

# PRINCIPLES OF FINANCE

WEEK 5

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# Outlook

## **Do you remember**

- The Efficient Frontier
- The Capital Market Line

## **Outline of today's lecture**

- The Capital Asset Pricing Model (CAPM)
- Estimation of the cost of equity

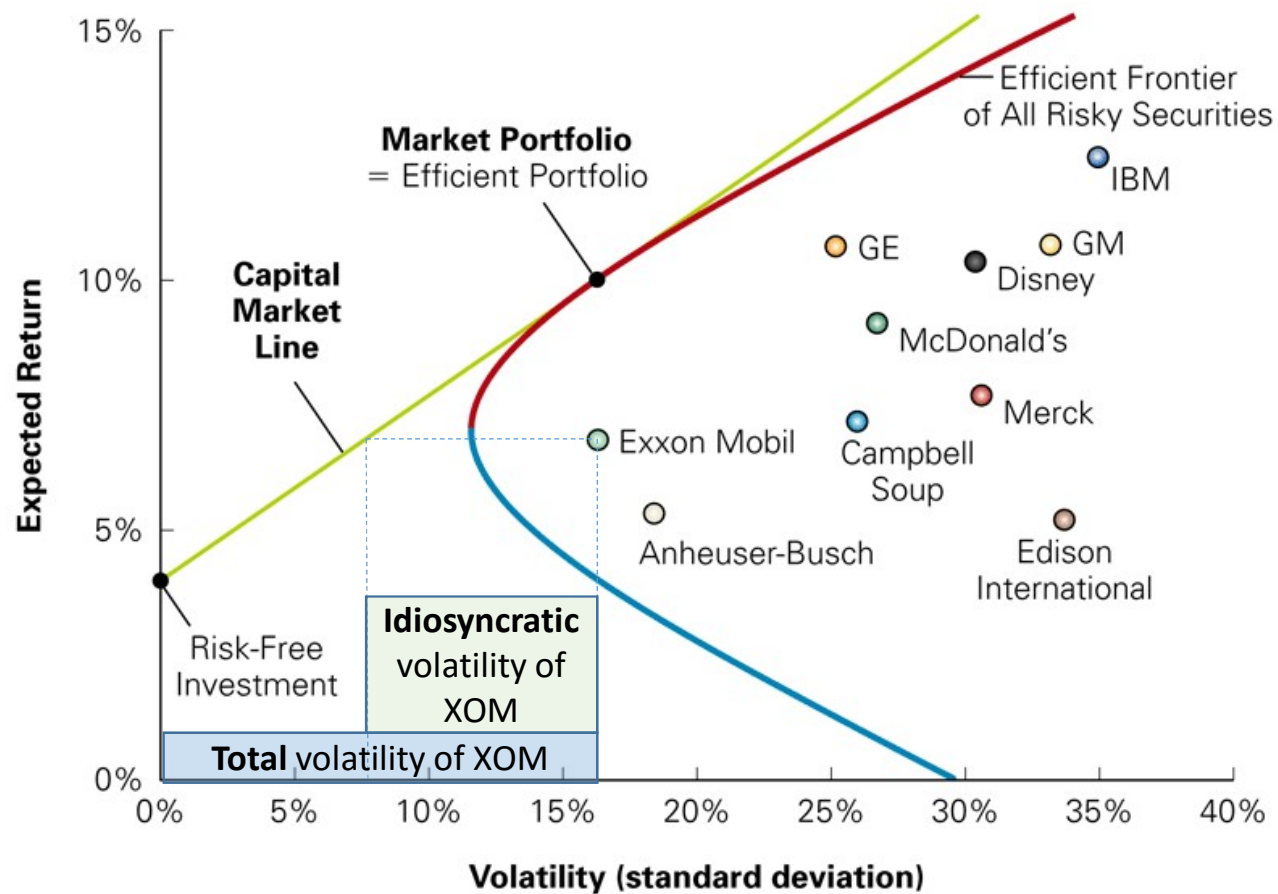
**Video:** Cost of capital and WACC

## **Applications:**

- Challenges of CAPM
- Search for alpha

# The CAPM

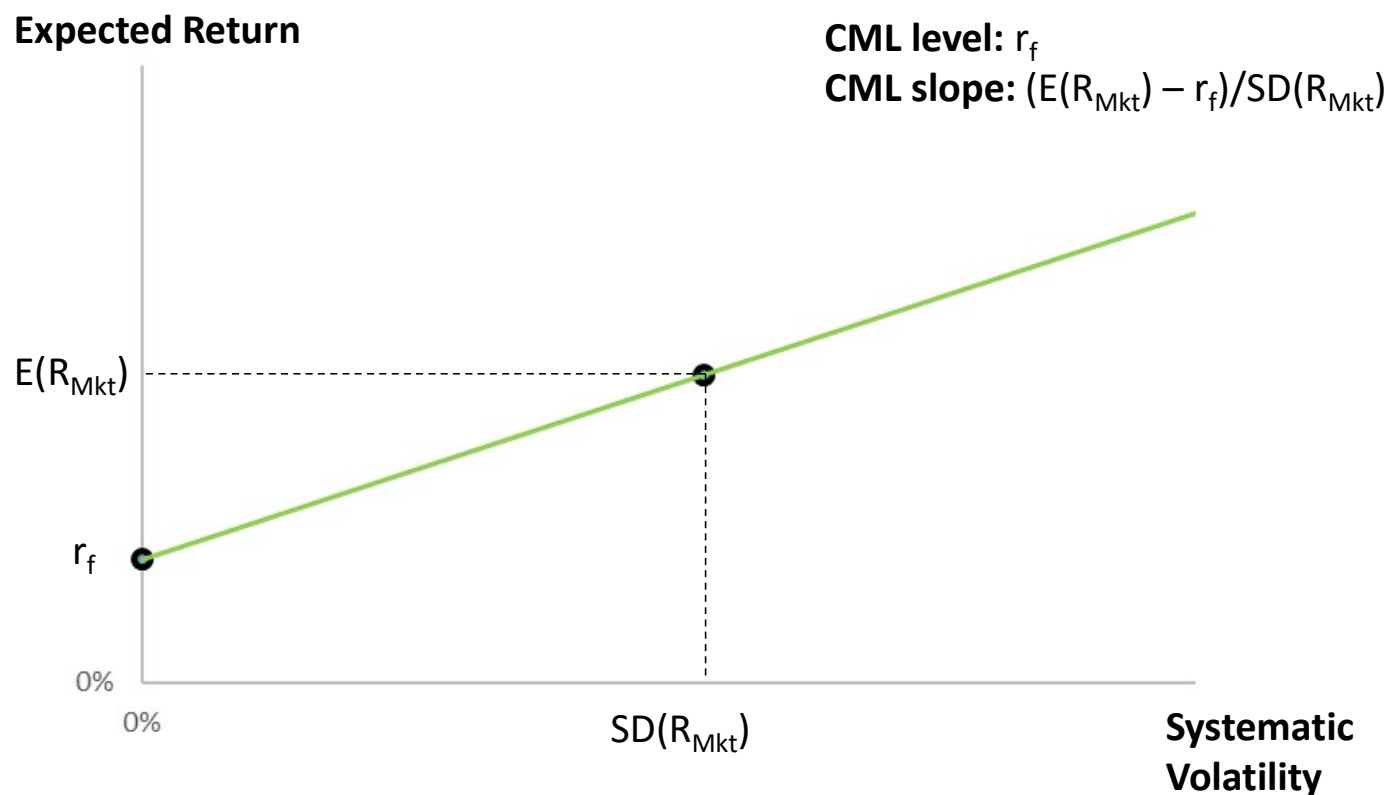
# The Capital Market Line



# Back from the Capital Market Line to the individual stock

- The Capital Market Line provides a benchmark for the return expected on any individual equity-funded project or stock
- Importantly, on the Capital Market Line, the volatility implied by any portfolio is systematic volatility
- This means that I only need to assess the systematic volatility of any stock before I can assess what the correct benchmark for its expected return is on the CML

# Capital Market Line as a benchmark



- We replaced total volatility with systematic vol. on the x-axis because all portfolios on the CML have fully systematic variance.
- Where should an individual stock fit on this line?

