INTERMEDIATE MICROECONOMICS

MONOPOLY

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Introduction

- Derive the equilibrium price and quantity under monopoly.
- Solve for the dead weight loss under monopoly.
- Understand the three types of price discrimination.
- Explain why perfect price discrimination by a monopolist leads to a socially optimal level of output.

1. Monopoly: Equilibrium

Monopoly: Equilibrium

- Derive the equilibrium price and quantity under monopoly.
- Solve for the dead weight loss under monopoly.
- Reading: pp. 841-876

Monopoly

- A **monopolist** is the only supplier of a good for which there is no close substitute.
- Monopoly occurs when
  - Exclusive control of inputs
  - Economies of scale
  - Patents
  - Networks
  - Government licenses

UPMC

Monopoly

- A monopolist does not take price as given:
  - Monopoly output is the market output.
  - Monopoly demand curve is the market demand curve.
  - Therefore, if a monopolist changes its output, it affects price.

Monopoly Profit Maximization

- Like all firms, monopolies maximize profits by setting price or output so that marginal revenue equals marginal cost.

- Profit function to be maximized by choosing output, Q:
  - $\pi(Q) = \overbrace{p(Q)Q}^{\text{revenue}} - \overbrace{C(Q)}^{\text{cost}}$
  - Inverse demand function: $p(Q)$
  - Cost function: $C(Q)$

Monopoly Profit Maximization

- A monopolist maximizes profits by choosing Q:

$$\pi(Q) = \underbrace{p(Q)}_{\text{negative}}Q - C(Q)$$

  - Taking the first derivative:

$$\overbrace{p(Q) + p'(Q)Q}^{MR} = \underbrace{C'(Q)}_{MC}$$

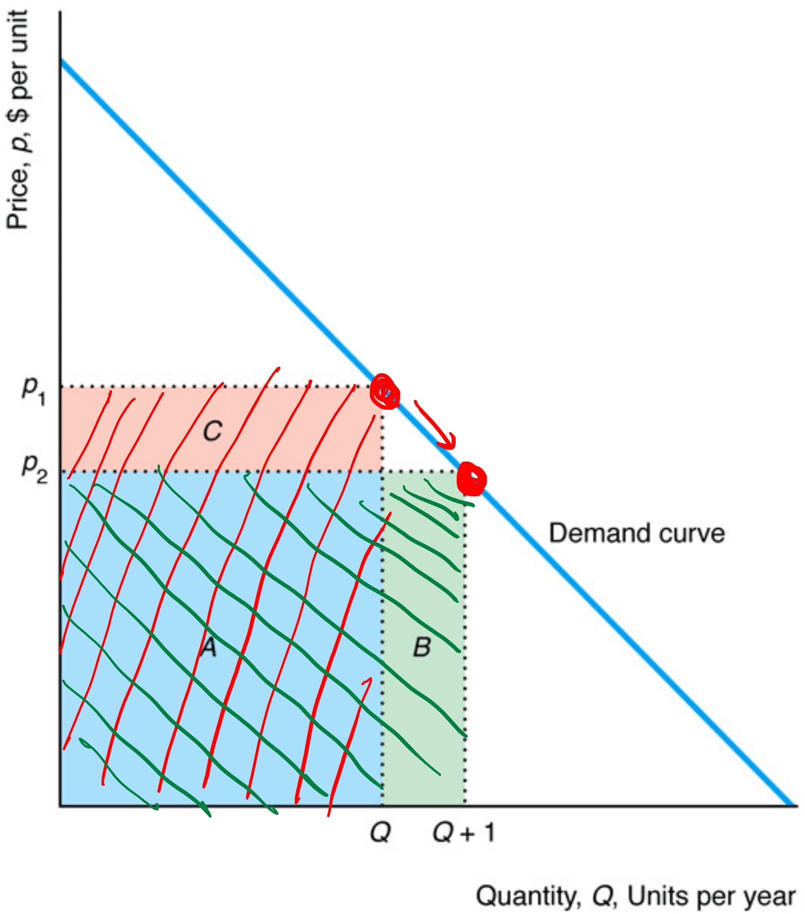
MR < P → under produce

$$\frac{d\pi(Q)}{dQ} = p'(Q) \cdot Q + p(Q) - C'(Q) = 0$$

  - Marginal revenue, $MR(Q)$: $p(Q) + p'(Q)Q$
  - Marginal cost, $MC(Q)$: $C'(Q)$
  - Profit maximization: $MR(Q) = MC(Q)$

Monopoly Profit Maximization

- MR lies below the demand curve.
- Selling one more unit requires the monopolist to lower the price.
- Moving from Q to Q+1, the monopoly's MR is less than the price by an amount equal to area C.



Example 1

- Demand function: $p(Q) = 24 - Q$
  - Marginal revenue function: $\boxed{MR(Q) = 24 - 2Q.}$
- Cost function: $C(Q) = Q^2 + 12$
  - Marginal cost function: $MC(Q) = 2Q.$
- Equilibrium quantity and price:
  - $MR(Q^*) = MC(Q^*)$: $\boxed{Q^* = 6}$
  - $p^* = p(Q^*)$: $p^* = 18.$

