FINAL QUIZ FINM32115

Tuesday 31 March

13h15-16h15

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The points and time indicated for each question serve as a guide.

Please rewrite word for word the following phrase and sign below: ”I hereby sign on my honor that I will not give or receive aid in this examination.”

Question 1 (3 points)

Capital Budgeting (30 minutes)

Roche is considering the possibility to try and produce a new vaccine for coronavirus. This project requires initially purchasing a new lab in year 0, for a price of \$2.5m (2.5 million). The equipment depreciates linearly from year 1 to 5. The salvage value and the resale value of the equipment are both zero. An investment in R&D in years 1 and 2, of \$2m each year, would also need to be made.

Roche considers two scenarios:

- Scenario 1 (probability 2/3): The team of scientists will come up with the new vaccine in year 1.
- Scenario 2: The team of scientists will not be successful at creating the new vaccine within the year. By then, a competing company will most likely have found one.

The project is assumed to last for five years.

In Scenario 1, the costs of production incurred are \$1m per year from year 2 to 5. Sales revenues amount to \$5m per year in years 2 to 5. Costs of inventory cause an increase in Net Working Capital (NWC) by \$0.5m in year 2. NWC will then remain stable. In Scenario 2, costs of production, sales revenues and costs of inventory are all zero.

The following stated annual rates are given: (1) risk-free rate: 5%; (2) corporate tax rate: 40% (if the taxable cash flow at year t is negative, assume that the tax credit is received the same year t); (3) cost of capital: 15%; (4) dividend rate: 8%.

(a) ($2\frac{1}{2}$ points) What are the expected Free Cash Flows of this project?

(b) ($\frac{1}{2}$ point) What is the NPV of this project? Is it profitable for Roche to undertake it? Justify your answer.

Solution:

The Free Cash Flows in scenario 1 are given by the following table.

Table 1: Free Cash Flows (FCF), everything in $ million

t	0	1	2	3	4	5
Revenues			5	5	5	5
Cost of production			-1	-1	-1	-1
Gross profit			**4**	**4**	**4**	**4**
R&D	0	-2	-2			
EBITDA		**-2**	**2**	**4**	**4**	**4**
After tax		-1.2	1.2	2.4	2.4	2.4
CapEx	-2.5					
Depreciation		-0.5	-0.5	-0.5	-0.5	-0.5
Tax shield		0.2	0.2	0.2	0.2	0.2
Operating cash flows		**-1**	**1.4**	**2.6**	**2.6**	**2.6**
CapEx	-2.5					
Delta NWC			-0.5			0.5
FCF	-2.5	-1	0.9	2.6	2.6	3.1
NPV	2.05					

The Free Cash Flows in scenario 2 are given by the following table.

Table 2: Free Cash Flows (FCF), everything in $ million

t	0	1	2	3	4	5
Revenues						
Cost of production						
Gross profit	**0**	**0**	**0**	**0**	**0**	**0**
R&D	0	-2	-2			
EBITDA		**-2**	**-2**	**0**	**0**	**0**
After tax		-1.2	-1.2	0	0	0
CapEx	-2.5					
Depreciation		-0.5	-0.5	-0.5	-0.5	-0.5
Tax shield		0.2	0.2	0.2	0.2	0.2
Operating cash flows		**-1**	**-1**	**0.2**	**0.2**	**0.2**
CapEx	-2.5					
Delta NWC						
FCF	-2.5	-1	-1	0.2	0.2	0.2
NPV	-3.78					

The expected Free Cash Flows (FCF) are the sum of FCF in each scenario, weighted by the probability of the scenario. In million $: -2.5, -1, 0.27, 1.80, 1.80 and 2.13, from year 0 to 5.

Accordingly, the NPV is equal to the expected Free Cash Flows (FCF), discounted using the cost of capital of 15%, i.e., $0.11m. The NPV is positive so it is profitable for the firm to try and produce the vaccine.

