INTERMEDIATE MICROECONOMICS

OLIGOPOLY

SPRING 2019, PROFESSOR ANH NGUYEN

Introduction

- Derive the equilibrium price and quantity and the deadweight loss under various market structures.
  - Oligopoly:

	Competing Over	Order of Move
Cournot	Quantity	Simultaneous
Stackelberg	Quantity	Sequential
Bertrand	Price	Simultaneous

  - ~~Monopolistic competition~~

1. Market Structures

Market Structures

- Markets differ according to:
  - Number of firms
  - Ease with which firms may enter and leave the market
  - Ability of firms to differentiate their products from rivals'

Market Structures

- **Oligopoly** is a market structure in which a small group of firms influence price and enjoy substantial barriers to entry

2. Cournot Competition

Cournot Competition

- Derive the Cournot equilibrium price and quantity in an oligopoly markets
  - Identical firms
  - Heterogeneous firms: Different marginal costs or different demands (differentiated goods)
- Compare the market equilibrium: Duopoly, monopoly, and perfect competition
- Reading: pp. 950-980

Cournot Oligopoly Model

- The Cournot model explains how oligopoly firms behave if they ***simultaneously*** choose ***how much they produce***.
- Four main assumptions:
  1. There are two firms and no others can enter the market
  2. The firms have identical costs
  3. The firms sell identical products
  4. The firms set their quantities simultaneously

Cournot Model of an Airline Market

- Consider American Airlines and United Airlines
  - Assume airlines simultaneously choose output levels
- The ***Cournot equilibrium*** in this model is a set of quantities chosen by firms such that, holding quantities of other firms constant, no firm can obtain higher profit by choosing a different quantity.

Best Responses and Nash Equilibrium

- The concept of equilibrium in Cournot model is Nash equilibrium.
  - Given other players' strategies, each player maximizes his/her own payoff.
  - Therefore, no player gains by changing his/her own strategy.

Cournot Model of an Airline Market

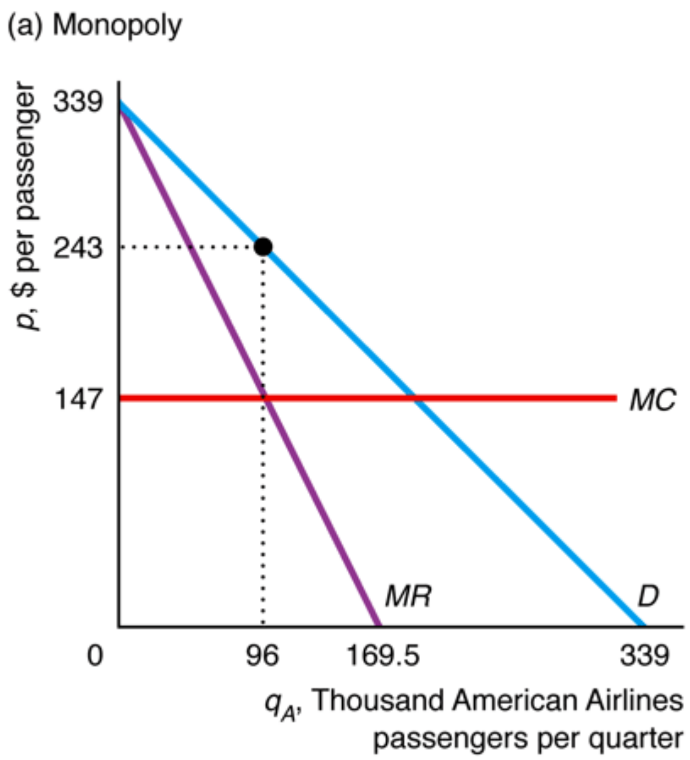
- Airline market demand:

$$Q = 339 - p$$

  - p = Dollar cost of one-way flight
  - Q = Total passengers flying one-way on both airlines (in thousands per quarter)
- Assume each airline has cost $MC = \$147$ per passenger

Cournot Model of an Airline Market

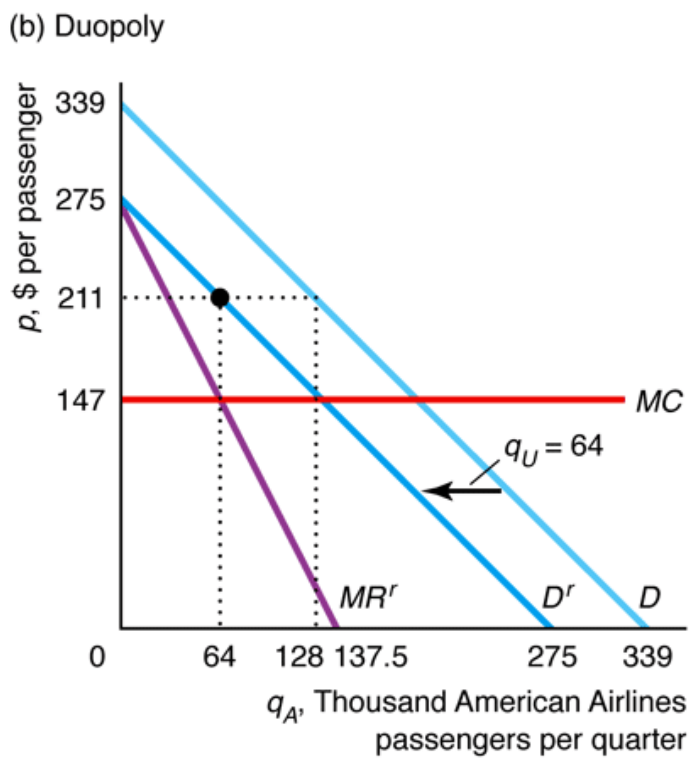
- Suppose American Airlines is the monopolist.



- ☐ Market Price: $243
- ☐ Market Quantity: 96
- ☐ AA's Profit: $\$(243 - 147)\times 96 - FC = \$9{,}216 - FC$

