PRINCIPLES OF FINANCE

WEEK 7

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Last lecture

Do you remember

- What financial derivatives are
- How they can be used for hedging or speculating
- How to value them?

What else do we need to know about risk and return?

- We have learned how to estimate the discount rate for a risky project when it is fully funded with <u>equity</u> contracts
  - CAPM formula
- In practice, most firms are funded with a mix of <u>debt</u> and <u>equity</u>
- We have seen that when this is the case, the discount rate is the WACC, which requires knowledge of the cost of debt
- How can one estimate the cost of debt?

Outline of today's lecture

In class

- The cost of debt-funded capital
- The impact of the choice of capital structure on the cost of capital and firm value (Modigliani Miller)

Recording: Deviations from Modigliani Miller assumptions

Application: Quick introduction to Private Equity

Cost of debt-funded capital

Debt cost of capital

- Debt cost of capital = expected return on the debt issued by the firm
- Debt yield
  - Yield to maturity is the IRR an investor will earn from holding the bond *to maturity* and *receiving all its promised payments*.
  - If there is little risk the firm will default: yield to maturity reasonable estimate of investors' expected rate of return
  - If there is significant risk of default: yield to maturity will *overstate* investors' expected return.
- Interest rate charged on existing debt has similar interpretation as the YTM

Relationship: Yield and expected return

Keywords:
Probability of default
Expected loss rate

- Consider a one-year bond with YTM of *y*. For each \$1 invested in the bond today, the issuer promises to pay \$(1+*y*) in one year.

- Suppose the bond will default with probability *p*, then bond holders receive \$(1+*y*-*L*)
  - *L* is the expected loss per \$1 of debt in the event of default = expected loss rate

- So the expected return of the bond is:

$$r_d = (1-p)y + p(y-L) = y - pL$$

$$= \textit{Yield to Maturity} - \textit{Prob(default)} * \textit{Expected Loss Rate}$$

- Importance of the adjustment (*p***L*) depends on the default risk of the bond

Annual default rates by debt rating (1983–2011)

Rating:	AAA	AA	A	BBB	BB	B	CCC	CC-C
Default Rate:								
Average	0.0%	0.1%	0.2%	0.5%	2.2%	5.5%	12.2%	14.1%
In Recessions	0.0%	1.0%	3.0%	3.0%	8.0%	16.0%	48.0%	79.0%

Source: "Corporate Defaults and Recovery Rates, 1920–2011," *Moody's Global Credit Policy*, February 2012.

- The average loss rate for unsecured (i.e. not guaranteed by a specific asset in the company) debt is 60%.
- According to the table above, during average times the annual default rate for B-rated bonds is 5.5%.
- So, for example, the expected return to B-rated bondholders during average times is 0.055*0.60=3.3% below the bond's quoted yield.

Keywords:
Credit spread /
Default spread

Debt betas

- Just as in the case of the equity cost of capital, you can calculate the debt cost of capital using the CAPM formula if you know the debt beta

$$r_D = r_f + Risk\ Premium = r_f + \beta_D(E[r_M] - r_f)$$

  - Risk premium for bonds: credit spread or default spread

- Debt betas for individual bonds are difficult to estimate because corporate bonds are traded infrequently and over a limited duration

- One approximation is to use estimates of betas of bond indices by rating category.

Debt betas

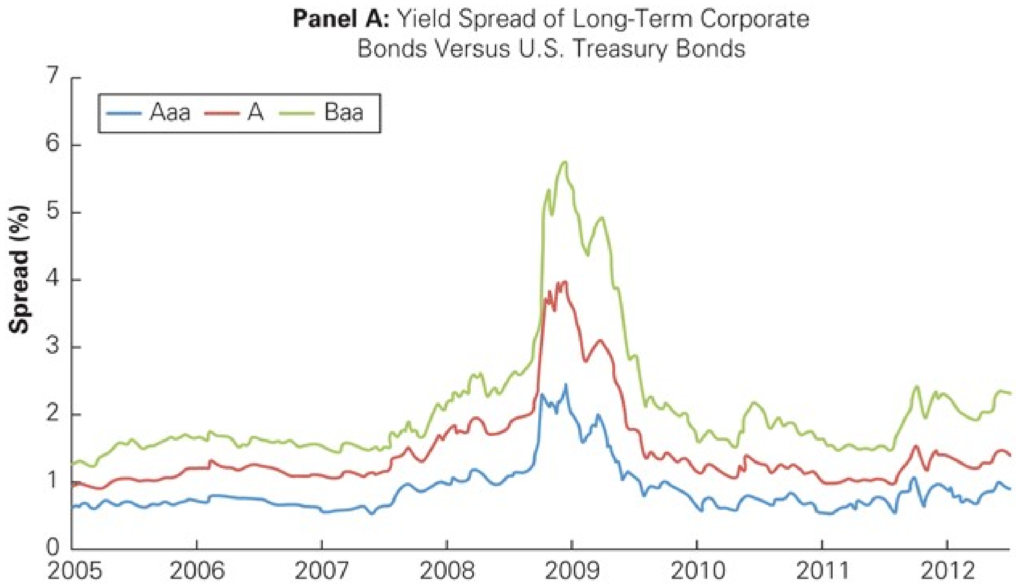
- Average debt Betas by rating and maturity:

By Rating	*A and above*	*BBB*	*BB*	*B*	*CCC*
Avg. Beta	< 0.05	0.10	0.17	0.26	0.31
By Maturity	(BBB and above)	*1–5 Year*	*5–10 Year*	*10–15 Year*	*> 15 Year*
Avg. Beta		0.01	0.06	0.07	0.14

Source: S. Schaefer and I. Strebulaev, "Risk in Capital Structure Arbitrage," Stanford GSB working paper, 2009.

Cost of debt

- If we don't know the debt beta but the firm has a bond rating from an established credit rating agency (e.g., S&P or Moody's)



- We can obtain an approximate credit spread based on the rating.
- In 2012, for instance, the default spread for BBB rated bonds was ~2.2% and could have been used as the spread for a BBB rated company in that year
- To get the cost of debt, add the credit spread to the risk-free rate

Cost of debt: example

Problem

- In early 2013, auto parts retailer Autozone had outstanding 10-year bonds
  - yield to maturity = 3%
  - BBB rating
- Risk-free rate = 1.5%
- Market risk premium = 8%
- Estimate the expected return of Autozone's debt!

Cost of debt: example

Solution

1. Average default rate for BBB debt from table above: 0.5%; expected loss rate: 60%

   $r_d = 3\% - 0.5\%(0.60) = 2.7\%$

2. Use CAPM: BBB Beta from table above = 0.10

   $r_d = 1.5\% + 0.10(8\%) = 2.3\%$

Both estimates are rough approximations and they both suggest that the expected return of Autozone's debt is below its yield-to-maturity of 3%.

Cost of equity versus cost of debt

- A company's cost of equity is 13% (r_E=13%) and its cost of debt is 10% (r_D = YTM = 10%, assuming the firm is not close to default).

- Does this mean that it is cheaper for this company to finance its projects with debt rather than equity?

- In the event the company is financing its project with both debt and equity, should it issue more debt and buy back some shares?

The Security Market Line again!

- The answer is : not necessarily!
- Basic idea is that equity is more risky than debt.
- This is why, the return on equity should be higher than that on debt
- So long as both the return on equity and the return on debt are on the SML, the firm should be indifferent to issuing debt versus equity.
- In other words: if investors are getting a “fair” return according to the SML, the firm should not care whether it issues debt or equity

Example

- An entrepreneur wants to sell *CapitalBudgeter*, an app to help business with capital budgeting.
- Project cash flows under two (equally-likely) scenarios

Date 0	Date 1	
	Strong Economy	Weak Economy
−$800	$1400	$900

- Expected cash flow in one year is:

$$\frac{1}{2}(\$1400) + \frac{1}{2}(\$900) = \$1150$$

Example

- Suppose the project is financed with equity only
- Suppose that equity holders are willing to pay $1000 for the project (value of equity)
  - They are willing to pay more than the project requires, because the entrepreneur must be rewarded for his idea
- This implies:
  - Expected return for equity holders: $\frac{\$1150}{\$1000} - 1 = 15\%$
  - NPV: $-\$800 + \frac{\$1150}{(1+15\%)} = \$200$

Cost of capital

Example

- Project cash flows and returns to equity holders

	Date 0	Date 1: Cash Flows		Date 1: Returns	
	Initial Value	Strong Economy	Weak Economy	Strong Economy	Weak Economy
Unlevered equity	\$1000	\$1400	\$900	40%	−10%

- Expected returns on the "**unlevered**" equity is

$$\frac{1}{2}(40\%) + \frac{1}{2}(-10\%) = 15\%$$

- Value of the "unlevered" firm is

$$V_U = PV(Unlevered\ Equity\ Cash\ Flow) = \frac{\$1150}{(1 + 15\%)} = \$1000$$

''Unlevered'' means that there is no debt used to finance the project.

