Intermediate Macroeconomics - 73240
November 6, 2019

MIDTERM II

Name:______________________

User Name:______________________

1. This exams contains **14** pages.
2. The final answer must be written in the space provided.
3. The exam lasts 80 minutes. You may leave after 70 min.
4. Define any notation you use that is different from lecture.
5. No calculators are allowed. The exam is closed book.
6. Answers must be written in pen.

Short Questions - 6 points each

Question 1:

The IMF estimates that GDP per capita (measured in terms of purchasing power parity) in the US in 2018 is about twice that of Malaysia. If the US grows at a constant rate of 2% per year, and Malaysia grows at a constant rate of 5% per year, calculate the time it takes for Malaysia to catch up to the US. You can leave your answer in the form of a formula with time t as the subject of the equation.

Answer:

Denote GDP_t^{US} as GDP of US at time t and GDP_t^{MY} as GDP of Malaysia at time t. Let t be the time it takes for Malaysia to catch up to the US

$$GDP_t^{US} = GDP_t^{MY}$$

$$GDP_0^{US}(1+g^{US})^t = GDP_0^{MY}(1+g^{MY})^t$$

Use the fact that GDP^{US} is initially twice that of Malaysia:

$$2GDP_0^{MY}(1+g^{US})^t = GDP_0^{MY}(1+g^{MY})^t$$

$$2(1+g^{US})^t = (1+g^{MY})^t$$

Take natural log everywhere:

$$\ln 2 + t\ln(1+g^{US}) = t\ln(1+g^{MY})$$

and solve for t

$$t = \frac{\ln 2}{\ln(1.05) - \ln(1.02)}$$

Question 2:

List two assumptions we make in the Malthus model. **Answer:**
Any of the two qualify:

- $Y = zF(L, N)$ and/or land is a fixed factor
- Households do not have utility from leisure
- households cannot save
- there is no government
- Population growth is increasing in consumption per person

Assumptions on the production function are also acceptable (i.e. CRS, more inputs = more output,

Question 3:

Suppose there are no shocks in the economy and unemployed job-seekers have a 24% probability of finding a job every month. Further suppose employed individuals lose their jobs with a 1% probability every month. Find the monthly steady state unemployment rate.

Answer:

The change in the unemployment rate is given by:

$$u_{t+1} - u_t = s_t(1 - u_t) - p(\theta_t)u_t$$

In steady state, $u_{t+1} = u_t = u$, and $s_t = s$, $p(\theta_t) = p$ Hence we have:

$$0 = s(1 - u) - pu$$

which in turn gives us:

$$u = \frac{s}{s+p} = \frac{0.01}{0.01 + 0.24} = 0.04$$

Question 4:

How would you summarize growth of world GDP per capita prior to the industrial revolution? Which growth model would you use to explain this pattern? Explain your choice by describing what the model predicts about consumption per person when there are technological improvements to farming activity .

Answer:

i Growth in world GDP per capita was stagnant prior to 1820.

ii Malthus model of growth rationalizes the pattern of stagnant growth

iii Under Malthus, an improvement in technology z leads to no sustained (long-run) improvements in consumption per person and only serves to increase population.

Question 5:

Suppose a country starts with $k_0 > k^{ss}$ where k^{ss} is the steady state capital per person. State what happens to GDP per capita over time. State what happens to the growth rate of GDP per capita over time.

Answer:

GDP per capita will decline. GDP per capita will decline at a decreasing rate, i.e. the negative growth rate of GDP per capita will become less negative over time

A country that initially starts out with $k_0 > k^{ss}$ will observe break-even investment $(n+d)k$ greater than actual investment per person. Since capital per person evolves according to:

$$k' - k = szf(k) - (n + d)k$$

capital per person must decline when $(n + d)k > szf(k)$. Intuitively, the economy is not investing enough so as to replace all the capital stock per person lost either due to depreciation or due to a growing workforce. As such, k shrinks towards k^{ss}. Since there is a one-to-one mapping between $y = zf(k)$ and k, GDP per capita will decline. As the gap between $(n + d)k$ and $szf(k)$ narrows, the fall in GDP per capita will slow down.

Question 6:

Consider the search model of unemployment. Write down the household's expected returns to search. You may assume that the household has income b when unemployed and earns wage w when employed.

Answer:

$$P(U) = p(\theta)w + (1 - p(\theta))b$$

Question 7:

Consider the search model of unemployment. Suppose all firms are taxed a lump-sum T regardless of whether they hire a worker. Write down the firm's value of creating a job. Argue how labor market tightness, θ, is affected by the tax.