# Back from the Capital Market Line to the individual stock

- What is the correct measure of systematic volatility we should use?

$$\text{Systematic volatility of stock } i = SD(R_i) \times \text{Corr}(R_i, R_{Mkt})$$

- There is a tight link with the beta of stock i

$$\begin{aligned}\beta_i &= \frac{\text{Cov}(R_i, R_{Mkt})}{\text{Var}(R_{Mkt})} = \frac{SD(R_i)\text{Corr}(R_i, R_{Mkt})}{SD(R_{Mkt})} \\ &= \frac{\text{Systematic volatility of stock } i}{\text{Volatility of market portfolio}}\end{aligned}$$

- This is why the beta is seen as a proxy for systematic risk of an asset

# Risk premium of a risky asset: CAPM formula

- We may now set the benchmark for stock  $i$ 's expected return by looking at the point on the Capital Market Line corresponding to its systematic volatility ( $\beta_i \times SD(R_{Mkt})$ )
- Provided markets are efficient, the expected return on stock  $i$  should correspond to this benchmark, so we obtain the formula:

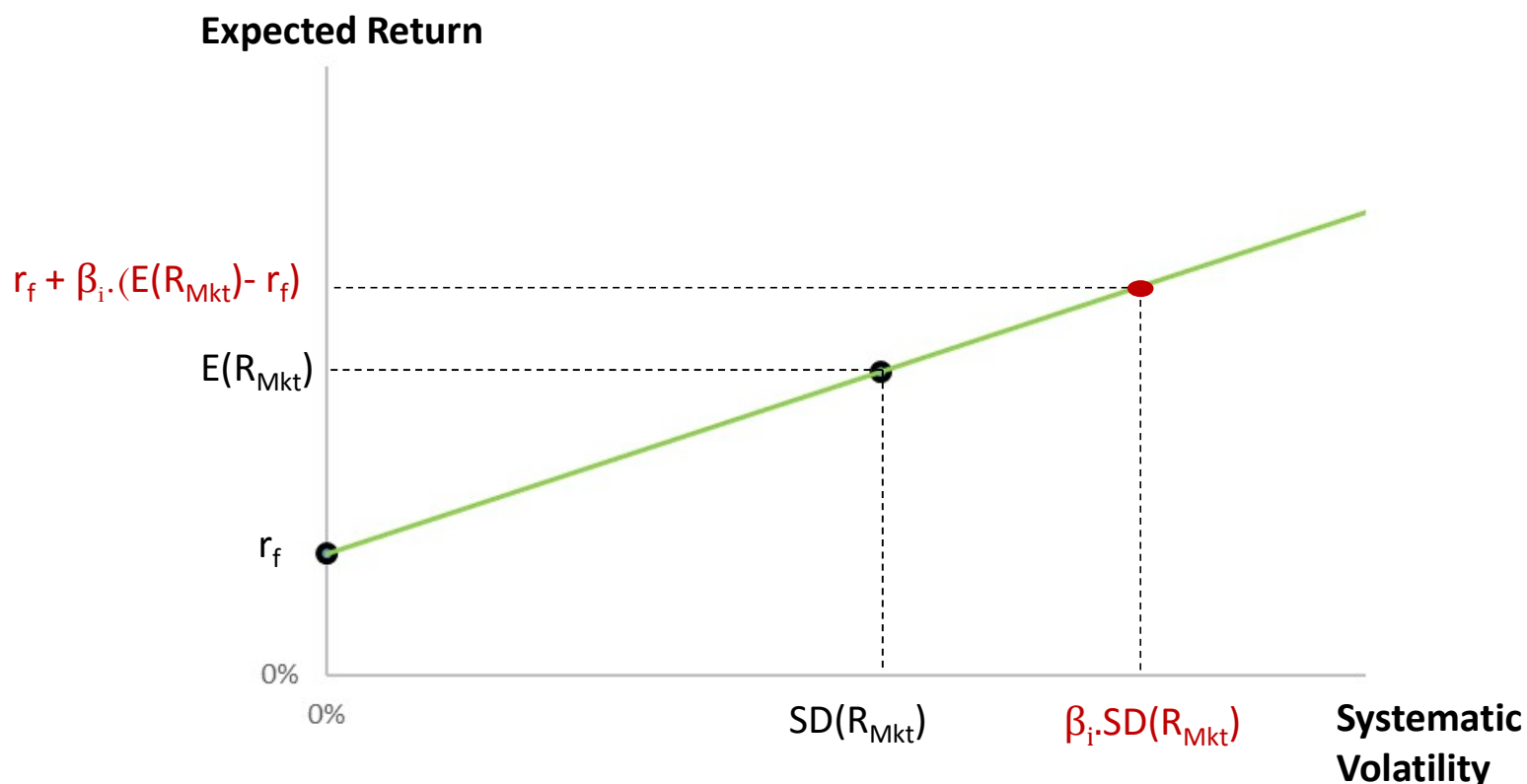
$$E[R_i] = r_i = r_f + \underbrace{\beta_i \cdot (E[R_{Mkt}] - r_f)}_{\text{Risk premium for security } i}$$

