MACROECONOMICS
73-240

Lecture 6

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U.S. Firms

U.S. Firms: Data

How many businesses?

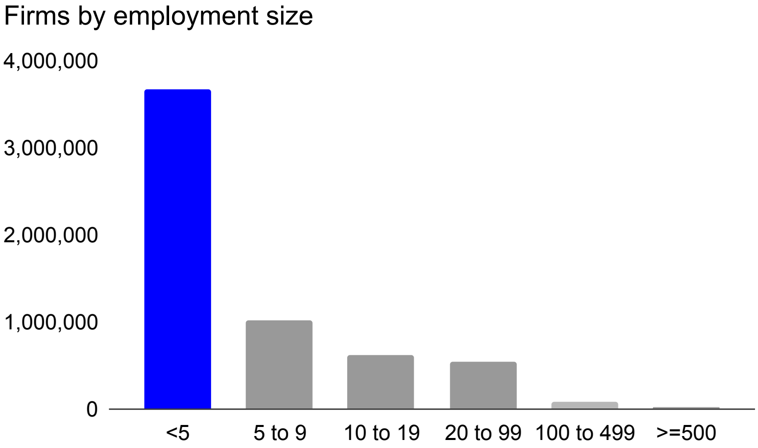
- **Non-employers** (firms with no payroll): 24,813,048 (US Census Bureau, 2016 Nonemployer Statistics)
- **Employers** (firms with payroll): (US Census Bureau, Business Dynamics Statistics 2016)

Firms	5,165,983
Establishments	6,886,453
Employment	124,231,335

Most individuals are not self-employed (firms with no payroll) but employed at a firm

U.S. Firms: Size Distribution

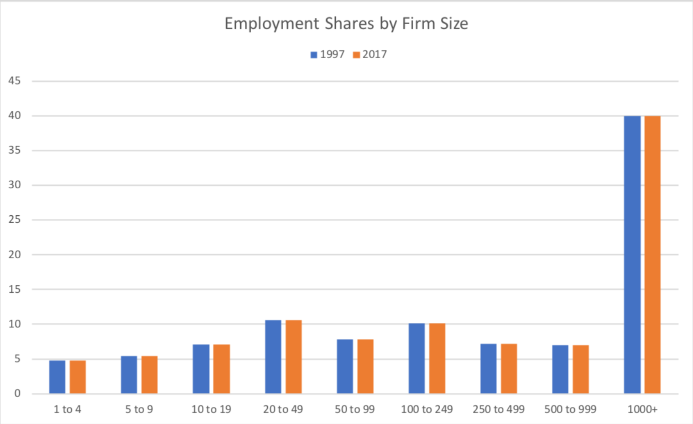
What is the distribution of firm size?



source: US Census Bureau, Statistics of US Businesses (SUSB) 2016

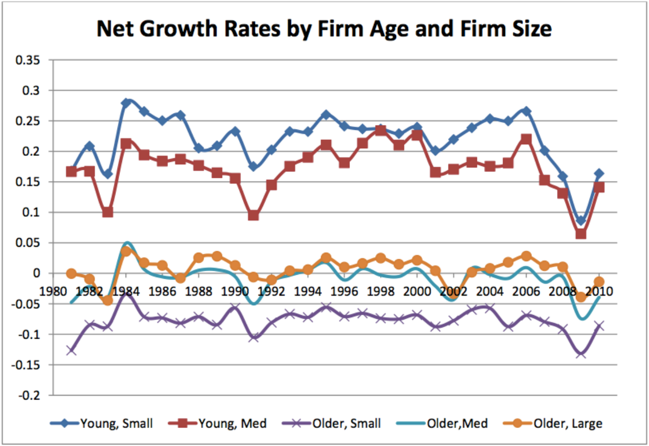
U.S. Firm Data: who affects employment?

Large firms account for the bulk of employment



Source: BLS

U.S. Firm Data: who affects employment?



Notes: Tabulations from BDS. See Notes to Figure 1.

