MACROECONOMICS
73-240

Lecture 4

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Building a model of the economy

The Household

Building a Model: Who demands and produces?

Who are the decision-makers in an economy?

- Households: decide how many commodities to consume, how much leisure to consume, how much to save
- Firms: how much to produce, how many inputs to purchase
- Government: How much tax/subsidies, any transfers

Today, we focus on the **Household**

The Average Household: Working

As of July 2019:

- from BLS: Civilian non-institutional population: 259,225,000
- From BLS: Civilian labor force: 163,351,000
- Unemployed: 6,063,000

  (http://www.bls.gov/news.release/empsit.a.htm)

- Average hours worked per week: 34.3

  Average hourly earnings: $ 27.98

  (http://www.bls.gov/news.release/empsit.b.htm)

The Average Household: Consuming

1. Excellent source: Consumer Expenditure Survey (CEX)

   http://www.bls.gov/cex/

2. Average income before taxes per consumer unit (2017-2018): $\$ 76,335$

3. Average annual expenditures per consumer unit (2017-2018): $\$ 60,815$

   Some spending categories:

Category	Expenses
Food	$ 7,869
Housing	$ 20,001
Transportation	$ 9,735
Healthcare	$ 4,924

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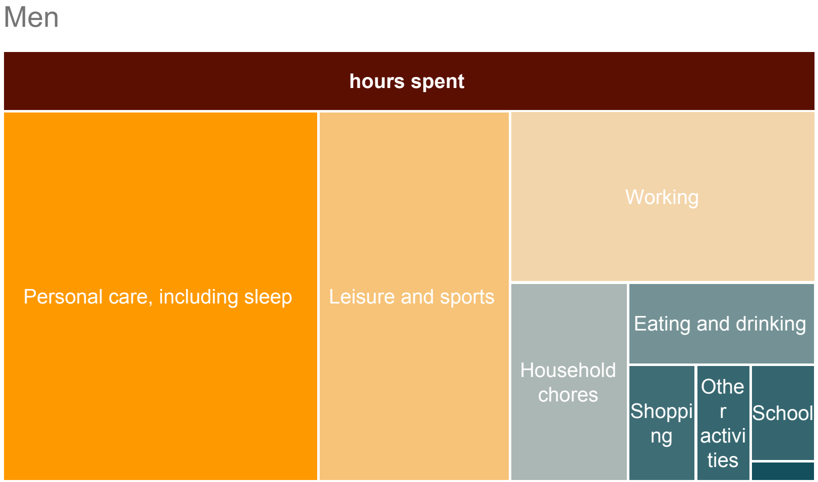
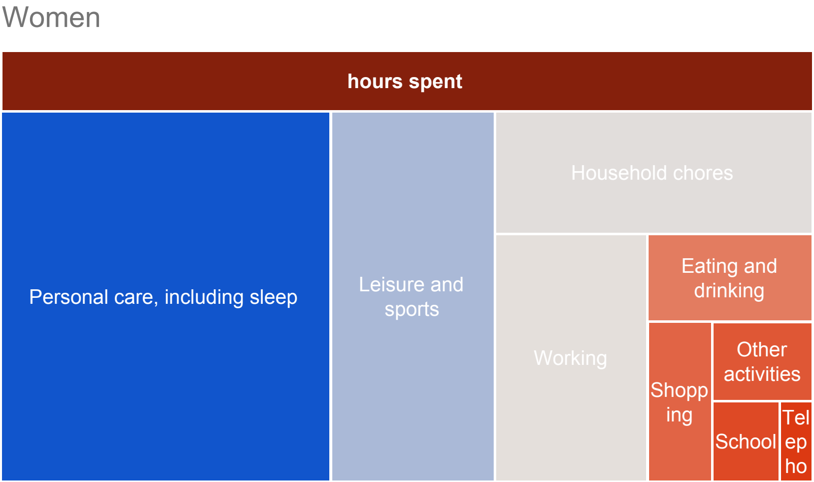
The Average Household: Time USE

1. Excellent source: American Time Use Survey (ATUS) http://www.bls.gov/tus/home.htm



The Average Household: Time USE

1. Similar result for women



Modeling the Household

Building a model of the economy

- When we think about the household's problem, we want to think about what the household would choose
- In the simplest form possible: the *endogenous* variables (choice variables) of a household are
  - consumption: c for an individual household, C for all households (summed up!)
  - leisure: ℓ for an individual household, L for all households (summed up!)

