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Informative Advertising with Differentiated Products

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In this paper we study the role of promotional expenditures by sellers in a model of product differentiation. Advertising conveys full and accurate information about the characteristics of products. Heterogeneous consumers, who have no source of information other than advertisements, seek to purchase the products that best fit their needs. Despite the roles played by advertising in improving the matching of products and consumers, and in increasing the elasticity of demand faced by each firm, we find that the market-determined levels of advertising are excessive, given the extent of diversity in the market.

We derive a promotional equilibrium based on a specific information transmission technology, paying explicit attention to the structure of consumer information and its impact on firms' demand curves. This allows us to study the effects of changes in the advertising technology, including an increased ability to target messages to specific groups of consumers, on the equilibrium in the product market. We find that decreased advertising costs may reduce profits by increasing the severity of price competition.

1. INTRODUCTION

An important aspect of competition in differentiated product markets is promotional competition. The presently rapid rate of change in the technology of advertising, and information transmission generally, suggests that the importance of this type of competition can only grow in the future.

The goal of this paper is to investigate the role of advertising in markets with product differentiation. One question of particular interest regards the effect that changes in advertising costs have on the market equilibrium. We hope to identify, for example, the various impacts that improvements in the ability to target messages to specific groups of consumers will have on consumer product markets. Continued improvement in targeting opportunities is likely to be an outgrowth of the current expansion in the set of available media, which makes narrow-casting more and more possible.

It is sometimes maintained that advertising levels are excessive; i.e. greater than those which would prevail in the socially optimal allocation. Our second major objective is to see whether, in the context of a formal model of a differentiated-product market, this view is borne out by a welfare analysis.

One reason that advertising levels are thought to be excessive is that advertising influences consumer preferences; i.e. it is "persuasive." Dixit and Norman (1978, 1981) have shown that advertising of this sort is carried out at an excessive level in the market under fairly weak conditions, even if the post-advertising demands are used to measure consumer surplus.

We do not wish to deal with persuasive advertising in what follows. Rather, we model advertising in a way that takes a very favourable view of its content. We assume that an advertisement provides full and truthful information about the product it promotes, while leaving consumers' tastes unchanged. Furthermore, we assume that the consumer has no alternative sources of information, and is unaware of the existence of a particular brand unless she sees an ad describing it.[1]

Our approach is to embed this view of advertising in a familiar model of product differentiation. In this setting, advertising is used by each firm as a competitive tool to attract customers away from other firms. The view that advertising redistributes consumers among firms underlies the popular perception of wasteful advertising. Where products are *homogeneous*, this socially unproductive expense of resources is undoubtedly the predominant effect of advertising, because once a consumer is aware of at least one brand, the social benefits of more ads to that consumer are zero (if all firms charge the same price) or small, and are most probably smaller than the private returns. But studying advertising in a homogeneous product setting is inappropriate and possibly misleading. If products are *heterogeneous*, we must take into account the social benefit deriving from the improved matching of consumers and brands that occurs when consumer information is more complete. In addition, advertising may play a role in making the market more "competitive" by increasing the elasticity of demand for each particular brand.

In the analysis below, we are able to compare the sizes of these various effects, and judge which is likely to be more important. We study the nature of the biases in the market equilibrium under the alternative market-structure assumptions of oligopoly (with a fixed number of firms) and free-entry (with zero profits). The oligopoly treatment may be viewed as a short-run analysis of a market in which there are no entry barriers.

A novel aspect of our analysis is the careful attention we pay to the transmission of information from the individual firm to the individual consumer. We consider the demand curve facing each firm as a function of the prices and advertising intensities of other firms. This demand curve is derived from the underlying preferences of consumers across brands, and the technology by which consumers receive messages (ads) from sellers. This approach, which is in contrast to the common reduced form specification in which advertising expenditures appear as a parameter in the demand function, allows us to study changes in advertising more directly.

In the next section we derive the individual firm's demand curve and compute the oligopoly and monopolistically competitive equilibria. In Section 3 we study the comparative static properties of these equilibria. Then we turn to the welfare analysis in Section 4. A brief conclusion follows.

2. A MODEL OF ADVERTISING AND PRODUCT DIFFERENTIATION

In this section we specify a model of advertising in a market for heterogeneous products. We choose a familiar type of product diffentiation in order to focus attention on the role played by advertising in the competitive equilibrium.

A. *A description of the model*

The product space and consumer preferences

We seek to study advertising in a context where it plays a role in matching heterogeneous consumers to products that suit their tastes.

To do so, we employ the product characteristics approach to product differentiation associated with Lancaster (1975), and developed by Salop (1979).[2] By analogy to spatial competition, we represent each product as a point on a circle of unit length.[3] Each consumer is identified by the point on the circle which corresponds to her most preferred brand. We assume that each consumer has use for at most one unit in the product class being studied, and attaches a gross dollar value of v to a unit of her most preferred brand. A product which is at a distance z along the circle from a consumer provides a dollar benefit of v-tz, where we refer to t as the transport cost (per unit distance), bearing in mind the locational competition interpretation of the circle model. The parameter t, then, measures the sensitivity of consumers to product characteristics.

The consumer surplus enjoyed by a consumer who purchases a product a distance z units away at price p is v-tz-p. Consumers purchase a unit in this product class only if they are aware of the existence of a brand offering positive surplus. If the consumer knows of several such brands, she selects that brand offering the greatest surplus. Finally, we assume that consumers are uniformly distributed around the circle with a constant density of δ consumers per unit length.

The production technology

The importance of economies of scale in the context of differentiated products is well understood; there is a basic tradeoff between scale economies and the value of diversity. We employ, therefore, a very simple cost structure exhibiting increasing returns to scale. A fixed cost, $F > 0$, is required to develop a product and to set up the facilities to produce, distribute and sell the brand. Actual production is subject to constant returns to scale with a marginal cost of $c > 0$. Each firm produces at most a single brand.

Information structure

We focus on product-specific information as supplied by sellers. To do so, we employ an information technology similar to that in Butters (1977). Consumers, who may or may not know the structure of the product market (that is, that there are n symmetrically-distributed brands on the unit circle of product characteristics), rely on information garnered from ads to locate specific brands in product space. In other words, a consumer may know that a product with certain characteristics exists in the market, but she does not know *a priori* which firm produces and sells goods of a given specification. An advertisement tells the consumer the characteristics and price of a particular brand. We assume that consumers remember all advertising messages that are successfully transmitted to them.

