MACROECONOMICS
73-240

Lecture 7

Shu Lin Wee

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Carnegie Mellon Tepper School of Business

The Government

The Government In Our Model

In our 1 period model the government

- Provides public good G (we will treat this as exogenous!)
    - Examples: Schools, Police, Fire, Military, Infrastructure
- Lives for 1 period!
    - Which implies that the Govt in the 1 period model does not issue debt. Why?

The Government In Our Model

In our 1 period model spending G

- can be a drain on resources
  - Example: (NPR, Sep 11 2019) Prison at Guantanamo Bay has cost >$6 bn to operate since opening nearly 18 years ago. Still costs more than $380mn/year despite housing only 40 prisoners today.
- an actual public good (Education, Healthcare)
  - spending G benefits the household:

$$U(c, l, G) = u(c) + u(l) + V(G)$$

- We will assume the first case in today's class.
  (Note: assumptions matter! we will return to this)

The Government: Budget Constraint

Let G the dollar value of the public goods. In the 1 period model, the *government budget constraint* is:

$$G = T$$

Where the money comes from (other examples):

- τ_y: income tax (appears in HH budget constraint)

$$C = (1 - \tau_y) w N^s + \pi$$

- τ_c: tax on consumption (appears in HH budget constraint)

$$(1 + \tau_c) C = w N^s + \pi - T$$

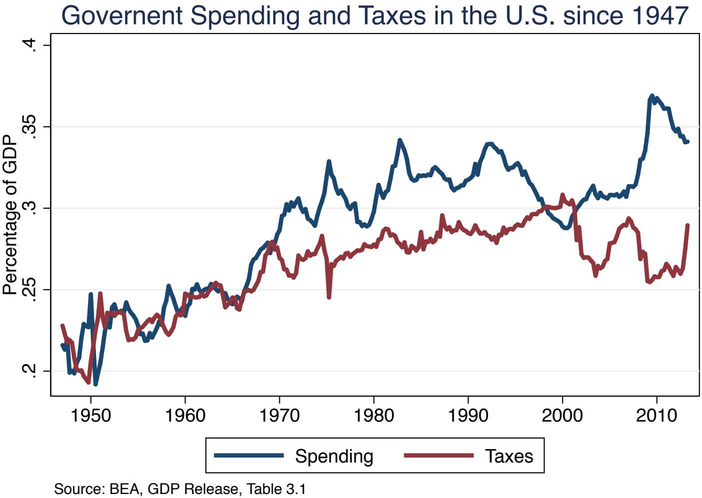
- τ_r: tax on revenues (appears in firm profits)

$$Y = (1 - \tau_r) z F(k, N^d) - w N^d$$

Government expenditure needs to be funded.

The Government in the Data

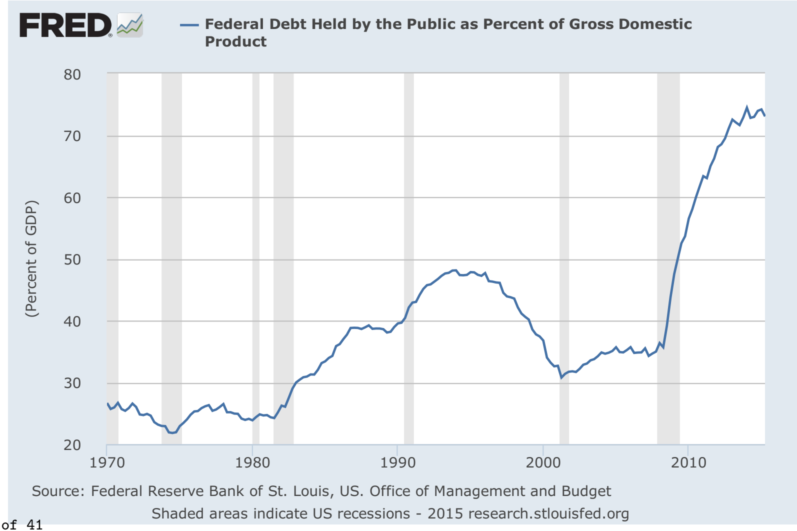
Question: Is G always $= T$?



