EXERCISES WITH SOLUTIONS FOR BUSINESS ECONOMICS

CHAPTER 1 TO 5

W. Sand-Zantman

(sand@essec.edu)

ESSEC DSBA - 2021

Chapter 1: On Perfect Competition - In class

We consider the milk market where the supply curve is given by $Q = 40P$ and the demand curve by $Q = 30 - 20P$ with Q the quantity (in millions of liters) and P the price per liter. We will assume that this market is competitive.

a Compute explicitly the equilibrium price and quantity.

Supply=Demand. If $40P = 30 - 20P$ *then* $P = 0.5$ *and* $Q = 20$*.*

b Analyze the price-elasticities of supply and demand at this equilibrium point.

The formula for elasticities is given as $\varepsilon = -\frac{dQ}{dP}\frac{P}{Q}$*. For both supply and demand, the ratio* $\frac{P}{Q}$ *is* $\frac{1}{40}$*. For supply* $\frac{dQ}{dP} = 40$*,so elasticity of supply is 1. For demand,* $\frac{dQ}{dP} = -20$*,so elasticity of demand is 0.5.*

We now assume that the government wants to support prices in this industry, through a policy of public purchase.

c How many units the government must buy to increase the price by 20%?

We want to obtain a price of 0.6*. This corresponds to a supply of* 24 *but the demand from consumers is given as* $30 - 12 = 18$*. The government should therefore buy* $24 - 18 =$ *6 millions of liters of milk.*

d What is the cost of such a policy for the government?

The cost for the government is 0.6×6 *= 3,6 millions.*

e Represent graphically the gains (or losses) for consumers and producers.

Producers gain 0.1 euros for the milk that is sold at the earlier price $(0.1 \times 20 = 2)$, *and gain* $\frac{0.1\times 4}{2} = 0.2$ *millions on the additional milk sold afterwards. For the consumers, the losses are* $0.1 \times 18 = 1.8$ *millions on that will be bought at the new price and* $\frac{0.1\times 2}{2} = 0.1$ *millions on the milk that will not be bought at this higher price. Accordingly, the producers gain 2.2 millions and consumers lose 1.9 millions.*

f What do you think of the global impact on efficiency of this public policy?

If we add the gains/losses of producers, consumers, and the government, we obtain a net loss of 3,3 millions. Therefore this is a globally inefficient policy.

Chapter 1: On perfect competition - Home

Market surveys show that there are two types of consumers for frozen yogurt. The first type like frozen yogurt and have an inverse demand of $P_1(q) = 12 - q$; the second type are crazy about yogurt and have an inverse demand of $P_2(q) = 18 - q$. In the town of Smallville there are only 2 consumers: one of them likes frozen yogurt and the other is crazy about frozen yogurt.

(a) Determine and plot the market demand for yogurt in Smallville.

Demands are $D_1(p) = 12 - p$ *for the first type,* $D_2(p) = 18 - p$ *for the second type. Therefore, if* $p \geq 18$ *nobody buys; if* $12 \leq p \leq 18$, *total demand is* $D(p) = D_2(p)$; *if* $p \leq 12$ *all types buy and total demand is* $D(p) = D_1(p) + D_2(p)$.

Suppose now that competitive firms supply the market and that the total supply curve is given by $S(p) = p$.

(b) Find the equilibrium price and quantity. How much does each consumer buy at the equilibrium price? Determine the surplus of each type of consumers.

Assume that all types buy a strictly positive quantity at the perfect competition equilibrium, that is, assume that the competitive price p^* *will be smaller than* 12, *an assumption that must be checked ex post. At the competitive equilibrium when both types consume, total supply equals total demand, or* $D(p^*) = S(p^*)$, *or* $30 - 2p^* = p^*$, *leading to* $p^* = 10$. *Type 1 consume* $q_1^* = 2$ *and derive a surplus equal to* $\frac{1}{2}4 = 2$; *Type 2 consume* $q_2^* = 8$ *and derive a surplus equal to* $\frac{1}{2}64 = 32$. *Firms produce a total quantity of* 10, *which yields an industry profit of* 50.

Chapter 2: On monopoly pricing - In class

You own a private parking lot near ESSEC with a capacity of 600 cars. The demand for parking at this lot is estimated to be $D(p) = 1000 - 2p$, where $D(p)$ is the number of customers with monthly parking passes and p is the monthly parking fee per car.

(a) Derive your marginal revenue schedule.

Solving for p *gives the inverse demand* $P(q) = 500 - q/2$. *Revenue function is* $R(q) = P(q)q$. *Marginal revenue associated is* $MR = 500 - q$.