Each consumer is passive in that she does not search for a brand which suits her taste, nor does she engage in other active information acquisition activities. Implicitly, we are assuming that the cost of search is high relative to the surplus offered by goods in this product group. Thus, each consumer has information about all and only those brands which she has seen advertised at least once.[4]

Advertising technology

Advertising messages are produced by the same firms that manufacutre the goods. Firms choose to make the content of their messages truthful, as our model provides no incentive for deceptive advertising.

Initially, we assume that a seller has no ability to target messages towards those consumers who find his product most attractive (i.e. those consumers nearby on the circle). The costs of advertising are left quite general. The per capita expenditure needed to achieve a reach of ϕ, $0 \leqq \phi \leqq 1$, is denoted by $A(\phi; \alpha)$. Here α is a shift parameter which we use below to investigate changes in the advertising technology. A reach of ϕ

means that a fraction ϕ of the target population is exposed to the message (at least once). Given our no targeting assumption, this implies that each consumer receives the message with probability ϕ. Since the total population size is δ, the advertising costs (for a given firm) are given by $\delta A(\phi; \alpha)$.

We assume that $A_\phi > 0$ and $A_{\phi\phi} > 0$. The latter assumption reflects the notion that it becomes increasingly expensive to reach higher fractions of the population, either because preferred media become saturated, or because the target population is heterogeneous along a second dimension, namely, the tendency to view ads. We also assume, without loss of generality, that $A(0; \alpha) = 0$.[5]

Advertising technology: An example

In order to understand the nature of the advertising cost function $A(\phi, \alpha)$ and to show that our assumptions about it are natural ones, we derive an example for a specific technology of message production. Consider the case where an advertiser can place ads in any of a set of magazines or newspapers. Assume that each magazine has a readership of δr in the target population of size δ, so that a fraction r of the population is exposed to an ad published in any given magazine.[6]

Let us assume that the probability that a given consumer sees an ad in one magazine is independent of the probability that she sees an ad in another magazine; that is, that different magazines have independent readerships. Then if the advertiser places ads in m magazines, the probability that a given consumer will see *none* of these ads is $(1-r)^m$. The reach of such an advertising campaign is $\phi = 1-(1-r)^m$. Equivalently, we can write the number of magazines in which ads must be placed in order to achieve a reach of ϕ as $m = \log(1-\phi)/\log(1-r)$.

What is the cost of achieving a given reach of ϕ? Let us suppose that each magazine charges a fixed amount, a, per reader. Then placing an ad in a single magazine will cost $a\delta r$, and the cost of a campaign running in m magazines is $a\delta rm$. Therefore, the cost of achieving a reach of ϕ, which requires $m = \log(1-\phi)/\log(1-r)$ ads, is

$$\delta A(\phi; \alpha) = \frac{a\delta r \log(1-\phi)}{\log(1-r)}. \tag{1}$$

We will refer to this advertising cost function as the constant-reach, independent-readership (CRIR) technology.[7] Note that this function has the properties we assume for $A(\phi; \alpha)$; that is, $A(0; \alpha) = 0$, $A_\phi > 0$, $A_{\phi\phi} > 0$. In this example, the shift parameter, α, can be identified with either a or r.

Equilibrium concept

The market equilibrium studied below is the non-cooperative Nash equilibrium in prices, p, and advertising intensities, ϕ. This is an extension of the classic equilibrium concept of monopolistic competition pioneered by Chamberlin (1931) to include advertising as a strategic variable. We study only symmetric equilibria. In such an equilibrium each firm takes as given the prices $\bar{p}$ and advertising levels $\bar{\phi}$ chosen by all other firms, and selects its own p and ϕ to maximize profits. We do not study location or product choice competition. Instead, we assume (again following Salop, 1979) that whatever the number of firms happens to be, they are equally spaced around the circle.

The symmetric equilibrium which takes as given the number of firms is called the oligopoly equilibrium. If free entry and exit are allowed, profits are driven to zero, and the number of firms is endogenously determined. We refer to this specification as the monopolistically competitive equilibrium.

B. *Equilibrium in the model*

We turn now to the actual computation of equilibrium, both with a fixed number of firms and with free entry. A necessary first step is the derivation of the demand curve facing the representative firm, $x(p, \phi)$, given the prices $\bar{p}$ and advertising levels $\bar{\phi}$ of the other firms. This calculation is more complex than under perfect information, because competition is no longer localized. A given firm may sell to a consumer far away from it on the circle, if that consumer has not received an ad offering a more attractive sale.

Taking the number of firms as given for the moment, and given the representative firm's price, p, and that of its $(n-1)$ rivals, $\bar{p}$, consumers can be partitioned into n groups, $k=1,\ldots,n$, where the kth group is that set of consumers to whom the representative firm would offer the kth highest surplus of the n firms, were all consumers perfectly informed. This partition is a useful one, because the probability that an ad results in a sale will depend below only on the identity of the group to which the consumer receiving the ad belongs. We now compute the sizes, N_k, of these n groups.

A consumer who is at distance z from the representative firm could achieve surplus v-tz-p by purchasing from that firm. For $0 \leqq z \leqq 1/n$, i.e. for consumers between the firm and its nearest neighbour, the best alternative to the representative firm is the neighbour, who offers surplus of $v-t(1/n-z)-\bar{p}$. Let z_1 be the location of the consumer who would be indifferent between these two alternatives if she were fully informed. z_1 is depicted in Figure 1, and is given algebraically by