Example

- Now consider that the entrepreneur raises $500 of the initial capital using debt, in addition to selling equity. Because the project's cash flow is enough to repay the debt, the debt is risk free. Then at date 1 the debt becomes 500x1.05 = 525.
  - Equity in a firm that also has debt outstanding is called **levered equity**. In this case, what the shareholders receive is

	Date 0	Date 1: Cash Flows	
	Initial Value	Strong Economy	Weak Economy
Debt	$500	$525	$525
Levered equity	$E = ?$	$875	$375
Firm		$1400	$900

Value of equity = value of the firm – value of debt

  - What price *E* should the levered equity sell for?
  - What is the firm value?

Keywords:
Levered / unlevered

The capital structure question

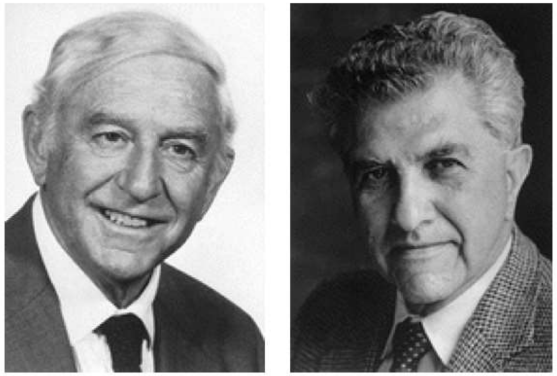
- Capital structure = choice of the mix between debt and equity contracts when funding a project
- Does capital structure affect firm value?
  - Can firm add value to equity and debt holders through decisions on the relative proportions of debt and equity?
- What determines capital structure choices of firms?
- Is there an "optimal" capital structure?

Keywords:
Capital structure

Capital Structure in Perfect Capital Markets

M&M Theorem

Franco Modigliani
Nobel Prize 1985



Merton Miller
Nobel Prize 1990

- Paper published by Modigliani and Miller in the American Economic Review (1958), “The Cost of Capital, Corporate Finance, and the Theory of Investment”
- **Punchline:** In perfect capital markets, a firm’s value is independent of its financial policy
- In other words, **the total value of a firm should not depend on its capital structure.**
- First steps towards understanding capital structure
- Birth of “modern” corporate finance

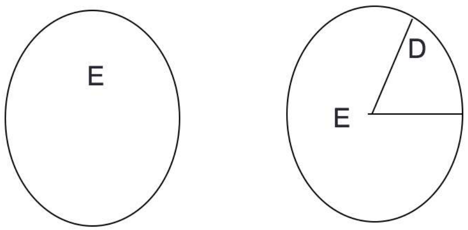
Assumptions behind MM’s theorem

1. Markets are complete: ability to buy, sell and borrow any security at current prices
2. Markets are efficient: securities trade at prices equal to the present values of their cash flows
    - No frictions (transaction costs, bankruptcy cost, etc.)
    - No asymmetric information
    - No moral hazard
3. Projects’ free cash flows are unaffected by capital structure (no taxes)
    - This is the strongest assumption. We will relax it a bit later

MM propositions (no tax)

Proposition 1: The total value of a firm is equal to the present value of the free cash-flows generated by its assets, and it is not affected by the choice of capital structure

- The size of the "pie" does not depend on how the pie is divided



V= E + D

The leverage of a firm does not matter:
$V_{Levered} = V_{Unlevered}$

Back to our example - Capital structure irrelevance – Financing with equity and debt

- Project cash flows for debt and equity holders are

	Date 0	Date 1: Cash Flows	
	Initial Value	Economy strong	Economy weak
Debt (D)	500	525	525
Levered Equity, E	?	875	375
Firm Value, V^L	?	1 400	900

- How much (levered) equity will the entrepreneur be able to raise?
- What is the total firm value of the levered firm?

Capital structure irrelevance – Financing with equity and debt

- What is the total firm value of the levered firm (V_L)?
  - Since the cash flow of the debt and equity sum to the cash flows of the project and financing does not change the project cash flow, by the Law of One Price:

$$V_L = V_U = \$1000$$

- How much (levered) equity will the entrepreneur be able to raise?
  - Since the value of the debt is \$500, the value of the levered equity must be

$$E = V_L - D = \$1000 - \$500 = \$500$$

Arbitrage opportunity if MM proposition 1 is false

- Suppose two companies have the same cash flow
  - $Cash\ Flow_{Strong} = \$1400, Cash\ Flow_{Weak} = \$900$
- Assume MM proposition I is violated, i.e.
  - Company U: $V_U = \$1000$
  - Company L: $\mathrm{D} = \$500, E = \$400, V_L = \$900; r_D = 5\%$
- Is there an arbitrage opportunity?