- Where, as mentioned on the previous slide, the beta is defined as:

$$\beta_i = \frac{SD(R_i) \times \text{Corr}(R_i, R_{Mkt})}{SD(R_{Mkt})} = \frac{\text{Cov}(R_i, R_{Mkt})}{\text{Var}(R_{Mkt})}$$



# CAPM Formula and Capital Market Line



- The correct benchmark portfolio on the CML for stock  $i$  is in red
- Its corresponding expected return gives the CAPM formula

# Decomposition of variance

- The CAPM formula:

$$E[R_i] = r_i = r_f + \underbrace{\beta_i \cdot (E[R_{Mkt}] - r_f)}_{\text{Risk premium for security } i}$$

can be rewritten as follows, with  $E[\varepsilon_i] = 0$

$$R_i = r_f + \beta_i(R_M - r_f) + \varepsilon_i$$

$$\underbrace{\sigma_i^2}_{\text{Total Risk}} = \underbrace{\beta_i^2 \sigma_M^2}_{\text{Market Risk}} + \underbrace{\sigma_\varepsilon^2}_{\text{Idiosyncratic Risk}}$$

# Intuition for the CAPM Formula

- Only systematic risk determines expected returns since firm-specific risk is diversifiable and therefore does not deserve extra return
- The expected return on any investment comes from two components:
  - A baseline **risk-free rate** of return that we would demand to compensate for the time value of money, even if there were no risk in the investment
  - A **risk premium** that varies with the amount of systematic risk in the investment
- This **Capital Asset Pricing Model** is the main method used to calculate the equity cost of capital in business practice

# Intuition for the CAPM Formula

- $\beta_i < 0 \Rightarrow E(R_i) < r_f < E(R_{Mkt})$ : Negative risk, valuable as a **hedge**
- $\beta_i = 0 \Rightarrow E(R_i) = r_f < E(R_{Mkt})$ : No systematic risk, regardless of  $\sigma_i^2$
- $0 < \beta_i < 1 \Rightarrow r_f < E(R_i) < E(R_{Mkt})$ : Less risky than the market portfolio
- $\beta_i = 1 \Rightarrow r_f < E(R_i) = E(R_{Mkt})$ : Same risk as in the market portfolio
- $\beta_i > 1 \Rightarrow r_f < E(R_{Mkt}) < E(R_i)$ : Riskier than the market portfolio

