73-240 – Problem set 1

Due Friday Sep. 13th

From the Syllabus:

1. Homework must be turned in on the day it is due (usually on a Friday) in the undergraduate program office in Tepper Quad 2400 by **430pm** sharp. Do NOT leave it under my office door. Late homework will NOT be accepted unless you are sick and have a doctor's note.

2. Homework regrading: There is a statute of limitations on regrades. If you believe a question has been incorrectly graded, please take your homework to your TA within 2 weeks of it being returned.

3. Working in groups: You may work in groups of up to 4. BUT: You MUST put names of other group members on your homework. You MUST write up your own set of answers. Do NOT simply copy some other person's work.

4. TYPE your work. Long equations may be hand written. Buy a stapler!

5. Write your first and last name on the title of each graph.

6. Carefully explain your work.

Further Notes: Unless explicitly stated, do not turn in/print out your excel sheets. Submit only the required charts and answers.

Problem 1: Getting Started, Using Logarithms

When calculating GDP growth rates, economists like to use the difference in the natural logarithm of GDP as an approximation for an economy's growth. In this question, we will examine the usefulness of describing economic variables in terms of its natural log.

Some background info: As a review, let's recall what happens with compound interest rates. Suppose you invest X_1 for one year at interest rate $r = 2\%$. At the end of the year, you get $X_2 = (1+r)X_1$, i.e. $X_2 = (1.02)X_1$. Interest can be compounded, for example, you might get a return every quarter. Suppose the annual interest rate is still $r = 2$, then compounded quarterly, your final return at the end of the year is $X_2 = (1 + \frac{r}{4})^4 X_1$ or $X_2 \approx 1.0202$. As the frequency of compounding becomes high, the formula $(1 + \frac{r}{t})^t$ converges to :

$$\lim_{t\to\infty}(1 + \frac{r}{t})^t = e^r$$

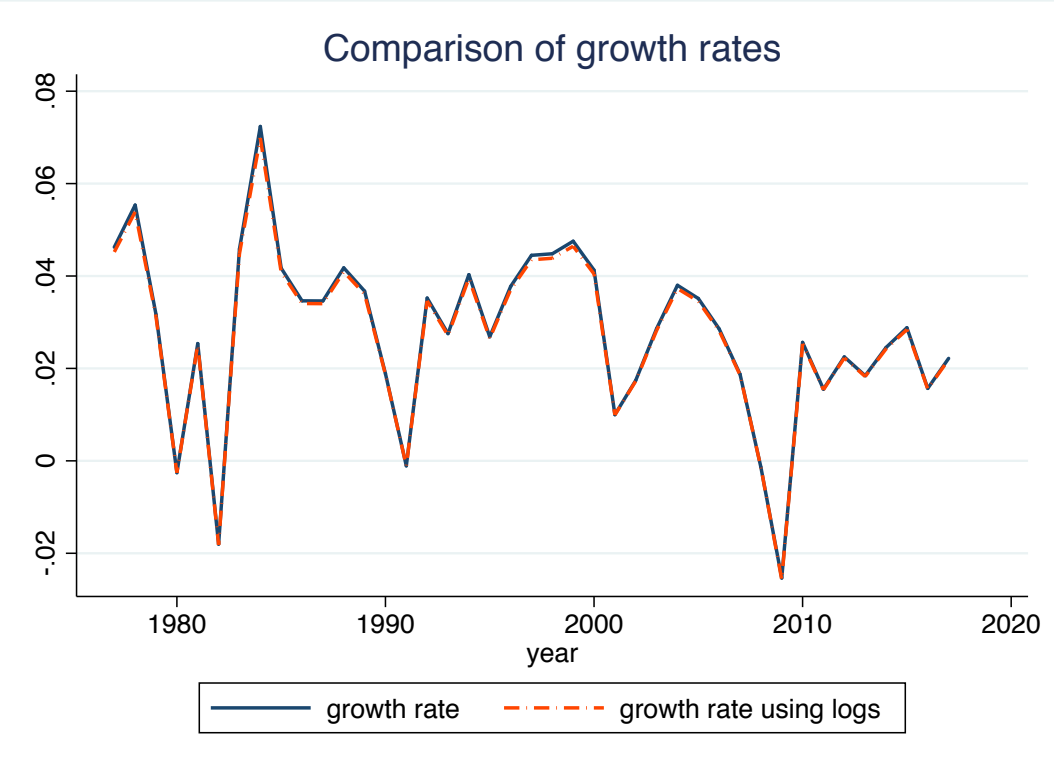
This implies that if your money gets compounded continuously, at the end of the year you would have $X_2 = e^r X_1$.
To get back the interest rate or growth rate of your investment, we can do a very simple transformation of the above equation by taking the natural logarithm on both sides