(b) What price generates the greatest revenue?

Revenues are maximized when marginal revenues equal zero or $q = 500$. *Then solving for price using the demand curve gives* $p = 250$.

Your fixed costs of operating the parking lot, such as the monthly lease paid to the landlord and the cost of hiring an attendant, are \$25,000 per month. In addition, your insurance company charges you \$20 per car per month for liability coverage, and the City charges you \$30 per car per month as part of its policy to discourage the use of private automobiles.

(c) What is your profit-maximizing price?

The (monthly) cost function here is $C(q) = 25,000 + 50q$. *Marginal cost per car is simply \$50. Setting* $MR = MC$ *gives* $500 - q = 50$, *or* $q^m = 450$. *Using the demand curve to solve for the price that goes along with this quantity gives* $p^m = \$275$.

(d) Is it a profitable business?

To confirm that this is indeed the profit-maximizing price, you also should check that it is not optimal to shut down, i.e., that your economic profits are positive in comparison with shutting down. This can be done by directly calculating profits, which are given by $\pi^m = p^m q^m - C(q^m) = \$76,250$. *You can also show that the price is above the average cost at the monopoly optimum.*

Chapter 2: peak-load pricing - Home

We consider a monopoly firm with marginal cost c. The demand varies with $D_1(p) = a_1 - b_1 p$ in period 1 and $D_2(p) = a_2 - b_2 p$ in period 2, with $a_1/b_1 > a_2/b_2$.

1. Characterize the profit-maximizing price for each period.

   We obtain $p_1 = \frac{a_1}{2b_1} + \frac{c}{2}$ *and* $p_2 = \frac{a_2}{2b_2} + \frac{c}{2}$.

2. Compare the two prices and discuss.

   Since $a_1/b_1 > a_2/b_2$ *we have* $p_1 > p_2$. *The difference comes from the different price-elasticities.*

We assume now that, before choosing its production, the firm must choose the size of the infrastructure Q with a marginal cost of investment C assuming that $C < \frac{(b_2-b_1)}{b_1}c$. To produce q at a given period, the initial investment should be at least $Q \geq q$. To keep things simple, we assume that $a_1 = a_2 = 1$.

3. For a given production q_1 in period 1 and q_2 in period 2, write down the total cost of production.

   The total cost is given by $cq_1 + cq_2 + C \max\{q_1, q_2\}$.

4. Assuming (and checking ex-post) that $q_1 > q_2$, characterize the profit-maximizing prices for the two periods. Comment.

   If $q_1 > q_2$, *the total marginal cost of production in period 1 is* $c+C$. *We thus obtain* $p_1 = \frac{1}{2b_1} + \frac{c+C}{2}$ *and* $p_2 = \frac{1}{2b_2} + \frac{c}{2}$. *So the total marginal costs are in fact different and play a role, on top if the elasticities, on the production choices. We can also check that, since* $C < \frac{(b_2-b_1)}{b_1}c$, *we indeed have* $q_1 = \frac{1}{2}(1-(c+C)b_1) > q_2 = \frac{1}{2}(1-(c)b_2)$.

5. What size Q should the firm choose at the initial stage?

   The firm should simply choose $Q = q_1$.

Chapter 3 - Monopoly two-part pricing - In Class

Suppose that the typical buyer of a vacuum cleaner has a straight-line (that is linear) individual demand curve for cleaner bags. At a price of \$11 per bag, he would buy zero, while at a price of \$1 per bag, he would buy 50 per year. He plans to use the vacuum cleaner for only one year. The costs of production are \$50 for the vacuum cleaner and \$1 per bag.

1. Derive the formula for the demand curve.

   $D = 55 - 5P$.

2. On a figure with price per bag on the vertical axis and quantity of bags per year on the horizontal axis, draw the inverse demand curve.

3. Suppose that the manufacturer sells the vacuum cleaner bundled [*that is sold together*] with 50 bags. What is the maximum that the manufacturer can charge for the bundle? What would be the manufacturer's profit per consumer?

   The maximum that the manufacturer can charge for the bundle is the consumer's area under demand curve up to 50 bags $= 1/2 \times (10) \times 50 + 1 \times 50 =$ *\$300. Then, the profit per consumer* $=$ *revenue - cost* $= 300 - 50 - (1 \times 50) =$ *\$200.*

4. Suppose that manufacturers sets a uniform price for bags. What would be the profit-maximizing price and the corresponding profit for the bundle?