Arbitrage opportunity if MM proposition 1 is false

- $V_L < V_U$ $\rightarrow$ arbitrage opportunity exists!
- Strategy:
  - Purchase 10% of company L's equity;
  - Purchase 10% of company L's debt;
  - Short sell 10% of company U's equity.

Arbitrage opportunity if MM proposition 1 is false

	Cash Flow to Arbitrageur		
	Now	End of Year	
		Weak Economy	Strong Economy
Short sell *U*'s equity	100	- 90	- 140
Purchase *L*'s equity	- 40	37.5	87.5
Purchase *L*'s debt	- 50	52.5	52.5
Net cash flow	10	0	0

- Note that
  - Cash flows from debt: $52.5 = 10\% \times 500 \times (1 + 5\%)$
  - Cash flows from equity: $37.5 = 10\% \times (\$900 - \$525); 87.5 = 10\% \times (\$1400 - \$525)$
- The trade is profitable as long as $V_L < V_U$
- Arbitrageurs will cause prices to adjust until $V_L = V_U$
- MM Proposition 1 holds

The MM theorem is the first application of the **absence of arbitrage argument** to corporate finance.

Cost of capital with and w/o leverage

	Date 0	Date 1: Cash Flows		Date 1: Returns		
	Initial Value	Strong Economy	Weak Economy	Strong Economy	Weak Economy	Expected Return
Debt	\$500	\$525	\$525	5%	5%	5%
Levered equity	\$500	\$875	\$375	75%	−25%	25%
Unlevered equity	\$1000	\$1400	\$900	40%	−10%	15%

- Without leverage, cost of capital (=cost of equity)=15%.

$$\frac{1}{2}\times 40\% + \frac{1}{2}(-10\%) = 15\%$$

- With leverage, cost of capital is the **weighted average of cost of debt and cost of equity**:
  - Cost of debt is 5%.
  - Cost of equity is 25%!

$$\frac{1}{2}\times 5\% + \frac{1}{2}\times 25\% = 15\%$$

- Debt financing increases the required return for equity (why?)…
- …but cost of capital remains the same as for the unlevered firm.

Leverage increases the risk of equity:

- Unlevered equity has a return of 40% or -10%;
- Levered equity has a return of 75% or -25%.

The effect of leverage on risk and return

- Leverage increases the risk of equity
  - Unlevered equity has a return of 40% or -10%;
  - Levered equity has a return of 75% or -25%.
- Returns to levered equity are particularly low (high) in “bad” (“good”) states of the economy (higher systematic risk)
  - Equity investors may therefore require higher expected returns on levered equity

	Return Sensitivity (Systematic Risk)	Risk Premium
	$\Delta R = R(\text{strong}) - R(\text{weak})$	$E[R] - r_f$
Debt	$5\% - 5\% = 0\%$	$5\% - 5\% = 0\%$
Unlevered equity	$40\% - (-10\%) = 50\%$	$15\% - 5\% = 10\%$
Levered equity	$75\% - (-25\%) = 100\%$	$25\% - 5\% = 20\%$

- While debt may be cheaper, its use raises the cost of equity
- Considering both sources of capital together, the firm’s average cost of capital remain unchanged

MM proposition 2:
Leverage, risk, and the cost of capital

- MM Proposition 1 states that the total market value of the firm's securities is equal to the market value of its assets, whether the firm is unlevered or levered

$$V_L = E + D = V_U$$

- By holding a portfolio of the levered firm's equity and debt, we can replicate the cash flow from the unlevered firm.

  - Weight for equity is $\frac{E}{E+D}$ and for debt is $\frac{D}{E+D}$

MM proposition 2: Leverage, risk, and the cost of capital

- The unlevered equity *return* (r_U), as well as the return on the asset (r_A) must be equal to the *returns* of debt (r_D) and levered equity (r_E) weighted according to the firm's capital structure.