$$\ln \frac{X_2}{X_1} = \ln X_2 - \ln X_1 = r$$

We can now do a very simple subtraction of two log variables to get out the growth rate!

a Download data on Annual Real GDP for the US economy from FRED (see instructions below) for the period covering 1976-2016. Calculate the growth rate of Annual Real GDP for each year by using the formula: $g_t = \frac{GDP_t - GDP_{t-1}}{GDP_{t-1}}$. Find the average growth rate across all years.
Answer
$g_{avg} = 0.0276$ or 2.76%.

b Now calculate the annual growth rate for each year as: $\hat{g}_t = \ln(GDP_t) - \ln(GDP_{t-1})$. Again find the average growth rate, $\hat{g}_{avg}$, across all years. Is there a large discrepancy between your answer in part (a) and part (b)? Plot the two calculated growth rate series, g_t and $\hat{g}$.
Answer
$\hat{g}_{avg} = 0.0270$ or 2.7%.



c Calculate the correlation between the growth rate g_t and $\hat{g}_t$ you found in part (a) and part (b)

Answer

Correlation ≈ 1

Final notes on logarithms: One more useful aspect about using logs is in terms of making comparisons. Suppose $GDP_t = 100$. Saying that GDP rose by 50% in period $t+1$ but then fell by 50% in $t+2$ does not give us back 100 in period $t+2$. However, if we said that GDP rose 50 log points in $t+1$ and dropped 50 log points in $t+2$, we would be back at the same level of GDP in period $t+2$ as in period t You can see this clearly from the formula $\ln GDP_t = \ln GDP_{t-1} + g_t$.

Problem 2: National Income and the Labor Share

This question requires you to examine how the labor share in the US has evolved over time.

a Download annual data 1976-2018 on the following items:

Item 1 : Compensation to employees

Item 2 : Proprietors' income

Item 3 : Rental income

Item 4 : Corporate profits

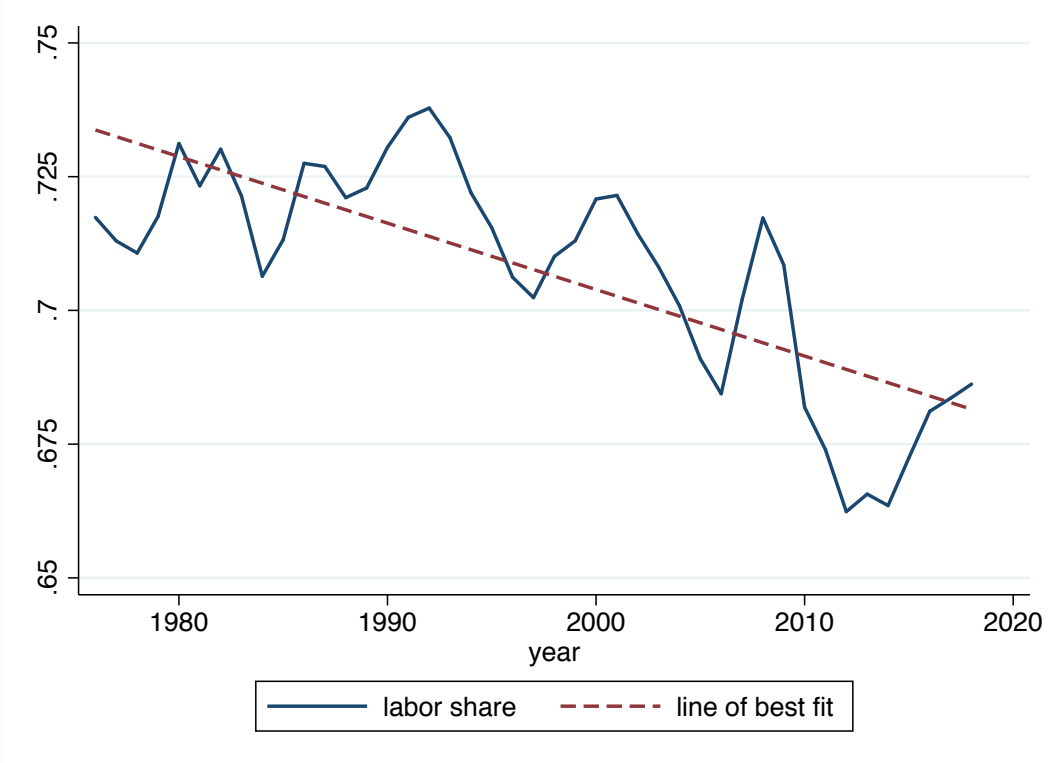
Item 5 : Net interest payments.

