

73230 Intermediate Microeconomics

Solution to Problem Set 2

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Due: In-class at 10:30am on February 20, 2019

Problem 1 (80 pts)

Martial consumes only two goods, food and leisure. Martial has \$100 to spend and can also choose to go to work to earn extra income. His hourly wage is \$20, and he can only work up to 16 hours per day (i.e, if Martial works for 4 hours, he can enjoy leisure for 12 hours).

Let L be the number of hours of leisure, and F be the unit of food. Martial's utility function is given by

$$u(L, F) = L^2 F^2$$

- (a) (10 pts) Write down Martial's constrained utility maximization.

Solution:

$$\max_{L, F} u(L, F) = L^2 F^2$$

$$\text{subject to } Fp_F \leq 100 + 20(16 - L) \iff Fp_F + 20L \leq 420$$

- (b) (10 pts) If the price of food is \$10, how many hours of work and how many unit of food will Martial consume?

Solution:

$$\frac{2LF^2}{2L^2F} = \frac{20}{10} \iff \frac{F}{L} = 2 \iff F = 2L$$

Since the budget constraint binds:

$$10F + 20L = 420 \iff 20F = 420 \iff F = 21, L = 10.5;$$

- (c) (40 pts) The price of food went up to \$20 due to adverse weather conditions. How many hours of work and how many unit of food will Martial consume now? Decompose the change into substitution effect and income effect and illustrate your answer on a graph.

Solution:

$$\frac{2LF^2}{2L^2F} = \frac{20}{20} \iff F = L$$

Since the budget constraint binds:

$$20F + 20L = 420 \iff 40F = 420 \iff F = L = 10.5;$$

To decompose the total effect into the substitution and income effect, we need to look for (F^*, L^*) such that $u(F^*, L^*) = u(21, 10.5)$, and $MRS_{F^*, L^*} = \frac{20}{20} = 1 \implies F^* = L^*$. Therefore

$$(F^*)^2 = 21 \times 10.5 \rightarrow F^* = L^* = 14.8$$

Therefore, the substitution effect is $(10.5, 21) \rightarrow (14.8, 14.8)$. The income effect is $(14.8, 14.8) \rightarrow (10.5, 10.5)$.

- (d) (10 pts) Martial's CEO cares about his wellbeing and decided to give him a bonus to help him cope with the increase in the price of food. The CEO would like to make sure Martial's utility level is the same as

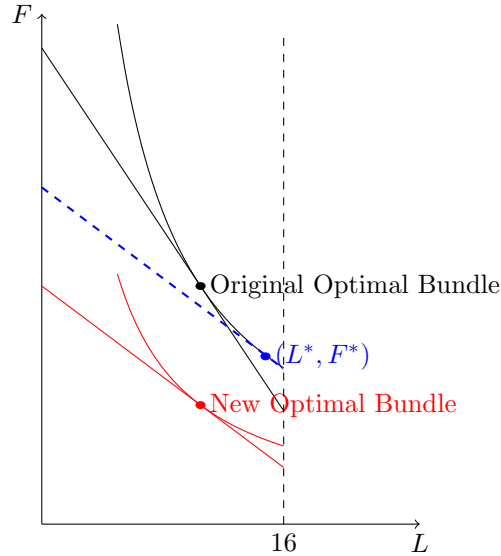


Figure 1: Illustration of Question 1(c)

before (in (b)). How much bonus should the CEO give to Martial?

Solution: Let the bonus be denoted by b . Similar to how we derived it in (c), with a bonus b :

$$40F = 420 + b \iff F = L = \frac{420 + b}{40}$$

The new utility level is:

$$\left(\frac{420 + b}{40} \right)^4$$

The old utility level was:

$$21^2 \times 10.5^2$$

Therefore, the bonus must satisfy:

$$\left(\frac{420 + b}{40} \right)^4 = 21^2 \times 10.5^2 \implies b = 174$$

- (e) (10 pts) The chairman of Martial's company disagrees with the CEO's bonus policy. The chairman thinks it's better to give Martial a food coupon of \$210 so that he can buy the same amount of food as under the old price. Which policy does Martial prefer? Illustrate your answer on a graph.

Solution: As we can see from (c), Martial's food expenditure under the new prices is $10.5 \times 20 = 210$, so he can spend all of the \$210 food coupon on food, leaving him with an extra \$210 to spend. This is greater than the bonus, so he strictly prefers the food coupon.

