Bus Econ HW7

1) Markets and pollution externalities

i) $P = MC = 2$ $2 = 10 - q$ $q = 8$ profit $= (2-2)q = 0$

customer surplus $= \frac{(10-2)8}{2} = 32$

ii) $P = MC = 2$ (+3 for pollution) $5 = 10 - q$ $q = 5$ profit $= (2-2)5 = 0$

producer surplus $= 15$ but must go towards environmental costs cust surplus $\frac{(10-5)5}{2} = 12.5$

The consumer surplus went down from $32 \to 12.5$ in order to bear the costs of pollution. Profit is still 0 bc firm sells at equilibrium $p = MC = 2$

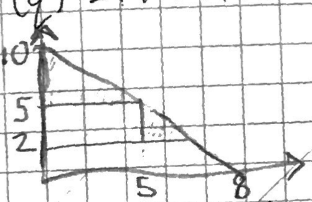
iii) now $MC = 5$ $5 = 10 - q$ $q = 5$ profit $= (5-5)5 = 0$

Customer surplus $= \frac{(10-5)5}{2} = 12.5$

iv) $\frac{d}{dq} P(q) = MR = MC \to 5 \Rightarrow \frac{d}{dq}(10q - q^2) = 10 - 2q = 5$ $\boxed{q = \frac{5}{2},\ p = \frac{15}{2}}$

$q = 4$ $p = 6$ profit $= (6-2)4 = 16$ C.S. $= 0$

if firm pays no taxes $\boxed{10 - 2q = 2}$



2) i) $\max_{q_1} \pi_1(q_1, q_2) = (1 - q_1 - q_2)q_1 - \frac{1}{2}(q_1 + q_2)^2$

$\hookrightarrow d/dx$ $1 - 2q_1 - q_2 = \frac{1}{2}(2q_1 + 2q_2) \Rightarrow 1 - 2q_2 = 3q_1 \Rightarrow \frac{1 - 2q_2}{3} = q_1$

$\hookrightarrow$ same for q_2: $q_2 = \frac{1 - 2q_1}{3}$, plug in: $q_1 = \frac{1 - 2\left(\frac{1-2q_1}{3}\right)}{3} \Rightarrow \frac{1 - \frac{2 - 4q_1}{3}}{3}$

$\hookrightarrow \frac{1}{3} - \frac{2}{9} + \frac{4}{9}q_1 = q_1 \Rightarrow \frac{5}{9}q_1 = \frac{1}{9} \Rightarrow \boxed{q_1 = \frac{1}{5}}$

$q_2 = \frac{1 - \frac{2}{5}}{3} = \frac{3}{15} = \boxed{\frac{1}{5} = q_2}$ $\boxed{p = \frac{3}{5}} = 1 - \frac{1}{5} - \frac{1}{5}$

q_1 profit: $\frac{1}{5}\frac{3}{5} - \frac{1}{5}(\frac{2}{5})^2\frac{1}{2} = 0.04$

q_2 profit: $(\frac{3}{5} - \frac{1}{2}(\frac{2}{5})^2)\frac{1}{5} = 0.104$

ii) $\max_{q_1} \pi_1 = (1 - q_1 - q_2) - \bar{p}q_1 \Rightarrow \frac{d\pi_1}{dq_1} = 1 - 2q_1 - q_2 - \bar{p}$

$\boxed{BR_1(q_2) = \frac{1 - q_2 - \bar{p}}{2},\ BR_2(q_1) = \frac{1 - q_1 - \bar{p}}{2}}$

similar logic for q_2 $\hookrightarrow q_1 = \frac{1 - q_2 - \bar{p}}{2} = q_2$

Nash Eq: $q_1 = \frac{1}{2}\left(1 - \left[\frac{1 - q_1 - \bar{p}}{2}\right] - \bar{p}\right) \Rightarrow 2q_1 = 1 - \left[\frac{1 - q_1 - \bar{p}}{2}\right] - \bar{p}$

$\hookrightarrow 2q_1 + \bar{p} = \frac{1 + q_1 + \bar{p}}{2} \Rightarrow 4q_1 + 2\bar{p} = 1 + q_1 + \bar{p}$

$\hookrightarrow 3q_1 = 1 - \bar{p} \Rightarrow \boxed{q_1 = \frac{1 - \bar{p}}{3} = q_2}$ similar logic

Profit: $\pi_1 = (1 - q_1 - q_2)q_1 - \bar{p}q_1 \Rightarrow \left(1 - \frac{1-\bar{p}}{3} - \frac{1-\bar{p}}{3}\right)\frac{1-\bar{p}}{3} - \bar{p}\left(\frac{1-\bar{p}}{3}\right)$