[See the last page of this assignment for information on how to get this data.]
No solution. Just instructions

b Calculate the labor share for each year as:

$$\text{Labor Share} = \frac{\text{Compensation to employees (item 1)}}{\text{sum of items 1 to 5}}$$

and present your findings in a graph with time on the x-axis and labor share on the y-axis. What do you observe about the labor share?
[Note: If the data series are in different units (e.g. billions of dollars vs. thousands of dollars), convert them to the same units.]
Answer:



c In a few sentences, suggest why rising GDP growth alongside a declining labor share may not be reflective of individuals' welfare improving over time.
Answer:
If a larger portion of the gains in income accrue to capital and business owners and if not all workers are also capital owners, then the rise in GDP may be accompanied by rising inequality. In other words, the gains in GDP are not equally shared by all and may not be representative of individuals' welfare.

Problem 3: Calculating GDP

A cocoa farmer in Country X exports $500 worth of his cocoa beans to a chocolate manufacturer in Country Y. The chocolate manufacturer keeps $300 worth of the cocoa beans in inventory for future production use. The chocolate manufacturer then refines the remaining beans and sells the refined cocoa beans to a chocolate company for $700. The chocolate company makes the beans into chocolate and sells that to households for $1000. If the economy in country Y only consumes chocolate, calculate GDP of country Y using the expenditure approach.
Answer:

- Consumption: 1000
- Investment: 300
- Government: 0
- Net Exports: -500

$$GDP = C + I + G + NX = 800$$

Problem 4: CPI and Substitution Bias

This question demonstrates why the CPI may overstate inflation. Consider the following economy where individuals only consume apples and bananas.

Prices per unit	Year 1	Year 2
Apples	1	1
Bananas	1	4

Suppose all consumers in this economy get utility (happiness) from consuming apples and bananas. Further assume that all consumers have a hard constraint of having 4 units of happiness (utils) every period. Consumers want to spend the smallest amount they can to achieve this constraint of 4 utils. Formally, the consumer's problem is given by:

$$\min_{Q_A,Q_B} E = P_A Q_A + P_B Q_B \tag{1}$$

s.t.

$$\overline{u} = Q_A^{1/2} Q_B^{1/2} = 4 \tag{2}$$

where Q_A is the quantity of apples, Q_B is the quantity of bananas and P_A and P_B are the prices of apples and bananas respectively. $\overline{u}$ is the target utility (or happiness) that individuals must achieve each period.

