MACROECONOMICS
73-240

Lecture 15

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Solow Growth Model:
Optimal Savings

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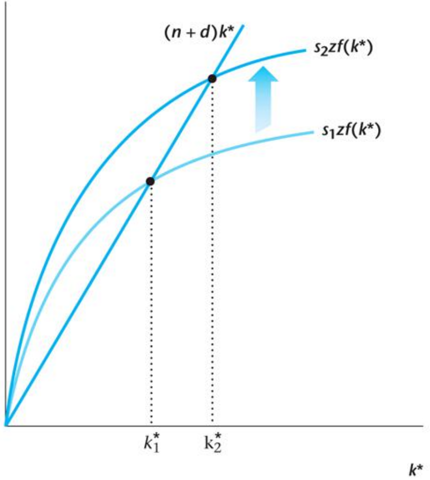
An increase in the saving rate

An increase in the saving rate raises investment



causing k_t to grow toward a new steady state:

Prediction:

- Higher s implies higher k^*
- and since $y = zf(k)$, higher k^* implies higher y^*
- Thus, the Solow model predicts that countries with higher rates of saving and investment will have higher levels of capital and income per worker in the long run.

Policy?

- Does this mean that the best policy is to promote saving as much as possible?
- Is the following policy a good one?
  - Tax consumption at rate 80% and use it to subsidize investment
- This means:
  - Always starve current generations for the hope that future generations have high income!

The Golden Rule: Introduction

- Different values of s lead to different steady states.
- How do we know which is the "best" steady state?
- -Definition- The "best" steady state has the highest possible consumption per person:

$$c^* = (1 - s)zf(k^*)$$

- An increase in s
  - Leads to higher k^* and y^*, which raises c^*
  - reduces consumption's share of income $(1 - s)$, which lowers c^*.
- So, how do we find the s and k^* that maximize c^*?

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The Golden Rule Capital per Worker

- Let

$$\begin{aligned} k_{gold} &= \textbf{the Golden Rule level of capital per work,} \\ &= \text{the steady state value of } k \text{ that maximizes consumption} \end{aligned}$$

- To find it, first express c^* in terms of k^*:

$$\begin{aligned} c^* &= (1-s)y^* \\ &= zf(k^*) - szf(k^*) \\ &= zf(k^*) - (d+n)k^* \end{aligned}$$

- The last equality is because in steady state $szf(k^*) = (d+n)k^*$

How to find golden rule Capital Per worker?

- Golden Rule level of Capital per worker solves

$$c^* = \max_{k^*} z f(k^*) - (d+n)k^*$$

- Solution:

$$MPK = d + n$$

or

$$z f_k(k^*) = d + n$$

let k_{gold} denote the solution

- Saving rate that achieves golden rule?

$$s_{gold} = \frac{(d+n)k_{gold}}{y_{gold}}$$

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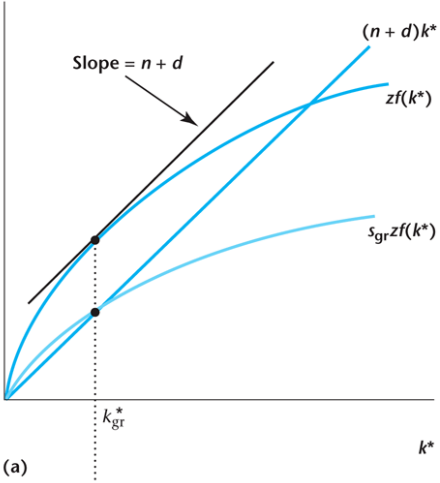
How to find golden rule Capital Per worker?

- Golden Rule level of Capital per worker solves

$$c^* = \max_{k^*} zf(k^*) - (d+n)k^*$$

- Let $zf(k^*) = zk^{*\alpha}$. Solve for golden rule savings rate.

Golden Rule Capital per Worker



Golden rule level, k_{gold} maximizes consumption per worker in the steady state

The transition to the Golden Rule steady state

- The economy does NOT have a tendency to move towards the Golden Rule steady state.
- Achieving the Golden Rule requires that policymakers adjust s.
- This adjustment leads to a new steady state with higher consumption.
- But what happens to consumption during the transition to the Golden Rule?

