Solution to Problem Set # 3

Problem 1

(a) $f(K,L) = K^{1/2} L^{1/4}$

- In the short run: $K = \bar{K} = 1$

$\longrightarrow q = f(\bar{K}, L) = 1^{1/2} L^{1/4} = L^{1/4}$

$\longrightarrow L = q^4 \longrightarrow TC^{SR}(q) = L.w + \bar{K}.r$

$= 5q^4 + 10$

- In the long run: $\min\ w.L + r.K$ subject to $f(L,K) = q$

This production function yields interior solution $\rightarrow MRTS = \frac{w}{r}$

$$MRTS = \frac{MP_L}{MP_K} = \frac{\frac{\partial q}{\partial L}}{\frac{\partial q}{\partial K}} = \frac{\frac{1}{4} \cdot L^{-3/4} K^{1/2}}{\frac{1}{2} \cdot K^{-1/2} L^{1/4}} = \frac{1}{2} \frac{K}{L}$$

$$\rightarrow \frac{1}{2} \frac{K}{L} = \frac{w}{r} = \frac{5}{10} \longrightarrow \frac{K}{L} = 1 \longrightarrow K = L$$

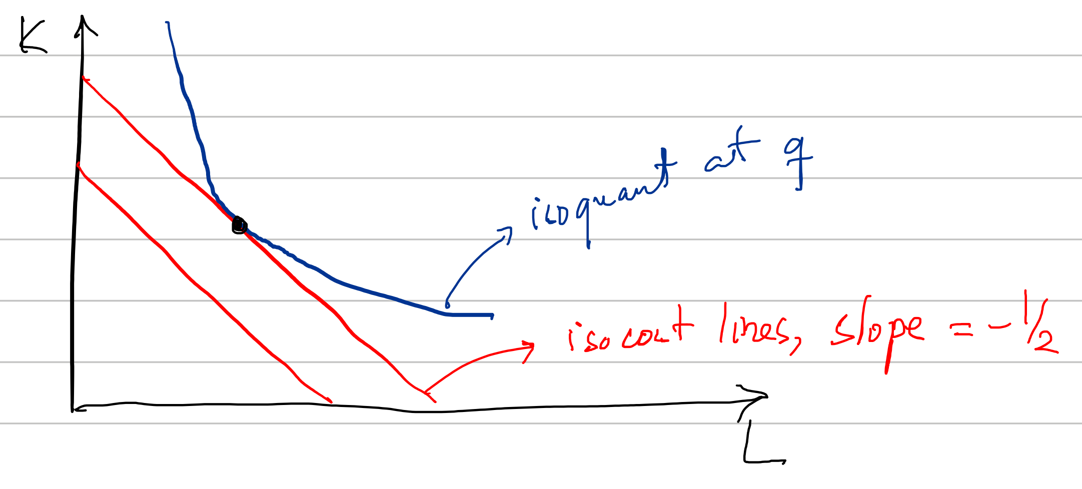
Therefore: $q = L^{1/2} L^{1/4}$

$= L^{3/4} \longrightarrow L = q^{4/3} \rightarrow K = q^{4/3}$

$\rightarrow TC^{LR}(q) = (q^{4/3})\ w + q^{4/3}.\ r$

$= 15\ q^{4/3}$

- Graph illustration



(b) $f(K, L) = \min(K, 3L)$

- In the short run: $f(\bar{K}, L) = \min(\bar{K}, 3L)$
$= \min(1, 3L)$

If $L < 1/3$; $q = \min(1, 3L) = 3L \rightarrow L = q/3$

$\rightarrow TC^{SR}(q) = \bar{K} \cdot r + L \cdot w$
$= 10 + 5/3\, q$ if $q < 3 \cdot 1/3 = 1$

If $L \geq 1/3$, $q = \min(1, 3L) = 1$

$\rightarrow$ The firm will choose $L = 1/3$ to minimize costs because any additional L beyond $1/3$ does not increase output

$\rightarrow TC^{SR}(q) = \bar{K} \cdot r + L \cdot w$
$= 10 + 5/3$ if $q = 1$

The firm cannot produce above 1 in the short run

$\rightarrow TC^{SR}(q) = \infty$ if $q > 1$

Final answer: $TC^{SR}(q) = \begin{cases} 10 + 5/3\, q & \text{if } q < 1 \\ 10 + 5/3 & \text{if } q = 1 \\ \infty & \text{if } q > 1 \end{cases}$

- In the long run: At the optimal allocation: $K = 3L$

$\rightarrow q = \min(K, 3L) = K \rightarrow L = 1/3 \cdot q$

$\rightarrow TC^{LR}(q) = w \cdot L + K \cdot r = 5/3\, q + 10q = 11\tfrac{2}{3}\, q$

- Graph illustration:

(c) $f(K,L) = K + 3L$

- In the short run, $\bar{K} = 1 \rightarrow q = f(\bar{K}, L) = 1 + 3L$

$$\rightarrow L = \frac{q-1}{3}$$

$$\rightarrow TC^{SR}(q) = r.\bar{K} + w.L = 10 + 5.\frac{q-1}{3}$$

$$= \frac{25}{3} + \frac{5}{3}q$$

- In the long run; note that $MRTS = \frac{MP_L}{MP_K} = \frac{3}{1} > \frac{w}{r}$