$MR(Q) = \frac{d}{dQ} \text{Revenue}(Q)$

$= \frac{d}{dQ}(P(Q) \cdot Q)$

$= \frac{d}{dQ}((24 - Q) \cdot Q)$

$= 24 - Q - Q$

$= 24 - 2Q$

$MR(Q) = MC(Q)$

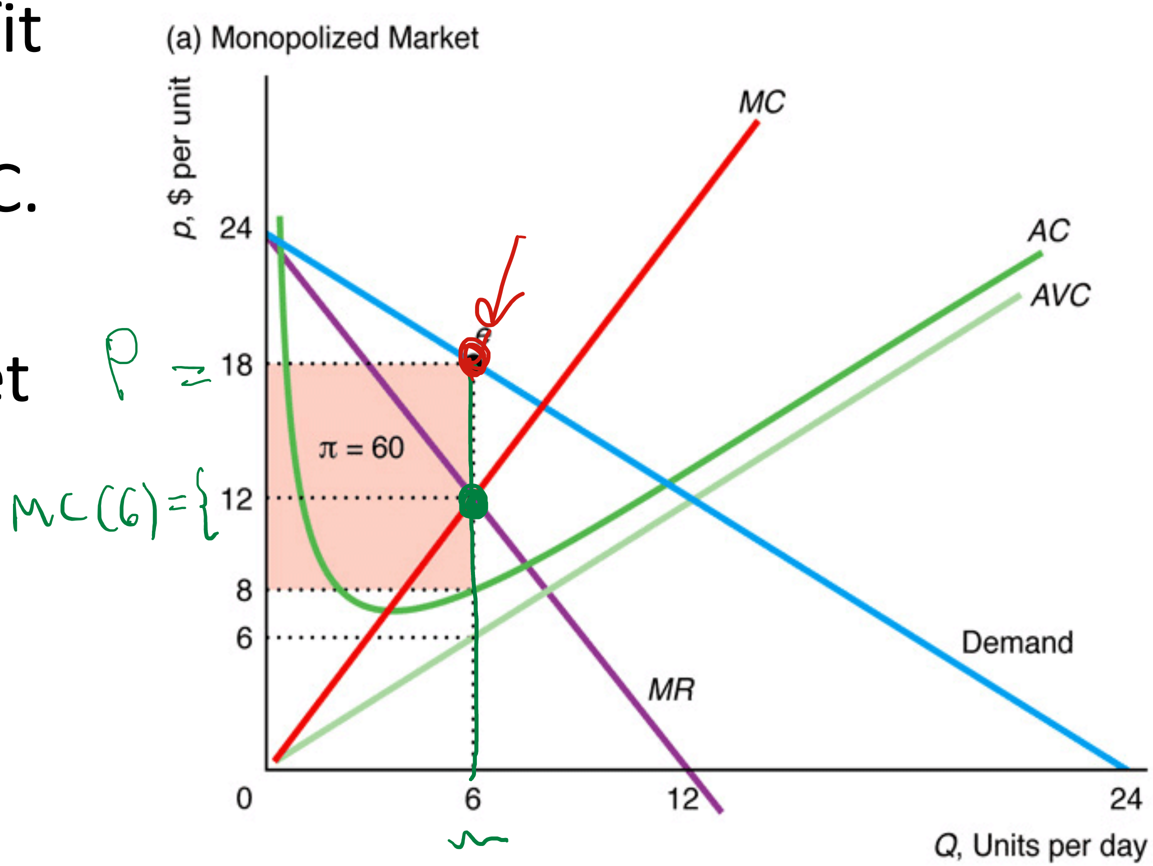
$\Rightarrow 24 - 2Q = 2Q$

$\rightarrow 24 = 4Q \rightarrow Q = 6 \longrightarrow P = 24 - Q$

$= 24 - 6 = 18$

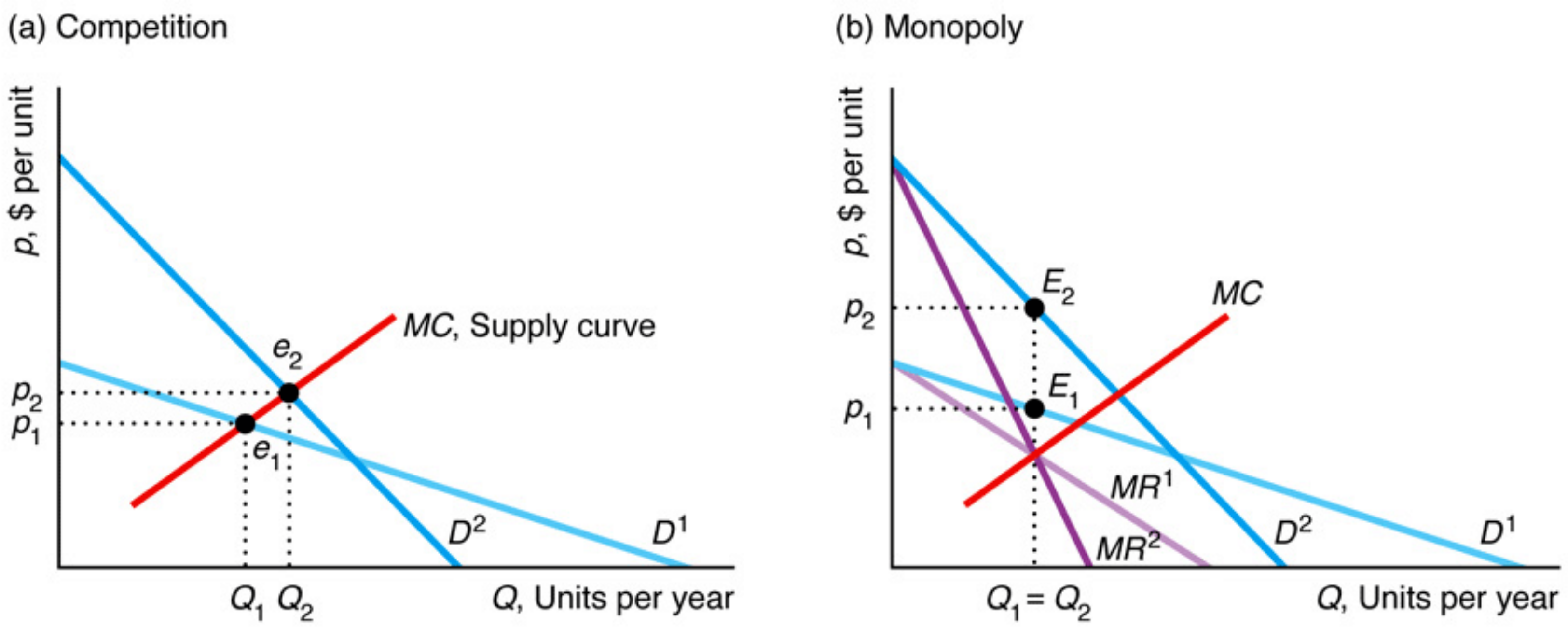
Example 1

- The monopolist's profit maximizing output is found where MR = MC.
- At the profit-maximizing output, set p according to inverse demand.



Effects of a Shift of Demand Curve

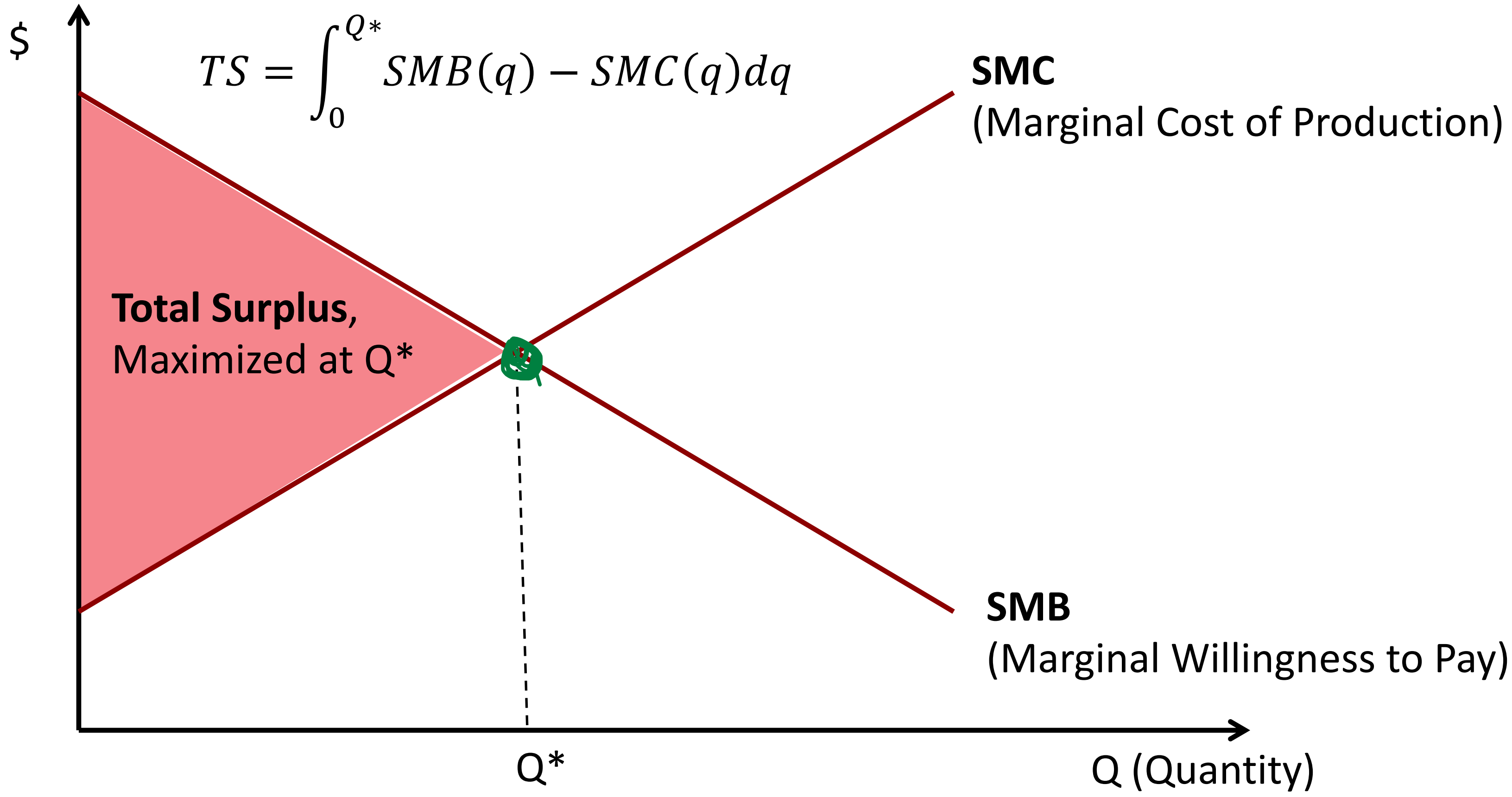
- Shifts in demand that would affect the equilibrium output in a competitive market need not affect monopolist's profit-maximizing output.



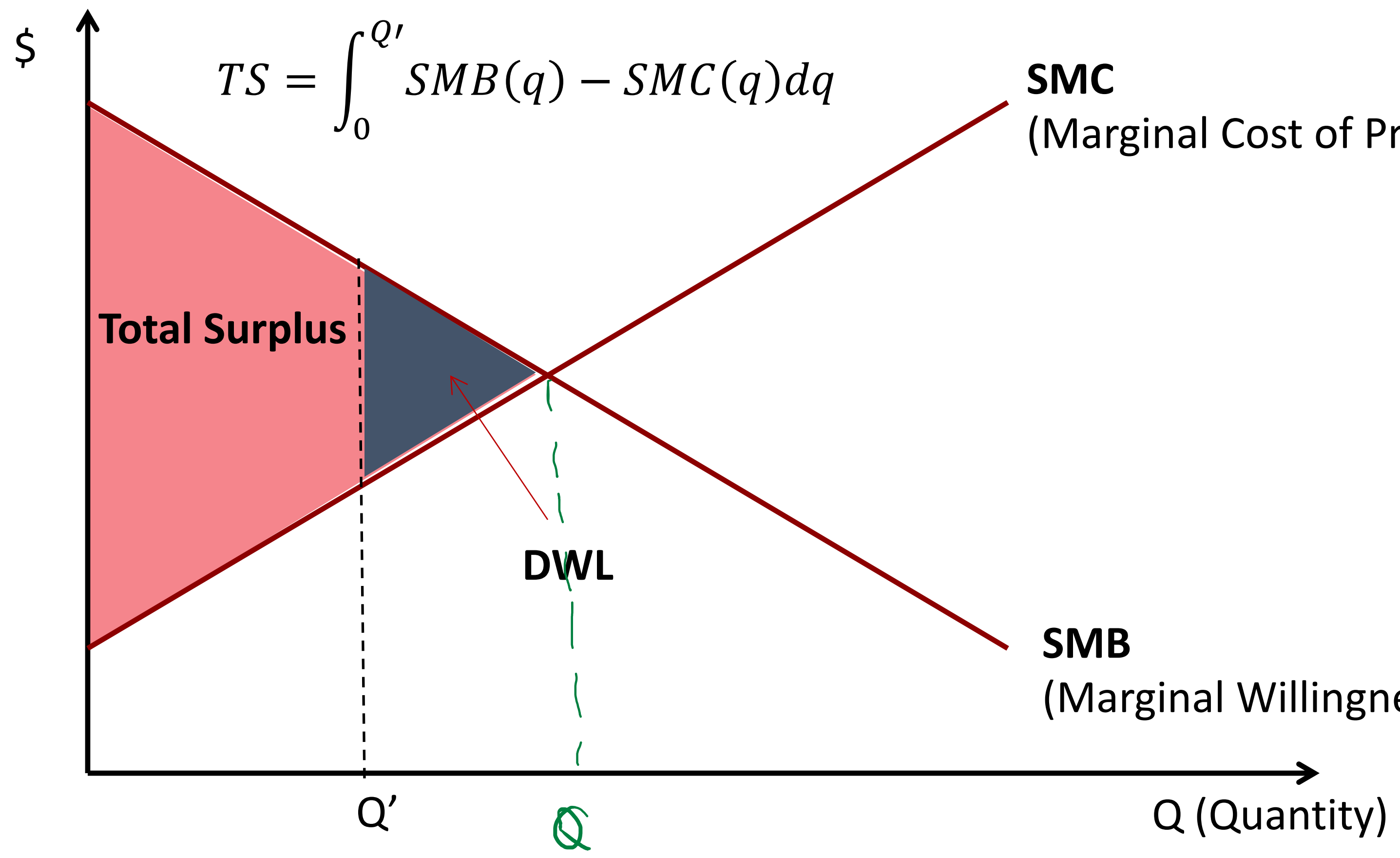
Deadweight Loss from Monopoly

- Competition maximizes welfare, which is the sum of consumer surplus and producer surplus, because price equals marginal cost.
- By contrast, a monopoly sets **price above marginal cost** and **causes consumer buy less** than the competitive level of output. This generates deadweight loss.

