Random variables - probability distribution

Exercise 1 (Binomial distribution)

During university registrations, each student completes a registration file. All checks carried out indicate that the probability that any registration is well filled in is equal to $p = 0.94$.

1) Introduce a random variable X which describes the two possible states for each file.

2) Provide its probability distribution

3) Calculate its expectation and its variance.

We now consider a batch of n files and we are interested in number of well-filled files among the n files.

4) Introduce a random variable X which represents the number of well-filled files.

5) What is its probability distribution ? Provide its expectation and its variance.

6) If $n = 5$, calculate the probability of the following events : $\{$ no file is well filled$\}$, $\{$all files are well filled$\}$, $\{X > 3\}$, $\{ 2 < X < 4\}$.

7) If $n = 100$, what probability distribution can we use to approximate the distribution of X ?

Exercise 2 (Distribution function)
Atmospheric ozone concentration (in $\mu g/m^3$ = microgram (one millionth of a gram) per cubic meter) is modeled by a Gaussian random variable X of mean m and variance σ^2, denoted by $\mathcal{N}(m, \sigma^2)$, where $m = 178$ and $\sigma^2 = 3.1$.

1) What are the units of measure of m and σ ? What do they represent ?

An ozone concentration greater than $180\mu g/m^3$ is considered dangerous for humans.

2) a) What is the probability that the concentration exceeds 180 ?
2) b) Assuming $m = 180$, find a real number δ such that the probability

$$\mathbb{P}(180 - \delta \leq X \leq 180 + \delta)$$

is larger than 95%.

3) Now assume that m and σ are unknown. For a fixed value x, calculate the probability that X is less than or equal to x. Deduce the distribution function of X.

4) Graph the distribution function of X.