MACROECONOMICS
73-240

Lecture 11

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Plan for this Lecture

- Search Model of Unemployment
    - Endogenizing the job-finding rate

Recap

At the end of last class:

- We showed how we could write down a law of motion for the unemployment rate
- We showed how sudden increases in the separation rate can lead to a spike in the unemployment rate
- And without any change in the job-finding rate, the unemployment rate would take time to go back to its mean level.

Key Question

- Where does $p(\theta)$ come from?

A Search Model of Unemployment

Key Ideas

- Unemployed workers and job openings/ vacancies do not instantaneously find each other
  - There exists search frictions in the labor market
  - Workers and firms undergo a search process to find each other
- because of the search process, unemployment as a stock variable adjusts slowly.

Simplified Search Model

- 1 period model
- A job is going to be firm-worker match
- Each consumer chooses whether to:
  - search for work in the market
  - do home production
- Consumers live one period, so assume they consume their wage.
- For simplicity: $u(c) = c$ where $c =$ income.

Model Overview: Workers

- Nobody is employed at start of period when economy begins
- A consumer searches for work if
  expected return from search > value of being out of labor force
- Let U = number of people search for a job (job-seekers)
- Let $P(U)$ = expected benefit of search as a function of U job seekers.

Note: In the textbook, Q is the number of job-seekers, here I have replaced U as the job-seekers.

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Expected benefit of search

But how do we calculate the expected benefit of search, $P(U)$?

- Possible outcomes of search
  - Unemployed: stay at home and produce b
  - Employed: payoff is wage w

$$P(U) = \underbrace{p(\theta)}_{\text{probability find job}} \times w + \underbrace{(1 - p(\theta))}_{\text{probability don't find job}} \times b$$

- But what are these probabilities?
- Will anybody choose to be out of the labor force? How does your answer depend on w?

Note: In the textbook, p_c is job-finding rate, here I have denoted $p(\theta)$ as the job-finding rate.

Model Overview: Firms

- Number of firms *endogenously* determined by model
- Firm needs a worker to produce output. (no capital!)
- One worker = 1 unit of labor.
- Output affected by productivity and labor. Hence output = $z \times 1 = z$
- Has to pay worker, so profits are:

$$\pi = z - w$$

Model Overview: Firms

- But a firm can't automatically hire a worker
- Firm must first post a vacancy and match with the worker to be able to produce
- Posting vacancies is costly, cost: κ
- A firm matched with a worker produces z units of output
- Let J be the value of creating a vacancy / job where

$$J = -\kappa + \underbrace{q(\theta)}_{\text{probability find a worker}} \times \pi$$

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Model Overview: Matching

- A match is between one firm and one worker
- No multiple offers or applications
- Timing of decisions (within a period)
  - 1. Search $\rightarrow$ 2. Match $\rightarrow$ 3. Produce
- Note: the more workers search, the easier for a firm to fill a job
- Note: the more vacancies posted, the easier for the unemployed to find a job.

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A Matching Function

- If there are U job seekers and V vacancies then

$$M = e\mathcal{M}(V, U)$$

  is the number of successful matches where $\mathcal{M}(V, U)$ is our matching function discussed in Lecture 10.

- A consumer who chooses to search for work will find a job with probability:

$$p(\theta) = \frac{M}{U} = \frac{e\mathcal{M}(V, U)}{U} = e\mathcal{M}(\frac{V}{U}, 1) = e\mathcal{M}(\theta, 1)$$

  where $\theta = \frac{V}{U}$

Optimization by Consumers

- Consumers who find jobs and work : receive w
- Consumers who don't find jobs: receive b unemployment benefit
- What is consumer's expected payoff to search?
  - Expected payoff to search:

$$P(U) = p(\theta)w + (1 - p(\theta))b = b + p(\theta)(w - b)$$

Optimization by Consumers

What can a worker choose?

- Can't choose b: exogenous
- Can't choose θ: can't affect how many vacancies and job-seekers there are
  - θ is endogenous but a single household and firm can't affect it
- Can't decide on wages unilaterally : w is endogenous.

Optimization by Consumers

What can a worker choose?

- Can choose whether to search for a job or stay out of the labor force
- Value of staying out of the labor force: home production b
- If $P(U) > b$, all households start out as unemployed job-seekers.
- If $P(U) < b$, no one wants to search for a job (this is uninteresting and not realistic)

$$P(U) = p(\theta)w + (1 - p(\theta))b = b + p(\theta)(w - b)$$

- if $w > b$, we always have $P(U) \geq b$

Optimization by Firms

- For firms, cost of vacancy is κ
- For firms, expected benefit of vacancy is (prob. of finding worker) $\times(z-w)$
- Prob of finding a worker: $q(\theta)=\frac{e\mathcal{M}(V,U)}{V}=e\mathcal{M}\left(1,\frac{1}{\theta}\right)$

Free Entry

How many firms choose to enter?

