INTERMEDIATE MICROECONOMICS

CHOICE UNDER UNCERTAINTY

SPRING 2019, PROFESSOR ANH NGUYEN

Choice Under Uncertainty

- Lottery
- Risk Averse/Risk Neutral/Risk Loving
- Insurance
- Certainty Equivalence
- Risk Premium

- Reading from the textbook: 578-584, 597-602

Choice with Risks

- ***Risky choice*** is often modeled as a ***choice over different lotteries***.

- Each ***lottery*** has at least two possible outcomes – with some ***probability*** associated with each outcome.

- How much we are willing to pay to reduce the risk we face depends on our degree of ***risk aversion*** – which is a feature of tastes that we have so far not modeled.

Choice with Risks

- Tastes will now be defined over ***outcome pairs*** – not over *consumption bundles*.

- *Markets like* ***insurance markets*** sell contracts that offer consumers the opportunity to alter the gambles they face – effectively allowing them to transfer resources from the *good state* to the *bad state.*

Two types of tastes with risks

- Anytime we model risky choices, we have the possibility of at least two "states of the world" arising.

- Typically, the "good state" comes with more consumption than the "bad state".

Expected Value

- I am selling a lottery ticket. You will either win a prize of $10000 with probability of 0.25 or nothing.
- What is the expected value of your winning if you participate in this lottery? $E(L) = 0.25 \times 10{,}000 + 0.75 \times 0 = 2500$
- What is the maximum amount you are willing to pay to participate in the lottery?

Example:

lottery:

10,000 w/ 10%

20,000 w/ 20%

50,000 w/ 50%

100,000 w/ 20%

Expected Utility

- Let's say your utility function has the form of $u(x) = \sqrt{x}$
  - Win the lottery with probability 0.25
  - Reward from winning: $10,000
  - No reward from losing

$E(x) = 2500$

$EU(x) = 0.25 \times \sqrt{10,000} + 0.75 \times \sqrt{0}$

$= 25$

- What is your expected utility of purchasing the lottery?

$EU(x) = p_1 \cdot u(x_1) + p_2 u(x_2) + \cdots$

p_1: prob associated w/ it

$u(x_1)$: lottery value in state 1

Risk Averse

- What's the expected cash reward from winning?

$$E(x) = 2500$$

- In case of of $u(x) = \sqrt{x}$, what is the utility you get from the expected value of the lottery?

$$EU(x) = 25 \quad ; \quad UE(x) = \sqrt{E(x)} = 50$$

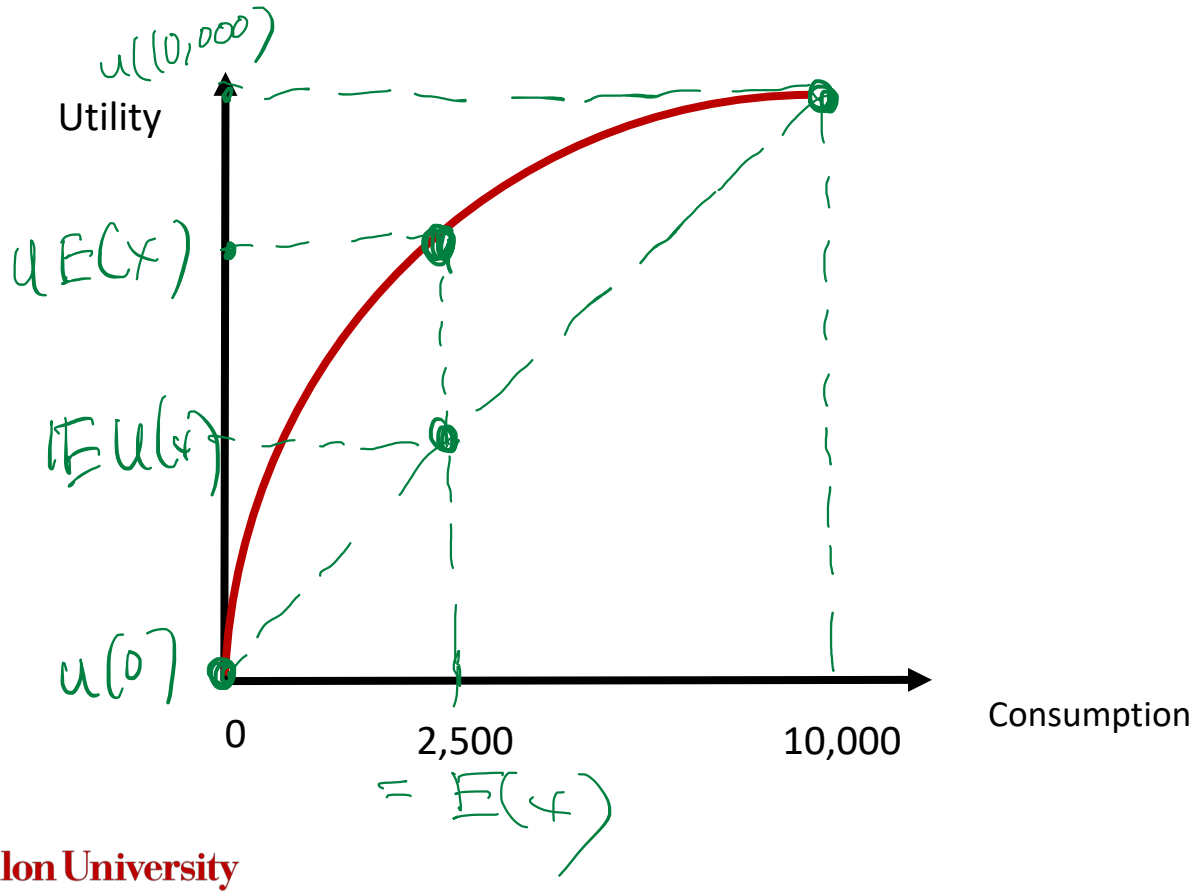
- We will call you a risk averse agent in this case. b/c $EU(x) < UE(x)$

- This is because the ***expected value of the lottery*** is **more** than the ***expected utility of the lottery***:

$$u(E[x]) > E[u(x)]$$

Risk Averse

$u(x) = \sqrt{x}$



$EU \leq u(Ex)$

$uE(x) \geq EU(x)$ ✓

E

Concavity $u''(x) < 0$

Risk Neutral

- Now assume that your $u(x) = x$, What is your expected utility of purchasing the lottery? (win $10,000 with probability 0.25)

$$\mathbb{E}u(x) = \mathbb{E}X = 2500$$

- What is your utility you get from the expected value of the lottery?

