Solution to Problem Set 4

Problem 1

$$P = 210 - Q$$
$$MC = 5$$

(a) Compute the profit maximizing price and quantity.

$$\max_{Q} \ \pi = (210 - Q) \cdot Q - 5 \cdot Q$$

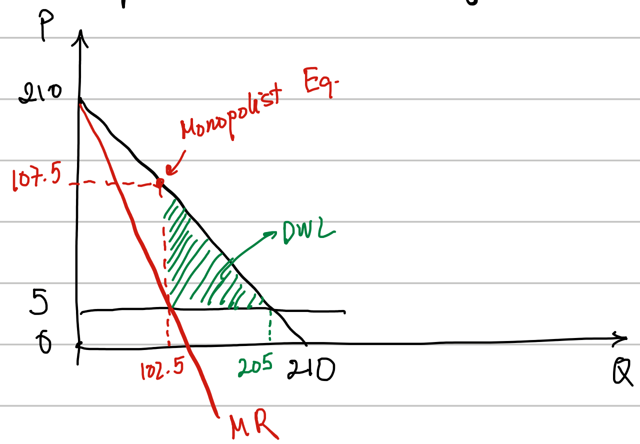
$$\rightarrow \frac{d\pi}{dQ} = 0 \rightarrow \underbrace{210 - 2Q}_{MR} \ \underbrace{-5}_{MC} = 0$$

$$\rightarrow 205 = 2Q$$

$$\rightarrow Q = 102.5$$

$$\rightarrow P = 210 - Q = 210 - 102.5 = 107.5$$

(b) Compute the deadweight loss



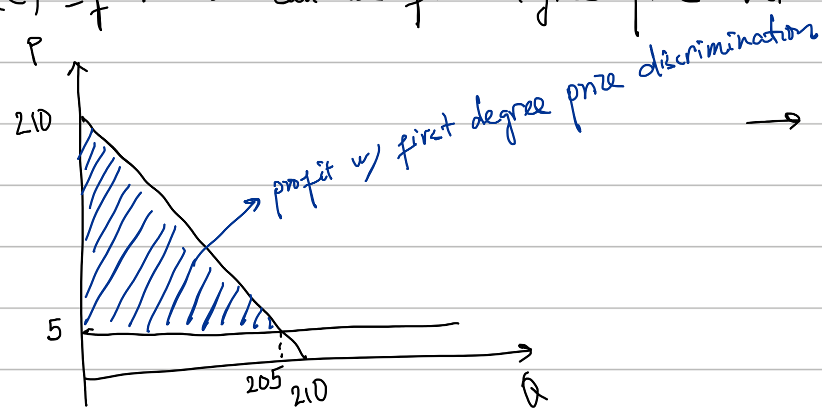
The socially optimal price is $P = MC = 5$

$\rightarrow$ socially optimal quantity is $Q = 210 - 5 = 205$

$$\rightarrow DWL = \frac{1}{2}(205 - 102.5)(107.5 - 5)$$

$$= 5253.125$$

(c) If Amazon can use first-degree price discrimination.



$$\rightarrow \pi = \frac{1}{2}(210 - 5) \cdot 205$$

$$= 21012.5$$

(d) (i) If Amazon can price discriminate between 2 markets.

$$\max_{Q^{us}, Q^{w}} \Pi = (210 - Q^{us}) Q^{us} + (70 - Q^{w}) \cdot Q^{w} - (Q^{us} + Q^{w}) \cdot 5$$

FOC: $[Q^{us}]$: $\dfrac{\partial \Pi}{\partial Q^{us}} = 210 - 2Q^{us} - 5 = 0 \rightarrow Q^{us} = 102.5$

$\rightarrow P^{us} = 210 - 102.5 = 107.5$

$[Q^{w}]$: $\dfrac{\partial \Pi}{\partial Q^{w}} = 70 - 2Q^{w} - 5 = 0 \rightarrow Q^{w} = 32.5$

$\rightarrow P^{w} = 70 - 32.5 = 37.5$

(ii) If Amazon cannot price discriminate:

- Case 1: If only 1 market is active (i.e., US market)

$\rightarrow P = 210 - Q$ if $P \geq 70 \rightarrow Q \leq 210 - 70 = 140$

In this case: $\max_{Q \text{ s.t. } Q \leq 140} \Pi = (210 - Q) \cdot Q - 5 \cdot Q$

FOC: $\dfrac{d\Pi}{dQ} = 210 - 2Q - 5 = 205 - 2Q$

At $Q = 140$; $\dfrac{d\Pi}{dQ}(140) = 205 - 2 \times 140 < 0 \rightarrow$ the firm has an incentive to price above 70 and produce <u>LESS</u> than 140.

