MACROECONOMICS
73-240

Lecture 18

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So far

We've focused on the 2 period household model

1) where income today and tomorrow, y and y', are exogenous

2) Used the model to understand how households' consumption *and* savings decisions are affected by

   - The path of income
   - Interest Rate changes

Missing Ingredients

What is the two period model missing so far?

1) Labor Supply and wages

2) Firms

3) Investment decisions

Plan for This Lecture

1) Extending the two period model
   - Representative household supplies labor and gets utility from leisure
   - Firms make investment

Extending the Two Period Model
The Consumer

The *Elastic* Consumer

Up to now y was exogenous $\Rightarrow$ now introduce labor supply:

- Quick Quiz: Suppose you know that households do not have exogenous income
- Instead they earn labor income by supplying labor and earning wage w for each hour worked.
- Suppose households earn dividend income each period and pay lump-sum taxes each period.
- How would you write down the first period budget constraint? Second period budget constraint?

The *Elastic* Consumer

The budget constraint of the Household with endogenous labor income

- Today BC

$$C + S^p = w(h - l) + \pi - T$$

- Tomorrow BC

$$C' = w'(h - l') + \pi' - T' + (1 + r)S^P$$

- Lifetime BC

$$C + \frac{C'}{1+r} = w(h - l) + \pi - T + \frac{w'(h - l') + \pi' - T'}{1+r}$$

Problem of the *Elastic* Consumer

Maximize utility subject to life-time budget constraint:

$$\max_{C,C',l,l'} u(C,l) + \beta u(C',l')$$

subject to

$$C + \frac{C'}{1+r} = w(h-l) + \pi - T + \frac{w'(h-l') + \pi' - T'}{1+r}$$

- 4 unknowns: (C, C', l, l')
- Need 4 equations for optimality!
- Hard to graph!

Problem of the *Elastic* Consumer

Maximize utility subject to life-time budget constraint:

$$\max_{C,C',l,l'} u(C,l) + \beta u(C',l')$$

subject to

$$C + \frac{C'}{1+r} = w(h-l) + \pi - T + \frac{w'(h-l') + \pi' - T'}{1+r}$$

- Make the problem unconstrained (Take the Lagrangian)
- 5 first order conditions with respect to (C, C', l, l', λ).

Consumer Optimality

Optimality conditions:

- First period consumption-leisure trade-off: $MRS_{l,C} = w$
- Second period consumption-leisure trade-off: $MRS_{l',C'} = w'$
- Optimal consumption saving: $MRS_{C,C'} = 1 + r$
- Q: What condition is missing?
  Life-time budget constraint holds with equality

- Notation: Let Non-Labor Income be denoted $NLI = \pi - T + \frac{\pi' - T'}{1+r}$

Example

- Assume utility is given by:

$$U(c, c', l, l') = \ln c + \ln l + \beta(\ln c' + \ln l')$$

and that $h = 1$, hence, $N^s + l = 1$ and the lifetime budget constraint is given by:

$$c + \frac{c'}{1+r} = wN^s + \frac{w' N^{s'}}{1+r} + NLI$$

- Solve for optimal c in terms of w, w', r, NLI, β
- Solve for optimal N^s in terms of w, w', r, NLI, β
- What can we say about how N^s reacts to w,NLI,r?
- What can we say about how c reacts to r and lifetime wealth?

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Interest Rates and Labor Supply

Let $N(w, r, NLI)$ denote Consumer labor supply:

- Current labor supply is increasing in **real wage**

$$\frac{dN(w, r, NLI)}{dw} > 0$$

  (if substitution larger than income effect, assume this from now on)