$$u(\mathbb{E}(x)) = 2500$$

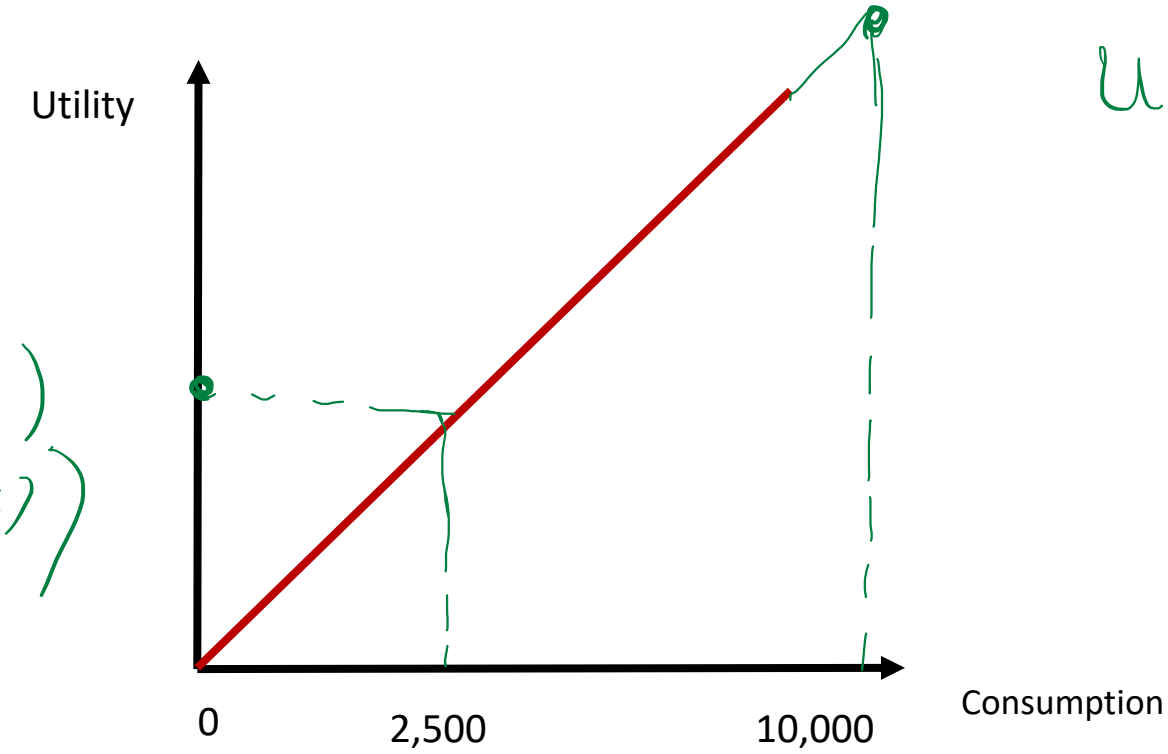
- We will call you a risk neutral agent in this case.

utility of the

$$u(\mathbb{E}(x)) = \mathbb{E}u(x)$$

- This is because the ***expected value of the lottery*** is **the same** as the ***expected utility of the lottery***.

Risk Neutral



$E(U(x)) = U(E(x))$

$U(x) = x$

$E U(x)$

$E U(x)$

Risk loving

- Now assume that your $u(x) = x^2$, What is your expected utility of purchasing the lottery? (win $10,000 with probability 0.25)

$$E(u(x)) = 0.25 \times 10{,}000^2 + 0.75 \times 0^2 = 25 \times 10^6$$

- What is your utility you get from the expected value of the lottery?

$$u(E(x)) = (2500)^2 = 6.25 \times 10^6$$

- We will call you a risk loving agent in this case.

$$E\,u(x) > u(E(x))$$

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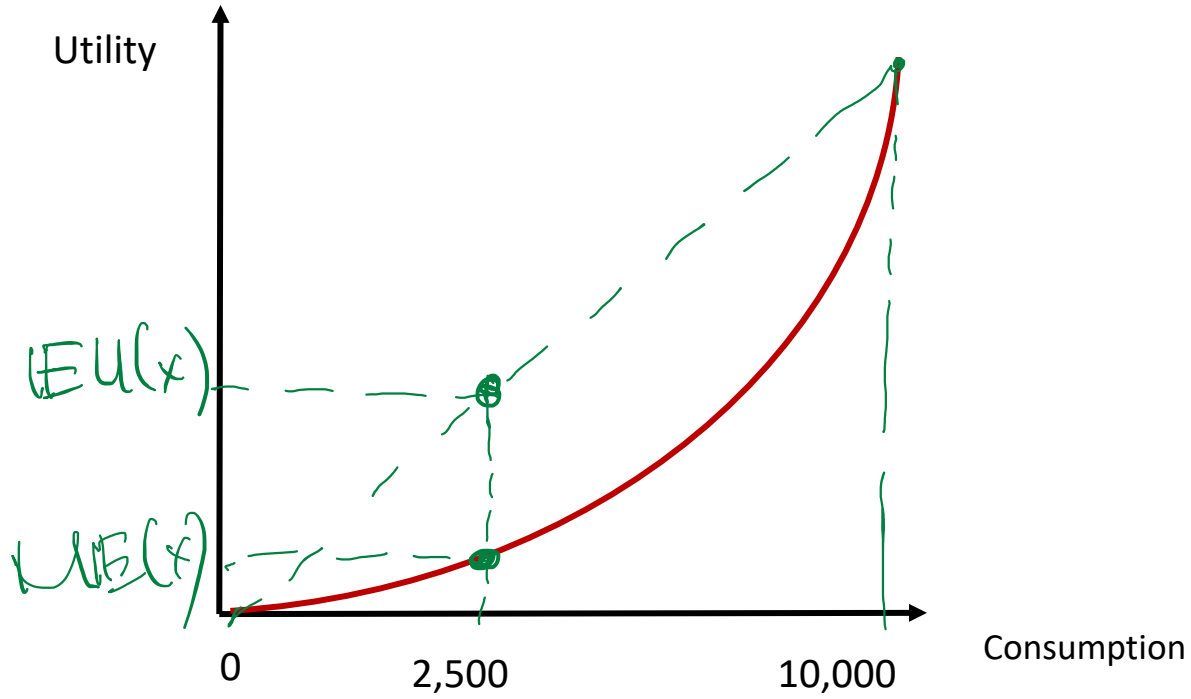
less

- This is because the ***expected value of the lottery*** is ~~more~~ than the ***expected utility of the lottery***.