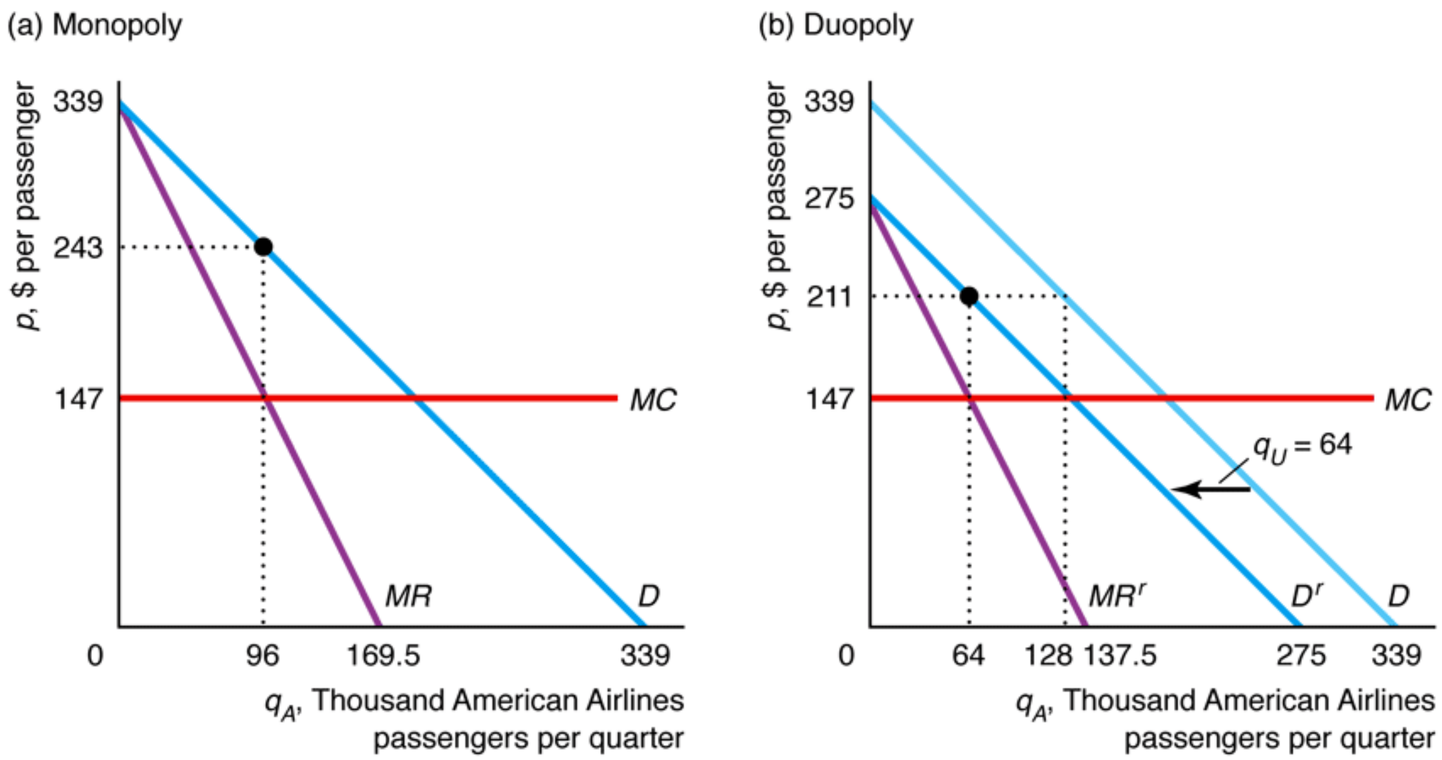
Cournot Model of an Airline Market

- Suppose United produces $q_U = 64$.
- American Airlines faces residual demand.
  - Price: $211
  - Quantity: 128
  - AA's Profit: $\$(211 - 147) \times 64 - FC = \$4{,}096 - FC$



Cournot Model of an Airline Market

- Comparison: Monopoly vs. Duopoly



Cournot Equilibrium: Math

- In duopoly, if United flies q_U passengers, American transports residual demand. American's residual demand:

$$q_A = Q(p) - q_U = (339 - p) - q_U$$

- What is American's best-response, profit-maximizing output if it believes United will fly q_U passengers?
  - American behaves as if it has a monopoly over people who don't fly on United (summarized by residual demand).

Cournot Equilibrium: Math

- Residual inverse demand function is useful for expressing revenue (and MR) in terms of rival's quantity.

$$p = 339 - q_A - q_U$$

$$\boxed{R^r(q_A)} = pq_A = (339 - q_A - q_U)q_A = 339q_A - (q_A)^2 - q_U q_A$$

Residual Revenue for AA

$$MR^r = \frac{dR^r(q_A)}{dq_A} = 339 - 2q_A - q_U$$

Cournot Equilibrium: Math

- Setting MR=MC yields American's best-response function:

MR= 339 -2q_A -q_U ; MC =147

$$\boxed{q_A = 96 - \frac{1}{2} q_U = B_A(q_U)}$$

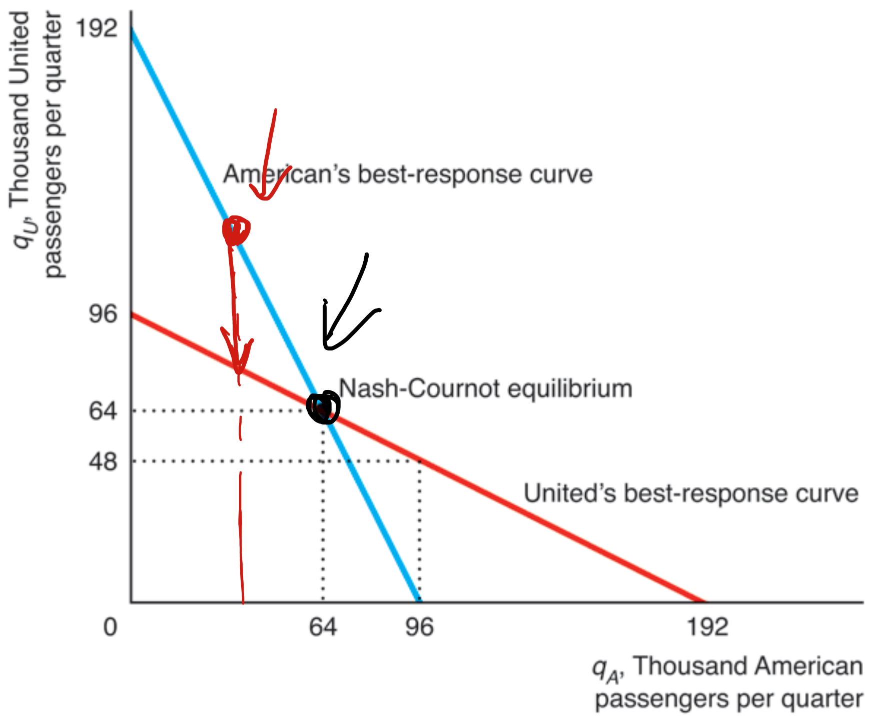
→ Best - response

- Given our assumptions, United's best-response function is analogous:

$$\boxed{q_U = 96 - \frac{1}{2} q_A = B_U(q_A)}$$

Cournot Equilibrium: Graph

- The Cournot equilibrium is the point where best-response functions intersect: $q_A = q_u = 64$



Cournot Equilibrium: Multiple Identical Firms

- With n firms, total market output is

$$Q = q_1 + q_2 + \ldots + q_n$$

- Firm 1 wants to maximize profit by choosing q_1:

$$\max_{q_1} \pi_1(q_1, q_2, \ldots, q_n) = q_1 p(\underbrace{q_1 + q_2 + \cdots + q_n}_{Q}) - C(q_1) = q_1 p(Q) - C(q_1)$$

$\to q_1 p(Q(q_1)) - C(q_1)$

- FOC when Firm 1 views the outputs of other firms as fixed:

$$\frac{\partial \pi}{\partial q_1} = \underbrace{p(Q) + q_1 \frac{dp(Q)}{dQ} \frac{\partial Q}{\partial q_1}}_{MR_1} - \underbrace{\frac{dC(q_1)}{dq_1}}_{MC_1} = 0$$

Cournot Equilibrium: Multiple Identical Firms

- Firm 1's best-response function found via $MR = MC$:

$$MR = p(Q) + q_1 \frac{\mathrm{d}p(Q)}{\mathrm{d}Q} = \frac{\mathrm{d}C(q_1)}{\mathrm{d}q_1} = MC$$