$\rightarrow$ FOC binds $\rightarrow 205 - 2Q = 0$

$\rightarrow Q = 102.5$

$\rightarrow P = 210 - 102.5 = 107.5$

$\rightarrow \Pi = 107.5 \times 102.5 - 5 \times 102.5$

$= 10506.25$

- Case 2: If both markets are active: $P \leq 70 \rightarrow Q \geq 140$

The aggregate demand is given by

$$Q = Q^{us} + Q^{w}$$

At $P \rightarrow \left.\begin{array}{l} Q^{us} = 210 - P \\ Q^{w} = 70 - P \end{array}\right\} \begin{array}{l} \rightarrow Q = 280 - 2P \\ \rightarrow P = \dfrac{280 - Q}{2} \end{array}$

$\Rightarrow$ Firm's profit maximization problem:

$$\max_{Q \text{ s.t. } Q \geq 140} \frac{280 - Q}{2} \cdot Q - 5Q$$

FOC: $[Q]: \dfrac{d\pi}{dQ} = 140 - Q - 5 = 135 - Q$

At $Q = 140$; $\dfrac{d\pi}{dQ}(140) < 0 \rightarrow$ the firm wants to <u>DECREASE</u> quantity to improve profit.

$\rightarrow$ The profit maximizing quantity is $Q = 140$ (the lowest possible)