- if $\kappa > q(\theta)(z - w)$
- Cost is greater than expected benefit: firms want to exit.
- if $\kappa < q(\theta)(z - w)$
- Cost is less than expected benefit: more firms want to enter.
- Free entry: firms "enter" or post vacancies until expected benefit equals the cost:
$$q(\theta)(z - w) = \kappa$$
- θ ratio of vacancies to job-seekers determined through free entry.

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Summing Up Optimization

Our goal is to find θ, w and unemployment rate u.

- From consumer optimality

$$P(U) = b + p(\theta)(w - b)$$

- From firm optimality

$$q(\theta)(z - w) = \kappa$$

where $p(\theta) = e\mathcal{M}(\theta, 1)\,, \quad q(\theta) = e\mathcal{M}\left(1, \frac{1}{\theta}\right)$

- Two equations, but 3 unknowns!
- Need another condition to find equilibrium
- Solution: Nash Bargaining

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Wage determination

- Previously, in our lectures on Competitive Equilibrium, labor demand = labor supplied at wage rate $w^* = MPN = MRS$.
- But here, there exists frictions to the labor market. (Even if you want to supply labor, you have some probability of not finding a job)
- Workers are not paid their marginal product here.
- Assume wages are set through bilateral bargaining

Nash Bargaining

- Bargaining takes place only when the firm and worker meet
- The outside option of each party is to not form a match
- The firm's outside option: no production
- The worker's outside option: home production b
- No other offers or applications
- Firm has bargaining power since worker has only one offer
- Worker has bargaining power since firm has only one application

Nash Bargaining Outcome

- In Nash Bargaining, the firm and worker will each receive a share of the total gain to matching
- Worker's gain to matching: $w - b$
- Firm's gain to matching: $z - w$
- Total gain to matching: $z - b$
- We call the sum of the worker and firm's gain to matching Total Surplus of a match

Nash Bargaining Outcome

- In Nash Bargaining, the firm and worker will each receive a share of the total surplus

- Nash bargaining determines a wage w that splits the total surplus according to bargaining weights

- Let $0 < \alpha < 1$ be bargaining weight of the worker, then choosing w^* to maximize the product of worker and firm gain to matching

$$\max_{w} (z - w)^{1-\alpha} (w - b)^{\alpha}$$

Solving, we get:

$$w = b + \alpha(z - b)$$

Observe that the firm gives the worker b PLUS a share of the surplus as her wage.

Nash Bargaining Outcome

- Let's look at the wage w that comes of out of bargaining

$$w = b + \alpha(z - b)$$

- Note that firm must give the worker at least b, otherwise if worker gets $< b$, better off at home.
- The bargaining weight α determines how much of the surplus the worker gets to take home.
- If the worker has all the bargaining weight, $\alpha = 1$, she takes home the whole surplus.
- If the worker has zero bargaining weight, she gets none of the surplus
- If the worker has bargaining weight $\alpha = 0.5$, she gets ...

Equilibrium

- Knowing w, we can go back to the firm's optimality condition (free entry condition), and figure out labor market tightness θ

$$\kappa = (z - w)q(\theta) = (z - w)\underbrace{e\mathcal{M}\left(1, \frac{1}{\theta}\right)}_{q(\theta)}$$

We can plug in for $w = b + \alpha(z - b)$ and we have:

$$\kappa = (1 - \alpha)(z - b)e\mathcal{M}\left(1, \frac{1}{\theta}\right)$$

Implicitly, can solve for θ given parameters α, κ and exogenous variables b, z, e.

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Equilibrium Outcomes

- Once we know θ, can figure out unemployment rate and other equilibrium outcomes

- At end of period, we have:
  - Unemployment Rate:

$$u = \frac{U(1 - p(\theta))}{U} = 1 - p(\theta) = 1 - e\mathcal{M}(\theta, 1)$$

    where U = labor force since all individuals were inititaly unemployed in our problem.

  - Output:

$$Y = p(\theta)Uz$$

    Only matched individuals can produce.

Search Model of Unemployment

- Congratulations! You just went through a basic search model of unemployment!
- The originators of this model (Peter Diamond, Dale Mortensen and Christopher Pissarides) won a Nobel prize in economics in 2010 for their work on labor market search!
- https://www.nobelprize.org/prizes/economics/2010/press-release/

What Happens to the Search Equilibrium when exogenous variables change?

Change in b

- Suppose unemployment benefit b increases.
- What happens to w, θ, u?

Change in b

- Focus first on w
- From Nash Bargaining: we know

$$w = b + \alpha(z - b)$$

- Rearrange to get wages:

$$w = \alpha z + b(1 - \alpha)$$

- which implies w is increasing in b:

$$\frac{\partial w}{\partial b} = 1 - \alpha$$

where $0 \leq \alpha \leq 1$.

