73-240 Practice Problem: Are firms more selective on who to hire in downturns?

1 Set-up

We will use the same set-up as in Lecture 11 of the search model of unemployment. We will assume the matching function is given by:

$$M = eV^{1-\gamma}U^{\gamma}$$

We will also assume that $0 < b < 1$.

All jobs are a single-firm worker pair. If a firm hires a worker, the worker supplies 1 unit of labor. Output from a match is given by zx, where z is total factor productivity and x is the match quality which we will define below. First, we will use movements in z to characterize the business cycle. A high z represents a boom, a low z realized in the economy represents a recession. Note that z is exogenous (nobody controls it). We will assume that the lowest value z can take is 1.

The second component that affects output is match quality x. We will assume that at the time where a firm meets a worker, the firm learns whether the worker is a good match. Note that this is similar to thinking about the interview process, the firm meets a worker, interviews the worker and then learns through the interview if the worker is someone it wants to hire. Match quality x is drawn from a continuous uniform distribution bounded between $[0, 1]$. Note that the probability density of a uniform distribution bounded between 0 and 1 is given by $f(x) = \frac{1}{1-0} = 1$ and the cumulative distribution function (in other words, probability that a worker draws a value less than or equal to x) is given by $F(x) = \frac{x-0}{1-0} = x$ (See en.wikipedia.org/wiki/Uniform_distribution_(continuous) if you are unclear about uniform distributions.)

Note that profits of a firm conditional on hiring are now given by:

$$\pi(x) = zx - w(x)$$

where we allow the wage $w(x)$ to potentially depend on x. Clearly, if the firm must always pay the worker at least its outside option b, then there are some values of x, where the firm would make losses if it hires the worker. To see this, observe that if $x = 0$, the firm always makes losses if it hires a worker with $x = 0$.

1.1 Firm's problem

The firm's value of creating a vacancy now becomes:

$$J = -\kappa + q(\theta) \int_0^1 \max\{zx - w(x), 0\} f(x) dx$$

Observe that the firm's problem is now slightly altered. Conditional on meeting a job-seeker, the firm can now choose to hire a job-seeker or reject the job-seeker. For given z and knowing the form of $w(x)$, the firm will always choose to reject the job-seeker if $zx - w(x) < 0$.

Let's define $\widehat{x}$ as the threshold where for any $x \geq \widehat{x}$, firms are willing to hire the job-seeker. Then the firm's value of creating a vacancy becomes:

$$\begin{aligned} J &= -\kappa + q(\theta) \int_{\widehat{x}}^1 [zx - w(x)] f(x) dx + \int_0^{\widehat{x}} 0 \cdot f(x) dx \\ &= -\kappa + q(\theta) \int_{\widehat{x}}^1 [zx - w(x)] f(x) dx \end{aligned}$$

Under free entry, we have:

$$\kappa = q(\theta) \int_{\widehat{x}}^1 [zx - w(x)] f(x) dx$$

Notice that the firm's problem requires us to now figure out how many vacancies does the firm create and also what is its cut-off (threshold) for hiring, $\widehat{x}$.

1.2 Household's problem

The household if it stays out of the labor force gets b. If the household chooses to search, the following outcomes can occur:

I Does not meet a vacancy, gets b (this happens with probability $1 - p(\theta)$)

II meets a vacancy (this happens with probability $p(\theta)$) and draws x from continuous uniform distribution with probability density $f(x)$

- If $x \geq \widehat{x}$, job-seeker is hired and gets $w(x)$
- If $x < \widehat{x}$, job-seeker remains unemployed and gets b.

This implies that the expected value of search for the household is:

$$\begin{aligned} P(U) &= p(\theta)\int_{\widehat{x}}^{1} w(x)f(x)dx + p(\theta)F(\widehat{x})b + (1-p(\theta))b \\ &= b + p(\theta)\int_{\widehat{x}}^{1} w(x)f(x)dx - p(\theta)\left[1 - F(\widehat{x})\right]b \\ &= b + p(\theta)\int_{\widehat{x}}^{1} \left[w(x) - b\right]f(x)dx \end{aligned}$$

Note the first term on the right hand side of the above equation is the expected wage the job-seeker gets if she is employed (its expected because the worker does not know what x she will receive prior to meeting a vacancy, and hence what wage she will get). The second term is the probability the job-seeker meets a vacancy but is rejected by the vacancy because she drew an $x < \widehat{x}$ and thus remains unemployed and consumes home production b. A job-seeker draws $x < \widehat{x}$ with probability $F(\widehat{x})$. Finally the last term is the probability the job-seekers failed to meet a vacancy, is unemployed and consumes home production b.

Note that so long as $P(U) > b$, the household will search for a job. So for any $\int_{\widehat{x}}^{1} w(x)f(x)dx > b$, the household will search for a job.

2 Wage Determination

Your task is to again define for a given draw of x:

- State what the worker's gain to matching is
- State what the firm's gain to matching is (i.e. what the firm gets if it hires the worker)
- State what is the total surplus of the match
- Using Nash-bargaining, write down for any given x, what the Nash-bargained wage $w(x)$ would be. (You don't have to worry about whether the firm would hire the worker or not. Just show what $w(x)$ would be assuming the firm hires the worker.)
- Given the Nash-bargained wage, write down what profits would be, $\pi(x)$.

Note that no firm wants to hire a worker if the profit from hiring that worker is negative. Use the fact that no match is formed if $\pi(x)$ is negative to figure out what is the lowest match quality firms are willing to accept. In other words, $\widehat{x}$ is pinned down when firms are exactly indifferent between hiring the worker and walking away from the match.

- Using the fact that $\pi(x) \geq 0$ for firms to hire, find the cut-off $\widehat{x}$ in terms of z and b.
- In a recession, z_{low} vs a boom, z_{high}, how does the cutoff change?