INTERMEDIATE MICROECONOMICS

PRODUCER THEORY

SPRING 2019, PROFESSOR ANH NGUYEN

Introduction

- We will study firms' optimization problem
  - Assume the firm is a price-taker: Input and output prices are fixed
  - Consider two inputs for production
  - If one input is variable while the other is fixed (in the short run), what is the optimal level of output?
  - If both inputs are variable (in the long run), what is the optimal choice of the output and the inputs?

1. Short-Run Production

Short-Run Production

- We consider a short-run model where one input is variable while the other is fixed.
- We study the short-run production function.
- Key concepts
  - Properties of a typical production function
  - Marginal product and average product
  - MP intersects AP at its maximum
- Reading: pp. 324-359

Production

- The various ways that a firm can transform inputs into the maximum amount of output are summarized in the **production function**.
- Assuming labor (L) and capital (K) are the only inputs, the production function is

$$q = f(L, K)$$

Production: Short vs. Long Run

- A firm can more easily adjust its inputs in the long run than in the short run.
  - The **short run** is a period of time so brief that at least one factor of production cannot be varied (the fixed input).
  - The **long run** is a long enough period of time that all inputs can be varied.

Short Run Production

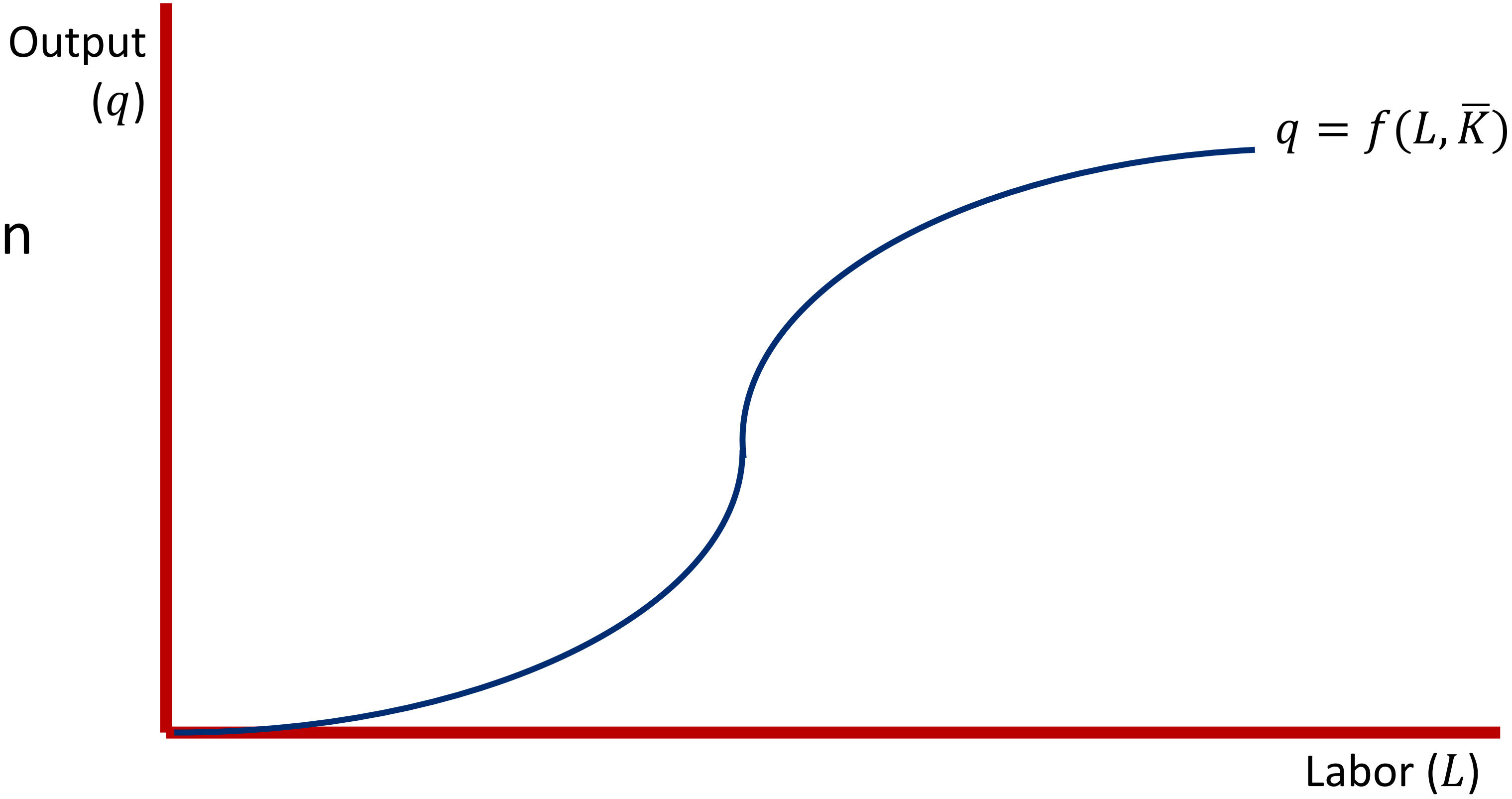
- In the short run, we assume that capital is a fixed input and labor is a variable input.

$$q = f(L, \overline{K})$$

  - q is output, or total product
  - L is labor
  - $\overline{K}$ is capital, **fixed** in the short run

Short Run Production: Graph

- At a given capital $(\bar{K})$
- $f(0, \bar{K}) = 0$
- Output is increasing in input



Marginal Product

- The ***marginal product of labor*** is the additional output produced by an additional unit of labor, holding all other factors constant.

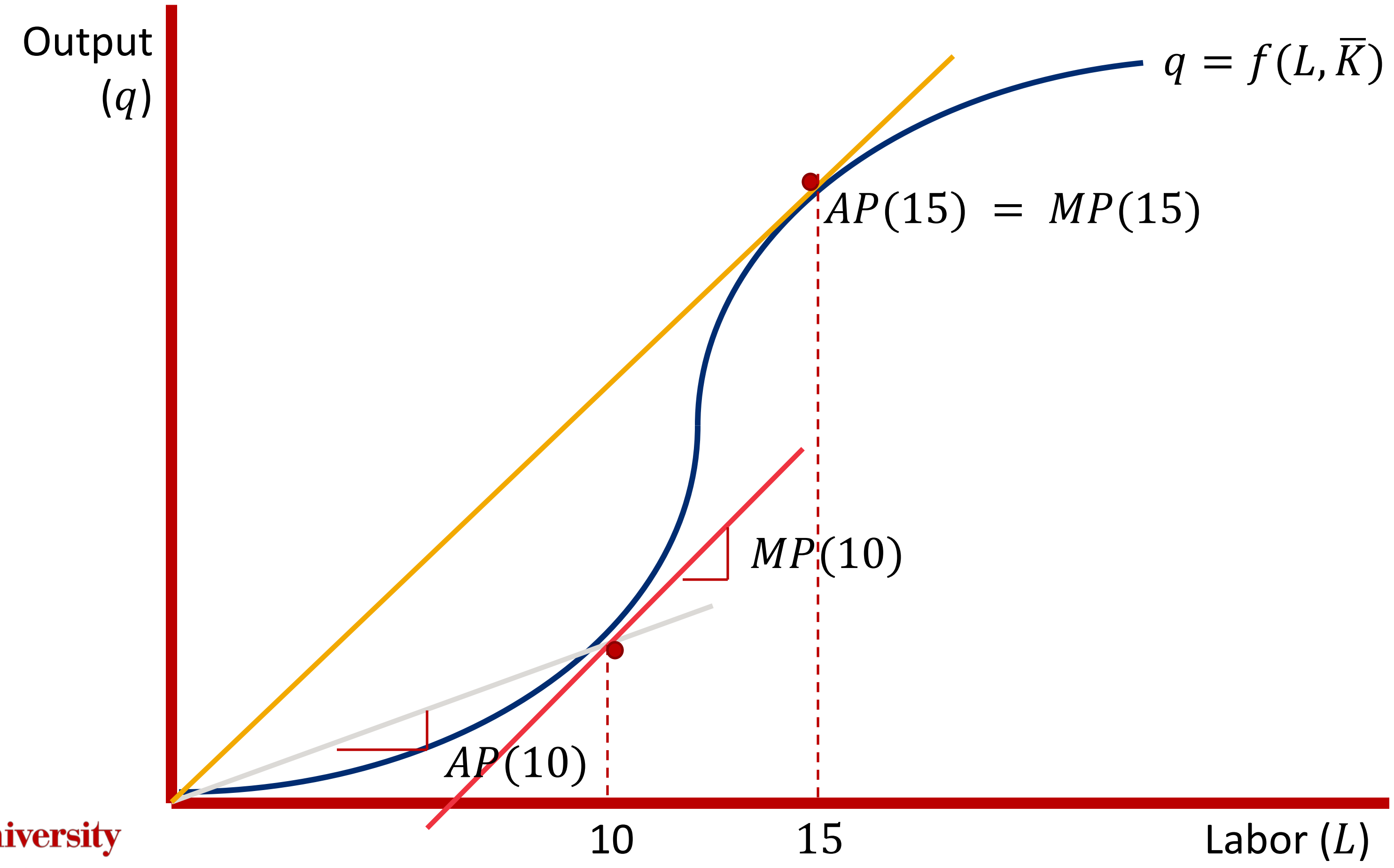
$$MPL = \frac{\partial q}{\partial L} = \frac{\partial f(L, \bar{K})}{\partial L}$$

Average Product

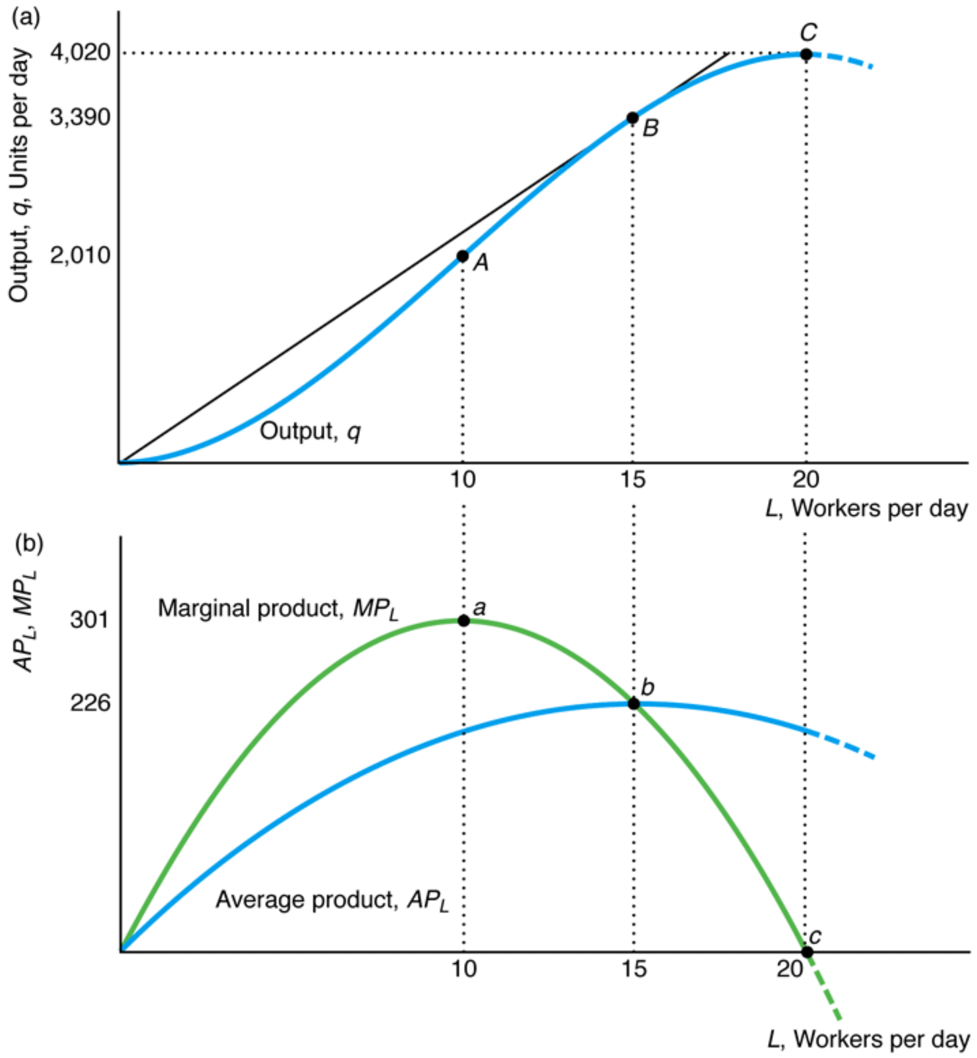
- The **average product of labor** is the ratio of output to the amount of labor employed.

$$APL = \frac{q}{L} = \frac{f(L, \bar{K})}{L}$$

Short Run Production: Graph



Short Run Production: Graph



- APL and MPL both first rise and then fall as L increases

- **MPL intersects APL at its maximum**

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APL and MPL

- When APL is the largest, APL = MPL. Why?
  - Calculus
  - Intuitively, assuming APL is decreasing:
    - If MPL>APL, then by using an additional unit of labor, you can increase APL.
    - If MPL<APL, then by decreasing the last unit of labor, you can increase APL.
    - Therefore, APL is maximized only if MPL=APL.

Diminishing Marginal Returns

- If a firm keeps increasing an input, holding all other inputs and technology constant, the corresponding increases in output will eventually becomes smaller.
  - Occurs at $L = 10$ in previous graph.
- Note that when MPL begins to fall, the total product is still increasing.

2. Short-Run Costs

Short-Run Costs

- We consider a short-run model where one input is variable while the other is fixed
- From a firm's production function, we derive the short-run cost function.
- Key concepts
  - Fixed cost and variable cost
  - Marginal cost and average cost
  - MC intersects AVC and AC at their **minimum** points
- Reading: pp. 324-359

Measuring Costs

- **Explicit costs** are direct, out-of-pocket payments for inputs such as labor, capital, energy, and materials.
- **Implicit costs** reflect a forgone opportunity.
- The **opportunity cost** of a resource is the value of the best alternative use of that resource.
  - Opportunity cost is the sum of implicit and explicit costs.

Measuring Costs

- Opportunity costs are not always easily observed, but should always be taken into account in production decisions.

- **Sunk costs**, past expenditures that cannot be recovered, are easily observed, but are never relevant in production decisions.
  - Sunk costs are NOT included in opportunity costs.
  - Example: Grocery store checkout line.

Short-Run Costs

- Recall that the short run is a period of time in which some inputs can be varied, while other inputs are fixed.

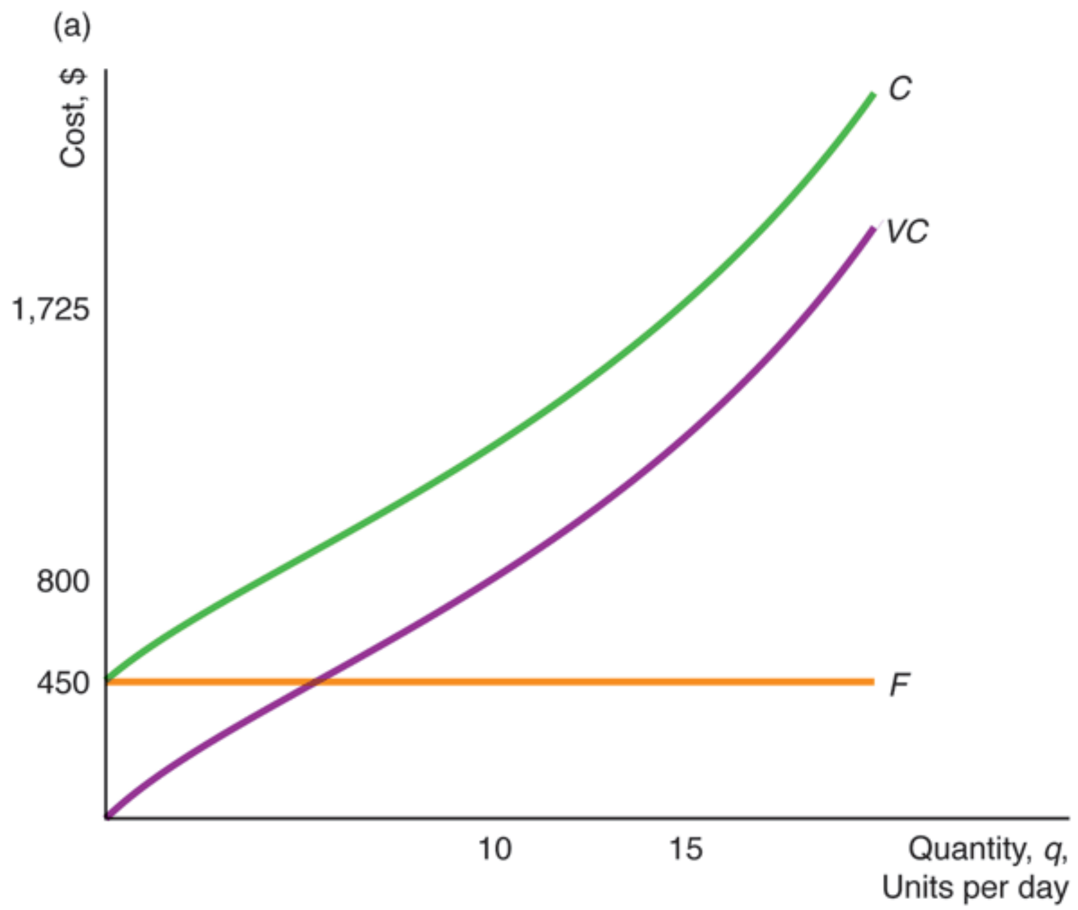
- In the short run, we assume labor is variable and capital is fixed.