Question 2 (3 points)

Career prospects (30 minutes)

Anthony has four career opportunities that equally interest him. The compensation schemes are as follows:

- Opportunity 1: Anthony has an offer in a consulting company. The base salary is \$50'000, plus a bonus of \$20'000 if he performs well and \$40'000 if he performs exceptionally well. He estimates the probability that he performs well to be 1/2, and the probability that he performs exceptionally well to be 1/5.
- Opportunity 2: Anthony has an offer in an audit company. The salary is \$60'000 irrespective his performance.
- Opportunity 3: Anthony becomes a self-employed consultant. He believes that if he does so, his salary will have a Normal (Gaussian) distribution with mean \$68'000 and standard deviation 10000.
- Opportunity 4: Anthony has an idea for a start-up, he wants to develop an app that will allow ESSEC students to connect more easily with alumni. Because the market of apps is a risky one, he thinks that this business has a 1/4 probability to not work, in which case he will not be paid, and a 3/4 probability to be very successful, in which case he expects to be paid \$150'000.

Anthony has the following utility function $U(W) = E(W) - 10^{-5} \times Var(W)$. Note that the questions of this exercise are independent of one another.

(a) (1 point) Show that Anthony is risk-averse.

Solution:

Anthony is risk-averse iff $E[U(W)] < U(E[W])$. With mean-variance utility,

$$\begin{aligned}
E[U(W)] &= E[E(W) - 10^{-5} \times Var(W)] = E[E(W)] - 10^{-5} \times E[Var(W)] \\
&= E(W) - 10^{-5} \times Var(W). \\
U(E[W]) &= E[E(W)] - 10^{-5} \times Var(E[W]) \\
&= E(W) - 0 = E(W) > E(W) - 10^{-5} \times Var(W) = E[U(W)].
\end{aligned}$$

(b) (2 points) Which opportunity should Anthony choose?

Solution:

Anthony chooses the possibility that maximizes his expected utility of wealth. We calculate it for each possibility the expected utility of wealth:

- Opportunity 1:

$$\begin{aligned}
E(W) =& 50'000 \times (1-\frac{1}{2}-\frac{1}{5}) + 70'000 \times \frac{1}{2} + 90'000 \times \frac{1}{5} = 68'000 \\
Var(W) =& (50'000-68'000)^2 \times (1-\frac{1}{2}-\frac{1}{5}) + (70'000-68'000)^2 \times \frac{1}{2} \\
& + (90'000-68'000)^2 \times \frac{1}{5} = 19'600 \times 10^4 \\
E[U(W)] =& E(W) - 10^{-5} \times Var(W) = 68'000 - 1'960 = 66'040.
\end{aligned}$$

- Opportunity 2:

$$\begin{aligned}
&E(W) = 60'000 \\
&Var(W) = 0 \\
&E[U(W)] = E(W) - 10^{-4} \times Var(W) = 60'000.
\end{aligned}$$

- Opportunity 3:

$$\begin{aligned}
&E(W) = 68'000 \\
&Var(W) = 10'000^2 = 1000 \times 10^5 \\
&E[U(W)] = E(W) - 10^{-5} \times Var(W) = 67'000.
\end{aligned}$$

- Opportunity 4:

$$\begin{aligned}
&E(W) = 0.75 \times 150'000 = 112'500 \\
&Var(W) = 0.75 \times (150'000 - 112'500)^2 + 0.25 \times (0 - 112'500)^2 \approx 42'187 \times 10^5 \\
&E[U(W)] = E(W) - 10^{-5} \times Var(W) = 70'312.5.
\end{aligned}$$

Anthony should choose possibility 4.

Question 3 (4 points)

Efficient portfolios (30 minutes)

Julia has an investment opportunity set that contains two stocks X and Y with expected returns $\mu_X = 0.2$ and $\mu_Y = 0.3$ and return variances $Var(R_X) = \sigma_X^2 = 0.04$ and $Var(R_Y) = \sigma_Y^2 = 0.08$. The correlation between returns is $\rho_{XY} = 0.5$. The risk-free rate is 15%.