$$v-tz_1-p=v-t\left(\frac{1}{n}-z_1\right)-\bar{p},$$

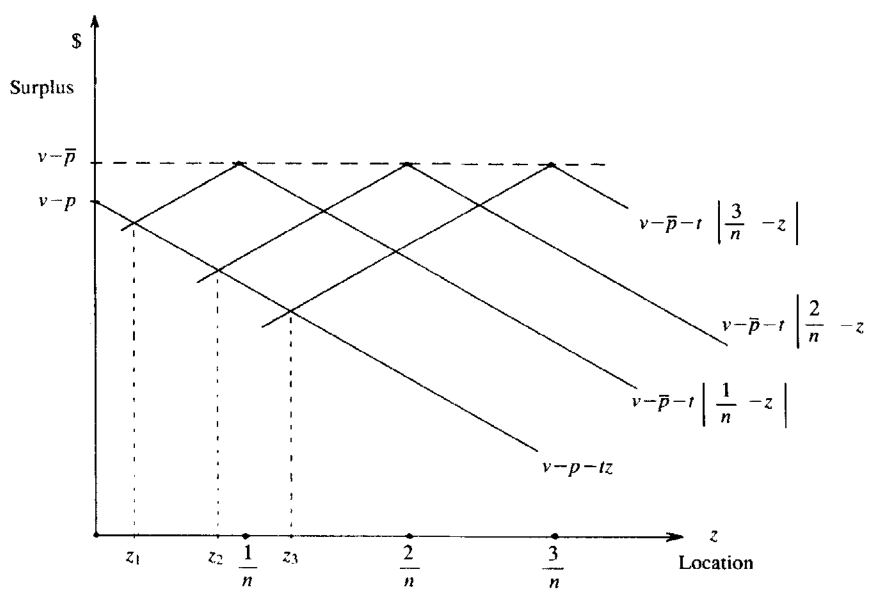

FIGURE 1

Surplus offered by representative firm and its nearest neighbours or

$$z_1=\frac{\bar{p}-p}{2t}+\frac{1}{2n}.$$

All those consumers with $z \leqq z_1$ would, if fully informed, find the product of the representative firm to be their first choice.[8] The total number of these consumers, counting consumers located on either side of the firms, is

$$N_1=2\delta z_1=\frac{\delta(\bar{p}-p)}{t}+\frac{\delta}{n}. \tag{2}$$

Next consider the location of the consumer that would be indifferent between purchasing the product of the representative firm and that of the next closest neighbour (in the same direction). This consumer is at distance z_2 from the firm (again, see Figure 1), where

$$v-tz_2-p=v-t\left(\frac{2}{n}-z_2\right)-\bar{p}.$$

Therefore,

$$z_2=\frac{\bar{p}-p}{2t}+\frac{1}{n}.$$

Those consumers between z_1 and z_2 would, with complete information, find the product of the representative firm to be their second-most preferred. The number of consumers in this second group is $N_2=2\delta(z_2-z_1)$, or, since $z_2-z_1=1/2n$, $N_2=\delta/n$. Repeating this procedure yields

$$z_k=\frac{\bar{p}-p}{2t}+\frac{k}{2n} \qquad k=1,2,\ldots,n-1,$$

and thus

$$N_k=\frac{\delta}{n} \qquad k=2,\ldots,n-1. \tag{3}$$

Finally, the n-th group comprises all those consumers not in the other $(n-1)$ groups, i.e. $N_n=\delta-\sum_{k=1}^{n-1} N_k$, or

$$N_n=\frac{\delta}{n}-\frac{\delta(\bar{p}-p)}{t}. \tag{4}$$

We are now prepared to compute the demand curve facing the representative firm. If the probability of making a sale to a consumer in the k-th group is ϕ_k, then demand is given by

$$x(p,\phi)=N_1\phi_1+N_2\phi_2+\cdots+N_n\phi_n.$$

A sale is always made to a consumer in the first group if she receives an ad, because no other firm offers as much surplus to consumers in this group (by definitior). Therefore, $\phi_1=\phi$. Likewise, an ad to a consumer in the second group results in a she if and only if she has not received an ad from her most preferred brand. This occurs an probability $(1-\bar{\phi})$, and, thus, $\phi_2=\phi(1-\bar{\phi})$. Now, if we assume that v is sufficier rge so that the constraint that consumer's surplus be non-negative is never binders then an ad transmitted to a member of any group results in a sale for the representative firm whenever the potential customer receives no better offer.[9] In general, we have

$$\phi_k = \phi(1-\bar{\phi})^{k-1} \qquad k = 1, 2, \ldots, n. \tag{5}$$

Substituting for the N_k's from (2), (3), and (4), and for the ϕ_k's from (5), into the expression for demand, and summing, gives

$$x(p, \phi) = \frac{\delta\phi(\bar{p}-p)}{t}[1-(1-\bar{\phi})^{n-1}] + \frac{\delta\phi}{n\bar{\phi}}[1-(1-\bar{\phi})^n]. \tag{6}$$

Finally, for reasonable parameter values such that $(1-\bar{\phi})^n$ is small,[10] an excellent approximation to (6) is

$$x(p, \phi) = \frac{\delta\phi(\bar{p}-p)}{t} + \frac{\delta\phi}{n\bar{\phi}}. \tag{7}$$

It is instructive to note that the elasticity of demand, ε, evaluated at $p = \bar{p}$, is given by $\varepsilon = \bar{p}n\bar{\phi}/t$. As usual, a greater number of firms raises the elasticity of demand faced by any given firm. What is new is the appearance of $\bar{\phi}$ in the expression for ε. This reflects the intuitive notion that improved information (i.e. a large $\bar{\phi}$), increases demand elasticities (and thus reduces prices).

The representative firm's optimal policy (i.e. choice of p and ϕ, given $\bar{p}$, $\bar{\phi}$ and n) is now readily computed. Profits for this firm are

$$\pi(p, \phi) = (p-c)x(p, \phi) - F - \delta A(\phi). \tag{8}$$

Using the approximation in (7), one first order condition (FOC) is

$$\pi_p(p, \phi) = (p-c)\left(\frac{-\delta\phi}{t}\right) + \frac{\delta\phi(\bar{p}-p)}{t} + \frac{\delta\phi}{n\bar{\phi}} = 0.$$

Solving this equation for p yields

$$p = \frac{\bar{p}+c}{2} + \frac{t}{2n\bar{\phi}}. \tag{9}$$

Note that this formula collapses to that derived by Salop (1979) under perfect information, when we replace $\bar{\phi}$ by 1. We find, therefore, that imperfect information, by reducing demand elasticities, raises markups.

The second FOC is given by

$$\pi_\phi(p, \phi) = \delta(p-c)\left[\frac{(\bar{p}-p)}{t} + \frac{1}{n\bar{\phi}}\right] - \delta A_\phi(\phi, \alpha) = 0.$$

In the symmetric equilibrium $p = \bar{p}$, so we get

$$\frac{p-c}{n\bar{\phi}} = A_\phi(\phi; \alpha). \tag{10}$$

This condition has an intuitive interpretation. Since $\phi\delta$ consumers receive the representative firm's message, a small increase in ϕ of $d\phi = 1/\delta$ will inform (on average) one more consumer. The probability that the consumer so informed will purchase from the firm is $(1/n)[1+(1-\bar{\phi})+(1-\bar{\phi})^2+\cdots+(1-\bar{\phi})^{n-1}] \approx 1/n\bar{\phi}$. Therefore, the small increase in ϕ will generate $1/n\bar{\phi}$ sales, which augments profits by $(p-c)/n\bar{\phi}$. Equation (10) states that this marginal private gain be equal to the marginal private cost, $A_\phi(\phi; \alpha)$.