  - $\frac{E}{E+D} r_E + \frac{D}{E+D} r_D = r_U = r_A \longrightarrow \boxed{r_E = r_U + \frac{D}{E}(r_U - r_D)}$

- *The cost of capital of levered equity is equal to the cost of capital of unlevered equity plus a premium that is proportional to the debt-equity ratio*

MM Proposition 2: The equity cost of capital in a levered firm increases with the firm's debt/equity ratio

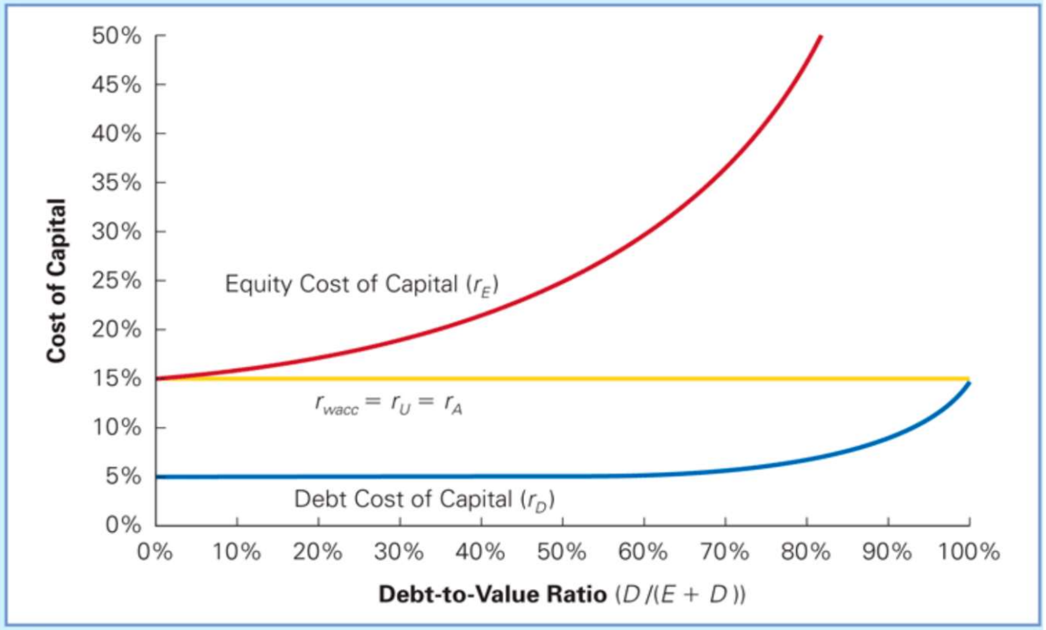
Cost of capital for a levered firm

- Cost of capital for a levered firm = expected return on portfolio of debt and equity = *weighted average cost of capital* (r_{WACC})
  - $r_{WACC} = \frac{E}{E+D} r_E + \frac{D}{E+D} r_D$

- Under MM assumptions, a firm's WACC is independent of its capital structure and is equal to its equity cost of capital if it is unlevered, which matches the cost of capital of its asset
  - $r_{WACC} = r_U = r_A$

Cost of capital and debt ratio in perfect markets

- Without debt, WACC is equal to the unlevered equity cost of capital.
- As the firm borrows at the low cost of capital for debt, its equity cost of capital rises, but the WACC is unchanged.



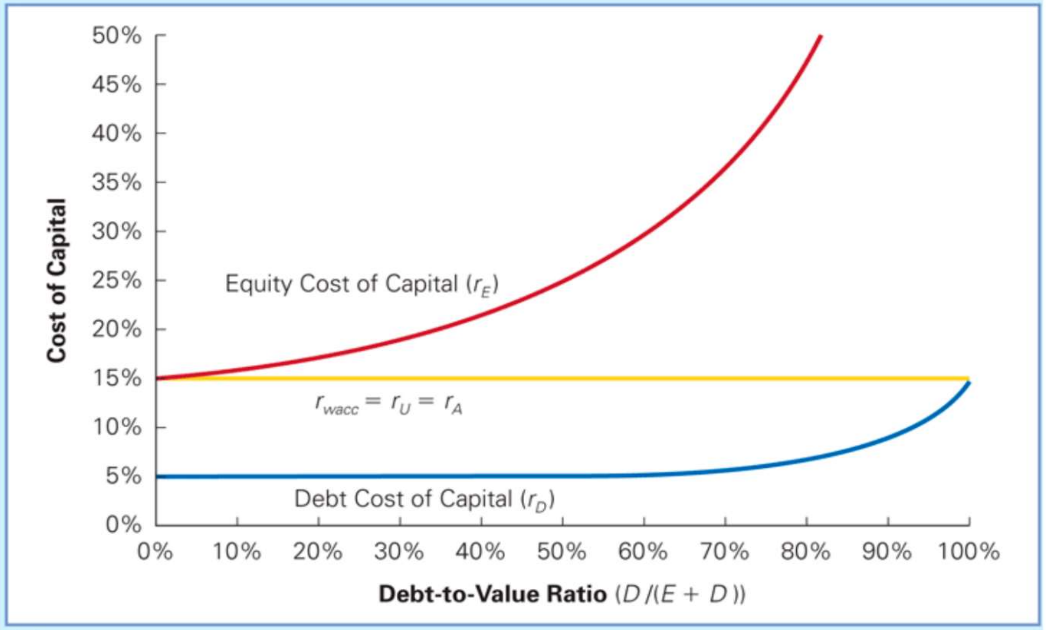
(a)

