

# 73230 Intermediate Microeconomics

## Practice Questions

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### Problem 1

Currently the price of consuming housing  $x_1$  is lowered by the fact that home mortgage interest is tax deductible. Suppose the government proposed to eliminate this implicit subsidy of your housing consumption, raising the price from  $p_1 - s$  to  $p_1$ . At the same time, the government lowers the tax on other consumption  $x_2$ , lowering the price from  $p_2 + t$  to  $p_2$ .

- (a) Write down your original budget constraint assuming the consumer has income  $I$ .
- (b) Suppose the utility function  $u(x_1, x_2) = \frac{1}{3} \ln x_1 + \ln x_2$  captures your tastes, and suppose  $I = 10,000$ . Write out the utility maximization problem for this consumer prior to any policy change.
- (c) Derive the demands for housing and other consumptions prior to any policy change. Show your work.
- (d) How much housing and other goods will this consumer consume prior to any policy change, if  $p_1 = 10$ ,  $p_2 = 1$ ,  $s = 6$  and  $t = 1$ ?
- (e) When the policy change goes into effect, will this consumer still be able to afford the bundle you derived in (c)? Explain.
- (f) When the policy change goes into effect, what bundle will the consumer consume?

### Problem 2

Suppose that there are two types of people: those who have a high demand for gas, and those who have a low demand for gas. For people with a high demand, the quantity they consume, as a function of price ( $P$ ), is given by:

$$Q_H(P) = 10 - 2P.$$

For people with a low demand, the quantity they consume, as a function of price, is given by:

$$Q_L(P) = 5 - 2P.$$

- (a) What is the elasticity of  $Q_H(P)$  with respect to  $P$ ?
- (b) What is the elasticity of  $Q_L(P)$  with respect to  $P$ ?
- (c) Suppose the market consists of 200 people with low demand for gas, and 100 people with high demand for gas. What is the market demand curve, denoted  $Q_M(P)$ ?
- (d) What is the elasticity of  $Q_M(P)$  with respect to  $P$ ?

### Problem 3

Suppose there are two goods,  $X$  and  $Y$ , with prices  $p_x$  and  $p_y$ . Consider a consumer with exogenous income  $M$ . This question will have you analyze the effects of a sales tax on the consumer's choices. To be clear,  $p_x$  and  $p_y$  refer to the pre-tax prices of  $X$  and  $Y$ .

- (a) First, let's characterize the budget constraint when there's no sales tax. What is the budget constraint? Draw the consumer's budget constraint and label it  $BC$ . Label the intercepts along each axis. What is the slope of  $BC$ ?
- (b) Suppose that Pennsylvania levies a sales tax on all goods purchased. For every dollar of goods (both  $X$  and  $Y$ ) that someone consumes, they must pay  $\tau$  dollars to the state. What is the budget constraint? On the same set of axes as part (a), draw the consumer's budget constraint and label it  $BC'$ . Label the intercepts along each axis. What is the slope of  $BC'$ ?
- (c) Assume that the consumer has a utility function given by  $u(X, Y) = \sqrt{XY}$ .
  - (i) What are the optimal choices of  $X$  and  $Y$  when the consumer maximizes her utility, subject to the budget constraint without sales taxes ( $BC$ )?
  - (ii) What are the optimal choices of  $X$  and  $Y$  when the consumer maximizes her utility, subject to the budget constraint with sales taxes ( $BC'$ )?
- (d) Now, suppose that the state sends a lump-sum rebate to each household to help alleviate the burden of the sales tax. In particular, the consumer gets  $\tau M$  dollars from the state, and this amount does not depend on how the consumer allocates her money between goods  $X$  and  $Y$ .
  - (i) What is the consumer's budget constraint with the rebate?
  - (ii) What are the consumer's optimal choices of  $X$  and  $Y$ ?

## Problem 4

Imagine a consumer who has preferences over two goods,  $X$  and  $Y$ . Assume that this consumer's preferences can be represented by the utility function  $u(X, Y) = \sqrt{XY}$ . Now, consider the following three alternative utility functions:

$$\hat{u}(X, Y) = 2(XY)^2 + 4$$

$$\tilde{u}(X, Y) = \log(X) + \log(Y)$$

$$\bar{u}(X, Y) = \sqrt{X} + \sqrt{Y}.$$

- (a) Do the utility functions  $u(X, Y)$  and  $\hat{u}(X, Y)$  represent the same preferences? Why or why not?
- (b) Do the utility functions  $u(X, Y)$  and  $\tilde{u}(X, Y)$  represent the same preferences? Why or why not?
- (c) Do the utility functions  $u(X, Y)$  and  $\bar{u}(X, Y)$  represent the same preferences? Why or why not?
- (d) Does  $u(X, Y)$  satisfy the monotonicity axiom? Why or why not?
- (e) Let  $U > 0$  be some fixed number. Derive the equation for the indifference curve corresponding to utility level  $U$ .