

*Intermediate Macroeconomics - 73240*  
October 7, 2019

MIDTERM I



Name:\_\_\_\_\_

User Name:\_\_\_\_\_

1. This exams contains **15** pages.
2. The final answer must be written in the space provided.
3. The exam lasts 80 minutes.
4. Define any notation you use that is different from lecture.
5. No calculators are allowed. The exam is closed book.
6. Answers must be written using pens not pencils.

## Short Questions - 6 points each

Before you begin, check that you have all the pages and write your name! Good luck!

### Question 1:

A country's trade balance is given by net exports. Use the following table to determine the size of Country A's trade deficit or surplus. **Answer:**

	value
GNP	80
I	10
G	10
C	70
NFP	60

$$GDP = GNP - NFP = 80 - 60 = 20$$

$$NX = GDP - I - G - C = 20 - 10 - 10 - 70 = -70$$

**Question 2:**

Suppose country A consumes only Apples and Bananas. Let  $P$  denote prices and  $Q$  denote quantities. Using the table below, calculate Nominal GDP for each year. **Answer:**

Item	Year 1 P	Year 1 Q	Year 2 P	Year 2 Q
Apples	2	20	1	50
Bananas	2	5	5	2

$$NGDP_1 = P_{A1}Q_{A1} + P_{B1}Q_{B1} = 2 \times 20 + 2 \times 5 = 50$$

$$NGDP_2 = P_{A2}Q_{A2} + P_{B2}Q_{B2} = 1 \times 50 + 5 \times 2 = 60$$

**Question 3:**

Following from Question 2, calculate the Consumer Price Index (CPI) for each year using year 1 as the base year. You may leave your final answer in improper fractions

**Answer:**

$$CPI_1 = 100 \times E_1/E_b = 100 \times NGDP_1/NGDP_1 = 100$$

$$E_2 = 1 \times 20 + 5 \times 5 = 45$$

$$CPI_2 = 100 \times E_2/E_b = 100 \times 45/50 = 90$$

**Question 4:**

Briefly argue if the following statement is true: “Employment would increase if workers were paid more per unit labor supplied.”

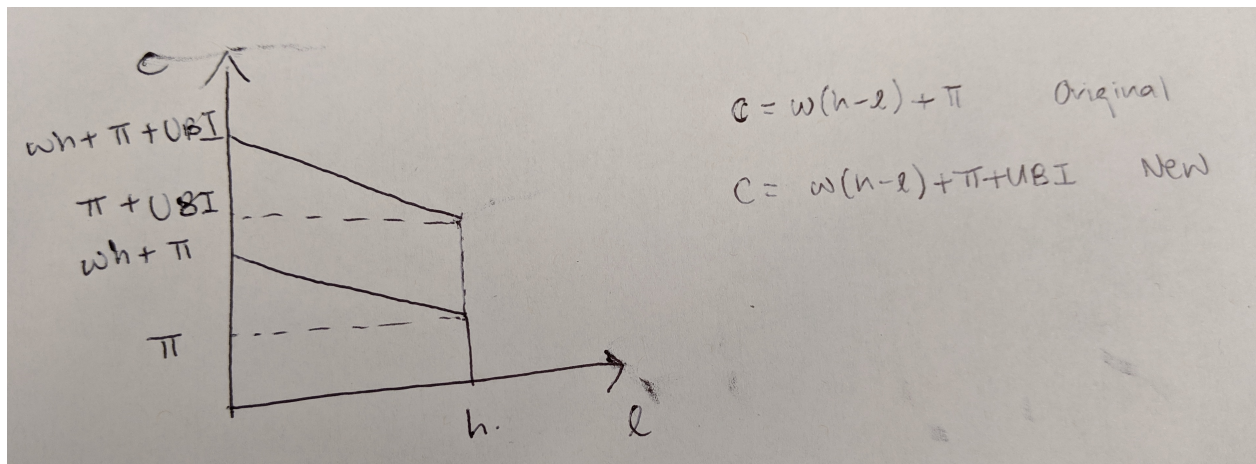
**Answer:**

This is not necessarily true and depends on income vs. substitution effects. Employment would increase if the substitution effect dominates.

### Question 5:

A recent proposal floated by a presidential candidate is to offer households universal basic income. Implementing this would imply that all households receive a minimum level of income regardless of whether they work. Denote universal basic income as UBI. Assuming households receive a wage for each unit of labor supplied, dividend income and pay zero lump-sum taxes, draw what the household budget constraint initially looks like if  $UBI=0$ . Show how the household budget constraint would change if UBI is positive.