E	D	r_E	r_D	$\frac{E}{E+D}r_E + \frac{D}{E+D}r_D$	$= r_{wacc}$
1000	0	15.0%	5.0%	$1.0 \times 15.0\% + 0.0 \times 5.0\%$	= 15%
800	200	17.5%	5.0%	$0.8 \times 17.5\% + 0.2 \times 5.0\%$	= 15%
500	500	25.0%	5.0%	$0.5 \times 25.0\% + 0.5 \times 5.0\%$	= 15%
100	900	75.0%	8.3%[4]	$0.1 \times 75.0\% + 0.9 \times 8.3\%$	= 15%

(b)

Cost of capital and debt ratio in perfect markets

- If the amount of debt increases further, because there is a chance that the firm defaults, the cost of debt increases.
- With 100% debt, the debt would be as risky as the assets themselves.



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100	900	75.0%	8.3%[4]	$0.1 \times 75.0\% + 0.9 \times 8.3\%$	= 15%

(b)

Example

Problem

NRG Energy, Inc. (NRG) is an energy company with a market debt-equity ratio of 2. Suppose its current debt cost of capital is 6%, and its equity cost of capital is 12%. Suppose also that if NRG issues equity and uses the proceeds to repay its debt and reduce its debt-equity ratio to 1, it will lower its debt cost of capital to 5.5%. With perfect capital markets, what effect will this transaction have on NRG's equity cost of capital and WACC? What would happen if NRG issues even more equity and pays off its debt completely? How would these alternative capital structures affect NRG's enterprise value?

Solution

We can calculate NRG's initial WACC and unlevered cost of capital using Eqs. 14.6 and 14.7:

$$r_{wacc} = r_U = \frac{E}{E+D} r_E + \frac{D}{E+D} r_D = \frac{1}{1+2}(12\%) + \frac{2}{1+2}(6\%) = 8\%$$

Given NRG's unlevered cost of capital of 8%, we can use Eq. 14.5 to calculate NRG's equity cost of capital after the reduction in leverage:

$$r_E = r_U + \frac{D}{E}(r_U - r_D) = 8\% + \frac{1}{1}(8\% - 5.5\%) = 10.5\%$$

The reduction in leverage will cause NRG's equity cost of capital to fall to 10.5%. Note, though, that with perfect capital markets, NRG's WACC remains unchanged at $8\% = \frac{1}{2}(10.5\%) + \frac{1}{2}(5.5\%)$, and there is no net gain from this transaction.

If NRG pays off its debt completely, it will be unlevered. Thus, its equity cost of capital will equal its WACC and unlevered cost of capital of 8%.

In either scenario, NRG's WACC and free cash flows remain unchanged. Thus, with perfect capital markets, its enterprise value will not be affected by these different capital structure choices.

Intuition behind MM

- Overall cost of capital cannot be reduced as debt is substituted for equity even though debt appears to be cheaper than equity.
- As debt is added, the remaining equity becomes riskier and therefore requires higher rate of return.
- The increase in the cost of the remaining equity capital offsets the higher proportion of the firm financed by low-cost debt.
- In fact, MM prove that, in a frictionless world without taxes, the two effects exactly offset each other, so that both the value of the firm and the firm's overall cost of capital are invariant to leverage.

Example 1

ABC Co., an all-equity firm with perpetual cash-flows, estimates its earnings per year to be 100 in recession, 150 in normal time, and 200 in expansion. Each of the three states are equally likely. The earnings will all be paid out as dividend. The current value of equity is 1000.

a) What is the expected return on equity?

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a) What is the expected return on equity?

Solution: Expected Cash flow: (100 + 150 + 200)/3 = 150. 150/1000 =15%, which is the expected return on equity.

