MACROECONOMICS
73-240

Lecture 16

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This version: November 13, 2019

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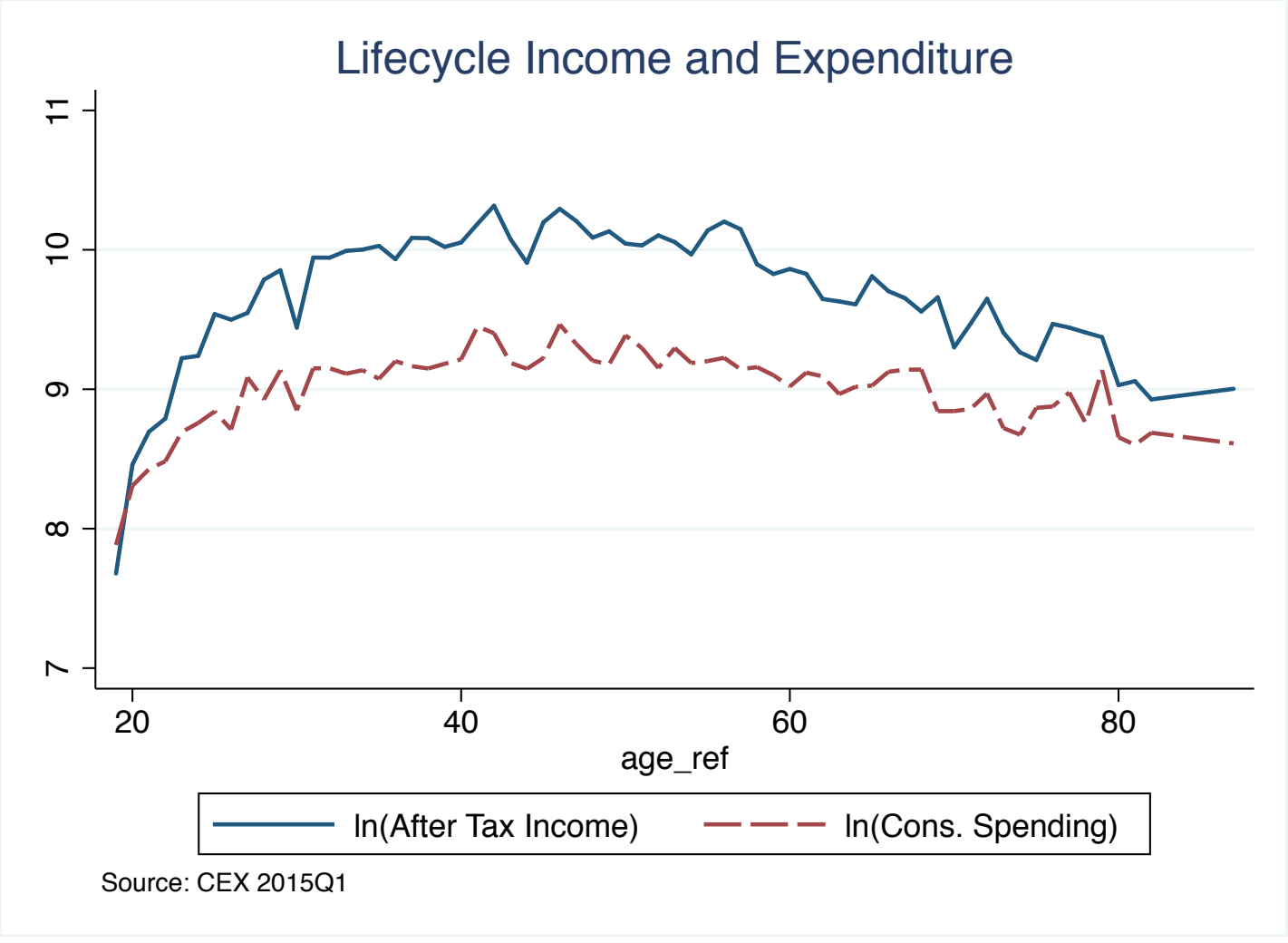
Plan for This Lecture

Goal: Understand optimal (micro!) savings decision of the representative household

- The two period model: the household
  - Inter-temporal budget constraints
  - Optimality
- Temporary changes in income vs. Permanent changes in income
- Changes in Interest Rate
- Fiscal Policy and Ricardian Equivalence

Lifecycle income and expenditures



Source: CEX 2015Q1

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Lifecycle income and expenditures

- Two things show up from the previous picture
  - Consumption spending is not exactly equal to income received
  - Consumption spending is smoother than income: households like a stable path of consumption
  - All this points to a role for savings.