Answer:

Firm's value of creating a job:

$$J = -\kappa - T + q(\theta)(z - w)$$

Under free entry, we have:

$$\kappa + T = q(\theta)(z - w)$$

OR

$$q(\theta) = \frac{\kappa + T}{z - w}$$

Using the fact that the job-filling rate is given by: $q(\theta) = e\mathcal{M}(1, 1/\theta)$, we have:

$$\mathcal{M}(1, 1/\theta) = \frac{\kappa + T}{e(z - w)}$$

Observe that labor market tightness is inversely related to $\kappa + T$ which is now the effective cost of creating a vacancy. Note that firms must pay T whether or not they hire a worker. So if they enter the labor market, it is as if they pay a cost $\kappa + T$. Since it is now more expensive to enter the labor market, firms are less incentivized to enter and create fewer vacancies. As such θ falls (This can be seen from the inverse relationship between θ and $\kappa + T$). Thus labor market tightness θ is falling when T rises.

Question 8:

Using the Solow model, offer an explanation as to why a country B that is initially poorer than country A, may never catch up to country A.

Answer:

If countries have different s, z, n, d, they need not catch up to the same point. If country B has a lower s or z than A (or a higher n and d than A), it may have a lower steady state capital per person $k_B^{ss} < k_A^{ss}$. Thus in the long-run, country B may never catch up to country A even if it starts out poorer because it converges to a lower equilibrium steady state capital per person. Since there is a one-to-one mapping between $y = zf(k)$ and k, GDP per capita in B in the steady state will always be than A's steady state GDP per capita.

Question 9:

Consider the Malthusian model of growth. Suppose at time T, there is a loss of land because part of it becomes submerged due to rising sea levels. Explain what happens to consumption per person at time T

Answer:

At time T, land is lost, so $l = \frac{L}{N}$ falls. This causes output per person to fall at time T and consumption per person to fall at time T.

Question 10:

Following from question 9, state what happens to consumption per person and population in the long run (when the economy arrives at steady state).

Answer:

A fall in c causes $g(c)$ to be less than 1, and population starts to shrink. The shrinkage in population will lead to higher land per person and therefore output and consumption per person will rise. This causes $g(c)$ to increase but so long as its less than 1, population will continue to shrink albeit at a slower rate. In the long run, when the economy arrives at steady state we have: 1. c restore to original levels, and 2. steady state population is lower.

Problem 1 (15 points) :

Consider the Malthus model of growth. Gross population growth depends on consumption per person. Assume there exists a feudal lord who taxes/grabs τ proportion of output every period. Output is still given by $Y = zL^{\alpha}N^{1-\alpha}$.

a Write down the equilibrium conditions in this economy. Write down what consumption per person is equal to in terms of τ, z, l, α where l is land per person.

Answer

In equilibrium, the labor market must clear:

$$N^s = N^d = N$$

the goods market must clear but now part of output goes to the feudal lord:

$$C + \text{feudal lord consumption} = Y = zL^{\alpha}N^{1-\alpha}$$

and population must grow:

$$N' = Ng(c)$$

In steady state, we have

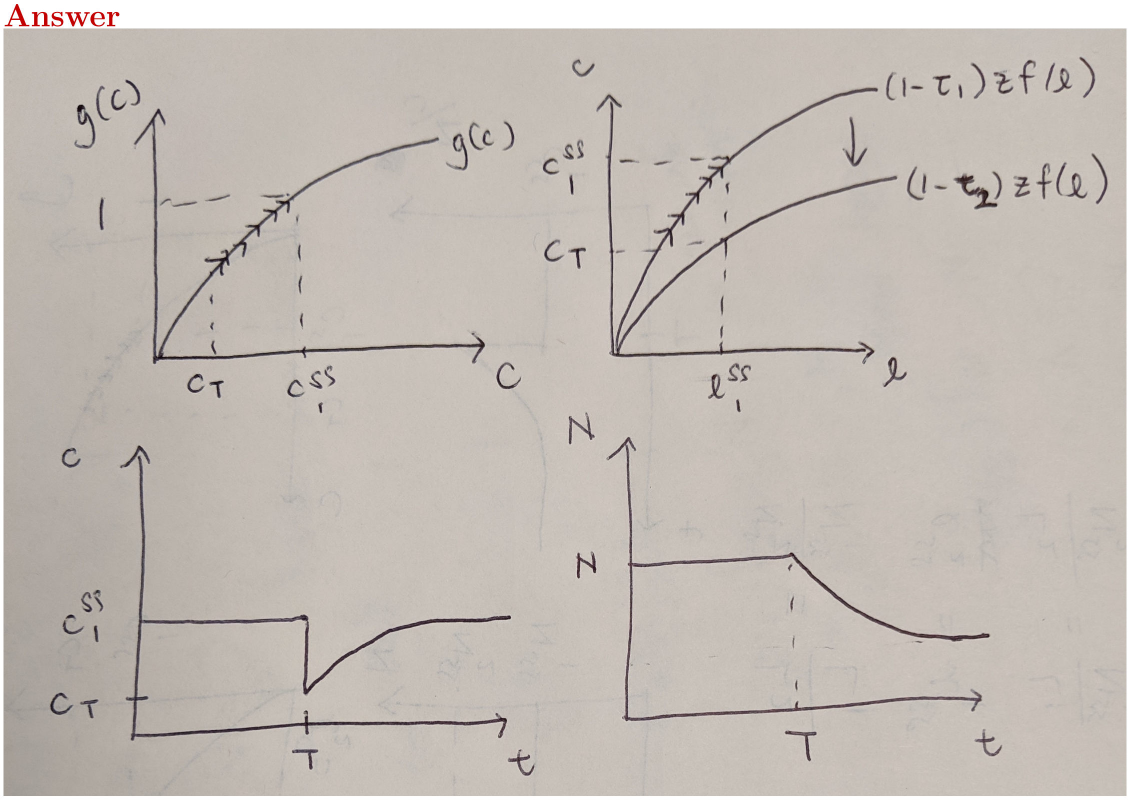
$$N' = N = N^{ss}$$

Consumption per person in this economy is given by:

$$c = \frac{C}{N} = \frac{(1-\tau)Y}{N} = (1-\tau)\frac{zL^{\alpha}N^{1-\alpha}}{N} = (1-\tau)zl^{\alpha}$$

b Suppose the economy was initially in steady state and the feudal lord raised the tax on his villages, i.e. τ increases. Show what happens to consumption per worker and population in the short and long run. Use graphs to accompany your explanation.

Answer



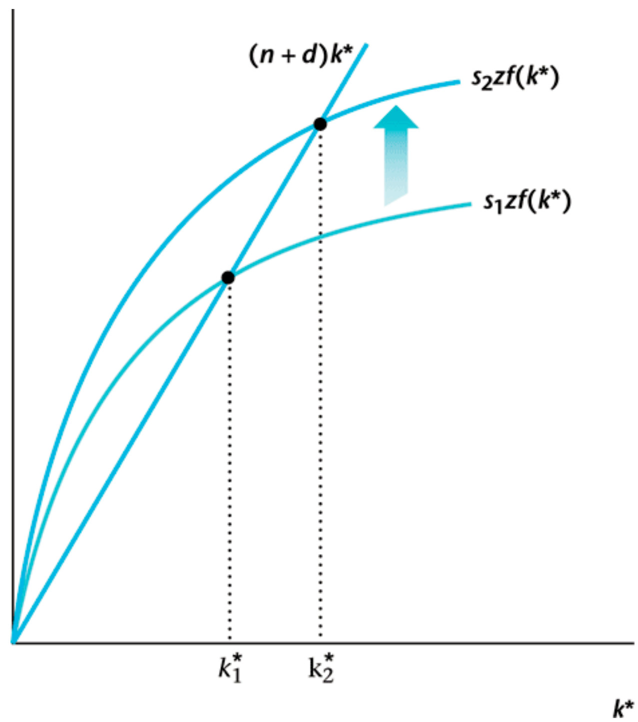
If the tax increases at time T, consumption per person falls at time T. This causes population growth rates to decline and be less than 1. This in turn causes population to shrink and land per person to be larger. A higher land per person raises output per person and consumption per person. This in turn causes population growth rates to be higher. So long as population growth rates are less than 1, the population will continue to decline but at a slower rate. In the transition to the steady state, population keeps declining, consumption per person keeps growing until we arrive at steady state where consumption per person is restored to its original level, gross population growth rates are equal to 1 and population is now lower than before.

Problem 2 (10 points):

Consider Solow Model of Growth. Let output be given by $Y = zK^{\alpha}N^{1-\alpha}$. Suppose the government raises the savings rate

a Show graphically how an increase in the new savings rate will affect steady state capital per person. Explain what it implies for output per person.