Short-Run Costs

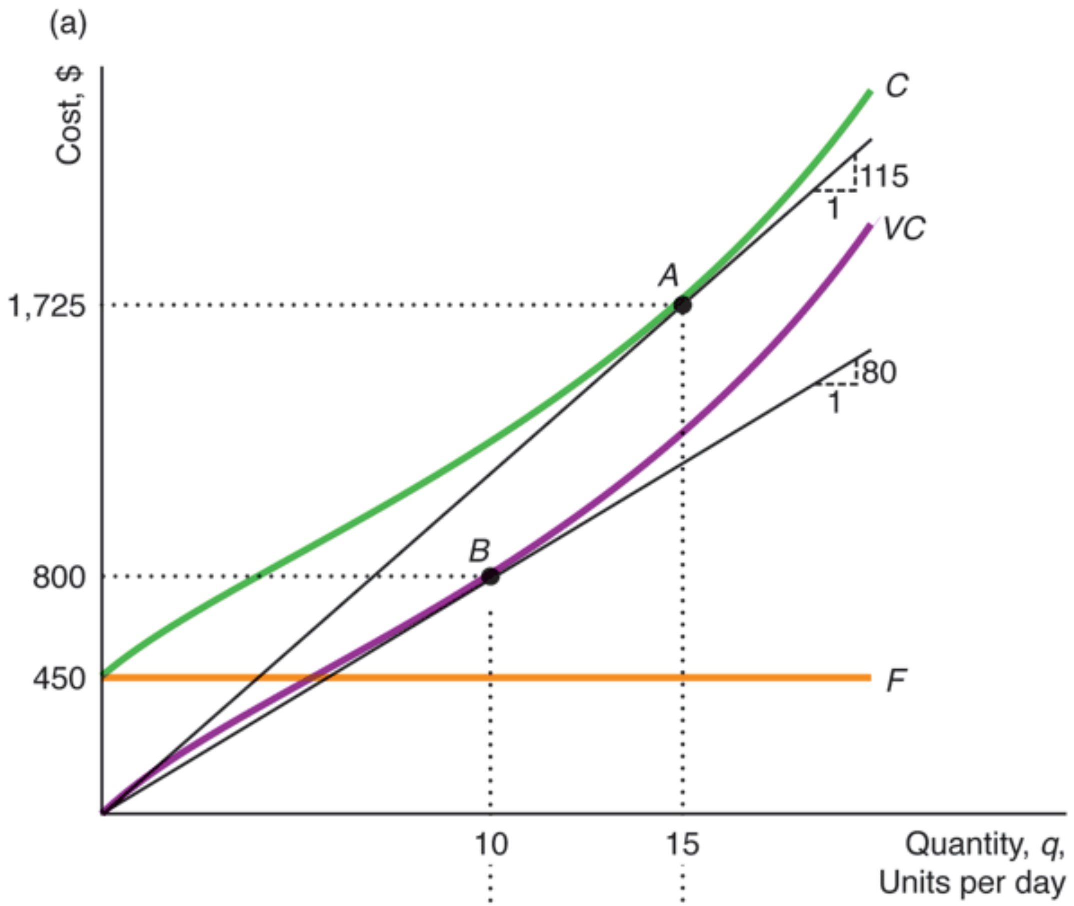
- **Fixed cost (F)**: Cost that doesn’t vary with the level of output (e.g. expenditures on land or production facilities).
- **Variable cost (VC)**:Production expense that changes with the level of output produced (e.g. labor cost, materials cost).
- **Total cost (C):** Sum of variable and fixed costs

$$C = VC + FC$$

Short-Run Cost Curves



Short-Run Cost Curves



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Short-Run Marginal Cost

- ***Marginal cost*** **(*MC*)**: Amount by which a firm's cost changes if it produces one more unit of output.

$$MC = \frac{\partial C(q)}{\partial q}$$

Short-Run Average Costs

- ***Average fixed cost (AFC)***: FC divided by output

$$AFC = \frac{FC}{q}$$

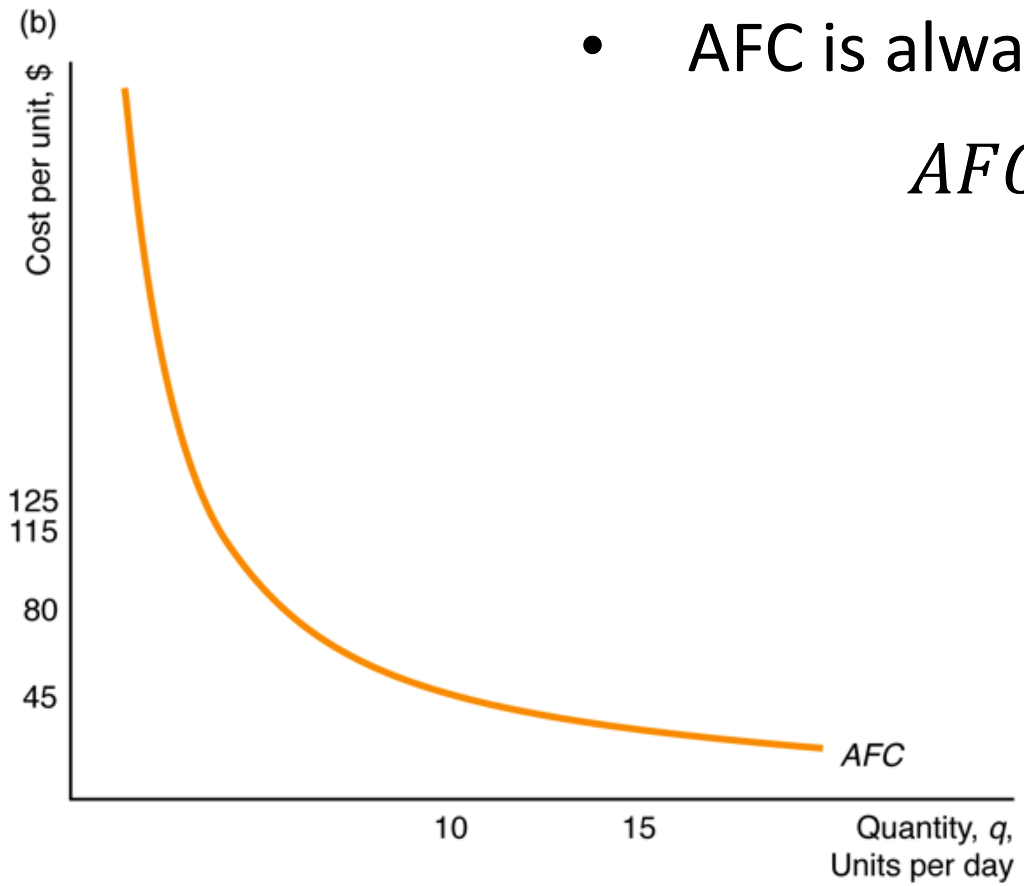
- ***Average variable cost (AVC)***: VC divided by output

$$AVC = \frac{VC}{q}$$

- ***Average cost (AC)***: C divided by output

$$AC = \frac{C}{q} = \frac{FC}{q} + \frac{VC}{q} = AFC + AVC$$

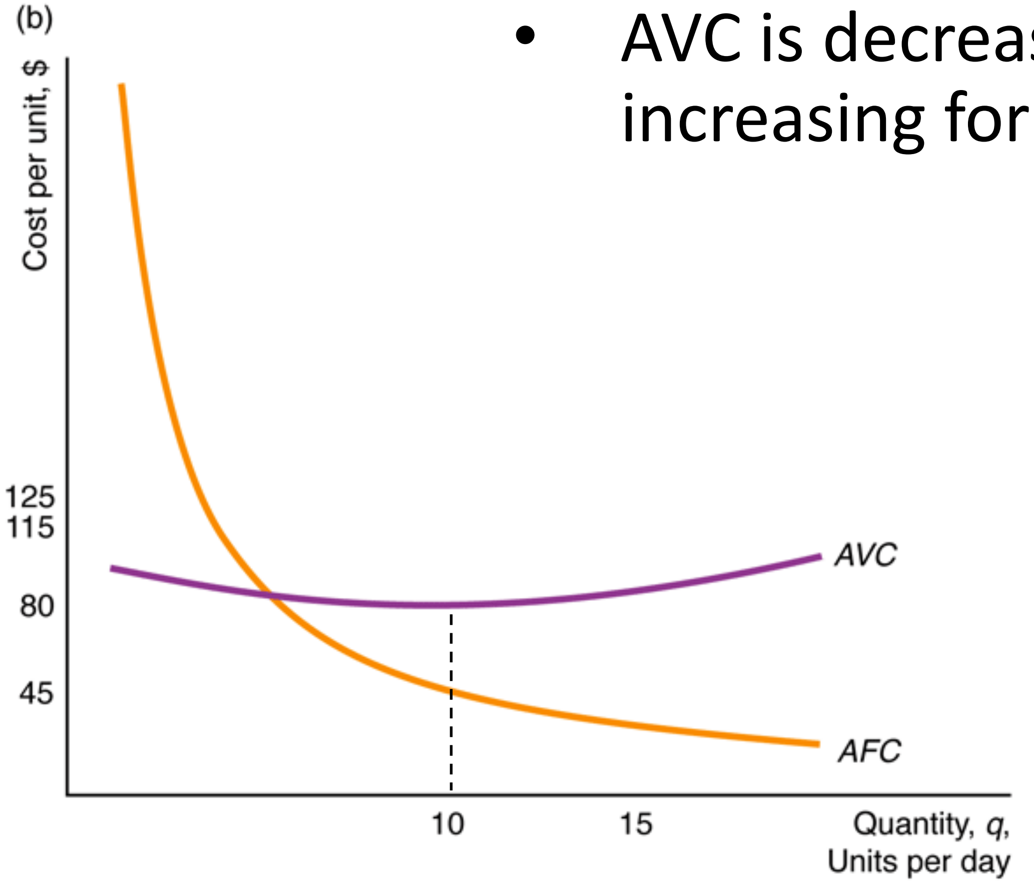
Average Fixed Cost



- AFC is always decreasing in q.

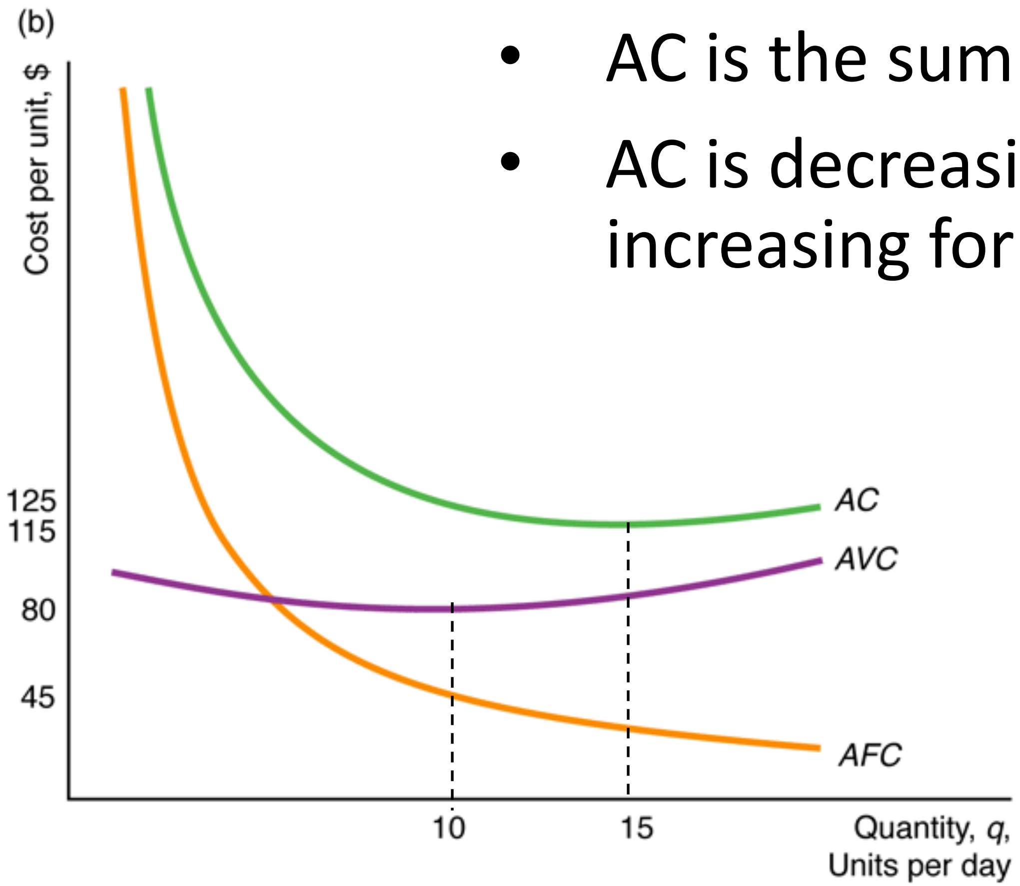
$$AFC = \frac{FC}{q}$$

Average Variable Cost



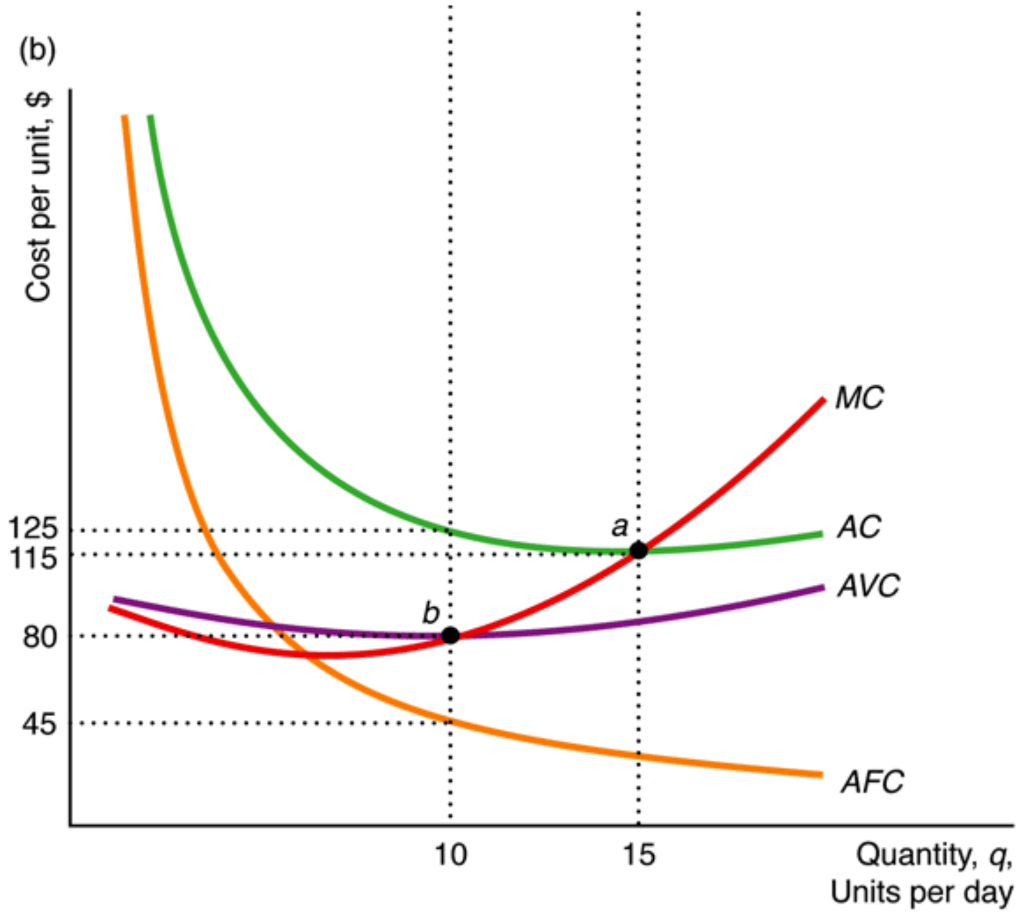
- AVC is decreasing for $q < 10$ and increasing for $q > 10$.

Average Variable Cost



- AC is the sum of AVC and AFC
- AC is decreasing for $q < 15$ and increasing for $q > 15$.

Short-Run Cost Curves



- AC and MC both first fall and then increase fall as q increases

- **MC intersects AC and AVC at its minimum**

AC and MC

- When AC is the smallest, AC = MC. Why?
  - Calculus
  - Intuitively, assuming that MC is increasing:
    - If MC<AC, then by increasing additional unit of output, you can decrease AC.
    - If MC>AC, then by decreasing the last unit of out, you can decrease APL.
    - Therefore, AC is minimized only if MC=AC.

Derivation of Cost Function

- The SR production function, $q = f(L, \bar{K})$, determines the shape of a firm's cost curves.

