Hypothesis testing

Introduction to hypothesis testing

Exercise 1: Hypothesis testing and biased coin

You are arguing with a friend about the movie you are about to watch. This friend decides to make this choice by tossing a coin. Before doing so, you want to check that the coin is not biased.

1. You propose to flip the coin 10 times and keep the coin if the number of heads is 5. Write the statistical model and the hypotheses you are testing. What is the rejection region and the acceptance region? Compute the significance level of this test and plot its power function.

2. You repeated the previous strategy twice on two of your four coins and you rejected the two first coins. Since the rejection rate under H_0 (0.75) is too high, you decide to change your strategy. You propose another rule for the two last coins: if the number of heads among 10 tosses is smaller or equal to 1 or larger or equal to 9 then you use another coin. What is the rejection region and the acceptance region? Compute the significance level of this test. Plot the power function. What is the probability that you accept the coin if it is actually biased with a probability of 3/4 (and 1/4) to obtain a head?

3. Your friend disagrees with your rule because he is superstitious and hates number 2. He would prefer to accept the coin if there are 1, 3, 4, 5, 6, 7, 8 or 9 heads. What is the rejection region and the acceptance region? Compute the significance level of this test. Plot the power function. What is the probability that you accept the coin if it is actually biased with a probability of 3/4 (and 1/4) to obtain a head?

4. You finally propose to throw 30 times the coin and reject the coin if the number of heads X is such that $|X - 15| > \delta$ for some $\delta > 0$. Propose a hypothesis testing at level smaller than and as close as possible to 0.05. Plot its power. What is the probability that you accept the coin if it is actually biased and with a probability of 3/4 (and 1/4) to obtain a head? What is your conclusion if you obtain 4 heads?

5. Comparing the power function of the four previous decision rules, which test would you choose and why?

Exercise 2: Various experiments

Formalize the hypotheses and statistical models in the following cases:

1. Defeated in his most recent attempt to win a congressional seat because of a large gender gap, a politician has spent the last two years speaking out in favor of women's rights issues. A newly released poll claims to have contacted a random sample of 120 of the politician's current supporters and found that 72 were men. In the election that he lost, exit polls indicated that 65% of those who voted for him were men. The politician wants to know if his campaign has a positive effect on women.

2. A herbalist is experimenting juices extracted from berries and roots that may have the ability to affect the Stanford-Binet IQ scores of students afflicted with mild cases of attention deficit disorder (ADD). A random sample of twenty-two children diagnosed with the condition have been drinking Brain-Blaster daily for two months. Past experience suggests that children with ADD score an average of 95 on the IQ test with a standard deviation of 15.

3. A company sold a sampler producing uniform random digits. Its client suspects the sampler to be biased towards small digits and wants to complain. The client pays an external control society to test this sampler. They obtain the following sample $1, 2, 1, 2, 0, 8, 6, 1, 2, 4, 5, 1, 2, 4, 8, 4, 4, 3, 0, 2$.