PRINCIPLES OF FINANCE

WEEK 6

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Last lecture

Do you remember

We have learned how to estimate the discount rate for a risky project when it is fully funded with **equity** contracts

- The Capital Asset Pricing Model (CAPM)

Real-life applications

- Challenges with CAPM
- Search for alpha

Video

Most projects are financed by a mix of debt and equity

- Weighted Average Cost of Capital (WACC)

Outline of today's lecture

- So far, we have seen how to value bonds and stocks.
- Today we will focus on **financial derivatives** (=products whose value depends on the value of an underlying security).
- Example: options

Video:

- Binomial option pricing model
- Very brief introduction to Black Scholes model

Real-life applications:

- Option strategies
- Gamestop

Financial markets

Who trades on financial markets

- Households
- Banks
- Insurance companies
- Funds (pension funds, mutual funds, private equity funds, hedge funds...)
- …

There are three broad categories of traders:

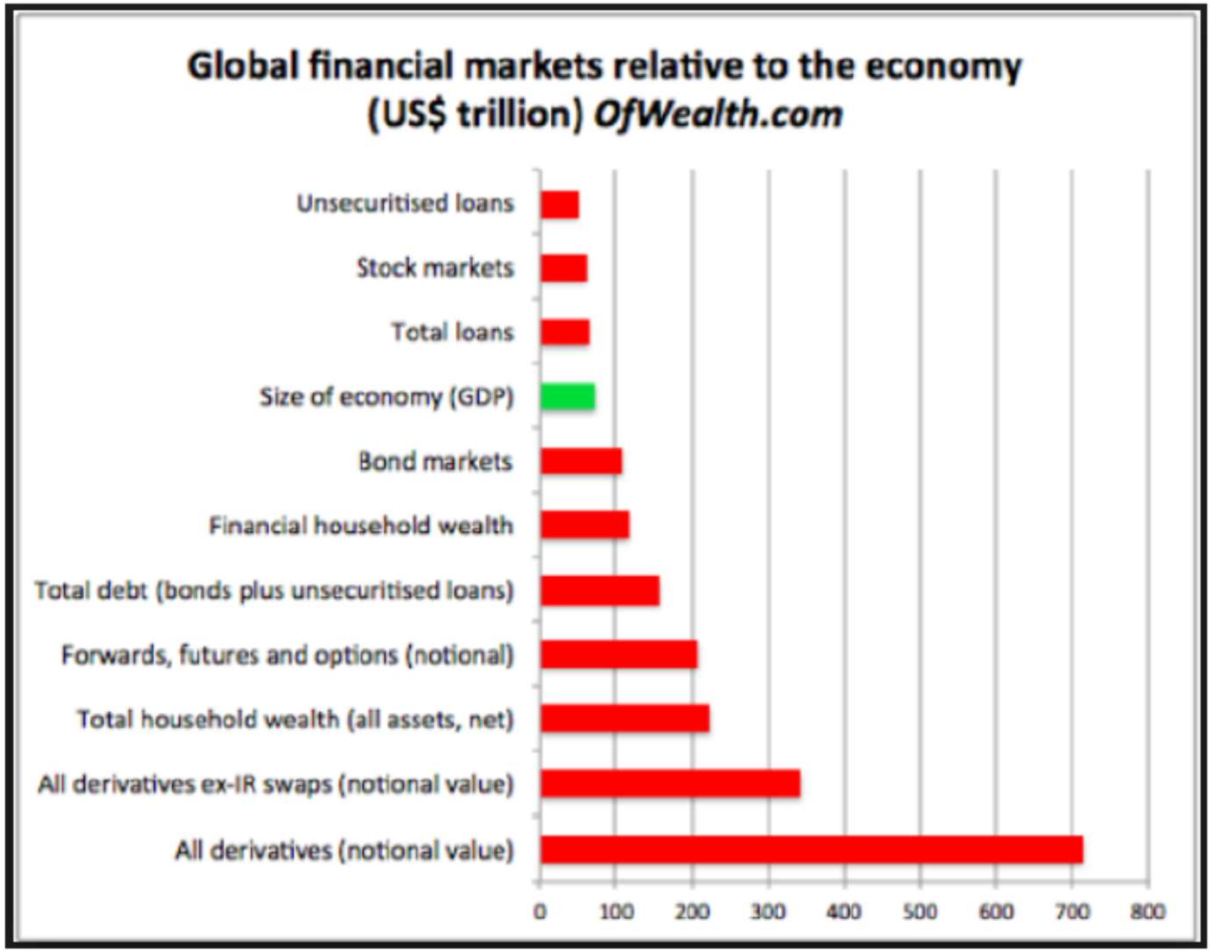
- **Hedgers:** use derivatives to reduce the risk that they face from potential future movements in a market variable.
- **Speculators:** use derivatives to bet on the future direction of a market variable.
- **Arbitrageurs:** take offsetting positions in two or more instruments to lock in a profit

Some examples of markets

- Capital markets
  - Stock markets (e.g. NYSE, NASDAQ, LSE)
  - Bond markets (mostly OTC)
- Money market (interbank lending, short-term loans...)
- Derivatives markets (mostly OTC, CME, Eurex...)
- Futures markets (e.g. CBOE, LIFFE)
- Foreign Exchange markets (OTC)
- Commodity markets (e.g. LCE, LME, CME)

The oldest and largest organized exchange is the Chicago Board Options Exchange (CBOE)

Market sizes as of 2018



Standardized versus OTC markets

Keywords:
OTC markets /
standardized markets

Standardized markets

- Enhanced price transparency but
- Limited choice because of regulatory requirements (price per share, disclosure...)

