73-240 – Problem set 3

Due Friday Oct 25th

From the Syllabus:

1. Homework must be turned in on the day it is due in the undergraduate program office in Tepper 2400 by **430pm** sharp. Do NOT leave it under my office door. Late homework will NOT be accepted unless you are sick and have a doctor's note.
2. Homework regrading: There is a statute of limitations on regrades. If you believe a question has been incorrectly graded, please take your homework to your TA within 2 weeks of it being returned.
3. Working in groups: You may work in groups of up to 4. BUT: You MUST put names of other group members on your homework. You MUST write up your own set of answers. Do NOT simply copy some other person's work.
4. TYPE your work. Long equations may be hand written. Buy a stapler!
5. Write your first and last name on the title of each graph. Graphs may be hand drawn.
6. Carefully explain your work.
7. Do not attach your excel worksheet. Just the graphs and write-up only.

Problem 1: Unemployment Dynamics (35 pts)

This question asks you to use unemployment dynamics to construct new measures of labor market frictions. Recall from Lecture 10 that our model of unemployment dynamics requires information on the percent of the labor force that experiences job separations ($s_t e_t$) and the percent of the labor force that experiences job accessions ($p(\theta_t)u_t$). We start by using raw data to determine the probability an individual leaves unemployment.

A) Obtain data for the period 2000m12-2019m8 on the number of unemployed individuals, the number of hires, and the number of job openings/vacancies. Also gather information on the separation rate and the unemployment rate. On the St. Louis FRED website, the mnemonics for these data are

- UNEMPLOY (number of individuals unemployed)
- JTSHIL (number of hires)
- JTSJOL (number of vacancies)
- JTSTSR (separation rate, remember to divide this by a 100 since the original data is represented in percent terms.)
- UNRATE (unemployment rate, remember to divide this by a 100 since the original data is represented in percent terms.)

No answer required, part A is instructions only.

B) We will treat all unemployed as the job-seekers in our question. We will treat all job-openings as vacancies in this question. Assume that matches are given by the following matching function

$$M = e\frac{U_t V_t}{(U_t^\gamma + V_t^\gamma)^{1/\gamma}}$$

Assume the matching efficiency $e = 3.5$ and that $\gamma = 0.5$. Show that the matching function exhibits constant returns to scale (CRS).
Answer: CRS implies that if we double the inputs, we can double the outputs:

$$\begin{aligned} e\frac{2U_t \times 2V_t}{[(2U_t)^\gamma + (2V_t)^\gamma]^{1/\gamma}} &= e\frac{4U_t V_t}{[2^\gamma(U_t^\gamma + V_t^\gamma)]^{1/\gamma}} \\ &= e\frac{4U_t V_t}{2\left[U_t^\gamma + V_t^\gamma\right]^{1/\gamma}} \\ &= 2e\frac{U_t V_t}{\left[U_t^\gamma + V_t^\gamma\right]^{1/\gamma}} = 2M \end{aligned}$$

C) Calculate the labor market tightness defined as the ratio of vacancies to job-seekers: $\theta_t = \frac{V_t}{U_t}$. Report the average labor market tightness.
Answer: Average labor market tightness, $\theta = 0.55$.

D) Derive an expression for the job-finding rate $p(\theta_t)$ in terms of γ, e, θ_t. Use information on e, γ and θ_t to calculate $p(\theta_t)$ for each period (DO NOT just take data on hires/unemployed in this question). Report the average job-finding rate.

Answer: We know that $p(\theta_t) = \frac{M_t}{U_t}$. This can further be expressed as:

$$\begin{aligned} p(\theta_t) &= \frac{M_t}{U_t} \\ &= \frac{eU_tV_t}{U_t\left[U_t^\gamma + V_t^\gamma\right]^{1/\gamma}} \\ &= \frac{eV_t}{\left[U_t^\gamma\left(1+\left[\frac{V_t}{U_t}\right]^\gamma\right)\right]^{1/\gamma}} \\ &= \frac{eV_t}{U_t\left[1+\left[\frac{V_t}{U_t}\right]^\gamma\right]^{1/\gamma}} \\ &= \frac{e\theta_t}{\left[1+\theta_t^\gamma\right]^{1/\gamma}} \end{aligned}$$

The average job-finding rate, $p(\theta)$ is 0.61.