- Recall Solow Model: Exogenous savings rate
- Household saved a constant fraction of income
- But in reality, how much to save is a choice!
- Today we want to think about how to model the savings decision

Two Period Model of Consumption and Saving

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Two Period Model: Household

- Consumers live two periods
- New trade-off: consumption today vs consumption tomorrow
- For this lecture, we *ignore* the consumption leisure trade-off we have already studied, i.e.
  - Assume Household has no utility from leisure.
- Households are perfectly rational and forward looking.
- There is no uncertainty (can relax this later)

Two Period Model: Household

Households can save in:

- New instrument: bonds (not capital goods!)
  1) All bonds are identical
  2) All bonds are safe
  3) No intermediaries
- New price: interest rate r (1 bond today, pays $1 + r$ tomorrow)
  4) Borrowers and lenders face the same r

Two Period Model: Consumer

- Consumer cares about consumption today *and* consumption tomorrow
- As before, we will need to think about what constraints a household faces
- And what preferences the household has

Two Period Model: Budget Constraints

- For now, we will assume that households are endowed with income, y, each period.
  - Notice households do not need to work for their income and are instead given income y. We call this the **endowment** economy.
- Each household has to pay lump-sum tax t each period
- Each household takes the interest rate on bonds, r, as given.

Question: What are the choice variables of the consumer?

Two Period Model: Budget Constraints

- Budget constraint today

$$c + s = y - t$$

- Budget constraint tomorrow (recall $'$ denotes tomorrow's variable)

$$c' = y' - t' + (1 + r)s$$

- Lifetime budget constraint (in terms of today's \$)

$$\underbrace{c + \frac{c'}{1+r}}_{\text{present value of consumption}} \leq \underbrace{y + \frac{y'}{1+r}}_{\text{...of income}} - \underbrace{\left(t + \frac{t'}{1+r}\right)}_{\text{...of taxes}}$$

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Lifetime Wealth Explained

- "Lifetime wealth" is just the present value of income less the present value of taxes
- Notation: Lifetime Wealth $\equiv we$

$$we = y + \frac{y'}{1+r} - \left(t + \frac{t'}{1+r}\right)$$

- Present Discounted Value of a stream of cash flow is the sum of cash flow in all dates with future cash flows discounted by the interest rate

Some Notes about Present Value

- Notice that future income, y', is discounted by the interest rate $\frac{1}{1+r}$
  - All income flows have to be brought to the same point in time before they can be compared and aggregated

- We call $\frac{y'}{1+r}$ the present value of y' goods delivered tomorrow

  - e.g. I wish to borrow from you today and return A units tomorrow.
  - Must ask how much value, B, you're willing to give up today.
  - Alternative: is to put B in bank and earn $(1+r)B$
  - So only make deal if (at least)

$$A = (1+r)B$$

$$\underbrace{B}_{\text{present value}} = \frac{A}{1+r}$$

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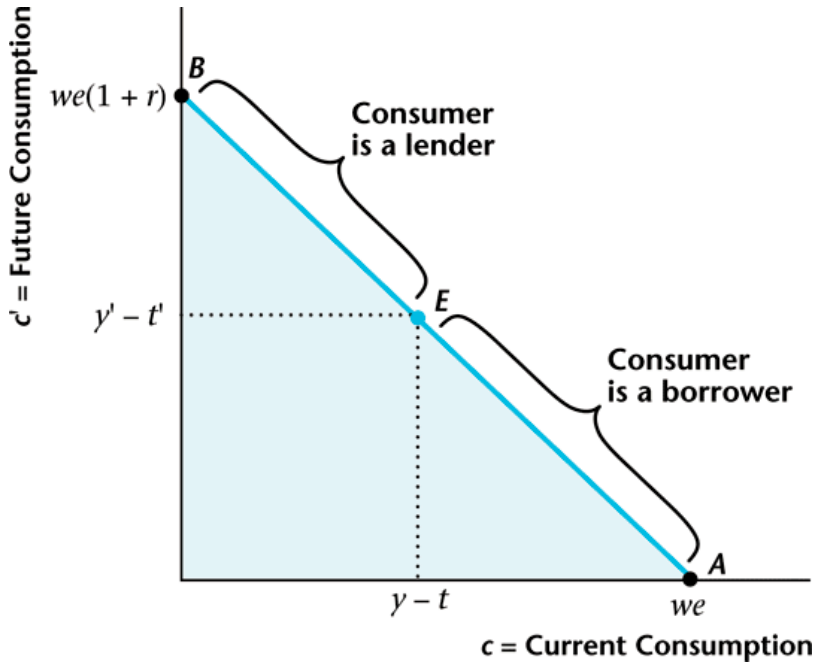
Two Period Model: Budget Constraints

- Can re-write the budget constraint as

$$c' = -(1+r)c + (1+r)we$$

and plot it:



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Interest rate and budget line

The interest rate is the relative price/opportunity cost of consumption today (represented by slope of budget constraint $\frac{dc'}{dc}$)

- If I save one unit today, I get $(1 + r)$ units of c'
- The present value of one unit of c' is worth $1/(1 + r)$ units of c
- If r increases, consumption today becomes more expensive (saving yields a higher return now.)

Two Period Model: Preferences

- Household's lifetime utility is represented by

$$U(c, c') = u(c) + \beta u(c')$$

- $u(c)$ is the utility from consuming c in current period
- $u(c')$ is the utility from consuming c' in future period
- β (beta) is the discount factor

Discount factor

- Parameter β (beta) is the subjective discount factor

$$0 < \beta \leq 1$$

- The utility of future periods is discounted.
- β tells us how much households prefer consumption today vs. tomorrow
- $\beta < 1$ means that households are impatient (they care less about utility flows in the future)
- If β increases, households become more patient. It is less painful for them to postpone consumption.