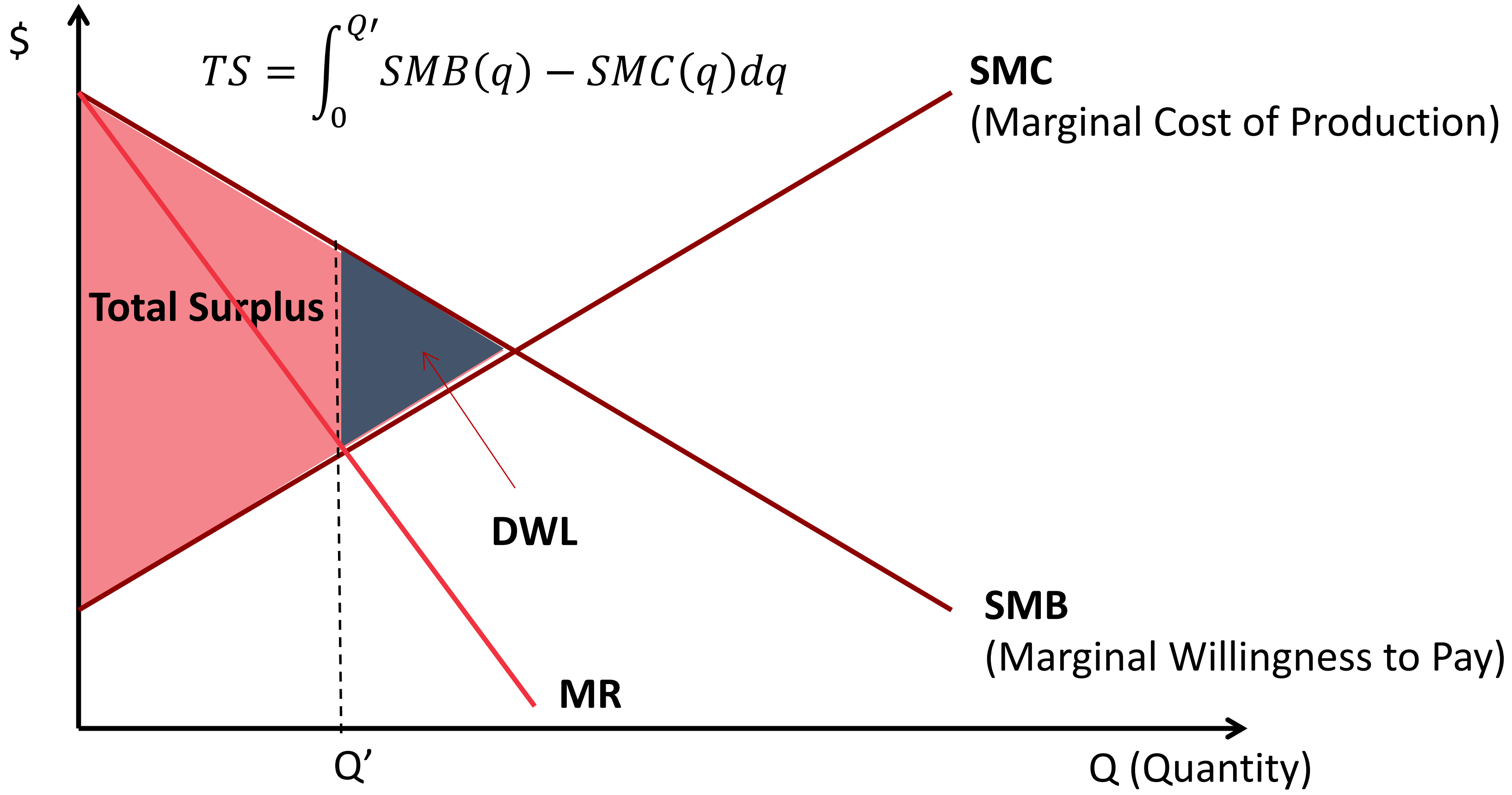
Social Welfare



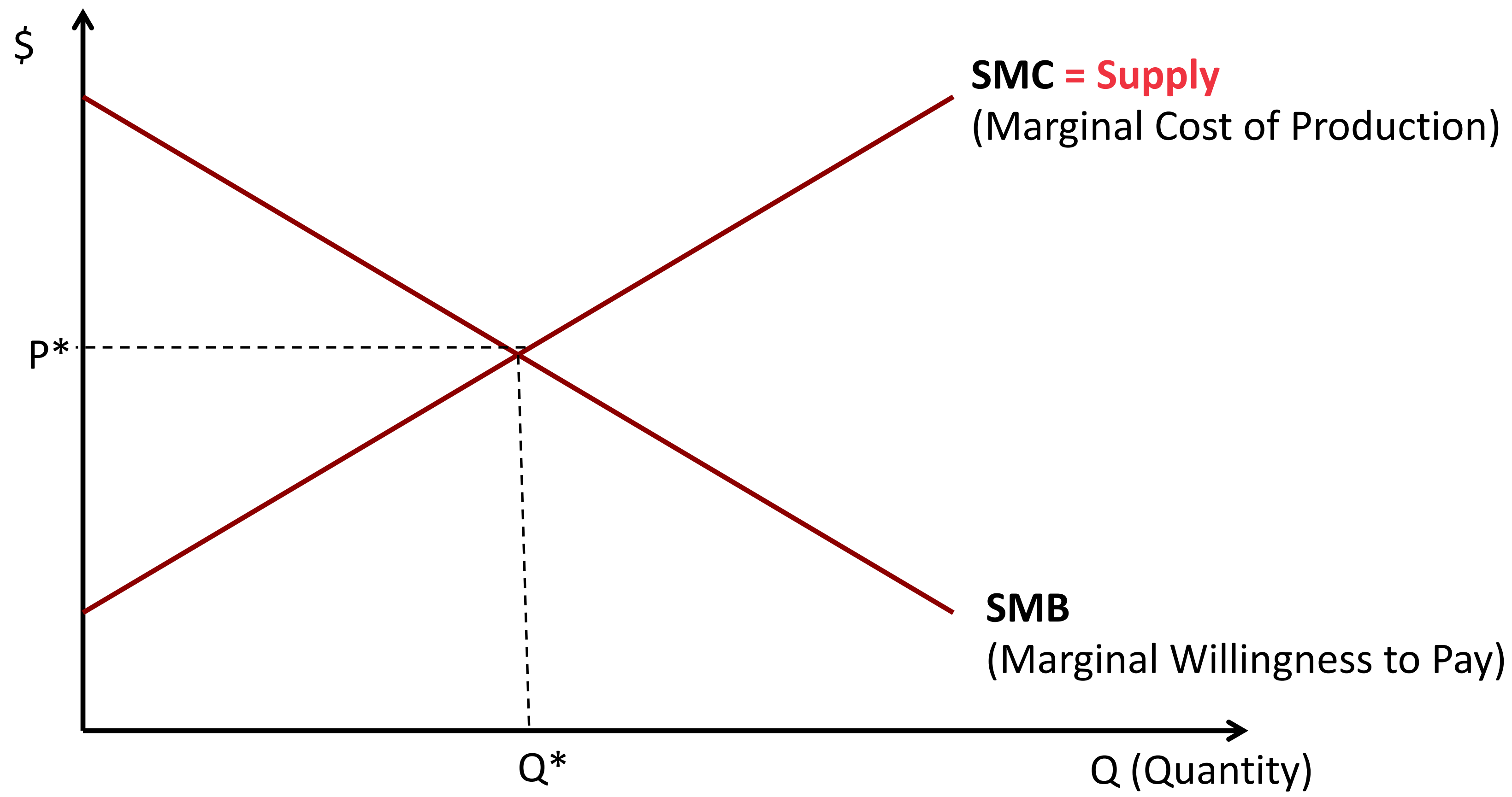
Deadweight Loss



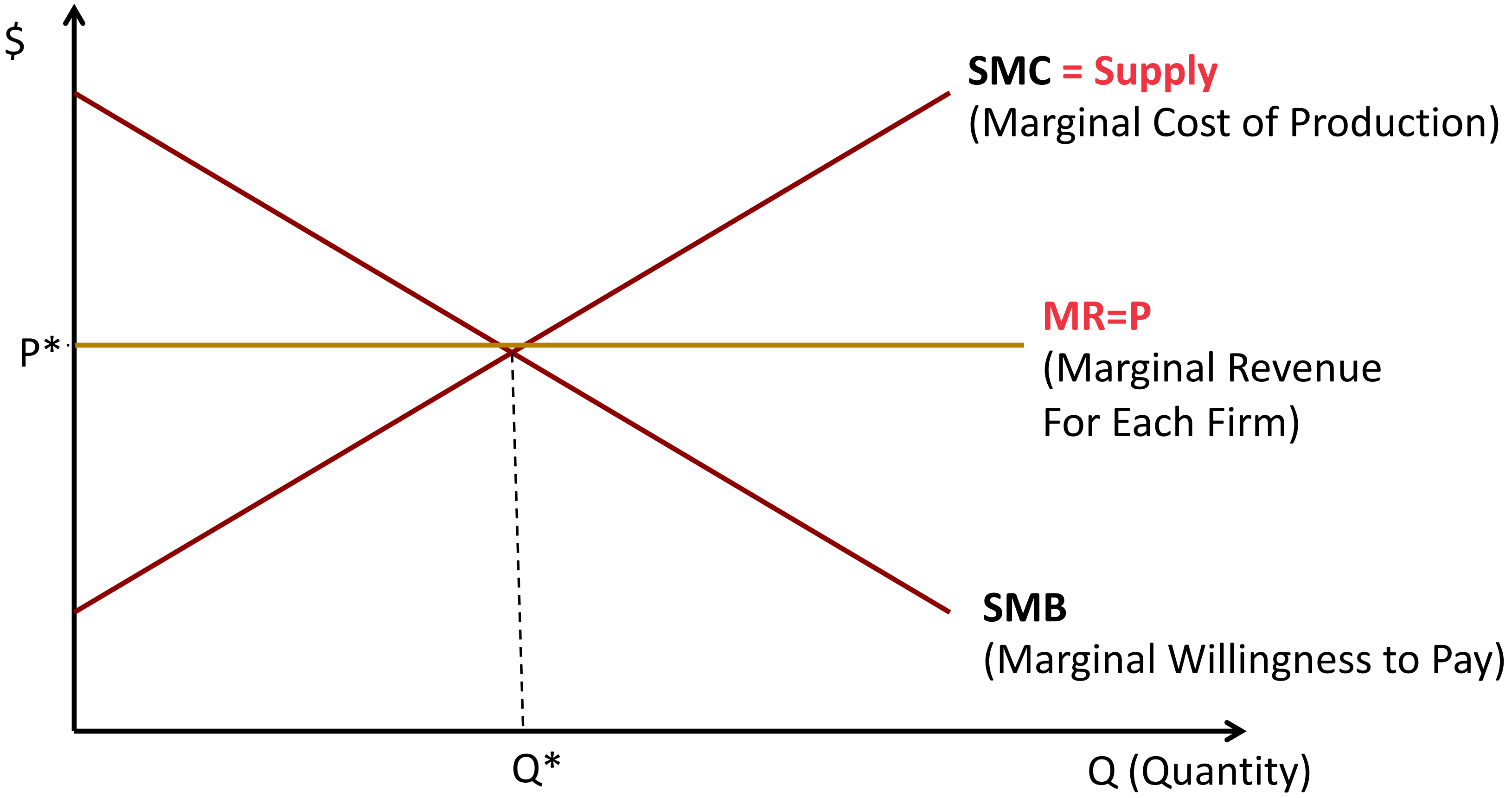
Deadweight Loss and Monopoly



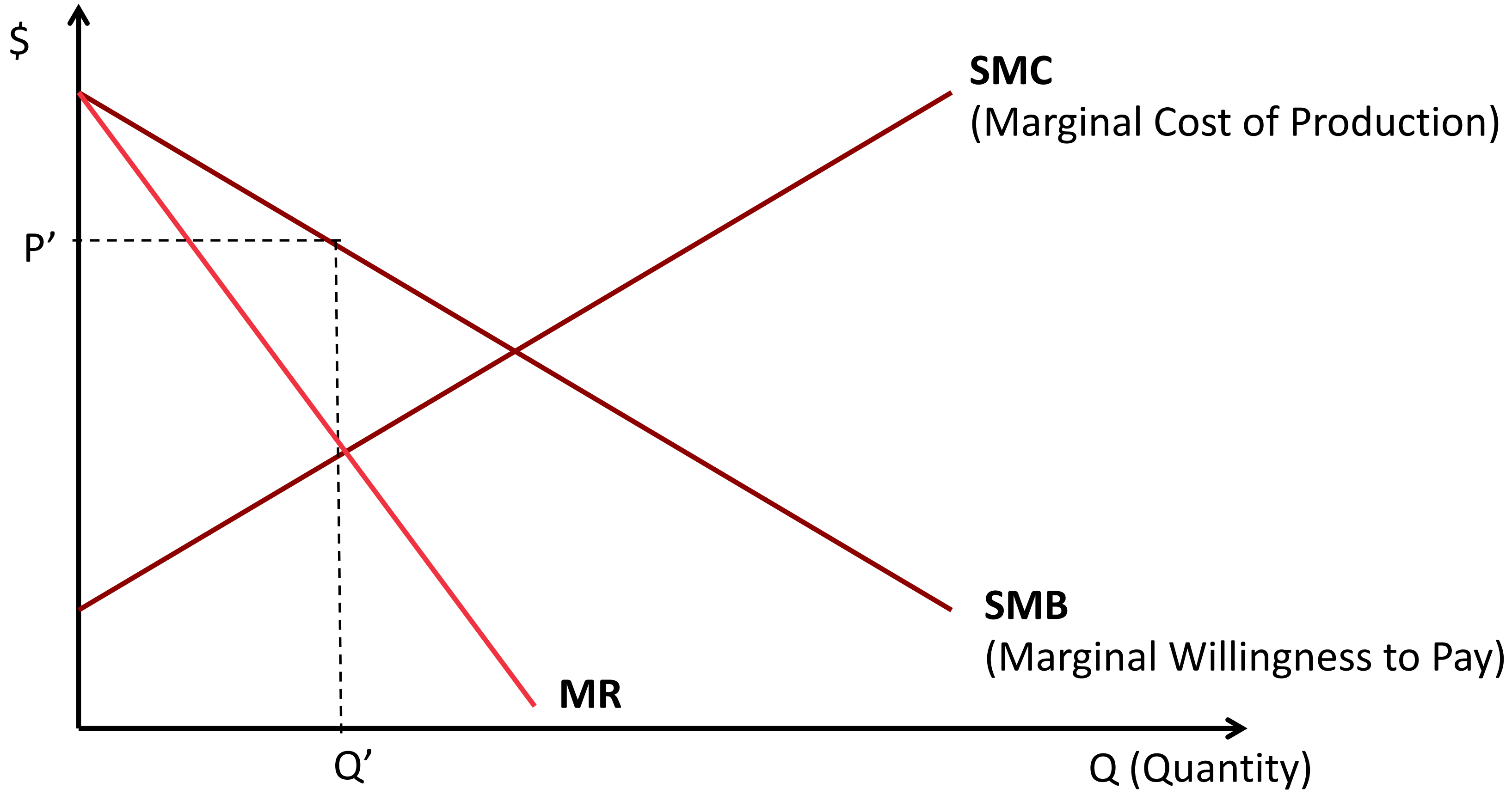