- Current labor supply decreases if **Non-labor income** increases

$$\frac{dN(w, r, NLI)}{dNLI} < 0$$

- Current labor supply increases when the **interest rate** increases

$$\frac{dN(w, r, NLI)}{dr} > 0$$

  - Consumption and leisure tomorrow becomes cheaper

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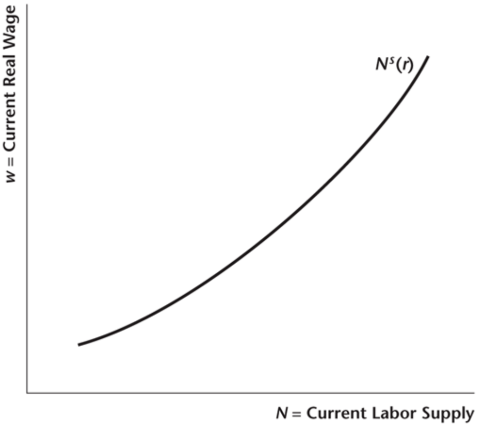
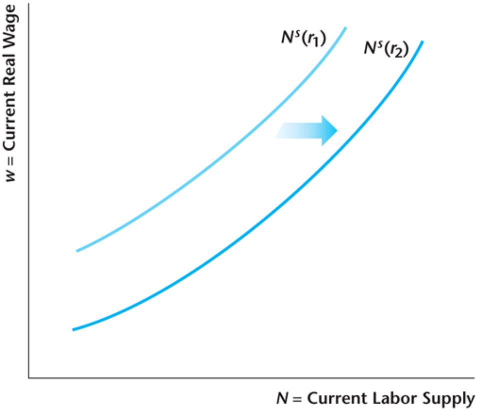
Labor Supply



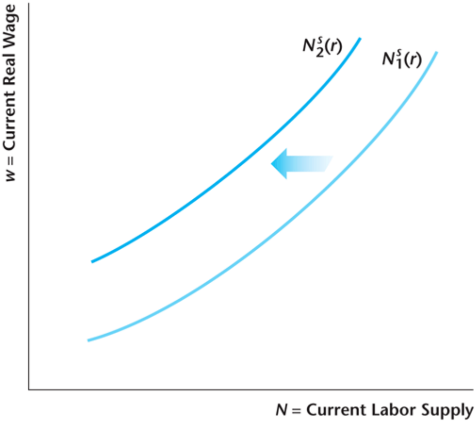
Interest Rates and Labor Supply

Following increase of $r_1 \uparrow r_2$ labor supply shifts to the right



Why? Think of price of leisure today vs tomorrow

Lifetime Non-Labor Income and Labor Supply



Example: tax cuts, increase in value of stocks, . . .

Demand for Current Consumption Goods

Let $C(\text{we}, r)$ denote consumption. Effects on the consumption of goods:

- Demand of current consumption increases when **lifetime wealth** increases (consumption is a normal good)

$$\frac{dC(\text{we}, r)}{d\text{we}} > 0$$

- Current consumption decreases with **interest rate** increases (assuming substitution effect dominating)

$$\frac{dC(\text{we}, r)}{d\text{r}} < 0$$

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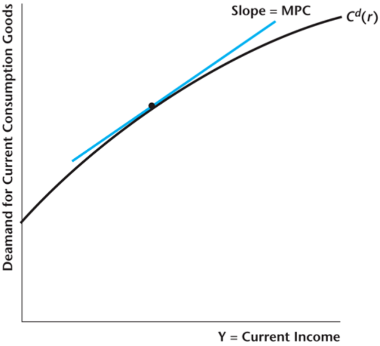
Current income and Demand for Current Consumption

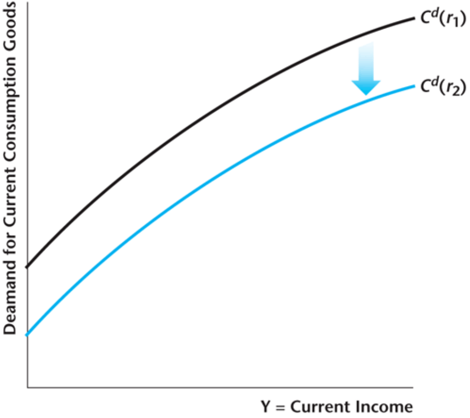
Demand for current consumption goods increases with current income. The slope of this curve is Marginal Propensity to Consume (MPC)