$\hookrightarrow \boxed{\left(\frac{1-\bar{p}}{3}\right)^2 = \pi_1}$ which by similar logic also $= \pi_2$

Q2)

2) $\Pi_M = \bar{p}Q - \frac{1}{2}Q^2$ where $Q = (q_1 + q_2) = 2\left(\frac{1-\bar{p}}{3}\right)$

↳ $\Pi = \bar{p}\left[\frac{2-2\bar{p}}{3}\right] - \frac{1}{2}\left(2\left(\frac{1-\bar{p}}{3}\right)\right)^2$

↳ $\left(\frac{2\bar{p}-2\bar{p}^2}{3}\right) - \frac{1}{2}\left(\frac{2-2\bar{p}}{3}\right)^2 \iff \left(\frac{2\bar{p}-2\bar{p}}{3}\right) - \frac{1}{2}\left(\frac{2-2\bar{p}}{3}\right)^2$

↳ $\frac{2\bar{p}-2\bar{p}^2}{3} - \frac{1}{2}\cdot\frac{4-8\bar{p}+4\bar{p}^2}{9} \Rightarrow \frac{2\bar{p}+2\bar{p}^2}{3} - \frac{2-4\bar{p}+2\bar{p}^2}{9}$

↳ $\frac{6\bar{p}-6\bar{p}^2 - 2 + 4\bar{p}^3 + 2\bar{p}^2}{9} \Rightarrow \frac{10\bar{p} - 8\bar{p}^2 - 2}{9}$

$\frac{d\Pi}{d\bar{p}}$ ↳ $\frac{10}{9} - \frac{16\bar{p}}{9} = 0 \Rightarrow \frac{10}{9} = \frac{16\bar{p}}{9} \Rightarrow 10 = 16\bar{p} \Rightarrow \boxed{\bar{p} = \frac{5}{8}}$

3) Quantities $q_1 = \frac{1-\bar{p}}{3} = q_2 \Rightarrow \frac{1-5/8}{3} \Rightarrow \boxed{\frac{1}{8}}$ $\quad p = 1 - 1/8 - 1/8 = \boxed{\frac{3}{4}}$

$Q = 1/8 + 1/8 = \boxed{1/4}$

4) $\Pi_1 = \Pi_2 = pq_1 + 2F = (1-q_1-q_2)q_1 + 2F$

$\frac{d\Pi_1}{dp_1} = \frac{1-\bar{p}-q_2}{2} = \frac{1-\bar{p}-q_2}{2} \Rightarrow q_1 = q_2 = \frac{1-\bar{p}}{3}$

$p = MC = \frac{d}{dQ}\left(\frac{1}{2}Q^2\right)$, max $F = \Pi_1 = \Pi_2 = \left(\frac{1-\bar{p}}{3}\right)^2 \Rightarrow \frac{1-2\bar{p}-\bar{p}^2}{9}$

5) $\Pi = \bar{p}Q + 2F - \frac{1}{2}Q^2$

↳ $\bar{p}\left(\frac{2-2\bar{p}}{3}\right) + 2\left(\frac{1-2\bar{p}-\bar{p}^2}{9}\right) - \frac{1}{2}\left(\frac{4}{3}\left(\frac{1-\bar{p}}{3}\right)^2\right)$

↳ $\frac{2\bar{p}-2\bar{p}^2}{3} + \frac{2\bar{p}-4\bar{p}-2\bar{p}^2}{9} - \frac{2\bar{p}-4\bar{p}-2\bar{p}^2}{9}$

$\frac{d\Pi}{d\bar{p}}$ ↳ $\frac{2\bar{p}-2\bar{p}^2}{3} = 0 \Rightarrow \frac{2-4\bar{p}}{3} = 0 \Rightarrow 4\bar{p} = 2 \Rightarrow \boxed{\bar{p} = 0.5}$

6) $\Pi = \left(\frac{2\bar{p}-2\bar{p}^2}{3}\right) \Rightarrow \frac{2(0.5)-2(0.5)^2}{3} \Rightarrow \frac{1}{3} - \frac{1}{6} \Rightarrow \boxed{\frac{1}{6}}$

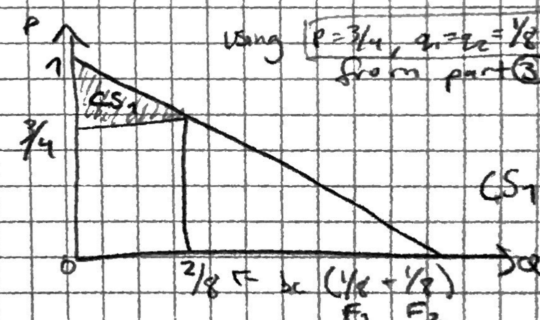
$Q = \frac{2}{3}(1-\bar{p}) \Rightarrow \frac{2}{3}(1-0.5) \Rightarrow \boxed{\frac{1}{3}}$

