Annuity and perpetuity (1 Euro):

Annuity

$$PV = \frac{1}{r} \times \left[1 - \frac{1}{(1+r)^N}\right]$$

Perpetuity

$$PV = \frac{1}{r}$$

Growing perpetuity at a rate equal to g

$$PV = \frac{(1+g)}{r-g} \qquad for \quad r > g$$

Statistics

Let $\tilde{X}$ be a discrete variable in the domain $\{x_1, x_2...x_n\}$ and the associate probabilities $P(\tilde{X} = x_i)$ with $\sum_{i=1}^{n} P(\tilde{X} = x_i) = 1$:

Mean

$$E(\tilde{X}) = \sum_{i=1}^{n} P_i \ x_i$$

- **Characteristics** : k is a real integer,

$$\begin{aligned} E(k) &= k \\ E(k\ \tilde{X}) &= k\ E(\tilde{X}) \\ E(k + \tilde{X}) &= k + E(\tilde{X}) \\ E\left(\sum_{i=1}^{n} k_i \tilde{X}_i\right) &= \sum_{i=1}^{n} k_i E(\tilde{X}_i) \end{aligned}$$

Variance

$$Var(\tilde{X}) \equiv \sigma_X^2 = \sum_{i=1}^{n} P_i \ \left(x_i - E(\tilde{X})\right)^2 = E(\tilde{X}^2) - E(\tilde{X})^2 \geq 0$$

- **Characteristics** : k is a real integer,

$$\begin{aligned} Var\,(k) &= 0 \\ Var(k\ \tilde{X}) &= k^2 Var(\tilde{X}) \\ Var(k + \tilde{X}) &= Var(\tilde{X}) \\ Var(\sum_{i=1}^{n} k_i \tilde{X}_i) &= \sum_{i=1}^{n} k_i^2 Var(\tilde{X}_i) + \sum_{i=1}^{n} \sum_{j \neq i}^{n} k_i k_j Cov(\tilde{X}_i, \tilde{X}_j) \end{aligned}$$

Portfolio Analysis: 2 assets

Variance

$$\sigma^2(r_p) = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \sigma_{12}$$

Covariance

$$Cov(r_1, r_2) = \sigma_{12} = E[(r_1 - E(r_1)) \times (r_2 - E(r_2))]$$

Coefficient of correlation

$$\rho_{1,2} = \frac{\sigma_{12}}{\sigma_1 \times \sigma_2}$$

Global minimum variance portfolio with 2 risky assets

$$w_1^g = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}} \qquad \text{with} \qquad w_2^g = 1 - w_1^g$$

CAPM

$$\beta_i = \frac{Cov(r_i, r_m)}{Var(r_m)}$$

$$\text{WACC} = \frac{D}{D+E}\, r_D(1-\tau) + \frac{E}{D+E}\, r_E \qquad \text{(With taxation)}$$

OPTIONS

Value of a Call and Put at maturity:

$$C_T = \text{Max}\ [0, S_T - K]$$

$$P_T = \text{Max}\ [0, K - S_T]$$

Modigliani & Miller with taxation (fixed debt schedule):

$$r_E = r_U + \frac{D}{E}[r_U - r_D](1-\tau)$$

$$\text{WACC} = r_U \left(1 - \tau\ \frac{D}{D+E}\right)$$

Levered beta (debt is assumed risk-free)

$$\beta_E = \left[1 + (1-\tau)\frac{D}{E}\right] \beta_U$$