**Answer:**



**Question 6:**

Continuing with Question 5, holding  $w, \pi$  constant, state what would happen to the household's consumption and leisure under the implementation of UBI, i.e. if  $(UBI > 0)$ . Holding  $w, \pi$  constant, argue if the household is happier with the implementation of UBI.

**Answer:**

The implementation of UBI increases the household's non-labor income, implying a pure positive income effect. As such, the household wants more consumption and leisure with the implementation of UBI. Since the household gets utility from consumption and leisure,  $U(c, \ell)$ , this implies the household would be happier with UBI being implemented.

**Question 7:**

Continuing with Question 5 and 6, the implementation of  $UBI$  needs to be funded. Suppose the government must spend exogenous  $G$ , and also has to spend UBI on households. The government can only levy a proportional tax,  $\tau$ , on firms' revenues. Write down the government budget constraint. You may assume the output of a firm is  $Y$ .

**Answer:**

$$G + UBI = \tau Y$$

**Question 8:**

Continuing with Question 5,6, and 7. Write down the firm's profit maximization problem if it observes a proportional tax,  $\tau$ , on revenues. You may assume the firm has production function  $Y = zK^\alpha N^{1-\alpha}$  and is born with capital. The firm can hire labor at wage rate  $w$ . Holding  $w, z, K$  constant, argue what may happen to the firm's labor demand as  $\tau$  increases.

**Answer:**

Firm's problem is:

$$\max_{N^d} \pi = (1 - \tau)zK^\alpha N^{d,1-\alpha} - wN^d$$

Optimality implies that

$$(1 - \tau)(1 - \alpha)zK^\alpha N^{d,-\alpha} = w$$

From the above, we can see that the marginal benefit of using labor in production is decreasing in  $\tau$  increases. So firms would hire less labor if  $\tau$  rises. To see this, observe that:

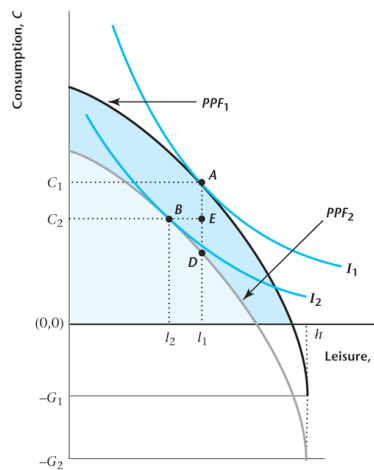
$$N^d = \left( \frac{(1 - \tau)(1 - \alpha)zK^\alpha}{w} \right)^{1/\alpha}$$

### Question 9:

Output is given by  $Y = zK^\alpha N^{1-\alpha}$  and  $G > 0$ . Draw the PPF. Show what happens to the PPF if  $G$  increases from  $G_1$  to  $G_2$  where  $G_2 > G_1$ . Write down the slope of the PPF. In your graph, label all your axes and intercepts clearly. You may assume total hours a household has is equal to  $h$ .

**Answer:**

Slope of PPF is  $-MPN = -(1 - \alpha)zK^\alpha N^{-\alpha}$ .



### Question 10:

State 1 reason why the price level as measured by the GDP deflator may not be the same as that measured by the CPI.

**Answer:**

Any of the 2 is correct:

CPI assumes fixed weights while GDP deflator assumes changing weights.

CPI covers only goods household consumes, GDP deflator covers all goods included in GDP.



## Problem 1: (10 points)

Consider the following economy. Households in the economy have utility given by

$$U(c, \ell) = \ln \left[ c + \frac{1}{1 - \gamma} \ell^{1 - \gamma} \right]$$

Households receive dividend income and pay lump-sum taxes. Households also receive a wage rate  $w$  for every unit of labor supplied. You may assume  $h = 1$  and  $0 < \gamma < 1$ .

A) Write down the household's problem.

**Answer(2pts):**

$$\max_{c, \ell} U(c, \ell) = \ln \left[ c + \frac{1}{1 - \gamma} \ell^{1 - \gamma} \right]$$

s.t.

$$c = w(1 - \ell) + \pi - T$$

- B) Characterize the household's optimality conditions. What can you say about the relative size of substitution vs. income effects in this model?