Risk loving



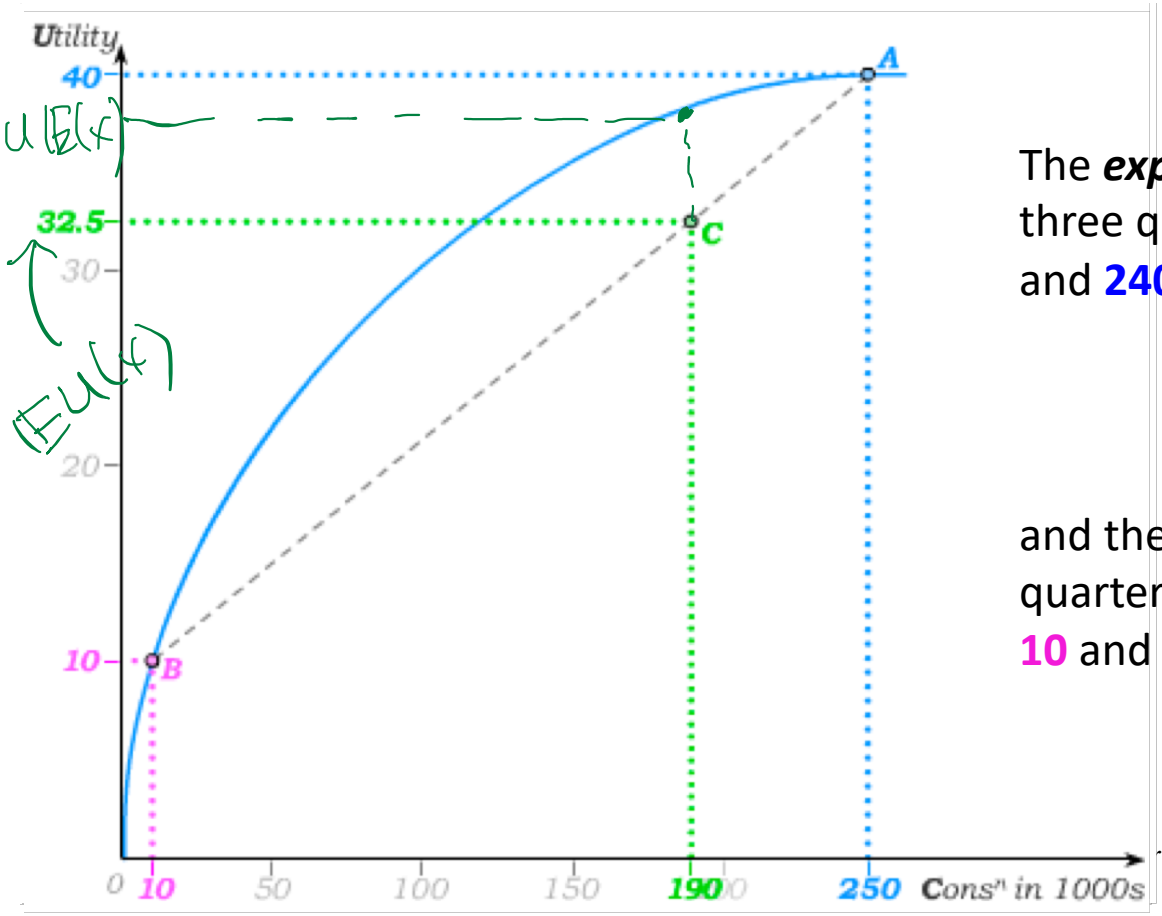
$u(x) = x^2$ (Convex: $u''(x) > 0$)

$u(x) = 2x + 5$

Example

There is a 75% chance of $250 in consumption (***A***) and a 25% chance of $10 in consumption (***B***).

The ***expected utility*** of this gamble is then read off the line connecting ***A*** and ***B*** three quarters of the way toward ***A***.

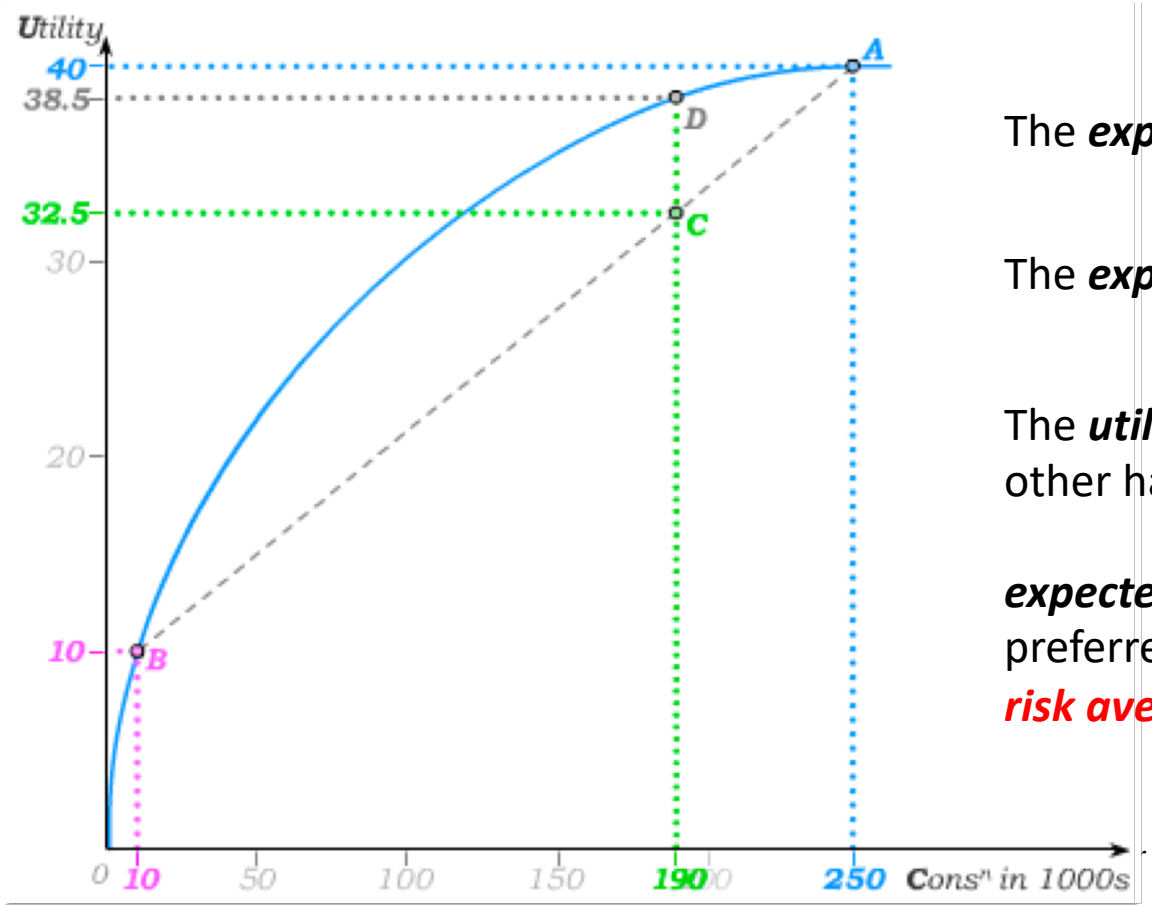


The ***expected value*** of the gamble then lies three quarters of the distance between **10** and **240** on the horizontal axis; i.e.

and the ***expected utility*** lies three quarters of the way between the utilities **10** and **40** on the vertical axis; i.e.

Expected Value of Expected Utility

There is a 75% chance of $250 in consumption (***A***) and a 25% chance of $10 in consumption (***B***).



The ***expected value*** =

The ***expected utility*** =

The ***utility of the expected value***, on the other hand, is read off ***D*** =

expected value of the gamble more preferred to staying with the gamble => ***risk averse***

Utility and Risk: Example

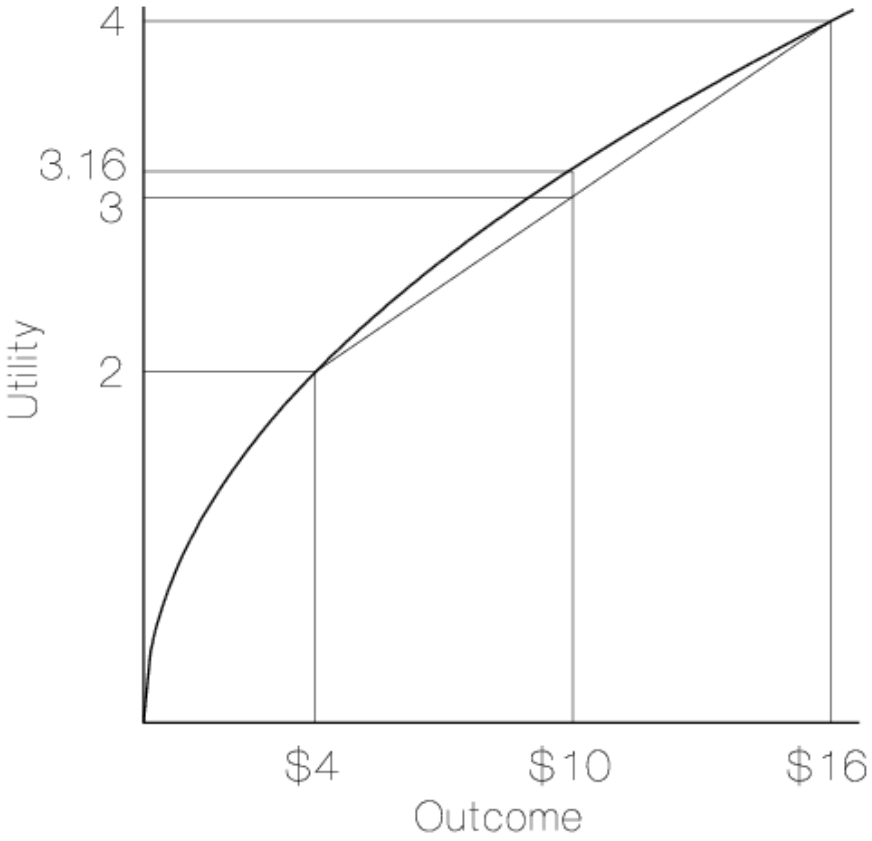
- Suppose you prefer $10 for sure to a 50-50 lottery between $4 and $16.
  - Are you risk averse, risk neutral, or risk loving?