Assumptions on utility functions

- Utility is increasing in consumption

$$u'(c) > 0$$

  - Household, in any period, prefers more consumption to less

- There is diminishing marginal utility

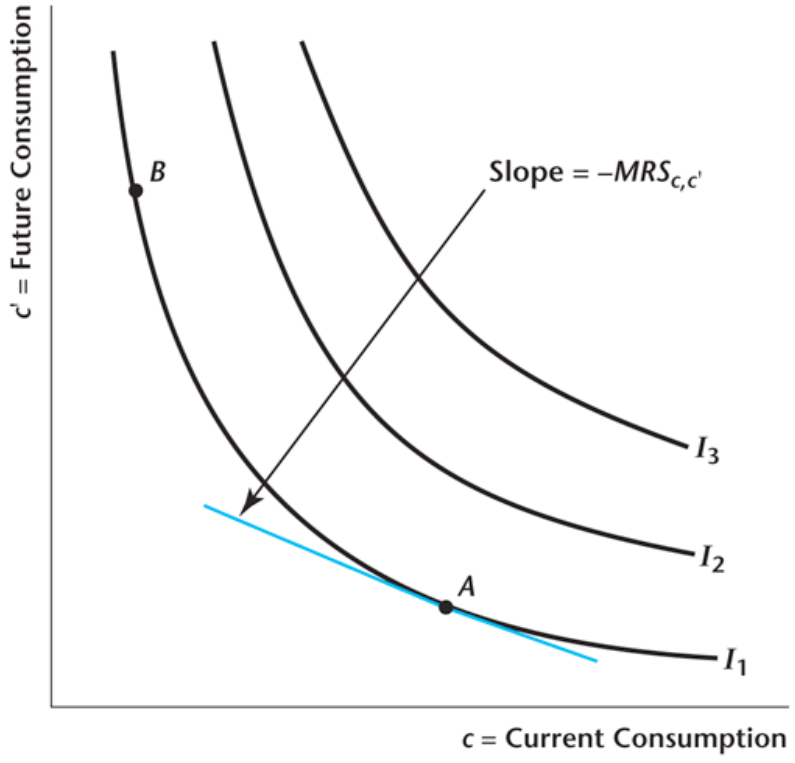
$$u''(c) < 0$$

  - As we increase consumption, extra satisfaction from consuming one more unit declines.

- Assume consumption today and consumption tomorrow are normal goods

Two Period Model: Preferences

- Minus the Slope of indifference curve is the marginal rate of substitution (between consumption today and consumption tomorrow)



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Two Period Model: Preferences

- The MRS between consumption now and in the future is the units of future consumption needed to compensate households for one unit reduction of consumption now so that they are just as happy.

$$MRS_{c,c'} = \frac{u'(c)}{\beta u'(c')}$$

$u'(c)$ is marginal utility of consumption

- $\beta u'(c') =$ marginal utility of consumption tomorrow, takes into account the weight put on consumption tomorrow.

Two Period Model: Optimality

- Problem of the household:

$$\max_{c,c'} u(c) + \beta u(c')$$

$$\text{Subject to} \qquad (1+r)c + c' = we(1+r)$$

- From first order conditions:

$$\underbrace{u'(c)}_{\text{Marginal Benefit of } c} = \underbrace{(1+r)\beta u'(c')}_{\text{Marginal Cost of } c}$$

Optimality:

$$\underbrace{\frac{u'(c)}{\beta u'(c')}}_{MRS_{c,c'}} = (1+r)$$

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Two Period Model: Optimality

To find the optimal consumption *choice* we need to solve the following two equations

1) Dynamic Optimality condition (also known as Euler Equation)

$$\frac{u'(c)}{\beta u'(c')} = 1 + r$$

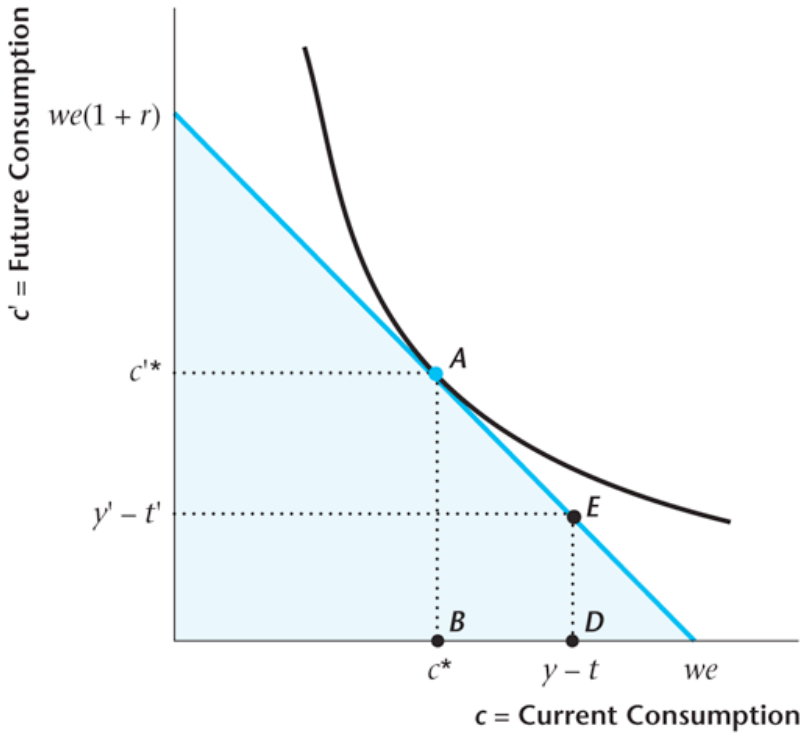
Intuition: Give up one consumption today (valued at $u'(c)$) for $(1 + r)$ units of consumption tomorrow (valued at $u'(c')$ and discounted by β)

