INTERMEDIATE MICROECONOMICS

CONSUMER DEMAND – PRICE ELASTICITY

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Demand Functions

Utility Function	$U(q_1, q_2)$	Solution	Demand Functions q_1	q_2
Perfect complements	$\min(q_1, q_2)$	interior	$Y/(p_1 + p_2)$	$Y/(p_1 + p_2)$
CES, $\rho \neq 0, \rho < 1, \sigma = 1/(\rho - 1)$	$(q_1^\rho + q_2^\rho)^{\frac{1}{\rho}}$	interior	$q_1 = \dfrac{Yp_1^\sigma}{p_1^{\sigma+1} + p_2^{\sigma+1}}$	$q_2 = \dfrac{Yp_2^\sigma}{p_1^{\sigma+1} + p_2^{\sigma+1}}$
Cobb-Douglas	$q_1^a q_2^{1-a}$	interior	aY/p_1	$(1 - a)Y/p_2$
Perfect substitutes, $p_1 = p_2 = p$	$q_1 + q_2$	interior	$q_1 + q_2 = Y/p$	
$p_1 < p_2$		corner	Y/p_1	0
$p_1 > p_2$		corner	0	Y/p_2
Quasilinear,	$aq_1^{0.5} + q_2$			
$Y > a^2p_2/[4p_1]$		interior	$\left(\dfrac{a}{2}\dfrac{p_2}{p_1}\right)^2$	$\dfrac{Y}{p_2} - \dfrac{a^2}{4}\dfrac{p_2}{p_1}$
$Y \leq a^2p_2/[4p_1]$		corner	Y/p_1	0

Price Elasticity

- *Price* elasticity of demand: Percentage change in the quantity demanded in response to a given percentage change in *price*

own-price elasticity

$$\varepsilon = \frac{\%\ change\ in\ demand}{\%\ change\ in\ price} = \frac{dx_1}{dp_1}\frac{p_1}{x_1}$$

- Elasticity is a unit-less measure, i.e. percentage differences

Price Elasticity

- The price elasticity of demand is also called the *own-price* elasticity of demand.

- There's another notion of price elasticity: **cross-price elasticity**

$$\epsilon_{12} = \frac{\%\ change\ in\ demand\ of\ good\ 1}{\%\ change\ in\ price\ of\ good\ 2} = \frac{dx_1}{dp_2}\frac{p_2}{x_1}$$

$$\epsilon_{21} = \frac{\%\ change\ in\ demand\ of\ good\ 2}{\%\ change\ in\ price\ of\ good\ 1} = \frac{dx_2}{dp_1}\frac{p_1}{x_2}$$

Cross-Price Elasticity

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$$\epsilon_{21} = \frac{\%\ change\ in\ demand\ of\ good\ 2}{\%\ change\ in\ price\ of\ good\ 1} = \frac{dx_2}{dp_1}\frac{p_1}{x_2}$$

Cross-Price Elasticity

- We have seen perfect substitutes:
  - Example: $u(x_1, x_2) = x_1 + x_2$
- We have also seen perfect complements:
  - Example: $u(x_1, x_2) = \min(x_1, x_2)$
- Why are they called “perfect” ?
  - What about imperfect substitutes and imperfect complements?

Cross-Price Elasticity

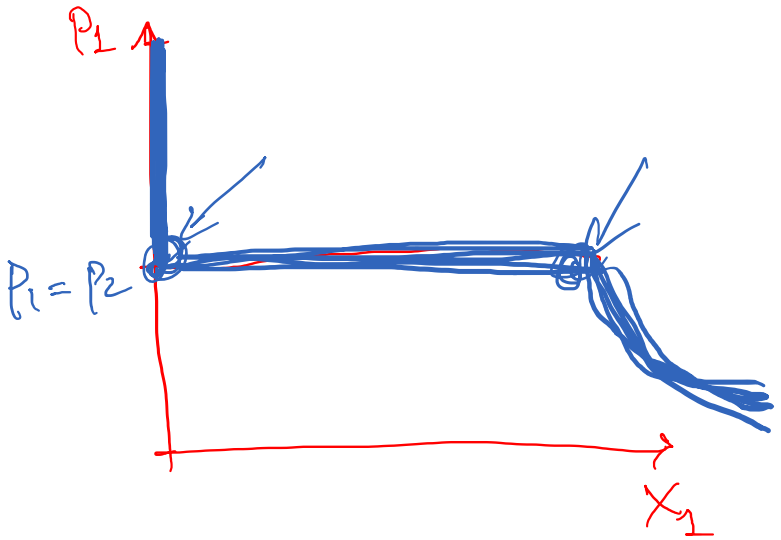
- Two goods are substitutes if $\epsilon_{12} > 0$
- Two goods are complements if $\epsilon_{12} < 0$
- ϵ_{12} & ϵ_{21} always have the same sign, so you don't have to check both.

