No distortionary taxes

Summing Up

-Formal Definitions of Welfare Theorems-

- First Welfare Theorem: Under certain conditions, a competitive equilibrium is Pareto Efficient.
  - In the absence of externalities, market frictions and distortionary taxes, markets allow for efficient outcomes
- Second Welfare Theorem: Under certain conditions, a Pareto Optimum is a competitive equilibrium.
  - To understand equilibrium outcomes, we need only study and understand pareto optima.
- Key Question for Policy: Taking G as given, how should the government raise revenue to pay for G?

Question: Does the US economy resemble a competitive equilibrium?

- Can you think of one assumption that may not hold?

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Distortionary Taxes and Equilibrium: An Example with Linear Production

Distiortionary Taxation

- So far we only talked about lump-sum taxation
  - Taxes that people pay which don't depend on their actions.
- However, lump-sum taxes almost do not exist.
  - In reality taxes are function of economic choices (e.g, income).
- Because these taxes are function of choices, they also affect choices.That is why we call them distortionary.

Simple Model

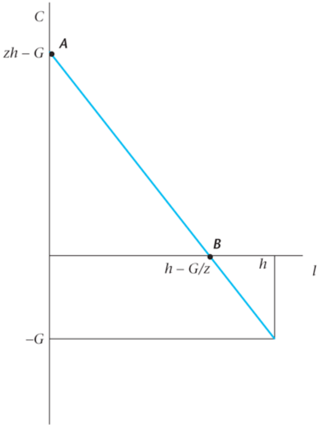
- We study an economy with a *linear production function* ($Y = zN^d$)
- Purpose of this example:
  - Study the incentive effects of the income tax rate
- Formally
  - Assume labor is the only production input

$$Y = zN^d$$

    Note: this production function satisfies constant returns to scale!
  - Take G as given
  - Tax rate, t must raise enough revenue to finance G

Production Possibility Frontier

PPF has a very simple form: $C = z(h - l) - G$



- What is the slope of the PPF?

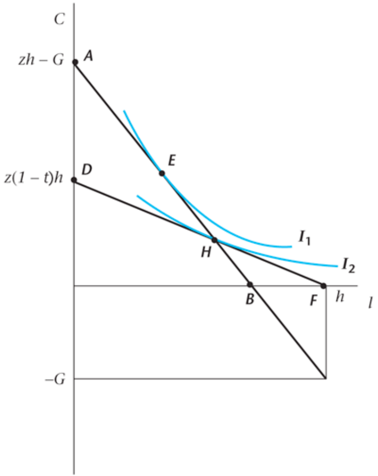
Consumer's budget constraint

- Consumers have to pay a flat rate tax (constant proportion) on each unit of wage income they earn

$$C = (1-t)w(h-l) + \pi$$

- What is the slope of budget constraint?
- In equilibrium
  - $w = z, \pi = 0$
  - HH optimality: (C, l) on budget constraint
  - Markets Clear: (C, l) on PPF

Flat rate taxes are distortionary



- Equilibrium is the point H.
- Note that the slope of budget constraint at this point is not equal to the slope of production possibility frontier
- That means equilibrium is inefficient (Note that point E is feasible and puts everyone on a higher indifference curve.)

Summary of Example So Far

- Equilibrium C, l on PPF and Budget constraint
- Implication: Equilibrium is inefficient!
- Intuition: Worker's gain from working is less than the social gain of work.