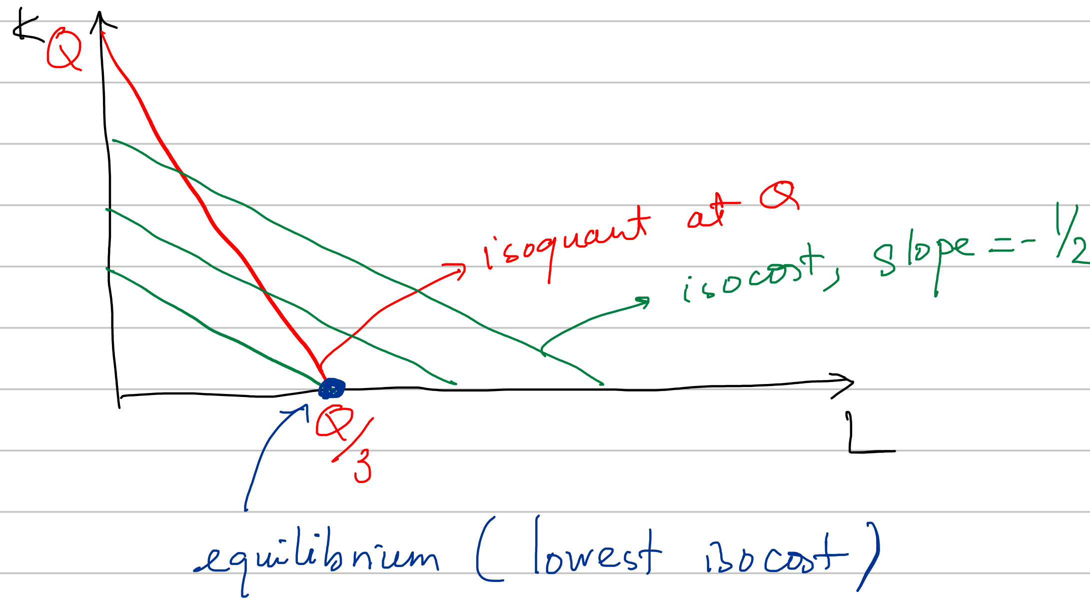
$\rightarrow$ The firm will only use labor to produce $\rightarrow K = 0$

Therefore:

$$q = F(0, L) = 3L \rightarrow L = \frac{q}{3}$$

$$\rightarrow TC^{LR}(q) = w.L + r.K = \frac{5}{3}.q$$

Graph illustration



Problem 2

$$TC(Q) = Q^3 - 10Q^2 + 100Q$$

(a) $$AC(Q) = \frac{TC(Q)}{Q} = Q^2 - 10Q + 100$$

$$\frac{d\,AC(Q)}{dQ} = 2Q - 10 \rightarrow \frac{d\,AC(Q)}{dQ} > 0 \text{ if } Q > 5$$

$$\frac{d\,AC(Q)}{dQ} < 0 \text{ if } Q < 5$$

$$\frac{d\,AC(Q)}{dQ} = 0 \text{ if } Q = 5$$

Therefore, the firm has decreasing returns to scale for $Q > 5$
increasing returns to scale for $Q < 5$
constant returns to scale at $Q = 5$

(b) The long-run shut down price is $p = \min AC^{LR}$

$$= \min\left(Q^2 - 10Q + 100\right)$$

$$= \min\left[(Q-5)^2 + 75\right]$$

$$= 75 \quad (\text{at } Q = 5)$$

Therefore, the long-run shutdown price is $p = 75$

(c) Suppose that $Q = 200 - P$. In the long run, all firms make zero profit and $P = \min AC =$ the shut down price $= 75$

$\rightarrow Q = 200 - 75 = 125$

When $P = 75$, each firm will produce $q = 5$ (see part (c))

$\rightarrow$ The number of firm is $n = \dfrac{125}{5} = 25$ firms.

$\boxed{\text{Problem 3}}$

Demand: $Q = 40 - 2P$

Short-run cost: $TC(Q) = Q^2 + 4Q + 10$

(a) We first derive the supply curve of individual firms. Since they are price takers, they will produce at q such that $MC(q) = p$. In other words, their supply curves are their marginal cost curves,

$$MC(q) = \frac{dTC(q)}{dq} = 2q + 4$$

$\rightarrow$ Supply curve of individual firm: $P = 2q + 4$

To derive the market supply curve; we need to aggregate the supply curves horizontally.

