Question 4.

a)

Tree for stock

$$\begin{tikzcd}[column sep=small]
 & & S_2^{uu} = 66.125 \\
 & S_1^{u} = 50 \times 1.15 = 57.5 \arrow[ur, dash] \arrow[dr, dash] & \\
S_0 = 50 \arrow[ur, dash] \arrow[dr, dash] & & S_2^{ud} / S_2^{du} = 51.75 \\
 & S_1^{d} = 50 \times 0.9 = 45 \arrow[ur, dash] \arrow[dr, dash] & \\
 & & S_2^{dd} = 40.5
\end{tikzcd}$$

Tree for Bond

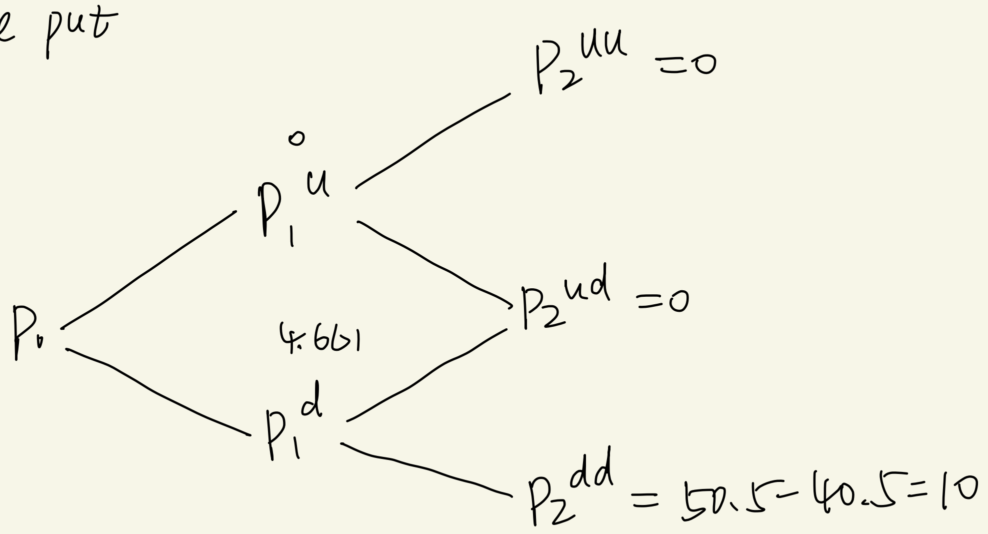
$$\begin{tikzcd}[column sep=small]
 & & B_2^{uu} = 1.0609 \\
 & B_1^{u} = 1.03 \arrow[ur, dash] \arrow[dr, dash] & \\
B_0 = 1 \arrow[ur, dash] \arrow[dr, dash] & & B_2^{ud} / B^{du} = 1.0609 \\
 & B_1^{d} = 1.03 \arrow[ur, dash] \arrow[dr, dash] & \\
 & & B_2^{dd} = 1.0609
\end{tikzcd}$$

b) for the call :

$$\begin{tikzcd}[column sep=small]
 & & C_2^{uu} = 66.125 - 56.125 = 10 \\
 & C_1^{u} \arrow[ur, dash] \arrow[dr, dash] & \\
C_0 \arrow[ur, dash] \arrow[dr, dash] & & C_2^{ud} = 0 \\
 & C_1^{d} \arrow[ur, dash] \arrow[dr, dash] & \\
 & & C_2^{dd} = 0
\end{tikzcd}$$

$K = 56.125$

for the put



$K = 50.5$

Since P_1^u will generate 0 in the end, so $P_1^u = 0$

for $\underline{\underline{P_1^d}}$, we have
$$\begin{cases} \Delta = \dfrac{V_1^u - V_1^d}{S_0(u-d)} = \dfrac{0-10}{51.75-40.5} = \dfrac{-10}{11.25} = -0.889 \\ B = \dfrac{V_1^u - S_0 u \Delta}{1+rf} = \dfrac{0 - 45 \times 1.15 \times (-0.889)}{1.03} \end{cases}$$
$$= 44.666$$