E) Write down an expression for how the unemployment rate evolves over time.
Answer:

$$u_{t+1} = (1 - p(\theta_t))u_t + s_t(1 - u_t)$$

F) Find the average separation rate. Assuming no shocks, let $s =$ the average separation rate you found, and $p(\theta) =$ the average job-finding rate you found in part D, calculate the steady state unemployment rate. How does this compare to the actual average unemployment rate in the US economy for the period 2000m12-2019m8?
Answer: Average s from the data is: 3.6% or 0.036
In steady state, the unemployment rate is given by:

$$u = \frac{s}{s+p}$$

Using average $p(\theta) = 0.61$, $s = 0.036$, we arrive at steady state unemployment rate $u = 5.6\%$ or $u = 0.056$.
The actual average unemployment rate for the period 2000m12-2019m8 in the data is: actual $u = 6.0\%$ or actual $u = 0.060$. Our model predicted steady state unemployment rate comes fairly close to the average unemployment rate observed in the data.

G) Using the data on the unemployment rate in 2000m12, the separation rates in the data and your calculated $p(\theta_t)$ for each period, calculate the predicted unemployment rates for 2001m1-2019m8 using the equation you wrote down in part E. Note the only unemployment rate you should use from the data series, UNRATE, is the unemployment rate in 2000m12. **No answer required, part G is instructions only.**

H) Plot your predicted unemployment rate from part G against the actual unemployment rate (UNRATE) observed in the data. How well does our predicted unemployment rate which only used information on employed and unemployed individuals match the unemployment rate in the data? Report the correlation between the actual and your predicted unemployment rates.

Answer: See Figure 1. Our predicted unemployment rate tends to line up fairly well with the actual unemployment rate although it tends to predict much lower unemployment rates post Great Recession (after Oct 2009) and higher unemployment rates pre Great Recession (prior to Jan 2008). The correlation between the two series is about 0.97.

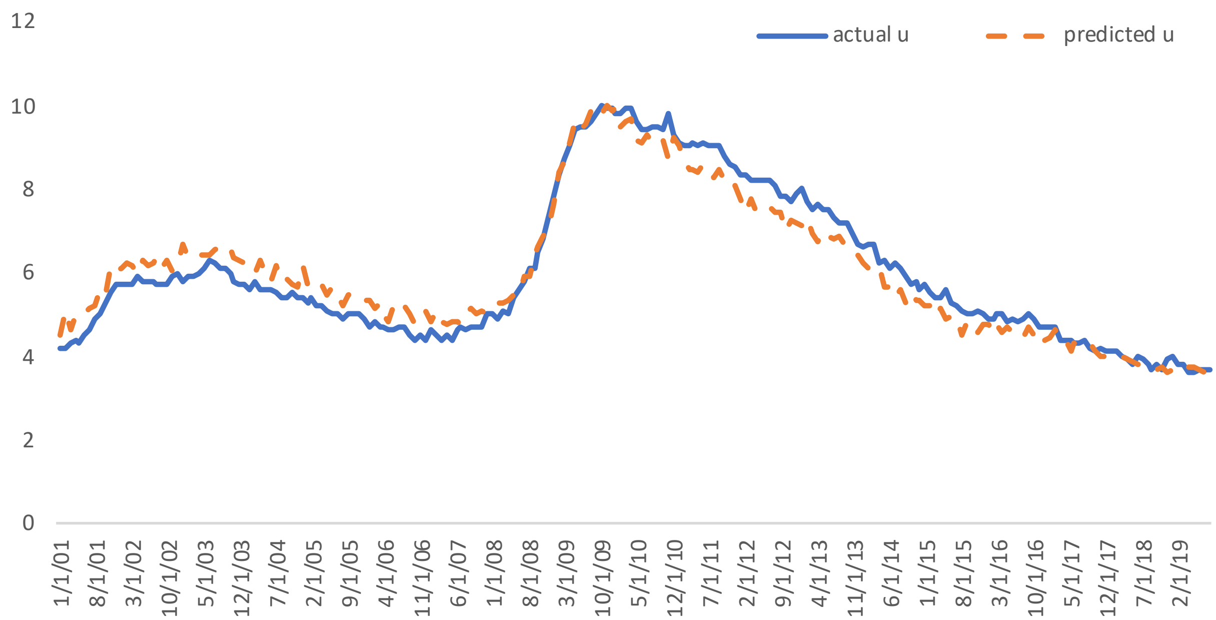

Figure 1: predicted vs. actual unemployment rates

Problem 2: Working With the Matching Function (30 pts)