→ Large stocks

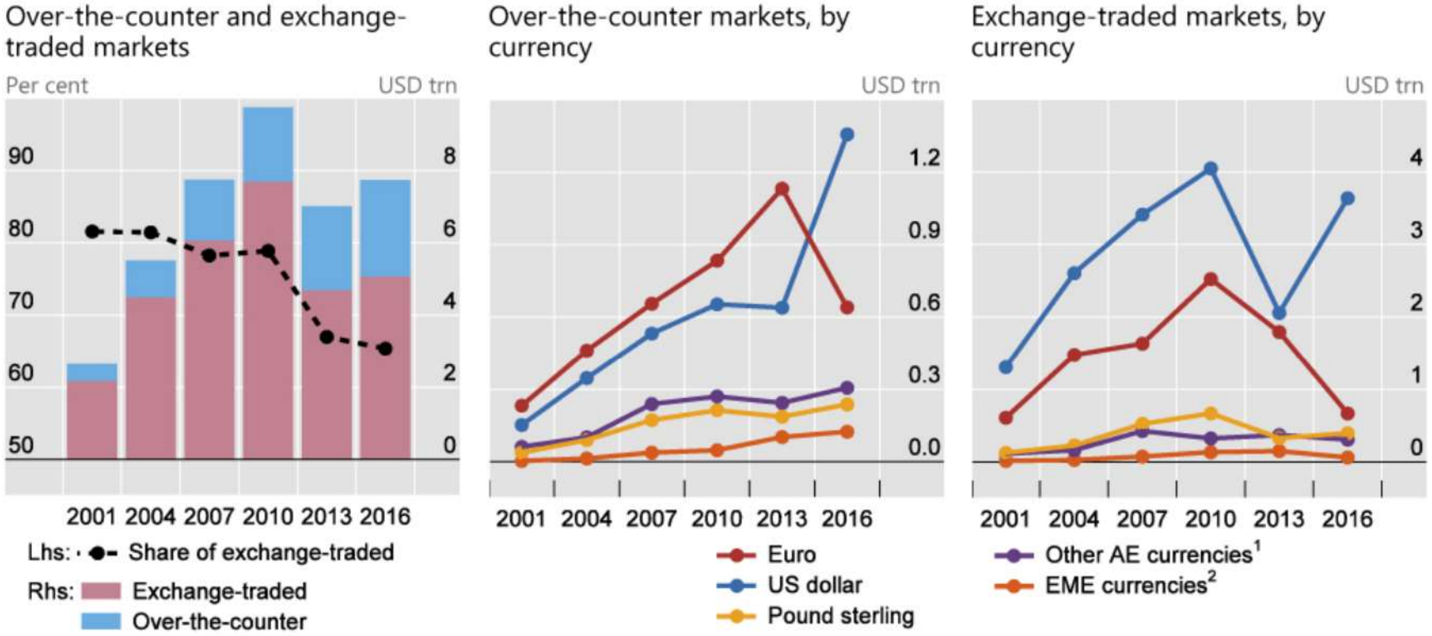
Over the counter markets

- Off-exchange / traded on markets with few requirements, e.g. OTCQX, OTCQB and Pink Sheets operated by the OTC Markets Group
- More than 80% of OTC firms with market cap above $1 million are traded on exchanges either before, concurrently, or after their OTC trading activity
- Many penny stocks / start-ups
- Notional amount outstanding of OTC derivatives equal USD 640 trillion in June 2019.
- More choice and more flexible than standardized contracts
- Issue of transparency

Standardized versus OTC markets

Turnover and currency composition in interest rate derivatives markets

Notional amounts, daily averages in April — Graph 1



[1] Other advanced economy (AE) currencies: AUD, CAD, CHF, DKK, JPY, NOK, NZD and SEK. [2] Currencies which joined the euro zone drop out as from the starting date of the euro. For exchange-traded markets, this comprises all currencies other than EUR, GBP, USD and other AE currencies. For over-the-counter markets, emerging market economy (EME) currencies: ARS, BGN, BHD, BRL, CLP, CNY, COP, CZK, EEK, HKD, HUF, IDR, ILS, INR, KRW, LTL, LVL, MXN, MYR, NLG, PEN, PHP, PLN, RON, RUB, SAR, SGD, SKK, THB, TRY, TWD and ZAR.

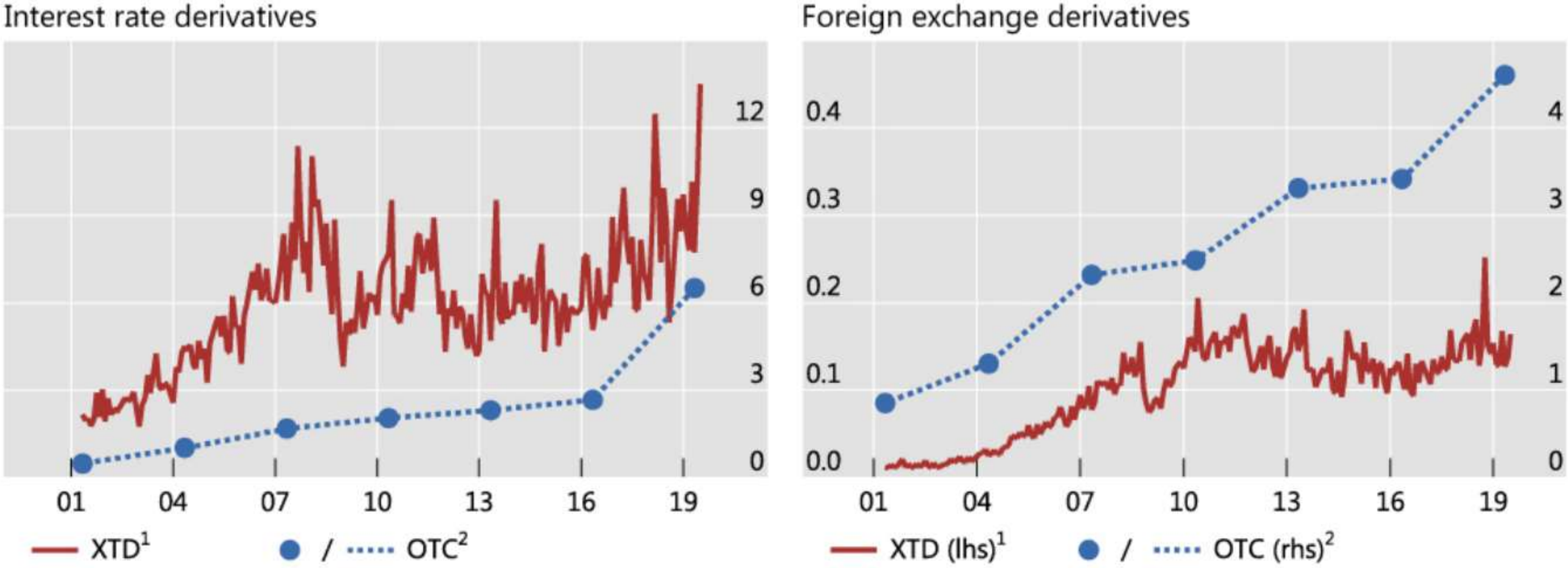
Sources: Euromoney TRADEDATA; Futures Industry Association; The Options Clearing Corporation; BIS derivatives statistics and Triennial Central Bank Survey.