2) Budget constraint (the choices must be affordable for the household)

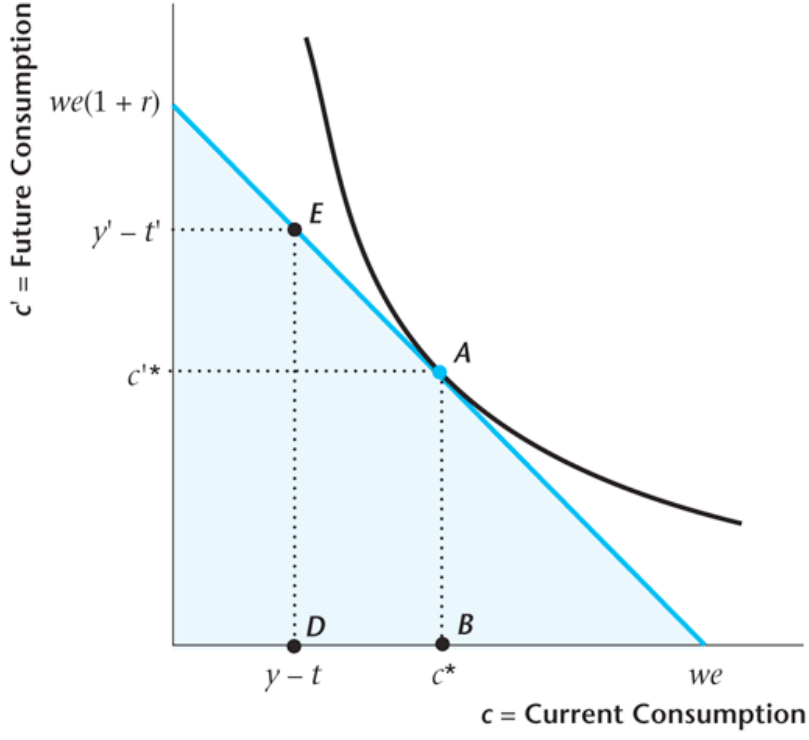
$$c + \frac{c'}{1+r} = y + \frac{y'}{1+r} - t - \frac{t'}{1+r}$$

Two Period Model: Optimality

- Optimality $\Rightarrow$ Slope $= -MRS_{c,c'} \Rightarrow MRS_{c,c'} = 1 + r$



(a) Lender



(b) Borrower

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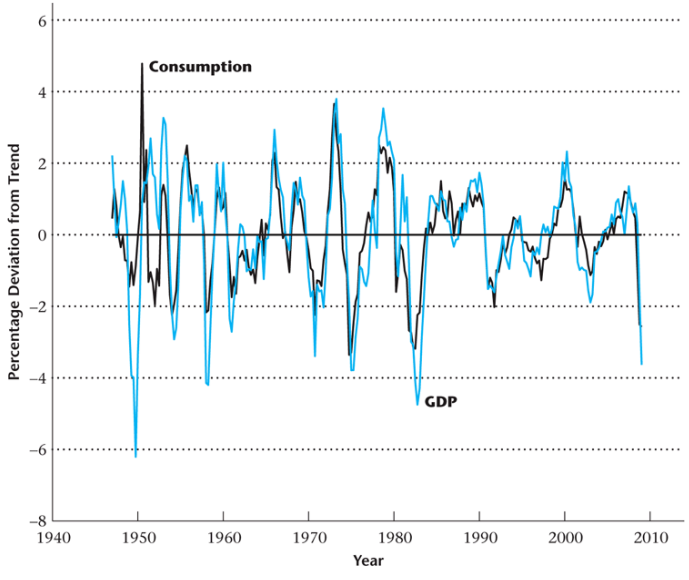
A Special Example

Special Example: What if $\beta = \frac{1}{(1+r)}$?

Optimality $\Rightarrow \frac{u'(c)}{u'(c')} = \beta(1+r) \Rightarrow u'(c) = u'(c')$

- Household tries to equalize marginal utility of consumption across periods
- Consumption -smoothing occurs!



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Working with the Special Example

Question: how much does a household save?

