· This lecture is purely theoretical. I don't want to deter from reading & enjoying it yourself. Thus, the notes will be stopped here.

$\boxed{\text{Lecture 10}}$

TSM competition derivation

$$U_i^1 = a_i n_j^1 - p_i^1 - \theta t = \delta_i^1 - \theta t$$

Indifferent customer of type i

$$\delta_i^1 - \theta t = \delta_i^2 - (1-\theta)t$$

$$(1-\theta)t - \theta t = \delta_i^2 - \delta_i^1$$

$$-2\theta t = \delta_i^2 - \delta_i^1 - t$$

$$\theta^* = \frac{t - \delta_i^2 + \delta_i^1}{2t}$$

$$n_i^1 = D_i^1 = \{\theta : \theta \le \theta^*\} = \theta^* \qquad D_i^2 = 1 - \theta^* = n_i^2$$

$$n_A^1 = \frac{t - (a_A n_B^2 - p_A^2) + (a_A n_B^1 - p_A^1)}{2t} =$$

$$= \frac{t - \alpha_A(1-n_B^1) + p_A^2 + \alpha_A n_B^1 - p_A^1}{2t} =$$

$$= \frac{1}{2} + \frac{\alpha_A(n_B^1 - 1 + n_B^1) - (p_A^1 - p_A^2)}{2t} =$$

$$= \frac{1}{2} + \frac{\alpha_A(2n_B^1 - 1) - (p_A^1 - p_A^2)}{2t}$$

$$\Pi_1 = D_A^1 p_A^1 - D_A^1 C_A^1 + D_B^1 p_B^1 - D_B^1 C_B^1 =$$

$$= D_A^1(p_A^1 - C_A^1) + D_B^1(p_B^1 - C_B^1) =$$

$$= \cancel{\left[\frac{1}{2} + \alpha_A(2\right.}$$

$$n_A^1 = \frac{1}{2} + \frac{\alpha_A(2n_B^1 - 1) - (p_A^1 - p_A^2)}{2t}$$

$$n_B^1 = \frac{1}{2} + \frac{\alpha_B(2n_A^1 - 1) - (p_B^1 - p_B^2)}{2t} =$$

$$= \frac{1}{2} - \frac{(p_B^1 - p_B^2)}{2t} + \frac{\alpha_B}{2t}\left[1 + \frac{\alpha_A(2n_B^1 - 1) - (p_A^1 - p_A^2)}{t} - 1\right]$$

$$= \frac{1}{2} - \frac{p_B^1 - p_B^2}{2t} + \frac{\alpha_B}{2t} \cdot \frac{\alpha_A(2n_B^1 - 1) - (p_A^1 - p_A^2)}{t}$$

$$= \frac{1}{2} - \frac{p_B^1 - p_B^2}{2t} + \frac{\alpha_B \alpha_A \cdot n_B^1}{t^2} + \frac{\alpha_B}{2t}\left[\frac{-\alpha_A - p_A^1 + p_A^2}{t}\right]$$

$$n_B^1 \cdot \left(1 - \frac{\alpha_A \alpha_B}{t^2}\right) = \frac{1}{2} - \frac{p_B^1 - p_B^2}{2t} + \frac{\alpha_B}{2t}\left[\frac{-\alpha_A - p_A^1 + p_A^2}{t}\right]$$

$$n_B^1 \cdot \frac{t^2 - \alpha_A \alpha_B}{t^2} = \frac{1}{2} - \frac{p_B^1 - p_B^2}{2t} + \frac{-\alpha_A \alpha_B - \alpha_B p_A^1 + \alpha_B p_A^2}{2t^2}$$

$$n_B^1 = \frac{t^2}{2(t^2 - \alpha_A \alpha_B)} - \frac{t(p_B^1 - p_B^2)}{2(t^2 - \alpha_A \alpha_B)} + \frac{-\alpha_A \alpha_B - \alpha_B p_A^1 + \alpha_B p_A^2}{2(t^2 - \alpha_A \alpha_B)}$$

$$n_B^1 = \frac{t^2 - t(p_B^1 - p_B^2) - \alpha_A \alpha_B - \alpha_B (p_A^1 - p_A^2)}{2(t^2 - \alpha_A \alpha_B)}$$

$$n_A^1 = \frac{t^2 - t(p_A^1 - p_A^2) - \alpha_A \alpha_B - \alpha_A (p_B^1 - p_B^2)}{2(t^2 - \alpha_A \alpha_B)}$$