Answer
Steady state capital per person will increase. Since output per person is given by zk^{α}, output per person will increase as k increases and in the long-run, output per person will also be higher.



b Some of the Asian economies that rapidly developed (e.g. Hong Kong, Singapore, South Korea, Taiwan) relied on a policy of heavy investment which in the Solow model would show up as having higher savings rates. Explain whether a policy of continually raising the savings rate is one that makes the household happy. In other words, argue if there is any trade-off to the household from pursuing a policy that seeks to raise GDP per capita by raising savings rates. You may assume that a single household only receives utility from consumption per person.

Answer

Observe that consumption per person is given by:

$$c = (1-s)y = (1-s)zk^{\alpha}$$

A policy that continually raises the savings rate while it increases k^{ss} and hence y^{ss} in the long run, need not necessarily increase c^{ss}. This is because there are two opposing effects on consumption when the savings rate is raised. First, higher savings rate raises consumption per person through higher output per person, but a higher savings rate also implies that a smaller fraction of output per person now goes towards consumption. As such, a policy that seeks to raise GDP per capita by only raising savings rates need not necessarily make consumers better off given the two opposing effects.

Problem 3 (15 points):

Using the search model of unemployment, suppose the government levies a lump-sum tax, T, only on individuals who are not employed. If individuals are employed, there is no tax. The government has in mind that it can encourage individuals to work if they are taxed while non-employed. Individuals who stay out of the labor force or who are unemployed produce home production goods valued at b.

a Write down the job-seeker's expected returns to search.

Answer

$$P(U) = p(\theta)w + (1 - p(\theta))(b - T)$$

b Suppose the tax collected is used to award firms a credit s for each vacancy they create. Write down the value of creating a job.

Answer

$$J = -\kappa + s + q(\theta)(z - w)$$

c Assume that the number of matches, $M = eV^{1-\gamma}U^{\gamma}$, where e is the matching efficiency, V is the number of vacancies and U is the number of job-seekers. Write down $p(\theta)$ (probability job-seeker finds a job) in terms of e, θ, γ where $\theta = \frac{V}{U}$

Answer

$$p(\theta) = \frac{M}{U} = \frac{eV^{1-\gamma}U^{\gamma}}{U} = e\theta^{1-\gamma}$$

d Show how equilibrium w is affected by T and s. You may assume that the worker has bargaining weight α.

Answer

Worker gets w if agree to match and if she disagrees to the match, she gets $b - T$. So the worker's gain to matching is:

$$w - (b - T)$$

The firm's gain to matching is given by:

$$z - w$$

The total surplus then is given by:

$$z - (b - T)$$

Under Nash-bargaining we have:

$$\max_{w} (z - w)^{1-\alpha}(w - [b - T])^{\alpha}$$

Solving we get:

$$w = b - T + \alpha(z - [b - T])$$

Note that the worker must be paid at least her outside option, here $b - T$ and receive her share of the total surplus. We can re-write w as:

$$w = \alpha z + (1 - \alpha)(b - T)$$

From the above it is clear that w is decreasing in T. w is not directly affected by s (only through T if the govt budget constraint is taken into account).

e Show how equilibrium θ and $p(\theta)$ are affected by T and s. Is the household necessarily better off in terms of expected returns to search?

Answer

Using w, one can show that the firm's profits are given by:

$$\pi = (1-\alpha)(z-[b-T])$$

Notice that as T rises, firm's profits rise. This is because when T rises, it worsens workers' outside options and makes unemployment less attractive. So firms can pay workers less now to make them agree to the match.

Under free entry, we have:

$$\kappa - s = q(\theta)(1-\alpha)(z-[b-T])$$

Plug in for the fact that $q(\theta) = e(1/\theta)^\gamma$

$$\kappa - s = e(1/\theta)^\gamma(1-\alpha)(z-[b-T])$$

which in turn gives us:

$$\theta = \left(\frac{e(1-\alpha)(z-[b-T])}{\kappa - s}\right)^{1/\gamma}$$

From the above equation, we see that θ is increasing in T and θ is also increasing in s. This implies that if the government wants to provide a larger subsidy s by raising taxes T, both a rise in T and s cause θ to rise. A rise in s makes posting vacancies cheaper while a rise in T makes profits larger. As such, firms are incentivised to create more vacancies, causing θ to go up.

The rise in θ in turn raises the worker's probability of finding a job $p(\theta)$ and causes unemployment rates to go down. The impact on worker's expected returns to search is ambiguous as wages are lower but job finding rates are higher.