$10 = \mathbb{E}(x)$

$U(\mathbb{E}(x)) > \mathbb{E}U(x)$

- Assume your utility function can be represented by the functional form: $\sqrt{x}$, draw your preferences, and show the expected utility of the lottery as well as the utility expected value of the lottery.

Utility and Risk: Example



$$U(outcome) = \sqrt{outcome}$$

Preferences toward risk: Example

- Suppose a person earns $15,000 and receives 13.5 units of utility from job
  - $u(15000)=13.5$
- Offered a new, **commission-based** job
  - 0.50 chance of $30,000
  - 0.50 chance of $10,000
- Person prefers commission-based job
- Are they risk averse or risk loving?

$$E(x) = 0.5 \times 30{,}000 + 0.5 \times 10{,}000 = 20{,}000$$

Risk averse: $u(E(x)) > E(u(x))$

$u(20{,}000) > E(u(x))$

Risk loving: $u(20{,}000) < E(u(x))$

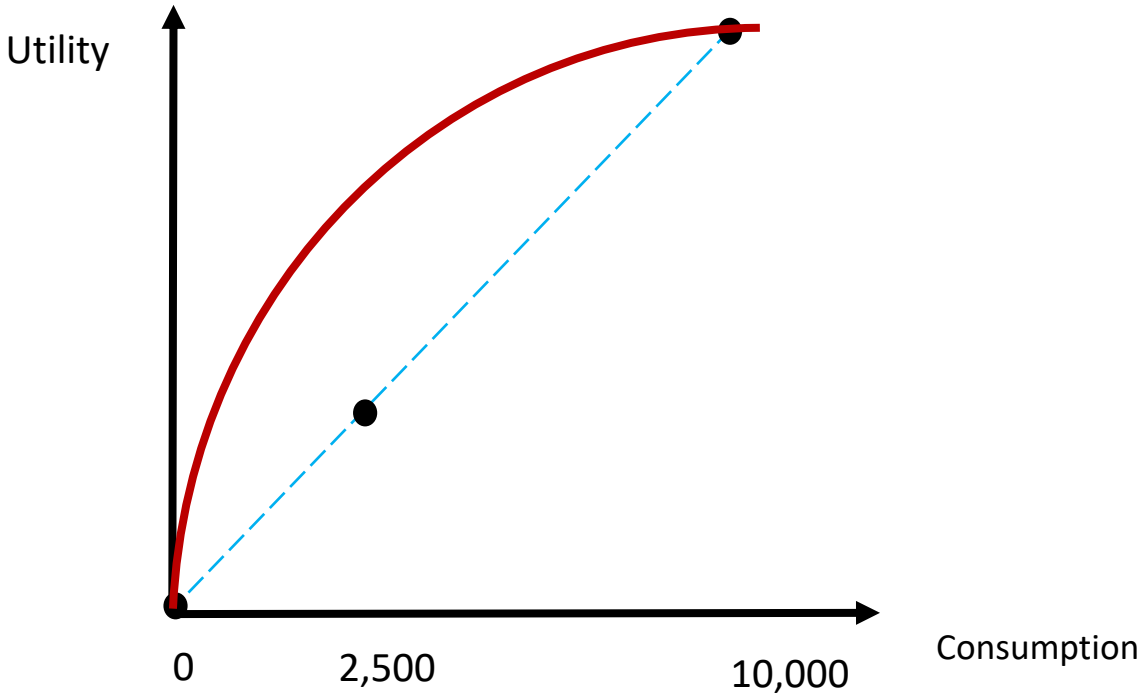
$u(15{,}000) < u(20{,}000)$

Certainty Equivalent

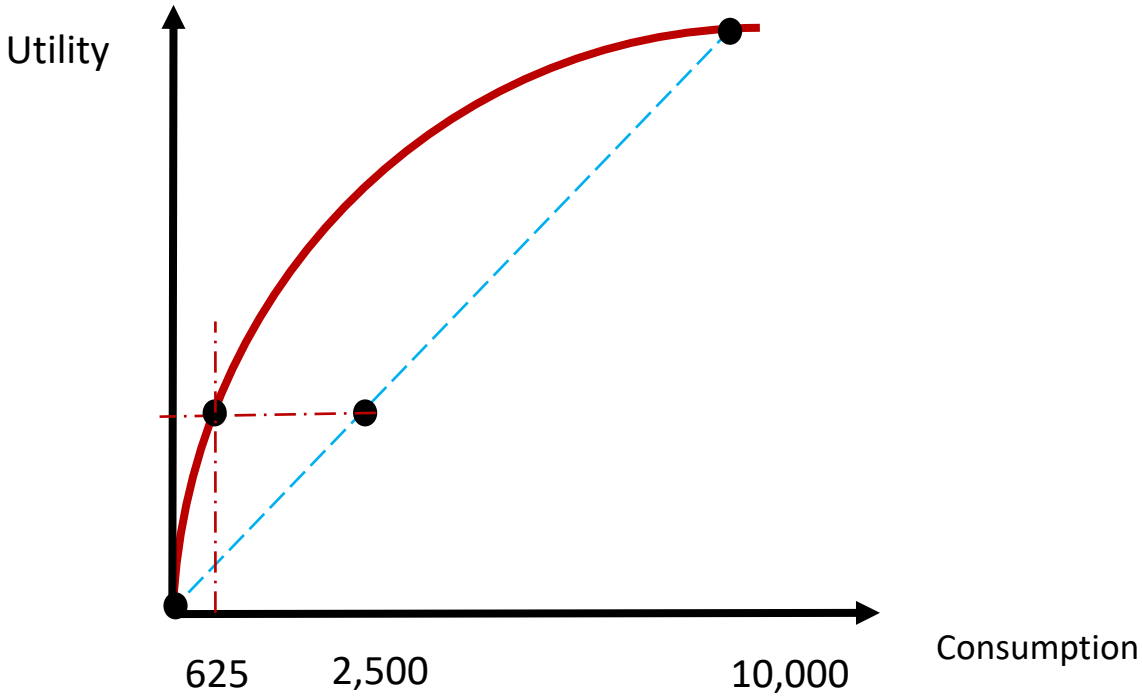
- The ***certainty equivalent*** of a lottery is the amount an individual would be willing to accept *without risk* in exchange for participating in the lottery.
- Example: $10,000 with probability of 0.25 or otherwise nothing
  - Utility function: $u(x) = \sqrt{x}$
  - What is your certainty equivalence?