If govt last more than 1 period, we will model $G = T + \Delta\text{debt}$

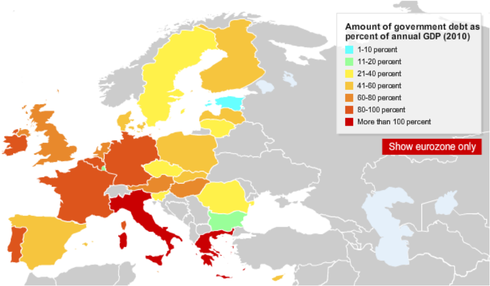
The Government in the Data

Government Debt: Debt held by public + Debt held in government accounts



Governments in Europe: Debt size

In 2012:



Source: The Economist

In 2014, Debt as a % of GDP

- US : 85.1%
- Greece: 151.9%
- Japan: 244.8%

RECAP

Putting things together

- At this stage, we have covered all the agents in the economy
    - Household
    - Firm
    - Government
- we are ready to put them together

A Note on Aggregation

Remember:

- Our consumer is a representative consumer: it represents ALL of the consumers in the US.
  Suppose there are X number of households:

$$X \times U(c,l) \underbrace{= U(Xc, Xl)}_{\text{utility is hod 1}} = U(C,l)$$

- Our firm is a representative firm: it represents ALL of the firms in the US
  Suppose there are M number of firms:

$$M \times y = M \times zF(k, n^d) \underbrace{= zF(Mk, Mn^d)}_{\text{because of CRS}} = zF(K, N^d)$$

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Putting things together

- Representative Household takes (w, π, T) as given chooses (C, ℓ, N^s)
  - Total hours available, h, is a parameter of the economy

$$\max_{C,\ell} U(C, \ell)$$

s.t.

$$C = wN^s + \pi - T$$

$$N^s = h - \ell$$

- Since each household owns shares in the firm, this means the firm is owned by representative HH

Putting things together

- Representative Firm takes (w, K, z) as given chooses N^d
  - Capital share, α, is a parameter of the economy

$$\max_{N^d} \pi = zF(K, N^d) - wN^d$$

Putting things together

- Government takes spending G as exogenously given (must provide roads)
- Government must balance budget (Govt chooses size of T):

$$G = T$$

Now we need markets to tie everything together!

Competitive Equilibrium

Why do we Need an Equilibrium Concept

We can answer how the economy responds when:

- Government consumption (G) Increases by 20%
  ...what happens to C ?

- Government subsidizes employment?
  ... what happens to wages?

Equilibrium

The idea:

1) Set some external conditions (exogenous variables)

2) Determine what happens to all of the other variables of interests (endogenous variables)

In our static model:

1) Exogenous variables: (K, G, z)

2) Endogenous variables: $(C, N^s, N^d, T, Y, \pi, w)$

N^d = labor demanded; N^s = labor supplied

Equilibrium

How do we know what is *going to happen*?

1) Must be optimal:
   everybody (household and firm) must like the decision it has taken

2) Must be feasible:
   total consumption (by household and government) must equal total goods produced

Competitive Equilibrium: Static

-KEY DEFINITION-

For a set of exogenous variables (K, G, z) A competitive equilibrium is a set of endogenous variables $(C, N^s, N^d, T, Y, \pi, w)$, so that:

1) The consumer chooses C (consumption) and N^S (labor supply) optimally, taking as given w (wage), T (taxes), π (dividends)

2) The firm chooses N^d (labor demand) to maximize profits, taking as given w (wage), K (capital stock), z (productivity)

Competitive Equilibrium: Static

[...] continued:

3) Government balances the budget: $G = T$

4) Labor market clears: $N^d = N^s = N^*$

5) Goods market clears: $Y = C + G$
(sometimes called the Income-Expenditure identity)

Working with the Model:
Graphical Approach

Working With the Model: Graphical Approach

- First question: what and how much can the economy produce?
    - To answer this question: we must ask who produces goods in the economy.