- Young firms: positive net growth in terms of employment (hires less separations)

The Representative Firm
And
The Production Function

The Firm

- Objective of the firm: maximize profits

- How? : A firm buys inputs (factors of productions) at some cost and converts them into output (consumption goods)

- A firm gets revenue from selling its output

- Profits are then given by:

$$\text{Profits} = \text{Revenue - Cost}$$

The Firm

Assumptions we will make:

1) Firms are very smart
2) All firms have the same technology $\Rightarrow$ **focus on a representative firm**
3) They use only two factors of productions: **capital and labor**
4) Live 1 period (relaxed later in the course)
5) No financing issues (relaxed later in the course)

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The Production Function

- Before we can talk about how the firm maximizes profits
- We need to know how it can convert inputs into output
- So we need to define our production function

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The Production Function

- Production Function: specifies how much output (y) can be produced given any number of inputs k and n
- Notation:
  - Labor: n^d for an individual firm, N^d for aggregate
  - Capital: k for individual firm, K for aggregate
  - TFP: z
  - Output: y for individual firm, Y for aggregate.
- Production Function: $y = zF(k, n^d)$

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Marginal Product

Definition

- Marginal product of labor (capital) MPN (MPK) is the additional output produced by increasing capital (labor) by one unit, keeping fixed the other input.

- Mathematically, given a production function, we define the marginal product of labor as:

$$MPN = \frac{dzF(k, n^d)}{dn^d}$$

- And the marginal product of capital as:

$$MPK = \frac{dzF(k, n^d)}{dk}$$

Marginal Product Explained

- Suppose La Prima keeps the number of espresso machines it has, but hires one more student worker

- Then the number of coffee drinks (output) it serves increases

- This <u>increase</u> in output is called *the marginal product of labor* (MPN for short)

- In other words **marginal product of labor** is the amount of increase in output, if labor input increase by one unit (and other inputs are held constant)

- Marginal product of capital is defined in a similar way.

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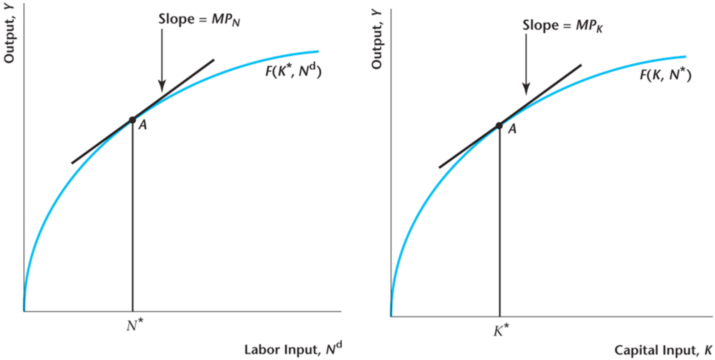
Assumptions on Production functions

We will assume that production has the following properties

1. **More input, more output** : Holding capital fixed, more labor produces more output
2. **Constant returns to scale**
3. **Diminishing marginal products**
4. **Complementarity** : More capital, makes labor more productive

Properties of the Production Function

1) **More Input, More Output** implies F is increasing



Properties of the Production Function

2) F exhibits constant return to scale
   "Double the inputs double the output"

Properties of the Production Function

Note: $MPK = \frac{\partial F}{\partial K}$ and $MPN = \frac{\partial F}{\partial N}$

3) **Diminishing Marginal Products** implies F is concave

4) **Complementarity** implies Marginal Product of Labor is increasing in capital or MPN increases as capital increases

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Example

- Which one of the following production functions exhibit diminishing marginal product of labor (or capital)

$$F(K,N) = 2K + 15N$$

$$F(K,N) = \sqrt{KN}$$

$$F(K,N) = 2\sqrt{K} + 15\sqrt{N}$$

- To answer we need to find the derivative with respect to N (or K) and check if it decreases with N (or K)

What about complementarity?

Cobb-Douglas Production Function

- It is a very common production function

$$F(K,N) = zK^{\alpha}N^{1-\alpha}$$

  because of nice properties it has

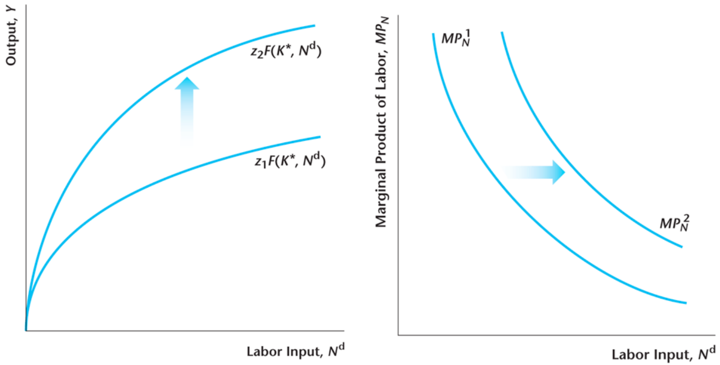
- α is called *capital share* parameter – more about it later