$$U(CE) = \sqrt{CE} = U(Lottery)$$

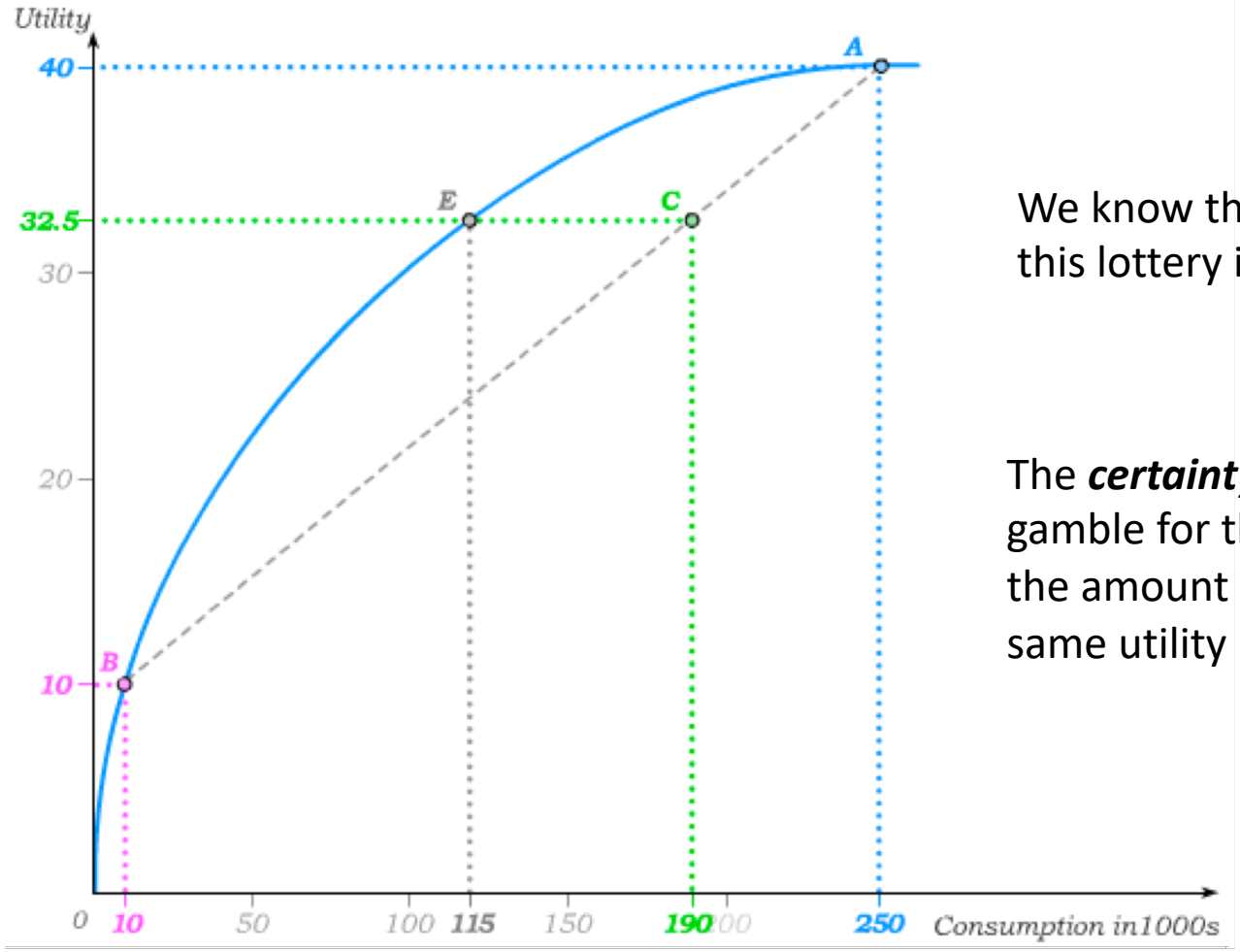
Certainty Equivalence



Certainty Equivalence



Certainty Equivalence: Example



We know that the expected utility of this lottery is **32.5**.

The ***certainty equivalent*** of the gamble for this individual is then the amount that results in the same utility level.

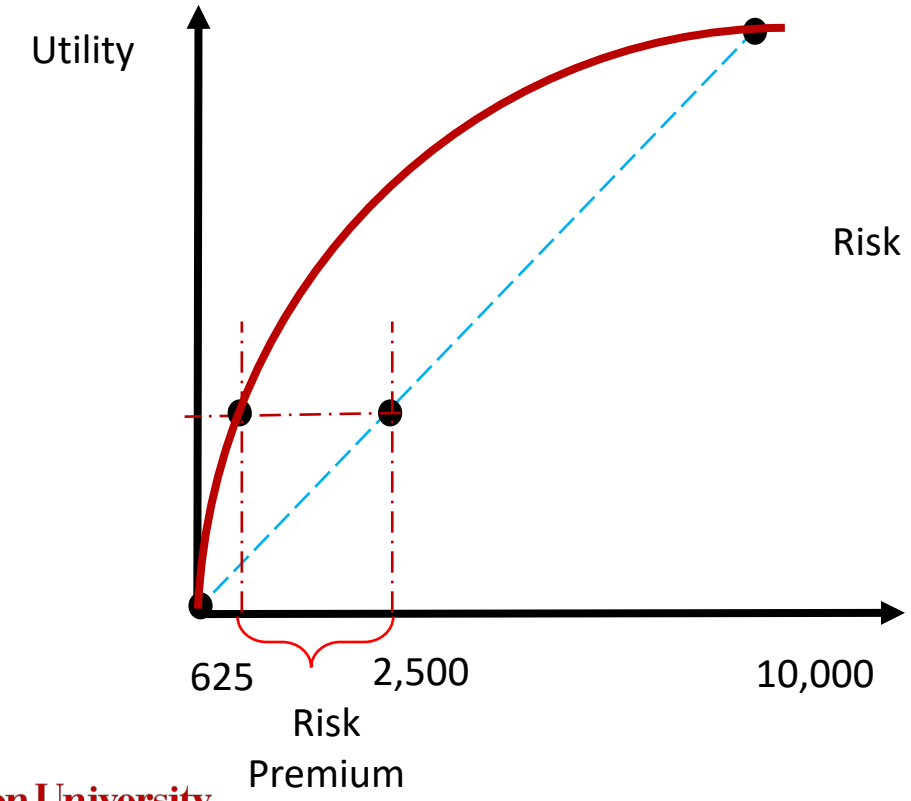
Risk premium

- The ***risk premium*** of a lottery is the *difference* between the expected value of the lottery and the certainty equivalent.