# Intuition for the CAPM Formula

## A Negative-Beta Stock

### Problem

Suppose the stock of Bankruptcy Auction Services, Inc. (BAS), has a negative beta of  $-0.30$ . How does its expected return compare to the risk-free rate, according to the CAPM? Does this result make sense?

$$E[R_{BAS}] = 4\% - 0.30(10\% - 4\%) = 2.2\%$$

This result seems odd: Why would investors be willing to accept a 2.2% expected return on this stock when they can invest in a safe investment and earn 4%? A savvy investor will not hold BAS alone; instead, she will hold it in combination with other securities as part of a well-diversified portfolio. Because BAS will tend to rise when the market and most other securities fall, BAS provides “recession insurance” for the portfolio, and investors pay for this insurance by accepting an expected return below the risk-free rate.

# Determinants of beta

## **1. Cyclicality of Revenues**

- The revenues of many firms are quite cyclical. That is, these firms do well in expansion and do poorly in recession.
- Empirical evidence suggests that high-tech, retailers, and automotive firms fluctuate with the business cycle.
- Firms in utilities, transportation, and food are less dependent upon the business cycle.

# Determinants of beta

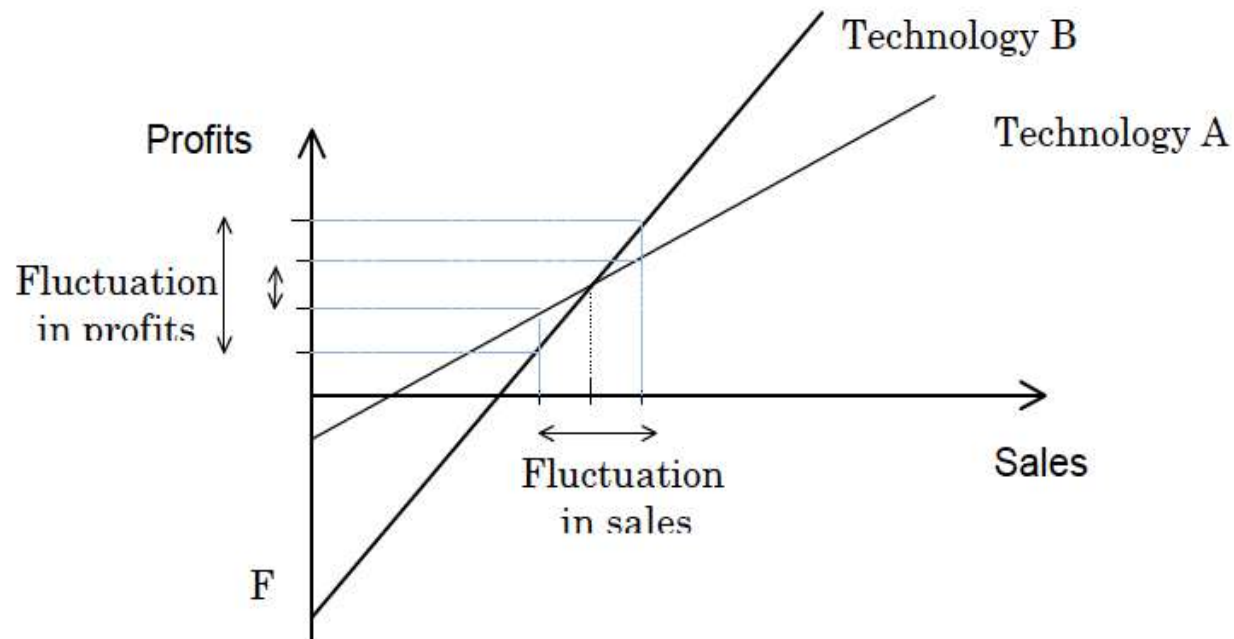
**2. Operating leverage:** measures the percentage change in EBIT for a given percentage change in sales or revenues.