Suppose $\beta = \frac{1}{(1+r)}$ (Special Example)

Then we want to perfectly smooth consumption, $c = c'$

- saving equals

$$s = y - t - c$$

- With $c = c'$, the budget constraint becomes

$$c(2 + r) = y(1 + r) + y' - t(1 + r) - t'$$

So that

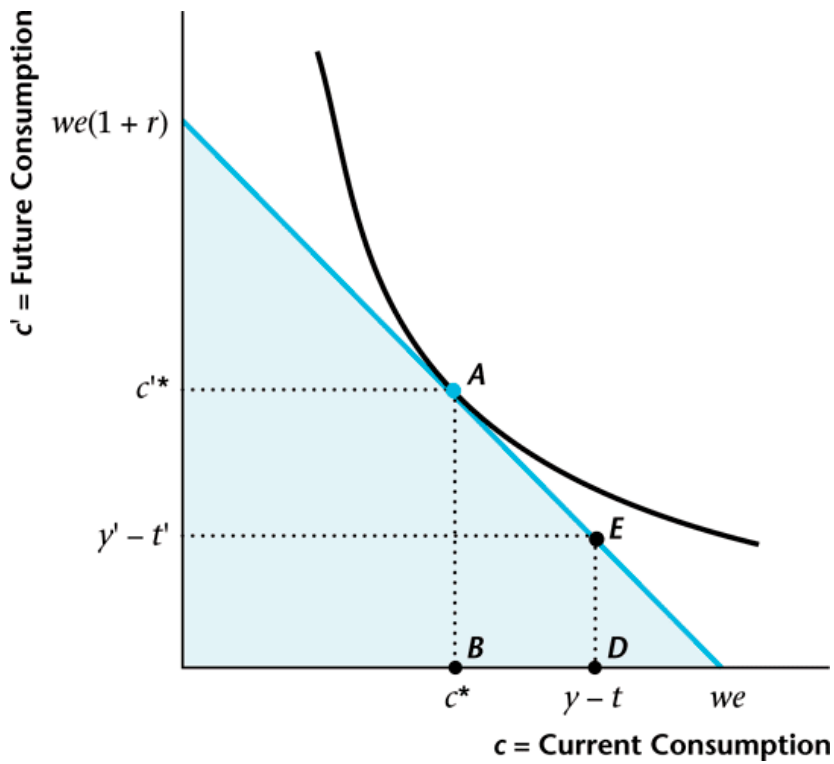
$$s = \frac{(y - t)(2 + r) - [y(1 + r) + y' - t(1 + r) - t']}{2 + r}$$

$$s = \frac{(y - t) - (y' - t')}{2 + r}$$

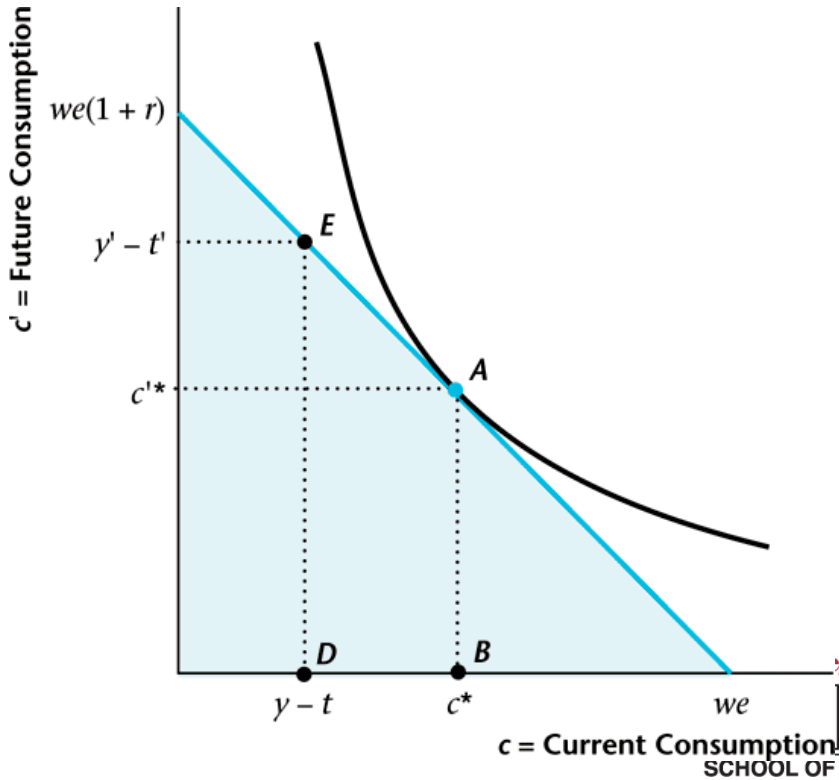
Two Period Model: Saving

What are the implication for saving?