Standardized versus OTC markets (2019)

OTC trading of interest rate and FX derivatives outpaced exchange trading

Daily average turnover, in trillions of US dollars

Graph A1

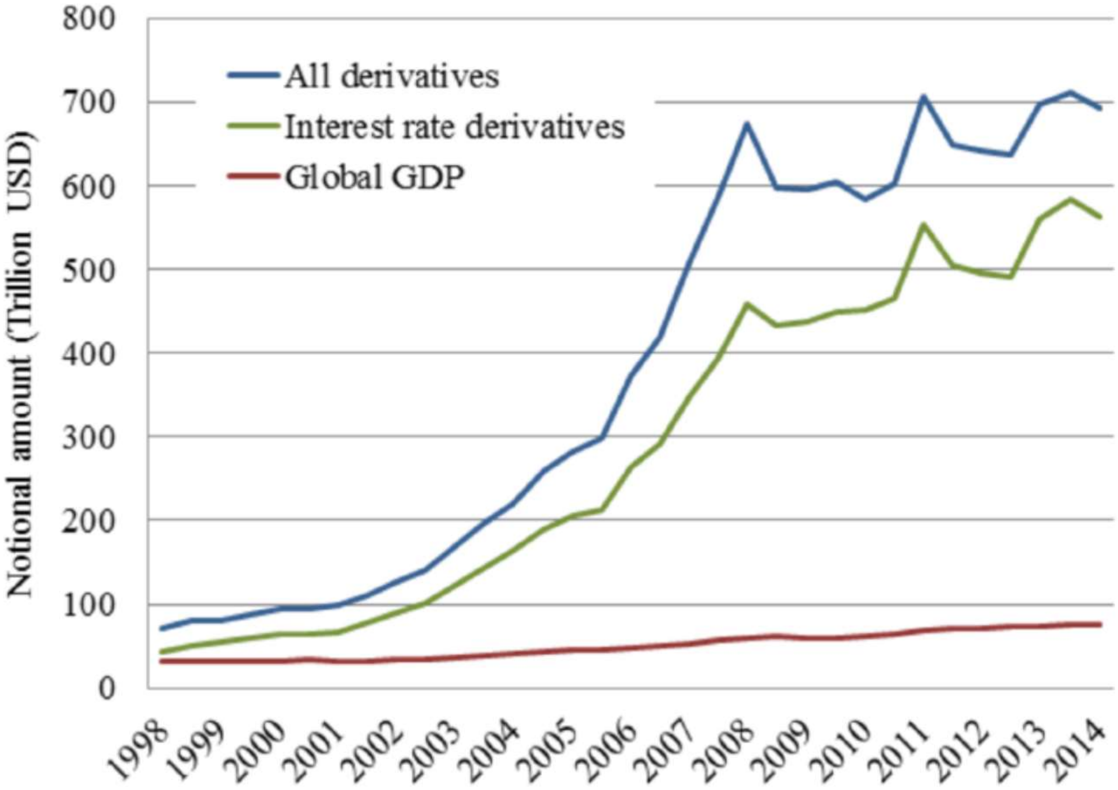


XTD = exchange-traded derivatives; OTC = over-the-counter derivatives.

[1] Turnover on exchanges worldwide, at monthly frequency. [2] Turnover in April, adjusted for local and cross-border inter-dealer double-counting. The dashed line shows a linear interpolation of data between Triennial Surveys.

Sources: Euromoney TRADEDATA; Futures Industry Association; The Options Clearing Corporation; BIS derivatives statistics.

Derivatives growth



Financial derivatives

Derivatives

Keywords:
Equity derivative
Forward contract

- An **equity derivative** is a financial instrument whose value is at least partly derived from one or more underlying equity securities.
  - Forward contracts
  - Options
  - Many structured products

- A **forward contract** on the stock S is an obligation to buy this stock for a predetermined price K at some fixed time in the future $T > 0$ called **maturity**. The **forward price** K remains the same whatever happens to the price of the stock before maturity.
  - Example: A producer who believes electricity prices will rise, and who knows that he/she will need to buy electricity in 3 months, may buy a forward contract on electricity.
  - Futures are similar to forward contracts (but sold on standardized markets while forwards are OTC).

Options

- A **European call option** on the stock S is a contract which gives the holder of the option the right but not the obligation to buy the stock at a fixed time in the future (called **exercise time** or **maturity**) T for a fixed price, called the **strike price** K.

- A **European put option** on the stock S is a contract which gives the holder of the option the right but not the obligation to sell the stock at **exercise time** (**maturity**) for a fixed price, the **strike price** K.

Keywords:
Call and put options
Exercise time
Strike price

Example

Price of different options on Danone on August 27, 2012. The price of the stock on that day is 50.82.

Option Type	Strike	Maturity 21/12/2012	Maturity 15/03/2013	Maturity 21/06/2013
Call	46	5.97	6.68	7.02
Call	50	3.17	4.04	4.44
Call	52	2.09	2.93	3.43
Call	60	0.22	0.60	0.94
Call	72	0.01	0.03	0.08
Put	46	0.98	1.65	2.68
Put	50	2.17	3.00	4.28
Put	52	3.08	3.89	5.35
Put	60	9.22	9.56	11.10
Put	72	21.01	21.03	22.30

Payoffs

- Payoff of a forward:

$$H_{Forward}(S_T, K) = S_T - K$$

- Payoff of a European call option:

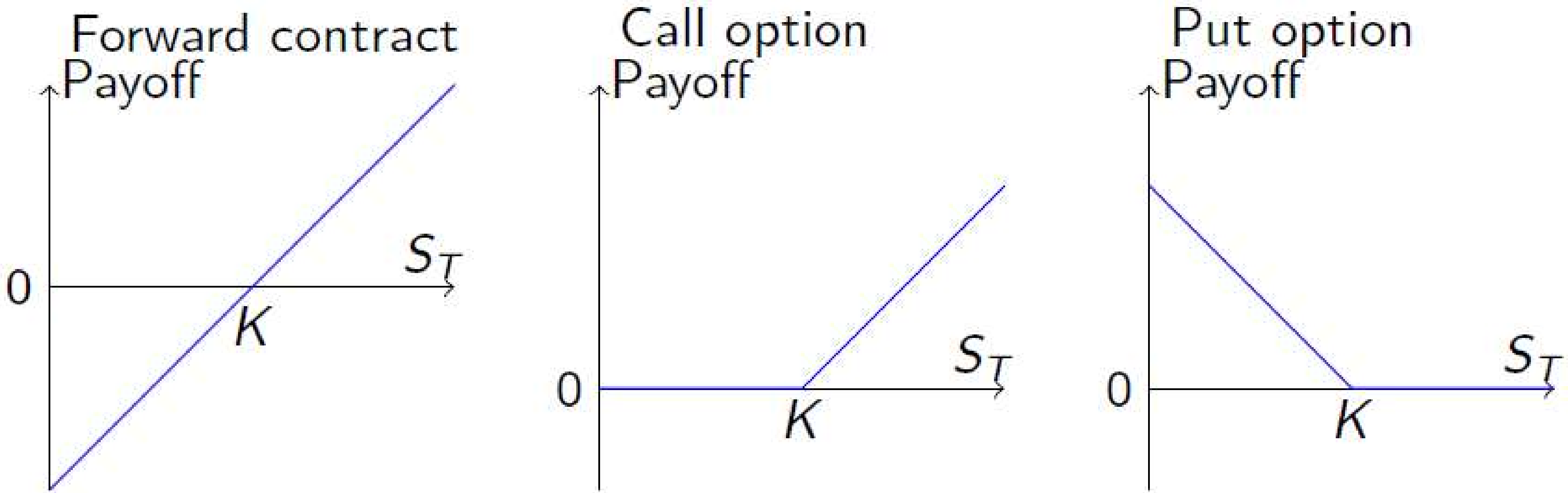
$$H_{E.Call}(S_T, K) = \max(0, S_T - K) = (S_T - K)^+$$