In the aftermath of the Great Recession, some have argued that the slow recovery in the unemployment rate was due to a decline in matching efficiency. In particular, people have focused on how the shifting out of the Beveridge Curve is a sign of decreasing matching efficiency in the labor market.

i Using data on the vacancy rate and unemployment rate, do a scatter plot of the Beveridge curve for the period 2000m12 - 2007m12. You should have the vacancy rate on the y-axis and unemployment rate on the x-axis. You can obtain data on the vacancy rate and unemployment rate from FRED.
[Go to http://research.stlouisfed.org/fred2/categories and download the series JTSJOR and UNRATE].
Answer: See combined graphs in Figure 2.

ii On the same graph, do a scatter plot of the Beveridge Curve for the period 2008m1 -2019m8.
Answer:
See Figure 2

iii What can you say about the Beveridge curve after 2007m12?
Answer:
The Beveridge curve shifted out post 2007m12. In most recent periods, it has started to shift back in.

iv Some commentators attribute the shift in the Beveridge curve to a decline in matching efficiency. Let's now try to back out matching efficiency from our matching function. Suppose you are told

$$M_t = e_t \frac{U_t V_t}{(U_t^\gamma + V_t^\gamma)^{1/\gamma}}$$

where $\gamma = 0.5$. Using information on hires (M_t), vacancies (V_t) and unemployed (U_t), back out e_t. Plot the graph of e_t for the period 2000m12-2019m8. What happened to e_t in the Great Recession?[Use JTSHIL for M, JTSJOL for V and UNEMPLOY for U].
Answer:
From Figure 3, we can see that matching efficiency declined during the Great Recession.

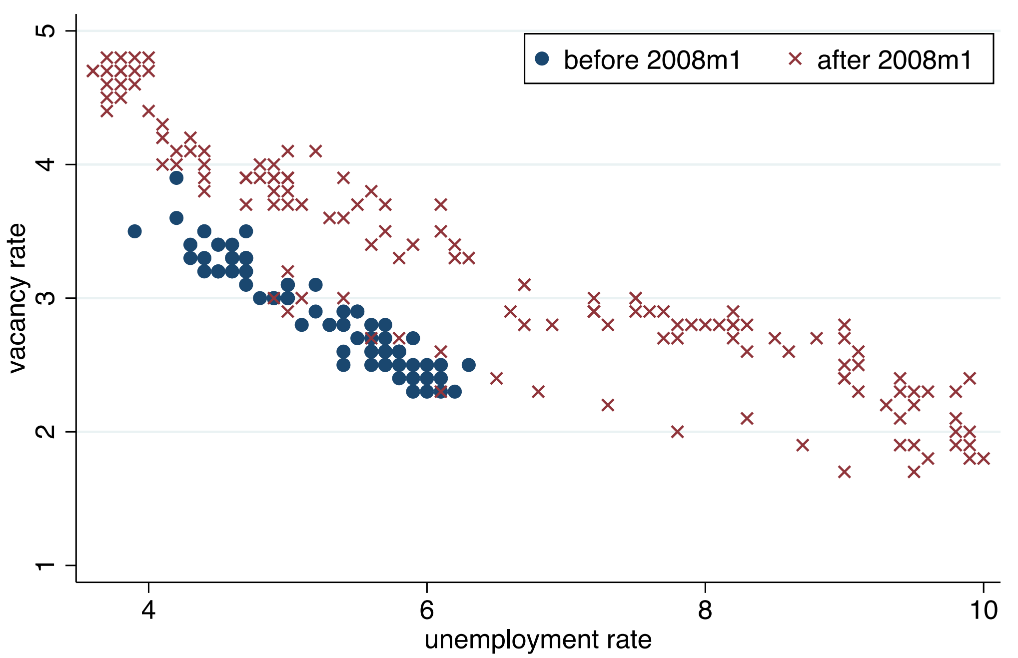

Figure 2: Beveridge Curve Over Time

v Plot for the period 2000m12-2019m8 the job-finding rate you calculated in 1D) where e was assumed to be constant at $e = 3.5$. Suppose instead matching efficiency was not constant, calculate and plot (on the same graph) the job finding rate when e_t is as your answer in part (iv). What do you notice about the two job-finding rates between the two graphs?