1) flat income profile $(y - t = y' - t')$: optimal saving is 0
2) increasing income $(y - t < y' - t')$: optimal saving is < 0
3) decreasing income $(y - t > y' - t')$: optimal saving is > 0



(c) Positive saving



(d) Negative saving

Comparative Statics

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Comparative Statics

We look at the following cases:

1) Temporary Change in Income: today (A) and tomorrow (B)

2) Permanent changes in income

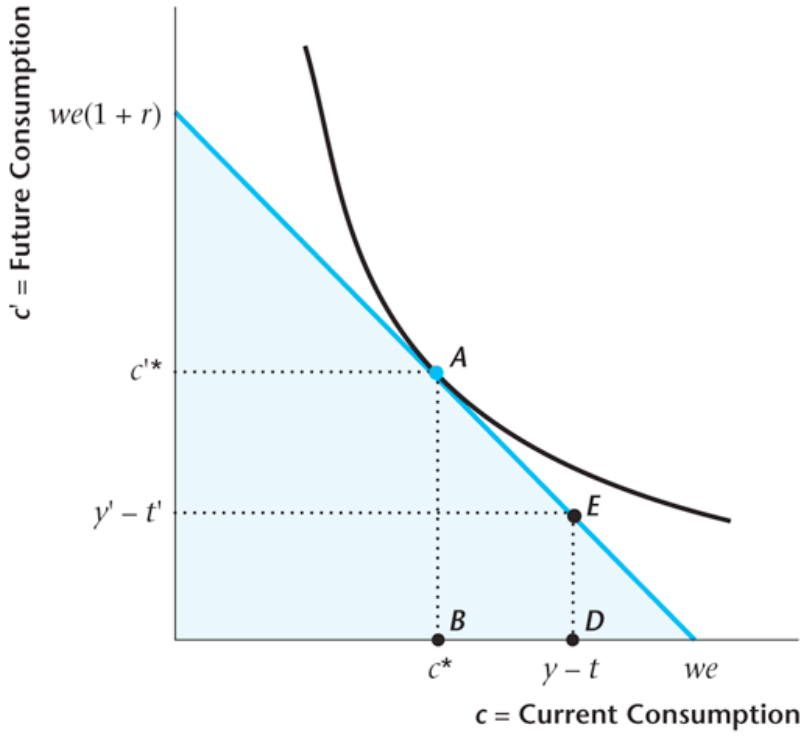
3) Changes in interest rate

1A) Temporary Change in Income Today

Increase in today's income (recall y' is tomorrow's income)

- $y_1 \Rightarrow y_2$, since: $we_i = y_i + \frac{y'}{1+r} - t - \frac{t'}{1+r}$

  we have $\Delta we = (we_2 - we_1) = y_2 - y_1 > 0$



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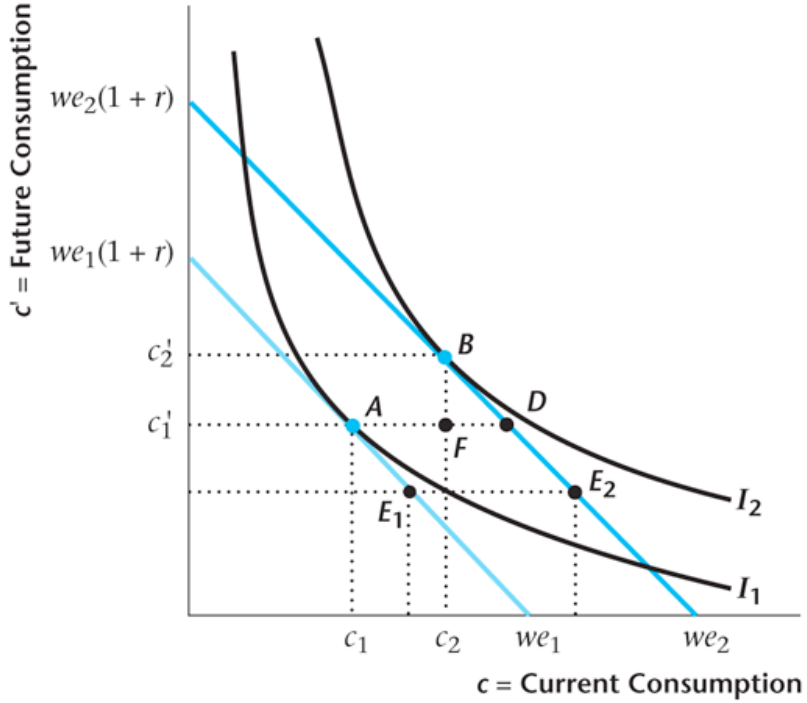
1A) Temporary Change in Income Today

Increase in today's income (recall y' is tomorrow's income)

- $y_1 \Rightarrow y_2$, since: $we_i = y_i + \frac{y'}{1+r} - t - \frac{t'}{1+r}$

  we have $\Delta we = (we_2 - we_1) = y_2 - y_1 > 0$



Remember income effects!