- Operating leverage increases as fixed costs rise and as variable costs fall. Firms with high fixed costs and low variable costs are generally said to have high operating leverage.
- Example: A firm can choose either technology A or B

Technology A	Technology B
Fixed Cost: \$1000/year	Fixed Cost: \$2000/year
Variable Cost: \$7/unit	Variable Cost: \$6/unit
Price: \$10/unit	Price: \$10/unit
Contribution Margin: \$3 (= \$10-7)	Contribution Margin: \$4 (= \$10-6)

# Determinants of beta

- Technology B has both higher fixed cost and lower variable cost, so its contribution margin and operating leverage are greater.





# Determinants of beta

- Business risk depends both on the responsiveness of the firm's revenues to the business cycle and on the firm's operating leverage.
- The cyclicalities of a firm's revenue is a determinant of the firm's beta. Operating leverage magnifies the effect of cyclicalities on beta.

# Determinants of beta

**3. Financial leverage:** measures the extent to which a firm relies on debt. It refers to the firm's fixed costs of *financing*.

- The beta of a *levered* firm's stock is the firm's **equity beta**. The beta of a levered firm's asset is the firm's **asset beta**, which is different from the beta of its equity.
- When there are no taxes,

$$\beta_{Asset} = \frac{Debt}{Debt + Equity} \times \beta_{Debt} + \frac{Equity}{Debt + Equity} \times \beta_{Equity}$$

# Determinants of beta

- In practice,  $\beta_{Debt}$  is very low. If we make the common assumption that  $\beta_{Debt} = 0$ , then

$$\beta_{Asset} = \frac{Equity}{Debt + Equity} \times \beta_{Equity} \quad \text{or} \quad \beta_{Equity} = \beta_{Asset} \left(1 + \frac{Debt}{Equity}\right)$$

- It follows that  $\beta_{Asset} < \beta_{Equity}$ , if  $Debt > 0$ .
- The beta of the unlevered firm must be less than the beta of equity in an otherwise identical levered firm.

# Example

Consider a tree-growing company, which is currently all equity and has a beta of 0.8. The firm has decided to move to a capital structure of one-part debt to two-part equity.

- What is the firm's asset beta?

Since the firm is staying in the same industry, its asset beta should remain at 0.8.

- What is its equity beta?

Assuming zero beta for its debt, its equity beta is given by

$$\beta_{Equity} = \beta_{Asset} \left(1 + \frac{Debt}{Equity}\right) = 0.8 \left(1 + \frac{1}{2}\right) = 1.2$$

# Example

**Important Caveat:** This does not say that the beta of the firm's assets is determined by  $\beta_{Debt}$ ,  $\beta_{Equity}$  and its debt-equity ratio. Instead, the beta of the firm's assets is determined by the risk of the revenues and costs accruing to the firm.

- The debt-equity ratio affects  $\beta_{Equity}$  and eventually  $\beta_{Debt}$  (when bankruptcy comes into play);
- $\beta_{Assets}$  is constant throughout.
- The higher the debt-equity ratio, the higher  $\beta_{Equity}$ .

# Intuition for the CAPM Formula

- The risk premium of individual stocks depends on the size of their systematic risk (beta) as well as the market risk premium.
- The market risk premium is increasing in the risk of the market portfolio and the degree of risk aversion of the average investor.
- The beta of a portfolio is the weighted average of the betas of the assets in the portfolio:

$$\beta_p = \sum_i w_i \beta_i$$

- The CAPM holds not only for single risky asset, but also holds for portfolios.

# Assumptions of the CAPM

- Investors can buy and sell all securities at competitive market prices (no taxes or transaction costs) and borrow and lend at the risk-free interest rate.
- Investors hold only the efficient portfolios of traded securities.
- Investors have homogeneous expectations regarding the volatilities, correlations, and expected returns of securities.