- Simultaneously solving for all firms' best-response functions yields Nash-Cournot equilibrium quantities, $q_1 = q_2 = \ldots = q_n = q$

Cournot Equilibrium: Multiple Identical Firms

- Liner demand: $p = a - b(q_1 + q_2 + \ldots + q_n)$
- Marginal cost: c
- Equilibrium output for each firm $q_i = q$:

$$q = \frac{a - c}{(n+1)b}$$

B/c firms are symmetric
→ In eq.: $q_1 = q_2 = \ldots = q_n = q$

$$q = \frac{a - b \cdot (n-1) \cdot q - c}{2b}$$

Firm 1's profit maximization

$$\max_{q_1} \pi_1 = q_1 \cdot [a - b(q_1 + q_2 + \ldots + q_n)] - c \cdot q_1$$

→ FOC: $[q_1]$ $\frac{\partial \pi_1}{\partial q_1} = 0$ → $[a - b(q_1 + q_2 + \ldots + q_n)] - b \cdot q_1 - c = 0$

→ $\boxed{q_1 = \frac{a - b \cdot \sum_{i=2}^{n} q_i - c}{2b}}$

Cournot Equilibrium: Multiple Identical Firms

- Equilibrium market output:

$$Q = nq = \frac{n(a-c)}{(n+1)b}$$

- Equilibrium market price is

$$p = \frac{a+nc}{(n+1)}$$

$p = a - bQ$

$= a - b \cdot \left[\frac{n(a-c)}{(n+1)b}\right]$

Cournot Equilibrium: Two Heterogeneous Firms (Cost)

- Linear demand: $p = a - b(q_I + q_A)$
- Marginal costs: c_I for Intel, $c_A > c_I$ for AMD.
- Best response functions:

$$q_I = \frac{a - c_I - bq_A}{2b}, q_A = \frac{a - c_A - bq_I}{2b}$$

- Equilibrium outputs are:

$$q_I = \frac{a - 2c_I + c_A}{3b} > q_A = \frac{a - 2c_A + c_I}{3b}$$

Firm I's problem:

$$\max_{q_I} \; q_I \cdot (a - b(q_I + q_A)) - c_I \cdot q_I$$

FOC:

$$a - 2b \cdot q_I - b \cdot q_A - c_I = 0$$

$$\max_{q_A} \pi_A$$

Cournot Equilibrium: Two Heterogeneous Firms (Cost)

- Equilibrium market output is

$$Q = q_I + q_A = \frac{2a - c_A - c_I}{3b}$$

- Equilibrium market price is

$$p = a - bQ = \frac{a + c_A + c_I}{3b}$$

Cournot Equilibrium: Two Heterogeneous Firms (Demand)

negative → substitutes

- Differentiated Products:

$$p_I = 490 - 10q_I - 6q_A$$
$$p_A = 197 - 15.1q_A - 0.3q_I$$

substitutes

$\max_{q_I} \pi_I$

- Identical MC's: $m = 40$

$\max_{q_A} \pi_A$

- Best response functions:

$$\boxed{q_I = \frac{450 - 6q_A}{20}}, \boxed{q_A = \frac{157 - 0.3q_I}{30.2}}$$

For Intel: $\max_{q_I} \pi_I = q_I \cdot (490 - 10q_I - 6q_A) - 40 \cdot q_I - FC$

→ FOC: $[q_I]$: $490 - 20q_I - 6q_A - 40 = 0$

→ $q_I = \frac{450 - 6q_A}{20}$

Cournot Equilibrium: Two Heterogeneous Firms (Demand)

- Equilibrium outputs are

$$q_I = 21 > q_A \approx 5$$

- Equilibrium prices per CPU are

$$p_I = \$250 > p_A = \$115.2$$

Comparison with Monopoly

- Assume identical marginal costs: $c_I = c_A = c$
- Linear demand: $\mathrm{p} = \mathrm{a} - b(q_I + q_A), a > c$
- Monopoly:

$$Q^M = \frac{a-c}{2b}, p^M = \frac{a+c}{2}$$

- Duopoly:

$$Q^D = \frac{2(a-c)}{3b} > Q^M$$

$$p^D = \frac{a+2c}{3} < p^M$$

Comparison with Perfect Competition

- Assume there are many identical firms with $MC = c$
- Linear demand: $\mathrm{p} = \mathrm{a} - b\sum q_j, a > c$
- Perfect competition:

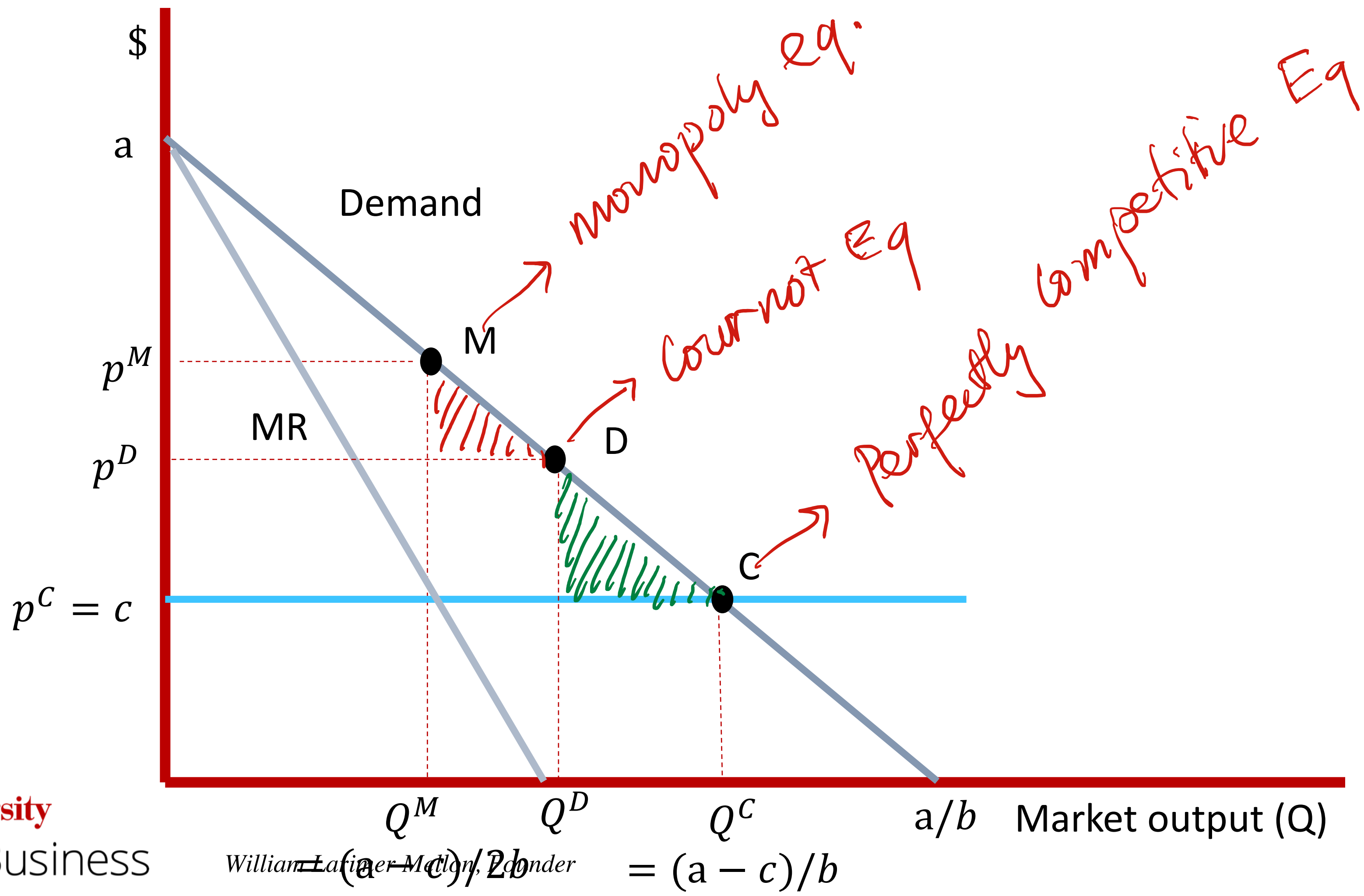
$$Q^C = \frac{a-c}{b}, p^C = MC = c$$

- Duopoly:

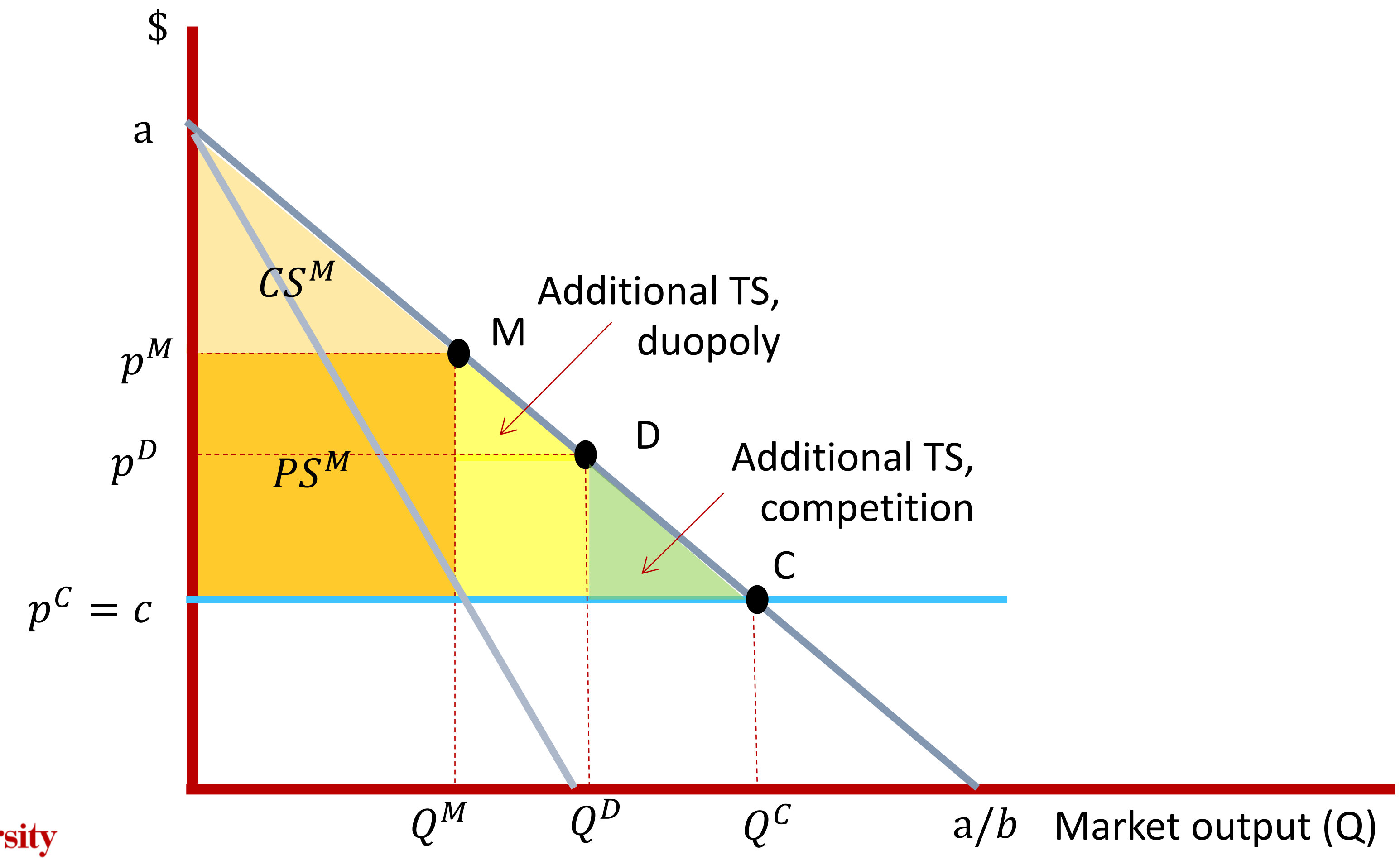
$$Q^D = \frac{2(a-c)}{3b} < Q^C$$

$$p^D = \frac{a+2c}{3} > p^C$$

Welfare Comparison



Welfare Comparison



Example

Consider two firms competing in a Cournot competition. The market demand is given by $\underbrace{P = 30 - 0.5Q.}_{\text{homogenous products}}$ The marginal cost of firm 1 is 4, and the marginal cost of firm 2 is 6.

(a) What are the equilibrium price and quantities?

(b) Compute the deadweight loss.

(c) If these firms were producing differentiated goods with the following demands :

$$P_1 = 20 - Q_1 - 0.5Q_2$$

$$P_2 = 20 - Q_2 - 0.5Q_1$$

What are the equilibrium prices and quantities?

Example: Answer

(a) $P = 30 - 0.5Q$; $MC_1 = 4$; $MC_2 = 6$

Consider firm 1's profit maximization problem:

$$\max_{q_1} \pi_1 = p \cdot q_1 - q_1 \cdot MC_1 - FC$$

$$= (30 - 0.5(q_1 + q_2))q_1 - 4q_1 - FC$$

$$\rightarrow \text{FOC: } [q_1]: \frac{\partial \pi_1}{\partial q_1} = 0 \rightarrow \underbrace{30 - q_1 - 0.5q_2}_{MR_1} - \underbrace{4}_{MC_1} = 0$$

$$\rightarrow \underbrace{q_1 = 26 - 0.5q_2}_{\text{Firm 1's best-response.}}$$

Example: Answer

Consider firm 2's profit maximization problem

$$\max_{q_2} \pi_2 = p \cdot q_2 - MC_2 \cdot q_2 - FC$$

$$= \left(30 - 0.5(q_1 + q_2)\right) \cdot q_2 - 6q_2 - FC$$

FOC: $[q_2]$: $\frac{\partial \pi_2}{\partial q_2} = 0 \rightarrow \underbrace{30 - 0.5q_1 - q_2}_{MR_2} \underbrace{-6}_{MC_2} = 0$

$\rightarrow q_2 = 24 - 0.5q_1 \rightarrow$ Firm 2's BR

Using firm 1's BR: $q_1 = 26 - 0.5 \cdot q_2$

$\rightarrow q_2 = 24 - 0.5(26 - 0.5 \cdot q_2) \Rightarrow q_2 = 11 + 1/4 \cdot q_2$