Perfect Competition



Perfect Competition



Monopoly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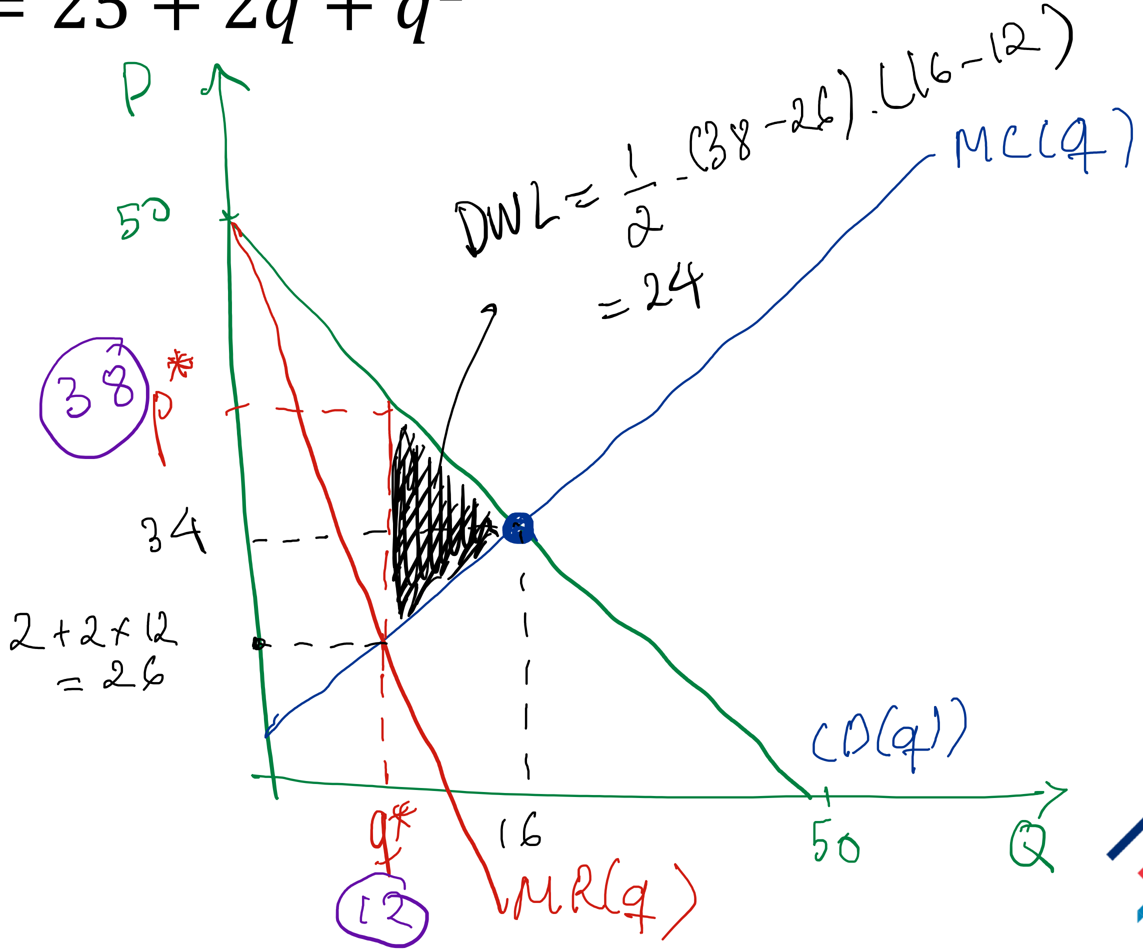
SMC
(Marginal Cost of Production)
P’
SMB
(Marginal Willingness to Pay)
MR
Q’
Q (Quantity)

