73-240 Handout: Search Model of Unemployment

1 Notation

- Aggregate Variables
  - u unemployment rate (endogenous)
  - U total job seekers (endogenous)
  - V total vacancies (endogenous)
  - M total matches (endogenous)
  - Y total output (endogenous)
- From Household
  - b home production (exogenous)
  - w wage rate (endogenous but determined through bilateral bargaining)
  - $p(\theta)$ probability of finding a job (endogenous but individual household cannot control)
  - $P(U)$ expected benefit of search (endogenous)
- From Firm
  - π = profits (endogenous)
  - z = productivity (exogenous)
  - w = wage rate (endogenous but determined through bilateral bargaining)
  - $q(\theta)$ probability of filling a job (endogenous but individual firm cannot control)
  - κ vacancy posting cost (parameter)
  - J expected value of creating a job (endogenous)
- Matches
  - M total matches, $M = e\mathcal{M}(V, U)$
  - e matching efficiency (exogenous)
  - $\mathcal{M}(V, U)$ matching function which takes as inputs job-seekers, U, and vacancies V.

- θ labor market tightness which is the ratio of vacancies to job-seekers, $\theta = \frac{V}{U}$

- Wage Determination
  - α bargaining weight of worker where $0 \leq \alpha \leq 1$. This is a parameter of the model

2 Policies to alleviate unemployment and raise wages

A key useful of the search model of unemployment is we can use it to predict how unemployment and wage rates would change if any of the exogenous variables in the economy change, e.g. a change in z or a change in e. Note that we can also use our search model of unemployment to figure out what policies the government should craft if its goal is to reduce unemployment and raise wages. The following outlines the outcomes from three different policies and explains to what extent each policy can achieve the government's objective.

For each policy scenario, we assume throughout that the government levies on all households regardless of their status, a lump-sum tax T to finance its policy. Note that under a balanced budget, this implies that the total tax collected must equal:

$$T \times \text{population of households } = \text{policy } \times \text{reciepients of policy}$$

Further, we assume throughout that the matching function in this economy is given by:

$$M = eV^{1-\gamma}U^{\gamma}$$

2.1 Policy 1: Tax all households lump-sum T, give unemployment benefit c

Policy 1 involved collecting a lump-sum tax T from all households and only giving the unemployed (i.e. those who searched but failed to find a job) benefit c in addition to what they already get from home production b.

2.1.1 Setting up the Household's problem

The household's decision each period is whether or not to search for a job. If the household chooses to stay out of the labor force, the household gets: $b - T$. If the household searches for a job, there are only two outcomes from searching for a job:

1 Find a job with probability $p(\theta)$ and receive after-tax wage $w - T$

2 Don't find a job with probability $1 - p(\theta)$ and receive after-tax home production and unemployment benefit $b + c - T$

Hence the expected value of search is:

$$P(U) = p(\theta)(w - T) + (1 - p(\theta))(b + c - T) = b + c + p(\theta)(w - b - c) - T$$

Notice that the unemployment benefit c makes it more attractive for the household to search, in fact so long as

$$w > b - \frac{1 - p(\theta)}{p(\theta)} c$$

all households will search for a job where from the above, where the above is a weaker condition than $w > b$ for workers to be willing to search for a job. Intuitively, this weaker condition arises because now even if you fail to find a job but searched for one, you get additional benefit c which a person out of the labor force would not have received.

2.1.2 Setting up the Firm's problem

The firm's problem is unchanged from the standard problem we considered in class under Policy 1. Since the tax is only levied on households and the unemployment benefit only given to households who fail to find a job, the firm's value of creating a vacancy takes the standard form of:

$$J = -\kappa + q(\theta)(z - w)$$

Under free entry, firms enter until cost is equal to expected benefit:

$$\kappa = q(\theta)(z - w)$$

2.1.3 Determining wages

We assume that wages are Nash-bargained after a firm and worker have met. Observe that the each agent's gain to matching and total surplus is given by:

	Gain to Matching
Worker	$(w - T) - (b + c - T) = w - (b + c)$
Firm	$z - w$
Total (Surplus)	$z - w + w - (b + c) = z - (b + c)$