Overview of the Solow Model

- Saving affects steady state **level** of capital per worker
  - And therefore, steady state **level** of output per worker
- Golden rule level of capital gives highest **level** of consumption per worker
  - It only depends on technology (production function and depreciation rate)
- Saving does <u>not</u> affect long run growth
  - Total output grows at the rate of population growth

Testing the Solow Growth Model:

Predictions of the Solow Model vs. Data

- Long run k^* and y^* depend on s, z, n, d
- Suppose production is given by $Y = zK^\alpha N^{1-\alpha}$

$$\ln(y) = \ln\left(\frac{Y}{N}\right) = \ln z + ln\left(\frac{K}{N}\right)^\alpha$$

- In steady state, $k^* = \left(\frac{sz}{n+d}\right)^{1/(1-\alpha)}$
- Plugging in k^*:

$$\ln(y) = \frac{1}{1-\alpha}\ln z + \frac{\alpha}{1-\alpha}\ln s - \frac{\alpha}{1-\alpha}\ln(n+d)$$

- If we know s, n, d,z, and α can predict y

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Predictions of the Solow Model vs. Data

- Mankiw Romer Weil (1992) use $Y = K^\alpha (zN)^{1-\alpha}$, so their form of productivity is labor-augmenting productivity.

TABLE I
ESTIMATION OF THE TEXTBOOK SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	5.48	5.36	7.97
	(1.59)	(1.55)	(2.48)
ln(I/GDP)	1.42	1.31	0.50
	(0.14)	(0.17)	(0.43)
$\ln(n + g + \delta)$	−1.97	−2.01	−0.76
	(0.56)	(0.53)	(0.84)
$\bar{R}^2$	0.59	0.59	0.01

Source: Mankiw, Romer and Weil (1992)

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Predictions of the Solow Model vs. Data

- Mankiw Romer Weil (1992): that s and n affect GDP per capita in the directions suggested by the Solow Model

- From R^2 of regression: over 50% of variation in GDP per capita explained by differences in s and n

- But magnitudes are off. Data suggests capital share, $\alpha = 0.33$, unrestricted regression requires an $\alpha \approx 0.6$

Predictions of the Solow Model vs. Data

- Restricted regression: model says that coefficient on s and $n + d$ should be the same but of opposite sign

$$\ln(y) = \frac{1}{1-\alpha} \ln z + \frac{\alpha}{1-\alpha} \ln s - \frac{\alpha}{1-\alpha} \ln(n+d)$$

- α restricted to be $1/3$.

- R^2 falls from 0.59 to 0.28

Predictions of the Solow Model vs. Data

- Mankiw Romer Weil (1992) solution? Put in human capital

$$Y = zK^{\alpha}H^{\beta}N^{1-\alpha-\beta}$$

  where $H =$ human capital.

- Now there is a savings rate for physical capital, s_k

$$K' = (1-d)K + s_k Y$$

- And a savings rate for human capital s_h

$$H' = (1-d)H + s_h Y$$

- Depreciation of H and K assumed to be the same.

Predictions of the Solow Model vs. Data

- In per-capita terms:

$$k'(1+n) = (1-d)k + s_k z k^\alpha h^\beta$$

$$h'(1+b) = (1-d)h + s_h z k^\alpha h^\beta$$

- In steady state:

$$0 = k' - k = s_k z k^\alpha h^\beta - (n+d)k$$

$$0 = h' - h = s_h z k^\alpha h^\beta - (n+d)h$$

- Two equations, two unknowns, can solve for k, h in terms of exogenous s_h, s_k, z, n, d and parameters α, β
- output per worker in steady state:

$$\ln y^* = \ln z + \alpha \ln k^* + \beta \ln h^*$$

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Predictions of the Solow Model vs. Data

- Mankiw Romer Weil (1992): how to proxy s_h

  1. measure fraction of aged 12-17 in secondary school (enrollment rate)
  2. Multiply enrollment rate by fraction of working age population that is of school age (15-19)

- The product of the two items above gives an approximate measure of the percentage of working age population in secondary school

- Notice that the definition of human capital used here assumes the only investment in human capital is in terms of education (ignores investment in health)