Example 1

Now the firm considers borrowing 500 of perpetual debt to repurchase half of the equity. The interest on this debt will be 10%.

b) What is the value of the equity after this transaction?

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Solution: Proposition 1 of MM says that $V_L = V_U$ => the value of equity = 1000 – 500.

c) What is the expected return on equity after the transaction?

Example 1

Now the firm considers borrowing 500 of perpetual debt to repurchase half of the equity. The interest on this debt will be 10%.

c) What is the expected return on equity after the transaction?

Solution: Proposition 2 of MM says*:*

$$r_E = r_U + \frac{D}{E}(r_U - r_D) = 0.15 + 1 \times (0.15 - 0.10) = 0.20.$$

Example 2

ABC co. is an all-equity firm and has expected earnings of \$10m per year in perpetuity. The firm pays all of its earnings out as dividends. There are 10m shares outstanding. The cost of capital for this unlevered firm is 10%. The firm will soon build a new \$4m-plant, which is expected to generate additional cash flow of \$1m per year.

a) What is the NPV of the new project?

Example 2

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Solution: NPV= -4 + 1/0.1 = \$6m.

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b) Before the market knows of the project, what is the stock price per share?

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b) Before the market knows of the project, what is the stock price per share?

Solution: Before the market knows the project, the value of firm is the value of a perpetuity: \$10m/0.1 = \$100m. Then the stock price is: \$100m/10m = \$10.

Example 2

c) (**Stock Financing**) Suppose that the firm announces that it will raise \$4m in equity to build the plant. What is the value of the firm? what is the price per share? and how many new shares will the firm issue?

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Solution: After the announcement, the stock price and the firm value should reflect the net present value of the plant immediately. That is, the firm value increases by the NPV of the new project to \$100m + \$6m = \$106m.

Since the new shares have not yet been issued, the number of outstanding shares remains 10m. Hence, the price per share is 106/10 = \$10.6.

Shortly thereafter, \$4m of stock is issued, which are 4m/10.6 = 377,358 shares.

Example 2

d) After stocks are issued, suppose the plant is immediately built. What is the firm value? And what is the price per share?

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Solution: The firm value includes the original value plus the present value of the new plant. Since the building expenditures of \$4m have already been paid, they no longer represent a future cost. That is, the firm value = 100m + \$1m/0.1 = \$110m.

The number of shares outstanding is 10m + 377,358 = 10,377,358, and the price per share is 110m/10,377,358 = \$10.60.

Then the expected return to equity holders is: r_E = \$(10 + 1)/\$110 = 10% $\equiv r_U$, since the firm is all-equity.

Example 2

e) (**Debt Financing**) Alternatively, imagine the firm announces that it will borrow \$4m at 6% to build the new plant. Then what are the firm value and stock price?

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e) (**Debt Financing**) Alternatively, imagine the firm announces that it will borrow $4m at 6% to build the new plant. Then what are the firm value and stock price?

Solution: As before, the stock price will change immediately after the announcement. The firm value is = $100m + 6m = $106 (same as before), and the stock price = 10.6. (MM)

Example 2

f) When the debt is issued and the money is immediately used to build the plant, what is the firm value? What is the return on equity?

Example 2

f) When the debt is issued and the money is immediately used to build the plant, what is the firm value? What is the return on equity?

Solution: The firm value = \$106 + 4 = \$110.

The equity holders expect yearly cash flow as follows: 10m + 1m - \$4m x 0.06 = \$10,760,000. The equity holders expect to earn a return of \$10,76000/\$106,000,000 = 10.15%.

Alternatively, according to the 2nd proposition of MM, $\mathrm{r_E} = \mathrm{r_U} + \frac{\mathrm{D}}{\mathrm{E}}(\mathrm{r_U} - \mathrm{r_D}) = 10\% + \frac{4}{106}(10 - 6)\% = 10.15\%$.

Capital structure and beta

- Suppose an individual owns all the firm's debt and equity. What is the beta of her portfolio?
- Remember that the beta of a portfolio is the weighted average of the betas of the individual securities in the portfolio
  - Weight for equity is $\frac{E}{E+D}$ and for debt is $\frac{D}{E+D}$

Capital structure and beta

- The unlevered beta (β_U), as well as the asset beta (β_A) must be equal to the *beta* of debt (β_D) and levered equity (β_E) weighted according to the firm's capital structure.

$$\frac{E}{E+D}\beta_E + \frac{D}{E+D}\beta_D = \beta_U = \beta_A \longrightarrow \boxed{\beta_E = \beta_U + \frac{D}{E}(\beta_U - \beta_D)}$$

- Since β_U and β_A do not change with capital structure, this allows us to calculate β_E after changes in capital structure.