1. Amount of L needed to produce q:
$$L = f^{-1}(q; \bar{K})$$
2. If the wage paid to labor is *w*, variable cost:
$$VC(q) = wL = w\,f^{-1}(q; \bar{K})$$
1. Total cost :
$$C(q) = VC(q) + FC = wf^{-1}(q; \bar{K}) + FC$$

Derivation of Cost Function: Example

- The SR production function: $q = L\bar{K}$
- Wage: \$10/unit
- Capital: 200
- Price of capital: \$1/unit
- Derive the cost function, $C(q)$

Derivation of Cost Function: Example

1. Amount of L needed to produce q:

$$L = q/200$$

1. Variable cost:

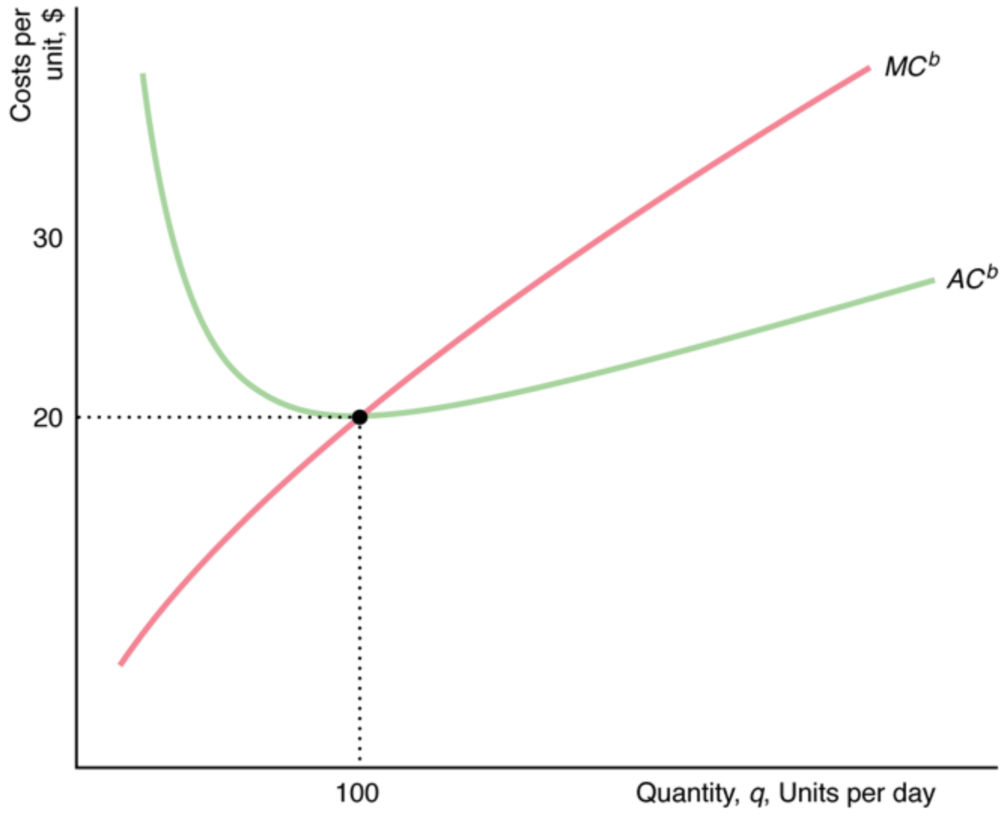
$$VC(q) = wL = 10 \times \frac{q}{200} = \frac{q}{20}$$

1. Total cost:

$$C(q) = VC(q) + F = \frac{q}{20} + 200$$

Effects of Taxes on Costs

- A $10 per unit tax increases firm costs, shifting up both AC and MC curves.



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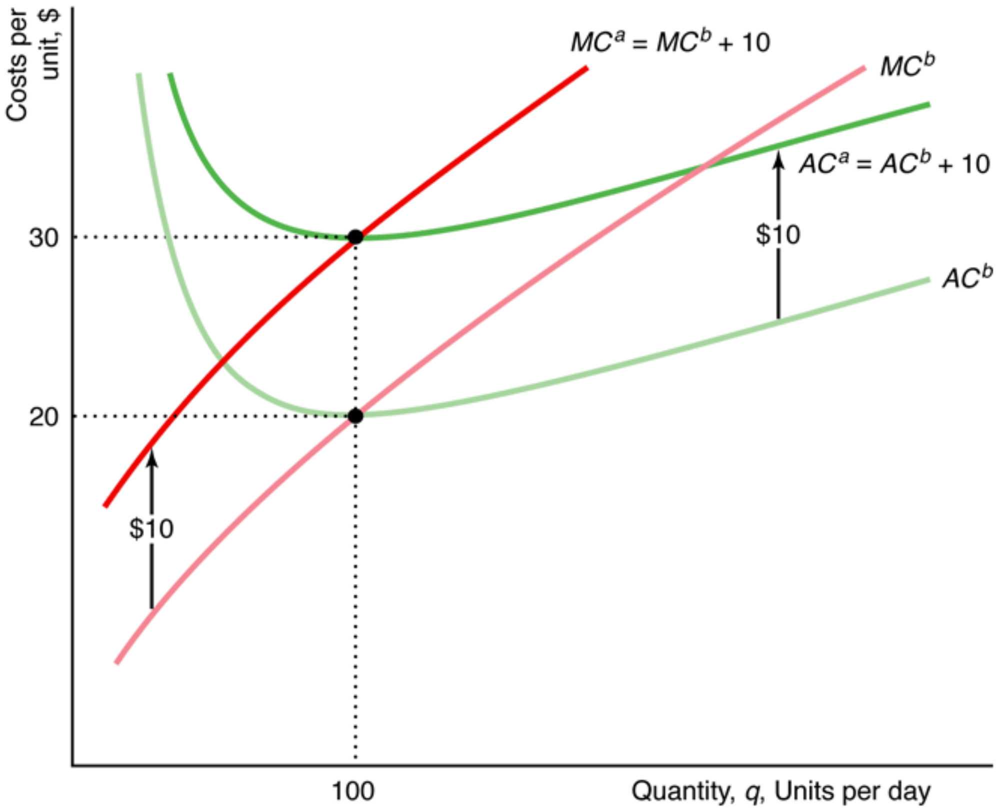


3. Long Run Production

Long-Run Production

- We consider a long-run model where both inputs are variable.
- We study the long-run production function.
- Key concepts
  - Isoquants
  - Marginal rate of technical substitution (MRTS)
  - Returns to scale
- Reading: pp. 367-404

Long Run Production

- In the long run (LR), we assume that both labor and capital are variable inputs.
- The freedom to vary both inputs provides firms with many choices of how to produce (labor-intensive vs. capital-intensive methods).

Long Run Production

- Consider a Cobb-Douglas production function:

$$q = L^{0.5}K^{0.5}$$

- This describes how different input combinations generate different levels of output.

Long Run Production: Isoquants

- Consider a Cobb-Douglas production function:

$$q = L^{0.5}K^{0.5}$$

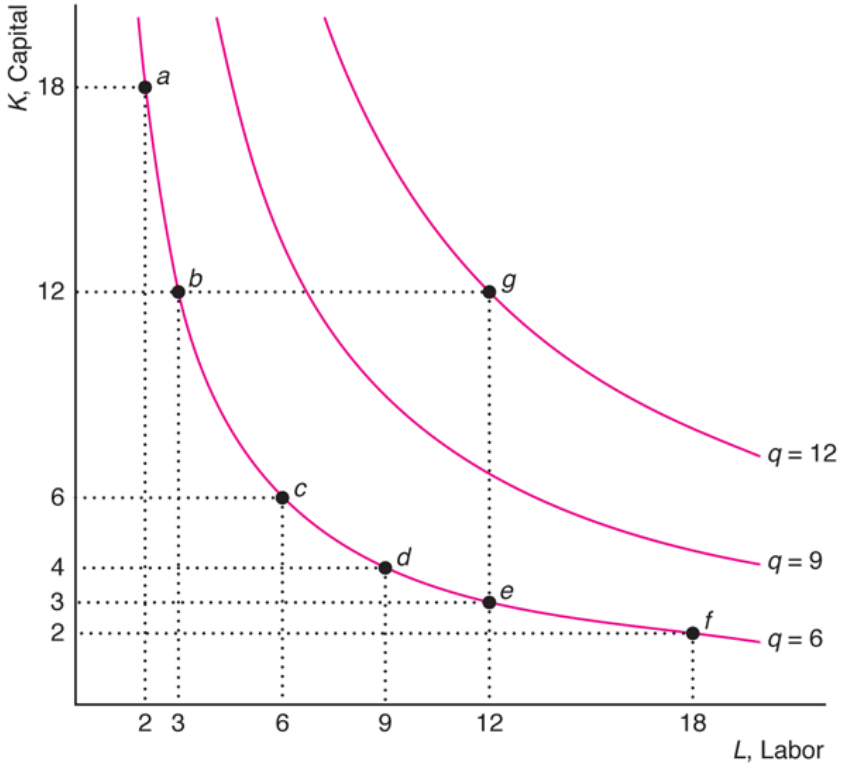
- For a given level of output, say $q = 6$:

$$6 = L^{0.5}K^{0.5} \rightarrow LK = 36 \rightarrow K = \frac{36}{L}$$

  - Based on this, we can say which input combinations produce $q = 6$.

Long Run Production: Isoquants

- A production **isoquant** graphically summarizes the efficient combinations of inputs that will produce a specific level of output.



Long Run Production: Isoquants

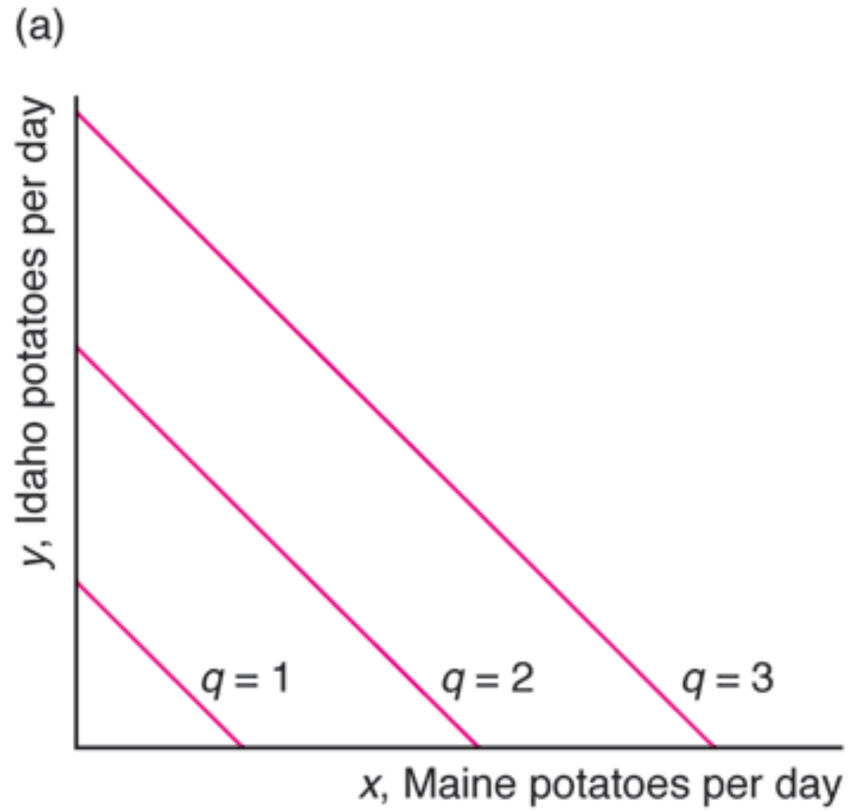
- Properties of isoquants:
  1. The farther an isoquant is from the origin, the greater the level of output.
  2. Isoquants do not cross.
  3. Isoquants slope downward.
  4. Isoquants must be thin.

Long Run Production: Isoquants

- The shape of isoquants (curvature) indicates how readily a firm can substitute between inputs in the production process.
  - Perfect Substitutes
  - Perfect Complements (Fixed-proportions)
  - Imperfect Substitutes

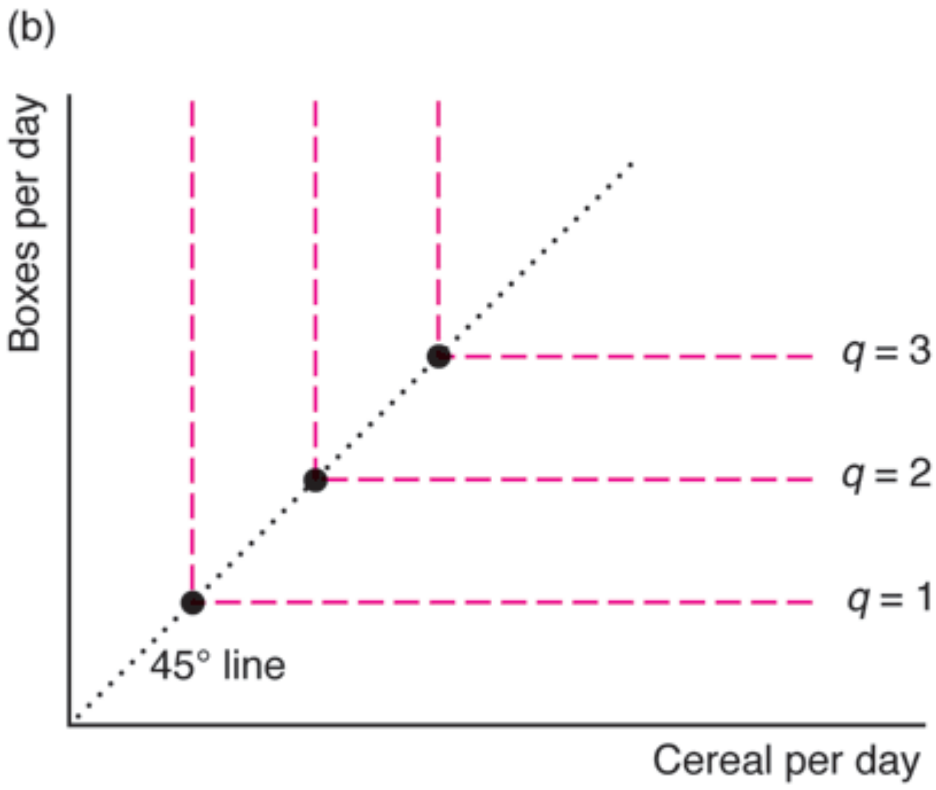
Long Run Production: Isoquants

- Perfect Substitutes
  - For example, $q = L + K$



Long Run Production: Isoquants

- Perfect Complements
  - For example, $q = \min\{L, K\}$



Substituting Inputs

- The slope of an isoquant shows the ability of a firm to replace one input with another (holding output constant).

- ***Marginal rate of technical substitution*** (***MRTS***) is the absolute value of the slope of an isoquant at a single point.

Substituting Inputs: MRTS

- ***Marginal rate of technical substitution*** (***MRTS***) is the absolute value of the slope of an isoquant at a single point.

$$MRTS = -\frac{Change\ in\ Capital}{Change\ in\ Labor} = -\frac{dK}{dL}$$

  - $MRTS$ tells us how many units of K the firm can replace with an extra unit of L (q constant)

Substituting Inputs: MRTS

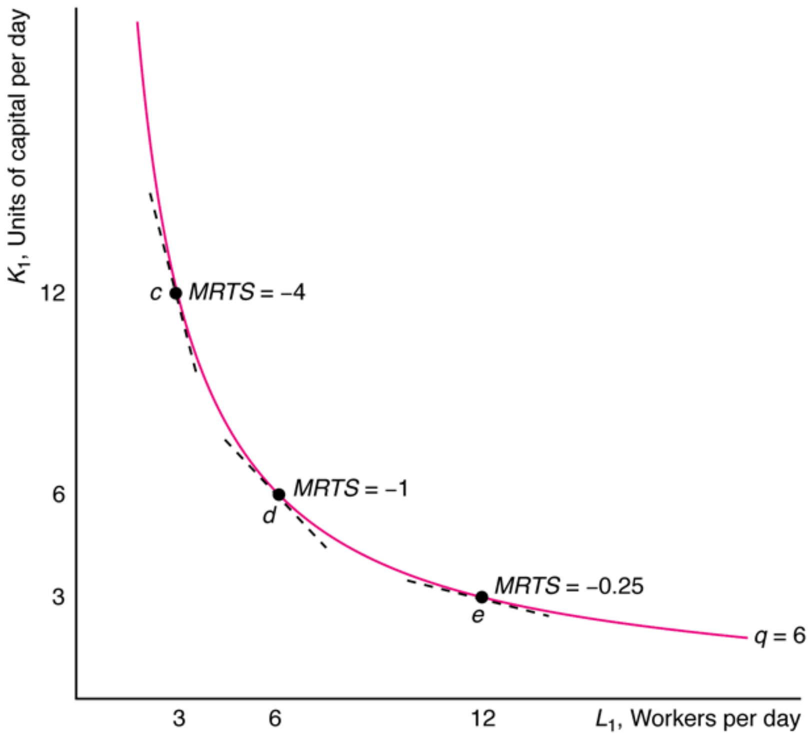
- ***Marginal rate of technical substitution*** (***MRTS***) is the ratio of the marginal products of each input.

$$MRTS = \frac{MPL}{MPK}$$

  - MPL is marginal product of labor
  - MPK is marginal product of capital

Substituting Inputs: MRTS

- MRTS diminishes along a convex isoquant
  - The more L the firm has, the harder it is to replace K with L.



Returns to Scale

- How much does output change if a firm increases all its inputs proportionately?