$$RP = EV - CE$$

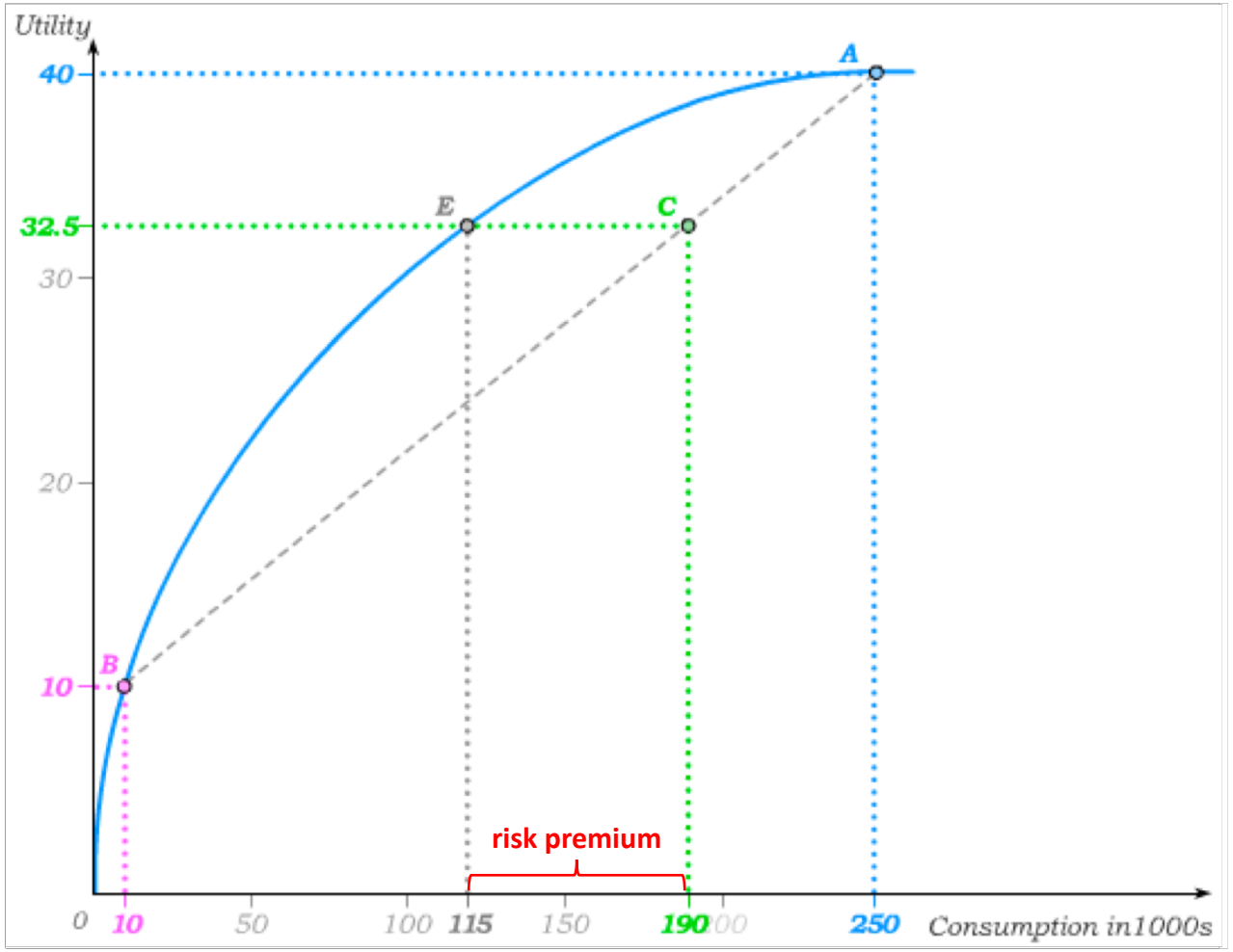
- It is ***positive*** for *risk averse tastes*, ***zero*** for *risk neutral tastes* and ***negative*** for *risk loving* tastes.

Risk premium



Risk Premium= RP=EV-CE

Risk premium: Example



Example

- The picture shows a 50-50 lottery between $10,000 and $30,000.

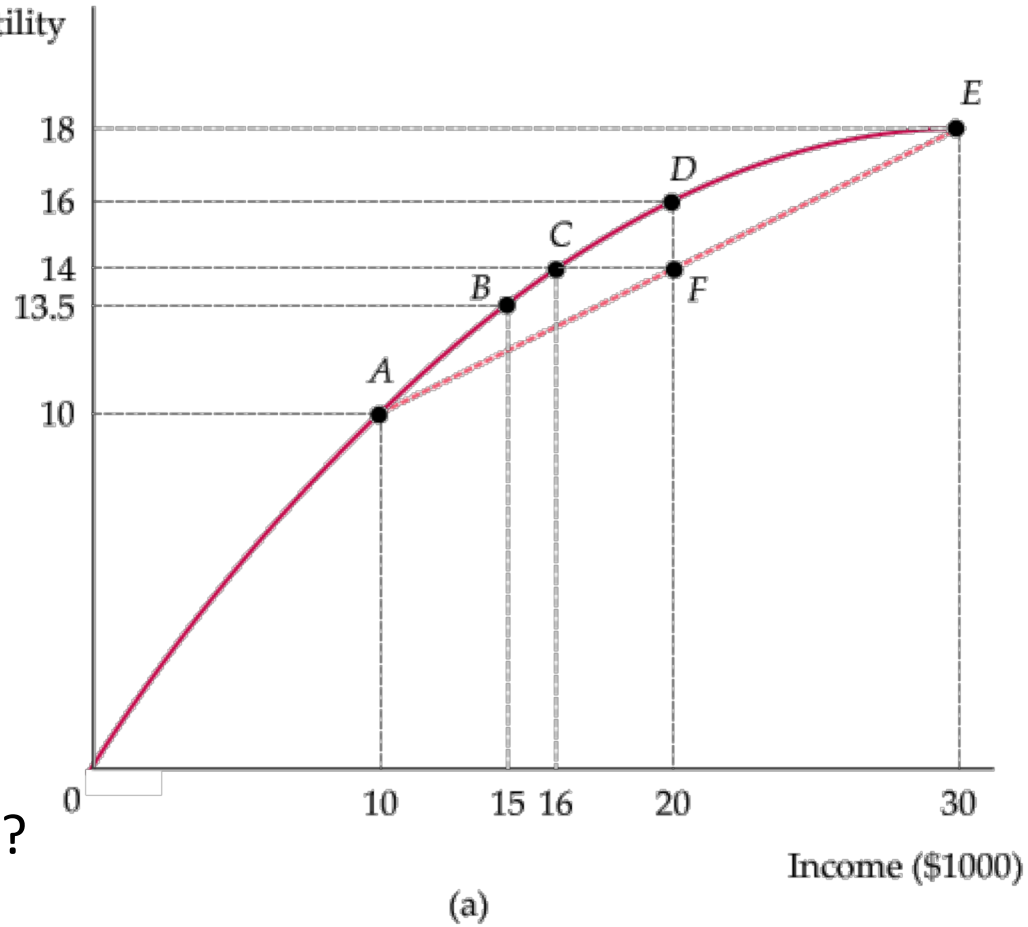
A. What is the expected value of the lottery?

B. What is the expected utility of the lottery?

C. Is this person is risk averse? Why?

D. What is his certainty equivalence?

E. What is his risk premium?



Example: Insurance

- Matthew owns a car which has a value of $2,500. There is a 10% chance that he might get into an accident that the car gets totaled and will have a value of zero. We know that his utility function is: $\sqrt{x}$. An insurance company sells an insurance that will fully replace Matthew's car in case of such accidents. How much is the maximum Matthew is willing to pay for such policy?