Capital structure and beta - Example

- MM.Inc is an all-equity financed company
  - $\beta_U = 0.5, r_f = 8\%,\ E(r_M) = 14\%$
  - What is the cost of capital now?

- MM.Inc repurchase 1/3 of its outstanding equity shares and finance the repurchase by issuing risk-free debt ($r_D = r_f = 8\%$).
  - What is the capital structure now?
  - What is the beta of debt now?
  - What is the new cost of equity?
  - What is the new WACC?

Capital structure and beta - Example

	E/(D+E)	D/E	β_U	β_E	r_U	r_E	r_D	r_{WACC}
Before recapitalization	1.0	0	0.50		11%		8%	11%
After recapitalization	2/3	1/2	0.50	0.75	11%	12.5%	8%	11%

$$r_U = r_D + \beta_U(E(r_M) - r_D) = 8 + 0.5 \times 6 = 11$$

$$\beta_E = \left(1 + \frac{D}{E}\right)\beta_U = \left(1 + \frac{1}{2}\right) \times 0.5 = 0.75$$

$$r_E = r_D + \beta_E(E(r_M) - r_D) = 8 + 0.75 \times 6 = 12.5$$

$$r_E = r_U + \frac{D}{E}(r_U - r_D) = 11 + 0.5 \times (11 - 8) = 12.5$$

$$r_{WACC} = \frac{2}{3} \times 12.5 + \frac{1}{3} \times 8 = 11$$

“Unlevering” and “re-levering” beta

- Take levered equity beta of a firm under the current capital structure (β_E) and calculate the beta of the firm as if it was unlevered (β_U).
  - $\beta_U = \frac{E}{E+D}\beta_E + \frac{D}{E+D}\beta_D$
  - With risk-free debt, $\beta_D = 0$;
  - With risky debt, can back out β_D if you know r_D (see earlier in this session)

- We can then “re-lever” the equity beta under the new (or proposed, or any) capital structure we want ($\beta_{E'}$).
  - $\beta_{E'} = \beta_U + \frac{D'}{E'}(\beta_U - \beta_{D'})$

“Unlevering” Beta - Example

- MM II.Inc is private (unlisted) company, so that we cannot estimate its equity Beta directly.
  - $r_D = r_f = 8\%, E(r_M) = 14\%, \quad \frac{D}{E} = 0.25, \rightarrow \frac{D}{D+E} = 0.2, \frac{E}{D+E} = 0.8$
- Strategy: obtain equity Beta of comparable firms (with similar riskiness in assets), un-lever using comparable firms’ capital structure, and then re-lever using MM II. Inc’s capital structure

- Two comparison companies:

Comparison firm	β_E	$\frac{E}{D+E}$	$\frac{D}{E}$	β_U
A	1.00	0.5	1.0	0.50
B	1.50	0.3	2.33	0.45
Average				**0.475**

- Re-lever: $\beta_E = \frac{D+E}{E}\beta_U = 1.25 \times 0.475 = 0.59$
- Cost of equity for Company X:

$$r_E = r_D + \beta_E\left[E(r_M) - r_f\right] = 8 + 0.59(14 - 8) = 11.54\%$$

WACC and firm valuation

- You now know how to estimate the discount rate for any risky project, no matter how it is funded using the WACC concept

- In the simple case where FCF grows at an expected rate g from this year's level CF, the value of the company V is like a perpetuity:

$$V = \frac{CF}{WACC - g}$$

- Usually, we know the market value of debt D (roughly equal to its book value if not close to bankruptcy), so we can obtain a value for the equity E:

$$E = \frac{CF}{WACC - g} - D$$

WACC and firm valuation

- It can be useful to compare the value of equity we reach with the method of discounting cash flows (E_{dcf}) we have used in last slide, with the listed valuation of equity (E_{mkt}):

$$E_{mkt} = E_{dcf}$$

$$E_{mkt} + D = E_{dcf} + D = \frac{CF}{WACC - g}$$

- This means that by computing the ratio of CF (this year's FCF) to current firm value as seen by the market, I can recover the implicit discount rate and future growth rate assumed by market participants:

$$\frac{E_{mkt} + D}{CF} = \frac{1}{WACC_{mkt} - g_{mkt}}$$