   $\Pi = (P-1)(55-5P) - 50$ so $P^* = 6$ and $\Pi = 755$ \$.

5. Suppose that the manufacturer sets a two-part pricing policy, comprising a price for the vacuum cleaner and a price of \$3 per bag. What is the maximum, \$X, that the manufacturer can charge for the vacuum cleaner? What would be the manufacturer's profit per consumer?

   If the price of the bag is \$3, the consumer would buy 40 bags. The maximum that the manufacturer can charge for the vacuum cleaner is the buyer surplus, $X = 1/2 \times [11-3] \times 40 = \$160$.

Chapter 3 - Price Discrimination - Home

A monopolist faces some consumers (called group-I consumers) with an inverse demand function for **each consumer** given by $P = 80 - Q$. The firm's cost function is given by: $C(Q) = 10Q$.
Suppose first that the firm only uses linear pricing.

1. Find the the price maximizing the firm's profits.

   The profit writes as $PQ - C(Q) = P(80-P) - 10(80-P)$ *and we get* $P = 45$ *or equivalently* $Q = 35$.

2. What are the corresponding profit and surplus per consumer?

   The profit is $(p-10)(80-p) = 1225$ *and consumer surplus by* $(80-p)^2/2 = 612,5$

Suppose now that the firm can use two-part tariffs (p, F), with p the unit price and F the fixed part.

3. For any value of p, compute the surplus of the consumer and thus the maximum value F the firm can set to induce consumption (i.e. the total consumer surplus cannot be negative).

   Consumer surplus can be easily computed as the area between the inverse demand curve and the price line. We obtain $(80-p)^2/2$ *therefore we need* $F \leq (80-p)^2/2$.

4. Assuming that the firm chooses this maximum value, compute the profit-maximizing unit price p and then the fixed part F.

   The profit is now $(p-10)(80-p) + (80-p)^2/2$. *This leads to an optimal of* $p = 10$ *(the marginal cost) and the firm sets* F *to capture the whole surplus* $(80-p)^2/2 = (70)^2/2 = 2450$.

5. Compute the firms profit per consumer and compare with the one found in the linear pricing case. Comment.

The firm does not gain anything on the variable part but only through the fixed part $F = 2450$. This is more than on the linear case because the maximum surplus has been generated so there is no social losses.

Suppose now that there is a new group of consumers, called group II, with an inverse demand function per consumer given by $P = 60 - Q$, and that the firm cannot tell whether a consumer belong to group I or II. Suppose that the firm proposes a new two-part tariff $(\hat{p}, \hat{F})$ with $\hat{p} = 10$ and $\hat{F} = 1250$.

6. Show that both types of consumers (group I and II) prefer this new tariff to the one found in question (4).

   Group I surplus is 1200 now instead of 0 before. Whereas group II does not gain anything now but the surplus was negative before. So the new tariff is Pareto Improving.

7. Explain how the firm should change the tariffs to maximize its profit and ensure that the group-I consumers choose the adjusted (p,F) while the group-II consumers choose the adjusted $(\hat{p}, \hat{F})$. NB: you are not required to compute formally the new profit-maximizing tariffs.

   No formal computation is required here. One should understand how the firm can induce each group to self select with the adequate tariff. This is very close to what has been studied in Chapter 3. In this setting, the firm will increase $\hat{p}$ and decrease F to make the option $(\hat{p}, \hat{F})$ less attractive. Here, group I consumers value marginal consumption more than group II consumers. If $\hat{p}$ is higher than p, group I agents consume less when they opt for the group II option, and therefore their utility is lower. But as long as their net utility is positive, they are better off with this option rather then choosing (p, F). To increase the attractiveness of the later tariff, the firm needs to lower F. Formally, the following conditions must hold

$$\begin{aligned} \frac{(80-p)^2}{2} - F &\geq \frac{(80-\hat{p})^2}{2} - \hat{F} \\ \frac{(60-\hat{p})^2}{2} - \hat{F} &\geq \frac{(60-p)^2}{2} - F \end{aligned}$$

   At then end, the first condition will be binding (but solving all that is not required)

Chapter 4 - The Battle of the sexes - In class

Peter and Mary must meet to go to a show. They must choose between either going to the stadium or going to the Opera, but they cannot coordinate (it was a long time ago, before the development of mobile telephony). Peter gets a utility of 2 if he goes to the stadium with Mary, while Mary has only a utility of 1 in this case. Conversely, if Peter and Mary meet at the Opera, Mary has a utility of 4 while Peter gets only 1. At last, if Peter and Mary choose different places, they both get 0 utility.