Example 2

- There is a monopolist for potatoes. The cost function for the firm is
$$C(q) = 25 + 2q + q^2$$
The demand is $P = 50 - Q$.
- Solve for
  - Equilibrium price and quantity
  - Deadweight loss



Socially optimal outcome:

$P = MC$

$\rightarrow 50 - q = 2 + 2q$

$\rightarrow q = 16 \; ; \; P = 50 - 16$
$= 34$

Example 2: Answer

$C(q) = 25 + 2q + q^2$

$P = 50 - q$

(a) $P^* = ?$, $Q^* = ?$

$MC(q) = \frac{d}{dq} C(q) = 2 + 2q$

$MR(q) = \frac{d}{dq}\left[P(q)\,q\right] = \frac{d}{dq}(50q - q^2) = 50 - 2q$

$\rightarrow \quad 2 + 2q = 50 - 2q \rightarrow 48 = 4q$

$\rightarrow \quad q = \boxed{12}$

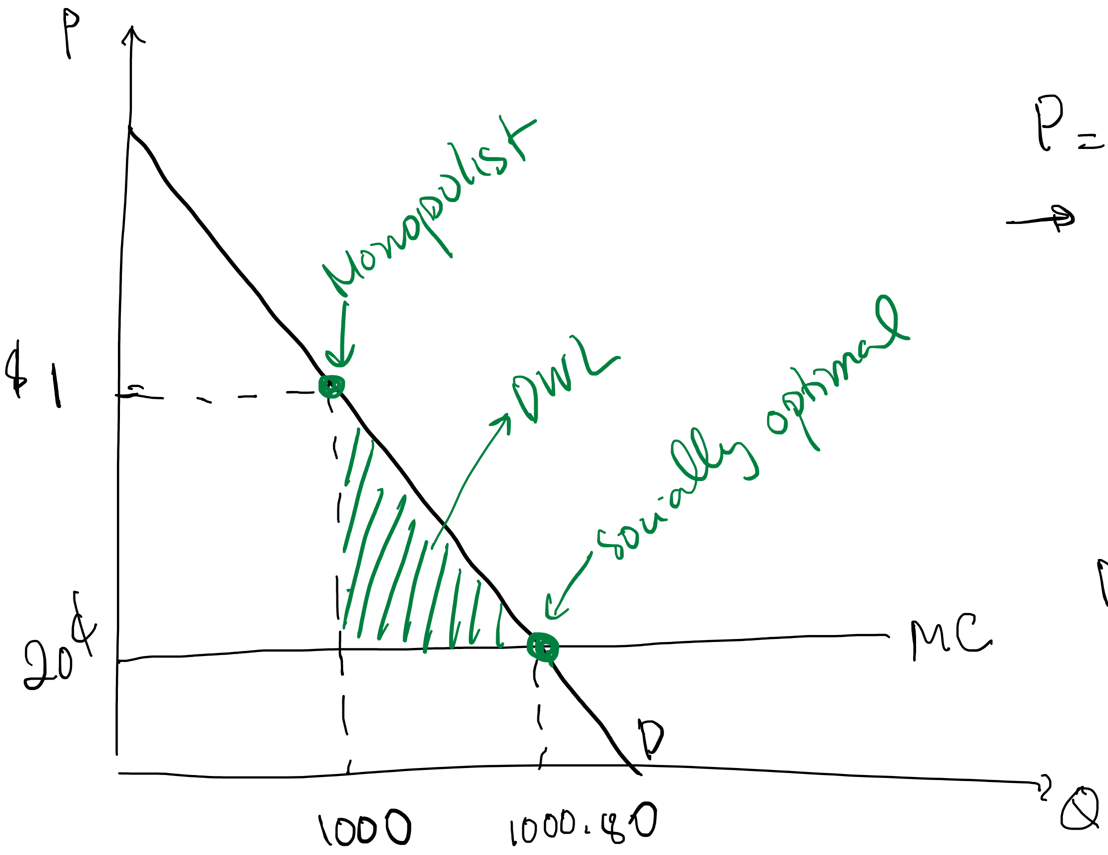
$\rightarrow \quad P = 50 - q = 50 - 12 = \boxed{38}$

Example 2: Answer

Example 3

- The SSS Co. has a patent on a particular medication. The medication sells for $1 per daily dose and marginal cost is constant at 20 cents.
- The firm currently sells 1,000 doses per day.
- The demand curve is decreasing and linear. $P = 1001 - Q$
- Question: *Compute the deadweight loss from monopoly pricing.*

Example 3: Answer



$P = 1001 - Q$

→ Socially optimal outcome is:

$q = 1001 - 20¢$

$= 1000.80$

$$DWL = \frac{.80 \times .80}{2}$$

$= 0.0032$

Example 4

A monopolist has the following short-run cost function:

$$TC(Q) = 20Q + \boxed{100}$$

The demand is given by: $D(Q) = 180 – 4Q$ $\rightarrow P = 180 - 4Q$

$\rightarrow Q = \frac{180-P}{4} \rightarrow \frac{dQ}{dP} = -\frac{1}{4}$

(a) Derive the firm's marginal revenue
(b) What is the equilibrium price and quantity?
(c) Calculate producer surplus, social surplus, and deadweight loss.
(d) What is the price elasticity of the demand?

$$\varepsilon = \frac{dQ}{dP} \cdot \frac{P}{Q} = -\frac{1}{4} \cdot \frac{180-4Q}{Q} = -\frac{180-4Q}{4Q}$$

Example 4: Answer

(a) $MR = \frac{d}{dQ}\left(Q \cdot \underbrace{(180-4Q}_{P(Q)})\right)$

Marginal Revenue is always a function of Q!

$= \frac{d}{dQ}(180Q - 4Q^2) = \boxed{180 - 8Q}$

$MC = \frac{d}{dQ} TC(Q) = \frac{d}{dQ}(20Q + 100) = 20$

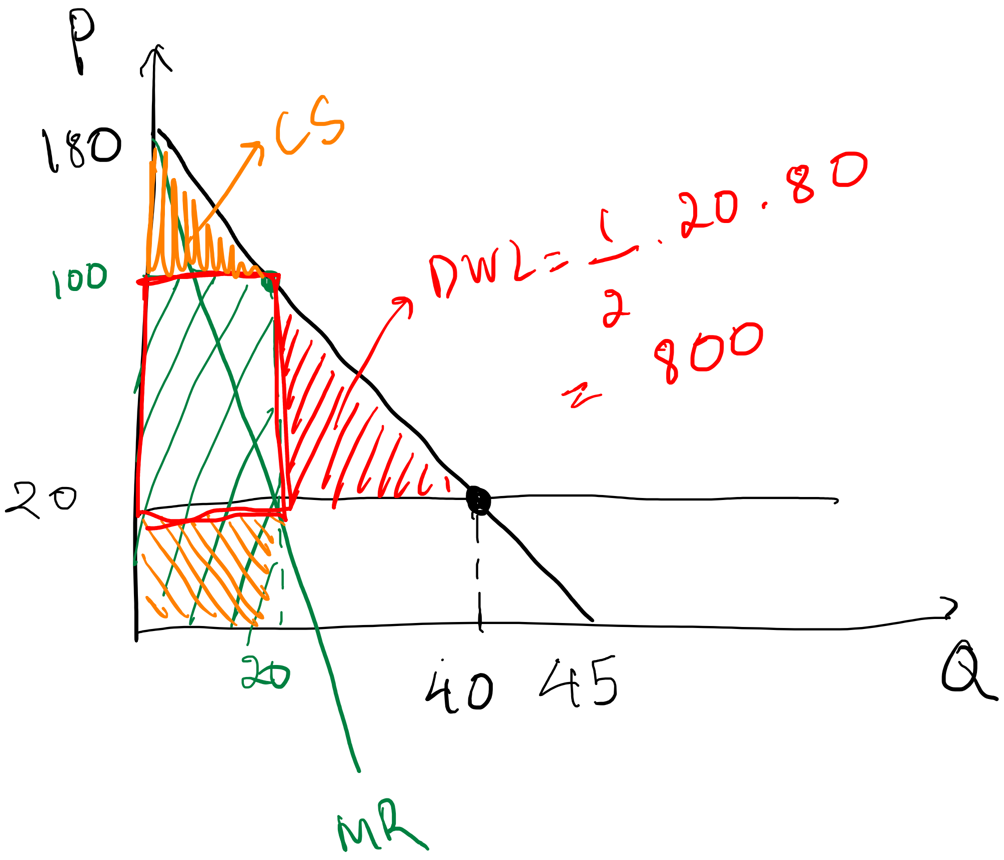
(b) $MC = MR$

$\rightarrow 20 = 180 - 8Q$

$\rightarrow 160 = 8Q \rightarrow Q = 20$

$\rightarrow P = 180 - 4 \cdot 20 = \boxed{100}$

Example 4: Answer



Demand: $P = 180 - 4Q$

$MC = 20$

$Q^{\text{socially optimal}} = \frac{180 - 20}{4} = 40$

Producer Surplus $= 80 \times 20 - FC$

$= 1600 - 100$

$= 1500$

Consumer surplus $= \frac{1}{2}(180 - 100) \times 20$

$= 800$

Social surplus $= 1500 + 800 = 2300$

2. Price Discrimination

Price Discrimination

- Why does Disneyworld charge less for local residents than out-of-towners?
- Firms use non-uniform pricing, where prices vary across customers, to earn higher profit.
- Understand the effect of price discrimination on the social welfare.
- Reading: pp. 841-876

Price Discrimination

- A firm engages in **price discrimination** by charging consumers different prices for the same good based on
  - Individual characteristics → Amazon (in the past)
  - Belonging to an identifiable sub-group of consumers
  - The quantity purchased
- It is not price discrimination if the different prices simply reflect differences in costs.

