Optimization

Tutorial 2

Problem 1. Model fitting

We have measured data of a given system illustrated by the Figure 1.

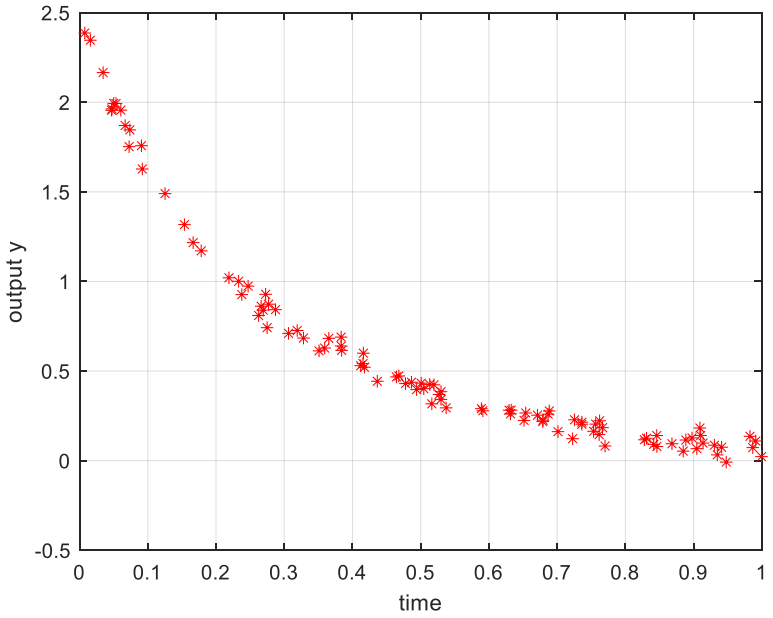

Figure 1. Measured data versus time.

We want to determine a model that represents this behavior. The chosen model is given by:

$$M(a_{1..3}, T_{1..3}) = a_1 \exp(-t/T_1) + a_2 \exp(-t/T_2) + a_3 \exp(-t/T_3) \quad (1)$$

Where T_1, T_2 and T_3 are strictly positive.

I. We assume first that the T_1, T_2 and T_3 are known. We want to find the best values for a_1, a_2 a_3 so that the total quadratic error between the model and measured data is minimized.

1. Give the corresponding optimization problem.
2. How to solve efficiently this problem ?
3. Solve numerically this optimization problem (complete the file *Problem1_I.m*).

II. Now, we assume that a_1, a_2 a_3 are known. We want to determine T_1, T_2 and T_3 so that the total quadratic error between the model and measured data is minimized.

1. Give the corresponding optimization problem.
2. Can we use the same algorithm as in question I.3 to solve this optimization problem ?
3. Solve numerically the new optimization problem (complete the file *Problem1_II.m*).

Problem 2. Portofolio optimization

An investor wants to invest into two stock types, to get at least 10% rate of return, while minimizing the risk of losses.

He has the previous rates of return of these two stock types over the last 5 years. They are given in the following table.

Stock type	Year 1	Year 2	Year 3	Year 4	Year 5
Type 1	16,1	15,7	10,1	11,5	10,3
Type 2	13,3	20,4	11,2	18,5	17,9

1. Formulate the considered problem as an optimization problem, by introducing the mean values of the rate of returns, and the covariance of these two types.
2. What is the kind of the determined optimization problem? Does a solution exist for this problem? Is it unique?
3. Solve this problem (complete the file *Problem2.m*).
4. Analyze the obtained solution (rate of return, variance).

Problem 3.

Consider the optimization problem:

$$\min_{x,y} x^2 + y^2 \ \ s.t.\ x + y = 1 \tag{2}$$

Solve the optimization problem:

a. Graphically (use the file *Problem3.m*).
b. Using the KKT conditions,
c. Numerically by using Matlab. Analyze the solution (active/inactive constrains, Lagrange multipliers…).

Problem 4.

Consider the optimization problem:

$$\min_{x,y} (x-3)^2 + (y-2)^2 \ \ s.t. \begin{cases} x^2 - y - 3 \le 0 \\ y \le 1 \end{cases} \tag{3}$$

- Solve the optimization problem:
  a. Using the KKT conditions,
  b. Numerically by using Matlab.
- Analyze the solution (active/inactive constrains, Lagrange multipliers…).

Problem 5.

Consider the optimization problem:

$$\min \quad y$$
$$s.c.\begin{cases}(3-x)^3-(y-2)\geq 0\\ 3x+y\geq 9\end{cases}$$

- Solve the optimization problem:
  a. Using the KKT conditions,
  b. Numerically by using Matlab.
- Analyze the solution (active/inactive constrains, Lagrange multipliers…).

Problem 6.

Consider the problem :

$$\min_{x_1,x_2} -3x_1-4x_2$$
$$s.t.\begin{cases}x_1+2x_2\leq 50\\ x_1\leq 20\\ x_2\leq 30\\ x_1,x_2\geq 0\end{cases}$$

1. What kind of optimization problem is this problem ?
2. Give the standard form of this problem.
3. Solve numerically the problem (using Linprog of Matlab). Analyze the result (active/inactive constraints, optimal cost, solution…).