1. Write down the payoff matrix .

   *The students should do a Standard 2*2 payoff matrix.*

2. What is (are) the Nash Equilibrium (equilibria)?.

   The student should first define (formally or not) the notion of Nash equilibrium, stressing the no-regret principle. There are two equilibria, (S,S) and (O,O), and the students should explained why there are equilibria (using the definition). Important point: an equilibrium is characterized by the strategies, but not by the gains.

3. When there are more than one equilibrium, which one is the more likely? Why?

   One possible answer would be to use an argument based on total gains, which would favor the (O,O) equilibrium. One could also select the one distributing the gains the more evenly, which would lead to the (S,S) equilibrium.

Suppose Mary has claimed she would never go the stadium. If Peter chooses to go to the stadium, he can send a message to Mary "I am at the stadium" with no way to get any answer.

4. What is then the most likely Nash equilibrium? Explain.

   The game is now a sequential one. To select the most likely Nash equilibrium, it is reasonable to focus on Subgame Perfect Nash Equilibria. In this new game, the announcement made by Mary can be understood as a threat, but it does not survive a serious analysis using backward induction. Indeed, if Peter chooses to go to the stadium, he can be his choice public and binding. As Mary cannot change Peter's action, she will act optimally from this point of the game, i.e. then choose to join Peter at the stadium. The most likely equilibrium is now (S,S) in term of action, or $(S, S0)$ *if we write Mary's contingent strategy with the first component corresponding to her choice when Peter chooses S and the second to her choice when Peter does not choose S.*

Chapter 4: Rebel without a Cause - Home

Two drivers drive towards each other on a collision course: one must swerve, or both may die in the crash, but if one driver swerves and the other does not, the one who swerved will be called a "chicken", meaning a coward.

Consider the game described by the matrix form below.

	Stay	Swerve
Stay	(-10 ; -10)	(1 ; -1)
Swerve	(-1 ; 1)	(0 ; 0)

(a) Briefly explain why this matrix is a good representation of the situation depicted above.

(b) Find the two Nash equilibria of the game. Does one equilibrium Pareto-dominate?

(c) In light of the game studied previously, discuss briefly the following historical episode (from the Wall Street Journal, 26/08/2017): "Following the Greco-Persian Wars (499-449 B.C.), Athens built up the Delian League, while Sparta led the Peloponnesian League. Each nation kept its respective allies in line through a combination of bribery and force. Facing off with rising stakes, both sides assumed the other would stand down first, but the strategy backfired. Around 432 B.C., the Athenian leader Pericles tried to isolate Corinth, a member of the Peloponnesian League, by declaring a trade embargo against one of Corinth's allies. Pericles assumed that the Spartans wouldn't have the stomach to go to war for the sake of one League member. But neither superpower was willing to lose face, and the miscalculation led to the Peloponnesian War and the eventual ruin of Athens."

[The principle of the game is that while it is to both players' benefit if one player yields, the other player's optimal choice depends on what his opponent is doing: if his opponent yields, the player should not, but if the opponent fails to yield, the player should. The name "chicken" has its origins in a game in which two drivers drive towards each other on a collision course: one must swerve, or both may die in the crash, but if one driver swerves and the other does not, the one who swerved will be called a "chicken," meaning a coward. There are two Nash equilibria in which players choose different strategies (Stay,Swerve) and (Swerve,Stay). None of these equilibrium Pareto-dominates the other. Moreover, in real-world situations, players may well end up on (Stay,Stay) as in the situation described by the Wall Street Journal.]

Chapter 5 - Duopoly and Innovation - In class

We consider a market where two firms, F1 and F2, compete *à la* Cournot. Both firms have the same marginal cost c (with $c < 1/2$), with respective output q_1 and q_2, and the inverse demand function is given by $P = 1 - q_1 - q_2$.

1. Find the equilibrium quantities chosen by both firms, and the corresponding profits.

   We first write the profit of each firm and obtain the reaction function $q_1 = \frac{1-q_2-c}{2}$ *and* $q_2 = \frac{1-q_1-c}{2}$. *We combine then and find the equilibrium quantities* $q_1 = q_2 = \frac{1-c}{3}$. *It yields the profits* $\pi = \frac{(1-c)^2}{9}$.

We assume that F1 can use a new technology that allows to reach the same production at a marginal cost of 0.