- Payoff of a European put option:

$$H_{E.Put}(S_T, K) = \max(0, K - S_T) = (K - S_T)^+$$

- Net profit of an option = payoff – initial price.

Payoffs



Forward contract
Payoff
0
K
S_T
Call option
Payoff
0
K
S_T
Put option
Payoff
0
K
S_T

Example

Open Interest = number of outstanding contracts

Daily volume of options traded

Bid and ask of the underlying stock

Volume of transactions in the underlying stock

Underlying stock: Amazon

AMZN **77.03 +1.40**

Jul 08 2009 @ 15:26 ET **Bid** 77.02 **Ask** 77.03 **Size** 1 x 3 **Vol** 6548487

Calls	Last Sale	Net	Bid	Ask	Vol	Open Int	Puts	Last Sale	Net	Bid	Ask	Vol	Open Int
09 Jul 70.00 (ZQN GN-E)	7.65	1.60	7.20	7.30	221	2637	09 Jul 70.00 (ZQN SN-E)	0.36	−0.18	0.36	0.38	684	11031
09 Jul 75.00 (ZQN GO-E)	3.35	0.86	3.20	3.30	943	6883	09 Jul 75.00 (ZQN SO-E)	1.30	−0.66	1.38	1.40	2394	15545
09 Jul 80.00 (QZN GP-E)	0.94	0.24	0.93	0.96	2456	9877	09 Jul 80.00 (QZN SP-E)	4.15	−1.05	4.00	4.10	700	10718
09 Jul 85.00 (QZN GQ-E)	0.22	0.07	0.19	0.21	497	26679	09 Jul 85.00 (QZN SQ-E)	8.25	−1.25	8.25	8.35	112	7215
09 Aug 70.00 (ZQN HN-E)	9.75	1.04	9.60	9.70	51	326	09 Aug 70.00 (ZQN TN-E)	2.77	−0.39	2.75	2.79	225	1979
09 Aug 75.00 (ZQN HO-E)	6.50	0.70	6.40	6.50	65	1108	09 Aug 75.00 (ZQN TO-E)	4.60	−0.55	4.55	4.60	2322	6832
09 Aug 80.00 (QZN HP-E)	4.00	0.50	3.90	4.00	172	2462	09 Aug 80.00 (QZN TP-E)	6.95	−0.95	7.05	7.15	145	2335
09 Aug 85.00 (QZN HQ-E)	2.15	0.15	2.22	2.26	833	5399	09 Aug 85.00 (QZN TQ-E)	10.15	−1.00	10.30	10.40	43	4599

Source: Chicago Board Options Exchange at www.cboe.com

Expiration dates and strikes of call options on Amazon

The bid-ask spread corresponds to the difference between the price at which there is a counterparty willing to buy the stock (bid) and the price at which there is a counterparty willing to sell it (ask).

Option moneyness

- If an option's payoff were positive, could the option be exercised immediately, this option is **in-the-money.**
  - A call option is in-the-money if $S_t > K$.
  - A put option is in-the-money if $S_t < K$.

Keywords:
In-the-money
At-the-money
Out-of-the-money

- If $S_t = K$, the option is **at-the-money**.

- If an option's payoff were zero, could the option be exercised immediately, and if $S_t \neq K$ this option is **out-of-the-money**.
  - A call option is out-of-the-money if $S_t < K$.
  - A put option is out-of-the-money if $S_t > K$.

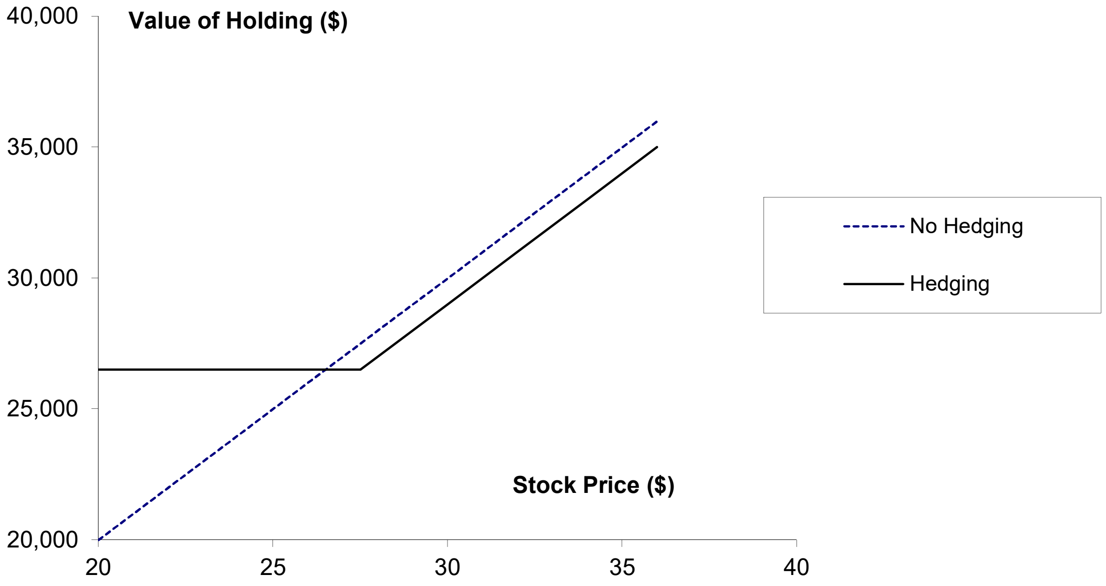
Moneyness and liquidity

- A call option that is far in-the-money resembles the underlying stock.
  - $S_t \gg K$ so $(S_t - K)^+ \approx S_t$

- An put option that is out-of-the-money ressembles an insurance product. It is cheap and insures the holder against a crash in the underlying stock/index.
  - $K \ll S_t$ so $(K - S_t) \ll 0$. The option will probably expire with a zero payoff, which justifies why it is cheap.
  - If there is a large decrease in S_t by the time the option expires, so that $(K - S_t) > 0$, the option expires in-the-money.