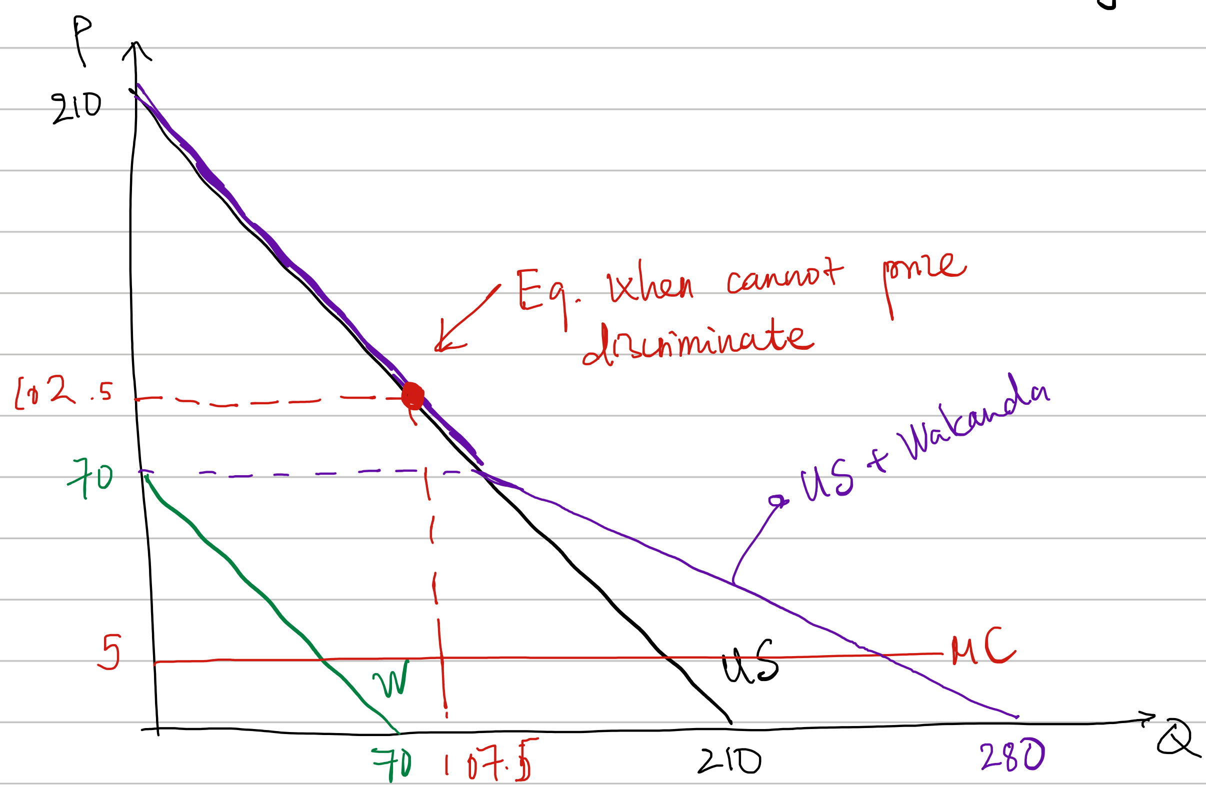
$P = 140 - \dfrac{Q}{2} = 70$

$\rightarrow$ In this case, $\pi = 70 \times 140 - 5 \times 140 = 9100 < \pi^{case\ 1}$

Therefore, the profit maximizing price and quantity is $P = 107.5$

$Q = 102.5$

(only selling in the US)



(iii) Wakanda is better off.
US is indifferent.

(iv) $TC(Q) = \frac{1}{2}Q^2$

With third-degree price discrimination

$$\max_{Q^{us}, Q^{w}} \pi = (210 - Q^{us})Q^{us} + (70 - Q^{w}) \cdot Q^{w} - \frac{1}{2}(Q^{us} + Q^{w})^2$$

FOC: $[Q^{us}]$: $\frac{\partial \pi}{\partial Q^{us}} = 210 - 2Q^{us} - (Q^{us} + Q^{w}) = 0$

$\longrightarrow 210 - 3Q^{us} - Q^{w} = 0 \quad (*)$

$[Q^{w}]$: $\frac{\partial \pi}{\partial Q^{w}} = 70 - 2Q^{w} - (Q^{us} + Q^{w}) = 0$

$\longrightarrow 70 - 3Q^{w} - Q^{us} = 0 \quad (**)$

2 linear equations with two unknowns:

$$\left.\begin{aligned} (*)&: 210 - 3Q^{us} - Q^{w} = 0 \\ 3(**)&: 210 - 9Q^{w} - 3Q^{us} = 0 \end{aligned}\right\} \rightarrow \begin{aligned} Q^{w} &= 0 \\ Q^{us} &= 70 \end{aligned}$$

$\rightarrow P^{us} = 210 - 70 = 140$

P^{w} is any value ≥ 70.

$\rightarrow$ Only sell to the US market because of decreasing returns to scale.

(e) Example: Quantity discount.

$\boxed{\text{Problem 2}}$

$P = 20 - Q = 20 - q_1 - q_2$

$MC_1 = MC_2 = 2$

(a) Firm 1's profit maximization problem:

$$\max_{q_1} \pi_1 = q_1 \cdot (20 - q_1 - q_2) - 2 \cdot q_1 - \text{Fixed cost}$$

FOC: $[q_1]$: $\frac{\partial \pi_1}{\partial q_1} = 0 \longrightarrow 20 - 2q_1 - q_2 - 2 = 0$

$\longrightarrow 18 - 2q_1 - q_2 = 0$

$\longrightarrow q_1 = \frac{18 - q_2}{2}$

(b) Firm 2's profit maximization problem

$$\max_{q_2} \pi_2 = q_2(20 - q_1 - q_2) - q_2 \cdot 2 - \text{Fixed Cost}$$

FOC: $[q_2]$: $\frac{\partial \pi_2}{\partial q_2} = 0 \rightarrow 20 - q_1 - 2q_2 - 2 = 0$

$\rightarrow 18 - q_1 - 2q_2 = 0$

$\rightarrow q_2 = \frac{18 - q_1}{2}$

Combining with firm 1's best response: $q_1 = \frac{18 - q_2}{2}$

$\rightarrow$ 2 linear equations, 2 unknowns

- First method: plug q_1 into q_2's best response

$\rightarrow q_2 = 9 - \frac{1}{2}\left(\frac{18 - q_2}{2}\right)$

$\rightarrow \frac{3}{4} q_2 = \frac{9}{2} \rightarrow q_2 = 6 \rightarrow q_1 = \frac{18 - 6}{2} = 6$