2. What are the new equilibrium quantities in this case?

   The reaction functions are now $q_1 = \frac{1-q_2}{2}$ *and* $q_2 = \frac{1-q_1-c}{2}$*. Combining them leads to* $q_1 = \frac{1}{3}(1+c)$ *et* $q_2 = \frac{1-2c}{3}$*.*

3. Compute the profits and compare with the one obtained in the first question.

   The profits is this case are given by $\pi_1 = \frac{(1+c)^2}{9}$ *and* $\pi_2 = \frac{(1-2c)^2}{9}$*. It can be easily seen that F1 benefits from this new technology, and the increase in profit is* $\frac{4c}{9}$*. On the contrary, F2 loses* $\frac{c(2-3c)}{9}$*.*

4. What price F1 will be ready to pay to benefit from this new technology? Discuss the case where only F1 can bid for the technology and the case where both firms can bid.

   There are two cases. If only F1 can bid for the new technology, it is ready to pay up to the difference between the profit earned with and without the new technology, that is up to $\frac{4a}{9}$*. If both firms can bid, F1 wants to avoid being in a situation where F2 has the new technology. F1 is therefore ready to pay up to the difference between the profit when it can benefit from the new technology and its profit when F2 can use the new technology. Formally, the maximum price F1 is ready to pay for the new technology is* $\frac{c}{3}(2-c)$*.*

Chapter 5: Limit pricing - Home

We consider a situation in which 2 firms, F1 and F2, compete proposing differentiated products, with the same marginal cost $c = 1$ and a fixed cost of $K = 49/128 \approx 0,38$. The consumers are uniformly distributed over the segment [0,1], and we assume that the differentiation (or taste intensity) parameter t is equal to 1.

1. As a function of the prices p_1 and p_2 set by the two firms, write down first the demand that is addressed to each of them and then their profit function.

   Following the course, we assume that the two firms are located at the two ends of the line. Therefore, we can characterize the location of the consumer indifferent between both firms. For given p_1 *and* p_2*, it is* $\tilde{x} = \frac{1+(p-2-p_1)}{2}$*. This implies that* $D_1 = \tilde{x}$ *and* $D_2 = 1 - \tilde{x}$ *and therefore* $\Pi_1 = (p_1 - 1)\frac{(1+p_2-p_1)}{2} - K$ *and* $\Pi_2 = (p_2 - 1)\frac{(1+p_1-p_2)}{2} - K$

2. For each firm, derive the reaction function, that is its best choice of price as a function of the price chosen by the other.

   Let us focus on firm 1. We can take the FOC that is the derivative of the profit wrt p_1*. This leads to* $p_1 = 1 + \frac{p_2}{2}$*. Using the same method for firm 2, we get* $p_2 = 1 + \frac{p_1}{2}$*.*

3. Assuming that both firms choose their price simultaneously, characterize the choice of price at the Nash equilibrium and derive the profit of each firm.

   Using the reaction functions derived above leads to $p_1 = p_2 = 2$. *This leads* $\Pi_1 = \Pi_2 = 1/2 - K = 0,12$.

We now assume that F1 chooses its price first, and then F2 reacts optimizing its own price. We want to show that in this situation where the choices are sequential, F1 may want to choose alow enough price to deter F2 from entering the market (this is called a limit pricing strategy). Hint: sequential games should be solved by backward induction, that is by solving first the last period, and then the before last, and so on.

4. Derive the price optimally chosen by F2, taking as given the price choice of F1. What is (as a function of p_1) the profit formula of F2?

   The reaction function for F2 does not change so get we $p_2 = 1 + p_1/2$. *We know that* $\Pi_2 = (p_2 - 1)(1 - \tilde{x}(p_1, p_2))$. *But now,* $(1 - \tilde{x}(p_1, p_2)) = \frac{(1+p_1-p_2(p_1)}{2} = \frac{p_1}{4}$ *so* $\Pi_2(p_1) = p_1^2/8 - K$

5. How should F1 set its price to induce zero profit for F2 (taking into in account the fixed cost)? Assuming that F2 leaves the market in this case, what is then F1's profit?

   We want to find p_1 *such that* $\Pi_2 = p_1^2/8 - K = 0$. *This leads to* $p_1 = \sqrt{K/8}$. *since* $K = 49/128$, *we get* $p_1 = 7/4$ *and F1's profit is therefore* $\pi = (7/4 - 1) \times 1 - K = 0,37$.

6. Show that this limit pricing strategy allows F1 to make a higher profit than in a standard situation in which both firms choose their price simultaneously. *If firm 1 covers the whole market, its profit is given by* $\hat{\Pi}_1 = (p_1 - 1) \times 1 - K = 3/4 - K$ *whereas it was* $1/2 - K$ *before. This implies that the limit pricing strategy is profitable for firm 1*