Hedging with options

Suppose you own 1000 shares of IBM stock. The share price is $28 per share. You are concerned about a possible share price decline in the next 2 months and want protection.



Assume a two-month put with a strike price of $27.50 costs $1.

Speculating with options

An investor with \$2,000 to invest feels that a stock price will increase over the next 2 months. The current stock price is \$20 and the price of a 2-month call option with a strike of 22.50 is \$1. What are the alternative strategies?

Speculating with options

An investor with \$2,000 to invest feels that a stock price will increase over the next 2 months. The current stock price is \$20 and the price of a 2-month call option with a strike of 22.50 is \$1. What are the alternative strategies? What is their net profit?

1. Buy 2000 call options. The profit at maturity is:

$$2000(S_T - 22.5)^+ - 2000$$

2. Buy 100 shares of stock. The profit at maturity is:

$$100S_T - 2000$$

Traders' jargon

- Long call = buy call options
- Long put = buy put options
- Short call = sell (=write) call options
- Short put = sell (=write) put options

Keywords:
Long/short positions

Exercise

What are the payoff and net profit of a short position in a call option?

Exercise

What is the payoff of a short position in a call option?

Solution: If you have a short position in a call option, you need, at maturity, to pay to the holder of the option (the person who bought it), the payoff of the option $(S_T - K)^+$. So your payoff is $-(S_T - K)^+$. There are 2 scenarios at maturity:

- $S_T > K$: the option expires in-the-money. Your payoff is

$$-(S_T - K) = K - S_T < 0$$

- $S_T \leq K$: the option expires worthless, your payoff is 0.

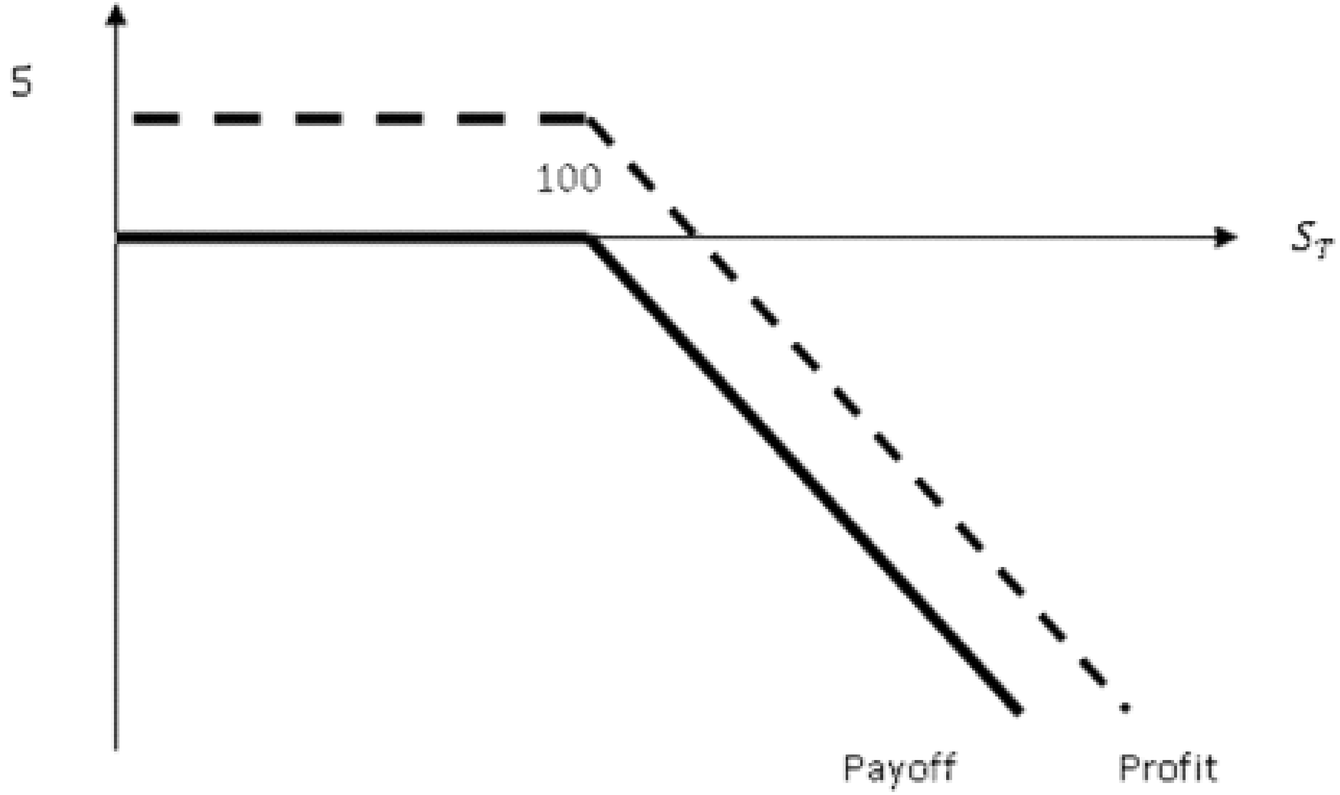
Therefore, your payoff can be written as

$$\min(K - S_T, 0)$$

The net profit is, with C_0 the initial price of the option:

$$\min(K - S_T, 0) + C_0$$

Profit from writing a call option



Strike price:

$$K = 100$$

Option price:

$$C_0 = 5$$

Exercise

What are the payoff and net profit of a short position in a put option?

Exercise

What is the payoff of a short position in a put option?

Solution: If you have a short position in a put option, you need, at maturity, to pay to the holder of the option (the person who bought it), the payoff of the option $(K - S_T)^+$. So your payoff is $-(K - S_T)^+$. There are 2 scenarios at maturity:

- $K > S_T$: the option expires in-the-money. Your payoff is

$$-(K - S_T) = S_T - K < 0$$

- $K \leq S_T$: the option expires worthless, your payoff is 0.