# CAPM Example

- Assume the risk-free return is 5% and the market portfolio has an expected return of 12% and a standard deviation of 44%.
- ATP Oil and Gas has a standard deviation of 68% and a correlation with the market of 0.91.
- **What is ATP's beta with the market?**
- **Under the CAPM assumptions, what is ATP's expected return?**



## CAPM Example

$$\beta_i = \frac{SD(R_i) \times \text{Corr}(R_i, R_{Mkt})}{SD(R_{Mkt})} = \frac{(.68)(.91)}{.44} = 1.41$$

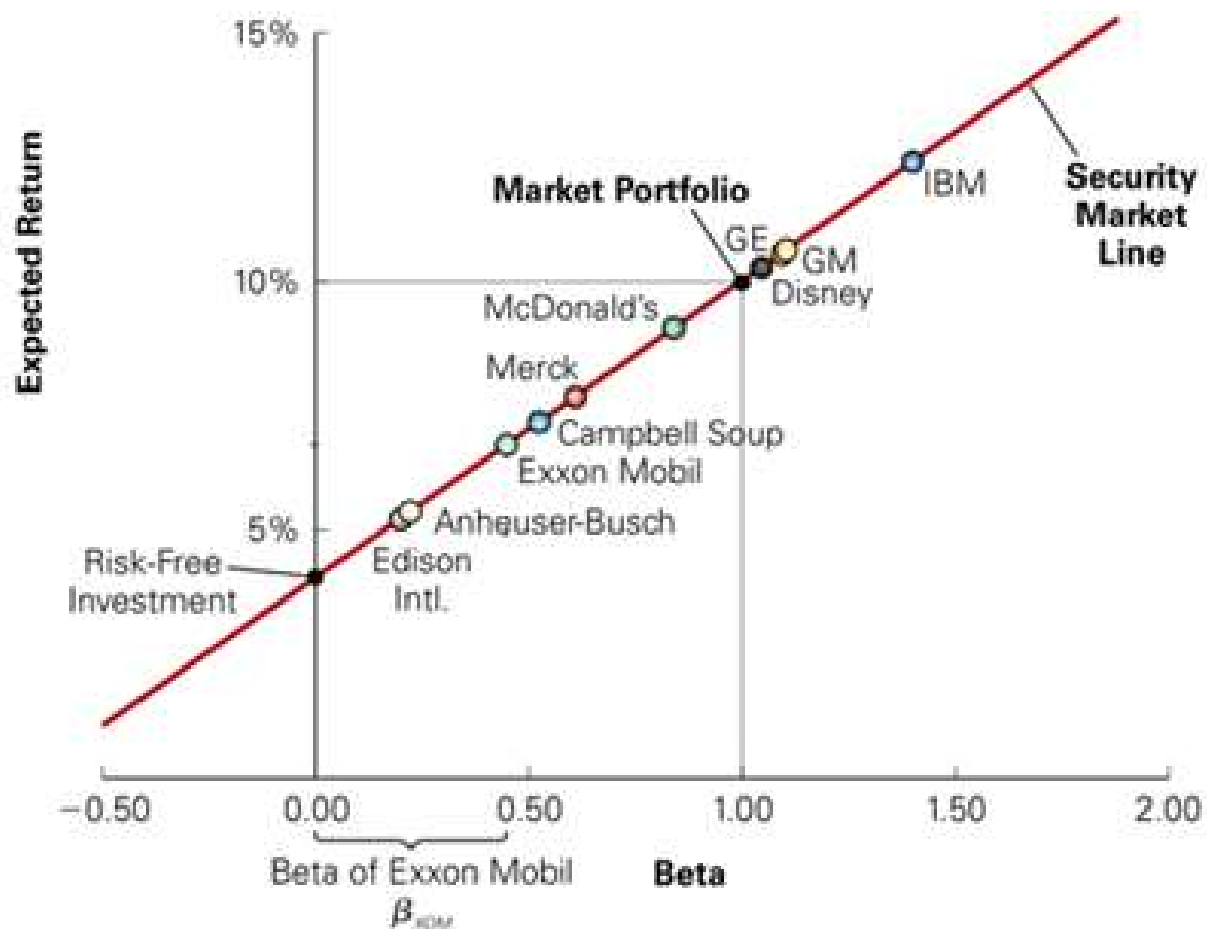
- Thus, the investors demand an expected return of 14.87% to compensate for the risk associated with holding this stock

$$E[R_i] = r_f + \beta_i \cdot (E[R_{Mkt}] - r_f) = 5\% + 1.41(12\% - 5\%) = 14.87\%$$