Change in b

- Rising w due to increase in $b \rightarrow$ firm has to give worker higher wages to make the worker willing to participte:

- This in turn implies firm's profits $\pi \downarrow$

- Return to firm's optimality condition:

$$\kappa = q(\theta)(z - w)$$

- plugging for w

$$\kappa = q(\theta)(1 - \alpha)(z - b)$$

- Holding all else constant, posting a job is less valuable: vacancies fall, job-filling rate rises:

$$q(\theta) = e\mathcal{M}\left(1, \frac{1}{\theta}\right) = \left(\frac{\kappa}{(1 - \alpha)(z - b)}\right)$$

Change in b

- Job-filling rate rises as b rises

$$e\mathcal{M}\left(1, \frac{1}{\theta}\right) = \left(\frac{\kappa}{(1-\alpha)(z-b)}\right)$$

- But job filling rate is negatively related to labor market tightness, θ

- Which implies θ falls.

  (To see this for yourself, assume $e\mathcal{M}(V, U) = eV^{\gamma}U^{1-\gamma}$, and write down what $q(\theta)$ is)

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Change in b

- θ falls: what happens to expected benefit of search?
- On one hand: $b \uparrow$, so direct effect is to increase $P(U)$.
- On the other hand: $b \uparrow \implies \theta \downarrow \implies p(\theta) = e\mathcal{M}(\theta, 1) \downarrow$
- Job-finding rate, $p(\theta)$, falls.
- Impact on $P(U)$ ambiguous

Change in b

- So far, we know if b rises: $w \uparrow$.
- Unemployment:

$$u = \frac{\text{Total Unemployed}}{\text{Labor Force}} = \frac{(1 - p(\theta))U}{U} = 1 - p(\theta)$$

- Since $p(\theta)$ falls, unemployment rate $u \uparrow$.

Change in z

- Suppose productivity z increases.
- What happens to w, θ, u?

Change in z

- Increase in z raises total surplus, i.e. it raises all parties' gain from matching:
- From Nash Bargaining, we know the wage is

$$w = b + \alpha(z - b)$$

- Wage w rises if z rises

$$\frac{dw}{dz} = \alpha$$

- Firm profits:

$$\pi = z - w = (1 - \alpha)(z - b)$$

- Profits are also rising:

$$\frac{d\pi}{dz} = (1 - \alpha)$$

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Change in z

- Since profits are rising, firms find it more attractive to create jobs.
- More vacancies created until under free entry we have again:

$$\kappa = q(\theta)(1-\alpha)(z-b)$$

Note if firms post more vacancies, compete more with each other, job-filling rate should fall.

$$q(\theta) = e\mathcal{M}\left(1, \frac{1}{\theta}\right) = \frac{\kappa}{(1-\alpha)(z-b)}$$

$$\frac{dq(\theta)}{dz} = -(1-\alpha)\frac{\kappa}{[(1-\alpha)(z-b)]^2} < 0$$

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Change in z

- Job-filling rate falls, $q(\theta)\downarrow$. $q(\theta)$ negatively related to labor market tightness, θ.
- so we have $q(\theta)\downarrow$, $\theta\uparrow$.
- which implies $p(\theta)\uparrow$

$$p(\theta) = e\mathcal{M}(\theta, 1)$$

Change in z

- Job-finding rate $p(\theta)$ increasing in labor market tightness θ.
- Then two effects on expected benefit of a job, $P(U)$.
- Direct effect: $w = b + \alpha(z - b)$. Increase in z means higher w.
- Indirect effect: increase in z also now means higher probability of getting a job, $p(\theta) \uparrow$

$$P(U) = b + p(\theta)(w - b)$$

- $\implies P(U) \uparrow$

Change in z

- z increase, w increases, θ increases.
- Unemployment: $u = \frac{(1-p(\theta))U}{U} = 1 - p(\theta)$
- Since $p(\theta) \uparrow \implies u \downarrow$

Recap

Search Model of Unemployment

- Households choose to search if

$$P(Q) \geq \text{value of staying at home}$$

where $P(Q) = p(\theta)w + (1 - p(\theta))b$

- Firms enter until value of creating a job is driven to zero:

$$J = -\kappa + q(\theta)(z - w)$$

Under free entry, $\kappa = q(\theta)(z - w)$

- Wages are determined by Nash Bargaining:

$$w = b + \alpha \times \text{Total Gain to Matching}$$

Choosing Between Policies

Suppose you have the following information

- Matches $M = eV^{1-\alpha}U^{\alpha}$, $p(\theta) = \frac{M}{U}$, $q(\theta) = \frac{M}{V}$
- Suppose you are a consultant hired by the government and you are tasked with the objective of making households better off by increasing wages and employment
  .
- The government says it will collect a lump-sum tax T from all households (regardless of employment status) and use T to finance transfers. [Note: $T =$ transfers, no other govt spending]
- The government says it has 3 policies to choose from:

Choosing Between Policies

3 policies to choose from:

- Policy 1: Collect T from all households. Give unemployed individuals (not non-employed!) extra benefit, c. Note that if you are unemployed, you can already produce home goods b.
- Policy 2: Collect T from all households. Give firms a subsidy s whenever they create a job (i.e. post a vacancy).
- Policy 3: Collect T from all households. Give firms a subsidy s whenever they hire a worker.

Suppose the government's goal is to raise wages and employment. Which policy achieves this?