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1A) Temporary Change in Income Today

- Only has income effect
- Increase in current income, increases consumption (current and future)
- For this to happen, current consumption has to increase less than current income does

$$\Delta c < \Delta y$$

- This implies that saving increases

$$\Delta s = \Delta y - \Delta c > 0$$

- Decrease in current income has opposite effect

1B) Temporary Change in Income *Tomorrow*

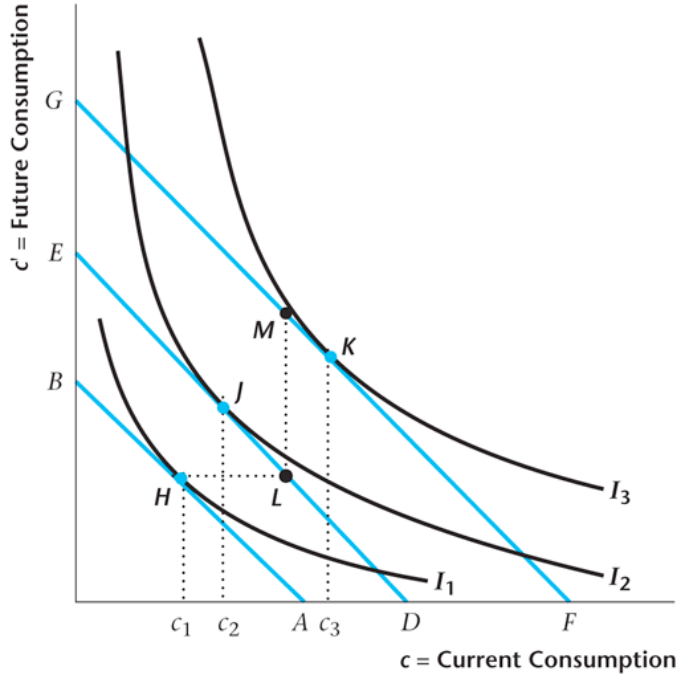
- Only has income effect
- Increase in future income, increases consumption (current and future)
- Since current income has not changed, increase in current consumption means decline in saving
- This implies that saving decreases

$$\Delta s = \Delta y - \Delta c = 0 - \Delta c < 0$$

- Decrease in future income has opposite effect

2) Permanent Change in Income

- With **permanent** income changes $y_1 \Rightarrow y_2$ and $y_1' \Rightarrow y_2'$
- In the picture we move from $H \Rightarrow K$



Difference: larger effect on consumption since savings may not increase

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Temporary vs. Permanent Increases in Income

- As a permanent increase in income will have a larger effect on lifetime wealth than a temporary increase, there will be a larger effect on current consumption

- Consumers respond to temporary income changes by saving or dis-saving. Response of current consumption dampened.

- Permanent Income Hypothesis: Household's decision on how much to consume depends *largely* on the present value of lifetime (permanent) income, not on current income

Temporary vs. Permanent Increases in Income

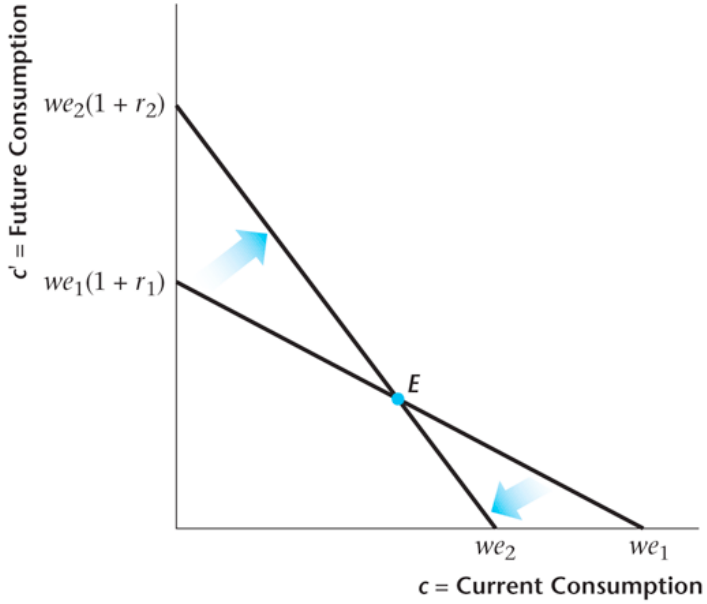
- Implications Household expenditure should not respond as much to transitory movements in income!

- It suggests that if you have a bonus today (temporary increase in income) or if you find \$100 on the floor today, you would not spend all of it immediately on consumption today.

What if the Interest Rate Changes?

3) Change in The Interest Rate

- Suppose r increases: $r_1 \Rightarrow r_2$
- Budget constraint pivots around the endowment point E (if $(y - t) > 0$, $(y' - t') > 0$)



Question: can you show why it pivots?