$\rightarrow 3/4 \, q_2 = 11 \rightarrow q_2 = 44/3$

3. Stackelberg Competition

Stackelberg Competition

- Suppose that one of the firms in our previous example was the ***leader*** and set its output before its rival, the ***follower***.
- Does the firm that acts first have an advantage?
- How does this model's outcome differ from the Cournot oligopoly model?
- Reading: pp. 950-980

Stackelberg Oligopoly Model

- Linear demand: $p = a - bQ$
- Two firms have identical marginal costs, c
- Firm 1 (American Airlines) is the Stackelberg leader and chooses output first
- Firm 2 (United Airlines) is the follower and chooses output using best-response function

Firm 1 chooses q_1

↓

Firm 2 chooses q_2 ⟶ Always use backward induction

↓

profits are realized

Stackelberg Oligopoly Model

- The Stackelberg leader knows the follower will use its best-response function and so the leader views the residual demand in the market as its demand.
  - The follower (United) chooses output as the best response to the leader’s (American) output
  - The leader (American) chooses output **anticipating** the follower’s (United) best response

Stackelberg Oligopoly: Math

- Follower's best response function:

$$q_U = B_U(q_A) = \frac{a - c - bq_A}{2b}$$

- Leader's profit function:

$$\pi_A = (a - b[q_A + B_U(q_A)] - c)q_A$$

- By taking FOC,

$$q_A = \frac{a - c}{2b}$$

$$\max_{q_2} \Pi_2 = q_2 \cdot (a - b \cdot (q_1 + q_2)) - c \cdot q_2 - FC$$

FOC$[q_2]$:

$$\frac{\partial \Pi_2}{\partial q_2} = \underbrace{a - 2b \cdot q_2 - b \cdot q_1}_{MR_2} \underbrace{- c}_{MC_2} = 0$$

$$\boxed{\rightarrow q_2 = \frac{a - c - bq_1}{2b}}$$

Leader's problem:

$$\max_{q_1} \Pi_1 = P \cdot q_1 - MC \cdot q_1 - FC$$

$$= (a - b \cdot q_1 - b \cdot q_2(q_1))q_1 - MC \cdot q_1 - FC$$

$$= \left(a - b \cdot q_1 - b \cdot \frac{a - c - bq_1}{2b}\right) \cdot q_1 - c \cdot q_1 - FC$$

FOC$[q_1]$: $a - 2bq_1 - \frac{a-c}{2} + b \cdot q_1 - c = 0$

Stackelberg Oligopoly: Math

- Plugging q_A into the follower's best response function:

$$\boxed{q_U = \frac{a-c}{4b}}$$

$$q_2 = \frac{a-c-b \cdot q_1}{2b}$$

$$q_1 = \frac{a-c}{2b}$$

- First mover's advantage:

$$q_U = \frac{a-c}{4b} < q_A = \frac{a-c}{2b}$$

$$\pi_U < \pi_A$$

$$q_2 = \frac{a-c}{2b} - \frac{a-c}{4b}$$

$$= \frac{a-c}{4b}$$

Comparison of Market Outcomes

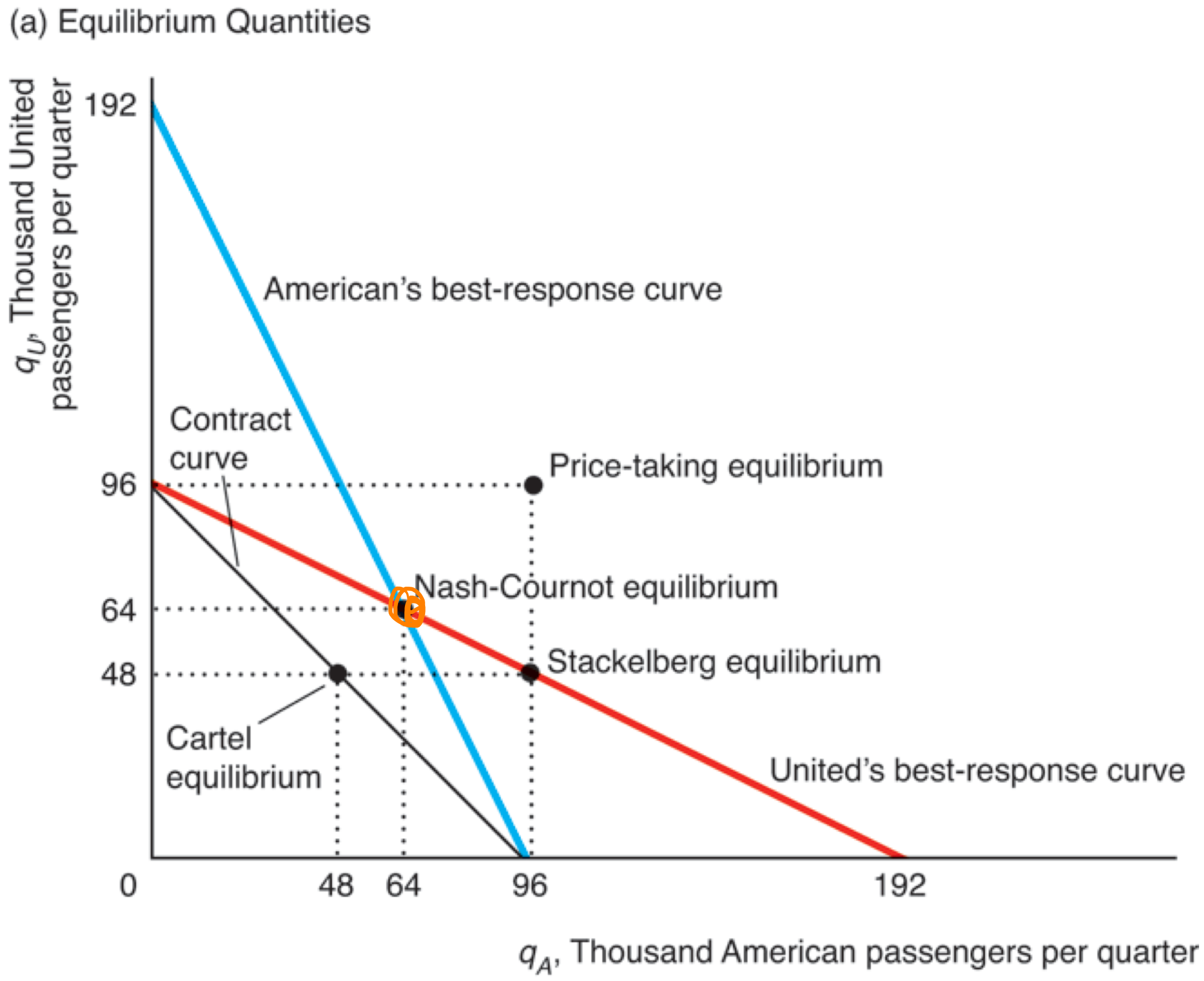
- Example:
  - Airline market demand:

$$Q = 339 - p$$

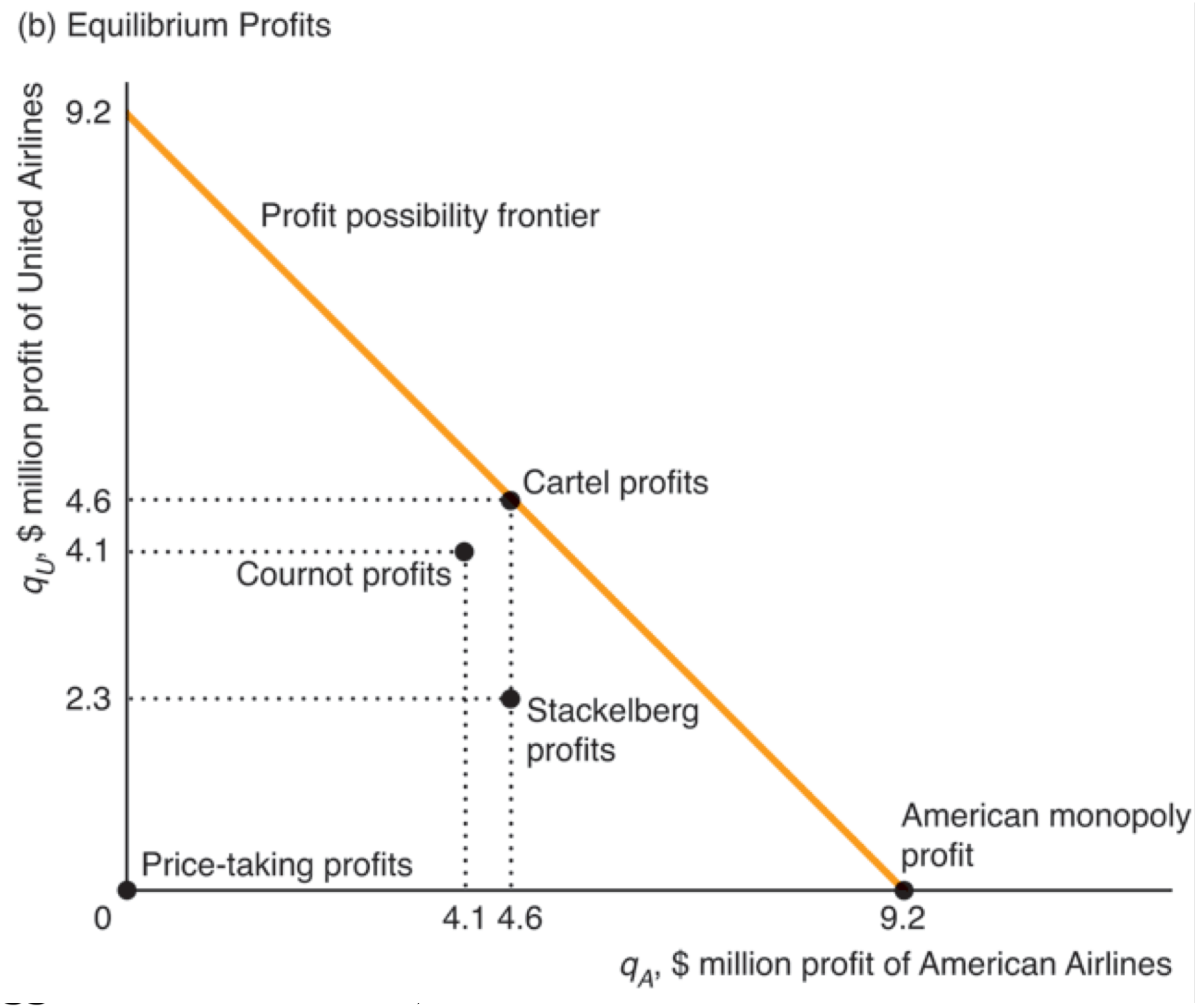
  - Identical $MC = \$147$ per passenger
- Cournot and Stackelberg equilibrium outcomes lie between competition and collusion.

2 firms merge together into a monopolist.

Comparison of Market Outcomes



Comparison of Market Outcomes



Example

Consider two firms competing in a Stackelberg competition. Firm 1 is the first mover. The market demand is given by $P = 30 – 0.5Q$. The marginal cost of firm 1 is 4, and the marginal cost of firm 2 is 6.

(a) What are the equilibrium price and quantities?
(b) Compute each firm's profit. Compared to the Cournot case, which firm earns higher profit?
(c) Compute the deadweight loss.

Example: Answer

(a) $P = 30 - 0.5Q$; $MC_1 = 4$; $MC_2 = 6$

- Consider firm 2's profit maximization problem:

move last

$$\max_{q_2} \pi_2 = (30 - 0.5 q_1 - 0.5 q_2) q_2 - 6 q_2 - FC$$

$$\text{FOC: } [q_2]: \frac{\partial \pi_2}{\partial q_2} = 0 \rightarrow 30 - 0.5 q_1 - q_2 - 6 = 0$$

$$\rightarrow q_2 = 24 - 0.5 q_1$$

- Consider firm 1's profit maximization problem:

$$\max_{q_1} \pi_1 = (30 - 0.5 q_1 - 0.5 (24 - 0.5 q_1)) \cdot q_1 - 4 q_1 - FC$$

$$\text{FOC: } [q_1]: \frac{\partial \pi_1}{\partial q_1} = 0 \rightarrow 30 - q_1 - 12 + 0.5 q_1 - 4 = 0$$

$$\rightarrow 14 = 0.5 q_1 \rightarrow q_1 = 28 ; q_2 = 10$$

Example: Answer

(b) Comparing to Cournot outcome : $q_2 = 44/3$

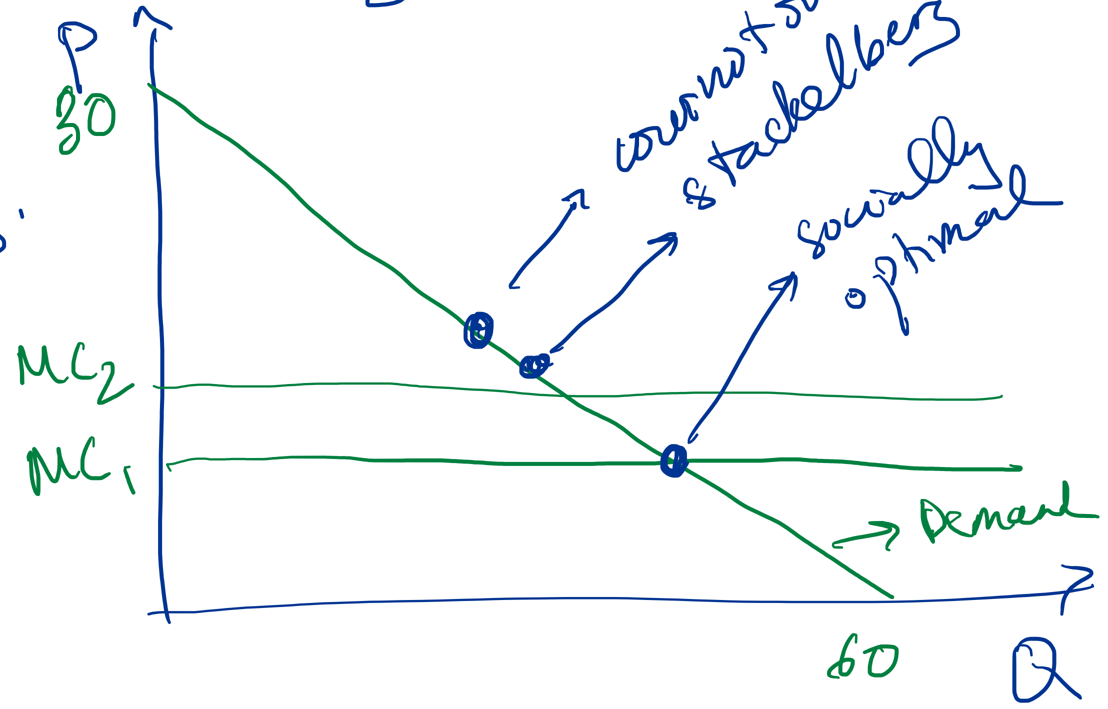
$q_1 = \boxed{?}$

→ Firm 2 produces less under Stackelberg

Firm 1 produces <u>more</u> under Stackelberg.