# The Security Market Line

- We can plot the main CAPM equation in a graph with now  $\beta_i$  on the x-axis
- When plotted in a graph, the linear relationship between a stock's beta and its expected return is called the **security market line (SML)**
  - According to the CAPM, if the expected return and beta for individual securities are plotted, they should all fall along the SML
  - If that does not hold, investors will buy (sell) the undervalued (overvalued) asset until it is back on the SML

# The Security Market Line



# Alpha

- The difference between a stock's expected return and its required return according to the CAPM is called the stock's **alpha** (or **Jensen's alpha**)

$$\alpha_i = E(R_i) - [r_f + \beta_i(E(R_{Mrk}) - r_f)]$$

- The CAPM predicts that all alphas should be zero.

# Alpha

- Stocks with positive alphas are attractive stocks that are relatively undervalued
  - Their expected returns are higher than what they should be according to the CAPM
  - I.e., their expected returns are higher than implied by their riskiness (as measured by the CAPM)
- Investors can improve the performance of their portfolios by buying stocks with positive alphas and by selling stocks with negative alphas
- Mutual fund managers try to identify stocks with positive alphas
- Over time, as more and more investors buy such stocks, their price increases and their expected return decreases, which means that their alpha goes to zero over time

# Example

- Suppose that the risk-free rate is 5% and the market portfolio has an expected return of 13% with a volatility of 18%.
- Monsters Inc. has a 24% volatility and a correlation with the market of 0.60
- Suppose that you are convinced that Monsters' expected return is 12%
- What is Monsters' expected return according to the CAPM? What is its alpha?

## Example

- The beta of Monsters' is:

$$\beta_{Monsters} = \frac{SD(R_{Monsters})Corr(R_{Monsters}, R_{Mkt})}{SD(R_{Mkt})} = \frac{0.24 \times 0.6}{0.18} = 0.80$$

Hence its expected return according to CAPM is:

$$\begin{aligned} E(R_{Monsters}) &= r_f + \beta_{Monsters}[E(R_{Mkt}) - r_f] \\ &= 0.05 + 0.8(0.13 - 0.05) = 0.114 = 11.4\% \end{aligned}$$

- Its CAPM-implied alpha is:

$$\alpha_{Monsters} = 12\% - 11.4\% = 0.6\%$$

# Example

- The beta of a portfolio P is just the weighted average beta of the securities in the portfolio.

$$\beta_P = \frac{\text{Cov}(R_P, R_{Mkt})}{\text{Var}(R_{Mkt})} = \frac{\text{Cov} \left( \sum_i x_i R_i, R_{Mkt} \right)}{\text{Var}(R_{Mkt})} = \sum_i x_i \frac{\text{Cov}(R_i, R_{Mkt})}{\text{Var}(R_{Mkt})} = \sum_i x_i \beta_i$$



## Beta of portfolio - Example

- Suppose the stock of the 3M Company (MMM) has a beta of 0.69 and the beta of Hewlett-Packard Co. (HPQ) stock is 1.77.
- Assume the risk-free interest rate is 5% and the expected return of the market portfolio is 12%.
- **What is the expected return of a portfolio of 40% of 3M stock and 60% Hewlett-Packard stock, according to the CAPM?**

## Beta of portfolio - Example

### **Solution**

$$\beta_P = \sum_i x_i \beta_i = (.40)(0.69) + (.60)(1.77) = 1.338$$

$$E[R_i] = 5\% + 1.338(12\% - 5\%) = 14.37\%$$

The expected return of the portfolio is 14.37%