- (f) (10 pts) The union head of Martial's company disagrees with the CEO's bonus policy as well. He thinks it's better to give Martial a wage raise of \$10 (so his new wage is \$30/hour) to help Martial cope with the more expensive food. Which policy does Martial prefer? Illustrate your answer on a graph.

Solution: Martial's new budget constraint under a wage raise is given by $20F \leq 100 + 30(16 - L) \implies 20F + 30L \leq 580$. The new budget constraint is strictly contained in the original budget constraint in (d), so Martial is worse off.

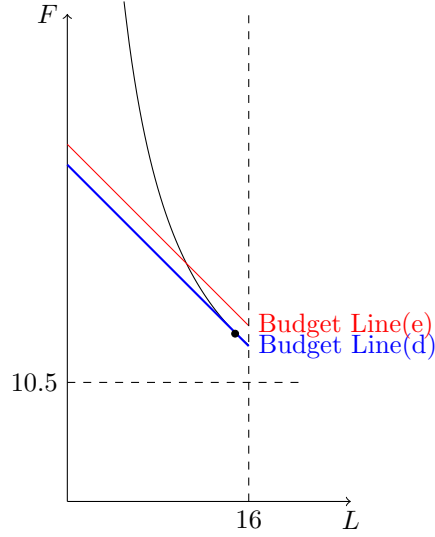


Figure 2: Illustration of Question 1(e)

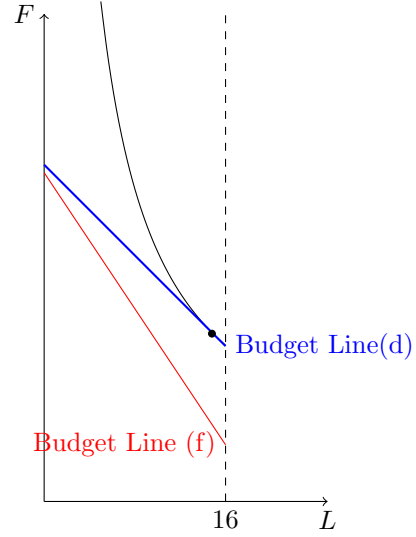


Figure 3: Illustration of Question 1(f)

Problem 2 (20 pts)

Imagine you are working for a consulting firm. Consumers in your world consume only fruit juice (F) and soft drinks (S). Your client sells fruit juice and would like to know more information about the consumer demand for fruit juice. After some surveys, you find out that each consumer the following utility function:

$$u(F, S) = \sqrt{F} + \sqrt{S}$$

Let p_F denote the price of fruit juice, and p_S denote the price of soft drinks. Let Y denote income of each consumer.

- (a) (10 pts) What is the demand of each consumer for fruit juice (in terms of p_F, p_S and Y)?

Solution:

$$MRS = \frac{\sqrt{S}}{\sqrt{F}} = \frac{p_F}{p_S} \implies S = \left[\frac{p_F}{p_S} \right]^2 F$$

Budget constraint binds:

$$Fp_F + Sp_S = Y \implies F\left(\frac{p_F^2}{p_S} + p_F\right) = Y \implies F = \frac{Y}{p_F\left(\frac{p_F}{p_S} + 1\right)} \implies S = \frac{Y}{p_S\left(\frac{p_S}{p_F} + 1\right)}$$

- (b) (10 pts) Your client would like to know whether consumers consider fruit juice and soft drinks are substitutes or complements. Calculate the cross-price elasticity $\left(\frac{\frac{\partial F}{\partial p_S}}{\frac{F}{p_S}} \right)$ and answer your client.

Solution:

$$\begin{aligned}\epsilon_{FS} &= \frac{\frac{\partial F}{\partial p_S}}{\frac{F}{p_S}} = \frac{\partial}{\partial p_S} \frac{Y}{p_F \left(\frac{p_F}{p_S} + 1 \right)} \times \frac{p_S}{F} \\ &= \frac{Y}{p_F \left(\frac{p_F}{p_S} + 1 \right)^2} \times \frac{p_F}{p_S^2} \times \frac{p_S}{F} > 0\end{aligned}$$

Since the cross-price elasticity is positive, fruit juice and soft drinks are substitutes.