(a) (1 point) What is the equation of Julia's mean-variance frontier if the investment opportunity only contains these two assets.

Solution:

We can express ω_X as a function of $E[R_P]$:

$$\omega_X = \frac{E[R_P] - \mu_Y}{\mu_X - \mu_Y} = \frac{0.3 - E[R_P]}{0.1}.$$

We plug this in the expression of the variance of R_P:

$$\begin{aligned}
Var(R_P) &= \omega_X^2 \sigma_X^2 + (1-\omega_X)^2 \sigma_Y^2 + 2\omega_X(1-\omega_X)\rho_{XY}\sigma_X\sigma_Y \\
&= \omega_X^2 \sigma_X^2 + (1 + \omega_X^2 - 2\omega_X)\sigma_Y^2 + 2(\omega_X - \omega_X^2)\rho_{XY}\sigma_X\sigma_Y \\
&= \omega_X^2(\sigma_X^2 + \sigma_Y^2 - 2\rho_{XY}\sigma_X\sigma_Y) + \omega_X(2\rho_{XY}\sigma_X\sigma_Y - 2\sigma_Y^2) + \sigma_Y^2 \\
&= \left(\frac{E[R_P] - \mu_Y}{\mu_X - \mu_Y}\right)^2 (\sigma_X^2 + \sigma_Y^2 - 2\rho_{XY}\sigma_X\sigma_Y) + \left(\frac{E[R_P] - \mu_Y}{\mu_X - \mu_Y}\right)(2\rho_{XY}\sigma_X\sigma_Y - 2\sigma_Y^2) + \sigma_Y^2 \\
&= (E[R_P] - \mu_Y)^2 \frac{\sigma_X^2 + \sigma_Y^2 - 2\rho_{XY}\sigma_X\sigma_Y}{(\mu_X - \mu_Y)^2} + (E[R_P] - \mu_Y)\frac{2\rho_{XY}\sigma_X\sigma_Y - 2\sigma_Y^2}{\mu_X - \mu_Y} + \sigma_Y^2 \\
&= \left(E[R_P]^2 + \mu_Y^2 - 2E[R_P]\mu_Y\right)\frac{\sigma_X^2 + \sigma_Y^2 - \rho_{XY}\sigma_X\sigma_Y}{(\mu_X - \mu_Y)^2} + (E[R_P] - \mu_Y)\frac{2\rho_{XY}\sigma_X\sigma_Y - 2\sigma_Y^2}{\mu_X - \mu_Y} + \sigma_Y^2 \\
&= \frac{\sigma_X^2 + \sigma_Y^2 - 2\rho_{XY}\sigma_X\sigma_Y}{(\mu_X - \mu_Y)^2} E[R_P]^2 + \left(\frac{2\rho_{XY}\sigma_X\sigma_Y - 2\sigma_Y^2}{\mu_X - \mu_Y} - 2\mu_Y \frac{\sigma_X^2 + \sigma_Y^2 - 2\rho_{XY}\sigma_X\sigma_Y}{(\mu_X - \mu_Y)^2}\right) E[R_P \\
&\quad + \frac{\sigma_X^2 + \sigma_Y^2 - 2\rho_{XY}\sigma_X\sigma_Y}{(\mu_X - \mu_Y)^2}\mu_Y^2 - \mu_Y \frac{2\rho_{XY}\sigma_X\sigma_Y - 2\sigma_Y^2}{\mu_X - \mu_Y} + \sigma_Y^2
\end{aligned}$$

We replace by the values of the return moments:

$$Var(R_P) = 6.34E[R_P]^2 - 2.77E[R_P] + 0.34$$

$$Vol(R_P) = \sqrt{6.34E[R_P]^2 - 2.77E[R_P] + 0.34}.$$

(b) (0.25 point) Can you create a portfolio P (with only X and Y) that replicates the risk-free asset? Justify your answer.