→ $\pi_2^{\text{Cournot}} > \pi_2^{\text{Stackelberg}}$

$\pi_1^{\text{Cournot}} < \pi_1^{\text{Stackelberg}}$.



4. Bertrand Competition

Bertrand Competition

- What if, instead of setting quantities, firms set prices and allowed consumers to decide how much to buy?
- A **Bertrand equilibrium** is a set of prices such that no firm can obtain a higher profit by choosing a different price if the other firms continue to charge these prices. [Also Nash Equilibrium]
- Understand how the Bertrand equilibrium differs from a quantity-setting equilibrium.
- Reading: pp. 950-980

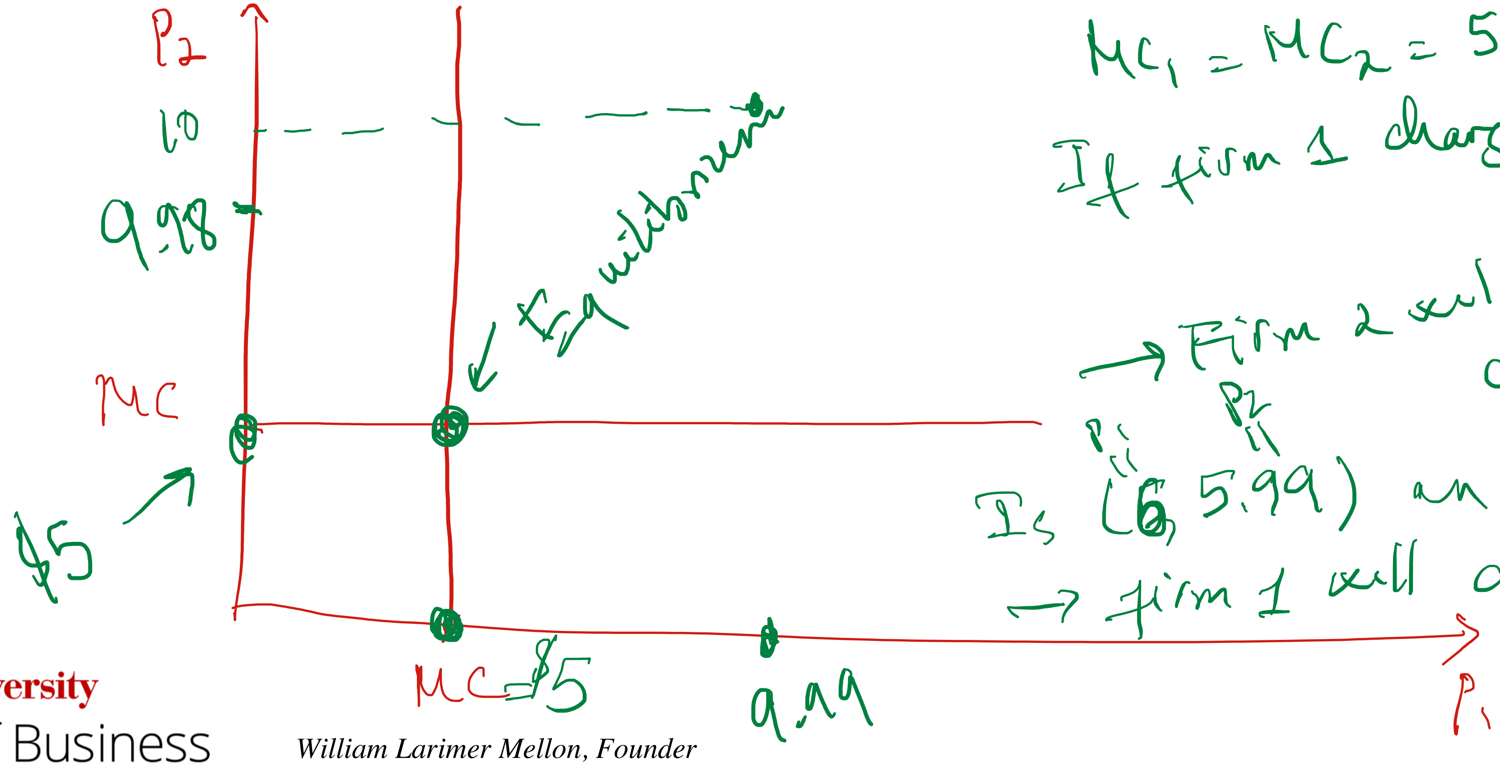
Bertrand Oligopoly Model

- Assumptions:
  - Firms have identical costs (and constant *MC*=\$5)
  - Firms produce identical goods
- Conditional on the price charged by Firm 2, p_2, Firm 1 wants to charge slightly less in order to attract customers.
- If Firm 1 undercuts its rival's price, Firm 1 captures entire market and earns all profit.

Bertrand Oligopoly Equilibrium

- Note that Firm 2 also has incentive to undercut Firm 1's price.
- Therefore, ***Bertrand equilibrium price equals marginal cost*** when the firms are identical, because of incentive to undercut.