- z is called *Total Factor Productivity* (or TFP for short) – more about it later

- As an exercise check that Cobb-Douglas production function has constant returns to scale and diminishing marginal product to labor (and capital)

Example: TFP and MPN

- Consider two economies 1 and 2 with $z_2 > z_1$



TFP increase, MPN increases (each unit of N produces more when z improves)

Can you think of an example of labor-improving technology?

TFP and Capital Examples

- Notice that if country A's TFP is higher than B's
- Then holding all else constant (i.e. α, K, N), MPN of country A $>$ B.
- Notice if country A has more K than B, holding all else constant, MPN of A $>$ B.

Suppose you knew:

$$z_{\text{Kenya}} < z_{\text{U.S.}}$$

$$K_{\text{Kenya}} < K_{\text{U.S.}}$$

and a is the same.

Question: where is labor more productive?

The Problem of the Firm

How does a firm decide how many workers (or capital) to hire?

Important Defnition: The *objective* of a firm is to maximize profits

Some additional assumptions:

a) A firm endowed with capital (later we will introduce the investment decision)

b) Firms takes the wage as given from the market

c) No taxes in the baseline environment

d) Productivity, z, is exogenous

The firm sets its own demand for labor: n^d
Summing across firms $\rightarrow$ aggregate labor demanded: N^d.

Profits

Profits = Revenue - Costs

Revenue: $zF(k, n^d)$

Variable cost: wn^d

$$\textbf{Profits} = \pi = zF(k, n^d) - wn^d$$

For now, think of k as fixed

Note: price of output normalized to 1

The firm solves:

$$\max_{n^d} \; zF(k, n^d) - wn^d$$

Question: Why doesn't the firm choose n^d VERY large?

Optimal Labor Choice

Choose n^d so that: $MPN = w$

Solution: Intuition

- Here, $MPN =$ marginal benefit of hiring one more unit of labor
- $w =$ marginal cost of one more unit of labor
- Suppose $MPN > w$
  - $\Rightarrow$ Then if raise n^d by a tiny amount: revenue raise faster than the costs
- Suppose $MPN < w$
  - $\Rightarrow$ Then if lower n^d by a tiny amount: revenue decrease slower than the costs

Optimality achieved at:

$$MPN = w$$

Example

Consider $F(k, n) = zk^{\alpha}n^{1-\alpha}$ (notice: I removed the ^{d})

The firm solves:

$$\max_n \pi(n) = \max_n zk^{\alpha}n^{1-\alpha} - wn$$

Find labor demand.

Solution: Labor demand curve

Solve for labor demand:

$$n^d(w) = \left[\frac{z(1-\alpha)k^\alpha}{w}\right]^{\frac{1}{\alpha}}$$

Comparative statics:

- z increases $\Rightarrow n^d$ increases
- k increases $\Rightarrow n^d$ increases
- w increases $\Rightarrow n^d$ decreases

Deriving the Aggregate Labor demand curve

- Two key assumptions we made help in terms of adding up across firms:
  1. Representative firm: all firms are identical

$$Y = \sum^{X} y = Xy$$

  2. Constant returns to scale.

$$Xy = XzF(k, n^d) = zF(Xk, Xn^d) = zF(K, N^d)$$

So now we can effectively solve the for aggregate labor demanded *as if* there is 1 firm choosing aggregate N^d.

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Firm Maximization
Calibration: What is α (capital share) and z (TFP) ?