Why Price Discrimination?

- A firm earns a higher profit from price discrimination than uniform pricing:
  - Price-discriminating firms charge higher prices to consumers who are willing to pay more than the uniform price.
  - Price-discriminating firms sell to some people who are not willing to pay as much as the uniform price.

Conditions for Price Discrimination

- A firm must have market power (otherwise it cannot charge a price above the competitive price).
- A firm must be able to identify which consumers are willing to pay relatively more.
- A firm must be able to prevent or limit resale from customers who are charged a relatively low price to those who are charged a relatively high price.

Types of Price Discrimination

- **First-degree** (Perfect price discrimination)
  - Each unit sold for each customer's maximum willingness to pay
- **Second-degree** (Nonlinear pricing)
  - Firm charges a different price for large quantities than for small quantities
- **Third-degree** (Market segmentation)
  - Firm charges different groups of customers different prices, but charges any one customer the same price for all units sold

Perfect Price Discrimination

First degree price discrimination.

- Under perfect price discrimination, the firm charges each consumer a price that is exactly equal to the maximum he is willing to pay.
- Therefore, each consumer gets zero consumer surplus.
- Firm profit is increased by the amount of consumer surplus that would exist in a competitive market → All CS is transferred to the firm.

Perfect Price Discrimination

- The discriminating monopoly's revenue is:

$$R = \int_0^Q D^{-1}(x)dx$$

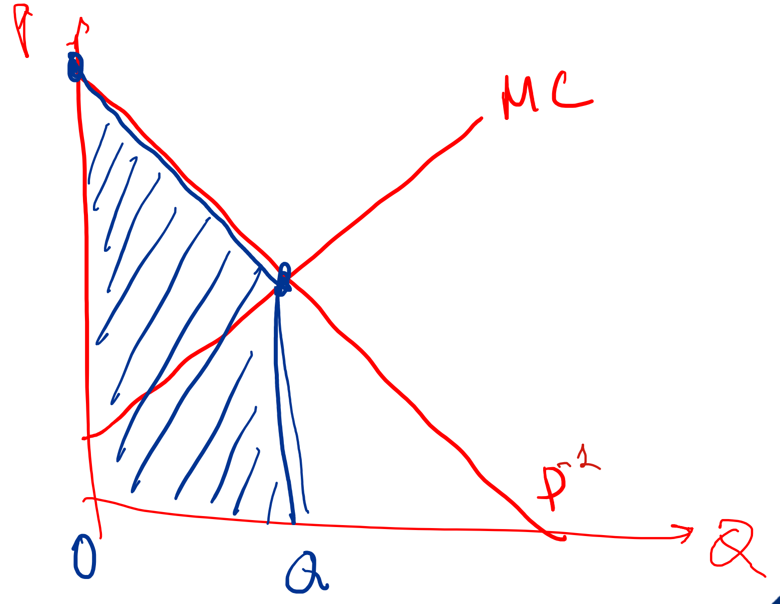
  - Equal to the area under the inverse demand curve up to Q.

$P = D^{-1}(Q) \leftarrow$ inverse demand

$Q = D(P) \leftarrow$ demand

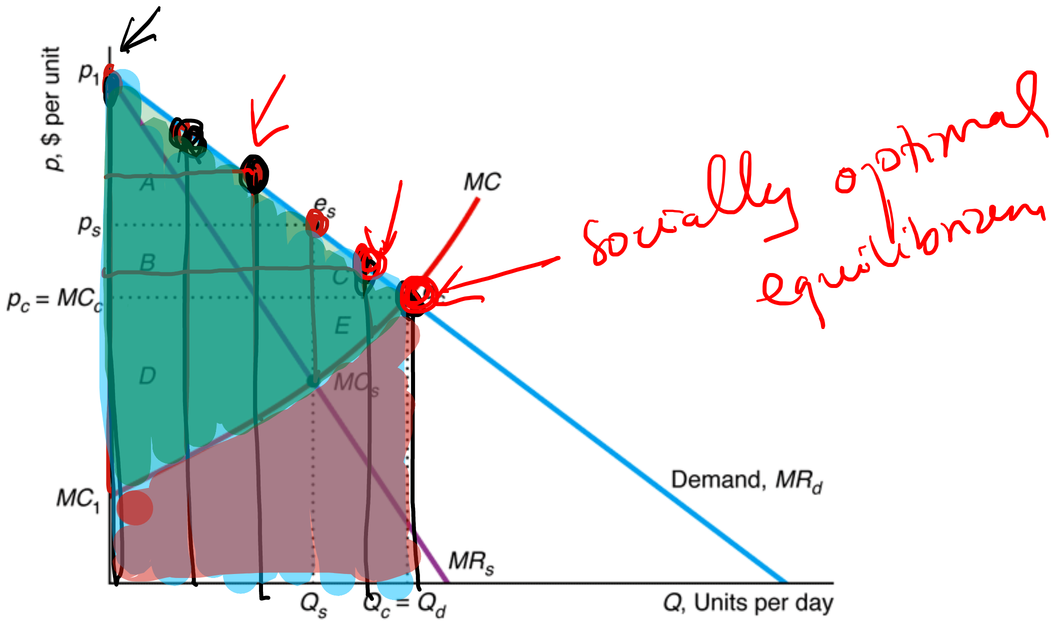
- Maximizing profit: $\boxed{\max_Q \int_0^Q D^{-1}(x)dx - C(Q)}$
  - FOC: $D^{-1}(Q) = MC(Q)$

- Result: produce where $D^{-1}(Q)$ equals MC.



Perfect Price Discrimination

- Producing where Demand = MC, all consumer surplus (*A+B+C*) is transformed into firm profit.

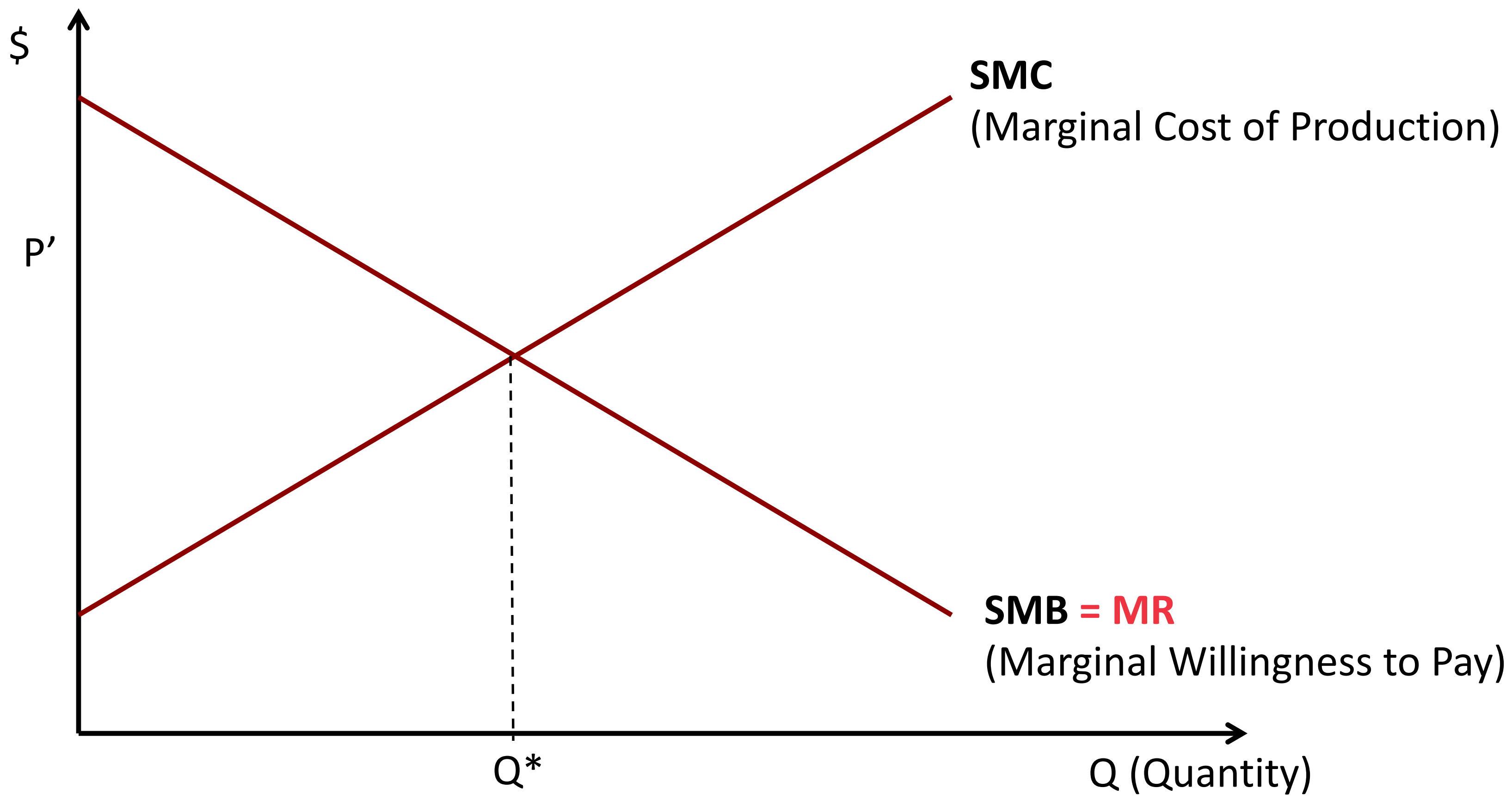


		Monopoly	
	Competition	Single Price	Perfect Price Discrimination
Consumer Surplus, *CS*	$A + B + C$	A	0
Producer Surplus, *PS*	$D + E$	$B + D$	$A + B + C + D + E$
Welfare, $W = CS + PS$	$A + B + C + D + E$	$A + B + D$	$A + B + C + D + E$
Deadweight Loss, *DWL*	0	$C + E$	0

Perfect Price Discrimination

- The perfect price discrimination result of producing where demand equals MC means that the competitive quantity of output gets produced.
- This outcome is efficient:
  - Maximizes total welfare
  - No deadweight loss is generated
- But the outcome is not palatable to consumers because all surplus is producer surplus!