Summary

Risk Averse	Risk Neutral	Risk Loving
U(EV)<E(U)	U(EV)=E(U)	U(EV)>E(U)
RP>0	RP=0	RP<0
Concave Utility Function	Linear Utility Function	Convex Utility Function

In-Class Quiz

Intermediate Microeconomics

Feb 20, 2019

Question 1

The labor supply is upward sloping when:

(a) The substitution effect is greater than the income effect (in absolute terms).

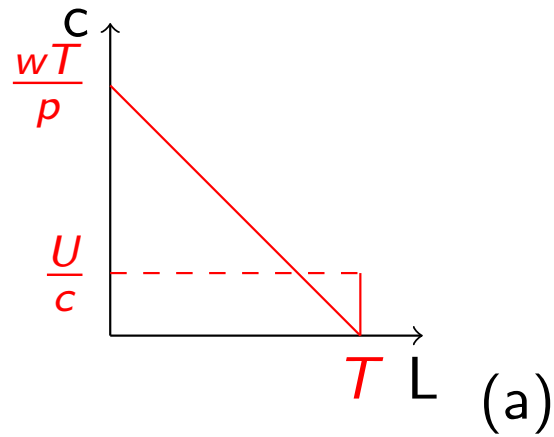
(b) The income effect is greater than the substitution effect (in absolute terms).

(c) Leisure is a normal good.

(d) Consumption is an inferior good.

Question 2

Consider the example of unemployment benefit we considered in class. Unemployment benefit is U. What does the budget constraint look like?



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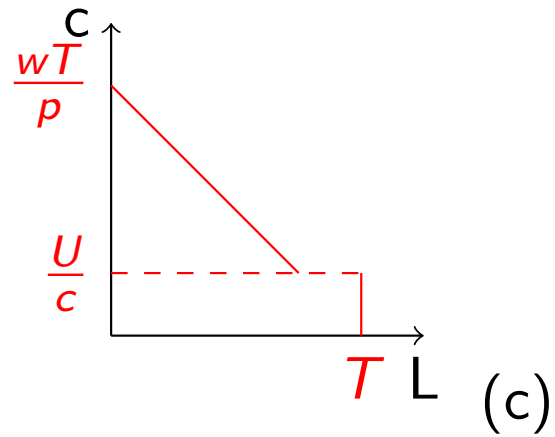

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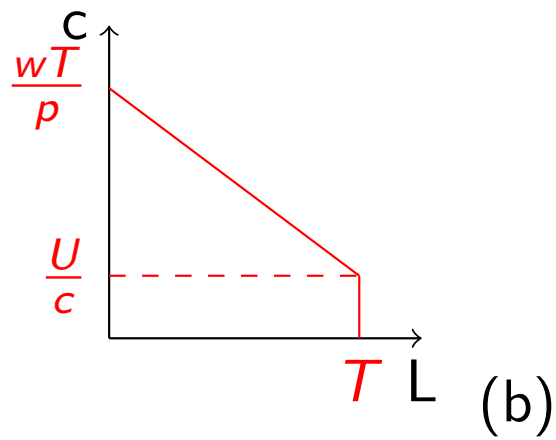

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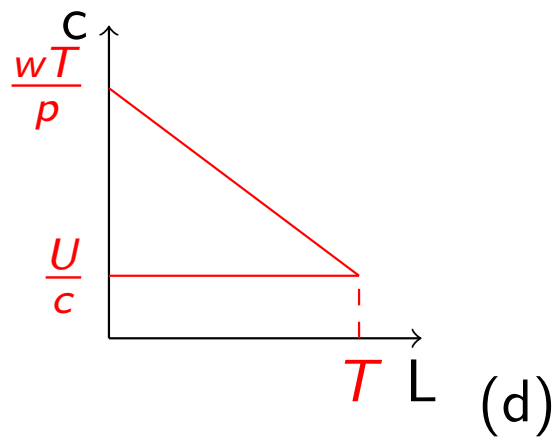
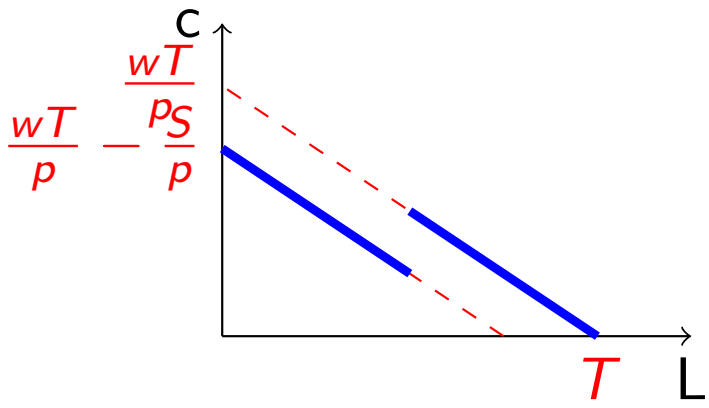


(d)

Question 3

Consider the following tax scheme: Everyone who earns $\geq \frac{wT}{2}$ has to pay $\$S$ of tax, and everyone who earns $< \frac{wT}{2}$ pays no tax. What does the budget constraint under this tax scheme look like?



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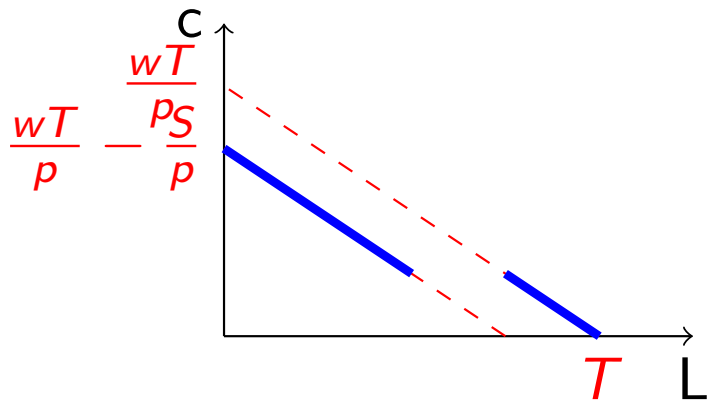

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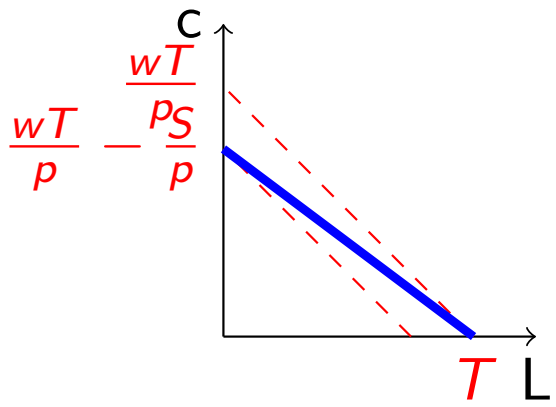

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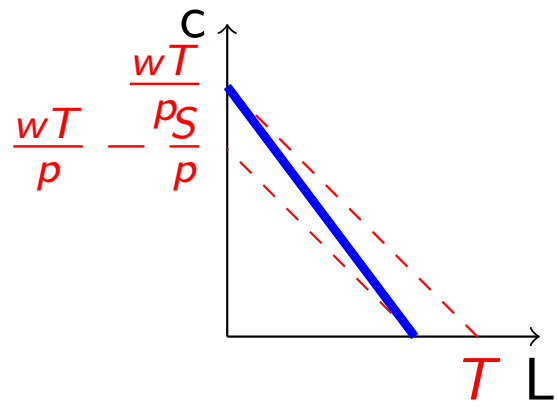

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Question 4

Consider the following lottery. The lottery gives you \$20 with probability 50%, -\$10 with probability 25%, and 0 with probability 25%. What is the expected value of the lottery?