Building a model of the economy

- Typically, in microeconomics, we treat prices as *exogenously* given
- And observe how agents respond to changes in the price.
- In macroeconomics, prices are *endogenous*.
- Although prices are endogenous, a single household (HH) cannot unilaterally choose prices and instead takes prices as given

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Building a model of the economy

How are prices endogenous?

- Prices are determined when markets clear and equilibrium demand = supply

What markets does the household participate in?

Building a model of the economy

- Households typically decide how much to consume and how much labor to supply
  - Goods Market
  - Labor Market
- Price of Consumption Good: P
- Nominal Wage (Price of Labor): W
- Relative Price of Labor : $\frac{W}{P} = w$
- Refer to the relative price of labor, w, as real wages.

Taking a Step Back

How Does Economics Approach Decision Making?

The Economics Approach

- Every agent in the economy has an objective he/she wants to achieve
- Agents however may face constraints that affect their decision making
  - Remember! Economics is about making decisions given scarce resources
- Approach in Economics: to achieve their objective, agents make decisions *on the margin*
  - e.g. what's the additional benefit of one hour more of studying vs. the additional cost?

The Household Problem

To write down the household problem, we must specify

- What is the Household's objective?
- What constraints does the Household face?

Given the Household's objective and constraint, we then ask

- What is the trade-off the household faces?
- How does he/she best achieve his/her objective?
  - What if he/she chooses a little more or a little less of a particular action

Household Constraints

HH Constraints

Consider a HH who lives for 1 period.

- What constraints does the HH face?

HH Constraints

In our economic model, HH have

- Limited amount of time available: time constraint
- let h be the total time available (time endowment)
- let n be the time allocated to working
- Household face the following time constraint

$$\ell + n = h$$

The Budget Constraint

- Limited disposable income: a budget constraint

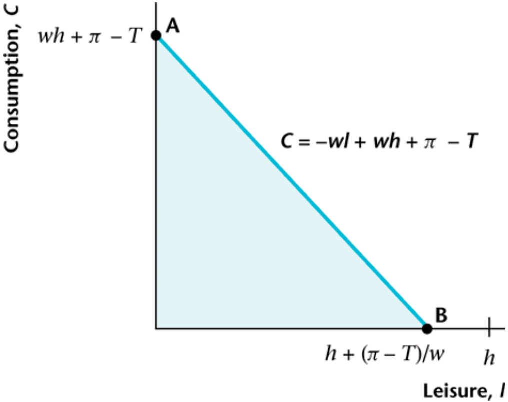
Let:

- $c =$ consumption
- $h =$ time endowment
- $\ell =$ leisure time
- $w =$ hourly wage
- $\pi =$ dividend income
- $T =$ lump sum taxes (rebates)

$$c = w(h - l) + \pi - T$$

Note: we have used the time constraint to write n in terms of h and ℓ.

The Budget Constraint

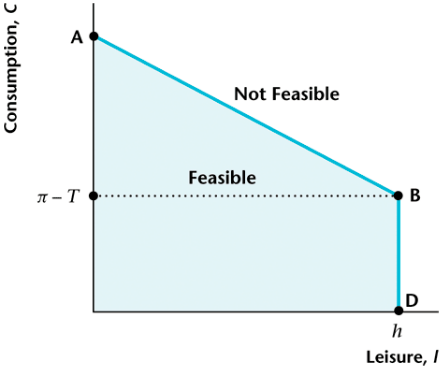


- Let's suppose $T > \pi$.
- Wages determines the slope of the budget constraint.
- π and T determine the intercept.

The Budget Constraint

Budget constraint determines what the consumer can afford

- Suppose now $T < \pi$: positive consumption even with no work



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The Household in our Model

PREFERENCES

The Household: Assumptions

Assume households are:

1. Representative: All the households have the same preferences
   - thinking of a typical decision maker to represent all households.
2. Smart: they optimize their preferences given some constraints
   - which means they choose some optimal combination of goods and leisure
3. Live one period (for now)

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The Household in our Model

- The Household cares about:
    1. Consumption: c
    2. Leisure: ℓ
- The household (HH) has an objective: be happy.