- MPC = marginal propensity to consume: the increase in demand for consumption goods induced by a one-unit increase in income

Interest Rate and Demand for Current Consumption

increase in the real interest rate shifts the demand for current consumption goods down



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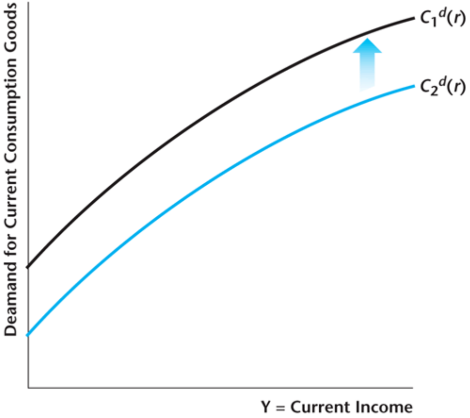
Lifetime Wealth and Demand for Current Consumption

increase in lifetime wealth shifts the demand for current consumption goods up



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Extending the Two Period Model
The Firm

The Firm

- Introduce production today and tomorrow, and investment decision
  - Output today: $Y = zF(K, N)$
  - Output tomorrow: $Y' = z'F(K', N')$
- (new!)Firm invests I so that $K' = (1 - d)K + I$

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Types of Investment

Back to Lecture 2!:

- Business Investment - includes the actual purchases of goods used in the production process.
- Changes in inventories - Firms invest in inventories, which are produced goods held in storage in anticipation of later sales.
- Also counted in investment: Residential Construction

The Firm

- Firm maximize present discounted value of the firm

$$V = \pi + \frac{\pi'}{1+r}$$

- **Profits today:** $\pi = Y - wN - I$
- **Profits tomorrow:** $\pi' = Y' - w'N' + \underbrace{(1-d)K'}_{\text{Liquidation Value}}$
- Price of investment assumed to be 1 (consumption and investment today are numeraire goods).

The Firm and Investment Decision

We now calculate the optimal investment decision

- The problem of the Firm is:

$$\max_{N,N',I} zF(K,N) - wN - I + \frac{z'F(K',N') - w'N' + (1-d)K'}{1+r}$$

subject to

$$K' = (1-d)K + I$$

- Optimality
  - Taking K, K' as given, optimality for N, N' is standard:

$$MPN = w \quad \text{and} \quad MPN' = w'$$

  - To find I, substitute for K' into profits tomorrow and take first order condition w.r.t. I!

The Firm and Investment Decision

- Re-write firm's problem as:

$$\max_{N,N',I} V = zF(K,N) - wN - I \tag{1}$$
$$+ \frac{z'F((1-d)K+I,N') - w'N' + (1-d)\left((1-d)K+I\right)}{1+r}$$

- Could also re-write the problem as a Lagrangian
- Now take first order conditions w.r.t N, N', I

The Firm and Investment Decision

We now calculate the optimal investment decision

- FOC for I:

$$-1 + \frac{1}{1+r}\big(z' F_K(K', N') + (1-d)\big) = 0$$

- Re-arrange terms:

$$\underbrace{z' F_K(K', N')}_{MPK'} - d = r$$

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The Firm and Investment Decision

- Cost of \$1 Investment? Each dollar spent on I reduces profit by 1 dollar
  $\Rightarrow$ marginal cost of investment $MC(I) = 1$
- Benefit of Investment? Increase tomorrow's capital stock, get more output & higher liquidation value in present value
  $\Rightarrow$ marginal benefit of investment

$$MB(I) = \frac{MPK' + 1 - d}{1 + r}$$

- Optimal Investment Equates marginal benefit and costs $\Rightarrow$ **Optimal investment rule**

$$MPK' - d = r \quad \text{or} \quad zF_K(K', N') - d = r$$

(net marginal product of capital tomorrow = interest rate)