$$\pi^1 = n_A^1(p_A^1 - c_A^1) + n_B^1(p_B^1 - c_B^1)$$

$$\frac{d\pi^1}{dp_A^1} = \frac{dn_A^1}{dp_A^1}(p_A^1 - c_A^1) + n_A^1 + \frac{dn_B^1}{dp_A^1}(p_B^1 - c_B^1)$$

$$= \frac{-t(p_A^1 - c_A^1)}{2(t^2 - \alpha_A\alpha_B)} + \frac{t^2 - t(p_A^1 - p_A^2) - \alpha_A\alpha_B - \alpha_A(p_B^1 - p_B^2)}{2(t^2 - \alpha_A\alpha_B)}$$

$$+ \frac{-\alpha_B(p_B^1 - c_B^1)}{2(t^2 - \alpha_A\alpha_B)} = 0$$

So, $-t(p_A^1 - c_A^1) + t^2 - t(p_A^1 - p_A^2) - \alpha_A\alpha_B - \alpha_A(p_B^1 - p_B^2)$
$-\alpha_B(p_B^1 - c_B^1) = 0$

And using $\frac{d\pi^1}{dp_B^1}$, we would have

$-t(p_B^1 - c_B^1) + t^2 - t(p_B^1 - p_B^2) - \alpha_A\alpha_B - \alpha_B(p_A^1 - p_A^2)$
$-\alpha_A(p_A^1 - c_A^1) = 0$

By symmetry, $p_A^1 = p_A^2$ and $p_B^1 = p_B^2$.

So, $-t(p_A - c_A) + t^2 - \alpha_A\alpha_B - \alpha_B(p_B^1 - c_B^1) = 0$

$$+t(p_A - c_A) = t^2 - \alpha_A\alpha_B - \alpha_B(p'_B - c'_B)$$

$$p_A - c_A = t - \frac{\alpha_B}{t}(\alpha_A + p_B - c_B)$$

$$p_A = c_A + t - \frac{\alpha_B}{t}(\alpha_A + p_B - c_B)$$

$$p_B = c_B + t - \frac{\alpha_A}{t}(\alpha_B + p_A - c_A)$$

$$p_A = c_A + t - \frac{\alpha_B}{t}(\alpha_A - c_B) - \frac{\alpha_B}{t}p_B =$$

$$= c_A + t - \frac{\alpha_B}{t}\cdot\left[c_B + t - \frac{\alpha_A}{t}(\alpha_B + p_A - c_A\right] =$$

$$= c_A + t - \frac{\alpha_B}{t}(\alpha_A - c_B) - \frac{\alpha_B}{t}\left[c_B + t - \frac{\alpha_A}{t}p_A\right.$$

$$\left. - \frac{\alpha_A}{t}(\alpha_B - c_A)\right] = c_A + t - \frac{\alpha_B\alpha_A}{t} + \cancel{\frac{\alpha_B c_B}{t}}$$

$$- \cancel{\frac{\alpha_B c_B}{t}} - \frac{\alpha_B \cdot \cancel{t}}{\cancel{t}} + \frac{\alpha_B\alpha_A}{t^2}p_A + \frac{\alpha_A\alpha_B}{t^2}(\alpha_B - c_A) =$$

$$= c_A + t - \frac{\alpha_B\alpha_A}{t} - \alpha_B + \frac{\alpha_B\alpha_A}{t^2}p_A + \frac{\alpha_A\alpha_B}{t^2}(\alpha_B - c_A)$$

$$P_A\left(1-\frac{\alpha_A\alpha_B}{t^2}\right)=C_A+t-\frac{\alpha_A\alpha_B}{t}-\alpha_B+\frac{\alpha_A\alpha_B}{t^2}(\alpha_B-C_A)$$

$$P_A=\frac{t^2}{t^2-\alpha_A\alpha_B}\left[C_A+t-\frac{\alpha_A\alpha_B}{t}-\alpha_B+\frac{\alpha_A\alpha_B}{t^2}(\alpha_B-C_A)\right]$$

$$=\frac{t^2}{t^2-\alpha_A\alpha_B}\left[C_A-\alpha_B-\frac{\alpha_A\alpha_B}{t^2}(C_A-\alpha_B)+\frac{t^2-\alpha_A\alpha_B}{t}\right]=$$

$$=\frac{t^2}{t^2-\alpha_A\alpha_B}\left[(C_A-\alpha_B)\left[1-\frac{\alpha_A\alpha_B}{t^2}\right]+\frac{t^2-\alpha_A\alpha_B}{t}\right]=$$

$$=C_A-\alpha_B+t$$

$P_A=C_A-\alpha_B+t$ So, $P_B=C_B-\alpha_A+t$