Example: Solving Analytically

We want to compare CE to SP solution

- Household utility:

$$U(c,l) = \ln C + \ln l$$

- BC:

$$C = (1-t)w(h-l) + \pi$$

- Output

$$Y = zN^d$$

- Government spending G

Example: Solving Analytically

From a SP, equilibrium requires:

- SP optimality:

$$MRS = \frac{C}{1-N} = z$$

$$zN = Y = C + G$$

- Suppose, $h = 1$, then solving:

$$N^* = \frac{1}{2} + \frac{G}{2z}$$

Example: Solving Analytically

From a CE, equilibrium requires:

- Household optimality:

$$MRS = \frac{C}{l} = (1-t)w$$

$$C = (1-t)w(h-l) + \pi$$

- Firm optimality

$$w = z \Rightarrow \pi = 0$$

- Government budget constraint

$$G = twN^s$$

- Market Clearing

$$C = Y - G$$

$$N^s = N^d$$

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Example: Solving Analytically

From a CE, equilibrium requires:

- From labor market clearing:

$$\frac{C}{1-N} = (1-t)z$$

- From goods market clearing:

$$C = zN - G$$

- Substitute for C, from govt budget constraint, $t = \frac{G}{zN}$, rearrange and solve:

$$N^* = \frac{1}{2} \quad \text{or } N^* = \frac{G}{z}$$

Why do we have proportional taxes?

Efficiency vs. Equity

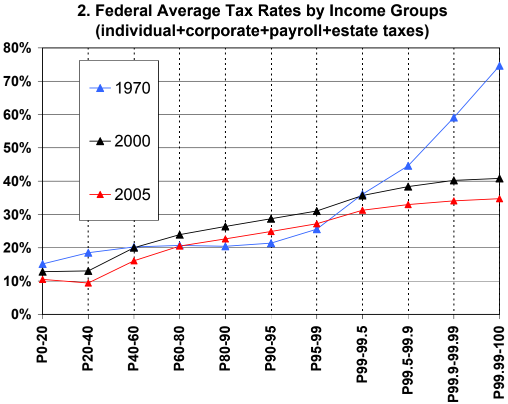
- Pareto Optimality tells us about efficiency. It does not inform about equity

- A bundle on the PPF that gives one agent (e.g. the firm) all of one commodity and the other (e.g. the consumer) nothing is pareto optimal.

- What are the tradeoffs for choosing taxes?

Federal Taxes

What is the profile of taxes in the data?



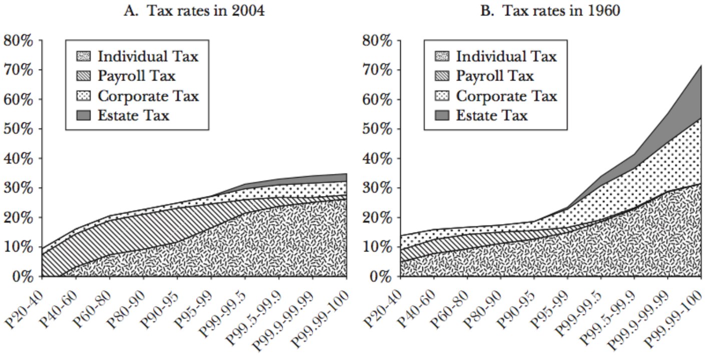
Source: Piketty and Saez (2007)

Note: Missing State and Local and FICA.

Federal Taxes

Figure 1

Federal Tax Rates in the United States in 2004 and 1960



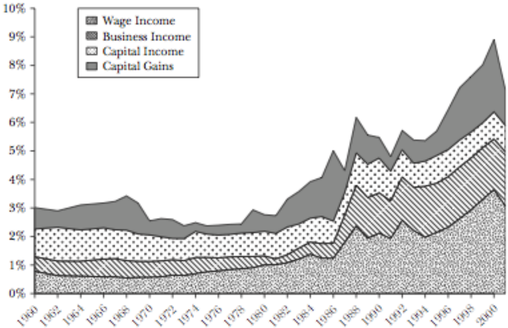
Notes: Figures display the tax rate for each of the four federal taxes for various groups of the income distribution in 2004 (based on 2000 incomes adjusted for economic growth) and in 1960. Tax rates are stacked.