Answer:

From Figure 4, the two job-finding rates, while close, particularly diverge in the aftermath of the Great Recession, a period where matching efficiency was observed to decline quite a bit. In particular, the job-finding rate was lower after the Great Recession when e was variable.

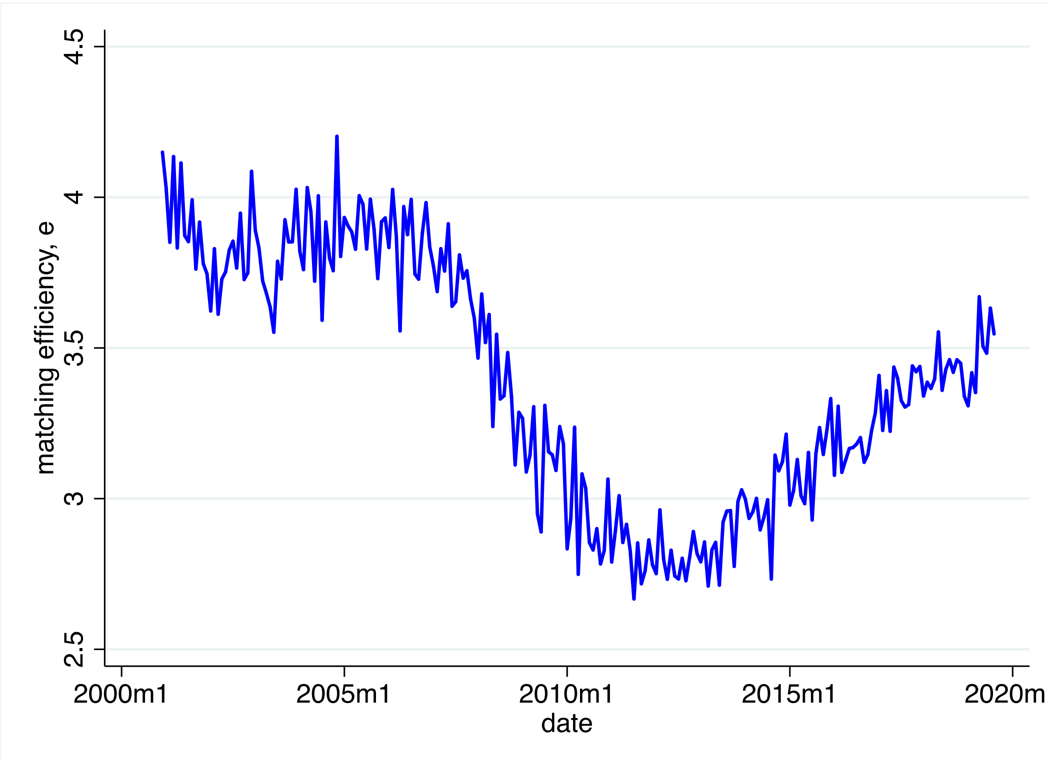

Figure 3: Matching efficiency over time

Problem 3: Unemployment Policy (35 points)

Use the search model of Unemployment from Lecture 11 to evaluate what happens to the unemployment rate when matching efficiency falls.
Answer:

The decline in matching efficiency lower labor market tightness which in turn leads to a decline in the probability of a worker finding a job and a rise in the unemployment rate. There is no change to wages. To see this, observe the following:

Starting with how wages are determined, we know the following:

	Gross Gain	Outside Option	Net Gain to Matching
Worker	w	b	$w-b$
Firm	$z-w$	0	$z-w$
Total	-	-	z-b

Total surplus in a match is given $z-b$. Under Nash bargaining, the wage is such that the