Solution: Example

Note also the following useful property of a Cobb-Douglas production function

$$
\begin{aligned}
MPN &= (1-\alpha)zK^{\alpha}N^{1-\alpha}N^{-1} \\
&= (1-\alpha)\underbrace{Y}_{=zK^{\alpha}N^{1-\alpha}}N^{-1}
\end{aligned}
$$

$$\Rightarrow MPN = (1-\alpha)\frac{Y}{N}$$

The Labor Share of Income

We now can define (and calculate from the data) a. Start from previous equation

$$MPN = w = (1-\alpha)\frac{Y}{N}$$

Solve for $1-\alpha$

$$1-\alpha = \frac{wN}{Y}$$

Where wN is the compensation of employees

and Y is the Gross Domestic Product = GDP

- Both can be obtained from the data! (See Lecture 2!)
- $wN/Y \approx \frac{2}{3}$ so $\alpha = \frac{1}{3}$

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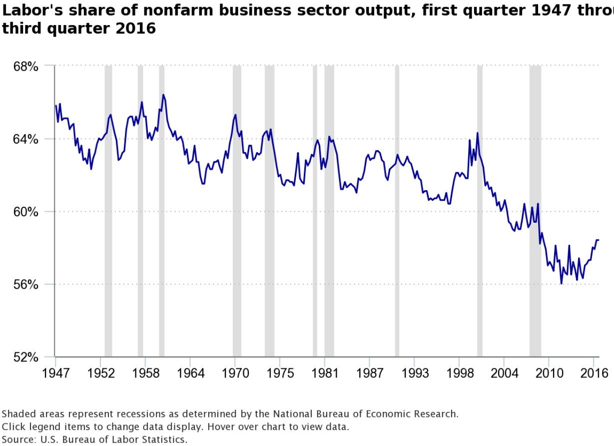
The Labor Share of Income

Apart from most recent drop, labor share in data fairly constant with $wN/Y \approx \frac{2}{3}$



$$1 - \alpha = \frac{wN}{Y} \implies \alpha \approx \frac{1}{3} \text{using US data}$$

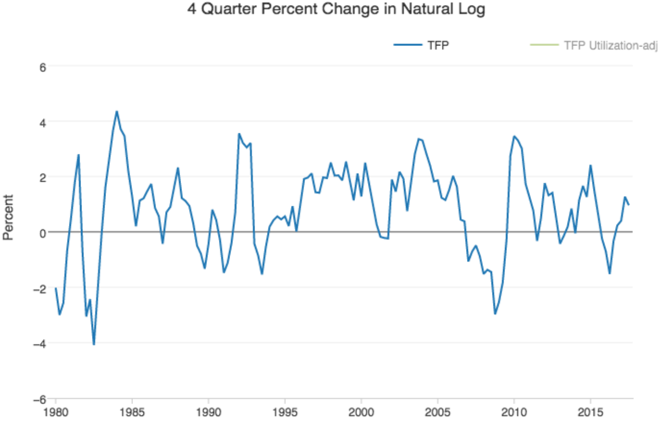
Calculating z From Data

Going from the same example:

- $Y = zK^{\alpha}N^{1-\alpha}$
- Take logs:
  $logY = logz + \alpha logK + (1-\alpha)logN$
- We have data on Y, K, N, and we now know α, can find z as the residual to the equation above.

Technical Progress as measured by z

- Total Factor Productivity, z, fell during periods of recessions, rose during booms



You can find the data here: https://www.frbsf.org/economic-research/indicators-data/total-factor-productivity-tfp/

Measures of productivity

- When you read the news, the word "productivity" is used to refer to many different objects

- Suppose the production function is $Y = zK^{\alpha}N^{1-\alpha}$
- Formally, we define:

$$\text{Total Factor Productivity, } TFP = z$$

$$\text{Avg. Product of Labor (Labor Productivity)} = Y/N$$

$$\text{Marginal Product of Labor, } MPN = \frac{\partial Y}{\partial N}$$

- Do the three measure move in the same direction?

Measures of productivity

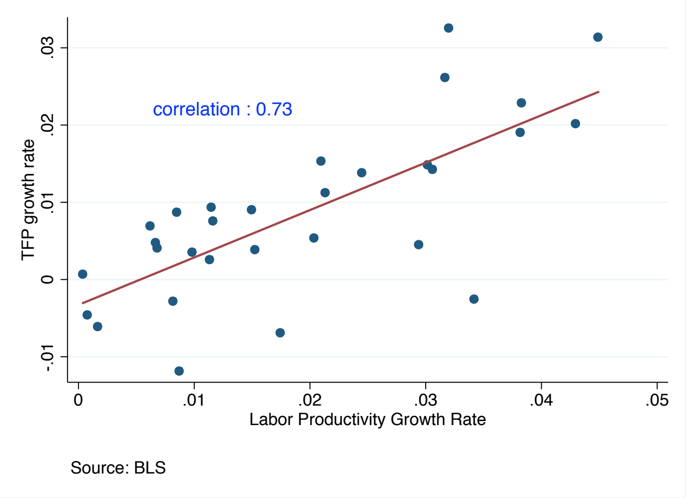
- Holding all else constant, if TFP z increases, Y/N increases and MPN increases