A) Solve for the quantities Q_A and Q_B the individual would choose to minimize her expenditure given prices in Year 1.
[Hint: At this point, it is useful to recognize that one can re-arrange Equation 2 to get $Q_A = 16/Q_B$. Plug this expression of Q_A into Equation 1. At this stage, the consumer's expenditure minimization problem should feature only prices and Q_B. You are now ready to solve for Q_B by taking first order conditions. Once you find Q_B, find Q_A.]
Answer:
Note that the utility constraint (individuals have a target utility) is given by:

$$\overline{u} = Q_A^{1/2} Q_B^{1/2} = 4$$

which implies $Q_A = 16/Q_B$. Plug this latter expression into the household's objective:

$$\min_{Q_B} E = P_A \frac{16}{Q_B} + P_B Q_B$$

and take first order conditions wrt Q_B

$$0 = -P_A \frac{16}{Q_B^2} + P_B$$

Re-arranging, we have:

$$Q_B = 4\sqrt{\left(\frac{P_A}{P_B}\right)}$$

Using Year 1 prices, we have:

$$Q_B = 4$$

This implies that

$$Q_A = \frac{16}{Q_B} = 4$$

B) Solve for the quantities Q_A and Q_B the individual would choose to minimize her expenditure given prices in Year 2.

Answer:

Recall, our expression for Q_B is given by:

$$Q_B = 4\sqrt{\left(\frac{P_A}{P_B}\right)}$$

Using Year 2 prices, we have:

$$Q_B = 4\sqrt{\left(\frac{1}{4}\right)} = 2$$

and for Q_A, we have:

$$Q_A = \frac{16}{Q_B} = 8$$

C) Calculate the percentage change in expenditures, E, between year 1 and year 2.

Answer:

Expenditure in Year 1 is given by:

$$E_1 = P_{A,1}Q_{A,1} + P_{B,1}Q_{B,1}$$

plugging in the values for Year 1, we have:

$$E_1 = 1 \times 4 + 1 \times 4 = 8$$

Expenditure in Year 2 is given by:

$$E_2 = P_{A,2}Q_{A,2} + P_{B,2}Q_{B,2}$$

plugging in the values for Year 2, we have:

$$E_2 = 1 \times 8 + 4 \times 2 = 16$$

So the expenditure doubled or in percentage terms, the increase in expenditure is 100%:

$$\%\text{ change in expenditure} = (\frac{16}{8} - 1) \times 100 = 100$$

D) Using year 1 as the base year, calculate the Consumer Price Index in years 1 and 2 and calculate the rate of inflation between years 1 and 2 using this price index. **Answer:** We need to calculate what is the expenditure in year 2 given year 1 quantities:

$$E_2^{base=1} = 1 \times 4 + 4 \times 4 = 20$$

Hence, the CPI in year 1 is given by:

$$CPI_1 = \frac{E_1^{base=1}}{E_1^{base=1}} \times 100 = 100$$

and the CPI in year 2 is given by:

$$CPI_1 = \frac{E_2^{base=1}}{E_1^{base=1}} \times 100 = \frac{20}{8} \times 100 = 150$$

Thus, we have that the inflation rate is 50%. Note that fixing the quantities, one would have predicted that expenditures were 150% higher.

E) Why is the percent change in actual expenditures less than the percent change in presumed expenditures under the CPI? Use this to explain why the CPI may overstate the cost of living.
Answer:
There is substitution bias in the CPI. The CPI does not account for how consumer expenditure patterns can change in response to changing prices.

Guide for Problem 1

All the data you will need for this question can be found at:
https://fred.stlouisfed.org.

- Use the following data series:
  - For GDP, use GDPCA
    (In the search bar in FRED, type GDPCA to retrieve the series)
- Using Excel organize the data into a single sheets (in the same file).

Guide for Problem 2

All the data you will need for this question can be found at:
https://fred.stlouisfed.org.

- Use the following data series:
  - for Compensation of employees, use COE
    (In the search bar in FRED, type COE to retrieve the series)
  - for Proprietors' income, use PROPINC
    (In the search bar in FRED, type PROPINC to retrieve the series)
  - for Rental Income, use RENTIN
    (In the search bar in FRED, type RENTIN to retrieve the series)
  - for Corporate Profits, use CPROFIT
    (In the search bar in FRED, type CPROFIT to retrieve the series)
  - for Net interest payments, type BOGZ1FU086130003A
    (In the search bar in FRED, type BOGZ1FU086130003A to retrieve the series)
- Using Excel organize the data into a single sheets (in the same file).