- KEY additional force: substitution effect between consumption today and tomorrow

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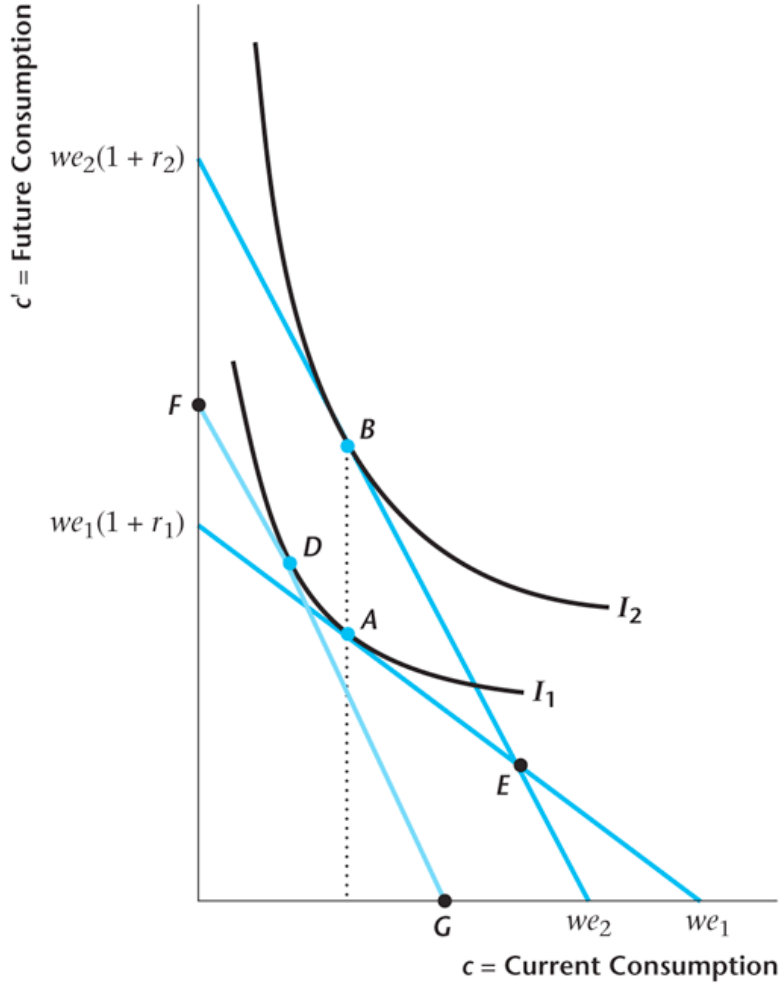
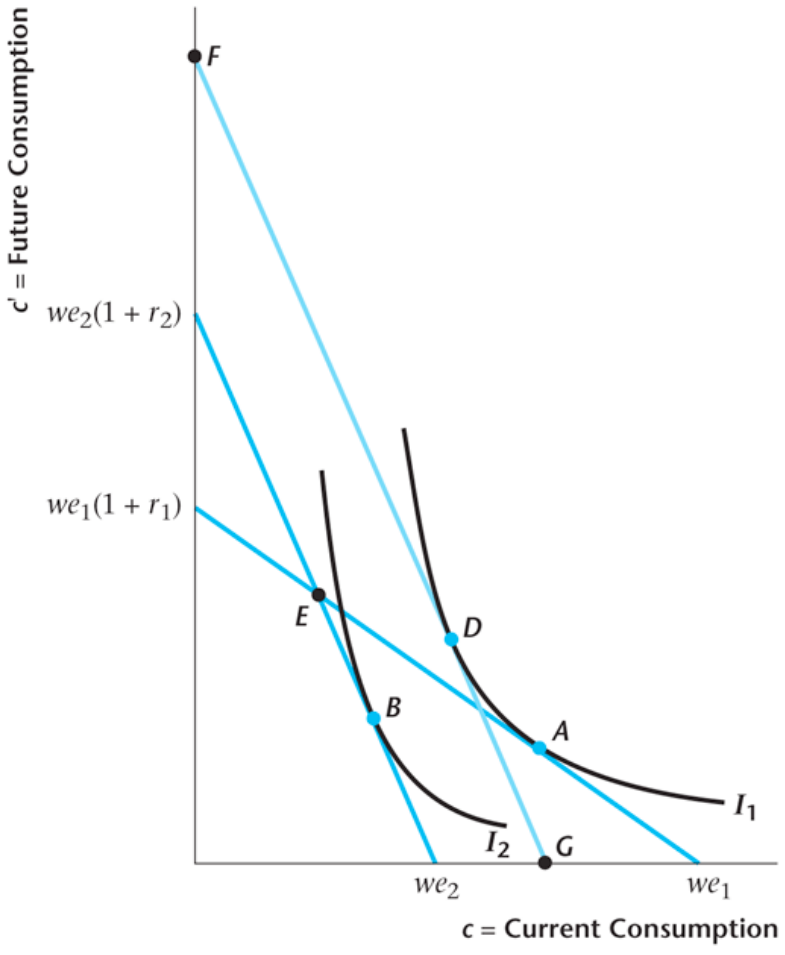
Change in The Interest Rate

- There are two effects on the budget line:
  - The slope changes
  - The permanent income (intercept on horizontal axis) changes
- It has both income and substitution effect.
- May have positive or negative income effect depending on whether the individual is borrower or lender (saver).

Increase in Interest Rate



(a) Lender



(b) Borrower

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Summary: interest rate increase

Table 8.2 Effects of an Increase in the Real Interest Rate for a Lender

Current consumption	?
Future consumption	Increases
Current savings	?

Table 8.3 Effects of an Increase in the Real Interest Rate for a Borrower

Current consumption	Decreases
Future consumption	?
Current savings	Increases

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