- Production function exhibits **constant returns to scale** when a percentage increase in inputs is followed by the same percentage increase in output.
  - Doubling inputs, doubles output
  - $f(2L, 2K) = 2f(L, K)$

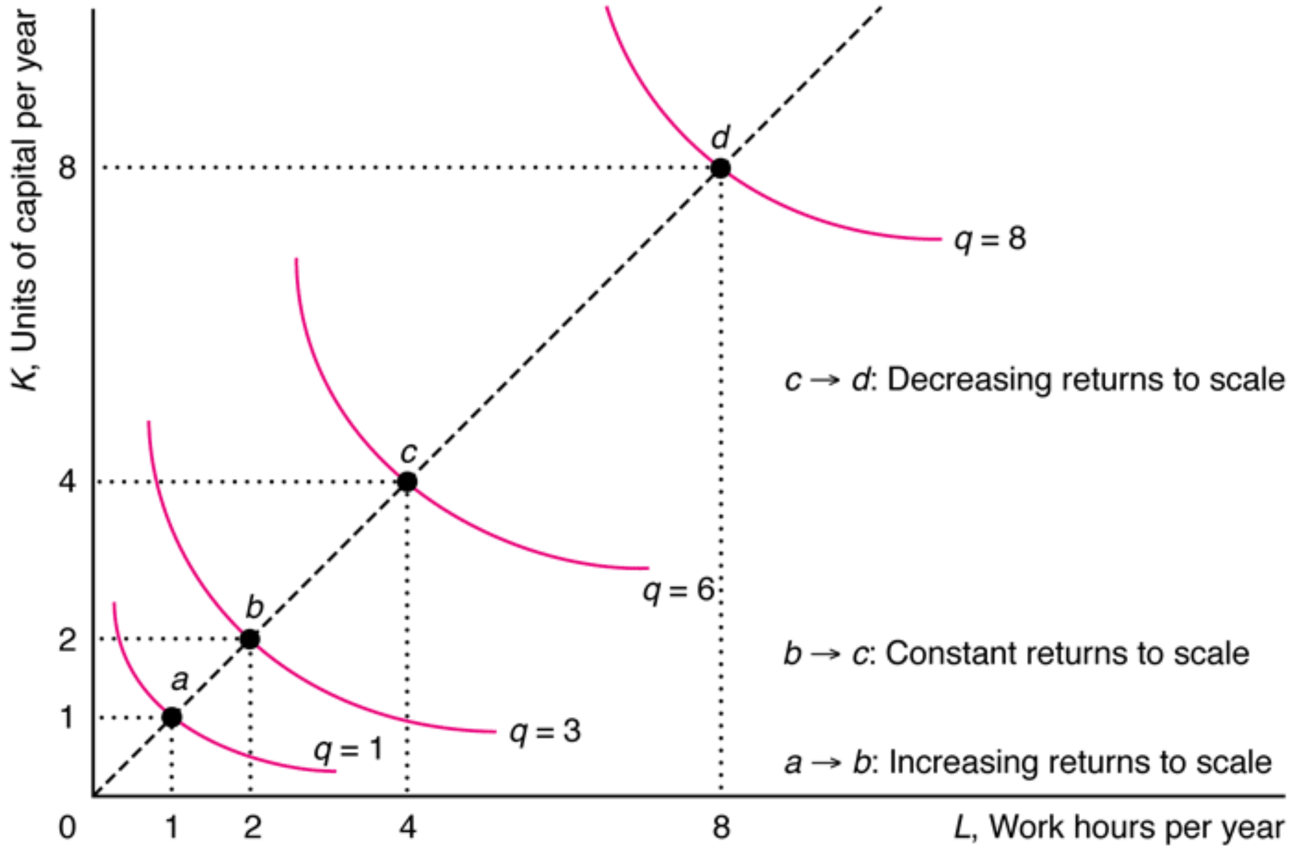
Returns to Scale

- Production function exhibits ***increasing returns to scale*** when a percentage increase in inputs is followed by a <u>larger</u> percentage increase in output.
  - $f(2L, 2K) > 2f(L, K)$
  - Occurs with greater specialization of L and K; one large plant more productive than two small plants

Returns to Scale

- Production function exhibits ***decreasing returns to scale*** when a percentage increase in inputs is followed by a <u>smaller</u> percentage increase in output.
  - $f(2L, 2K) < 2f(L, K)$
  - Occurs because of difficulty organizing and coordinating activities as firm size increases

Varying Returns to Scale



4. Long-Run Costs

Long-Run Costs

- We consider a long-run model where both inputs are variable.
- From a firm's production function, we derive the long-run cost function by choosing the optimal combination of two inputs.
- We study the relationship between the long-run cost functions and the short-run cost functions.
- Reading: pp. 367-404

Long-Run Costs

- Recall that the long-run is a period of time in which all inputs can be varied.
- In the LR, firms can change plant size, build new equipment, and adjust inputs that were fixed in the SR.
- We assume LR fixed costs are zero ($FC = 0$).

Long-Run Costs and Input Choice

- **Isocost line** summarizes all combinations of inputs that require the same total expenditure

$$\bar{C} = wL + rK$$

  - If the firm rents K hours of machine services at a rental rate of r per hour, total capital cost is rK.
  - Cost is fixed at a particular level, $\bar{C}$, along a given isocost line

- Rewrite the isocost line for easy graphing:

$$K = \frac{\bar{C} - wL}{r}$$

Isocost Lines

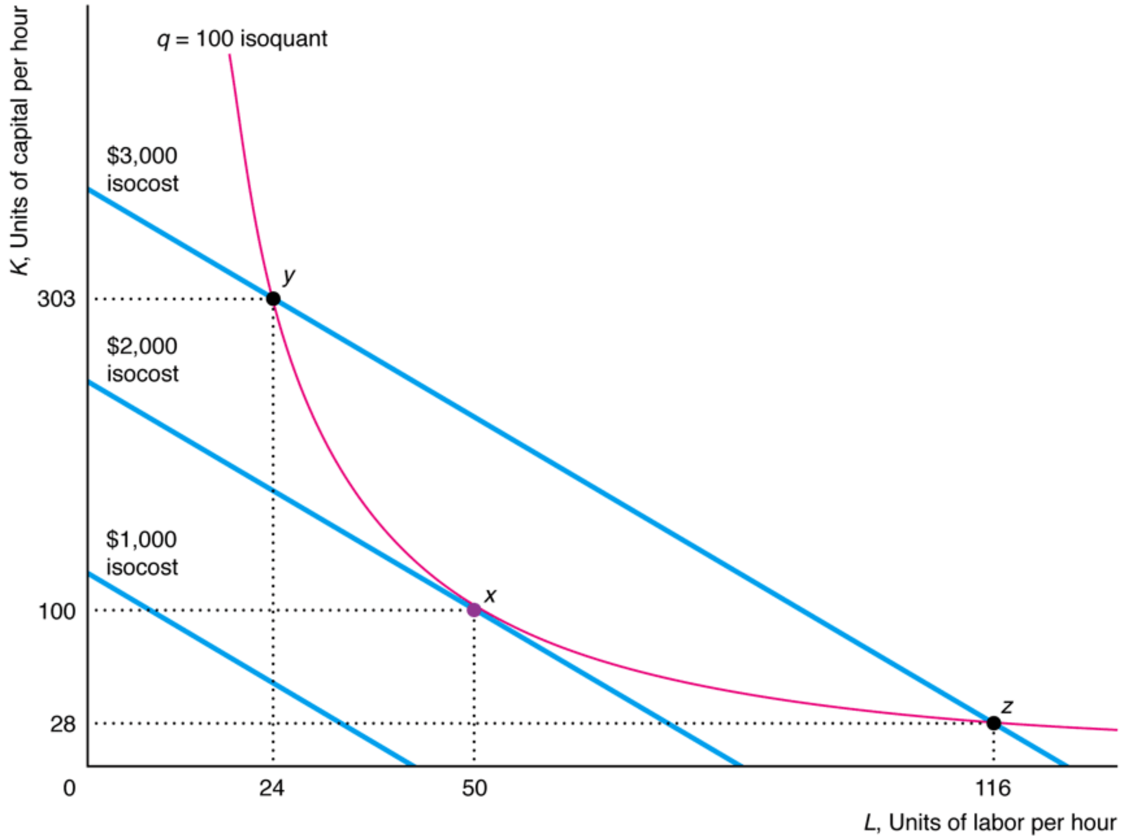
- Three properties of isocost lines:
  1. The firm’s costs, C, and input prices determine where the isocost line hits the axes.
  2. Isocosts farther from the origin have higher costs than those closer to the origin.
  3. The slope of each isocost is the same and is given by the relative prices of the inputs.

Comparison between Consumer and Producer Theories

Consumer	Producer
Two goods	Two inputs
Utility	Output
Marginal utility	Marginal productivity
Indifference curve, MRS	Isoquant curve, $MRTS$
$MRS = MU_1/MU_2$	$MRTS = MPL/MPK$
Budget line, p_1/p_2	Isocost line, w/r
Maximize utility subject to a budget constraint	Minimize cost subject to a production constraint

Cost Minimization

- This firm is seeking the least costly way of producing 100 units of output.



Using Graph to Minimize Cost

- Two equivalent approaches to minimizing cost:
  1. **Lowest-isocost rule**: Pick the bundle of inputs where the lowest isocost line touches the isoquant associated with the desired level of output.
  2. **Tangency rule**: Pick the bundle of inputs where the desired isoquant is tangent to the budget line.

$$MRTS = -\frac{dK}{dL} = \frac{w}{r}$$

Using Calculus to Minimize Cost

- In producing q, the firm minimizes the total cost:

$$\min_{L,K} wL + rK$$
$$subject\ to \quad f(L,K) = q$$

  - Consider the production technology given by
$$f(L,K) = L^{0.5}K^{0.5}$$

Using Calculus to Minimize Cost

- In producing q, the firm minimizes the total cost:

$$\min_{L,K} wL + rK \ \ s.t.\, L^{0.5}K^{0.5} = q$$

  - Substitution Method:
    1. Using the constraint, solve for K: $K = q^2/L$
    2. Plug $K = \frac{q^2}{L}$ in the objective function: $\min_{L} wL + r\frac{q^2}{L}$
    3. Take the first order condition: $w - r\frac{q^2}{L^2} = 0.$
    4. Solve for L: $\mathrm{L} = q\sqrt{r/w}$
    5. Plug $K = \frac{q^2}{L}$: $K = q\sqrt{w/r}$

Using Calculus to Minimize Cost

- In producing q, the firm minimizes the total cost:

$$\min_{L,K} wL + rK \;\; s.t. L^{0.5}K^{0.5} = q$$

  - Use-the-Graph Method:
    1. Solve for $MRTS$, $MRTS = \frac{0.5L^{-0.5}K^{0.5}}{0.5L^{0.5}K^{-0.5}} = \frac{K}{L}$.
    2. Using the optimality condition, $MRTS = \frac{w}{r}$, $\frac{K}{L} = \frac{w}{r}$.
    3. Using the constraint: $L^{0.5}K^{0.5} = q$.
    4. Solve two equations with two unknowns:

$$\mathrm{L} = q\sqrt{r/w}, K = q\sqrt{w/r}$$

Cost Minimization and Cost Functions

- Given the minimization, we have

$$\mathrm{L} = q\sqrt{r/w}, K = q\sqrt{w/r}$$

  - These are called **conditional factor demands**, as a function of input prices (w, r) and output (q).

Cost Minimization and Cost Functions

- Given the minimization, we have

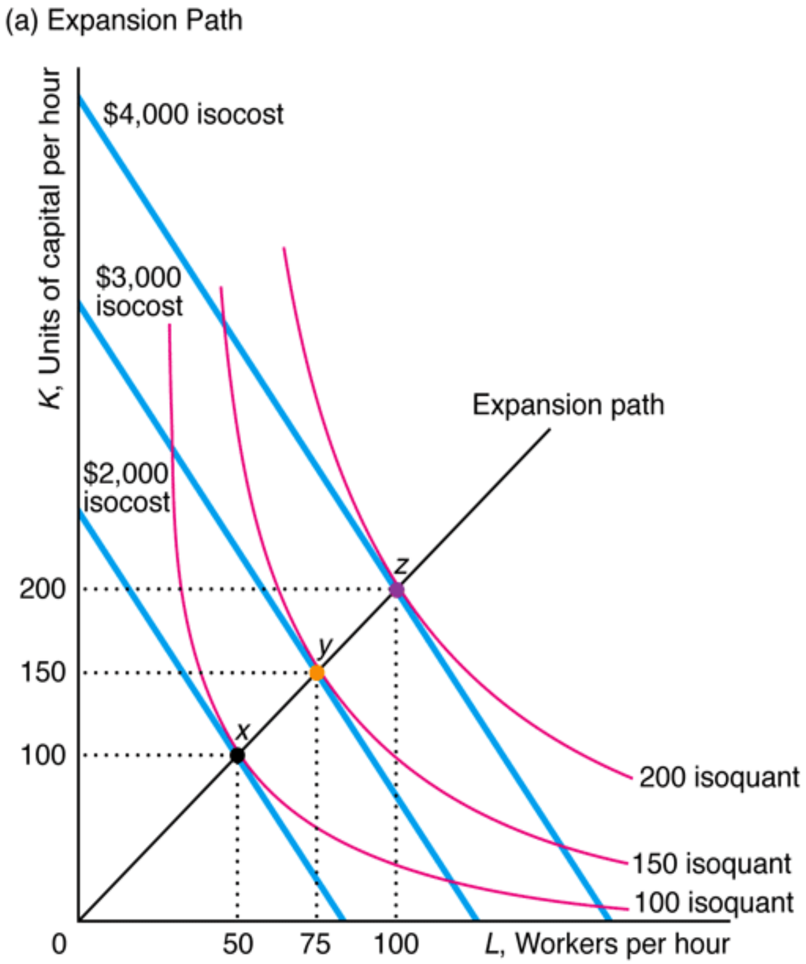
$$\mathrm{L} = q\sqrt{r/w}, K = q\sqrt{w/r}$$

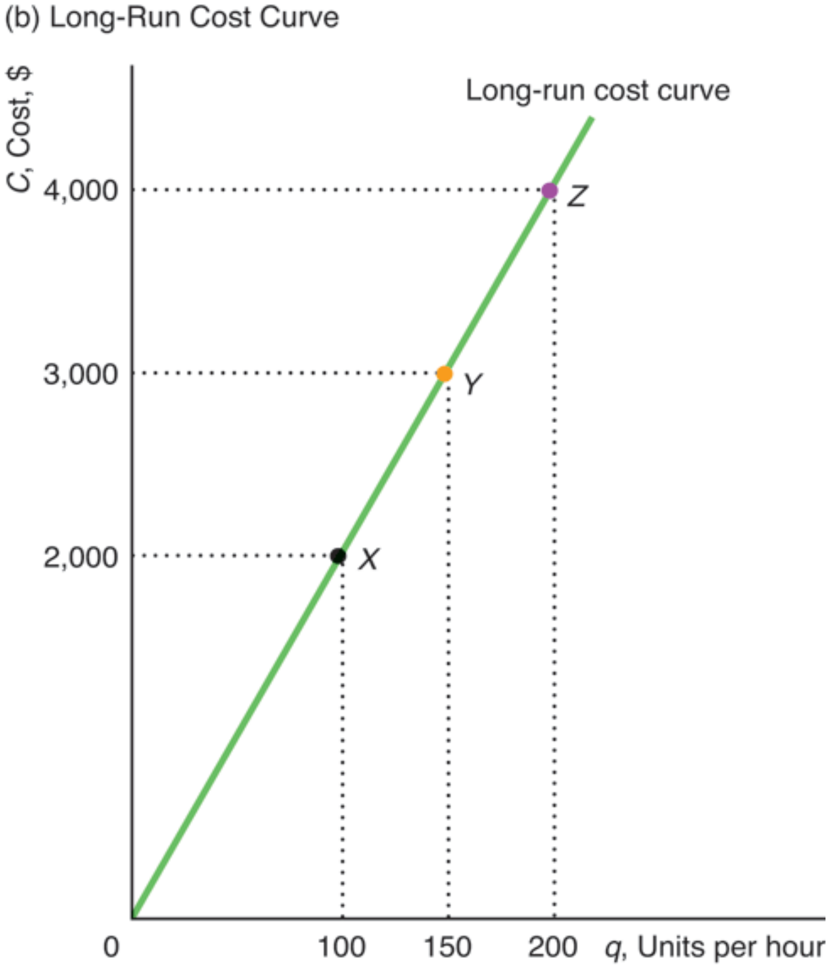
- Long-Run cost function:

$$C(q) = wL + rK = 2q\sqrt{wr}$$

  - Marginal cost: $MC(q) = 2\sqrt{wr}$
  - Average cost: $AC(q) = 2\sqrt{wr}$

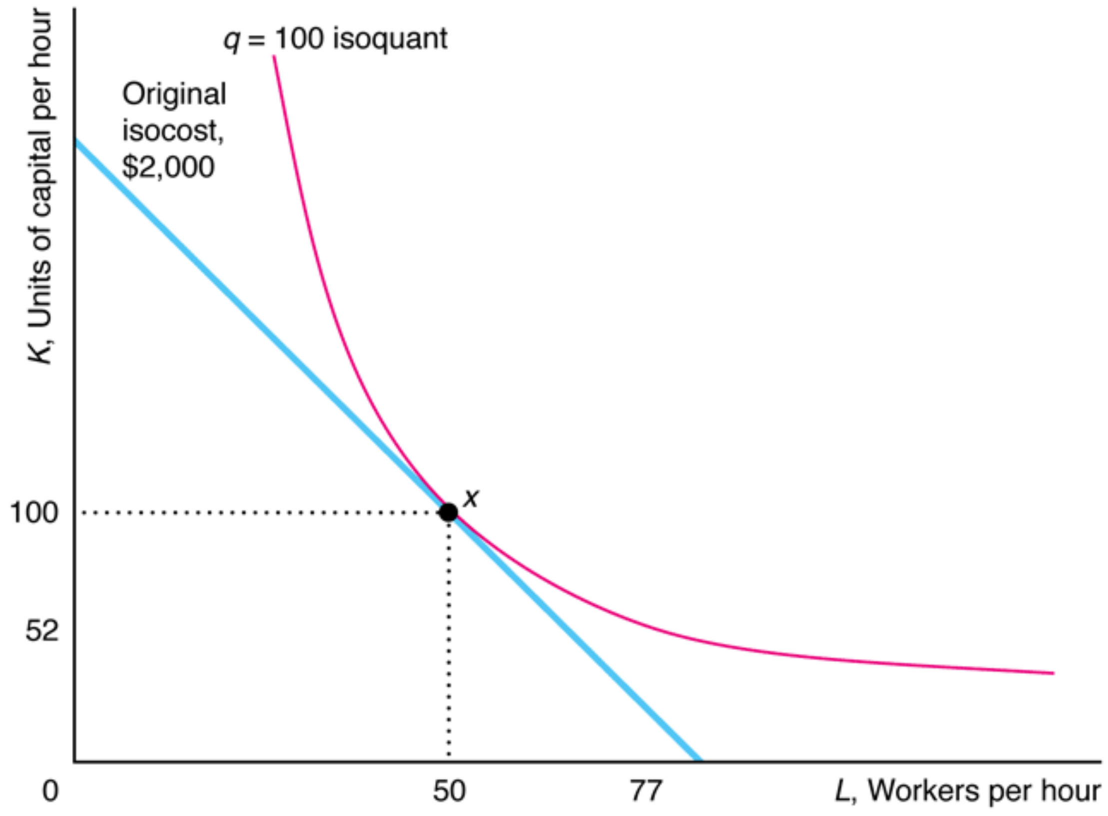
How LR Cost Varies with Output





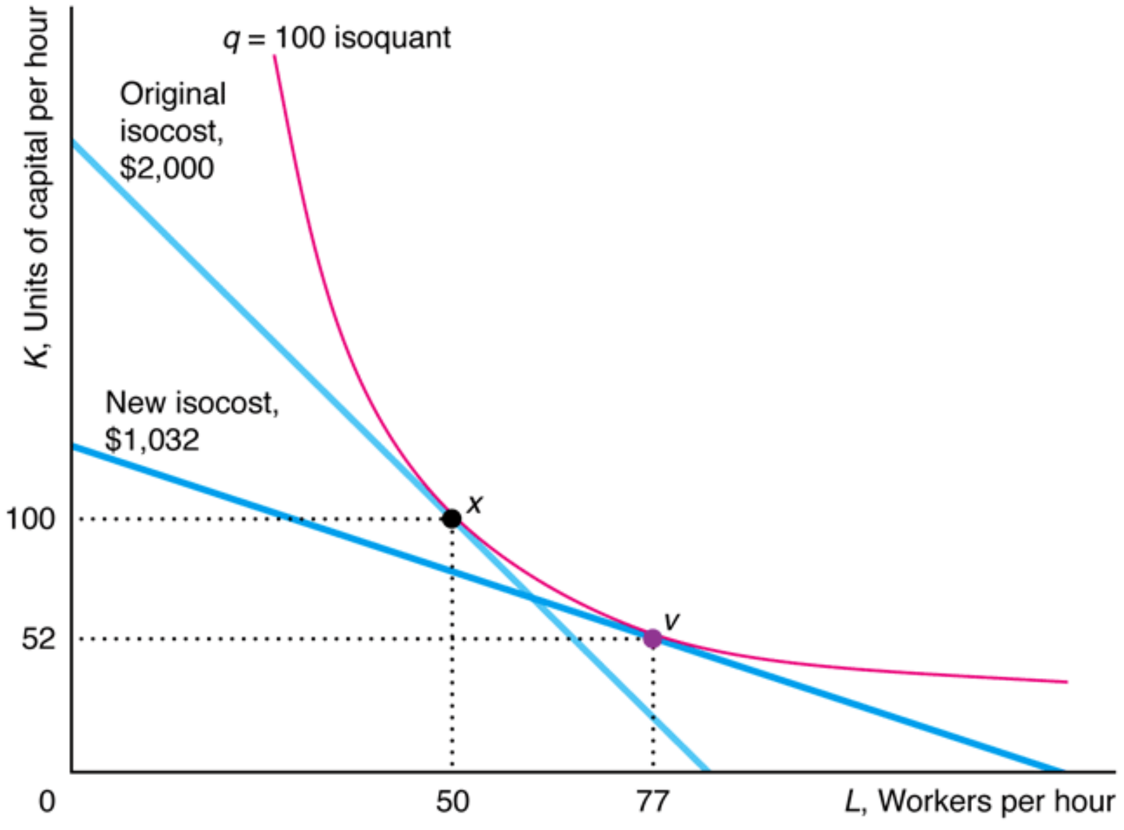
Factor Price Changes

- *w* = \$24 and *r* = \$8, the cost of producing 100 units of output is \$2,000.



Factor Price Changes

- w = \$8 and r = \$8, the cost of producing 100 units of output is \$1,032.



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The Shape of LR Cost Curves

- The LR AC curve may be U-shaped
  - Not due to downward-sloping AFC or diminishing marginal returns, both of which are SR phenomena.
  - Shape is due to economies and diseconomies of scale.

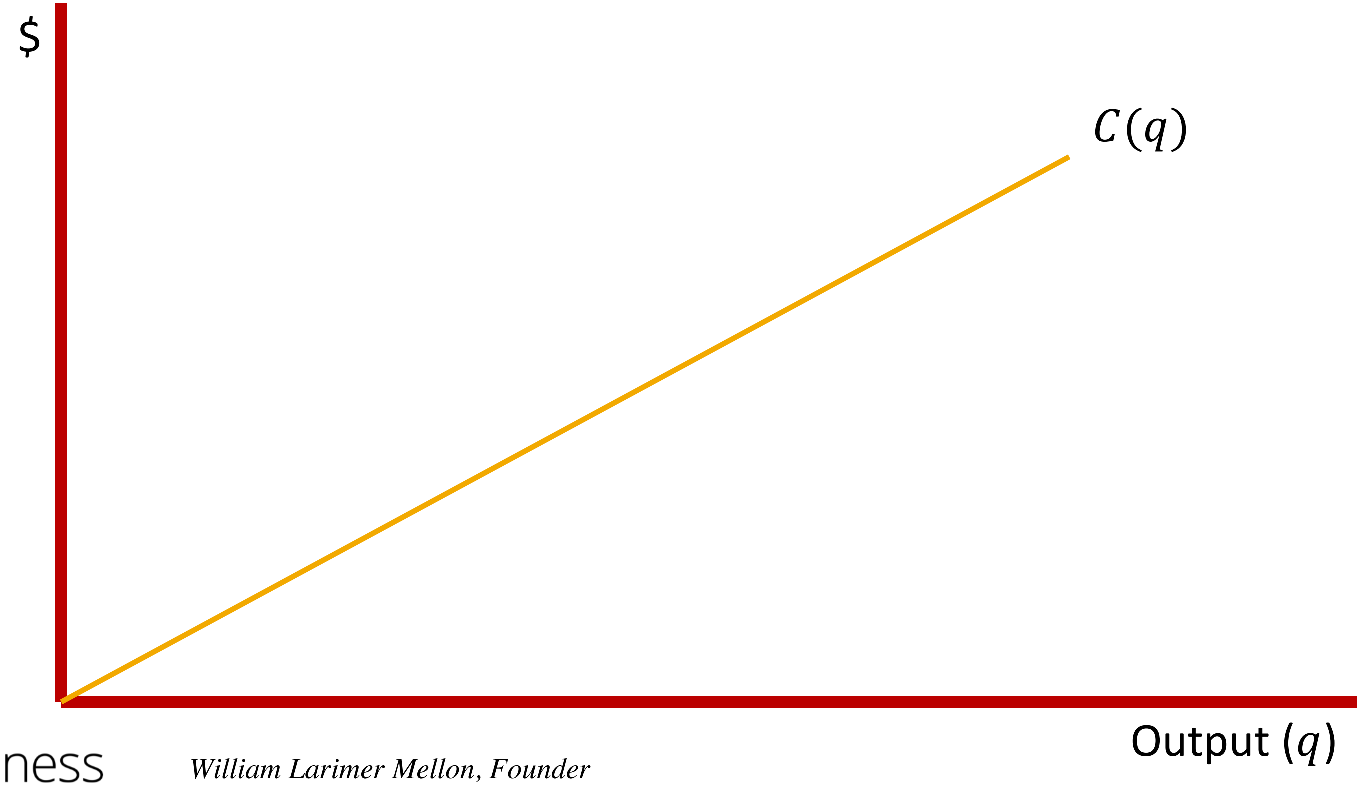
The Shape of LR Cost Curves

- A cost function exhibits ***economies of scale*** if the average cost of production falls as output expands.
  - Doubling inputs more than doubles output, so AC falls with higher output.

- A cost function exhibits ***diseconomies of scale*** if the average cost of production rises as output expands.
  - Doubling inputs less than doubles output, so AC rises with higher output.

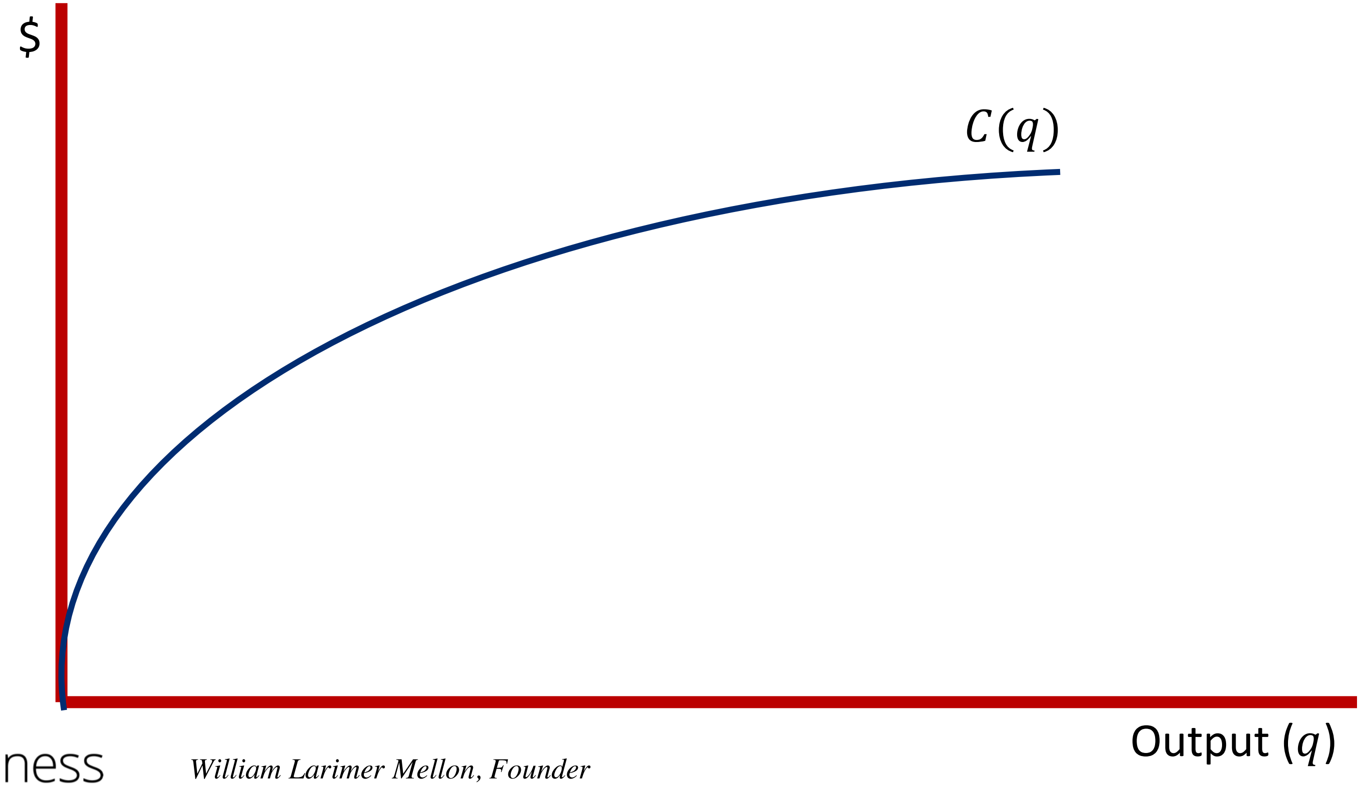
LR Cost Curves and Returns to Scale

- Constant Returns to Scale
  - $MC(q) = AC(q) = C(1)$ for all q



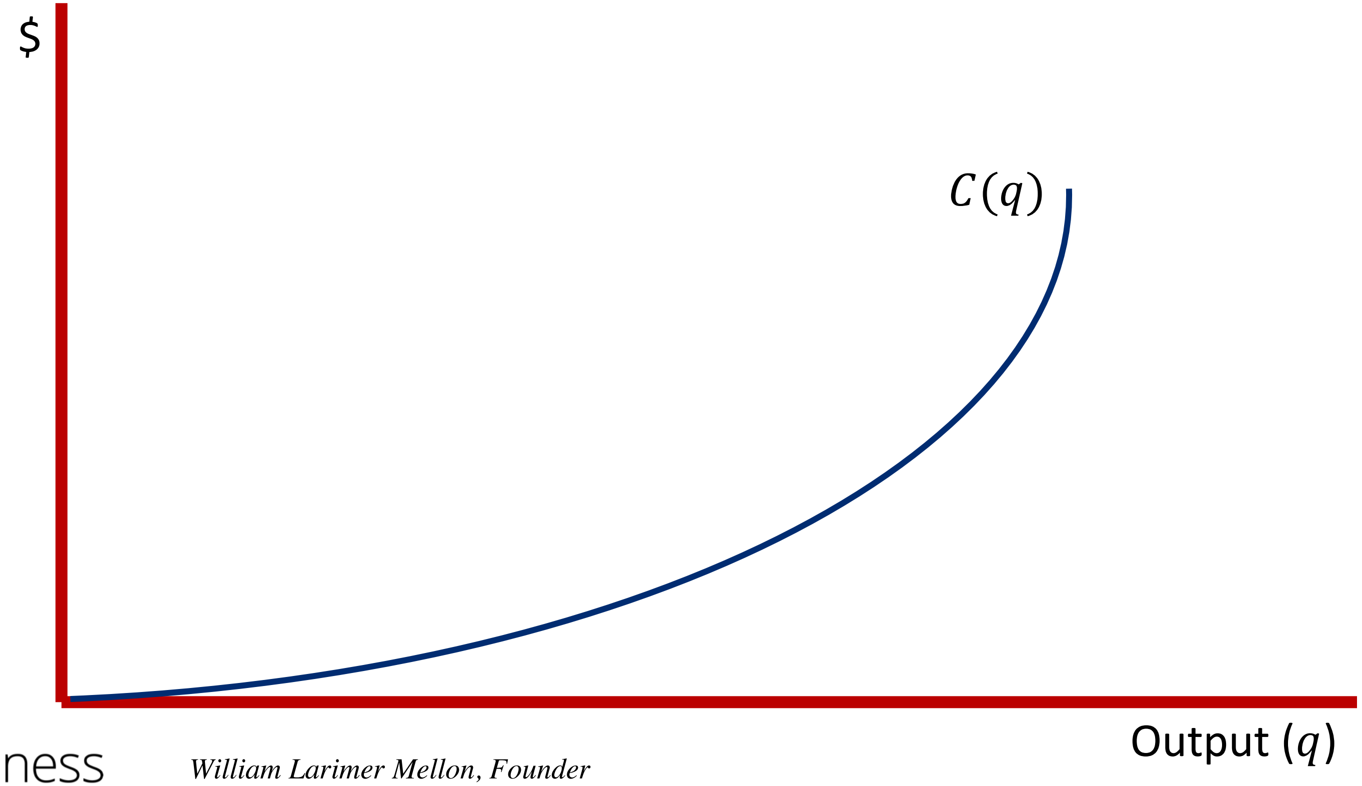
LR Cost Curves and Returns to Scale

- Increasing Returns to Scale
  - $MC(q)$ and $AC(q)$ decrease as q increases



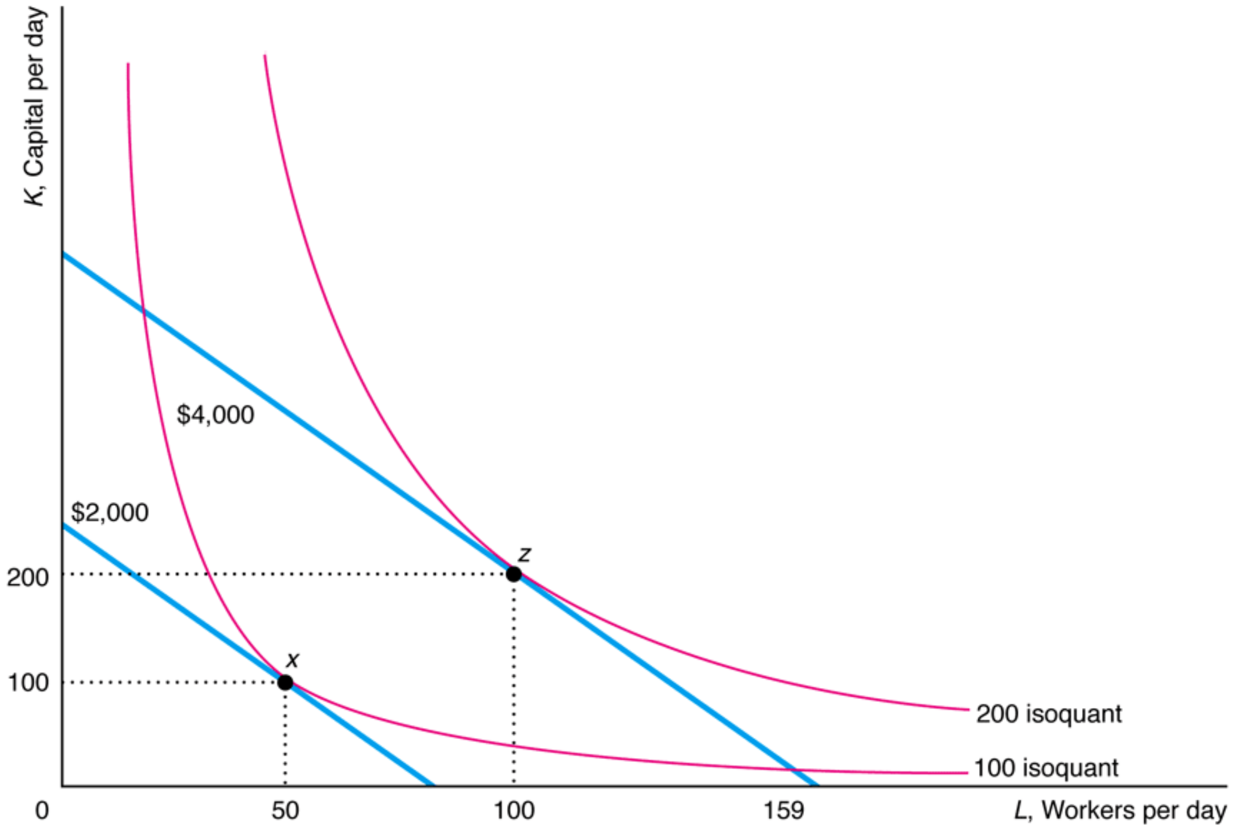
LR Cost Curves and Returns to Scale

- Decreasing Returns to Scale
  - $MC(q)$ and $AC(q)$ increase as q increases



SR and LR Costs

- In the LR, expanding output from 100 to 200 costs $2,000.