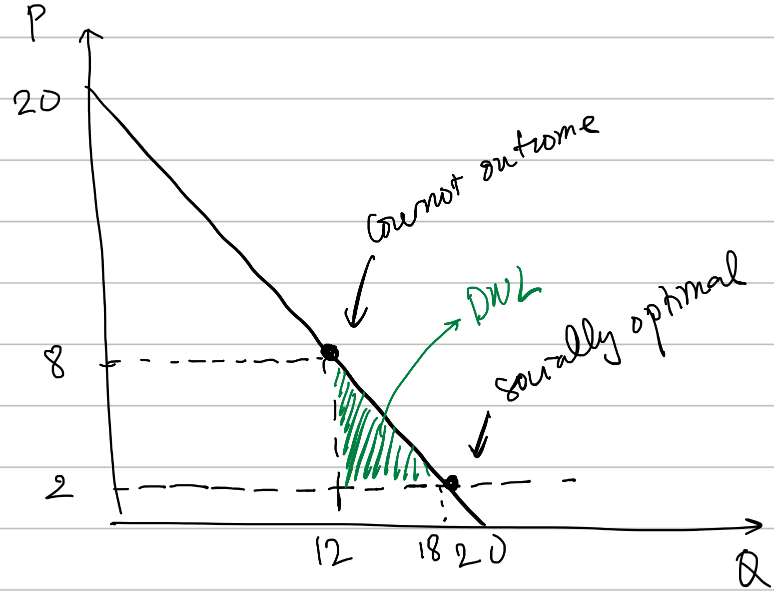
$\rightarrow P = 20 - (6 + 6) = 8$

- Second method: Note that firms are symmetric

$\rightarrow$ In equilibrium; $q_1 = q_2$ (because they are symmetric)

$\rightarrow q_1 = \frac{18 - q_1}{2} \rightarrow \frac{3}{2} q_1 = 9 \rightarrow q_1 = 6 = q_2$

$\rightarrow P = 8$



$$DWL = \frac{1}{2} \cdot (18 - 12) \cdot (8 - 2)$$

$$= 18$$

(c) Suppose that firm 1 moves first, firm 2 moves second.
<u>ALWAYS</u> solve the problem backward.

- Firm 2's profit maximization problem: Identical to (b)

$$q_2 = \frac{18 - q_1}{2}$$

- Firm 1's profit maximization problem:

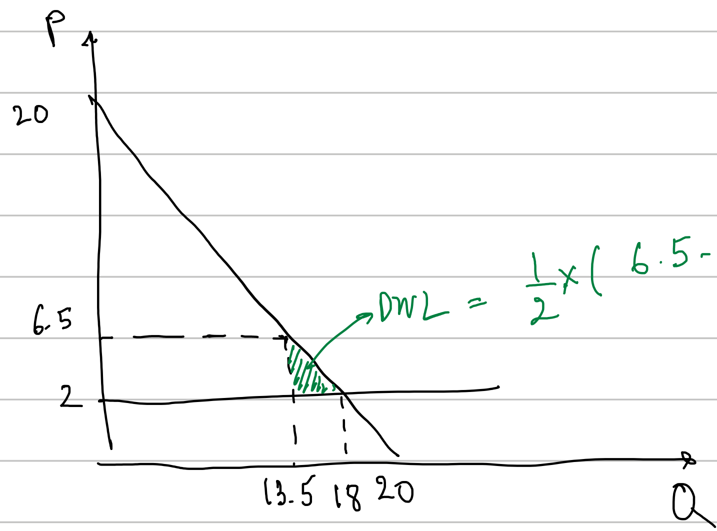
$$\max_{q_1} \; q_1 \left(20 - q_1 - \underbrace{\frac{18 - q_1}{2}}_{} \right) - 2q_1$$

this is q_2. Firm 1 anticipates q_2 will respond to its choice of q_1

$\rightarrow$ FOC: $[q_1]$: $\frac{\partial \pi}{\partial q_1} = 0 \rightarrow 20 - q_1 - \frac{18 - q_1}{2} - q_1 + \frac{q_1}{2} - 2 = 0$

$\rightarrow 9 - q_1 = 0 \rightarrow q_1 = 9 \rightarrow q_2 = 4.5$

$\rightarrow P = 20 - 9 - 4.5$
$= 6.5$



$\text{DWL} = \frac{1}{2} \times (6.5 - 2) \times (18 - 13.5) = 10.125$

Problem 3

(a) $P^{WF} = P^{TJ} = MC = 3.$

(b) $P^{WF} = 4 - \varepsilon$; $P^{TJ} = 4$

$\rightarrow$ Only whole foods sell; Trader Joe's will not be able to match the price and sell below its cost.