Working With the Model: Graphical Approach

First step: derive a relation (production possibility frontier - PPF) so that given (K, G, z) we can determine all the feasible (C, l) pairs

$$Y = zF(K, N^d)$$

since market clear $N^d = N^s$:

$$Y = zF(K, N^s)$$

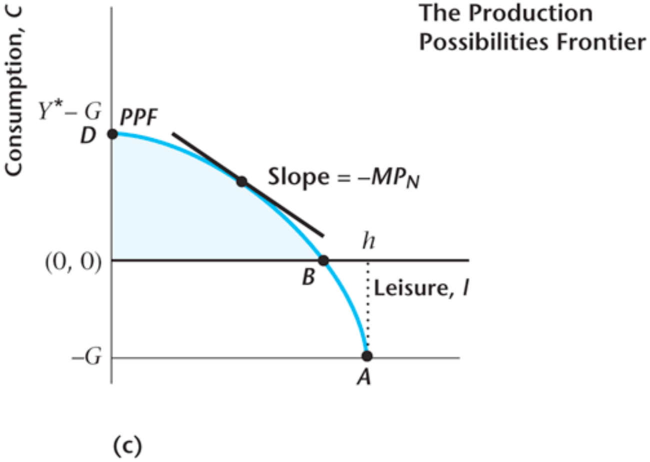
substitute feasibility of hours of household: $N^s = h - l$

$$Y = zF(K, h - l)$$

substitute the goods market clearing: $Y = C + G$

$$C = zF(K, h - l) - G$$

The Production Possibilities Frontier



The Production Possibilities Frontier

Some properties of the PPF:

$$C(l) = zF(K, h - l) - G$$

1. Equilibrium consumption decreasing in leisure: $\frac{dC(l)}{dl} = -z\frac{dF()}{dN} < 0$
2. Decreasing returns - PPF is concave: $\frac{d^2C(l)}{dl^2} < 0$

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The Marginal Rate of Transformation

Marginal Rate of Transformation (MRT) measures the extra amount of good 1 that can be obtained per unit reduction of good 2.

Another way of thinking about it is the opportunity cost of producing good 1.

$$C(l) = zF(K, h - l) - G$$

this implies

$$MRT_{l,C} = \frac{dC(l)}{dl} = \frac{dzF(k, h - l)}{dl} = -MPN$$

or in words:

Marginal rate of transformation = the slope of PPF =
- marginal product of labor

Moving Towards the Competitive Equilibrium

To the production possibilities frontier we need to add:

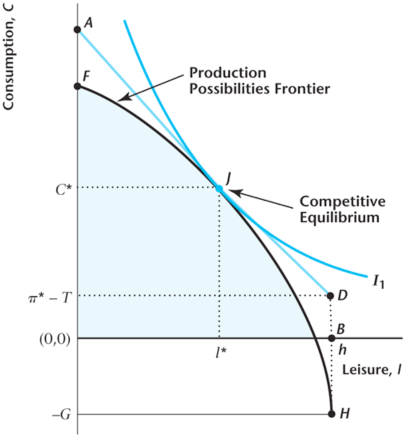
1. Add the household's budget constraint $C = w(h - l) + \pi - T$

   Find the slope: done

   Find the intercept: If $l = 0$, $C = ?$

2. Add indifference curves.

Adding The Consumer



We must verify that J is an equilibrium

Adding The Consumer

- Recall: competitive equilibrium implies that each agent (household, firm) must be
  - maximizing their objective (preferences, profits)
  - subject to their constraints or production possibilities
- and trades must be compatible such that at those prices, all markets clear

Adding The Consumer

With the Household's problem:

- In equilibrium we must have

$$MRT_{l,C} = -MPN = -w = -MRS_{l,C}$$

  Otherwise there is excess demand (or supply)

- And consumption-leisure choice must be feasible

$$C^* = zF(K, h - l^*) - G$$

- Also, the consumption allocation must be affordable for household

$$C^* = w(h - l^*) + \pi - T$$

Graphing A Competitive Equilibrium: In Summary

Steps to graphing:

1. Start with the production function
2. Given the production function, draw the PPF
3. Add the HH budget constraint and indifference curves
4. Eqm is achieved when HH indifference curve (IC) is tangent to its budget constraint (BC) and the PPF

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Graphing A Competitive Equilibrium: In Summary

Points about eqm:

1. Anything on the PPF satisfies goods-market clearing:

$$Y = C + G$$

   - Notice that if inside the PPF, we have instead $Y > C + G$.