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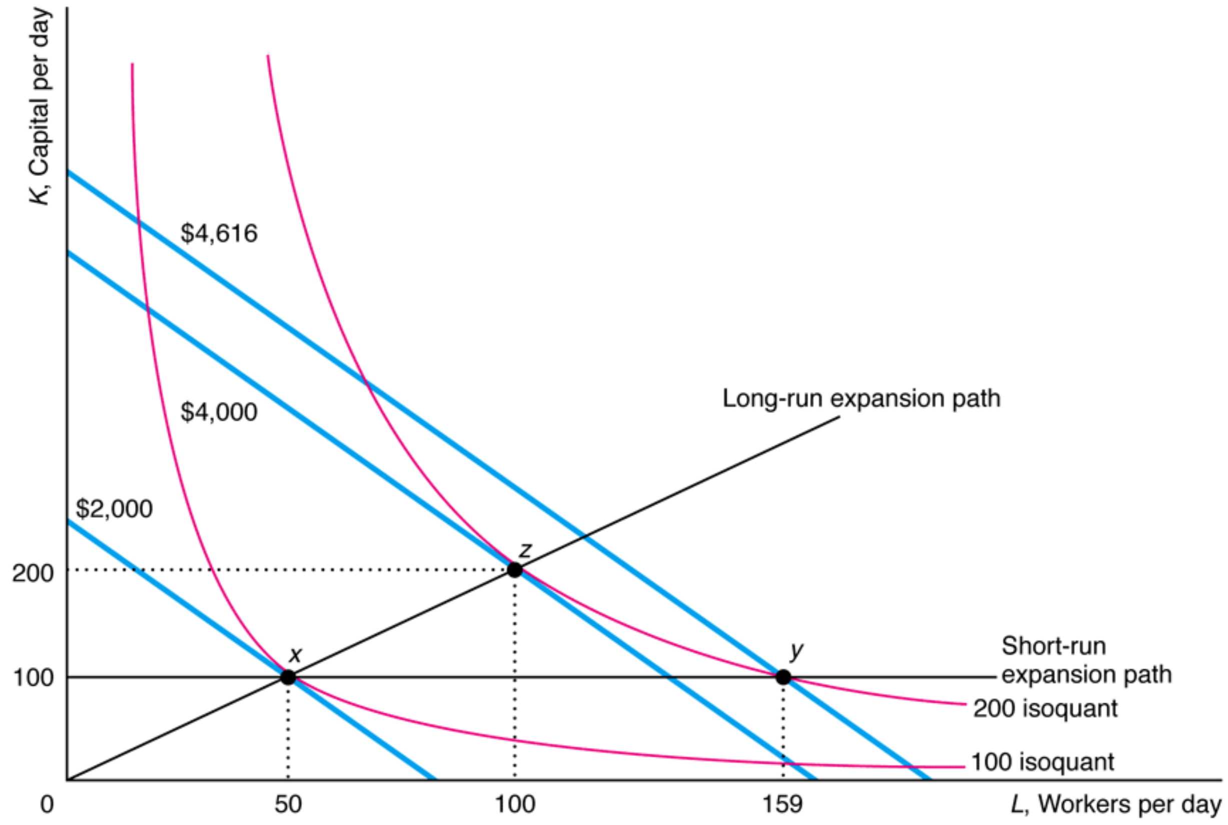
William Larimer Mellon, Founder

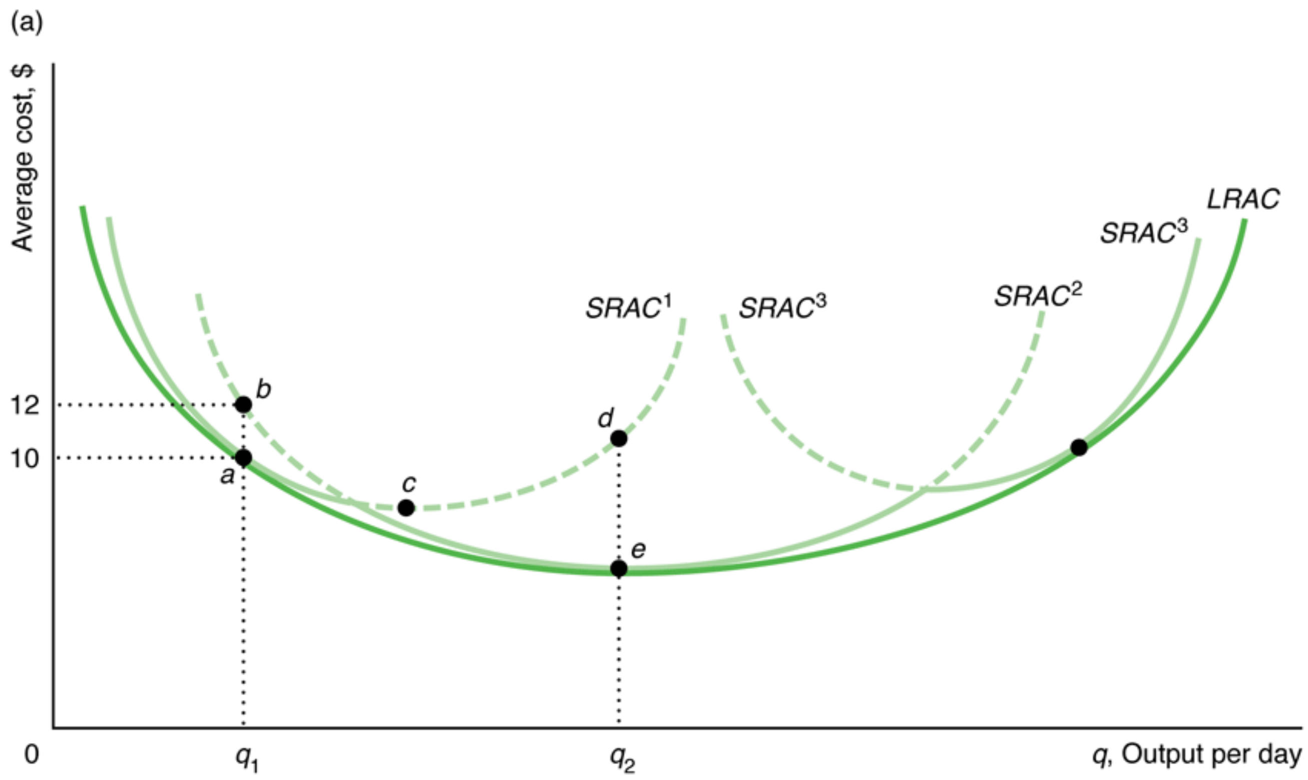
SR and LR Costs

- In the SR, the level of capital is fixed, for example at 100.
- In the SR, expanding output from 100 to 200 costs $2,616.



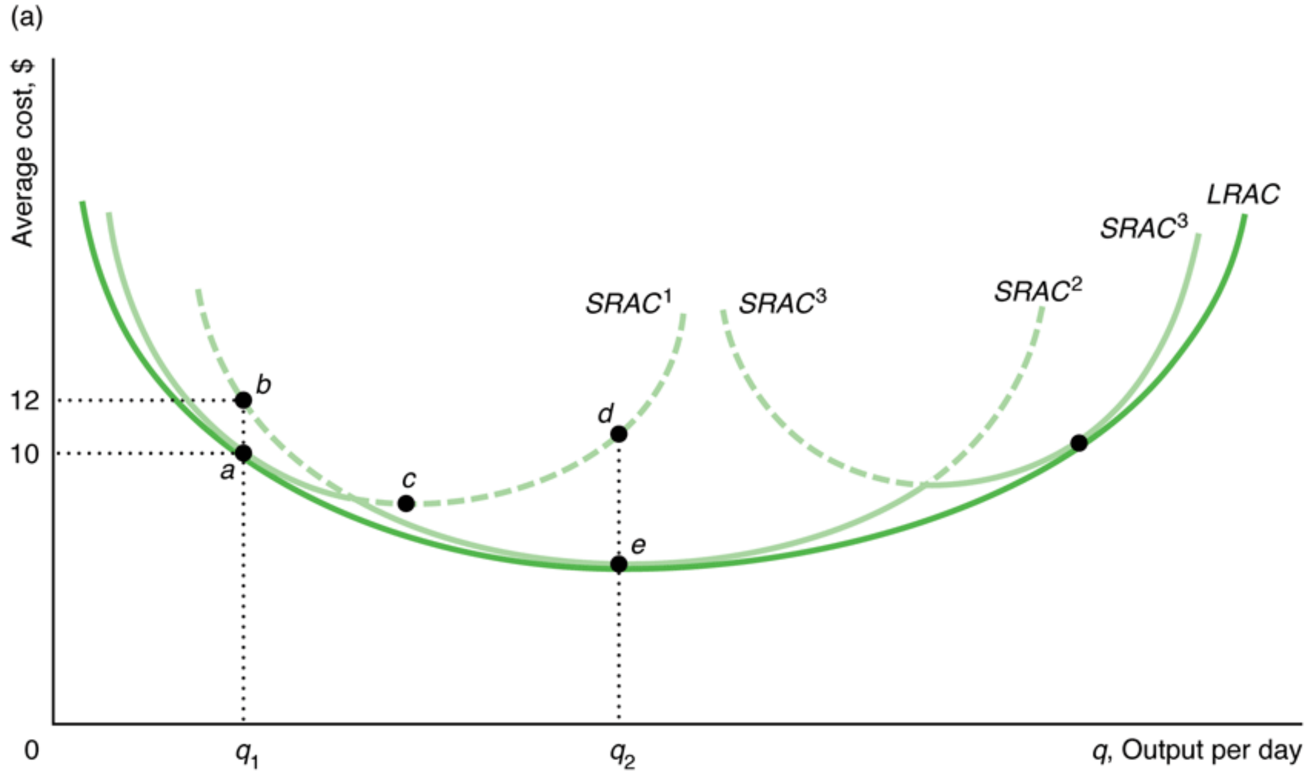
Lower Costs in the Long Run

- Because a firm cannot vary *K* in the SR but it can in the LR, SR cost is as least as high as LR cost.

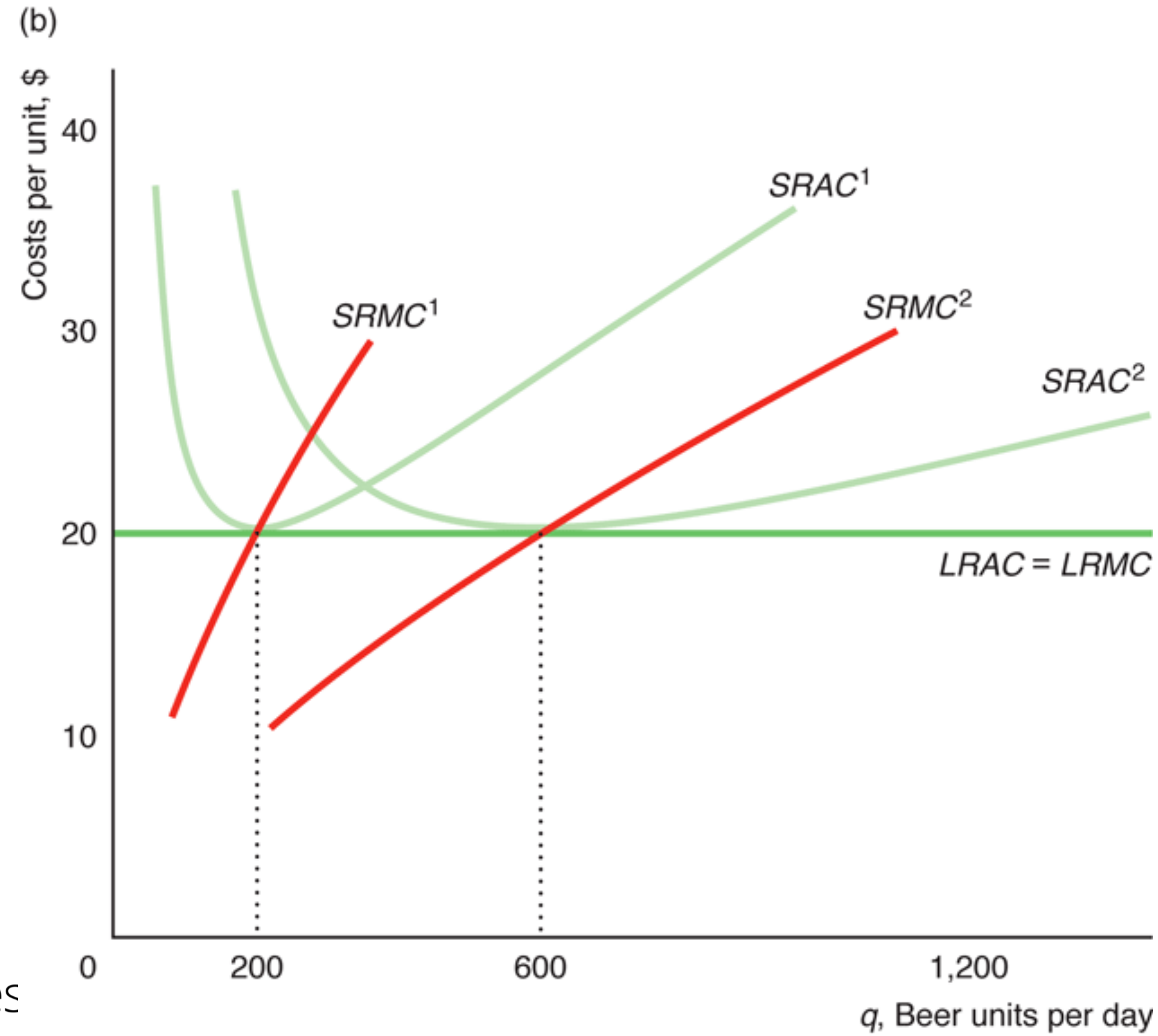


Lower Costs in the Long Run

- Short-Run ACs are tangent to the Long-Run AC
- LRAC is not always the minimum of SRAC!



