Carnegie Mellon Tepper SCHOOL OF BUSINESS

73-240 – Problem set 4

Due Monday Nov. 18th

Notes on HW submission:

1. Homework must be turned in on the day it is due. In this case, you will submit your HW 4 **in class**.
2. Late homework will NOT be accepted unless you are sick and have a doctor's note.
3. Homework regrading: There is a statute of limitations on regrades. If you believe a question has been incorrectly graded, please take your homework to your TA within 2 weeks of it being returned.
4. Working in groups: You may work in groups of up to 4. BUT: You MUST put names of other group members on your homework. You MUST write up your own set of answers. Do NOT simply copy some other person's work.
5. TYPE your work. Long equations may be hand written. Buy a stapler!
6. Write your first and last name on the title of each graph.
7. Carefully explain your work.

Problem 1: Growth Accounting

In the early 1990s, the high growth rates experienced in the Hong Kong and Singapore led these economies to be dubbed as part of the East Asian Growth Miracle. Looking at these economies, a student argues that all that is needed for rapid continuous growth is heavy investment in capital and maintaining a high capital to output ratio. In this question, you will investigate if this statement is true.

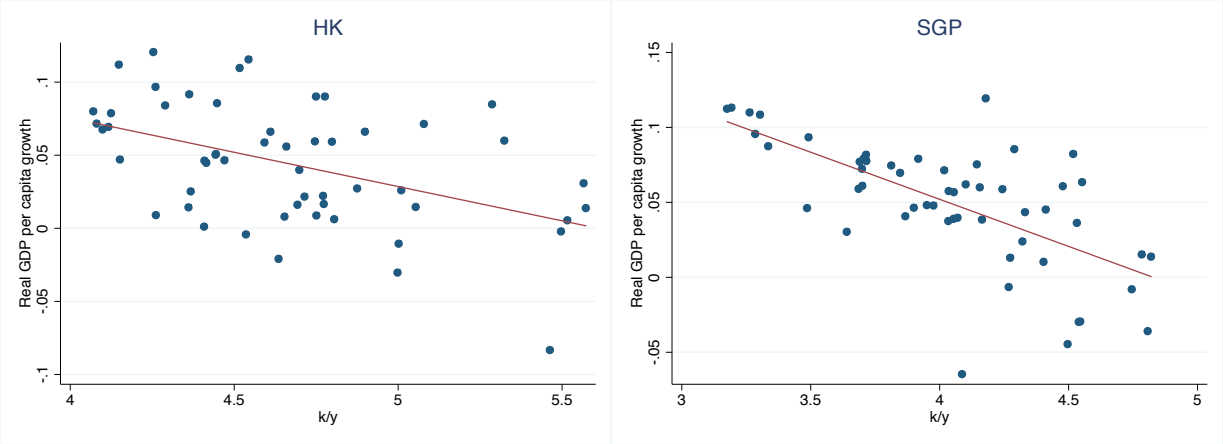
a Download the dataset `hkg_sgp_data.xls` from Canvas. Calculate GDP per capita and capital per person for each of the two economies in the dataset for the years 1960-2014. You do not need to print out or submit your answer for part a).

b For each economy, do a scatterplot of the growth rate in GDP per capita against the capital to output ratio. [Note you can calculate the capital to output ratio as: $\frac{K}{Y} = \frac{K/N}{Y/N} = \frac{k}{y}$ where the letters in small caps refer to per capita terms]. What trend

do you observe?

Answer:

In general, there exists a downward trend between growth rate in GDP per capita and the capital to output ratio.



c Let's try to understand the trend in part b). Consider the following production function, $Y = zK^{\alpha}N^{1-\alpha}$. First write down what output per capita, y is in terms of z, k, α where $k = K/N$. Suppose z is constant. At low levels of k, is the rate of change in output per capita wrt capital per capita, i.e. $\frac{dy}{dk}$ small or large? What about at high levels of k?