Solution:

The two assets should be perfectly correlated with correlation -1 so that the risk of the first asset can be offset using the second asset. Here, the correlation between the two assets is 0.5 so it is not possible to replicate the risk-free asset.

(c) (0.25 point) We add a third risky asset to Julia's investment opportunity set. This asset has correlation 0.7 with X and 0.3 with Y. Describe, qualitatively and briefly, how her efficient frontier will change.

Solution:

The new asset is imperfectly correlated with the other two. The efficient frontier will move to the left without touching the y-axis.

(d) ($1/2$ point) One can show that for this new investment opportunity set, efficient portfolios lie on the parabola with equation

$$x = \sqrt{6y^2 - 2.8y + 0.35}.$$

What is the idiosyncratic and the systematic variance of a portfolio with expected returns 0.23 and volatility 18.3%?

Solution:

The idiosyncratic variance is the difference between the variance of this portfolio and the variance of the efficient portfolio with the same expected return. The efficient portfolio with the same expected return has variance:

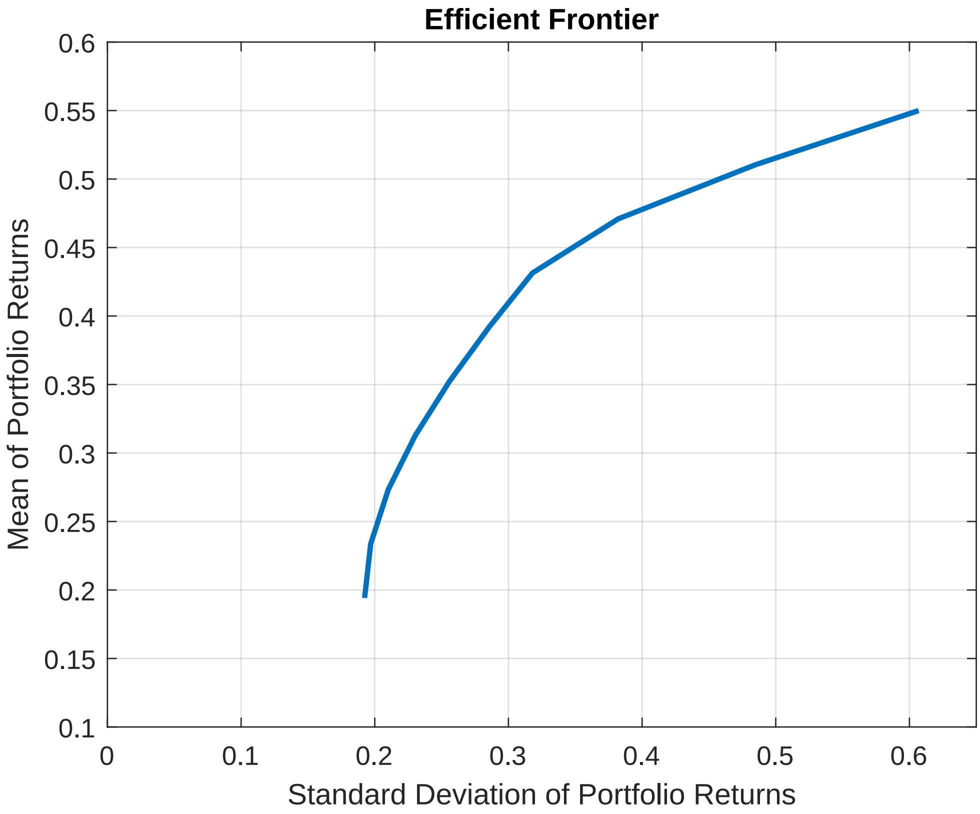
$$Var(R_{eff}) = 6 \times (0.23^2) - 2.8 \times 0.23 + 0.35 = 0.0234.$$

So the systematic variance is 2.34%, and the idiosyncratic variance is $18.3\%^2 - 0.0234 = 1.01\%$.

