

73-240 – PRACTICE FINAL

Sample Short Questions

Question 1

Consider the household that lives for two periods in an endowment economy. The household receives exogenous income y and y' in period 1 and 2 respectively. The household faces a proportional tax τ_y on its exogenous income in every period. Write down the household's lifetime budget constraint.

Answer:

$$c + \frac{c'}{1+r} = (1 - \tau_y)y + \frac{(1 - \tau_{y'})y'}{1+r}$$

Question 2

State the conditions under which Ricardian Equivalence holds.

Answer:

1. Taxes are equal for all household types
2. Debt repaid while you are alive
3. Lump sum taxes
4. No credit markets frictions

Question 3

Suppose the firm lives for two periods. Let the production function be given by $Y = zK^\alpha N^{1-\alpha}$. Show that optimal labor demand in period 1 is only a function of current period variables and parameters of the model.

Answer:

Firm's problem is

$$\max_{N, N', I} V = Y - wN - I + \frac{Y' - w'N' + (1-d)K'}{1+r}$$

s.t. $K' = (1-d)K + I$ Since we are interested in optimal N , we can take first order conditions with respect to N :

$$(1 - \alpha)zK^\alpha N^{-\alpha} = w$$

re-arrange to find optimal N

$$N^* = \left(\frac{(1 - \alpha)zK^\alpha}{w} \right)^{1/\alpha}$$

The above shows that N only depends on current period objects.

Question 4

Consider the consumption-savings problem of a household who lives for 2 periods. Assume the Household has utility given by $U(c, c') = u(c) + \beta u(c')$. The household gets exogenous net income $y - t$ and $y' - t'$ in periods 1 and 2 respectively where t, t' are the lump-sum taxes. Suppose the interest rate rises. What can you say about the change in the household's c, c', s when the household is a borrower?

Answer:

If the interest rate rises, the relative price of consumption increases. Under the substitution effect (SE), consumption is more expensive today, so households want to reduce c and increase s and c' . The household who is a borrower experiences a negative income effect (IE) when r rises. As such, it wants to reduce both c and c' . Overall, c unambiguously falls (while s rises). The effect on c' is ambiguous and depends on the size of IE vs SE.

Question 5:

Consider the **2 period production economy**. Write down the definition of a competitive equilibrium for this economy. In your answer, state what are the exogenous variables and which are the endogenous variables.

Answer:

Given a set of exogenous parameters/variables $\{h, K, z, z', d, G, G'\}$ a *competitive equilibrium* is a set of endogenous variables $\{C, C', l, l', \pi, \pi', N, N', I, T, T', w, r\}$ such that

- 1) Given w, r, T, T', π, π' , the consumer's choices (C, C', l, l') are optimal
- 2) Given w, r , the firm's choices (N, N', I) are optimal
- 3) The Government's lifetime budget constraint is satisfied
- 4) Labor Markets Clear ($N = h - l$ and $N' = h - l'$)
- 5) Goods Markets Clear ($C + I + G = zF(K, N)$ and $C' + G' - (1 - d)K' = z'F(K', N')$)

6) Credit Market Clears

Question 6:

Consider the government in a two period economy. Write down its first period budget constraint, second period budget constraint and lifetime budget constraint.

Answer:

first period

$$G = T + B$$

second period

$$G' + (1 + r)B = T'$$

lifetime

$$G + \frac{G'}{1 + r} = T + \frac{T'}{1 + r}$$

Question 7:

Consider the **2 period production economy**. Write down what aggregate output demand is equal to in terms of each of its components. Show analytically that aggregate output demand is decreasing in the interest rate.

Answer:

Output demand is given by:

$$Y^d(r) = C^d(we, r) + I(r, z', K) + G$$

Differentiate wrt r

$$\frac{dY^d(r)}{dr} = \underbrace{\frac{dC^d(we, r)}{dY^d(r)}}_{MPC} \frac{dY^d(r)}{dr} + \frac{dC^d(we, r)}{dr} + \frac{dI(r)}{dr}$$

Re-arrange:

$$\frac{dY^d}{dr} = \frac{1}{1 - MPC} \left[\underbrace{\frac{dC^d(we, r)}{dr}}_{<0} + \underbrace{\frac{dI(r, z', K)}{dr}}_{<0} \right] < 0$$

Question 8:

Consider the two period household problem. Suppose the household has utility given by $U(c, c') = \ln(c) + \beta \ln(c')$. Assume that the household receives exogenous income every period and pays zero lump-sum taxes every period. Assume $\beta = 1/(1+r)$. Suppose the household has a decreasing income stream, show whether optimal savings is positive or negative.

Answer:

When $\beta = 1/(1+r)$, this is the special case where we have perfect consumption smoothing, $c = c'$ (you can prove this by solving the problem as well). In this case, from the budget constraint we have:

$$c + \frac{c}{1+r} = y + \frac{y'}{1+r}$$

re-arranging:

$$c = \frac{(1+r)y}{2+r} + \frac{y'}{2+r}$$

savings is then given by:

$$s = y - c = \frac{y - y'}{2+r}$$

A decreasing income stream implies $y' < y$, so optimal savings is then positive. Individuals anticipating less income when they are older, save for old age.

Sample Long Questions

Problem 1: Inheritance

Consider the household that lives for 2 periods in an endowment economy. Suppose households care about the future of their children and would like to leave behind an inheritance for them. Denote the bequests left for their children once the household dies as x . Let the household's utility be given by:

$$U(c, c', x) = \ln c + \beta \ln c' + \gamma \ln x$$

where $0 < \beta < 1$ and $0 < \gamma < 1$. Assume that there is a government that has to spend exogenous G and G' , and finances its lifetime government spending by only collecting a proportional inheritance tax, τ .