The Decision of The Household

The household (HH) has an objective: be happy

- The household gets happiness from consuming c and ℓ
- The utility function U represents the happiness of the HH:
  - $U(c, \ell)$ **represents the level of utility** associated with a bundle (c, l)
  - **We say that** (c_1, ℓ_1) **is preferred to** (c_2, ℓ_2) **if and only if**

$$U(c_1, \ell_1) > U(c_2, \ell_2)$$

Thinking about the Household's preferences

- We represent the household's preferences with a utility function
- Some properties we will summarize using the utility function:
  - Does the household like having more of a particular good?
  - Does the household get increasing or decreasing gains from having more of a particular good?

Some Math Notes

- Marginal Utility
  - The extra utility resulting from consumption of an additional unit of the good
  - Mathematically, the slope of utility with respect to that good
  - Marginal Utility with respect to consumption, $MU_c = \frac{\partial U(c,\ell)}{\partial c}$

Properties of the Utility Function

How would we represent the following?

1. The household prefers more to less
   Answer:

2. Each additional unit of consumption and leisure adds less utility (or less gain in happiness)
   Answer:

- Assume that HH likes diversity (1 apple + 1 orange is better than 2 oranges)
- Assume HH dislikes risk (later in the course)

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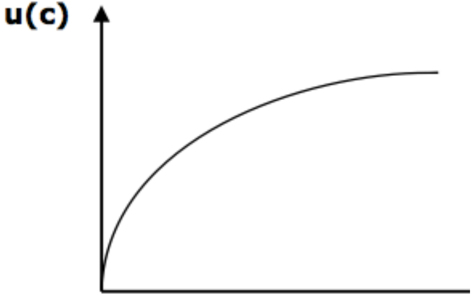
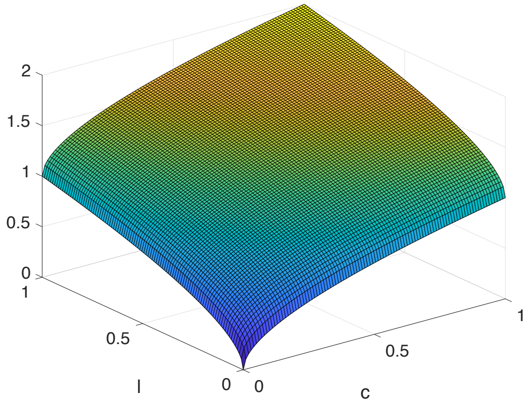
Concave Utility functions

- Easy to graphically think of the one good increasing utility function with diminishing marginal returns



From U to Indifference Curves

- We look at 2 dimensional utility functions (HH cares about c AND ℓ)
- How can we summarize preferences graphically?



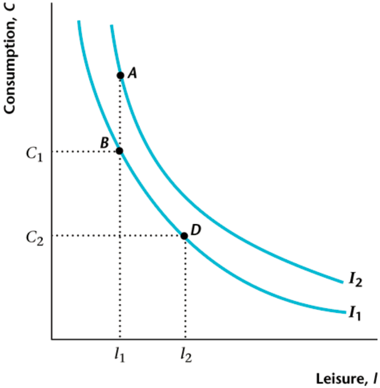
From U to Indifference Curves

- HH cares about c AND ℓ
- Summarize preferences over c and l using indifference curves (The pair of (c, ℓ) that provide the same level of utility)

Definition An indifference curve is a curve connecting all values of consumption and leisure among which the consumer is indifferent.

Indifference Curves

- HH like diversity $\Rightarrow$ Indifference curves are convex



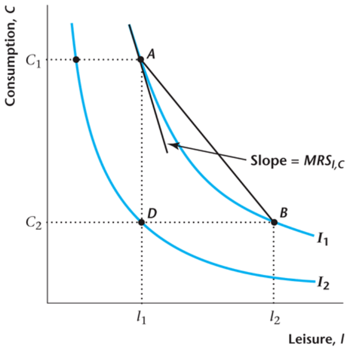
Question: can indifference curves cross each other?

The MRS of Leisure for Consumption

- Definition: The Marginal Rate of Substitution of Leisure for Consumption is the amount of consumption the HH would give up for 1 additional unit of leisure while maintaining the same utility
- Value the additional gain in leisure from giving up consumption in terms in terms of marginal utilities
- this implies:

$$MRS_{\ell,c} = \frac{\partial U(c,\ell)}{\partial \ell} \Big/ \frac{\partial U(c,\ell)}{\partial c} = -\frac{dc}{d\ell}$$

The MRS of Leisure for Consumption



$\Rightarrow$ Minus the Slope of indifference curve $= MRS_{\ell,c}$

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For those interested...