If we replace $\bar{\phi}$ and $\bar{p}$, in (9) and (10) by ϕ and p, we obtain two equations that determine the symmetric oligopoly equilibrium, given the exogenous variables (including n). These two equations are

$$p-c=\frac{t}{n\phi}, \tag{11a}$$

$$p-c=n\phi A_{\phi}(\phi, \alpha). \tag{11b}$$

The symmetric, monopolistically competitive equilibrium is determined by these two equations, and the free-entry condition, which says that profits are zero. This equation, which makes n now an endogenous variable, is given by[12]

$$\frac{(p-c)\delta}{n}=F+\delta A(\phi; \alpha). \tag{11c}$$

This completes the derivation of the oligopoly and monopolistically competitive equilibria. We turn now to an investigation of their comparative static properties (which are summarized in Table I below).

3. COMPARATIVE STATICS

A. *Oligopoly*

The effects of changes in the exogenous parameters on the oligopoly equilibrium prices and advertising levels are most easily derived by first substituting (11a) into (11b) to give a single equation in ϕ:

$$\phi^2 A_{\phi}(\phi; \alpha)=t/n^2. \tag{12}$$

An increase in the number of firms (and brands) leads to a reduction in equilibrium advertising intensities (since $A_{\phi\phi}>0$, the left-hand side of (12) is increasing in ϕ). This is because an increase in competition tends to reduce the yield on a given firm's ads, as customers are more likely to get word of a brand better suited to their tastes. This is consistent with the observation that advertising is especially important in industries that are relatively concentrated.

The effect of product diversity (n) on markups is as expected: $d(p-c)/dn<0$. This can be seen by rewriting (12) as

$$(n\phi)^2 A_{\phi}(\phi, \alpha)=t, \tag{12'}$$

and noting that as n increases, since ϕ and therefore $A_{\phi}(\phi; \alpha)$ fall, $n\phi$ must rise. Inspection of (11a) reveals that a rise in $n\phi$ implies a decline in the markup. In other words, the direct, positive effect on competition caused by an increase in the number of firms dominates the induced negative effect associated with the consequent reduction in advertising intensities.

Increases in t, which reflect increased sensitivity of consumers to product specifications, lead to increases in ϕ (by (12)). Thus, when product differentiation is more important, so too is advertising. Markups also increase with transport costs, due to the fact that the elasticity of demand falls as t rises.

Finally, we consider changes in the advertising technology. By the usual convention, we choose our parameterization such that $A_{\alpha}>0$ and $A_{\phi\alpha}>0$. Increases in α correspond to increases in total and marginal advertising costs. In our example of the CRIR advertising technology, α might represent the cost per exposure (which we called a). Then $A(\phi, \alpha)$

would be of the form $\alpha\tilde{A}(\phi)$. For future reference, we define the elasticity of A_α with respect to ϕ, $\beta = \phi A_{\phi\alpha}/A_\alpha$, and the elasticity of A_ϕ with respect to ϕ, $\eta = \phi A_{\phi\phi}/A_\phi$. In the case of $A(\phi, \alpha) = \alpha\tilde{A}(\phi)$, $A_{\phi\phi} > 0$ implies that $\beta > 1$. In all cases, $\beta > 0$ and $\eta > 0$.

Changes in the cost of advertising are particularly relevant today due to the introduction of new media (e.g. cable television), which permit advertisers to send messages only to those consumers who are most likely to purchase the product. Suppose that consumers around the circle constitute only a fraction of the overall consumer population, and that consumers not located on the circle attach no value to goods in this product group. If we interpret α as the fraction of messages which go to consumers who are *not* interested in the product group, then an increased ability to target advertisements to the relevant population is captured by a decrease in α. Targeting leads to a decline in the effective cost of getting a message to the relevant consumers. Alternatively, we can think of targeting as the ability of a firm to send ads specifically to those consumers who lie within a distance $\lambda < \frac{1}{2}$ of the firm's product on the circle. This is also formally equivalent to a decrease in α, because ads sent to consumers far away on the circle, which were formerly wasted so long as λn is moderately large, are no longer sent. We are therefore able to study the effect on product markets of media proliferation (and of improved information about the correspondence between demographic characteristics and demand, as needed for targeting) via changes in the advertising cost function.

Totally differentiating equation (12′), we find that $d\phi/d\alpha = -A_{\phi\alpha}/(2A_\phi + \phi A_{\phi\phi}) < 0$; i.e. if the marginal cost of reach rises, the reach (per firm) in the market falls. Then, differentiating (11a) and substituting (12) gives $d(p-c)/d\alpha = -nA_\phi(d\phi/d\alpha) > 0$; increases in advertising costs cause markups to rise. This reflects the pro-competitive effect of advertising via improved information.

Do profits per firm rise or fall with increases in the cost of advertising? Writing the change in profits per firm as

$$\frac{d\pi}{d\alpha} = (p-c)\frac{dx}{d\alpha} + x\frac{dp}{d\alpha} - \delta A_\phi \frac{d\phi}{d\alpha} - \delta A_\alpha,$$

and recognizing that across symmetric, large-group equilibria, $dx/d\alpha = 0$, it follows that

$$\frac{d\pi}{d\alpha} = \frac{\delta A_\alpha}{\eta + 2}[2(\beta - 1) - \eta]. \tag{13}$$

The expression in the brackets cannot be signed in general. On the one hand, an increase in α directly increases costs, and thus tends to reduce profits. On the other hand, an increase in advertising costs reduces the degree of competition as measured by demand elasticities and hence increases markups. For example, if $A(\phi, \alpha)$ can be written as $\alpha\tilde{A}(\phi)$, then $\beta > 1$, and it is possible to have profits *increase* with advertising costs. This is an example of a potential cost-based facilitating practice; that is, it may be in the interests of oligopolists to raise advertising costs.[13] It is sometimes argued, for example, that cigarette manufacturers have benefitted from their exclusion from television advertising.

Equation (13) makes intuitive sense, because a large value of η means that it is expensive at the margin to change the reach. Therefore, when α rises, ϕ falls only slightly, and the direct cost effect dominates the decreased competition effect. We note that when α specifically represents the cost per exposure, $\boldsymbol{a}$, in the CRIR advertising technology, one can show that $d\pi/da < 0$, unambiguously, so the "normal" result applies.