Monopoly with Perfect Price Discrimination



Market Segmentation

(Third degree price discrimination)

- Firms divide potential customers into two or more groups based on some easily observable characteristic and set a different price for each group.
  - Example: Senior or student discounts

Market Segmentation

P1 ↓ P2 ↓

- Firm chooses quantities sold to each group, Q_1 and Q_2 such that

revenue from market 1 ↓ revenue from market 2 ↓ total cost of production ↓

- Taking FOC's: same for a monopolist

$$\max_{Q_1, Q_2} \pi = R_1(Q_1) + R_2(Q_2) - C(Q_1 + Q_2) \quad \cancel{C(Q_1) + C(Q_2)} \text{ incorrect}$$

↑ ↑ instead of P1 & P2

$$\frac{\partial \pi}{\partial Q_1} = \frac{dR_1(Q_1)}{dQ_1} - \underbrace{\frac{dC(Q)}{dQ}}_{MC} \underbrace{\frac{\partial Q}{\partial Q_1}}_{=1} = 0$$

- MR from each group should equal to marginal cost:

$$\frac{\partial \pi}{\partial Q_2} = \underbrace{\frac{dR_2(Q_2)}{dQ_2}}_{MR_2} - \underbrace{\frac{dC(Q)}{dQ}}_{MC} \underbrace{\frac{\partial Q}{\partial Q_2}}_{=1} = 0$$

MR_2 MR_1 MC

$$MR^1 = MC = MR^2$$

Market Segmentation

- Because marginal revenue is a function of elasticity, we can write:

$$MR^A = p_A\left(1 - \frac{1}{\epsilon_A}\right) = MC = p_B\left(1 - \frac{1}{\epsilon_B}\right) = MR^B$$

price Elasticity of market A

- Therefore, we have:

$$\frac{p_A}{p_B} = \boxed{\frac{\left(1 - \frac{1}{\epsilon_B}\right)}{\left(1 - \frac{1}{\epsilon_A}\right)}}$$

$$p_A\left(1 - \frac{1}{\varepsilon_A}\right) = p_A\left(1 - \frac{1}{\varepsilon}\right)$$

$$\varepsilon = \frac{\frac{dQ}{Q}}{\frac{dP}{P}} = \frac{dQ}{dP}\cdot\frac{P}{Q}$$

$$= P\left(1 + \frac{1}{\frac{dQ}{dP}\cdot\frac{P}{Q}}\right)$$

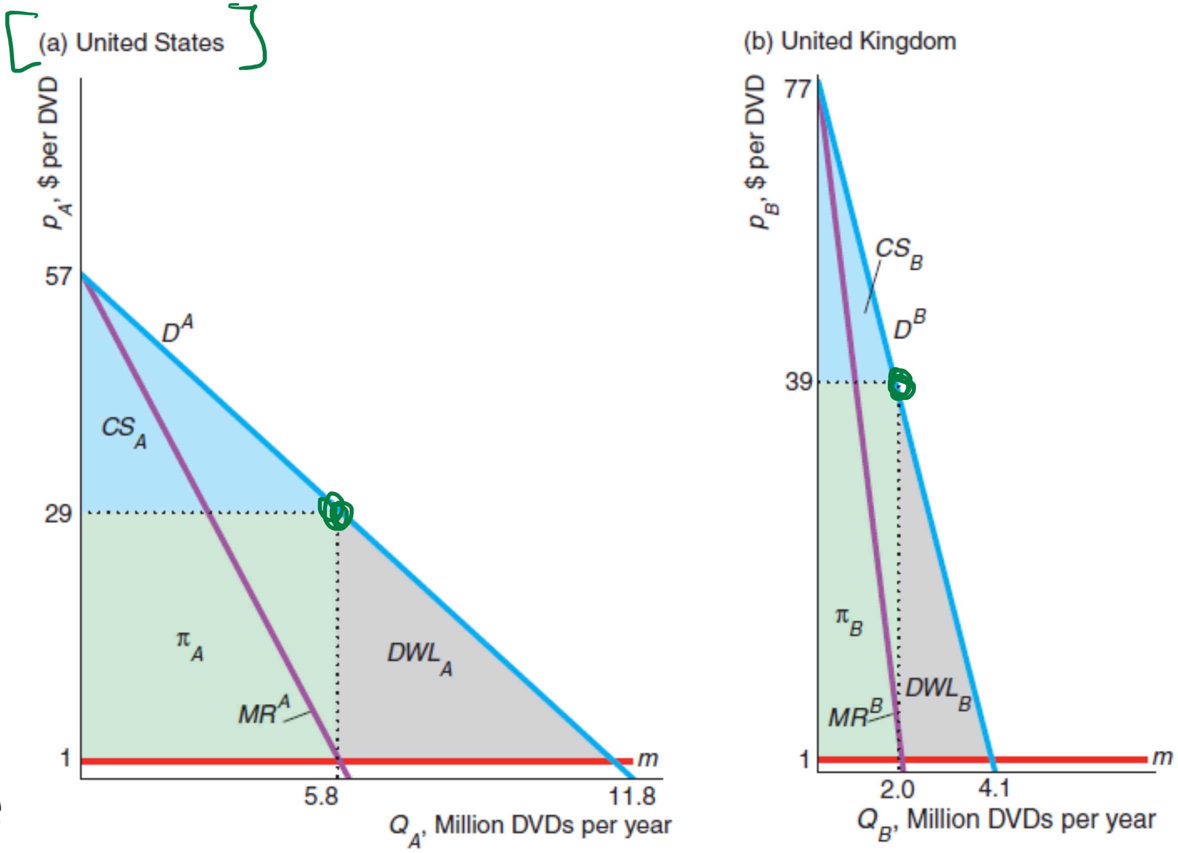
- Thus, the higher price will be charged in the less elastic market segment.

$$= P + Q\cdot\frac{dP}{dQ}$$

$$MR = \frac{d}{dQ}\left(P(Q)\cdot Q\right) = P + Q\cdot\frac{dP}{dQ}$$

Market Segmentation

- A higher price will be charged in the market with the more price-inelastic demand. more elastic



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Market Segmentation

- Welfare under market segmentation is lower than it is under either competition or perfect price discrimination.
  - Under competition, more output is produced and CS is greater.
- The welfare effects relative to uniform price monopoly are indeterminate.
  - Both types of monopolies set price above marginal cost, so output is lower than in competition.

Example 1

Students' demand for CMU sweatshirt is given by: P = 80 – 10Q, Faculties' demand for CMU sweatshirt is given by: $\boxed{P = 70 - 10Q.}$ $\rightarrow Q = \frac{80-P}{10}$

CMU is the monopolist producer of CMU sweatshirt, the marginal cost of producing sweatshirt is MC = 10.

(a) What are the equilibrium prices and quantities if CMU can price discriminate between students and faculties?

(b) What are the equilibrium price and quantity if CMU cannot price discriminate between students and faculties? Students are better off w/ price discrimination
faculties are worse off

(c) Who are better off and who are worse off between (a) and (b)?

(d) How would your answers to (a),(b), and (c) change if the faculties' demand is given by P = 20 – 10Q?

Example 1: Answer

(a) $P_S = 80 - 10Q_S$

$P_F = 70 - 10Q_F$

$MC = 10$

With third degree price discrimination:

$$\max_{Q_S;\,Q_F} \pi = P_S \cdot Q_S + P_F \cdot Q_F - MC \times (Q_S + Q_F) - \text{Fixed cost}$$

$$= (80 - 10Q_S)Q_S + (70 - 10Q_F)Q_F - 10(Q_S + Q_F) - FC$$

FOC: $[Q_S]$: $\frac{\partial \pi}{\partial Q_S} = 0 = 80 - 20Q_S - 10 \rightarrow Q_S = 3.5$

$\rightarrow P_S = 80 - 10 \times 3.5 = 45$

$[Q_F]$: $\frac{\partial \pi}{\partial Q_F} = 0 = 70 - 20Q_F - 10 \rightarrow Q_F = 3$