- Proof: Start by taking the total differential of $U(c, \ell)$:

$$dU(c,\ell) = \underbrace{\frac{\partial U(c,\ell)}{\partial c}}_{MU_c} dc + \underbrace{\frac{\partial U(c,\ell)}{\partial \ell}}_{MU_\ell} dl$$

- Divide by $d\ell$

$$\frac{dU(c,\ell)}{d\ell} = MU_c \frac{dc}{d\ell} + MU_\ell$$

- Recall along the indifference curve, utility is constant $\implies \frac{dU(c,\ell)}{d\ell} = 0$

$$0 = MU_c \frac{dc}{d\ell} + MU_\ell$$

$$-\frac{dc}{d\ell} = \frac{MU_\ell}{MU_c} = MRS_{\ell,c}$$

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Consumer Maximization

Consumer Maximization

A consumption-leisure bundle is:

- Affordable if it lies on or within the budget set.
- Optimal if it is affordable and is on the highest indifference curve.

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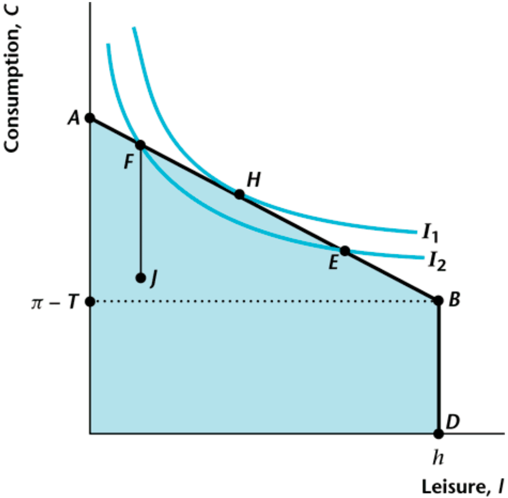
Optimal and Sub-optimal Bundles



Only H is optimal!!

Optimality (Seen graphically)

Graphically, we observe that:

- Optimality required we were on the budget constraint
  - No wastage! all household resources used up!

- The slope of the indifference curve must be equal to the slope of the budget constraint (tangency!) (more on this in a bit)

Consumer Maximization: Math!

The household problem:

$$\max_{c,l} U(c,\ell)$$
$$\text{s.t.} \quad w(h-\ell)+\pi-T=c$$

- This is a constrained optimization problem
- A few ways to solve:
  - Substitution method
  - Lagrangian method (This is what we will focus on today!)

The Lagrangian Method

- The Lagrangian method transfers a constrained optimization problem into a
  - unconstrained optimization problem
  - with a pricing problem
- The new function to be optimized is called a Lagrangian
- Each constraint has a shadow price in utils, called a Lagrange Multiplier (denoted by λ)
- In the new unconstrained optimization problem, λ prices the additional value one receives from relaxing that constraint.

Consumer Maximization: Lagrangian Method

The household problem:

$$\max_{c,l} U(c,\ell)$$
$$\text{s.t.} \quad w(h-\ell)+\pi-T=c$$

Write the lagrangian:

$$\mathcal{L}(c,\ell,\lambda)=U(c,\ell)+\lambda[w(h-\ell)+\pi-T-c]$$

where λ is our lagrange multiplier. First order conditions

$$\begin{aligned}
(C): &\quad \frac{\partial U(c,\ell)}{\partial c}-\lambda &= 0\\
(l): &\quad \frac{\partial U(c,\ell)}{\partial \ell}-w\lambda &= 0\\
(\lambda): &\quad w(h-\ell)+\pi-T-c &= 0
\end{aligned}$$

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Consumer Maximization: More Math!