(a) -7.5

(b) 7.5

(c) 2.5

(d) None of the above.

Question 5 (continue)

Consider the following options: (1) a lottery that gives you an expected value of $10 but involves some risks, and (2) an $8 that you can get without any uncertainty/risks.

(a) A risk-averse consumer will always choose (1) over (2)

(b) A risk-averse consumer will always choose (2) over (1)

(c) A risk-loving consumer will always choose (1) over (2)

(d) None of the above.

End of Quiz

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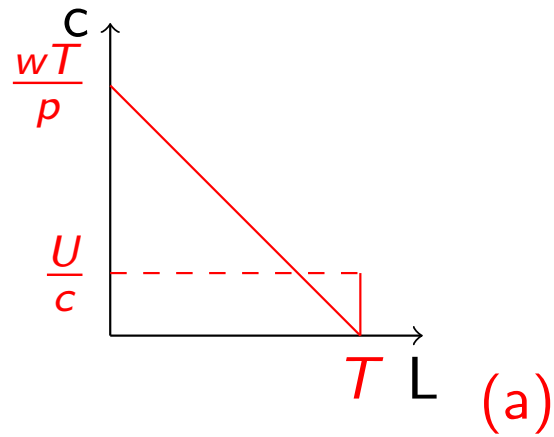

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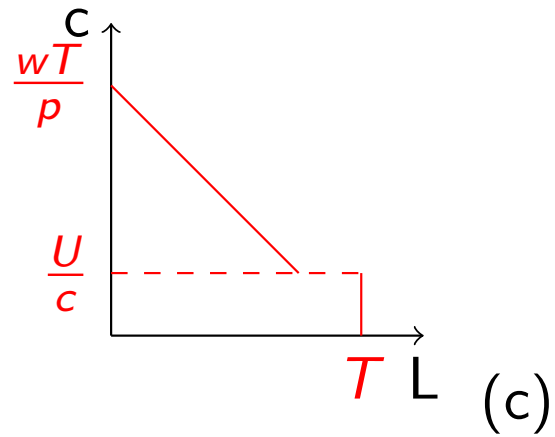

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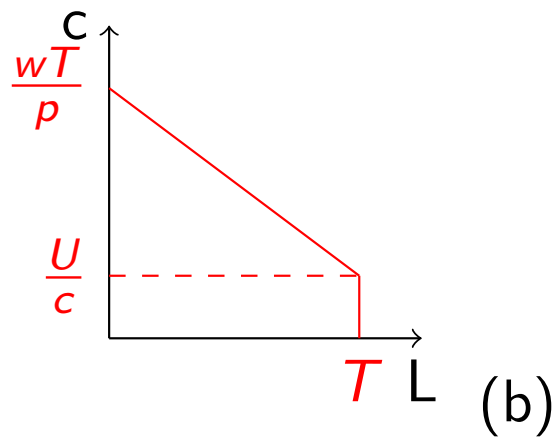

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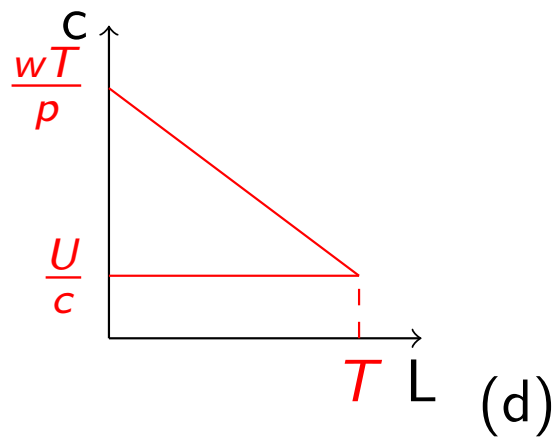

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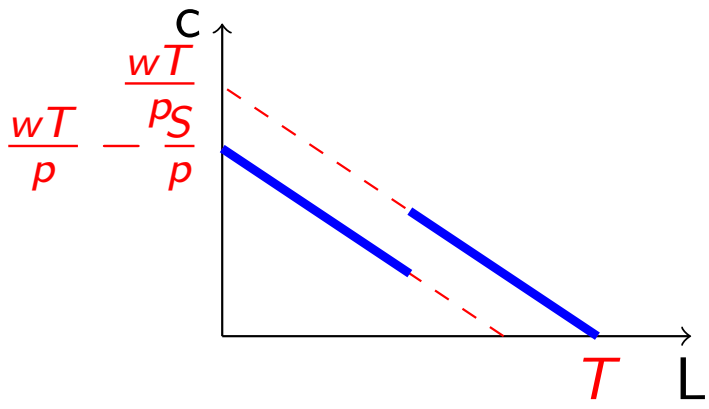

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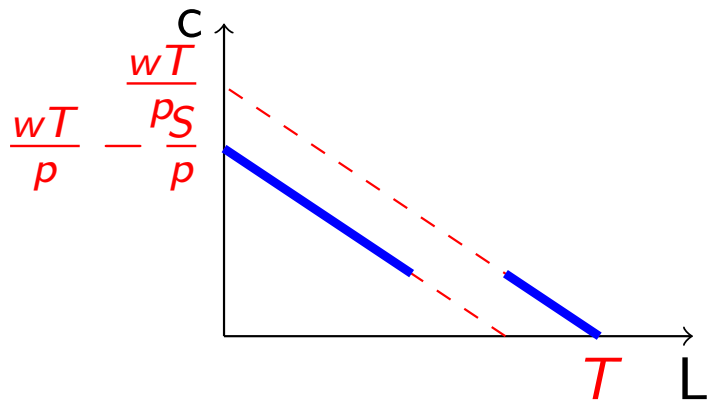

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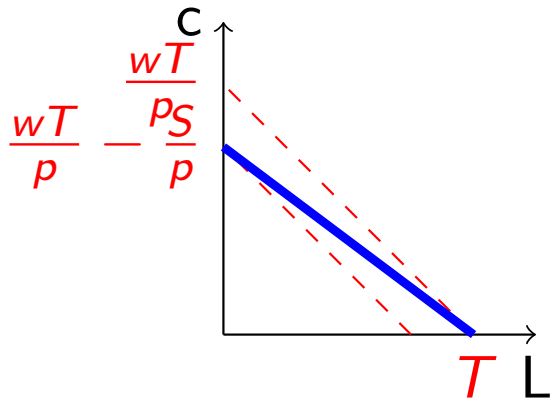

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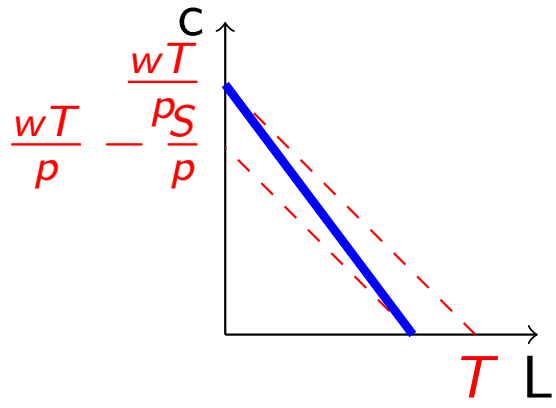

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