Source: Piketty and Saez (2007)

Note: Progressivity in the past largely from estate and corporate tax

Federal Taxes

Figure 2
Income Share and Composition for the Top 0.1 Percent, 1960–2001

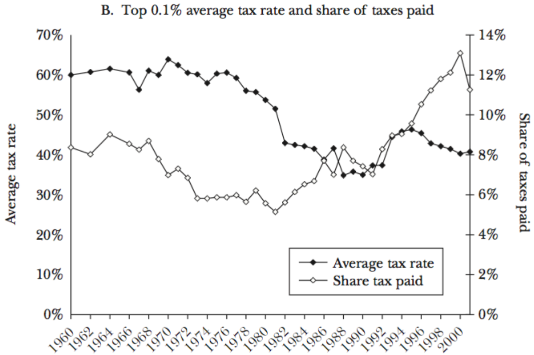


Source: Piketty and Saez (2007)

After tax income share of top 0.1 percent

- in 1970: 1.2%
- in 2000: 7.3%

Federal Taxes

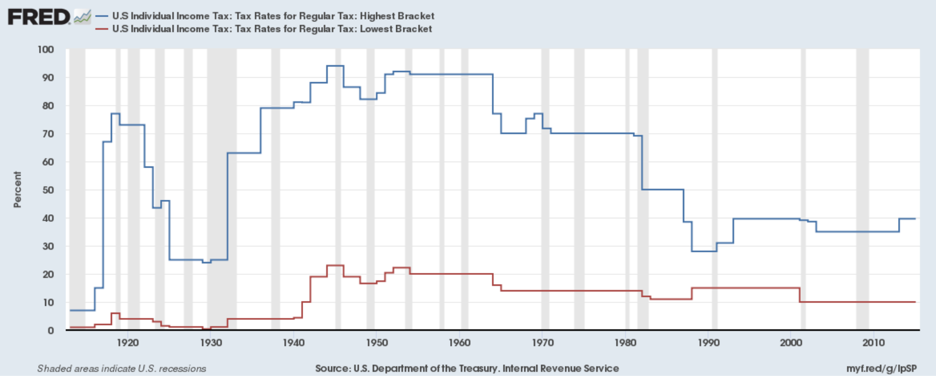


Source: Piketty and Saez (2007)

- Avg. tax rate paid by top 0.1% declining over time

Federal Taxes

- What about a longer view and with more recent data?



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Federal Taxes

- Suppose we want to finance G
- Want to pick a suitable proportional tax rate
- Are there behavioral responses to our choice of a tax rate?

Behavioral Effects

GDP per capita in France ≈ 0.75 of GDP per capita in the US

- Are the French less productive?

	1993-1996			1970-1974		
	Hours/Person	GDP/Person	GDP/hour	Hours/Person	GDP/Person	GDP/hour
Germany	75	74	99	105	75	72
France	68	74	110	105	77	74
Italy	64	57	90	82	53	65
Canada	88	79	89	94	86	91
United Kingdom	88	67	76	110	68	62
Japan	104	78	74	127	62	49
United States	100	100	100	100	100	100

- Not really, GDP per hour $\left(\frac{Y}{L}\right)$ actually higher.
- Do the French work less? Why?

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Behavioral Effects

In Data do we observe behavioral responses from taxation?

	1993-1996			1970-1974		
	Hours/week	GDP/hour	Tax Rate	Hours/week	GDP/hour	Tax Rate
Germany	19.3	99	0.59	24.6	72	0.52
France	17.5	110	0.59	24.4	74	0.49
Italy	16.5	90	0.64	19.2	65	0.41
Canada	22.9	89	0.52	22.2	91	0.44
United Kingdom	22.8	76	0.44	25.9	62	0.45
Japan	27.0	74	0.37	29.8	49	0.25
United States	25.9	100	0.4	23.5	100	0.4

Source: Prescott (2004)

Seems like higher the tax, the lower the labor supply.

Take home points

- Proportional tax rates can distort household marginal decision.
- When it distorts household's marginal decisions, there are welfare losses due to this distortion.
- There are also output losses because of this distortion.