Clearly, the total surplus is affected by the worker's larger outside option. Under Nash-bargaining, the wage can be solved from the following problem:

$$\max_w (z - w)^{1-\alpha}(w - (b + c))^{\alpha}$$

Solving, we get:

$$w^* = b + c + \alpha(z - b - c) = \alpha z + (1 - \alpha)(b + c)$$

Notice the Nash-bargained wage requires the firm to give the worker at least his/her outside option $b + c$ and his/her share of the total surplus. Observe that the wage is increasing in unemployment benefit c. Intuitively, this is because higher unemployment benefits make unemployment more attractive and expands the worker's outside option. For the firm to make employment attractive to the worker, it must now offer a higher wage. Hence w rises when c rises.

2.1.4 Equilibrium labor market tightness

Note that the higher wage w (due to c) comes at the expense of lower profits for the firms. To see this, plug w^* into $\pi = z - w$. In this case we get:

$$\pi = (1 - \alpha)(z - b - c)$$

Since profits are lower when unemployment benefit is higher because firms must now pay more to workers to make employment attractive, the lower profits make fewer firms want to enter the labor market, causing vacancies to fall.

$$\kappa = q(\theta)(1 - \alpha)(z - b - c)$$

and using the fact that $q(\theta) = e\theta^{-\gamma}$, we have:

$$\theta = \left(\frac{e(1 - \alpha)(z - b - c)}{\kappa} \right)^{1/\gamma}$$

So clearly, as $c \uparrow, \theta \downarrow$ as profits $\downarrow$. Fewer vacancies are created.

The decline in the ratio of vacancies-to-job-seekers makes it harder for workers to find jobs. The unemployment rate is given by:

$$u = \frac{(1 - p(\theta))U}{U} = 1 - p(\theta)$$

where U is all the job-seekers at the start of the period (notice that given our equilibrium wage , w^*, all households are job-seekers, so the labor force at the beginning of the period is equal to U and $U =$ population of households in this problem). Since $p(\theta) = e\theta^{1-\gamma}$, as $c \uparrow, \theta \downarrow$ implies that $p(\theta) \downarrow$. Again this makes sense since there are fewer vacancies for unemployed job-seekers to be matched to, job-finding rates are lower and unemployment rates are higher.

So Policy 1 achieves a higher wage rate but does not help to reduce unemployment. Note that Policy 1 also has the perverse effect of potentially causing a recession. Here output falls since z is constant, but the number of matched individuals (employed) falls:

$$\text{from } Y = p(\theta)Uz \quad \Longrightarrow \quad p(\theta) \downarrow \rightarrow Y \downarrow$$

2.2 Policy 2: Tax all households lump-sum T, give firms who create vacancy subsidy s

Policy 2 again levies a lump-sum tax T on all households, but gives firms who post a vacancy a subsidy s. Again we can set-up each agent's problem and check how the policy affects wages and unemployment.

2.2.1 Setting up the Household's problem

The household who chooses to stay out of the labor force gets: $b - T$. If the household searches for a job, there are only two outcomes from searching for a job:

1 Find a job with probability $p(\theta)$ and receive after-tax wage $w - T$

2 Don't find a job with probability $1 - p(\theta)$ and receive after-tax home production and unemployment benefit $b - T$

Hence the expected value of search is:

$$P(U) = p(\theta)(w - T) + (1 - p(\theta))(b - T) = b - T + p(\theta)(w - b)$$

Note that as long as $w > b$, the household prefers to search for a job.

2.2.2 Setting up the Firm's problem

The firm's now receives a subsidy whenever it posts a vacancy. The firm's value of creating a vacancy takes the form of:

$$J = -\kappa + s + q(\theta)(z - w)$$

Under free entry, firms enter until cost is equal to expected benefit:

$$\kappa = s + q(\theta)(z - w)$$

2.2.3 Determining wages

We assume that wages are Nash-bargained after a firm and worker have met. Observe that the each agent's gain to matching and total surplus is given by: Observe that since total surplus and worker's outside options are unchanged from the original standard problem considered in class, we have that the Nash-bargained wage is:

$$w = b + \alpha(z - b)$$

Since both z and b are not changing, Policy 2 has no impact on wages in our 1 period model.