- a Write down the period by period government budget constraints. Write down the lifetime government budget constraint

Answer:

With only a proportional tax on inheritance, the government budget constraint in the first period is:

$$G = B$$

and in the second period:

$$G' + (1 + r)B = \tau x$$

and so the lifetime government budget constraint is:

$$G + \frac{G'}{1 + r} = \frac{\tau x}{1 + r}$$

- b Write down the household's lifetime budget constraint

Answer:

$$c + \frac{c'}{1 + r} + \frac{(1 + \tau)x}{1 + r} = y + \frac{y'}{1 + r}$$

- c Solve for optimal c today given τ, y, y', r . If the inheritance tax is lowered, how does that affect consumption today?

Answer:

The household's problem is given by:

$$\max_{c, c', x} \mathcal{L}(c, c', x, \lambda) = \ln c + \beta \ln c' + \gamma \ln x + \lambda \left(y + \frac{y'}{1 + r} - c - \frac{c'}{1 + r} - \frac{(1 + \tau)x}{1 + r} \right)$$

first order conditions:

$$\begin{aligned} \frac{1}{c} &= \lambda \\ \frac{\beta}{c'} &= \frac{\lambda}{1 + r} \\ \frac{\gamma}{x} &= \frac{\lambda(1 + \tau)}{1 + r} \end{aligned}$$

and the budget constraint. Our optimality conditions include the budget constraint, the trade-off between consumption today and tomorrow:

$$\frac{1}{c} = (1 + r) \frac{\beta}{c'} \implies c' = \beta(1 + r)c$$

and the trade-off between consumption today and how much inheritance to leave behind:

$$\frac{1}{c} = \frac{1 + r}{1 + \tau} \frac{\gamma}{x} \implies x = \gamma \frac{1 + r}{1 + \tau} c$$

Plug expression of x and c' into budget constraint:

$$c + \frac{\beta(1+r)c}{1+r} + \frac{1+\tau}{1+r}\gamma\frac{1+r}{1+\tau}c = y + \frac{y'}{1+r}$$

and simplifying, we have:

$$c = \frac{1}{1+\beta+\gamma} \left[y + \frac{y'}{1+r} \right]$$

Notice that here the inheritance tax does not directly affect consumption today. Instead a lower inheritance tax merely makes the amount of inheritance left behind larger since

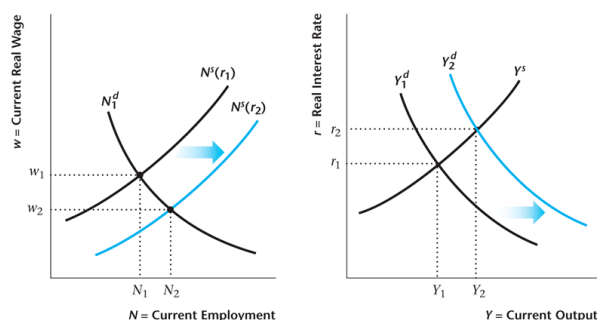
$$x = \gamma \frac{1+r}{1+\tau} c$$

Intuitively, the household has more resources available to allocate towards inheritance when the tax is lowered. (Note that this is not a case of Ricardian equivalence because here the only tax was an inheritance tax. Implicitly, the only way to lower the inheritance tax was to also have lower lifetime government spending. Ricardian equivalence is a concept about how, under certain conditions, the timing of taxes don't matter when there is no change in government spending plans.)

Problem 2: Good news about the future

Consider the 2 period production economy. The stock market is typically seen as a leading indicator of the business cycle; market analysts typically view a strong stock market as indicative of good news about future business conditions in the future. Given this information, a student argues that a strong stock market can actually stimulate economic activity today. Using graphs, show what happens to the key economic variables, Y, C, I, N, w and r when z' increases. State, if any, which variables might have an ambiguous outcome. Given your answer, argue if the student's statement is right or wrong.

When productivity increases tomorrow, investment rises as firms want to take advantage of higher productivity tomorrow and would invest to increase the amount of capital tomorrow available for production. As such the output demand curve shifts out.



A rise in output demand causes there to be excess demand at the old interest rate. As such interest rates rise. Since the interest rate is the relative price of consumption today, when the substitution effect dominates, we want to increase labor supply today so as to have more income to consume from tomorrow. Thus, the labor supply curve shifts out and overall output supply increases with the higher interest rate (this is captured as a movement along the output supply curve). Overall we see equilibrium labor increase and a fall in wage rates. A rise in interest rate reduces the incentive to invest since higher interest rates imply greater discounting of future values. **IF** the change in investment due to a higher interest rate is less than the original change in investment due to an increase in productivity tomorrow, then we would have overall investment increasing.

Finally higher interest rates and output, have 2 effects on consumption. 1) a higher interest rate depresses consumption today as consumption today is now more expensive. 2) A higher income today however, raises consumption as this is a positive income effect.

Overall we see, interest rates and output rise, labor increase, wages fall. Depending on the size of the income vs. substitution effect, consumption can be ambiguous. What we do know for sure, is that since equilibrium output rises, and since $\Delta Y = \Delta(C + I)$ (G is exogenous and so does not respond to z' or r), then the sum of investment and consumption spending must have also increased.

The student's statement is correct to the extent that if a strong stock market represents news that z' is higher, then news about a higher z' can lead to equilibrium output increasing and hence higher economic activity.