Cross-Price Elasticity Perfect Substitutes

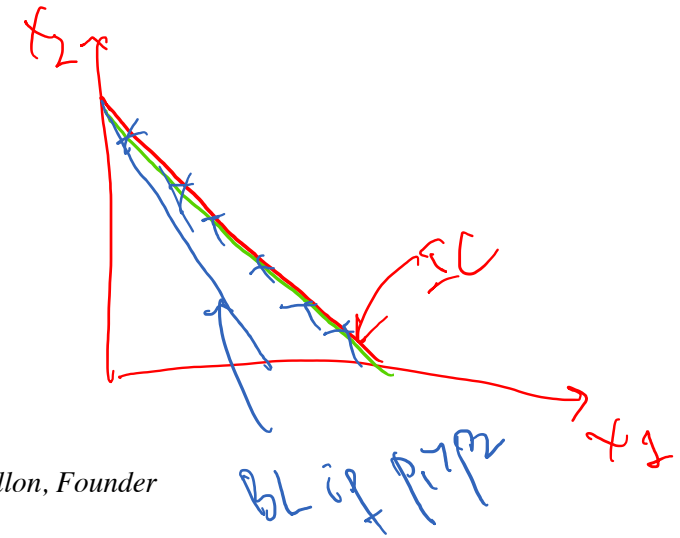
- Consider the following consumer's optimization problem:

$$u(x_1, x_2) = x_1 + x_2$$

Subject to: $Y \geq p_1 x_1 + p_2 x_2$



- If currently $p_1 = p_2,\ \epsilon_{12} = ?$
  - Be careful, because the demand is not differentiable!



Cross-Price Elasticity Perfect Substitutes

- Consider the following consumer's optimization problem:

$$u(x_1, x_2) = x_1 + x_2$$

Subject to: $Y \geq p_1 x_1 + p_2 x_2$

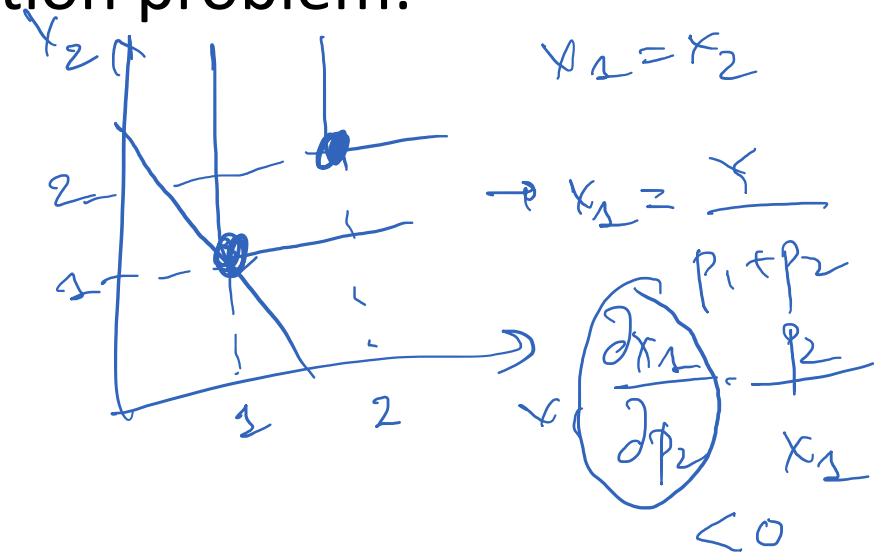
- If currently $p_1 = p_2$, $\epsilon_{12} = ?$
  - Be careful, because the demand is not differentiable!
  - Answer: $\epsilon_{12} = \infty$

Cross-Price Elasticity Perfect Complements

- Consider the following consumer's optimization problem:

$$u(x_1, x_2) = \min(x_1, x_2)$$

Subject to: $Y \geq p_1 x_1 + p_2 x_2$



- If currently $p_1 = p_2$, $\epsilon_{12} = ?$

Cross-Price Elasticity Perfect Complements

- Consider the following consumer's optimization problem:

$$u(x_1, x_2) = \min(x_1, x_2)$$

Subject to: $Y \geq p_1 x_1 + p_2 x_2$

- $\epsilon_{12} = ?$
  - $x_1 = x_2 = \frac{Y}{p_1 + p_2}$ (check that you can derive this)
  - $\epsilon_{12} = \varepsilon$

Cross-Price Elasticity More Example

- Consider the following consumer's optimization problem:

$$u(x_1, x_2) = \sqrt{x_1} + \sqrt{x_2}$$

Subject to: $Y \geq p_1 x_1 + p_2 x_2$

What is the own-price elasticity ? What is the cross-price elasticity ϵ_{12}?

Step 1: Find the demand function.

$Y = p_1 x_1 + p_2 x_2$

$\rightarrow x_2 = \frac{Y - p_1 x_1}{p_2}$

$\frac{\partial u}{\partial x_1} = \frac{1}{2\sqrt{x_1}}$

$\frac{\partial u}{\partial x_2} = \frac{1}{2\sqrt{x_2}}$

$\frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = \frac{2\sqrt{x_2}}{2\sqrt{x_1}} = MRS = \frac{p_1}{p_2}$

$\frac{\sqrt{x_2}}{\sqrt{x_1}} = \frac{p_1}{p_2}$

$\frac{x_2}{x_1} = \frac{p_1^2}{p_2^2}$

$x_1 = \frac{Y}{p_1 p_2}$

$(Y - p_1 x_1) p_2^2 = p_2 x_1 \cdot p_1^2$

$\frac{Y - p_1 x_1}{p_2 \cdot x_1} = \frac{p_1^2}{p_2^2}$

$\rightarrow x_1(Y, p_1, p_2)$