(e) (2 points) Julia's friends have some savings to invest. They have access to an investment opportunity set that includes Julia's opportunity set and the risk-free asset. They would like to follow the strategies described below:

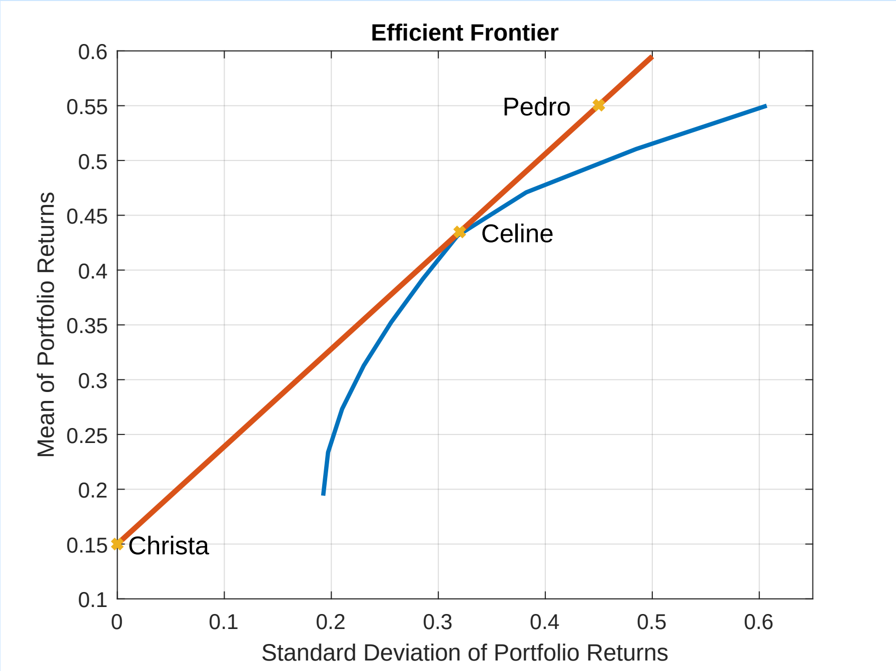
- Pedro wants to borrow money and invest the totality of his savings plus the borrowed amount in the market portfolio.
- Christa wants to be on the safe side and invest all her savings in the risk-free asset.

- Luca is a technology enthusiast and wants to invest his savings in companies of the tech industry.
- Céline wants to invest her savings in the risky portfolio with the highest Sharpe ratio.



Solution:

We do not have enough information to plot Luca's portfolio, it is under-diversified because very concentrated in tech firms, it should be somewhere below the blue line. It is an inefficient portfolio. The others are efficient.



Question 4 (3 points)

Options (20 minutes)

Stock ABC has initial price $S_0 = \$100$ at time $t = 0$. Each year, its price either increases by 20% or decreases by 20%.

The value of the risk-free bond at time $t = 0$ is 1 and the annual risk-free rate is 5%.

(a) (½ point) Draw the binomial trees for stock ABC and the bond, considering time steps $t = 0$, $t = 1$ and $t = 2$ years.

Solution:

The binomial tree of the stock is:

$$S_0 = 100 \xrightarrow{p_u} S_1^u = 100(1+20\%) = 120, \qquad S_0 = 100 \xrightarrow{p_d} S_1^d = 100(1-20\%) = 80$$

$$S_2^{uu} = 120(1+20\%) = 144$$

$$S_2^{ud} = 120(1-20\%) = 96$$

$$S_2^{dd} = 80(1-20\%) = 64$$

The binomial tree of the bond is:

$$B_0 = 1 \xrightarrow{p_u} B_1^u = 1.05, \qquad B_0 = 1 \xrightarrow{p_d} B_1^d = 1.05$$

$$B_2^{uu} = 1.05^2 = 1.1025$$

$$B_2^{ud} = 1.05^2 = 1.1025$$

$$B_2^{dd} = 1.05^2 = 1.1025$$

(b) (½ point) Three European put options on this stock are available to investors, all expiring in two years. Option 1 has strike 110, option 2 has strike 100 and option 3 has strike 96. Sort the three options from the least expensive to the most expensive, and from the least liquid to the most liquid. Justify your answer.