SR and LR Cost Curves



5. Profit Maximization

Profit Maximization

- Firm's problem: To maximize its profit under
  1. Technological constraints: Production function
  2. Market constraints: Fixed input/output prices
- We solve for the optimal output given prices.
- We derive the supply curve from this optimization problem.
- Reading: pp. 413-427, pp.439-449

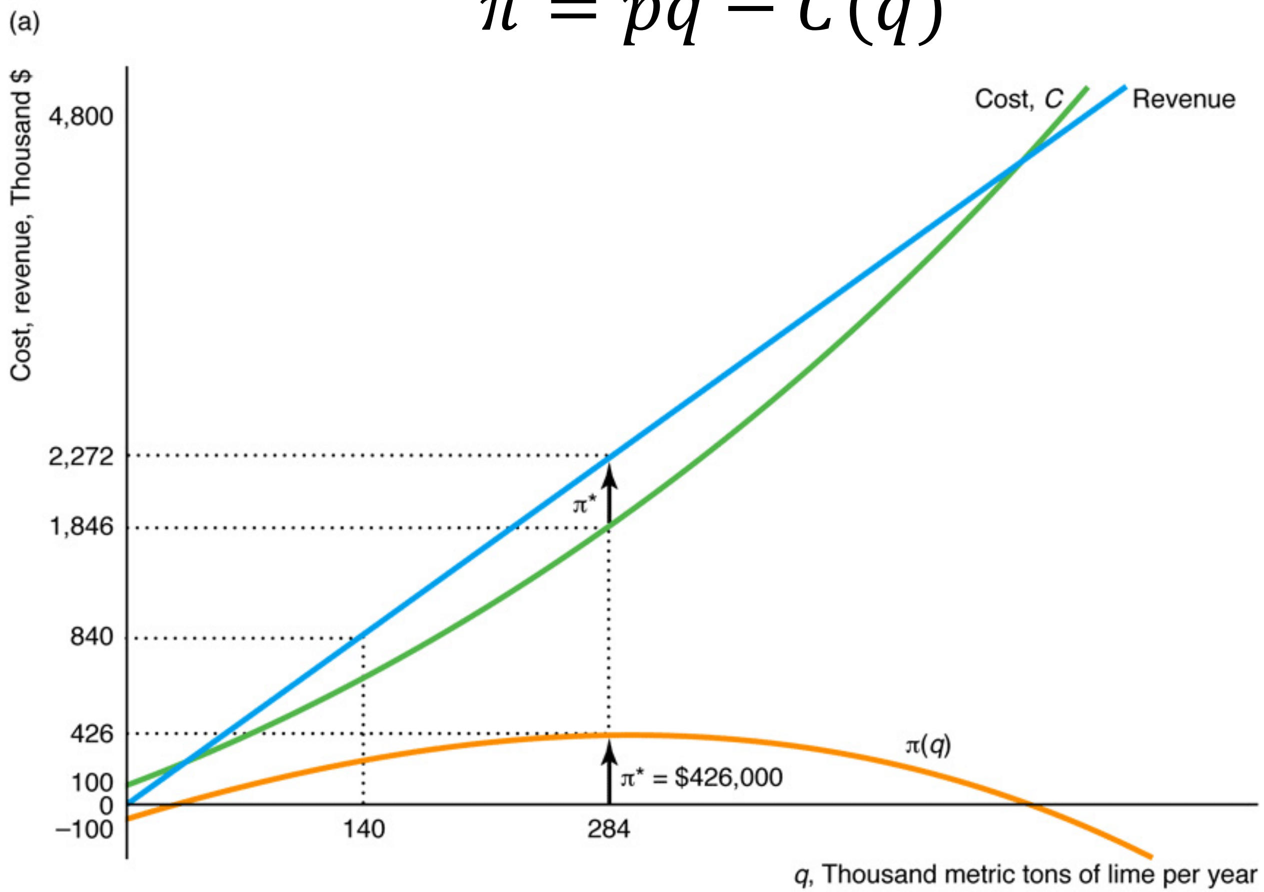
Profit Maximization

- Maximizing profit involves two important questions:
  1. **Output decision**: If the firm produces, what output level (q^*) maximizes its profit or minimizes its loss?
  2. **Shutdown decision**: Is it more profitable to produce q^* or to shut down and produce no output?

Profit

- Profit: Revenue – Cost

$$\pi = pq - C(q)$$

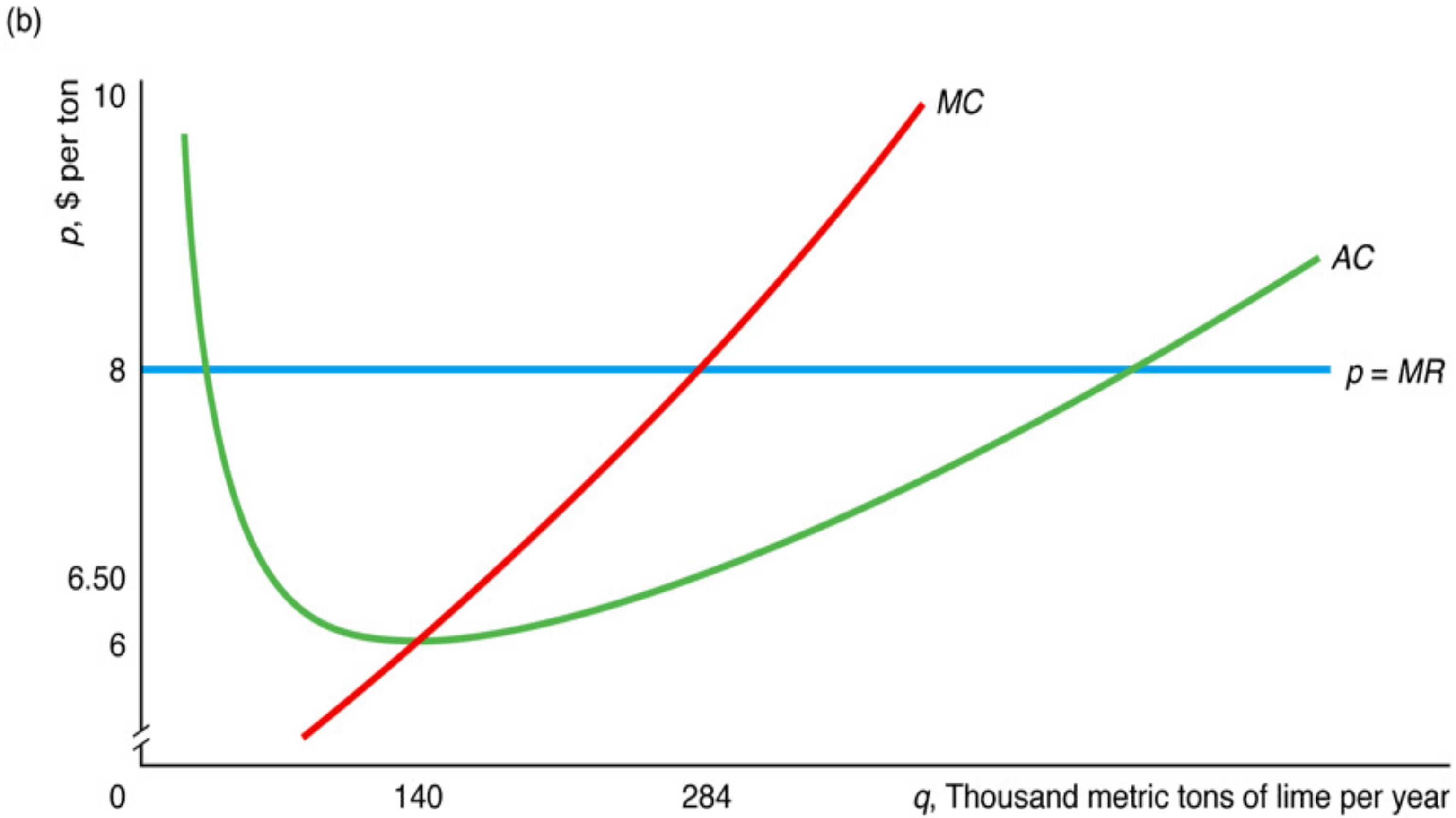


Output Decision

- Profits are maximized when $(TR - TC)$ is the largest, which occurs when the slope of TR is equal to the slope of TC
  - Slope of TR: $MR = p$
  - Slope of TC: MC
  - Profits are maximized when $MR = p = MC$

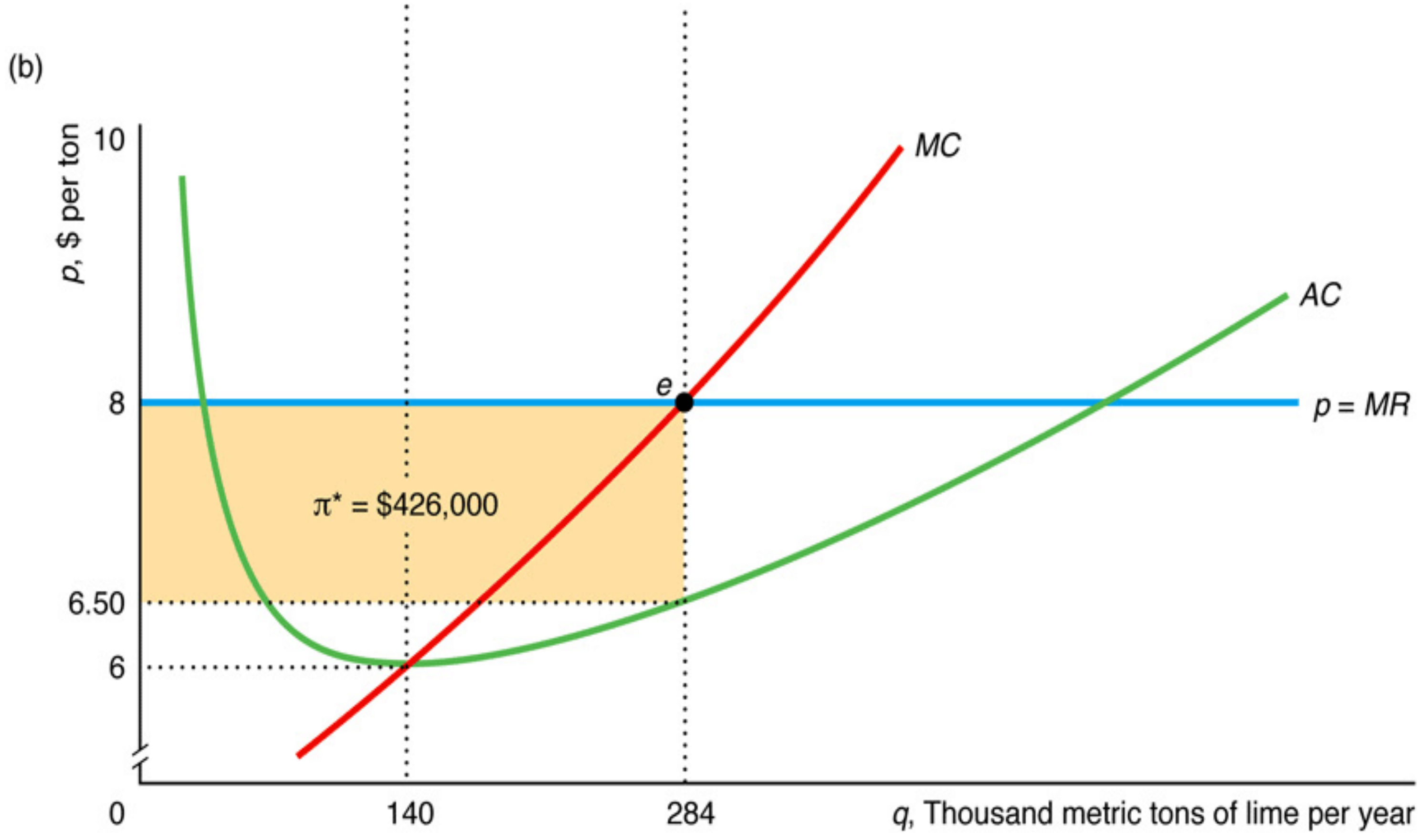
Output Decision: Graph

- Profit is maximized at $MR = MC$.



Output Decision: Graph

- Profit is maximized at $q = 284$ and $\pi = \$426{,}000$.



Output Decision: Math

- Profit: Revenue – Cost

$$\pi = pq - C(q)$$

  - In the short run, K is fixed

$$C(q) = wf^{-1}(q;\bar{K}) + r\bar{K}$$

  - In the long run, the optimal (L, K) is chosen

$$C(q) = wL^*(q) + rK^*(q)$$

Output Decision: Math

- Firm maximizes its profit:

$$\pi = pq - C(q)$$

  - Take the FOC:

$$p - MC(q^*) = 0$$

  - Take the SOC:

$$\frac{dMC(q^*)}{dq} > 0$$

Short-Run Shutdown Decision

- A firm shuts down only if it can reduce its loss by doing so.
  - Shutting down means that the firm stops producing (and thus stops receiving revenue) and stops paying avoidable costs.
  - Only fixed costs are unavoidable.
  - Firms compare revenue to **variable cost** when deciding whether to stop operating.

Short-Run Shutdown Decision

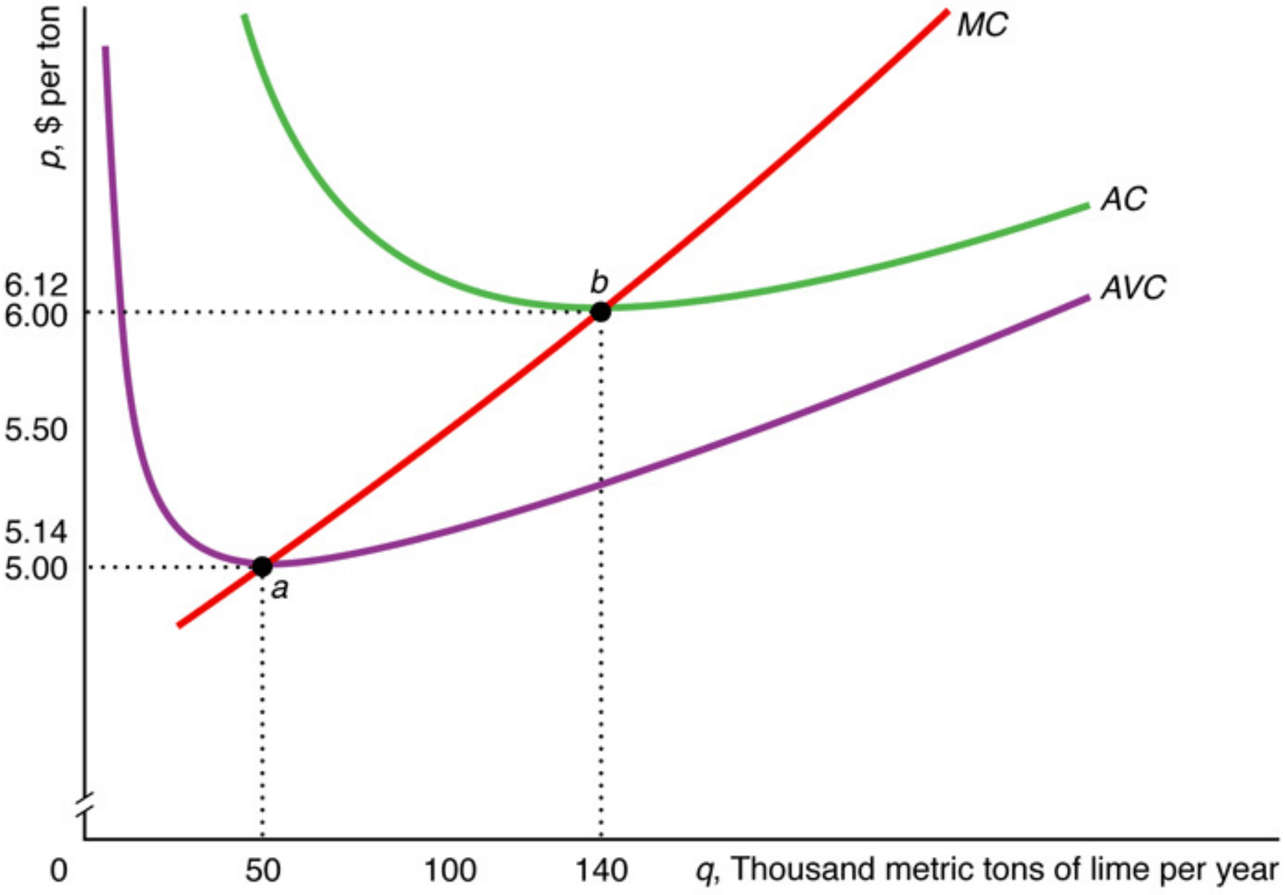
- Recall that firms compare revenues to **variable costs** to determine shutdown:

$$pq - VC(q) < 0 \rightarrow p < AVC(q)$$

- Shut down if market price is less than the minimum of its SR average variable cost curve.
  - If $p < \min_q AVC(q)$, the firm supplies zero.
  - If $p \geq \min_q AVC(q)$, the firm will produce $q > 0$, where $\mathrm{MC}(q) = p$ and $\frac{dMC(q)}{q} > 0$.