B. *Monopolistic competition*

The free-entry equilibrium values of p, ϕ and n are defined by equations (11a), (11b), and (11c). It is convenient to eliminate $(p-c)$ from these equations to reduce the equilibrium conditions to the following two equations in the variables n and ϕ:

$$\phi^2 A_\phi(\phi, \alpha) = t/n^2 \qquad (\pi_\phi = 0), \tag{14a}$$

$$\phi f + \phi A(\phi, \alpha) = t/n^2 \qquad (\pi = 0). \tag{14b}$$

Here we have also introduced $f \equiv F/\delta$.[14] These curves are depicted in Figure 2 below. It is not hard to prove that the two curves intersect as shown in the figure.

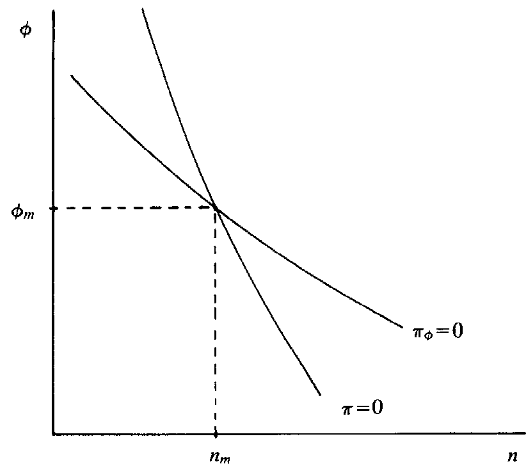

FIGURE 2
Monopolistically competitive equilibrium

It is also possible to eliminate n to obtain a single equation for the market level of advertising, ϕ_m, which is[15]

$$\phi A_\phi(\phi, \alpha) = f + A(\phi, \alpha). \tag{15}$$

The effect on the equilibrium of an increase in fixed costs (f), for example, can be found using the diagram. As fixed costs increase, the zero-profits locus shifts to the left. This causes ϕ to rise and n to fall. An increase in fixed costs causes firms to exit. With fewer firms, the benefits of advertising grow (i.e. the probability that an ad results in a sale increases), and advertising intensity (per firm) rises. It can also be shown that $d(p-c)/df > 0$; i.e. that with fewer firms left to compete markups must increase.

We are especially interested in the impact of changes in the advertising technology. An increase in α effects a downward shift in the $\pi_\phi = 0$ schedule; at a given n increased marginal costs of advertising reduces advertising levels. At the same time a rise in α causes the zero-profits curve to move to the left; at a fixed advertising intensity per firm increased advertising costs reduce profits and cause exit from the industry. The net effect of these two shifts must be derived analytically.

Using (15) it is straightforward to show that sgn $d\phi/d\alpha = \text{sgn}\,(1-\beta)$, where β is the elasticity of A_α with respect to ϕ as defined above. In the case where $A(\phi, \alpha) = \alpha\tilde{A}(\phi)$, as, for example, when α measures cost per exposure, we have $\beta > 1$ and therefore

$d\phi/d\alpha < 0$. This is certainly the most plausible result whereby increases in the marginal cost of advertising reduce advertising levels.

The effect of increased advertising costs on prices is unambiguously positive. This is due to the direct cost effect; prices must rise to cover added advertising expenses. This effect is reinforced when $d\phi/d\alpha < 0$, for in that case the reduction in reach reduces elasticities of demand. Even when $d\phi/d\alpha > 0$, however, $dp/d\alpha > 0$.

Finally, we compute $dn/d\alpha$ to see how changes in advertising costs affect diversity in the product market. As is intuitive, an increase in advertising costs causes entry exactly in those circumstances when it effects an increase in profits in the oligopoly equilibrium; i.e.

$$\operatorname{sgn} dn/d\alpha = \operatorname{sgn}[2(\beta-1)-\eta] = \operatorname{sgn} \left.\frac{d\pi}{d\alpha}\right|_n .$$

Again we have the possibility that improvements in advertising technology will result in exit, and thus fewer brands in the monopolistically competitive equilibrium.

The comparative static results for both the oligopoly and monopolistically competitive equilibria are summarized in Table I. To check the validity of these results in the absence of the large-group approximation, we have simulated the model using the functional form for $A(\phi, \alpha)$ derived from the CRIR technology and choosing parameter values such that n falls in the range of five to ten. In all cases we found the comparative static results indicated in the table to be valid.[16]

TABLE I

Comparative static results

		Endogenous variables					
		Oligopoly			Monopolistic competition		
		P	ϕ	π	p	ϕ	n
Exogenous variables	F	0	0	−	+	+	−
	t	+	+	+	+	0	+
	α	+	−	?[1]	+	?[2]	?[1]
	n	−	−	−			

Notes:
1. $\operatorname{sgn}(2(\beta-1)-\eta)$.
2. $\operatorname{sgn}(1-\beta)(<0 \text{ if } A(\phi,\alpha)=\alpha\tilde{A}(\phi))$.

4. WELFARE ANALYSIS

In this section, we investigate the nature of the biases that arise in the market equilibrium, relative to the outcome that would be socially optimal. We begin, in the first subsection, by deriving the socially optimal allocation of resources. Then, in subsection B, we compare the level of advertising that obtains in the market to the level that maximizes social welfare, *given* the (same) extent of diversity in each case. First we note that the monopoly case (that is, $n=1$) of our model is a special case of Shapiro (1980). Therefore we know that the monopolist always under-provides informative advertising. This finding serves as a useful basis for comparison with our result for the oligopoly equilibrium, given parameter values such that $(1-\phi)^n$ is small. Under this large-group assumption, advertising levels for each firm are always excessive.

In subsection C, we compare the free-entry monopolistically-competitive equilibrium to the full social optimum (with the number of firms endogenous). The Salop (1979) finding that the market overprovides diversity is extended to our model with imperfect information.

A. *The socially optimal allocation*

The welfare standard used here is the conventional one of consumer surplus plus profits, or gross benefits to consumers less production and marketing costs. Aggregate welfare is given by

$$W=(v-c)\delta[1-(1-\phi)^n]-nF-n\delta A(\phi)-T.$$

For notational ease, we now suppress α and write simply $A(\phi)$, and $A'(\phi)$ for A_ϕ.