Solution:

Option 1 is the only one which is in-the-money. It is therefore the most expensive and the least liquid. Option 2 is at-the-money. It is therefore not too expensive and very liquid. Option 3 is out-of-the-money. It is therefore the cheapest and also very liquid. It is unclear whether option 2 or option 3 is the most liquid (both answers accepted).

(c) ($\frac{1}{2}$ point) Denote by P_0 the initial price of option 3 (strike = 96). Draw the payoff and net profit of this option. Give a possible incentive to buy this option.

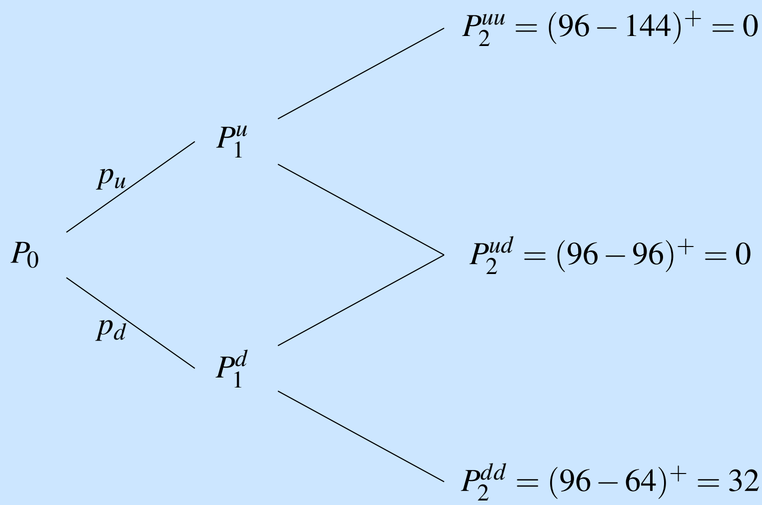
Solution:

See slides for payoff and net profit of a put option. This option will have a positive payoff if the stock price falls below 96. An investor who wants to be hedged (=insured) against such possibility could buy this option.

(d) (1 point) What are the payoffs of option 3 (strike = 96) in the 3 states, at maturity? Derive the price P_1^u of option 3 at time t=1, given that you are in the up state. Derive the price P_1^d of option 3 at time t=1, given that you are in the down state.

Solution:

Option 3 has the following payoff: $(96 - S_T)^+$. We calculate it in each state:



To find P_1^u, we apply the Law of One Price: the payoff is 0 in both up and down state so the initial price must be 0 as well: $P_1^u = 0$.

To find P_1^d, one can either build the replicating portfolio in stocks and bonds, or use the martingale approach. With the method of the replicating portfolio, we need to hold Δ units of stock and B bonds to match the payoffs P_2^{ud} and P_2^{dd}. The replicating portfolio is given by

$$\Delta = \frac{P_2^{ud} - P_2^{dd}}{S_1^d(u-d)} = \frac{0-32}{80(1+20\% - 1 + 20\%)} = -1$$

$$B = \frac{P_2^{ud} - S_1^d u \Delta}{1+r_f} = \frac{0+96\times 1}{1+5\%} = 91.4286$$

By the Law of One Price, the price of the option is given by $P_1^d = \Delta S_1^d + B = -1 \times 80 + 91.4286 = 11.4286$.

With the martingale approach, we need to calculate the risk-neutral probability of going up and down:

$$q_u = \frac{1+r_f-d}{u-d} = \frac{1+5\% - 1 + 20\%}{1+20\% - 1 + 20\%} = 0.6250$$

$$q_d = 1 - q_u = 0.3750$$

The price of the option is now given by the martingale property:

$$P_1^d = \frac{q_u \times 0 + q_d \times 32}{1+r_f} = 11.4286.$$

(e) (½ point) Assume now that the option is American. Assuming you are at time $t = 1$, in the down state. Would you exercise the option? Justify your answer.