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Predictions of the Solow Model vs. Data

ESTIMATION OF THE AUGMENTED SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985

Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	6.89	7.81	8.63
	(1.17)	(1.19)	(2.19)
ln(I/GDP)	0.69	0.70	0.28
	(0.13)	(0.15)	(0.39)
$\ln(n + g + \delta)$	−1.73	−1.50	−1.07
	(0.41)	(0.40)	(0.75)
ln(SCHOOL)	0.66	0.73	0.76
	(0.07)	(0.10)	(0.29)
$\overline{R}^2$	0.78	0.77	0.24

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Predictions of the Solow Model vs. Data

- Correct sign on all key variables
- Coefficients sum to 1 (recall we assume $Y = zK^{\alpha}H^{\beta}N^{1-\alpha-\beta}$
- Implied α from regression ≈ 0.3 (implied $\beta \approx 0.3$)
- R^2 about 0.77, close to 80% of differences in $\ln(y)$ explained by s_k, s_h, n, d.

Endogenous Growth

Changes in z

- The Solow Model predicts that if z continually grows
- then countries will continually grow as their steady state k^{ss} keeps being pushed further out (increasing)
- But how do we get this perpetual growth in z?
- Solow treats z as exogenous.
- But surely changes in z must come from somewhere.

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Changes in z

- There are many different types of endogenous growth models.
- We will only look at one simple type of endogenous growth model: Learning-by-doing

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Learning by doing

- Based on Romer (1989)
- Knowledge is accumulated during production or knowledge is a by-product of production
- Simple assumption: usage of capital creates more knowledge
- Example: using a computer helps me to build and design better computers.
- Knowledge is non-rival (everyone shares!)

Model

- Production function is:

$$Y = K^{\alpha}(zN)^{1-\alpha}$$

but now z is stock of economy wide knowledge that augments labor (makes labor more productive)

- Knowledge z is affected by stock of capital

$$z = AK$$

- Now we have the higher the capital K, the higher z is, $\implies$ both add to output

$$Y = K^{\alpha}(AKN)^{1-\alpha} = K(AN)^{1-\alpha}$$

Note: no diminishing marginal product in K!

Model

Equilibrium looks similar to Solow:

- Markets clear:
  - Labor: $N^d = N^s = N$
  - Goods: $C + I = Y$
  - Assets: $S = I$
- Population still grows:

$$N' = N(1+n)$$

- Capital accumulation is still given by:

$$K' = (1-d)K + I$$

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Model

- Starting from capital accumulation equation:

$$K' = (1-d)K + I$$

which we can re-write as:

$$K' - K = I - dK$$

substitute for $I = sY = sK(AN)^{1-\alpha}$

$$K' - K = sK(AN)^{1-\alpha} - dK$$

And use fact that $g_k = g_K - g_N$

$$k' - k = kg_k = k\left(\frac{sK(AN)^{1-\alpha} - dK}{K} - n\right)$$

which implies:

$$k' - k = sk(AN)^{1-\alpha} - (n+d)k$$

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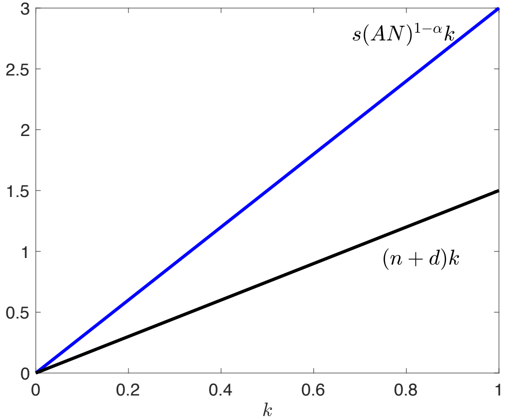
Model

- But now exists no steady state in k since N growing

$$k' - k = s(AN)^{1-\alpha}k - (n+d)k$$

- Even if N is constant, (case where $n = 0$), as long as $s(AN)^{1-\alpha} > (n+d)$, perpetual growth.



Implication of endogenous growth models

- If countries have different policies and institutions, they should have different *long-run* growth rates.
- Heavy investment here can have sustained long-run consequences
- Suggests there will never be convergence