Statistical modeling

Exercise 1. Small questions

1. Give a possible sample of size 4 from each of the following populations:
   - all daily newspapers published in the world,
   - all grades of students at the probability course,
   - all distances that may result when you throw a football,
   - all possible daily numbers of cars going through a crossing.

2. In 1882, Michelson and Newcomb measured the traveling time of light going from and to their lab through a mirror. Their first measurements were: 28, 26, 33, 24, 34, -44, 27, 16, 40, -2, 29, 24, 21, 25 ($*0.001 + 24.8$ in millionths of a second). Why are these measurements not identical? How do we model this variability in statistics?

3. Are the following statistical models identifiable
   - $\big((\mathcal{X} = \mathbb{R}, \mathcal{B}(\mathbb{R})), \{\mathcal{N}(\mu + a, \sigma^2), (\mu, a, \sigma) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}_+\}\big)$,
   - $\big((\mathcal{X} = \mathbb{R}, \mathcal{B}(\mathbb{R})), \{\mathcal{N}(\mu, \sigma^2), (\mu, \sigma) \in \mathbb{R} \times \mathbb{R}_+\}\big)$,
   - $\big((\mathcal{X} = \mathbb{R}, \mathcal{B}(\mathbb{R})), \{\mathcal{N}(\mu, \sigma^2), (\mu, \sigma) \in \mathbb{R} \times \mathbb{R}\}\big)$?

Exercise 2. Gaussian measurements
For this exercise, you can use the table of the cdf of the standard normal distribution in Figure 1. The measurement of atmospheric ozone concentration (in $\mu g/m^3$) is modeled by a random variable X with distribution $\mathcal{N}(m, \sigma^2)$ with $\sigma^2 = 3.1$.

1. Write the statistical model.

2. In many applications, data are often modeled with the Normal distribution, while often the observed values are by definition positive (e.g. weight, size, speed, duration). Can you explain why?

3. Some day, we make some measurements and we assume that this day the ozone concentration is $178\mu g/m^3$ (yet the experimenter doesn't know this concentration otherwise he wouldn't need measurements).
   (a) Compute the probability that a unique measurement is greater than 180?
   (b) What is the probability that the mean of three measurements is greater than 180 ?
   (c) How many measurements are necessary for the probability that the mean of these measurements is greater than 180 being less than 1%?

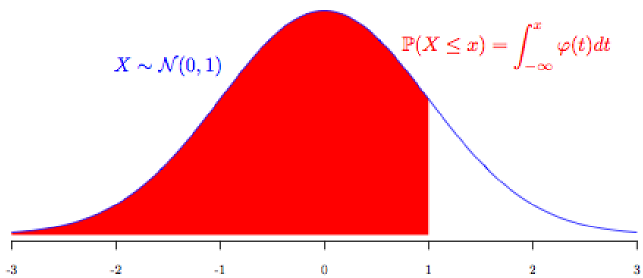
Exercise 3. We are in front of a black urn which contains N balls which are numbered from 1 to N. We don't know N the number of balls but we can draw as many balls as we wish if we put it in the urn before drawing another one.

1. Write the statistical model.

2. How can you guess the value of N with the observed numbers $x_1, \ldots, x_n$ during the sampling?

3. What is the distribution of the greatest number $\hat{N} = \max(X_1, \ldots X_n)$?

4. How many balls do you want to draw? Help: you can compute the probability that $\hat{N} = N$.



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Figure 1: Table of the cdf of the standard normal distribution