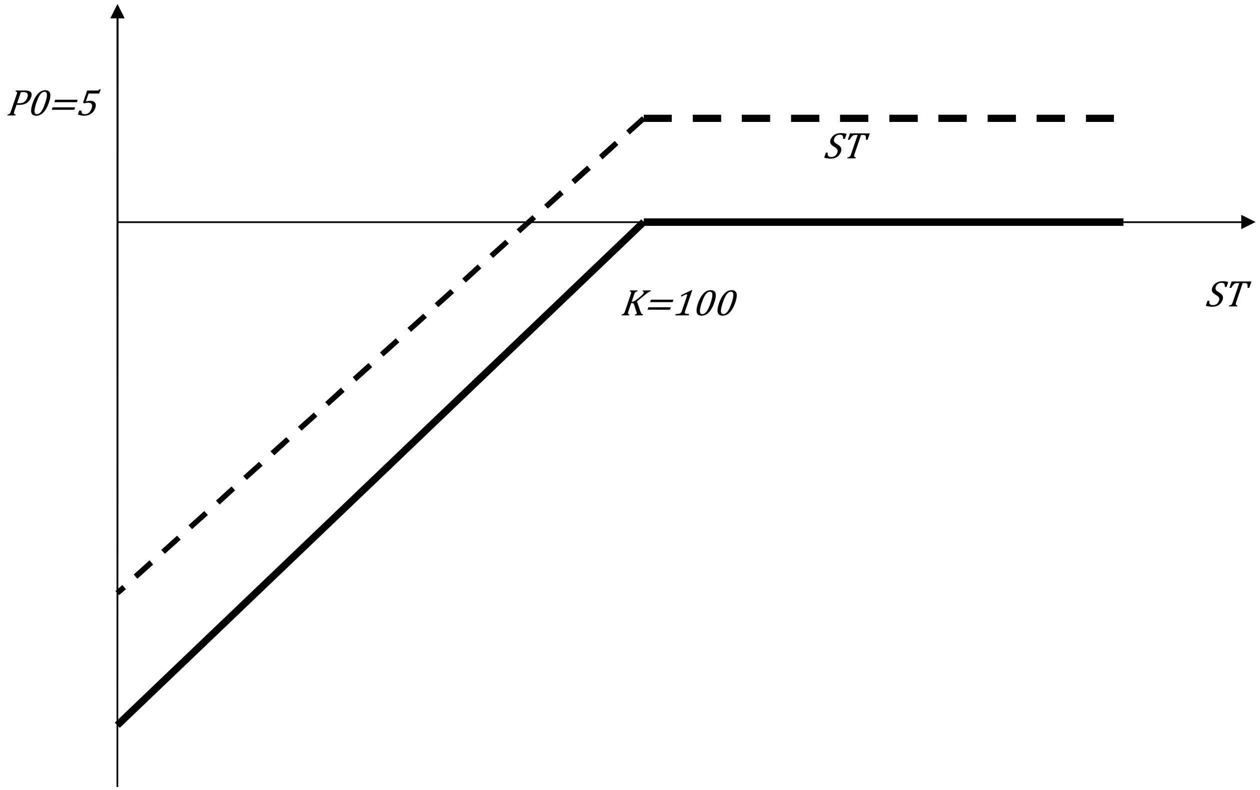
Therefore, your payoff can be written as

$$\min(S_T - K, 0)$$

The net profit is, with P_0 the initial price of the option:

$$\min(S_T - K, 0) + P_0$$

Profit from writing a put option



Strike price:

$$K = 100$$

Option price:

$$P_0 = 5$$

Example

Assume you decided to purchase each of the August put options quoted in the table on July 8, 2009, and you financed each position by shorting a two-month bond with a yield of 3%.

AMZN **77.03 +1.40**

Jul 08 2009 @ 15:26 ET **Bid** 77.02 **Ask** 77.03 **Size** 1 x 3 **Vol** 6548487

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Source: Chicago Board Options Exchange at www.cboe.com

Plot the profit of each position as a function of the stock price on expiration.

Example

					77.03	**+1.40**	
	Bid 77.02	**Ask** 77.03	**Size** 1 x 3			**Vol** 6548487	

Open Int	Puts	Last Sale	Net	Bid	Ask	Vol	Open Int
2637	09 Jul 70.00 (ZQN SN-E)	0.36	−0.18	0.36	0.38	684	11031
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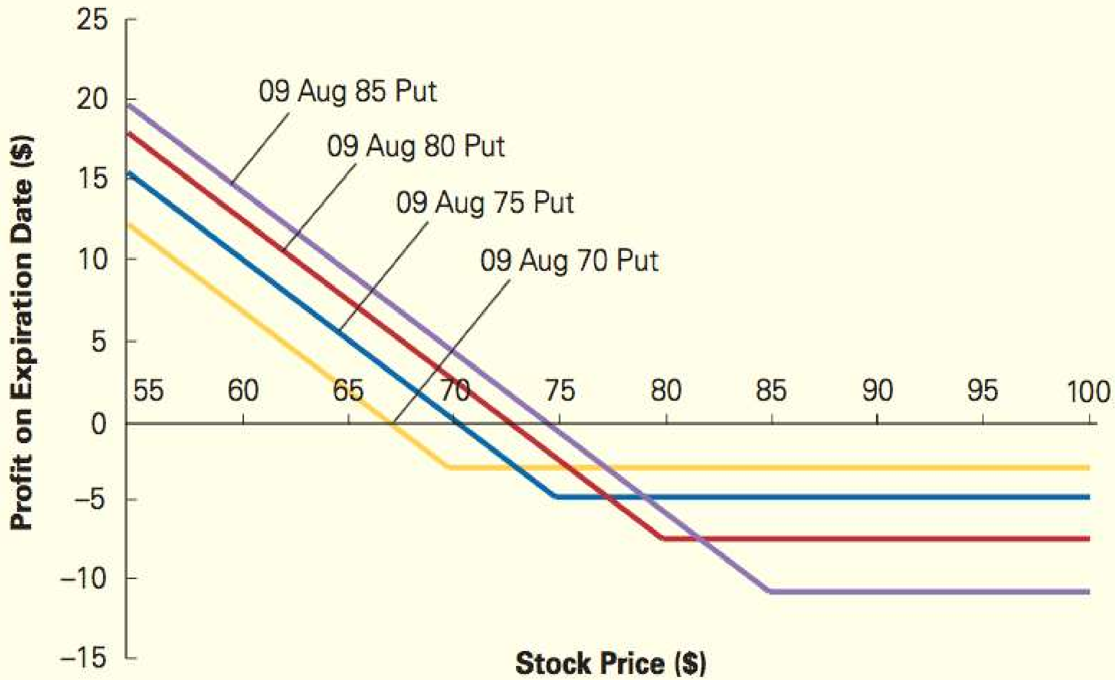
Solution

Suppose S is the stock price on expiration, K is the strike price, and P is the price of each put option on July 8. Then your cash flows on the expiration date will be

$$\max(K - S, 0) - P \times 1.03^{45/365}$$

The plot is shown below. Note the same trade-off between the maximum loss and the potential for profit as for the call options.



Portfolios of stocks and options

- What is the payoff of a portfolio with a put and the underlying stock?