The first term in W represents benefits net of variable production costs, but gross of transport costs. The term $(1-(1-\phi))^n$ is the fraction of the population that consumes the product. The second and third terms are fixed costs and marketing costs, respectively. The final term, T, represents aggregate transport costs, and reflects the losses due to imperfect product matching. We turn now to the computation of these costs.

To calculate T, we again partition the population into n groups, where the k-th group is those consumers who would find a given, representative brand to be their k-th most preferred under perfect information. The average travelling distance among consumers who actually receive an ad from their closest brand is $\bar{z}_1=1/4n$. (These consumers vary in distance from zero to $1/2n$ from their favourite brand.) Similarly, $\bar{z}_2$, the average transport distance of those who hear of their second closest (but not first closest) brand is $3/4n$. In general, $\bar{z}_k=(2k-1)/4n$, for $k=1,2,\ldots,n$.

As we have shown above, the fraction of consumers in the first group is $\phi_1=\phi$; in the second group is $\phi_2=\phi(1-\phi)$; and in the k-th group is $\phi_k=\phi(1-\phi)^{k-1}$. A fraction $\phi_0=(1-\phi)^n$ receives no messages. Thus, the average distance travelled by all consumers can be written as

$$\bar{z}=\textstyle\sum_{k=1}^n \phi_k\bar{z}_k=\sum_{k=1}^n \dfrac{(2k-1)}{4n}\,\phi(1-\phi)^{k-1}.$$

Summing this series,[17] we have

$$\bar{z}=\frac{1}{4n\phi}[(2-\phi)+(1-\phi)^n(\phi-2(1+\phi n))]$$

or, for $(1-\phi)^n$ small, the approximation $\bar{z}=(2-\phi)/4n\phi$. Aggregate transport costs are thus approximately $T=\delta t\bar{z}=\delta t(2-\phi)/4n\phi$.

Returning to the expression for social welfare, and assuming henceforth that $(1-\phi)^n$ is sufficiently small, we now have

$$W=(v-c)\delta-nF-n\delta A(\phi)-\frac{\delta t}{4n\phi}(2-\phi). \tag{17}$$

The socially optimal n and ϕ are defined by the two first-order conditions for the maximization of W, which after rearranging (recall $f\equiv F/\delta$) can be written as

$$\phi^2A'(\phi)=\frac{t}{2n^2} \qquad (W_\phi=0) \tag{18a}$$

$$\phi f+\phi A(\phi)=\frac{t}{2n^2}\left(1-\frac{\phi}{2}\right). \qquad (W_n=0) \tag{18b}$$

It is possible to eliminate n from these two equations to arrive at the following single equation in ϕ:

$$\phi\left(1-\frac{\phi}{2}\right)A'(\phi)=f+A(\phi). \tag{19}$$

This equation indicates that the optimal reach per firm, ϕ^*, depends only on the size of fixed costs relative to the market, f, and on the advertising technology. In fact, using the second-order conditions for the social optimum, it can be shown that $d\phi^*/df>0$; i.e. as fixed costs rise each firm should advertise more. This goes along with the expected result that $dn^*/df<0$; i.e. increased fixed costs reduce the optimal extent of diversity. Finally, it is worth noting that $W_{n\phi}<0$; i.e. the marginal social benefits of advertising (per firm) are smaller when the extent of diversity is greater.

B. *Comparison of the equilibrium and optimal levels of advertising: Diversity given*

In this subsection we compare the level of advertising per firm in the oligopoly equilibrium with the level that would be socially optimal, given the extent of diversity (n). There are four distinct effects which will in general cause divergences between the equilibrium and the optimum:

First, to the extent which advertising reaches consumers who would otherwise be uninformed, (that is, to the extent advertising increases the total market size), it tends to be undersupplied. The reason is easiest to see in the case of a monopoly seller ($n=1$). When considering an increase in his advertising budget, the monopolist sees the benefit as arising from additional sales, each of which he values according to its marginal contribution to profit: $p-c$. Yet an additional sale typically generates consumer surplus as well $(v-p-tz)$ so the social benefits of advertising $(v-c-tz)$ exceed the private benefits $(p-c)$. This effect implies that a monopolist will always underprovide purely informative advertising, and was proven in Shapiro (1980). We call this the market-size effect.

Second, the private and social benefits of sending a message to a consumer who has already received a message from another firm generally differ. The social benefit arises due to the improved matching of consumers with brands due to improved information. It is easy to verify that aggregate transport costs are declining with ϕ. Call this the matching effect.

The private benefit from advertising, however, comes in the form of the profit contribution made by an additional customer. An individual firm does *not* account for the profit reduction at other firms as it increases its advertising intensity and captures customers from its rivals. We call this the customer-capture effect.

It is far from obvious *a priori* which of the matching and the customer-capture effects is the dominant one. If the capture effect is the larger, advertising will tend to be oversupplied.

Finally, increases in advertising levels promote price competition by raising firms' demand elasticities. This is generally a beneficial effect of informative advertising, but does not have direct welfare effects in our model due to our assumptions of inelastic demand and no dropping out by consumers.

In the monopoly case, only the market size effect applies, and advertising is undersupplied. As the number of firms increases, however, the market size effect rapidly diminishes, and the matching and capture effects become the dominant ones. By looking at the large group case, such that $(1-\phi)^n$ is small,[18] we therefore focus on these latter two effects.

To see which of the effects is dominant, we need only compare the socially-optimal level of advertising per firm, as given implicitly in equation (18a), with the equilibrium level determined by (14a). On the left-hand side of each of these equations is the expression $\phi^2 A'(\phi)$. This term is an increasing function of ϕ. The right-hand side of (14a) is, *given n*, unambiguously larger than the right-hand side of (18a). It follows that for the large-group oligopoly case, $\phi_m > \phi^*$, that is, advertising intensity per firm in the market equilibrium is excessive.

What we have shown is that when there are a sufficiently large number of firms in a differentiated-product industry such that a marginal increase in reach per firm has no significant effect on the total number of consumers informed about at least one brand, then the beneficial effect of improved matching of consumers and products is outweighed by the "wasteful" aspect of merely shuffling consumers among firms. The capture effect outweighs the matching effect. Even though advertising is purely informative and has positive social value, the private returns to advertising always exceed the social returns. Of course, when n is not large, this statement must be qualified, since the market may then leave too many consumers totally uniformed. However, we have found in our simulations conducted over a wide range of parameter values that, even for n on the order of four or five, the market equilibrium exhibits excessive advertising per firm.