$\rightarrow P_F = 70 - 10 \times 3 = 40$

Example 1: Answer

(b) w/o price discrimination

$$\max_{P} \pi = Q_S \cdot P + Q_F \cdot P - MC \times (Q_S + Q_F) - FC$$

$$P_S = 80 - 10Q_S \rightarrow Q_S = \frac{80 - P_S}{10}$$

$$P_F = 70 - 10Q_F \rightarrow Q_F = \frac{70 - P_F}{10}$$

if $P > 70 \rightarrow$ only students buy

if $P \leq 70 \rightarrow$ both students and faculties buy

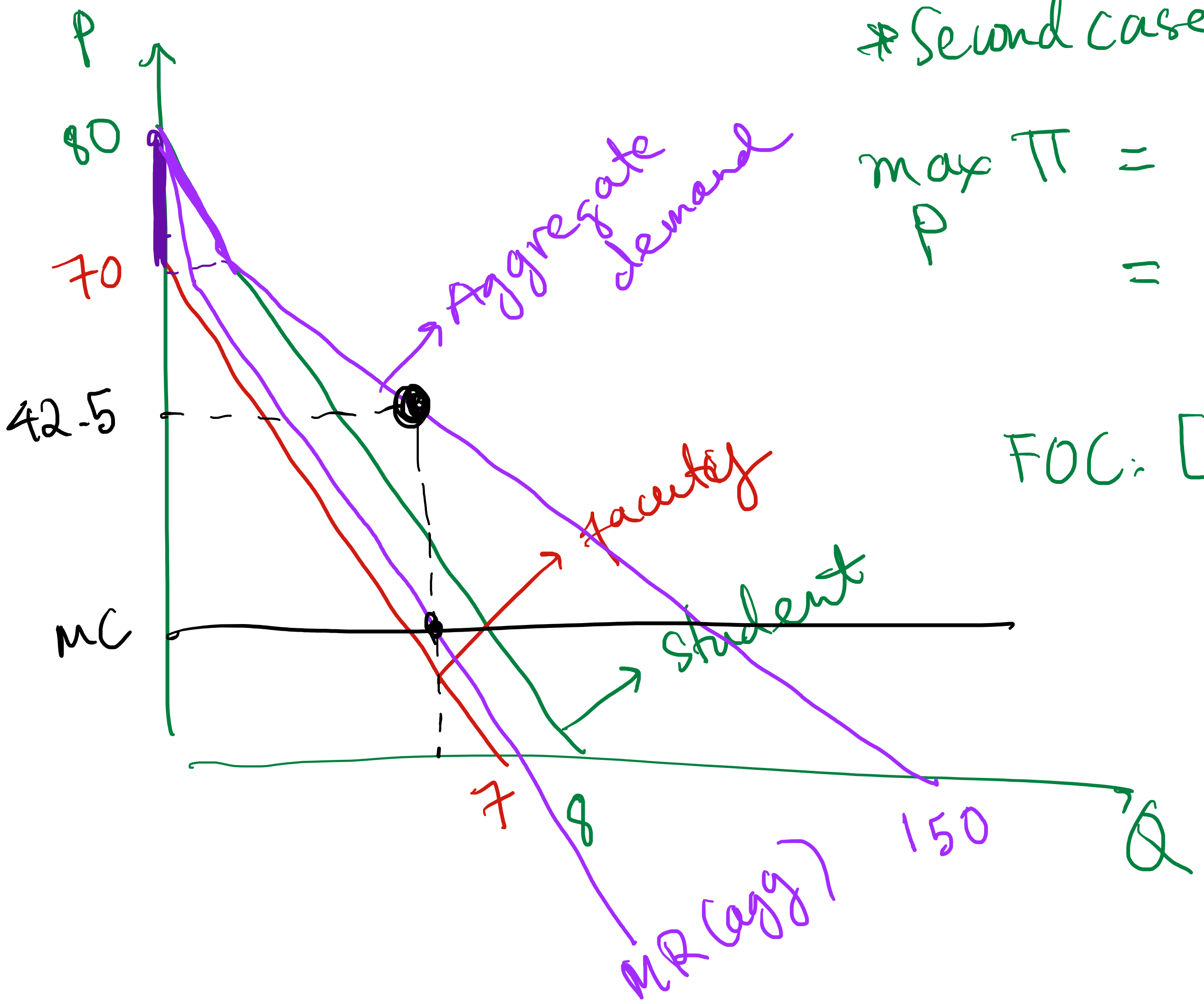
- First case: if $P \geq 70$

$$\max_{P \geq 70} \pi = \frac{80 - P}{10} \cdot P - 10 \cdot \left(\frac{80 - P}{10} \right)$$

$$\text{FOC: } \frac{d\pi}{dP} = \frac{80 - P}{10} - \frac{P}{10} + 1 \rightarrow \frac{d\pi}{dP}(P = 70) = -5 < 0$$

$\Rightarrow P = 70$

Example 1: Answer



*Second case: Both markets are active:

$$\max_{P} \Pi = Q_S \cdot P + Q_F \cdot P - MC(Q_S + Q_F) - FC$$

$$= \frac{80-P}{10} \cdot P + \frac{70-P}{10} \cdot P - 10\left(\frac{150-2P}{10}\right) - FC$$

$$\text{FOC: } [P]: \frac{d\Pi}{dP} = \frac{80-2P}{10} + \frac{70-2P}{10} + 2 = 0$$

$$\to 8 - 0.4P + 7 + 2 = 0$$

$$\to P = \frac{17}{0.4} = 42.5\ (?)\ (<70)$$

$$\to \text{Optimal price } P = 42.5$$

$$Q_S = \frac{80 - 42.5}{10} = 3.75;\ Q_F = 2.7$$

Example 2

A monopolist is facing the following demand: $P = 200 - Q$. The monopolist marginal cost is given by $MC = 20 + 2Q$.

(a) Calculate the equilibrium price and quantity if the monopolist cannot price discriminate. What is the monopolist's profit?

(b) If the monopolist can use first-degree price discrimination, what is the monopolist's profit?

Example 2: Answer

$P = 200 - Q$

$MC = 20 + 2Q$

(a) w/o price discrimination.

$$\max_{Q} \Pi = (200 - Q) \cdot Q - TC(Q)$$

$$= (200 - Q) \cdot Q - \int_0^Q (20 + 2Q)\, dQ - FC$$

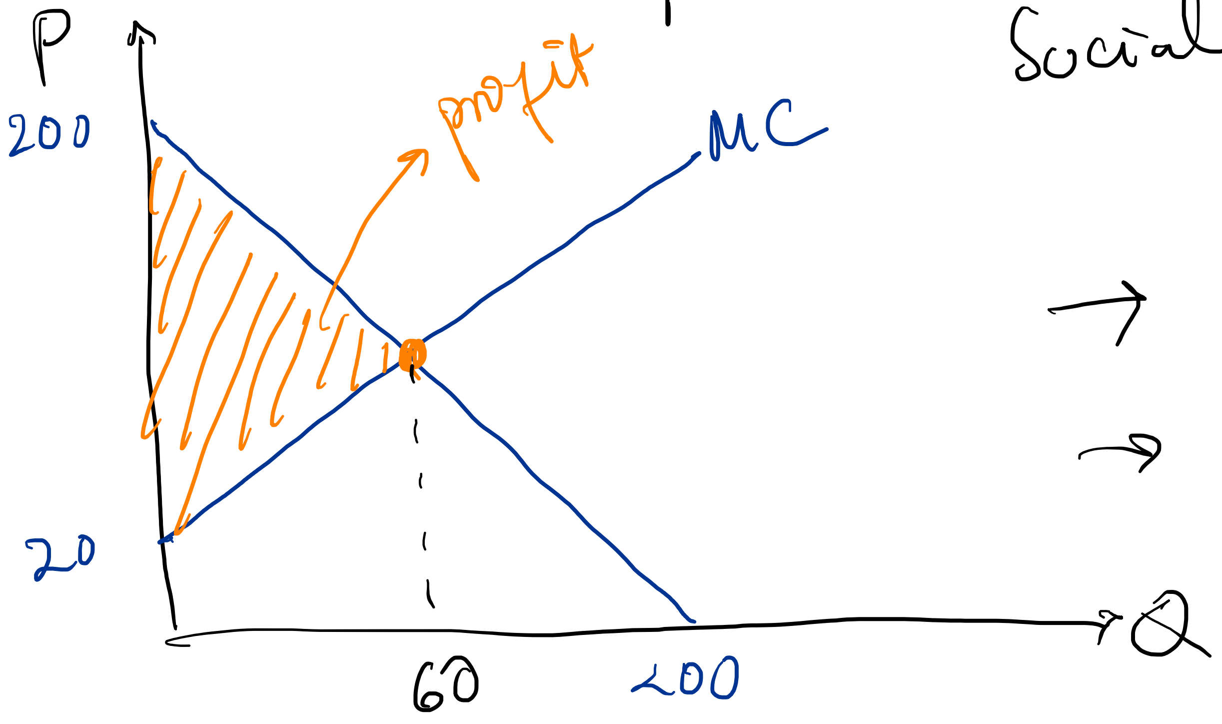
$$= (200 - Q) \cdot Q - [20Q + Q^2] - FC$$

$$\rightarrow \frac{d\Pi}{dQ} = 0 \rightarrow \underbrace{200 - 2Q}_{MR} - \underbrace{(20 + 2Q)}_{MC} = 0 \rightarrow Q = \frac{180}{4}$$

Example 2: Answer

$$Q = \frac{180}{4} \rightarrow P = 200 - Q = 200 - 45 = 155.$$

(b): First degree price discrimination:



Socially optimal outcome:

$$200 - Q = 20 + 2Q$$

$$\rightarrow 180 = 3Q$$

$$\rightarrow 60 = Q$$

$$\rightarrow \pi = \frac{1}{2} \times 60 \times 180$$

$$= 5400$$

Example 3: Starbucks

like coffee but not too much

ppl who don't quite like coffee

Starbucks have three menu options:

1. Tall (12 oz)
2. Grande (16 oz)
3. Venti (24 oz)

consumers who love Starbucks

(second degree PS)

(a) Which price discrimination scheme is this? What is Starbucks trying to obtain with this pricing scheme?

(b) There is one option that is not listed, it's called short (8 oz). You can still ask for it though. Why do you think Starbuck does not list it on the regular menu?