Let's substitute out λ from the two first order conditions:

$$\frac{\partial U(c,\ell)}{\partial \ell} - w\frac{\partial U(c,\ell)}{\partial c} = 0$$

which we can re-write as:

$$\underbrace{\frac{\partial U(c,\ell)}{\partial \ell}}_{\text{marginal benefit of 1 more unit of leisure}} = \overbrace{w\frac{\partial U(c,\ell)}{\partial c}}^{\text{marginal cost of 1 more unit of leisure}}$$

By having 1 more unit of ℓ, HH loses out in earning w wages, which could have $\uparrow c$ and made him/her happier by $\partial U(c,\ell)/\partial c$

Optimization

Observe that the household makes decisions on the margin!
Optimal choice requires marginal benefit (MB) $=$ marginal cost (MC)

$$\underbrace{\frac{\partial U(c,\ell)}{\partial \ell}}_{\text{MB of 1 more unit of leisure}} = \overbrace{w\frac{\partial U(c,\ell)}{\partial c}}^{\text{MC of 1 more unit of leisure}}$$

- If MB of 1 more unit of $\ell >$ than MC incurred $\Longrightarrow$ HH should have more ℓ (adds more to benefit than costs)
- If MB of 1 more unit of $\ell <$ than MC incurred $\Longrightarrow$ HH should cut back on ℓ (adds more to costs than benefits)

Optimality

Let's re-arrange the equation:

$$\frac{\partial U(c,\ell)}{\partial \ell} = w\frac{\partial U(c,\ell)}{\partial c} \quad \Longrightarrow \quad \underbrace{\frac{\partial U(c,\ell)/\partial \ell}{\partial U(c,\ell)/\partial c}}_{MRS_{\ell,c}} = w$$

Solving the household's problem, we arrive at two optimality conditions:

- A consumption-leisure bundle is optimal when:
  - Is on the budget line (Walras law)

    $$c = w(h-\ell) + \pi - T$$

  - Slope of indifference curve = Slope of budget line

    $$\underbrace{MRS_{\ell,c}}_{\text{Marginal rate of substitution}} = \underbrace{w}_{\text{opportunity cost of leisure}}$$

  Exactly like our graphical solution!

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Consumer Maximization: Try it yourself!

The household problem:

$$\max_{c,l} U(c,\ell) = \ln c + \ln \ell$$
$$\text{s.t.} \quad w(h-\ell) + \pi - T = c$$

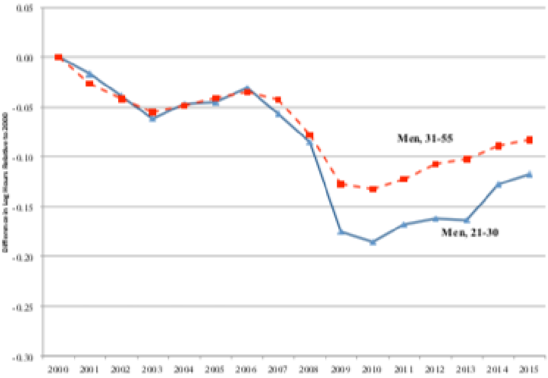
What can you say about c as $\pi - T$ increases?
In this example, what can you say about l as w increases? What about c?

Declining hours worked by Men

Taking a supply side perspective, how would you explain this graph?

Figure 1: Market Hours

(a) Log Annual Hours (Index)

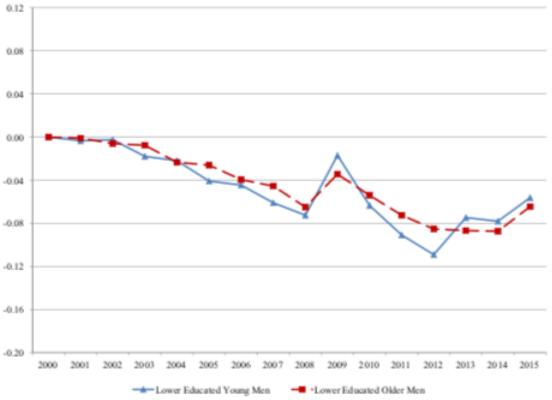


Source: Aguiar et al (2017)

Declining hours worked by Men

Hourly wage for men with less than 16 years of schooling (i.e. 4 year college degree) has fallen somewhat

(b) Men Ed<16



Source: Aguiar et al (2017)

Declining hours worked by Men

Where is the time being reallocated to?

Table 3: Leisure Activities for Men 21-30, Hours per Week

Activity	2004-2007	2012-2015	Change
Total Leisure	61.0	63.4	2.3
Recreational Computer	3.3	5.2	1.9
Video Game	2.0	3.4	1.4
ESP	24.3	24.9	0.6
TV/Movies/Netflix	17.3	17.1	-0.2
Socializing	7.8	7.9	0.1
Other Leisure	8.3	8.2	-0.1

Source: Aguiar et al (2017)

Alternative explanations?