$(p_1 = 5, p_2 = 5)$
where 5 is MC



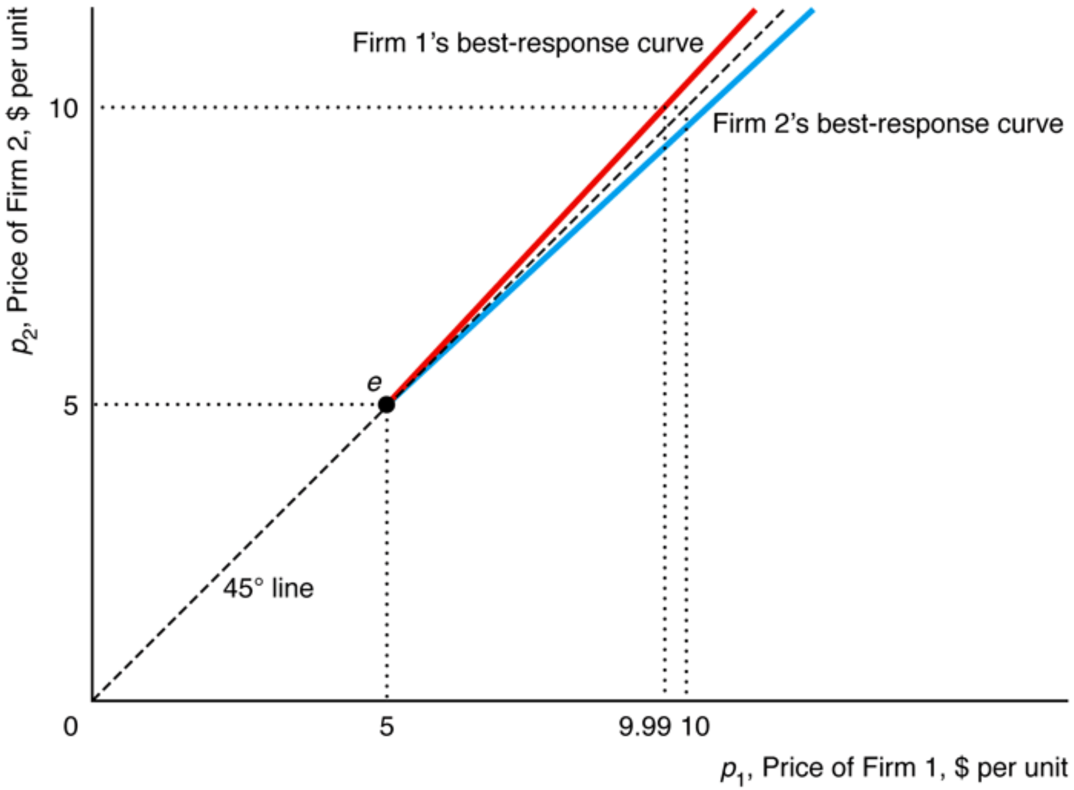
$MC_1 = MC_2 = 5$

If firm 1 charges $p_1 = \$6$
$p_2 = \$7$
→ Firm 2 will deviate and charge $p_2 = 5.99$

Is $(6, 5.99)$ an equilibrium? (p_1, p_2)
→ firm 1 will deviate and set $p_1 = 5.98$

Bertrand Oligopoly Equilibrium

- Bertrand equilibrium price equals marginal cost because of incentive to undercut.



Duopoly with Identical Products & MC

- Cournot competition:
  1. $P > MC$
  2. $q_1 = q_2$
- Bertrand competition:
  1. $P = MC$
  2. $q_1 = q_2$
- Stackelberg competition:
  1. $P > MC$
  2. $q_1 > q_2$ if firm 1 is the leader

Example: REAL LIFE EXAMPLE!

https://www.nytimes.com/2012/03/31/nyregion/in-manhattan-pizza-war-price-of-slice-keeps-dropping.html

Summary:

1. There are two pizza stores literally next to each other.

How low can they go in the price war on Avenue of the Americas between 37th and 38th? "We might go to free pizza soon," one combatant said. Hiroko Masuike/The New York Times

Example: REAL LIFE EXAMPLE!

https://www.nytimes.com/2012/03/31/nyregion/in-manhattan-pizza-war-price-of-slice-keeps-dropping.html

Summary:

2. 6 Avenue Pizza has been selling Pizza at 1$ per slice.

3. Then 2 Bros. Pizza slashes price to 75c per slice.

4. 6 Ave. Pizza has to cut its price to 75c as well to be competitive.

"I'm thinking, God help me," Mr. Patel said.
They said that 2 Bros. was trying to drive them out of business, that 2 Bros., unprovoked, slashed the price to 75 cents, forcing them to follow, that things were miserable, that Ramanlal Patel has serious kidney problems, that property in India had to be sold to keep the place going.
"We're angry," Bravin Patel said.
Depicting the battle as "small guy" (Bombay) against "big guy" (2 Bros.), Mr. Patel said: "He comes in and he thinks he's king."

Example: REAL LIFE EXAMPLE!

https://www.nytimes.com/2012/03/31/nyregion/in-manhattan-pizza-war-price-of-slice-keeps-dropping.html

Summary:

This is 2 Bros' version of the event:

On Thursday evening a week ago, Bombay/6 Ave. — unprovoked, and without warning — cut its pizza price to 79 cents. The next morning, 2 Bros. retaliated by moving to 75 cents (its owners felt it was easier to make change from a dollar than at 79 cents). Bombay/6 Ave. matched the 75 cents, and that's where everything sits.
"We don't sell pizza at 75 cents," Eli Halali said. "But if they think they're going to sit next to us and sell at 75 cents, they've got another think coming."

Example: REAL LIFE EXAMPLE!

https://www.nytimes.com/2012/03/31/nyregion/in-manhattan-pizza-war-price-of-slice-keeps-dropping.html

NY times follow-up:
https://www.nytimes.com/2012/09/06/nyregion/two-manhattan-pizza-parlors-end-price-war.html

But after several months and increasingly long lines, the proprietors of two pizza parlors on Avenue of the Americas have agreed to end a price war that saw the cost of their slices plummet to 75 cents each.

…Mr. Patel and 2 Bros. employees recalled as a critical sidewalk meeting between the two proprietors in recent weeks.
Mr. Patel said the negotiation was brief. "I said, 'We lose money, and the customer wins,' " he recalled. "He said, 'You're right.' "

collusion

Example: REAL LIFE EXAMPLE!

According to legal experts, a premeditated, simultaneous price change almost certainly qualifies as collusion — quaint, perhaps, but still technically illegal under antitrust laws.

"Any agreement to fix prices is illegal per se," Harry First, a law professor at New York University and an antitrust expert, said. The action could violate not only state law, but also federal law if it involved interstate commerce. "Who knows?" Professor First said. "Maybe the cheese is from out of state."

The office of the New York State attorney general, Eric T. Schneiderman, declined to comment on whether the price change was legal or whether it would intervene.

Takeaway: If you are interested in these strategic interactions between firms, do more economics!
The questions economists look at go beyond Pizza price wars!

Example: Back to math

Bertrand competition

There are only two gas stations in Pittsburgh, and they compete by simultaneously choosing prices. The demand for gas is given by P = 4 – Q.

(a) If both stations have the same marginal cost, MC = 2. What are the equilibrium prices and quantity?

(b) Assume now that one station has a lower marginal cost, MC = 1. What are the equilibrium prices and quantity?

(a): Eq.: $p_1 = p_2 = MC = \$2$

$\rightarrow Q = 4 - P = 2$

$\rightarrow q_1 = q_2 = \frac{Q}{2} = 1$

No DWL

Example: Answer

(b) $MC_1 = 1$ $[p_1 = MC_2 - \varepsilon,\ p_2 = MC_2]$ Equilibrium

$MC_2 = 2$ $P = 4 - Q$

- Is $(p_1 = \$3,\ p_2 = \$3)$ an equilibrium? No: both firms will deviate

- Is $(p_1 = \$1,\ p_2 = \$1)$ an equilibrium?
  $p_1 = \$1 \rightarrow \pi_1 = 0$; $p_2 = \$1 \rightarrow \pi_2 < 0$ b/c $1 < MC_2$ → firm 2 will deviate to $p_2 > 1$

- Is $\boxed{(p_1 = \$1.99,\ p_2 = \$2)}$ an equilibrium? Yes
  $p_1 = \$1.99 \rightarrow$ Firm 1 is selling everything, firm 2 is selling nothing; $\pi_2 = 0$ → Firm 2 doesn't want to deviate