Solution:

If you exercise the option, you get a payoff of $96 - 80 = 16$. This is higher than the continuation value of the option, i.e., the value of waiting for the next time step to exercise the option. Therefore, it is optimal to exercise the option at $t = 1$, if you are in the down state.

Question 5 (4 points)

CAPM (40 minutes)

Microsoft, Apple and Google are the three largest market capitalizations of the S&P 500. You have the following information on these companies. $E[R]$ and $Vol(R)$ represent the expected returns and volatility over next year.

Table 3: Market information

	Market cap ($ billion)	E[R]	Vol(R)
1.Microsoft	1,428	0.14	30%
2.Apple	1,400	0.18	40%
3.Google	1,035	0.10	20%

Assume that the correlation between any pair of these stock returns is 0.8.

Together, these 3 firms account for close to 14% of the total capitalization of the S&P 500. Consider the S&P 3, which contains these 3 stocks, and assume that it is a good proxy for the market portfolio.

(a) (1 point) What are the expected returns and volatility of the market porfolio?

Solution:

The total market capitalization (assuming the 3 companies proxy the market well) is the sum of the 3 market caps: $1,428+1,400+1,035 = 3'863$ billion dollars.

We calculate the fraction of the market that each stock represents: $\omega_1 = 0.3697$, $\omega_2 = 0.3624$ and $\omega_3 = 0.2679$.

Expected returns on the market portfolio are given by the weighted sum of expected returns of each stock:

$$E[R_M] = \sum_{i=1}^{3} \omega_i E[R_i] = 0.1438.$$

The volatility of the market is the square root of the variance:

$$Var(R_M) = \sum_{i=1}^{3}\sum_{j=1}^{3} \omega_i \omega_j \sigma_{ij},$$

where σ_{ij} is the covariance between asset i and asset j: $\sigma_{ij} = Correl(R_i, R_j)Vol(R_i)Vol(R_j)$.

The covariances of the assets are represented in the covariance matrix below. On the diagonal we have the variances of returns. The matrix is symmetric.

$$Cov(R_i, R_j) = \begin{bmatrix} 0.0900 & 0.0960 & 0.0480 \\ & 0.1600 & 0.0640 \\ & & 0.04 \end{bmatrix}$$

So the variance of the market is 0.0838. Its volatility is 28.95%.

(b) (1 point) What are the betas of the 3 assets?

Solution:

$$\beta_i = \frac{Cov(R_i, R_M)}{Var(R_M)}.$$

The covariance between each asset and the market is

$$Cov(R_i, R_M) = Cov(R_i, \sum_{j=1}^{3} \omega_j R_j) = \omega_i Var(R_i) + \sum_{j \neq i} \omega_j Cov(R_i, R_j).$$

We obtain that

$$Cov(R_1, R_M) = 0.0809$$
$$Cov(R_2, R_M) = 0.1106$$
$$Cov(R_3, R_M) = 0.0517.$$

The resulting betas are

$$\beta_1 = 0.9651$$
$$\beta_2 = 1.3194$$
$$\beta_3 = 0.6161.$$

(c) (½ point) How much of the total variance of each asset can be diversified?

Solution:

Systematic risk cannot be diversified, only idiosyncratic risk can. The percentage of idiosyncratic variance, for each asset, is:

$$Idio\,var_1 = Var(R_1) - \beta_1^2 Var(R_M) = 1.19\%$$
$$Idio\,var_2 = 1.40\%$$
$$Idio\,var_3 = 0.82\%.$$

(d) ($^1\!/_2$ point) What is the risk-free rate? Round to the nearest integer.