**Answer(8pts):**

We can set up the Lagrangian:

$$\max_{c, \ell, \lambda} \mathcal{L}(c, \ell, \lambda) = \ln \left[ c + \frac{1}{1-\gamma} \ell^{1-\gamma} \right] + \lambda [w(1-\ell) + \pi - T - c]$$

take first order conditions wrt  $c$

$$\frac{1}{c + \frac{1}{1-\gamma} \ell^{1-\gamma}} = \lambda$$

wrt  $\ell$

$$\frac{\ell^{-\gamma}}{c + \frac{1}{1-\gamma} \ell^{1-\gamma}} = \lambda w$$

wrt  $\lambda$

$$c = w(1-\ell) + \pi - T$$

The above is the budget constraint which is one of the household's optimality conditions. The other one comes from the optimal trade-off between consumption and leisure. Using the first order conditions with respect to  $c$  and  $\ell$ , we have:

$$\ell^{-\gamma} = w$$

OR

$$\ell = (w)^{-1/\gamma} = \frac{1}{w^{1/\gamma}}$$

Notice that in this case, leisure is declining whenever the wage rate rises, implying that the substitution effect dominates the income effect.

## Problem 2: (15 points)

Following from Problem 1, government spending is exogenously given by  $G > 0$ . The government only levies a lump-sum tax on households.

A) Write down the government budget constraint

**Answer(2pts):**

$$G = T$$

B) Output is given by  $Y = zN$ . Firms pay  $w$  per unit labor demanded. Write down the firm's problem. Characterize the firm's optimality conditions.

**Answer(3pts):**

firm's profit maximization problem:

$$\max_{N^d} \pi = zN^d - wN^d$$

taking first order conditions, we get the firm's optimality condition:

$$z = w$$

which also implies  $\pi = 0$ .

- C) Using your answer from Problem 1 and your answer to A) and B) of Problem 2, solve for equilibrium  $N^*$  and  $C^*$ .

**Answer(4pts):**

From labor market clearing, we have:

$$MRS_{\ell,c} = w^* = MPN$$

which from our optimality conditions for the household and the firm give us:

$$\ell^{-\gamma} = w = z$$

which re-arranging and using the fact that  $\ell = 1 - N$ , gives us:

$$N^* = 1 - \frac{1}{z^{1/\gamma}}$$

Knowing  $N^*$ , we know  $C^*$ . From goods market clearing:

$$C^* = Y^* - G = zN^* - G = z \left( 1 - \frac{1}{z^{1/\gamma}} \right) - G$$

- D) In macroeconomics, we typically treat productivity shocks as drivers of the business cycle. Use your findings from part C) to complete the following table. In particular, state how each variable in the model co-moved with output under a shock to  $z$ . In each cell of the column titled “Model Predictions”, write “pro-”, “counter-”, or “a-” for pro-cyclical, counter-cyclical, or a-cyclical.

Separately, also report how each variable co-moves with GDP in the data. (In the data column, write “pro-”, “counter-”, or “a-” depending on whether the variable is pro-, counter-, or a-cyclical in U.S. post-war data as we saw in class.)

**Answer(6pts):**

*Briefly* describe any discrepancies you notice.

	Model Prediction	Observed in Data
Employment	pro-	pro-
Consumption	pro-	pro-
Wage Rate	pro-	pro-

No discrepancies.

### Problem 3: (15 points) :

Following from question 1, we want to solve the Social Planner's problem. Set-up the social planner's problem using the information from Problem 1. Solve for optimal  $N^*$  and  $C^*$ . Given your answer, explain whether the equilibrium outcomes from the free-market economy in Problem 1 are pareto optimal.

**Answer:**

$$\max_{C, \ell} U(C, \ell) = \ln \left[ C + \frac{1}{1-\gamma} \ell^{1-\gamma} \right]$$

s.t.

$$C = z(1 - \ell) - G$$

We can set up the Lagrangian:

$$\max_{C, \ell, \lambda} \mathcal{L}(C, \ell, \lambda) = \ln \left[ C + \frac{1}{1-\gamma} \ell^{1-\gamma} \right] + \lambda [z(1 - \ell) - G - C]$$

Take first order conditions wrt  $c$ :

$$\frac{1}{C + \frac{1}{1-\gamma} \ell^{1-\gamma}} = \lambda$$

take first order condition wrt  $\ell$

$$\frac{\ell^{-\gamma}}{C + \frac{1}{1-\gamma} \ell^{1-\gamma}} = \lambda z$$

take first order condition wrt  $\lambda$ :

$$C = z(1 - \ell) - G$$

observe that the above is one of the social planner's optimality condition, it must respect the PPF.

Combining FOC wrt  $C$  and  $\ell$ , we get:

$$\ell^{-\gamma} = z$$

OR

$$N^{sp} = 1 - \frac{1}{z^{1/\gamma}}$$

and from the PPF, we have:

$$C^{sp} = zN^{sp} - G = z \left( 1 - \frac{1}{z^{1/\gamma}} \right) - G$$

So yes, the competitive equilibrium is pareto optimal since it chooses the same allocations as the social planner.