- Labor productivity:
$$\frac{Y}{N} = zK^{\alpha}N^{-\alpha}$$
as $z \uparrow, Y/N \uparrow$

- MPN:
$$\frac{\partial Y}{\partial N} = (1-\alpha)zK^{\alpha}N^{-\alpha}$$
as $z \uparrow$, $\frac{\partial Y}{\partial N} \uparrow$

Measures of productivity

- Holding all else constant, if TFP z increases, Y/N increases



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What happens if capital becomes more productive?

- Consider the following production function:

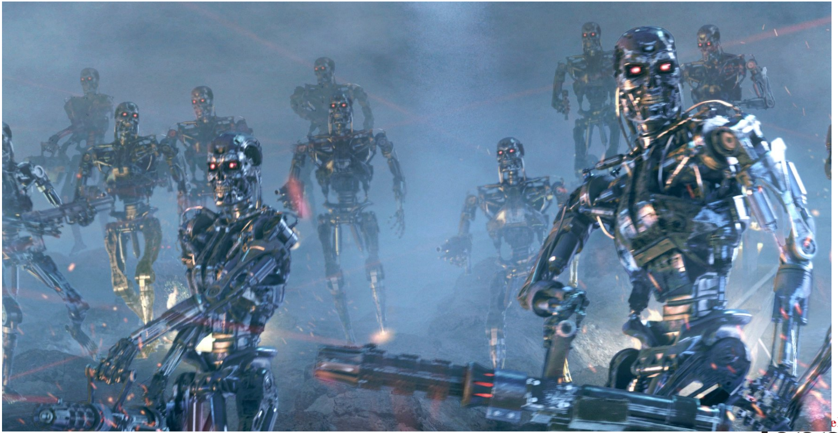
$$Y = (AK)^{\alpha} N^{d,1-\alpha}$$

  where A is capital-augmenting technology.
- Firms are born with capital and if they hire labor, must pay a wage of w to each unit of labor demanded.

- How is N^d affected by an increase in A?

The Firm, Technology, and Labor Demand

- Rise of the machines



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The Firm, Technology, and Labor Demand

- Why didn't our model predict a decline in N^d with a rise in A?
- What key assumption did we make that gives rise to this result?

The Firm, Technology, and Labor Demand

- A different kind of production function

$$Y = [(AK)^\gamma + N^\gamma]^{1/\gamma}$$

- As good practice, prove that this function satisfies:
  - More inputs$\rightarrow$ more output
  - Diminishing marginal product
  - Constant returns to scale

- Suppose $\gamma > 1$. Are capital and labor complements?

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The Firm, Technology, and Labor Demand

- Which jobs are likely to be affected by improvements in capital or technology?

Employment Classification Grid

	Routine	Nonroutine
Manual (blue collar)	Production Crafts Operative Repair	Food service Personal care Protective service
Cognitive (white collar)	Clerical Administrative Sales	Professional Technical Managerial

Low-skill Middle-skill High-skill

Source: Cheremukhin (2014)

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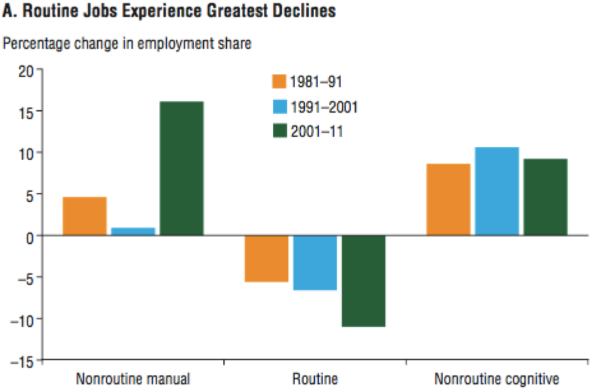
The Firm, Technology, and Labor Demand

- Whether improvements in capital or technology lead to labor losses depends on whether capital and labor are substitutes or complements.
- Job polarization in the US: routine jobs have been on decline



Source: Cheremukhin (2014)