2. HH chooses what is affordable (indifference curve is tangent to budget constraint)

3. All agents choose what is desirable. (tangency between PPF, BC and IC)
   - Slope of PPF =-MPN
   - Firm optimality: $MPN = w$
   - HH consumption-leisure tradeoff: $MRS_{l,c} = w$

Working with the Model:
Solve a system of equations

The Competitive Equilibrium

Algorithm to find a Competitive Equilibrium:

1. Find the values of capital (K), government expenditures (G) and productivity (z): these are the exogenous variables.
2. Given exogenous variables determine PPF,
3. Find point of tangency between PPF and preferences,
4. Recover endogenous variables (C, N^s, N^d, T, Y, w).
5. Use the constructed equilibrium to determine relationship between exogenous and endogenous variables:

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Mathematical Approach Explained

- Endogenous Objects to find: C, N^s, N^d, T, Y, w (6 objects!)

- Equilibrium Conditions:
  - HH optimality: 2 Conditions ($MRS_{l,c} = w$ and the Budget Constraint)
  - Firm optimality: 1 Condition ($MPN = w$)
  - Gov't Budget Constraint: 1 Condition ($G = T$)
  - Market Clearing: 2 conditions ($N^d = N^s$ and $C + G = Y$)

- Note: We must solve for 6 objects and we have 6 equilibrium conditions!

- Note π is endogenous, but we know $\pi = Y - wN^d$.

Example with no government

- No Government: $G = T = 0$
- Production function: $Y = 2K^{0.5}N^{0.5}$ with $K = 1$
- Utility function: $u(C, l) = \ln(C) + \ln(l)$ and $l = 1 - N^s$ (with $h = 1$)
- Strategy:
  - Solve for firm's labor demand and profits (as function of w)
  - Solve for optimal labor supply (as function of w and π)
  - Equate labor supply and labor demand (substituting for profits) and compute equilibrium wage, w^*
  - Use w^* to compute equilibrium labor supply and demand, output, and consumption

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Firm's problem

- Firm's optimal decision:

$$w = N^{-0.5}$$

- Labor demand function:

$$N^d(w) = \frac{1}{w^2}$$

- Supply of consumption good:

$$Y(w) = 2\left(\frac{1}{w^2}\right)^{0.5} = \frac{2}{w}$$

- Profit:

$$\pi(w) = Y - wN = \frac{2}{w} - w\frac{1}{w^2} = \frac{1}{w}$$

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Household's problem

- Household's optimal decision:

$$w = \frac{C}{1-N} \text{ and } C = wN + \pi$$

- Solving for labor supply:

$$N^s(w) = \frac{w-\pi}{2w}$$

and consumption

$$C = \frac{w+\pi}{2}$$

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Equilibrium wage

- Demand for labor = Supply of labor:

$$\frac{1}{w^2} = \frac{w - \pi(w)}{2w}$$

- Recall:

$$\pi(w) = \frac{1}{w}$$

Equilibrium wage

- That means:

$$\frac{1}{w^2} = \frac{1}{2} - \frac{1}{2w^2}$$

- Solve this to get w: $w = \sqrt{3}$

- Hence:

$$N^d = \frac{1}{3}, \quad N^s = \frac{1}{2} - \frac{1}{2 \times 3} = \frac{1}{3}, \quad \pi = \frac{1}{\sqrt{3}}$$

- Consumption and output are:

$$C = Y = \frac{2}{\sqrt{3}}$$

Roadmap

Hence to solve, we

1) Start with the HH's problem: solve for C, N^s in terms of π, w, T, h.

2) Move to the firm's problem: solve for N^d as a function of z, K, w

3) Use the fact that $G = T$ and substitute T for G

4) Labor market clears: $N^d = N^s = N^*$ at w^*. Solve for w^* by equating $N^s = N^d$.

5) Knowing $w^* \rightarrow$ know N^* from $N^d \rightarrow$ know π^* and Y^*

6) Knowing $Y^* \rightarrow$ know C^* from goods market clearing:

$$C = Y - G$$

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