Answer

To get GDP per capita, we divide Y by N

$$y = \frac{Y}{N} = \frac{zK^{\alpha}N^{1-\alpha}}{N} = \frac{zK^{\alpha}N^{1-\alpha}}{N^{\alpha}N^{1-\alpha}} = zk^{\alpha}$$

where $k = \frac{K}{N}$. Observe that the rate of change in y wrt k is given by its derivative:

$$\frac{dy}{dk} = \frac{\alpha z}{k^{1-\alpha}}$$

We can see that since k is in the denominator, when k is low, a small increment in k yields a large increase in y. Conversely when k is already high, a small increment in k leads to smaller and smaller increases in y. This comes from the concavity of our production function and the fact that we have diminishing marginal product.

d Now let's do some growth accounting. We want to understand the percentage of growth that can be explained by growth in k. Observe that we can write $\ln y = \ln z + \alpha \ln k$. Recall from HW1 that we can approximate the growth rate in GDP per capita with the change in logs, i.e. $g_y = \ln y_{t+1} - \ln y$. Using your production function, back out the growth rate in z for each economy. In a table, report the average growth rates of y, k, z for the periods 1960-1969, 1970-1979, 1980-1989, 1990-1999, 2000-2009 and for the years ≥ 2010. You may assume $\alpha = 1/3$.

Answer

We can decompose the growth rate as

$$g_y = \alpha g_k + g_z$$

Using this information, we can back out the growth rates of z given growth rates of k and y. See Table 1 on last page.

e What percentage of growth can be explained by capital growth for each of those time periods? What percentage of growth can be explained by z for each of those time periods? Together with your answers in part b) and c), do you think the student's statement is true?

Answer

See Table 1 on last page. Note to calculate the contribution due to capital per worker growth, we must include α(you can see this based on the equation $g_y = \alpha g_k + g_z$. The share of GDP per capita growth due to capital per worker growth is then $\frac{\alpha g_k}{g_y}$. We can see from Table 1 that percentage growth due to capital per worker is generally declining over time across the four economies. This suggests that as these economies became more industrialized and had higher capital-to-output ratios, capital per worker growth is a less important factor towards sustaining high growth.

f Many of the East Asian economies followed a pattern of high savings rate and heavy investment to grow their economies. Increases in the savings rate have an ambiguous effect on long run consumption per person. Following Lecture 15, find the golden rule capital per person by maximizing steady state consumption per person with respect to capital per person. Given your answer for the golden rule capital per person, state what the golden rule savings rate would be.

Answer

To find the golden rule capital per person, we want to maximize consumption per person in steady state:

$$\max_k c = zk^\alpha - (n+d)k$$

taking first order conditions wrt k, we have:

$$\alpha z k^{\alpha-1} = (n+d)$$

which re-arranging gives us:

$$k^{gold} = \left(\frac{\alpha z}{n+d}\right)^{1/(1-\alpha)}$$

this also implies that the golden rule savings rate is:

$$s^{gold} = \frac{(n+d)k^{gold}}{z k^{gold,\alpha}}$$

which plugging in for k^{gold} gives us $s^{gold} = \alpha$

Hong Kong					
	g_y	g_k	g_z	% due to k	% due to z
1960	0.055	0.05	0.04	0.30	0.70
1970	0.061	0.05	0.05	0.25	0.75
1980	0.056	0.06	0.04	0.36	0.64
1990	0.020	0.05	0.004	0.77	0.23
2000	0.037	0.03	0.03	0.24	0.76
2010	0.029	0.02	0.02	0.19	0.81
Singapore					
1960	0.059	0.02	0.05	0.09	0.91
1970	0.075	0.09	0.04	0.40	0.60
1980	0.053	0.06	0.03	0.41	0.59
1990	0.044	0.05	0.03	0.40	0.60
2000	0.025	0.02	0.02	0.34	0.66
2010	0.041	0.03	0.03	0.28	0.72

Table 1: Decomposing Growth