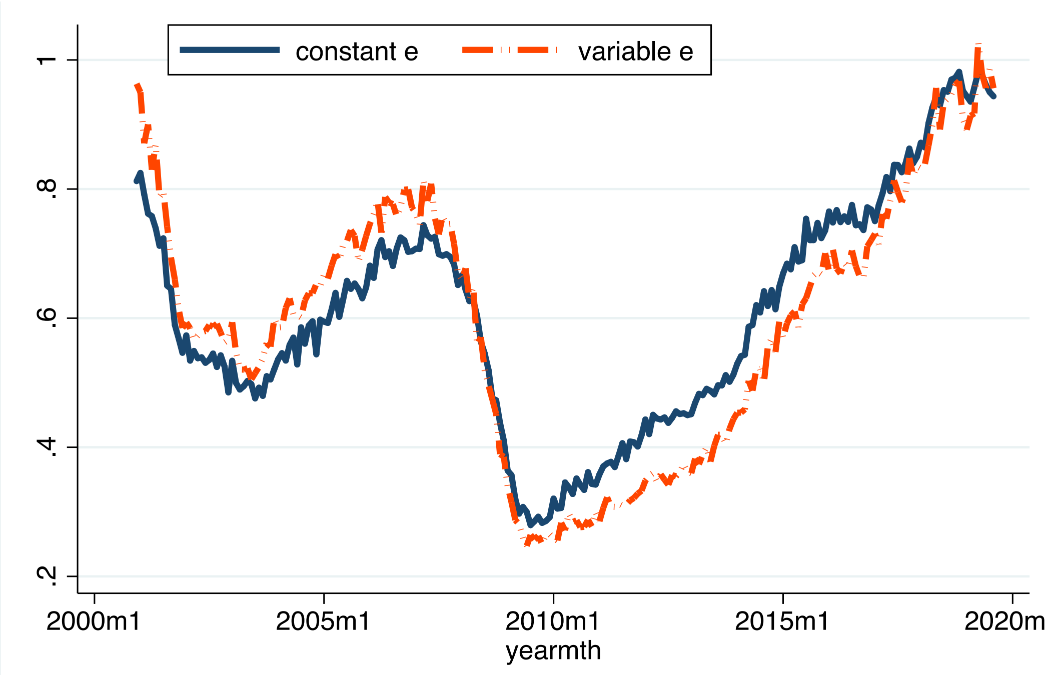

Figure 4: Job-finding rate under constant vs. variable e

worker always receives at least her outside option and a share of the total surplus, where her share is determined by her bargaining weight.

$$w = b + \alpha(z - b)$$

Observe that the matching efficiency e does not affect the wage. So wages are unchanged and profits are unchanged.

In equilibrium, under free entry, firms enter until the value of creating a job $J = 0$. Accordingly, we have:

$$(z - w)q(\theta) = \kappa$$

which is the same as:

$$(1 - \alpha)(z - b)e\mathcal{M}(1, \frac{1}{\theta}) = \kappa$$

Observe that from above, we know that κ has not changed and profits have not changed. Hence, this tells us that in equilibrium, we need $q(\theta)$ to be unchanged even when e falls. The only way this can occur is if θ falls with the fall in e. To see this, you can use a Cobb-Douglas matching function and show that when e falls, θ also falls.

To understand intuititvely why this occurs, consider the following argument. Holding all else constant, if e falls, for the same ratio of vacancies to job-seekers as before, i.e. same

θ as before, $q(\theta)$ falls, which means that it is now harder for firms to meet a worker since matching efficiency has declined. Accordingly, fewer firms will want to enter the labor market since now it is harder to meet a worker. This reduced incentive to enter the labor market is precisely what makes causes θ to adjust and fall, resulting in fewer vacancies per job-seeker. In equilibrium, fewer firms post vacancies and the fall in θ which is the ratio of vacancies to job-seekers declines up to the point that it completely counteracts the effect of a drop in matching efficiency (when there are fewer vacancies per job-seeker, there is less competition for a single firm looking to hire a worker). Put differently, the decline in matching efficiency made it less attractive for firms to create vacancies since its harder to meet a worker. This decline in vacancies per job-seeker in turn means that firms that do choose to create a vacancy now have less competition. Hence the fall in θ buffers the decline in e and keeps $q(\theta)$ constant.

The decline in θ however negatively affects the job-finding probability of a worker. Observe that the probability a worker finds a job is given by $p(\theta) = \frac{M}{U} = \frac{e\mathcal{M}(V,U)}{U} = e\mathcal{M}(\theta, 1)$. The fall in e has both a direct **and** indirect impact through θ on workers' job-finding rates. As both e and θ fall, $p(\theta)$ falls. This implies that the job-finding probability of workers declines and since it is now harder for workers to find jobs, the unemployment rate rises. One can see this from the equation that links unemployment rates to the job-finding probability:

$$u = 1 - p(\theta)$$