C. *Comparison of the monopolistically competitive equilibrium and the full social optimum*

In this subsection, we compare the market equilibrium to the socially optimal allocation of resources, when the number of firms, and therefore the extent of diversity, is taken to be endogenous in each case. Salop (1979) has shown that the market provides excessive diversity in a differentiated-product model with circular product space and complete information. Our purpose here is to show that his results extend to our model with imperfect information. This is so despite the fact that competition is no longer localized in our model.

The analysis, which assumes once again that $(1-\phi)^n$ is small, is performed by comparing the two equations that determine ϕ^* and n^*, equations (18a) and (18b), with those that define ϕ_m and n_m, equations (14a) and (14b). The results are displayed in Figure 3 below.

In the figure, (18a) is located by an equiproportionate leftward shift of (14a), for the reasons that were discussed in subsection B. Likewise, a comparison of (18b) and (14b) reveals that the former lies everywhere to the southwest of the latter. Finally, to show that the relative positions of the curves are as drawn, it is also necessary to prove that $\phi^* > \phi_m$. This is accomplished as follows. Subtract equation (15) from equation (19), and rearrange terms, to get

$$[A(\phi^*)-\phi^* A'(\phi^*)]-[A(\phi_m)-\phi_m A'(\phi_m)] = \frac{-\phi^{*2} A'(\phi^*)}{2} < 0. \tag{20}$$

Define $H(\phi)=A(\phi)-\phi A'(\phi)$, and differentiate to see that $H'(\phi)=-\phi A''(\phi)<0$. Therefore, $H(\phi^*)-H(\phi_m)<0$ in equation (20) implies that $\phi^*>\phi_m$. Finally, the knowledge that $\phi^*>\phi_m$ allows us to compare n^* and n_m. The fact that $n^*<n_m$ can be seen from the figure, or shown analytically.[19]

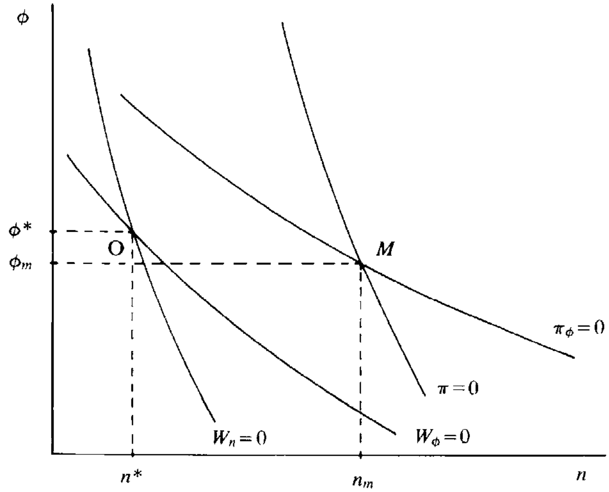

FIGURE 3

A comparison of the social optimum and the equilibrium

The fact that reach per firm in the full social optimum exceeds the level which obtains in the monopolistically-competitive equilibrium should be interpreted with care. Since increased diversity decreases the marginal social value of advertisements, ($W_{n\phi} < 0$), the finding that $\phi^* > \phi_m$ is largely due to the fact that $n^* < n_m$. Nonetheless, it is interesting to note that, while the social planner would want to cut back on the extent of diversity, he would also wish to offset somewhat the deleterious effect of this reduction in n on the average distance travelled by raising the informational reach of each of the remaining products.

5. CONCLUSIONS

We have constructed a model of purely informative advertising with heterogeneous goods. In our model, advertising does serve a useful social function; it informs customers about brands' characteristics, and improves the matching of consumers and products. Nevertheless, we find that the advertising levels that prevail in the large-group oligopoly equilibrium are always excessive.

We have also studied the effects of changes in the advertising technology on equilibrium in product markets. This question has particular relevance at a time when the availability of new media is decreasing the cost of reaching the target population. By explicitly modelling the information transmission associated with advertising messages, we found that improved efficiency of advertising (e.g. a reduction in the cost per exposure) does indeed increase the competitiveness of the market (as measured by demand elasticities) and causes prices to fall. We also derived a condition under which decreases in advertising costs augment the degree of diversity in the market. This condition is satisfied by the specific advertising cost function that we discussed in some detail, namely, that associated with the constant-reach, independent readership technology.

Further research is necessary to test whether our results generalize to alternative models of product differentiation. The circle model we have used is special, because it implies that the extent of diversity in the market is always excessive under full information. The incorporation of search behaviour or other information acquisition activities on the part of consumers would also be an important extension of this analysis.

APPENDIX

Simulation results

We have simulated the model developed in the text in order to determine whether our results apply for equilibria and optima with relatively few firms. In addition, the simulations allow us to compare the relative sizes of some of the different effects we have identified.

The equations for the monopolistically competitive equilibrium, which apply when $(1-\phi)^n$ is not assumed to be small, are

$$(p-c)[1-(1-\phi)^{n-1}]=\frac{t}{n\phi}[1-(1-\phi)^n]; \tag{A1}$$

$$(p-c)[1-(1-\phi)^n]=n\phi A_\phi(\phi,\alpha); \tag{A2}$$

$$(p-c)[1-(1-\phi)^n]=n[f+A(\phi,\alpha)]. \tag{A3}$$

The equations that determine n^* and ϕ^* in the social optimum are

$$f+A(\phi,\alpha)+(v-c)(1-\phi)^n\log(1-\phi)=\frac{1}{4n}(1-\phi)^n\log(1-\phi)\left[\frac{2}{\phi}+(2n-1)\right] + \frac{t}{2n}[1-(1-\phi)^n]. \tag{A4}$$

$$n(v-c)(1-\phi)^{n-1}=nA_\phi(\phi,\alpha)-\frac{t}{2n\phi^2}[1-(1-\phi)^n]-\frac{t}{4}(1-\phi)^{n-1}\left(\frac{2}{\phi}-1-n\right).$$

We used the constant-reach, independent readership technology in equation (1) for $A(\phi,\alpha)$ in our simulation. The parameter values selected for the base run are shown in Table AI. These values were chosen to yield an equilibrium with an intermediate number of firms, and a markup of somewhat less than 100% of variable cost. Note that the total size of the market has been normalized at one, so each firm produces about 0·07 units of the good. Variable costs are thus about five times fixed costs. In this base run, promotional expenditures per firm are 1·86, or roughly one-half variable costs.