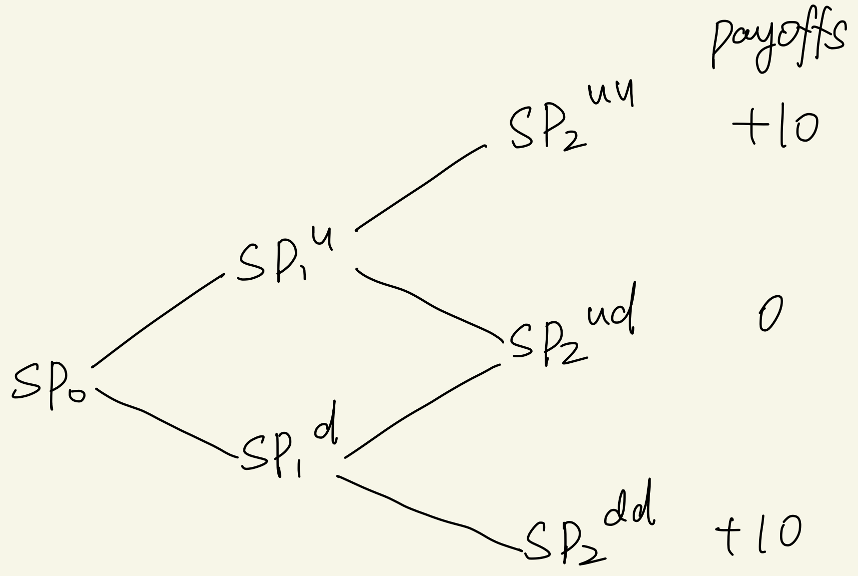
$$\text{So } V_1^d = V(P_1^d) = \Delta S_1^d + B$$
$$= -0.889 \times 45 + 44.666 = 4.661$$

So for $\underline{\underline{P_0}}$, we have
$$\begin{cases} \Delta = \dfrac{V_1^u - V_1^d}{S_0(u-d)} = \dfrac{-4.661}{50(1.15-0.9)} = -0.3728 \\ B = \dfrac{V_1^u - S_0 u \Delta}{1+rf} = \dfrac{0 - 50 \times 1.15 \times (-0.3728)}{1.03} \end{cases}$$
$$= 20.812$$

$$\text{So } V_0 = V(P_0) = \Delta S_0 + B = -0.3728 \times 50 + 20.812$$
$$= 2.172$$

So P_0's price is $\boxed{2.172}$ at T_0

c)



The incentive should be: the customer believes there will be great fluctuations in this market in the future, (a large variance of the market), thus he would buy it.

Similar to the option pricing:

P

Question 3.

a) $E(r_A) = 21\%$, $E(r_T) = 15\%$

$E(P) = 21\% \times 35\% + 15\% \times 65\% = 0.0735 + 0.0975 = 0.171$
$= 17.1\%$

$Var(P) = Var(w_1 r_1 + w_2 r_2)$

$= w_1^2 Var(r_1) + w_2^2 Var(r_2) + 2 w_1 w_2 cov(R_1, R_2)$

$= 0.35^2 \times (0.315)^2 + (0.65)^2 \times (0.174)^2 + 2 \times 0.35 \times 0.65 \times cov(R_1, R_2)$

Since $\beta_A = \dfrac{cov(R_A, R_{market})}{Var(market)} = 1.05 = \dfrac{cov(R_A, R_{market})}{(0.15)^2}$

$\Rightarrow cov(R_A, R_{market}) = 0.0236 = \rho_{A \cdot market} \times \sigma_A \times \sigma_{market}$

$= \rho \times 0.315 \times 0.15$

so $\rho_{A \cdot market} = 0.4995$

Since $\rho_{A \cdot market} = \rho_{A \cdot Tsla} = 0.4995$

we have $cov(R_A, R_T) = \rho_{A \cdot T} \times \sigma_A \cdot \sigma_T = 0.4995 \times 0.315 \times 0.174 = 0.0274$

Thus $Var(P) = 0.01216 + 0.01279 + 0.01267$

$= 0.03747$

so $Volatility(P) = \sqrt{Var(P)} = 0.1934 = 19.34\%$

b) For any two assets with a $\rho \neq 1$, there would be a diversification effect.

$\sigma(p) = 0.1934$

$w_T \sigma(T) + w_A \sigma(A) = 0.35 \times 0.315 + 0.65 \times 0.174 = 0.11025 + 0.1131$

So we find that $\sigma(p) < w_T \sigma(T) + w_A \sigma(A)$ $= 0.22335$

it proves the existence of diversification effect.

c) $\beta_p = w_1 \beta_1 + w_2 \beta_2 = w_A \beta_A + w_T \beta_T = 0.35 \times 1.05 + 0.65 \times$

$E(r_T) = r_f + \beta_T (E(\text{market}) - r_f)$

$= 3\% + \beta_T (10\% - 3\%) = 15\%$

$\Rightarrow \beta_T = \frac{12\%}{7\%} = 1.714$

So $\beta_p = 0.35 \times 1.05 + 0.65 \times 1.714 = 0.3675 + 1.1141$

$= 1.4816$

So, $\beta_T = 1.714$

$\beta_p = 1.4816$ $\beta_A = 1.05$

So from here we can see that, for every 1 point increase of the market, Tesla's stock would increase by 1.714, Apple's would increase by 1.05, they are all positively correlated to the market.

Also, for the beta of a portfolio, it's always smaller than the largest β in it and larger than the smallest β in it.

d) Yes, because the portfolio is not completely negatively correlated, thus their efficient frontier can never reach the y-axis, thus there must be idiosyncratic risk.

Yes, the portfolio contains systematic risk.