$P = (1-q_1-q_2) = (1-Q) = \boxed{2/3}$

7) $CS_1 = \frac{b \cdot h}{2} = \frac{(1-3/4)\cdot(2/8)}{2}$

↳ $\frac{1/4 \cdot 1/4}{2} \Rightarrow \boxed{\frac{1}{32}}$

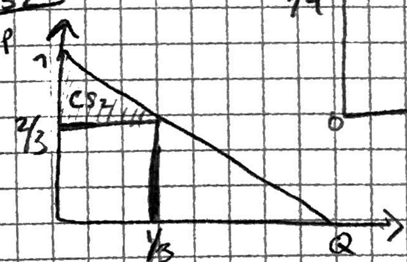
using $\boxed{p = 3/4,\ q_1 = q_2 = 1/8}$ from part ③



CS_1

from part ⑥ $\boxed{P = 2/3,\ Q = 1/3}$

$CS_2 = \frac{(1-2/3)(1/3)}{2} \Rightarrow \boxed{\frac{1}{18}}$



CS_2

3) Competition w/ limited attention

1) $\overset{\xleftrightarrow{d}}{0 \quad\quad x} \quad 1$ — 0 Firm 1, x, Firm 2, 1

$U_1 = \bar{S} - p_1 - dt = \bar{S} - p_1 - (x-0)t$

$U_2 = \bar{S} - p_2 - dt = \bar{S} - p_2 - (1-x)t$

Market Share: $\mu_1 = \mu_2 \Rightarrow \bar{S} - p_2 - t + xt = \bar{S} - p_1 - xt$

$\hookrightarrow -p_2 - t + xt = -p_1 - xt \Rightarrow p_2 + t - xt = p_1 + xt$

$\hookrightarrow 2xt = p_2 - p_1 + t \Rightarrow x = \frac{p_2 - p_1 + t}{2t}$

Firm 1 = x, Firm 2 = 1 − x market share

2) $\pi_1 = (p_1 - \text{cost}) \cdot x$

$\hookrightarrow = \boxed{(p_1 - c) \cdot \frac{p_2 - p_1 + t}{2t}}$

$\frac{d\pi_1}{dp_1} = 0 = p_2 - 2p_1 + t + c = 0$

$\boxed{p_1 = \frac{p_2 + t + c}{2} = f_2}$

→ similar logic for $\pi_2 = (p_2 - c)\left(1 - \frac{p_2 - p_1 + t}{2t}\right)$ ← one minus

Best Response function Firm 1 $\left(\frac{p_2 + t + c}{2}\right)$

Best Response function Firm 2 $\left(\frac{p_1 + t + c}{2}\right)$

3) Equilibrium $p_1 = \frac{p_2 + t + c}{2} \Rightarrow p_2 = \frac{\left(\frac{p_2 + t + c}{2}\right) + t + c}{2}$

$\hookrightarrow 2p_2 = \frac{p_2 + t + c}{2} + t + c \Rightarrow 2p_2 = \frac{p_2 + t + c + 2t + 2c}{2}$

$\hookrightarrow 4p_2 = p_2 + 3t + 3c \Rightarrow 3p_2 = 3t + 3c \Rightarrow \boxed{p_2 = t + c = p_1}$

Since prices are equal, the consumer is indifferent; 0 —— 0.5 —— x, $x = 0.5$

Demand of Firm 1 $= x = \frac{1}{2} = 1 - x$ = Demand of Firm 2

$\pi_1 = \pi_2 = (p_1 - c)\left(\frac{1}{2}\right) = (p_2 - c)\left(\frac{1}{2}\right) = (t + c - c)\left(\frac{1}{2}\right) = \boxed{\frac{t}{2}}$ so satisfying

4) F1's Demand: $D(p_1, p_2, \lambda_1, \lambda_2, \alpha) =$ [redacted]

$\lambda_1 \lambda_2 B$ = those who understand

$\lambda_1 \lambda_2 \frac{(1-B)}{2}$ = randomly split

$\hookrightarrow D_1 = \lambda_1(1 - \lambda_2) + \lambda_1\lambda_2 \frac{(1-B)}{2} + \lambda_1\lambda_2(B)\left(\frac{p_2 - p_1 + t}{2t}\right)$

$\pi_1 = D_1 p_1 - \frac{\alpha \lambda_1^2}{2}$

$\hookrightarrow \left[\lambda_1(1-\lambda_2) + \lambda_1\lambda_2\frac{(1-B)}{2} + \lambda_1\lambda_2(B)\left(\frac{p_2 - p_1 + t}{2t}\right)\right]p_1 - \frac{\alpha\lambda_1^2}{2}$