At P, each firm will produce $q = \frac{P-4}{2}$

$\rightarrow$ 8 firms will produce $Q = 8q = 4(P-4)$
$= 4P - 16$

$\rightarrow$ The short run supply curve for the market is:

$$Q = 4P - 16.$$

(b) In equilibrium, supply = demand

$$\rightarrow 4P - 16 = 44 - 2P$$
$$\rightarrow 6P = 60$$
$$\rightarrow P = 10$$
$$\rightarrow Q = 44 - 20 = 24$$

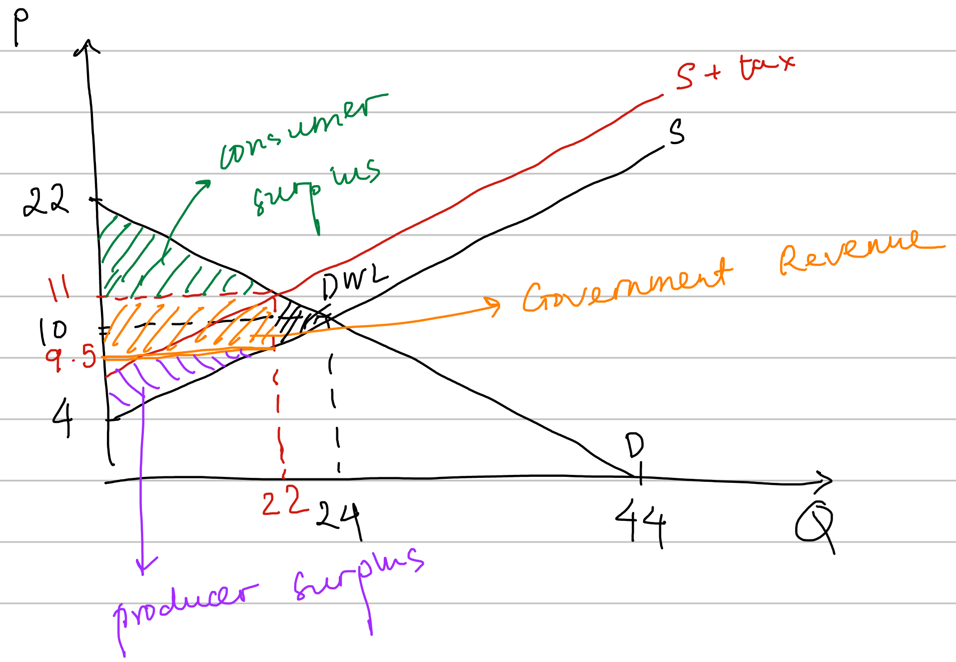
The short run equilibrium is then $P = 10$, $Q = 24$

(c) If the government imposes a \$2 tax per unit sold, it's as if the supply curve shifts upward!

$$\rightarrow \tilde{Q} = 4(\tilde{P} - 1.5) - 16$$
$$= 4\tilde{P} - 22 \text{ (where } \tilde{P} \text{ is the after tax price)}$$

Supply = demand $\rightarrow 4\tilde{P} - 22 = 44 - 2\tilde{P}$
$\rightarrow 6\tilde{P} = 66$
$\rightarrow \tilde{P} = 11$
$\rightarrow Q = 44 - 22 = 22$

$\rightarrow$ the equilibrium is $Q = 22$, after-tax price is $\tilde{P} = 11$



The after-tax consumer surplus is:

$$\frac{1}{2} \times 22 \times 11 = 121$$

The after-tax producer surplus is:

$$\frac{1}{2} \times 22 \times 5.5 = 60.5$$

Government Revenue is: $1.5 \times 22 = 33$

The deadweight loss due to tax is $\frac{1}{2} \times 2 \times 1.5 = 1.5$

$\boxed{\text{Problem 4}}$

$$u(m) = \frac{1}{100}\sqrt{m}\,; \quad 50\%: \; m = 160{,}000$$
$$50\%: \; m = 40{,}000$$

(a) $E(m) = \frac{1}{2} \times 160{,}000 + \frac{1}{2} \times 40{,}000$

$= 100{,}000$

(b) $\mathbb{E}[U(m)] = \frac{1}{2}\left[\frac{1}{100}\sqrt{160{,}000}\right] + \frac{1}{2}\left[\frac{1}{100}\sqrt{40{,}000}\right]$

$= \frac{1}{2}\cdot 4 + \frac{1}{2}\cdot 2 = 3$

(c) CE is the certain amount that makes Jane indifferent between facing the risk and getting the CE for certain.

$\rightarrow U(CE) = \frac{1}{100}\sqrt{CE} = \mathbb{E}[U[m]]$

$\rightarrow \frac{1}{100}\sqrt{CE} = 3$ (from question (b))

$\rightarrow CE = 90{,}000$

(d) Jane's risk premium is $\mathbb{E}(m) - CE$

$= 100{,}000 - 90{,}000$

$= 10{,}000$

(e) Let w denote the maximum willingness to pay for insurance. At w, Jane is indifferent between buying the insurance and staying uninsured

$\mathbb{E}[U(m)] = \frac{1}{2}\cdot\left[\frac{1}{100}\sqrt{160{,}000-w}\right] + \frac{1}{2}\left[\frac{1}{100}\sqrt{160{,}000-w}\right]$

$\leftrightarrow 3 = \frac{1}{100}\cdot\sqrt{160{,}000-w}$

$\leftrightarrow 300 = \sqrt{160{,}000-w} \rightarrow 160{,}000 - w = 90{,}000$

$\rightarrow w = 70{,}000$