	Gain to Matching
Worker	$(w-T)-(b-T)=w-b$
Firm	$z-w$
Total (Surplus)	$z-w+w-b=z-b$

2.2.4 Equilibrium labor market tightness

Note that the issuance of a subsidy whenever the firm creates a vacancy effectively lower the firm's cost of creating a vacancy. When firms find it cheaper to create a vacancy, more firms will enter the labor market to create vacancies until again under free entry, we have cost equal to expected benefit of a job restored. We can re-write the free entry condition as:

$$\kappa - s = q(\theta)(z-w)$$

Plug in for w and solve for θ:

$$\theta = \left(\frac{e(1-\alpha)(z-b)}{\kappa - s}\right)^{1/\gamma}$$

Clearly, one can show from the above that as $s\uparrow$, creating vacancies becomes cheaper and more firms enter, i.e. the ratio of vacancies-to-job-seekers, θ, rises.

Since $p(\theta)=e\theta^{1-\gamma}$, a higher vacancies-to-job-seekers ratio makes it easier for workers to find a job, so as $\theta\uparrow$, $p(\theta)\uparrow$. If its easier for workers to find a job, then the unemployment rate must fall.

Output in this world is also higher:

$$Y = p(\theta)Uz$$

2.3 Policy 3: Tax all households lump-sum T, give firms who hire workers subsidy s

Policy 2 again levies a lump-sum tax T on all households, but gives firms who hired a worker a subsidy s. Again we can set-up each agent's problem and check how the policy affects wages and unemployment.

2.3.1 Setting up the Household's problem

The household who chooses to stay out of the labor force gets: $b-T$. If the household searches for a job, there are only two outcomes from searching for a job:

1 Find a job with probability $p(\theta)$ and receive after-tax wage $w-T$

2 Don't find a job with probability $1 - p(\theta)$ and receive after-tax home production and unemployment benefit $b - T$

Hence the expected value of search is:

$$P(U) = p(\theta)(w - T) + (1 - p(\theta))(b - T) = b - T + p(\theta)(w - b)$$

Note that as long as $w > b$, the household prefers to search for a job.

2.3.2 Setting up the Firm's problem

The firm's now receives a subsidy whenever it hires a worker. A firm can only hire a worker if it met a worker. The firm's value of creating a vacancy takes the form of:

$$J = -\kappa + q(\theta)(z - w + s)$$

Under free entry, firms enter until cost is equal to expected benefit:

$$\kappa = q(\theta)(z - w + s)$$

2.3.3 Determining wages

We assume that wages are Nash-bargained after a firm and worker have met. Observe that the each agent's gain to matching and total surplus is given by: Observe that since total

	Gain to Matching
Worker	$(w - T) - (b - T) = w - b$
Firm	$z - w + s$
Total (Surplus)	$z - w + s + w - b = z - b + s$

surplus increases under the implementation of the subsidy. Essentially, the government by giving firm's a subsidy when it hires a worker improves the total gain to matching. Under Nash-bargaining, one can show that the Nash-bargained wage becomes:

$$w = b + \alpha(z - b + s)$$

Clearly, an increase in s by increasing total surplus, also raises w.

2.3.4 Equilibrium labor market tightness

Observe that the subsidy which is issued whenever firms hire a worker, raises total surplus and raises firm's profits:

$$\pi = z - w + s = (1-\alpha)(z-b+s)$$

Since the subsidy raises total surplus which in turn raises profits, more firms are incentivized to create vacancies, causing θ to go up.

$$\kappa = q(\theta)(1-\alpha)(z-b+s)$$

Solve for θ:

$$\theta = \left(\frac{e(1-\alpha)(z-b+s)}{\kappa}\right)^{1/\gamma}$$

Clearly, one can show from the above that as $s \uparrow$, profits are higher and more firms enter, i.e. the ratio of vacancies-to-job-seekers, θ, rises.

Since $p(\theta) = e\theta^{1-\gamma}$, a higher vacancies-to-job-seekers ratio makes it easier for workers to find a job, so as $\theta \uparrow$, $p(\theta) \uparrow$. If its easier for workers to find a job, then the unemployment rate must fall.

Note that Policy 3 is the only policy that raised w, and lowered unemployment rates.

Output in this world is also higher:

$$Y = p(\theta)Uz$$