Solution:

We use the CAPM, for any asset:

$$E[R_1] = r_f + \beta_1(E[R_M] - r_f),$$

i.e.,

$$r_f = \frac{E[R_1] - \beta_1 E[R_M]}{1 - \beta_1} \approx 3\%$$

(e) ($^1\!/_2$ point) Consider an investor holding portfolio P. One third of portfolio P is invested in Microsoft, one third in Apple and one third in the risk-free asset. Is this portfolio efficient? Justify your answer.

Solution:

Efficient portfolios are portfolios that contain the market portfolio and the risk-free asset. Portfolio P does not contain the market portfolio therefore it is not efficient.

(f) ($^1\!/_2$ point) A risk-averse investor targets an efficient portfolio with an expected return of 8%. What should his/her portfolio contain?

Solution:

A fraction ω of the portfolio is invested in the risk-free asset and the rest is invested in the market. We infer ω from

$$E[R] = \omega r_f + (1 - \omega)E[R_M]$$

which gives $\omega = 56\%$.

Question 6 (3 points)

Facebook (30 minutes)

As of end of 2019, Facebook's market value of equity was \$465,593m (million). The market value of its debt was estimated to be equal to \$6,108m (million). Its annual interest expense was \$20m. Assume that this debt level is permanent. Facebook's income tax rate is 19.15%.

The current risk-free interest rate is 0.23%. The market risk premium is 6%.

(a) (1 point) Briefly describe how you would estimate the beta of Facebook's equity, based on historical return data? Assume that you find a beta of 1.29. Calculate the cost of equity and the cost of debt of Facebook. Infer the WACC of Facebook.

Solution:

To estimate the beta of Facebook, you can regress the excess returns of Facebook on the market returns. The slope of the fitted line is the beta.

The cost of equity of Facebook can be calculated with the CAPM:

$$r_E = E[R_{Facebook}] = r_f + \beta(E[R_{Mkt} - r_f) = 0.23\% + 1.29 \times 6\% = 7.97\%.$$

The cost of debt (before tax) is the interest expense, per year, divided by the amount of debt: $r_D = 20/6,108 = 0.33\%$.

The WACC is therefore:

$$\begin{aligned} WACC &= \frac{E}{D+E} r_E + \frac{D}{D+E} r_D (1-\tau_c) \\ &= \frac{465,593}{465,593+6,108} 7.97\% + \frac{6,108}{465,593+6,108} 0.33\% (1-19.15\%) = 7.87\%. \end{aligned}$$

(b) (1 point) Facebook is considering initiating a project that would be all equity financed. What is the cost of capital of this project?

Solution:

The cost of capital of this project is r_U, the cost of unlevered equity. r_U satisfies:

$$r_E = r_U + (1-\tau)\frac{D}{E}(r_U - r_D).$$

Hence

$$r_U = \frac{r_E + (1-\tau)\frac{D}{E} r_D}{1+(1-\tau)\frac{D}{E}} = \frac{7.97\% + (1-19.15\%)\frac{6{,}108}{465{,}593} 0.33\%}{1+(1-19.15\%)\frac{6{,}108}{465{,}593}} = 7.89\%.$$

(c) (1 point) Suppose that the proportion of debt in total financing increases to 1/3. Determine the impact of this policy on the cost of levered equity and the WACC of Facebook (assuming that the cost of debt remains constant).

Solution:

The new cost of levered equity r_E is

$$r_E = r_U + (1-\tau)\frac{D}{E}(r_U - r_D).$$

We need to calculate the debt to equity ratio:

$$\frac{D}{D+E} = \frac{1}{3} \Rightarrow D = \frac{1}{3}(D+E) \Rightarrow D(1-\frac{1}{3}) = \frac{1}{3}E \Rightarrow \frac{D}{E} = \frac{1/3}{2/3} = 1/2.$$

The cost of equity of Facebook is:

$$r_E = 7.89\% + (1-19.15\%)\frac{1}{2}(7.89\% - 0.33\%) = 10.95\%.$$

The WACC becomes

$$WACC = \frac{2}{3}10.95\% + \frac{1}{3}0.33\%(1-19.15\%) = 7.39\%.$$