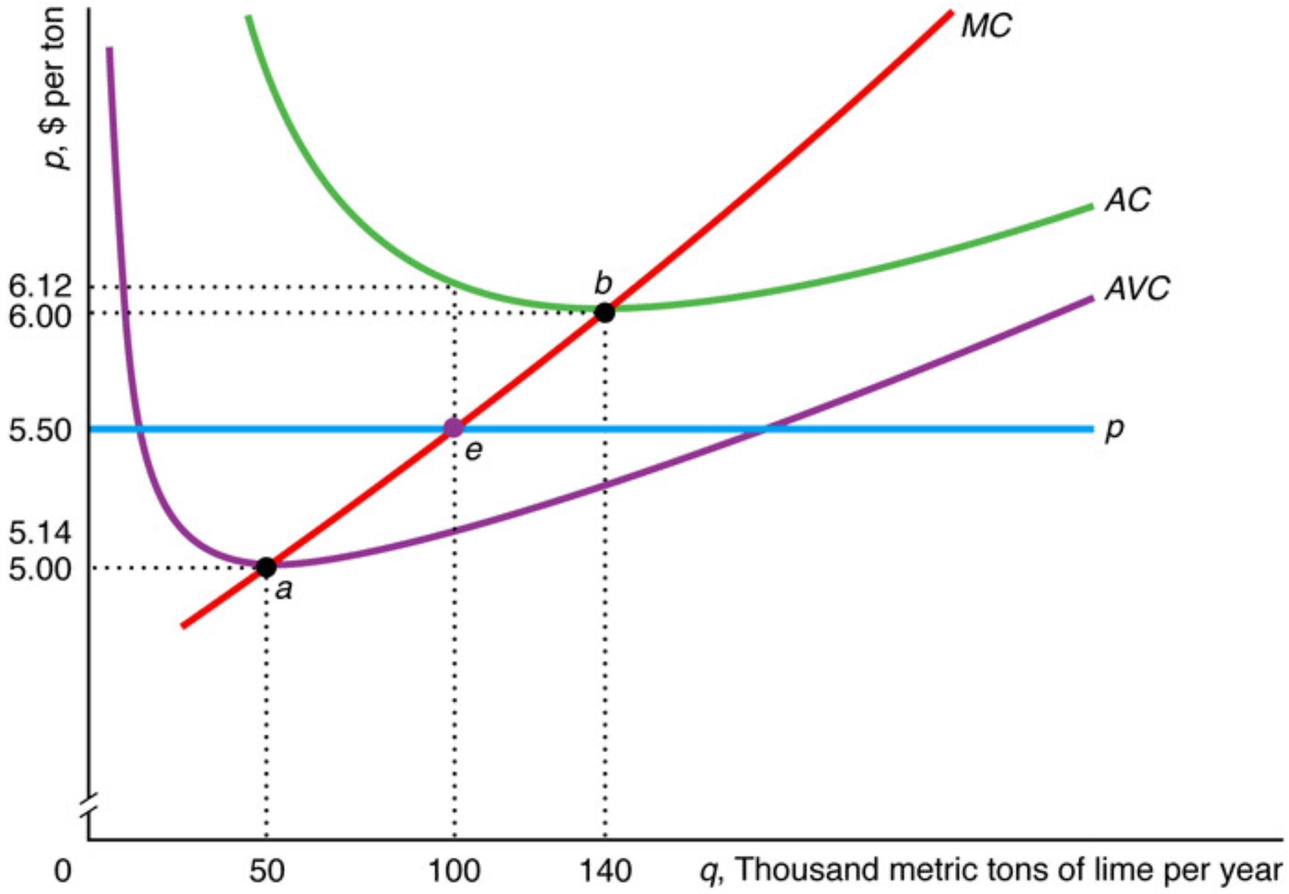
Short-Run Shutdown Decision

- Short-Run cost curves



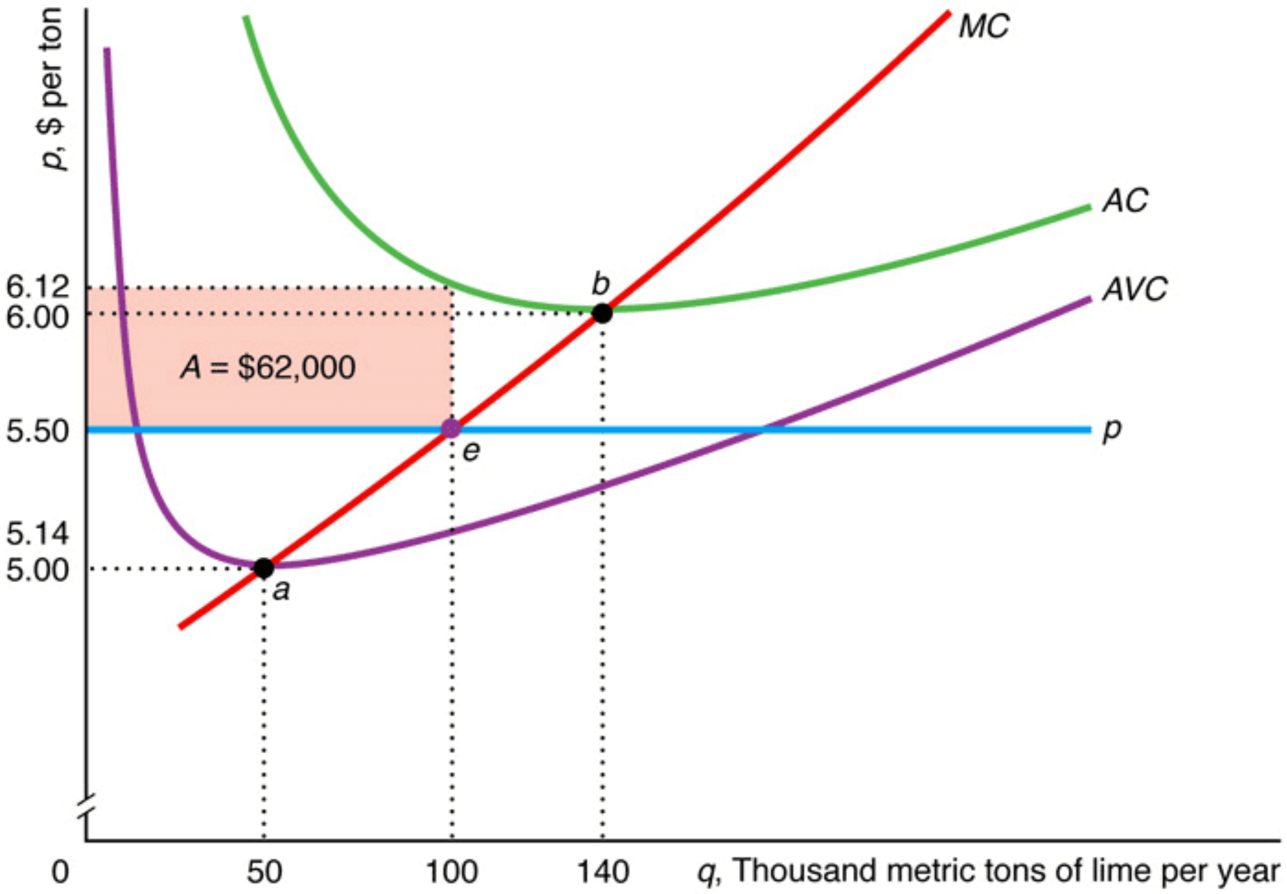
Short-Run Shutdown Decision

- Output price (p): $\min_q AVC(q) < p < \min_q AC(q)$



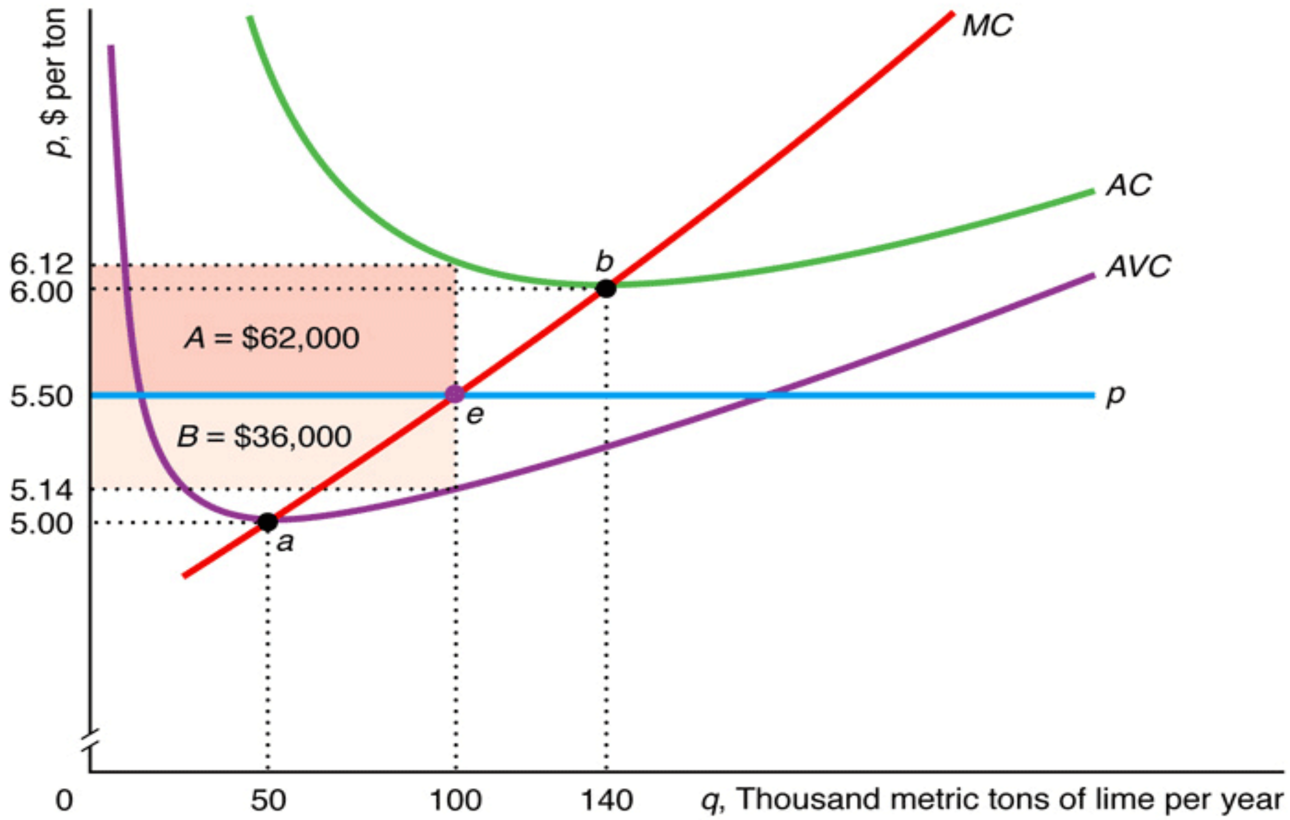
Short-Run Shutdown Decision

- By producing 100, the loss is $62,000.



Short-Run Shutdown Decision

- By shutting down, the loss is $98,000 (total fixed cost).
- Therefore, firm does not shut down, but at a loss.



Short-Run Shutdown Decision

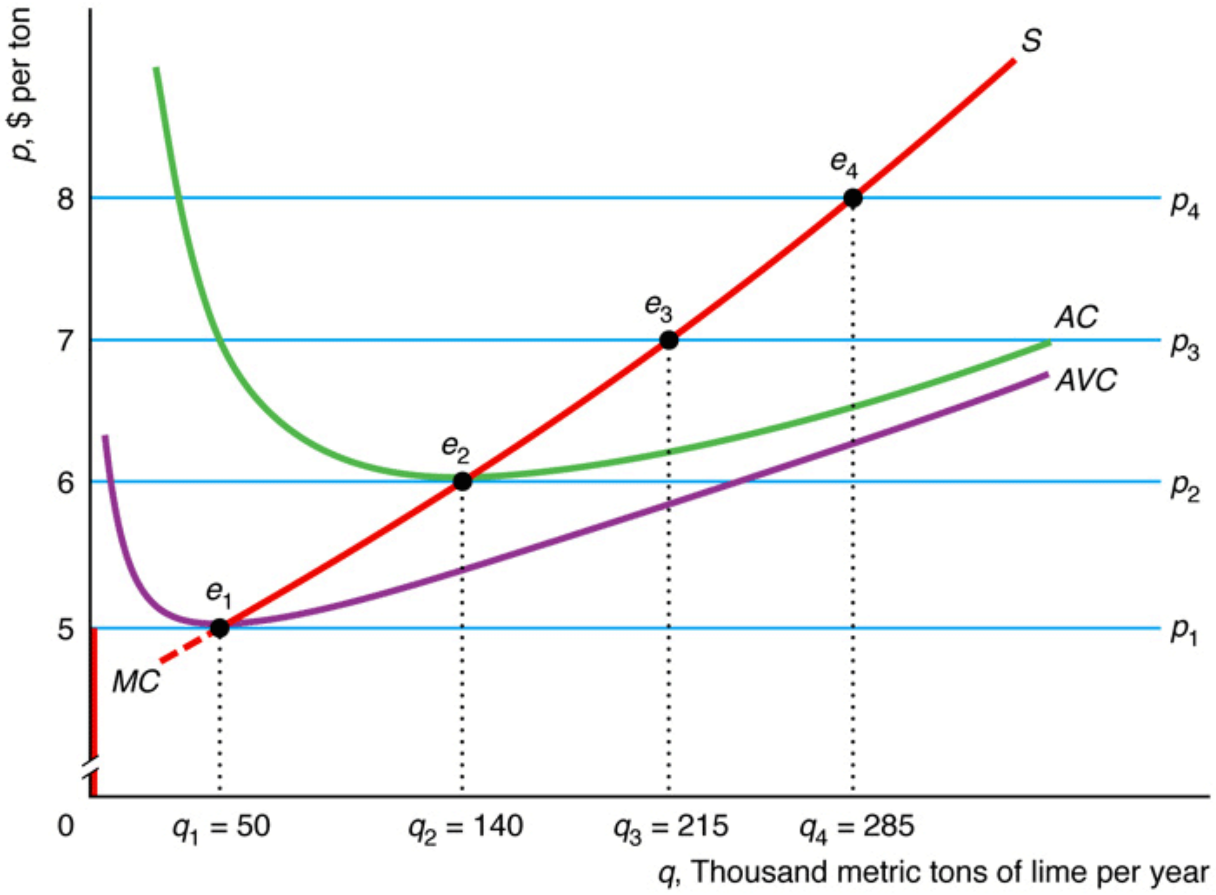
- $p < \min_q AVC(q)$: Firm shuts down.
- $p \geq \min_q AVC(q)$: Firm does not shut down.
  - When $\min_q AVC(q) \leq p < \min_q AC(q)$, the firm makes losses.
  - However, it doesn't shut down because it can still pay some of the fixed costs.

Short-Run Firm Supply Curve

- Firms will choose to produce as long as market price is above the *AVC* minimum, so that is where a firm's supply curve begins.

- As we consider higher and higher market prices, the horizontal firm demand curve rises and intersects *MC* at higher and higher quantities.

Short-Run Firm Supply Curve



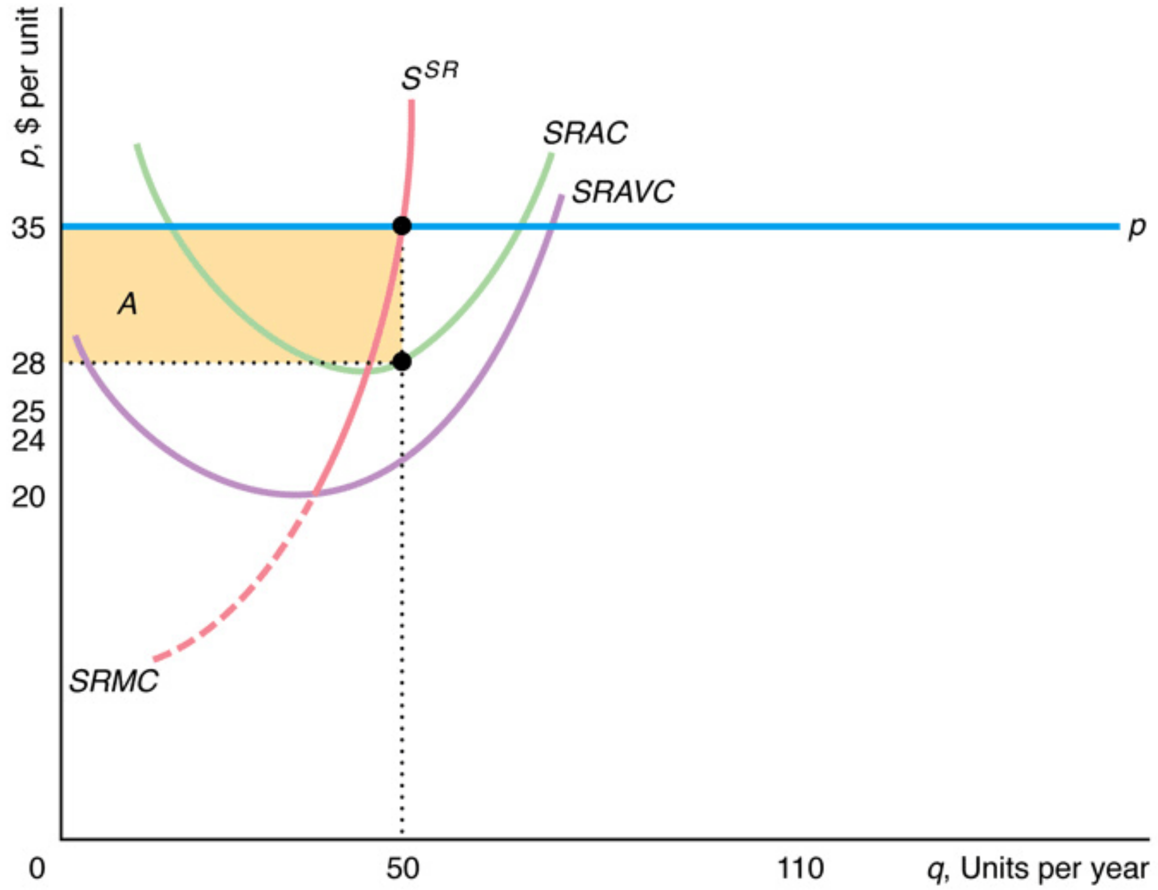
Long-Run Shutdown Decision

- Because all costs are variable in the LR, the firm shuts down if it would suffer an economic loss by continuing to operate.

- Graphically, relevant shutdown point is the minimum of the LR average cost curve.

Long-Run Firm Supply Curve

- In the Short-Run, firm produces 50 and earns A.



Long-Run Firm Supply Curve

- In the Long-Run, firm produces 110 and earns A+B.