5) $\frac{d\pi}{dp_1} = \lambda_1(1-\lambda_2) + \lambda_1\lambda_2\left(\frac{1-B}{2}\right) + \lambda_1\lambda_2 B\left(\frac{p_2 - 2p_1 + t}{2t}\right) = 0$

$\hookrightarrow \lambda_1 - \lambda_1\lambda_2 + \lambda_1\lambda_2\left(\frac{1-B}{2}\right) + \lambda_1\lambda_2 B\left(\frac{p_2 + t}{2t}\right) - \lambda_1\lambda_2 B\left(\frac{2p_1}{2t}\right) = 0$

$\hookrightarrow \lambda_2 B\left(\frac{p_1}{t}\right) = 1 - \lambda_2 + \lambda_2\left(\frac{1-B}{2}\right) + \lambda_2 B\left(\frac{p_2 + t}{2t}\right)$

$\boxed{p_1 = \frac{t - tB}{2B} + \frac{t}{\lambda_2} - \frac{t}{B} + \frac{p_2 + t}{2}}$

$\frac{d\pi}{d\lambda_1} = p_1(1-\lambda_2) + p_1\lambda_2\left(\frac{1-B}{2}\right) + p_1\lambda_2 B\left(\frac{p_2 - p_1 + t}{2t}\right) - \alpha\lambda_1 = 0$

$\hookrightarrow \alpha\lambda_1 = p_1\left[1 - \lambda_2 + \lambda_2\left(\frac{1-B}{2}\right) + \lambda_2 B\left(\frac{p_2 - p_1 + t}{2t}\right)\right]$

$\hookrightarrow \alpha\lambda_1 = p_1\left[\underbrace{1 - \lambda_2 + \lambda_2\frac{(1-B)}{2} + \lambda_2 B\left(\frac{p_2 - 2p_1 + t}{2t}\right)}_{\Rightarrow 0} + \lambda_2 B\left(\frac{p_1}{2t}\right)\right]$

$\hookrightarrow \alpha\lambda_1 = p_1\left[\lambda_2 B\left(\frac{p_1}{2t}\right)\right] \Rightarrow \alpha\lambda_1 = \frac{\lambda_2 B p_1^2}{2t}$

$\hookrightarrow \boxed{\lambda_1 = \frac{\lambda_2 B p_1^2}{2\alpha t}} = \frac{\lambda_2 B}{2\alpha t}\left[\frac{t - tB}{2B} + \frac{t}{\lambda_2} - \frac{t}{B} + \frac{p_2 + t}{2}\right]^2$

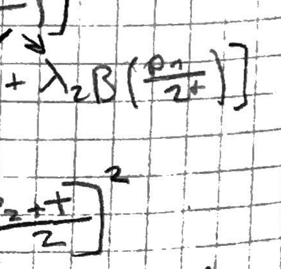
6) $p_1 = p_2 = p = \frac{t - tB}{2B} + \frac{t}{\lambda_2} - \frac{t}{B} + \frac{p_2 + t}{2} \Rightarrow p - \frac{p + t}{2} = \frac{t - tB}{2B} + \frac{t}{\lambda_2} - \frac{t}{B}$

$\hookrightarrow p - t = 2\left[\frac{t - tB}{2B} + \frac{t}{\lambda} - \frac{t}{B} + \frac{p + t}{2}\right] \Rightarrow p = \frac{2t}{B\lambda} - \frac{t}{B} \Rightarrow \boxed{\frac{t}{B}\left(\frac{2}{\lambda} - 1\right)}$

$\lambda_1 = \lambda_2 = \lambda = \frac{\lambda B}{2\alpha t}\left[\frac{t}{B}\left(\frac{2}{\lambda} - 1\right)\right]^2 \Rightarrow \frac{\lambda}{2\alpha}\left(\frac{t}{B}\right)\left(\frac{4}{\lambda^2} - \frac{4}{\lambda} + 1\right) = \lambda$

using calculator $\hookrightarrow \lambda = \frac{2(-t + \sqrt{2}\sqrt{B\alpha t})}{2B\alpha - t}$ or $\lambda = \frac{2(t + \sqrt{2}\sqrt{B\alpha t})}{-2B\alpha + t}$

$p = \frac{t}{B}\left(\frac{2B\alpha + \sqrt{2}\sqrt{B\alpha t}}{t - \sqrt{2}\sqrt{B\alpha t}}\right)$ or $p = \frac{t}{B}\left(\frac{2B\alpha + \sqrt{2}\sqrt{B\alpha t}}{t + \sqrt{2}\sqrt{B\alpha t}}\right)$



7) a change in B ↑ would cause λ ↓ and P ↓ or vice-versa because the variable are *inversely influenced* — the denominator grows more rapidly B^n