The Firm and Investment Decision: Formal

Suppose that the production function is Cobb-Douglas:

$$Y = zK^{\alpha}N^{1-\alpha} \quad Y' = z'K^{',\alpha}N^{',1-\alpha}$$

Then the firm's profit maximization problem is:

$$\max_{N,N',I} V = Y - wN - I + \frac{Y' - w'N' + (1-d)K'}{1+r}$$

s.t.

$$K' = (1-d)K + I$$

Solve for optimal N^* in terms of z, K, α, w.
Solve for optimal I^* in terms of z', K, α, r, d, N'

The Firm and Investment Decision: Formal

Suppose that the production function is Cobb-Douglas:

$$zF(K,N) = zK^{\alpha}N^{1-\alpha}$$

Then optimality implies

$$z'\alpha\left(\frac{K'}{N'}\right)^{\alpha-1} = r + d$$

substituting $K' = (1-d)K + I$ we get

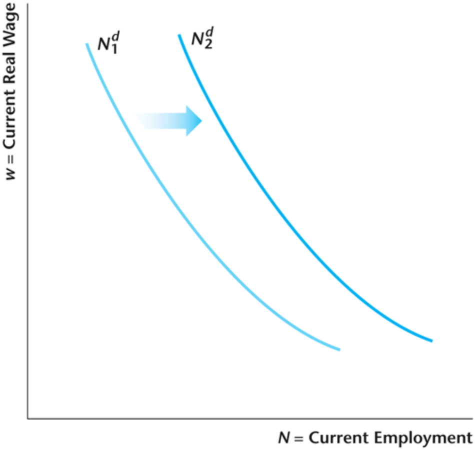
$$I = \left[\frac{z'\alpha}{r+d}\right]^{\frac{1}{1-\alpha}} N' - (1-d)K$$

Note that I is increasing in z' and decreasing in r and K. Solving for optimal N^d today, we arrive at:

$$N^d = \left[\frac{(1-\alpha)zK^{\alpha}}{w}\right]^{1/\alpha}$$

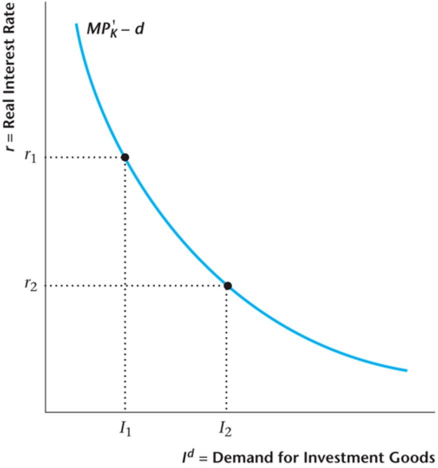
The Firm and Labor Demand

- Optimality $MPN = w$ (wage goes up labor demand goes down)
- If z or K increase $\Rightarrow$ labor demand increases



The Firm and Investment Decision

- If $r_1 \downarrow r_2$ investment increases $I_1 \uparrow I_2$



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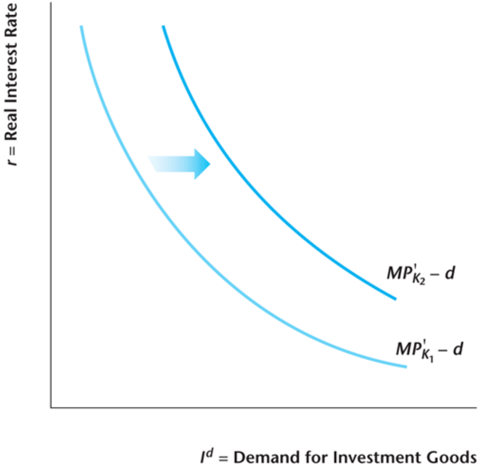
The Firm and Investment Decision

- If z' increases or if K decreases, investment curve shifts to the right (marginal benefit increases since MPK' increases)



Roadmap

- Today... ⇒ Elastic Consumers and Optimal Investment
- Next .. ⇒ Bringing the household and firm together