So what Should Taxes Be?

Going back to our earlier example

Suppose Govt's goal is to maximize revenue. Wants to figure out what level of τ to charge.

- Technology: $Y = zN^d$

- Household is taxed on labor income: $C = (1-\tau)wN^s + \pi$

- Government's revenue is $\tau w N^s$

Profits for the firm

- Firm's profit is

$$\pi = Y - wN^d = (z - w)N^d$$

- in the equilibrium $w = z$ and hence $\pi = 0$

- Therefore, consumer's budget constraint can be simplified

$$C = (1 - t)w(h - l)$$

- Gov't Revenue *in equilibrium*: $tw(h - l) = tz(h - l)$

Government's Revenue

- Given the tax rate t government's revenue is

$$Rev(t) = \underbrace{t}_{\text{tax rate}} \underbrace{z[h - l(t)]}_{\text{tax base}}$$

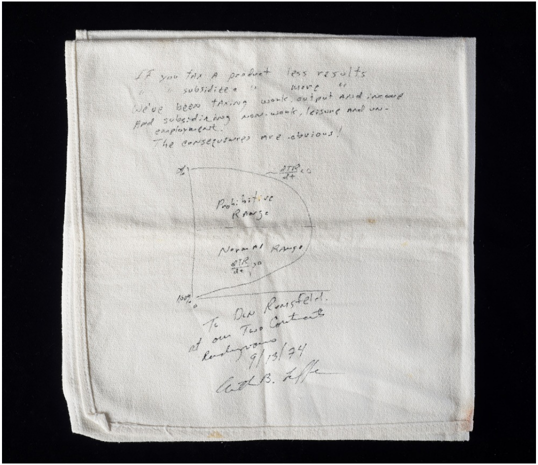
  in which $l(t)$ is household' leisure choice if the tax rate is t.

- We want to study how government's revenue changes with the tax rate t

On the one hand, raising t can lead to higher govt revenue
On the other hand, raising t could reduce tax base and lower govt revenue

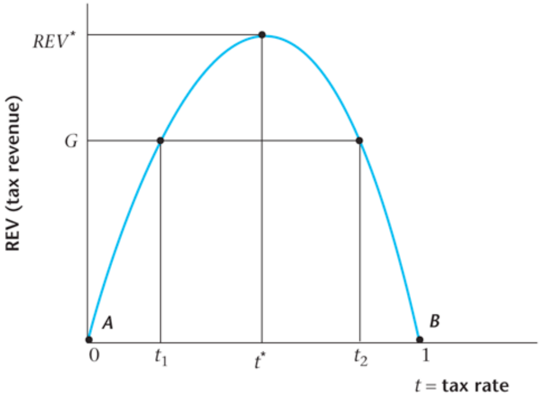
Laffer Curve as described on a napkin

The Laffer Curve: Describes a non-linear relationship between income tax revenue and the income tax rate



Laffer Curve

A clearer picture of the Laffer curve



Laffer's argument: raising tax rates beyond a certain level can be counter-productive and may not generate more tax revenue.

Laffer Curve

What economists agree on:

- There is a non-linear relationship between tax revenue and the tax rate.
- Raising the tax rate could affect the tax base (depending on whether SE or IE dominates).

What economists don't always agree on

- The laffer curve may not be a quadratic relationship (non-linear need not imply quadratic!)
- Which side current tax rates (high vs. low) are on even if we assumed a quadratic relationship

Taking Stock

Some lessons learned from optimal taxation:

1. In the U.S. and most countries taxes on income are proportional. (sometimes referred as marginal taxes)
2. Proportional taxes make the equilibrium inefficient.
3. If we ignore individual responses, then optimal taxes should be completely redistributive.
4. Quantitatively taxes at the top should be fairly high!
5. But not so high as being on the "wrong side" of the Laffer Curve.