73-240 The Borrowing Constrained Household

1 The Household who can borrow freely

Suppose we have a household whose optimal decision is to borrow against her future income. In class, for the case of log utility, i.e.

$$U(c, c') = \ln c + \beta \ln c'$$

and for the case where households only face lump-sum taxes, making the lifetime budget constraint take the form of:

$$c + \frac{c'}{1+r} = y - t + \frac{y' - t'}{1+r}$$

We showed that optimal consumption takes the form of:

$$c^* = \frac{y-t}{1+\beta} + \frac{y'-t'}{(1+\beta)(1+r)}$$

and optimal savings takes the form of:

$$s^* = \frac{\beta(y-t)}{1+\beta} - \frac{y'-t'}{(1+\beta)(1+r)}$$

(By now, it should be clear once you have found optimal c^* and s^*, you obviously also know what c'^* is.)

Observe that for $y' - t' = \beta(1+r)(y-t)$, optimal consumption becomes :

$$c^* = \frac{y-t}{1+\beta} + \frac{\beta(1+r)(y-t)}{(1+\beta)(1+r)} = \frac{1+\beta}{1+\beta}(y-t) = y - t$$

So for any $y' - t' > \beta(1+r)(y-t)$, the household wants to have consumption above its after-tax income, $c^* > y - t$, and as such the household wants to be a borrower, $s^* < 0$. We will consider this case where $y' - t' > \beta(1+r)(y-t)$.

Let's first observe graphically what optimality looks like for the household who optimally decides to be a borrower: Figure 1 shows that the household optimal consumption today is given by c^*, the consumption today associated with point A (the point where the highest indifference curve the household can achieve is tangent to its lifetime budget constraint.)

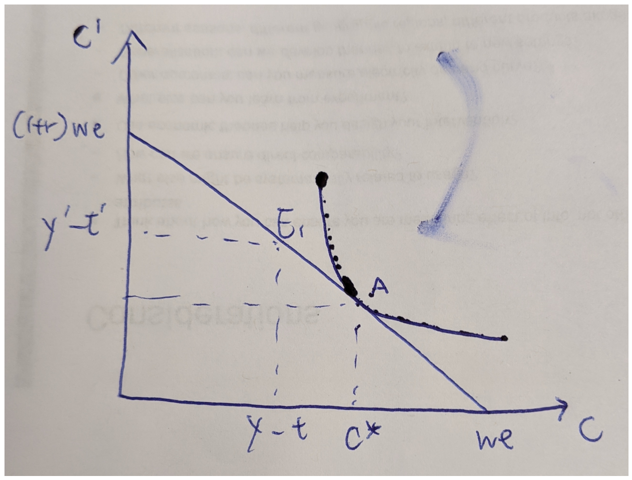

Figure 1: Household who can borrow

2 The Household who cannot borrow

If now we suppose there is a no-borrowing constraint, i.e. $s \geq 0$, then again for $y' - t' > \beta(1+r)(y-t)$, now we have a household who would like to consume

$$c^* = \frac{y-t}{1+\beta} + \frac{y'-t'}{(1+\beta)(1+r)} > y - t$$

However, in this case the household cannot borrow. This implies that we now have a kinked budget constraint where the kink appears at the endowment. Since no borrowing is allowed, any consumption $c > y - t$ is not possible. The highest consumption an individual can have today is just $c = y - t$. Thus, even though the household would like to achieve c^*, it cannot borrow and the best it can do is to have zero savings. In this case $c^{constrained} = y - t$.

Graphically, let's show what this household's borrowing constraint looks like and what this implies for the household's choice of consumption today:

Figure 2[1] shows that for the household who cannot borrow, the best she can do is have zero savings. This gives her the consumption (today) that is closest as possible to her optimal consumption c^*. Note that Figure 2 makes clear that the best the household can do at this point is to eat exactly her endowment each period, i.e. $c = y - t, c' = y' - t'$. Figure 2 also makes clear that eating her endowment each period provides the household with lower utility than what the household could achieve if she could borrow freely.

[1] Apologies for the skewness in the picture.

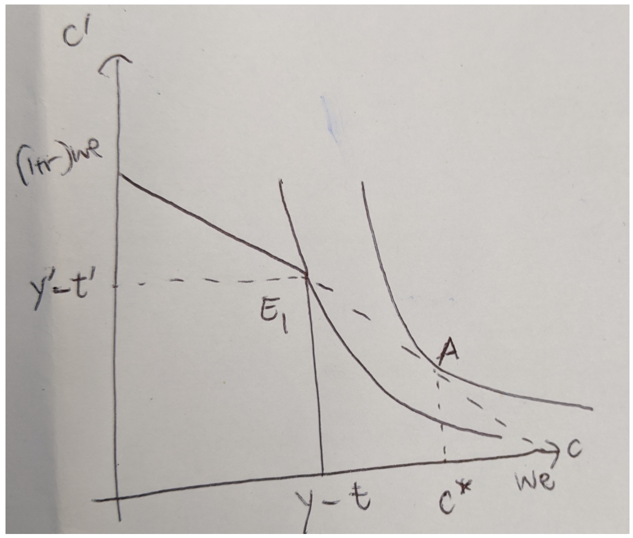

Figure 2: Household who cannot borrow

3 The Household who cannot borrow and a lump-sum tax cut today

Before we examine what happens to the household who cannot borrow when there is a lump-sum tax cut today. Let's recall what this implies about taxes tomorrow.

If the government is not changing its spending plans, G, G' constant, then a lump-sum tax cut today must be repaid by higher taxes tomorrow. From the government budget constraints, we know that (we proved this in class before, so show it to yourself again as practice!):

$$\Delta T' = -(1+r)\Delta T$$

Since a tax cut today requires a higher tax tomorrow, the implementation of a tax cut today must shift the household's endowment point down (lower after-tax income tomorrow) and to the right (higher after-tax income today).

$$y_2 - t_2 = y_1 - (t_1 + \Delta t) > y_1 - t_1$$

where Δt is negative since its a tax cut . And

$$y_2' - t_2' = y_1' - (t_1' + \Delta t') < y_1' - t_1'$$

where $\Delta t'$ is positive since $\Delta t' = -(1+r)\Delta t$.

So now, let's see graphically what happens to the household who cannot borrow:

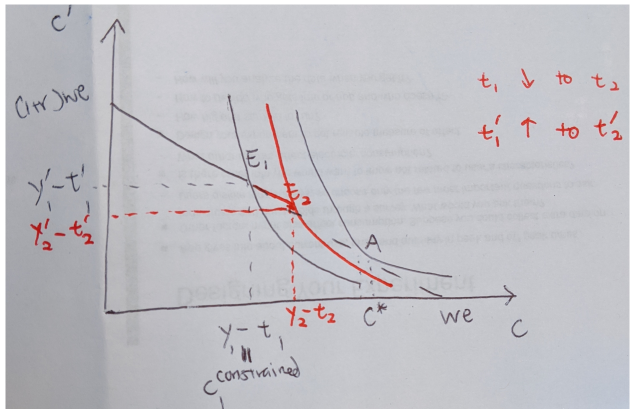

Figure 3: Household who cannot borrow under tax cut

In this case, the household who cannot borrow experiences an increase in her after-tax income today when there is a tax-cut today. Because this gets her closer to her optimal consumption point c^*, she responds fully to the tax cut and increases her consumption. The household does this because it allows her to achieve higher utility as evidenced by the higher indifference curve the household achieves in Figure 3.