TABLE AI

Base case simulations

Parameters	$a=3$	$t=250$	$v=250$
	$r=0{\cdot}1$	$f=0{\cdot}75$	$c=50$
Equilibrium	$\phi_m=0{\cdot}48$	$n_m=14$	$p_m=87$
Optimum	$\phi^*=0{\cdot}68$	$n^*=6$	

The effects of changes in f, with the other parameters held fixed at their base case values, are presented in Table AII. The comparative static properties derived in the text under the large n assumption are validated for n relatively small. We find that market diversity and reach per firm are both quite sensitive in fixed costs, while markups are effected relatively less.

TABLE AII

Variations in f

f	Equilibrium ϕ_m	n_m	p_m	Optimum ϕ^*	n^*
0·25	0·32	24	82	0·55	8
0·75	0·48	14	87	0·68	6
2·0	0·63	9	94	0·79	4
3·0	0·69	8	98	0·83	3

Changes in the advertising technology, as captured by the cost per exposure ***a***, are reported in Table AIII. (Change in the reach per issue of the magazine, *r*, would enter just like ***a***, except that large values of *r* correspond to small values of ***a***.) The comparative static properties are again in line with those reported in Table I. Here, the most pronounced effects of the changes in advertising technology are on advertising levels and equilibrium prices. The effect on product diversity is almost negligible.

TABLE AIII

Variations in a

a	Equilibrium ϕ_m	n_m	p_m	Optimum ϕ^*	n^*
1	0·65	15	76	0·80	6
3	0·48	14	87	0·68	6
5	0·40	14	95	0·63	5
10	0·31	14	109	0·55	5

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NOTES

1. With this view of advertising, the Dixit and Norman result is reversed for a monopolist, as is shown in Shapiro (1981). Monopolists engage in too little informative advertising of this type, because they fail to appropriate the consumer surplus associated with informing additional consumers about their products.
2. A commonly used, alternative model of product differentiation is the representative-consumer approach associated with Spence (1976) and Dixit and Stiglitz (1977). However, the notion of a representative consumer is not well-suited to the introduction of an information technology in which firms send messages to individual consumers who differ in their tastes across brands.
3. The spatial competition model is due originally to Hotelling (1929). Here we follow Salop (1979) very closely, except that we do not assume perfect consumer information.
4. The absence from our model of search, word-of-mouth and experience as sources of information is an important omission which we hope to rectify in future research. In principle, consumer search could be incorporated in a manner similar to that in Butters (1977). See also Wolinsky (1982), who studies optimal search behaviour in a model of differentiated products and imperfect information.

5. Any fixed costs of marketing can be incorporated in F.

6. Equivalently, we can imagine that everyone in the population "sees" the ad, but only a fraction r remembers it.

7. An "issue of the magazine" can always be reinterpreted as a mailing of size $r\delta$. When $r=1/\delta$ this is just the sending of a single letter. As δ grows, $r \to 0$, and since

$$\lim_{r\to 0} r/\log(1-r) = -1,$$

$A(\phi)$ then collapses to $A(\phi) = -a\delta \log(1-\phi)$. This is the technology used in Butters (1977), which is seen to be a limiting case of ours.

8. This formula, and the ones which follow, hold only in the range $\bar{p}-t/n < p < \bar{p}+t/n$. We are assuming that the parameters of the problem are such that it is neither profitable to reduce price to absorb a neighbour (what Salop terms the supercompetitive region) nor to raise price so high so as to drive N_1 to zero. The former assumption can also be justified by the non-Nash behavioural assumption that a rival will reduce price rather than have his prime group eliminated, while the latter assumption amounts to putting an upper bound on v. Without these assumptions, an equilibrium may fail to exist.

9. In order to ensure that consumers will not drop out of the market if they do not receive an ad from a brand that suits their tastes particularly well, it is necessary and sufficient that $v-p-t/2>0$ be satisfied in equilibrium, since the largest possible distance to any firm is $\frac{1}{2}$

10. Dixit and Stiglitz (1977) made a similar "large-group" assumption in their investigation of monopolistic competition and product differentiation. To check the closeness of our approximation, we have simulated the equilibrium in this model for a wide range of parameter values and found that the error involved in going from (6) to (7) is quite small and is generally several orders of magnitude smaller than the error involved in permitting n to take on non-integral values. Analytically, our approximation becomes exact as n gets large if (and only if) $A'(0)=0$. The approximation is most accurate for n large (when n is taken as exogenous) and $A'(0)/t$ small. In the monopolistically-competitive equilibrium, a large n is generated by t large and F/δ small.

We will, on occasion, refer to our simulations (described in detail in the appendix) to support the general validity of our propositions even when the number of firms is small.

11. It is straightforward to check that the second-order conditions for profit maximization are satisfied for each firm in the neighborhood of the symmetric equilibrium. The second-order conditions are also satisfied for the ϕ and p that solve equations (A1) and (A2) in the appendix, where we do not use the assumption that $(1-\phi)^n$ is small.

12. This equation also assumes that $(1-\phi)^n$ is small enough to make (7) a good approximation. For $(1-\phi)^n$ small, almost all customers are served in equilibrium, so each firm makes approximately δ/n sales.

13. Salop and Scheffman (1981) discuss cases when it is in the interest of a given firm or firms to raise their own costs.

14. In fact, it should now be clear that F and δ enter only as the ratio F/δ; i.e. fixed costs should be measured relative to the size of the market.

15. Equation (15) can be derived directly from equations (A1) and (A2) in the Appendix, where we do not make the assumption that $(1-\phi)^n$ is small. Consequently, the value of ϕ in the monopolistically-competitive equilibrium is determined *exactly* by equation (15), and the comparative static results relating to this variable are valid irrespective of the number of firms in the market.

16. See the Appendix for more detail on these simulations.

17. The derivation makes use of the fact that

$$\sum (k+1)(1-\phi)^k = \frac{d}{d(1-\phi)}[\sum (1-\phi)^{k+1}].$$

18. Recall, from footnote 10, that $(1-\phi)^n$ will become small as the number of firms in the industry becomes large provided that $A_\phi(0;\alpha)/t$ is relatively small.

19. $n^{*2} = t/2\phi^{*2}A'(\phi^*)$ and $n_m^2 = t/\phi_m^2 A'(\phi_m)$. Since $\phi^2 A'(\phi)$ is increasing in ϕ and $\phi^* > \phi_m$, it follows that $n_m > n^*$.

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