Portfolios of stocks and options

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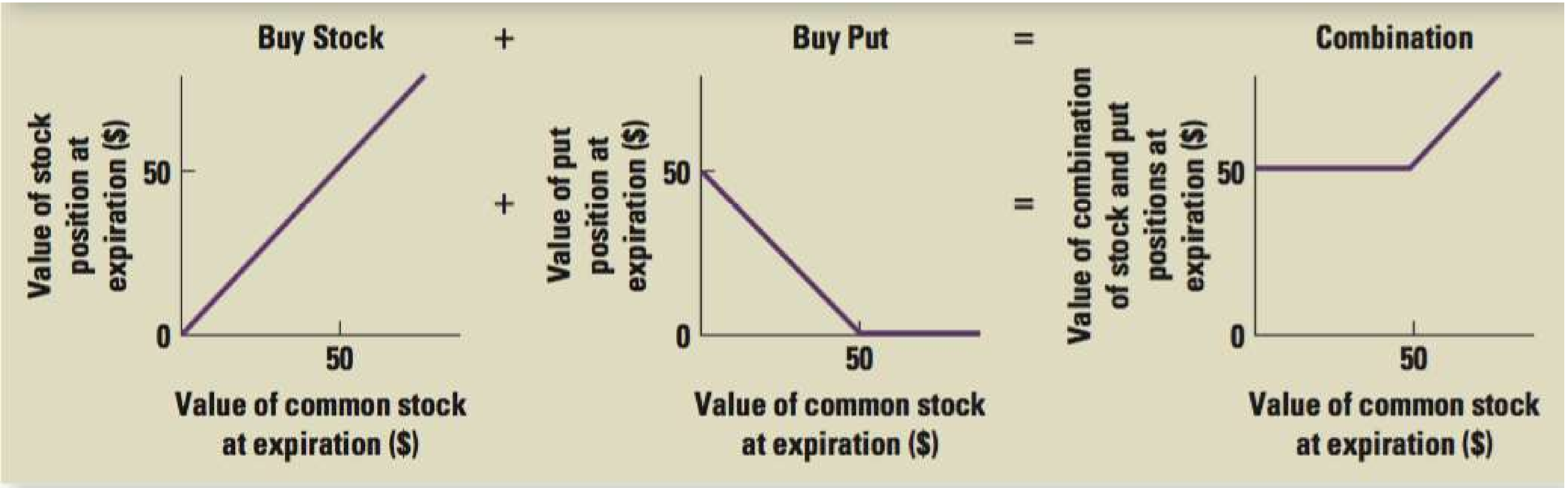

Portfolios of stocks and options

- What is the payoff of a portfolio with a short position in a call and the underlying stock?

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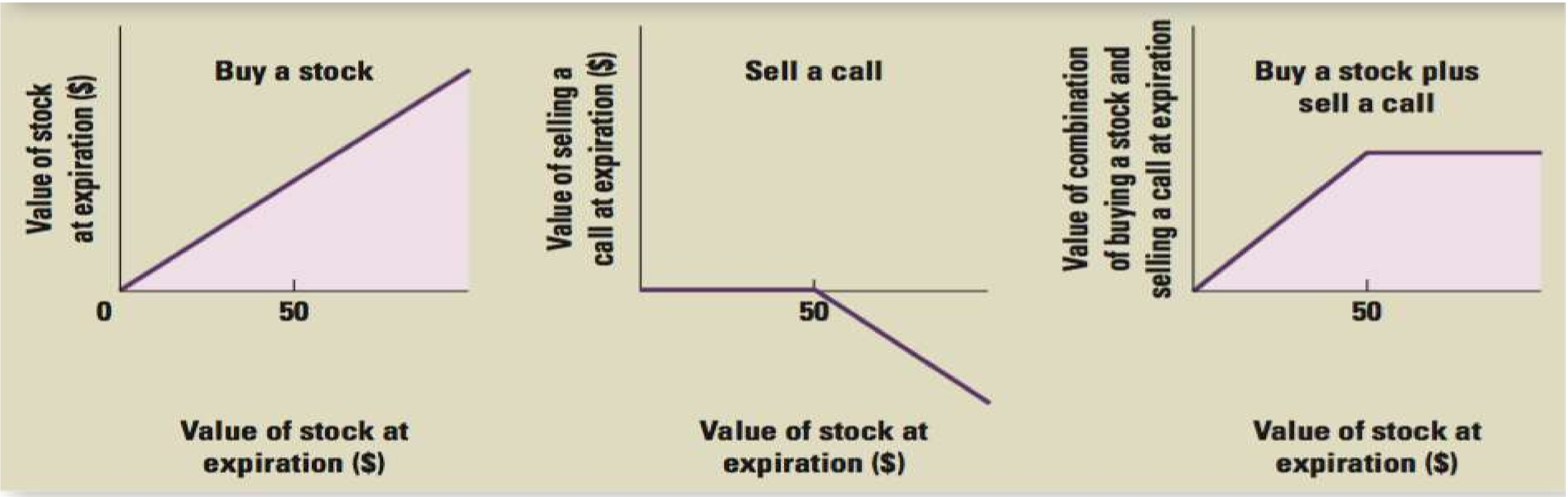
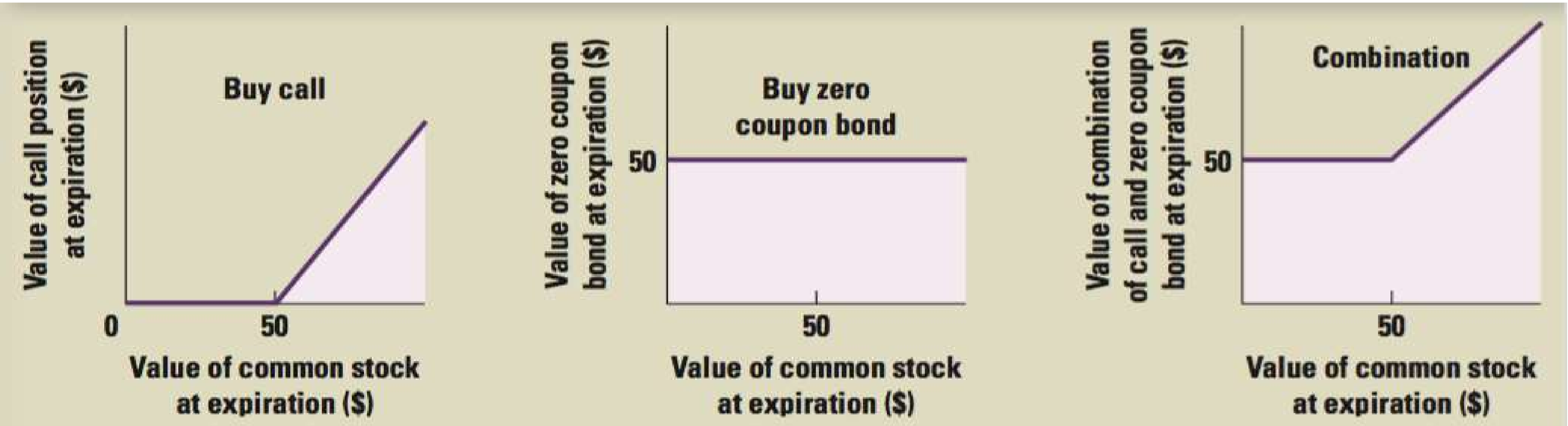


Portfolios of stocks and options

- What is the payoff of a portfolio that contains a call and a zero coupon bond?

Portfolios of stocks and options

- What is the payoff of a portfolio that contains a call and a zero coupon bond?



The graph of buying a call and buying a zero coupon bond is the same as the graph of buying a put and buying the stock in Figure 22.4

American vs. European options

Keywords: American/European options

- An **American call option** on the stock S is a contract which gives the holder of the option the right but not the obligation to buy the stock at any point in time in the future up to maturity T for a fixed price (strike price) K.

$$Price(American\ option) \geq Price(European\ option)$$

- One can prove that it is optimal for the holder of American call to exercise at maturity, indicating that American calls have the same price as European calls.
- However, early exercise matters for American puts, whose prices are at least as much as European puts.

Option valuation principles

Determinants of option value

1. The value of the option depends on the value of the underlying security:

Call option: For a given strike, if the value of the underlying stock increases,

- The probability of achieving a high payoff on exercise increases
- The value of the call option increases.

Put option: For a given strike, if the value of the underlying stock increases,

- The probability of achieving a high payoff on exercise decreases
- The value of the put option decreases.

Determinants of option value

2. The value of the option depends on the strike:

Call option: For a given price of the underlying stock, if the value of the strike increases,

- The probability of achieving a high payoff on exercise decreases
- The value of the call option decreases.

Put option: For a given price of the underlying stock, if the value of the strike increases,

- The probability of achieving a high payoff on exercise increases
- The value of the put option increases.

Determinants of option value

3. The value of the option depends on the maturity of the option:

American options:

- When the maturity increases, the holder has more opportunities exercising the option.
- The value of the option (call or put) increases.

European options:

- When the maturity increases, it usually increases the probability to achieve a higher payoff and therefore increases the value of the option.
- However, if a dividend is expected to be paid (in which case holding the stock would be desirable), increasing the maturity may decrease the value of the option.

Determinants of option value

4. The value of the option depends on the volatility of the underlying stock returns:

- When the volatility increases, it increases the probability that the underlying will be higher (call), or lower (put) at maturity
- Therefore it increases the probability of reaching a higher payoff
- The value of the option (call or put) increases.

Determinants of option value

5. The value of the option depends on the interest rate:

Call options: Holding all the other variables constant, when the interest rate increases

- The present value of the strike (which the holder of the option will pay upon exercise of the option) decreases
- The call option value will increase.

Put options: When the interest rate increases

- The present value of the strike (which the holder of the option will receive upon exercise) decreases.
- The put option value decreases.

Determinants of option value

6. The value of the option depends on the amount of future dividends:

- The stock price decreases on the ex-dividend day.
- The value of the call option will therefore decrease.
- The value of the put option will increase.

Determinants of option value

Variable	European Call	European Put	American Call	American Put
S_0	+	–	+	–
K	–	+	–	+
T	?	?	+	+
σ	+	+	+	+
r	+	–	+	–
D	–	+	–	+

Intrinsic value versus time value

- The **intrinsic value** of an option is the value of the option were it immediately exercised or discarded. This value is always ≥ 0.
  - The intrinsic value of a call option is $Max(S_t - K, 0)$.
  - The intrinsic value of a put option is $Max(K - S_t, 0)$.
  - If the intrinsic value of an option is strictly positive, it is equivalent to the option being in-the-money.
- **The time value** of an option is the difference between the value of the option and its intrinsic value.
  - For a call it is $C_t - Max(S_t - K, 0)$.
  - For a put it is $P_t - Max(K - S_t, 0)$.
- At expiration, the time value of an option is equal to zero and its price (= premium) is equal to the intrinsic value.

Put-call parity

Put-call parity for European options for non-dividend paying stocks

The put-call parity links the price of a European put to the price of a European call:

$$C_0 + Ke^{-rT} = P_0 + S_0$$

Value of call option with strike K and maturity T

Continuously compounded interest rate

Value of put option with strike K and maturity T

Value of underlying stock

The put and the call options need to have the same underlying, i.e., the stock with value S_t.

Example

A Synthetic T-Bill Suppose shares of stock in Smolira Corp. are selling for \$110. A call option on Smolira with one year to maturity and a \$110 strike price sells for \$15. A put with the same terms sells for \$5. What's the risk-free rate?

To answer, we need to use put–call parity to determine the price of a risk-free, zero coupon bond:

$$\text{Price of underlying stock} + \text{Price of put} - \text{Price of call} = \text{Present value of exercise price}$$

Plugging in the numbers, we get:

$$\$110 + \$5 - \$15 = \$100$$

Because the present value of the \$110 strike price is \$100, the implied risk-free rate is 10 percent.

Example

Problem

You are an options dealer who deals in non-publicly traded options. One of your clients wants to purchase a one-year European call option on HAL Computer Systems stock with a strike price of \$20. Another dealer is willing to write a one-year European put option on HAL stock with a strike price of \$20, and sell you the put option for a price of \$3.50 per share. If HAL pays no dividends and is currently trading for \$18 per share, and if the risk-free interest rate is 6%, what is the lowest price you can charge for the option and guarantee yourself a profit?

Solution

Using put-call parity, we can replicate the payoff of the one-year call option with a strike price of \$20 by holding the following portfolio: Buy the one-year put option with a strike price of \$20 from the dealer, buy the stock, and sell a one-year risk-free zero-coupon bond with a face value of \$20. With this combination, we have the following final payoff depending on the final price of HAL stock in one year, S_1:

	Final HAL Stock Price	
	$S_1 < \$20$	$S_1 > \$20$
Buy Put Option	$20 - S_1$	0
Buy Stock	S_1	S_1
Sell Bond	-20	-20
Portfolio	0	$S_1 - 20$
Sell Call Option	0	$-(S_1 - 20)$
Total Payoff	0	0

Note that the final payoff of the portfolio of the three securities matches the payoff of a call option. Therefore, we can sell the call option to our client and have future payoff of zero no matter what happens. Doing so is worthwhile as long as we can sell the call option for more than the cost of the portfolio, which is

$$P + S - PV(K) = \$3.50 + \$18 - \$20/1.06 = \$2.632$$