Computer Vision Notes
Created: 2024-09-06
Updated: 2024-09-08

References:
- Introduction to Image Understanding course at the University of Toronto

Linear Filters (TODO: Tb ch 3.2)
Digital Image: a map $f: \mathbb{R}^2 \to \mathbb{R}$ or a matrix I of integer intensity values $\in [0,255]$, I is $m \times n$ in a grayscale image, $m \times n \times 3$ in a color image.

Problem: want to locate object in image.
Solution: slide and compare the image of the object.

Problem: noise in image.
Solution: modify pixel by applying function on a neighborhood of pixels e.g., average neighbors (assumes neighbors similar, noise independent) using moving average with (non-)uniform weights.

Correlation (cv2.filter2D, 2D moving average with (non-)uniform weights): Given input I, $G = F \otimes I$ where

$$G(i,j) = \sum_{u=-k}^{k} \sum_{v=-k}^{k} F(u,v) \cdot I(i+u, j+v)$$

where size of the weight **kernel/mask** F is $(2k+1)^2$ and its entries $F(u,v)$ are **filter coefficients**.
where $\sum\sum F(u,v) = 1$.

Let $\vec{f} = F(:), \overrightarrow{t_{ij}} = T_{ij}(:)$ where $T_{ij} = I(i-k: i+k, j-k: j+k)$, then

$$G(i,j) = \vec{f}^T \cdot \overrightarrow{t_{ij}}$$

Normalized Cross-Correlation: exact match of image crop and filter results in 1. Normalized prevents $\overrightarrow{t_{ij}}$ that is all or almost all white (255) to generate large response.

$$G(i,j) = \frac{\vec{f}^T \cdot \overrightarrow{t_{ij}}}{\|\vec{f}\| \|\overrightarrow{t_{ij}}\|}$$

Types of Filters
Sharpening Filter:

$$F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Gaussian Filter: smooth/blur, reduce noise, neighbors closest to a center have the most influence.

$$h(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$

Generic Gaussian Filter: anisotropic (asymmetric), $x \in \mathbb{R}^d$.

$$\mathcal{N}(x; \mu, \Sigma) = \frac{1}{(2\pi)^{d/2} |\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\right)$$

Effect of Size of Filter and Variance

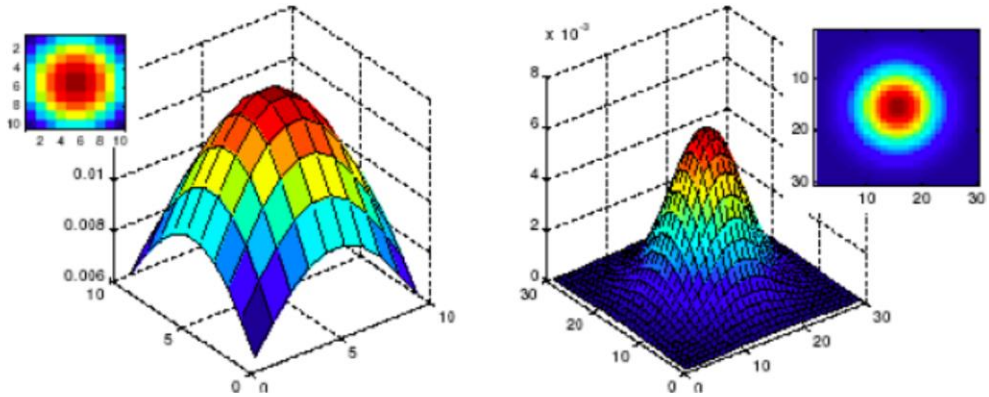

Figure: same $\sigma = 5$ different filter/mask/kernel size 10x10 vs 30x30.

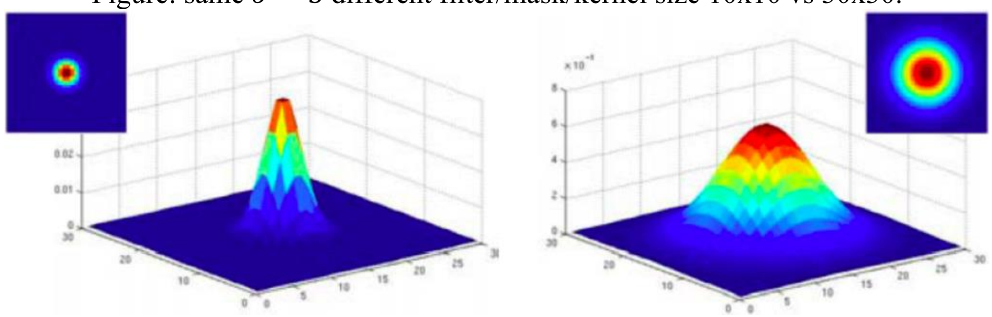

Figure: same size 30x30, $\sigma = 2$ (left) $\sigma = 5$ (right), larger is more smoothing.

Properties of Smoothing

- All values positive
- Sum to 1; prevents rescaling image
- Low-pass filter; removes high frequency (rate of change in pixel intensity values) components which include edges.

Convolution: operator that flips filter horizontally and vertically then applies correlation. Given input I, $G = F * I$ where

$$G(i,j) = \sum_{u=-k}^{k} \sum_{v=-k}^{k} F(u,v) \cdot I(i-u, j-v)$$

Properties of Convolution

Commutative	$f * g = g * f$
Associative	$f * (g * h) = (f * g) * h$
Distributive	$f * (g + h) = f * g + f * h$
Associative with scalar multiplier	$\lambda \cdot (f * g) = (\lambda \cdot f) * g$
Convolution Theorem ($\mathcal{F}$ is Fourier Transform)	$\mathcal{F}(f * g) = \mathcal{F}(f) \cdot \mathcal{F}(g)$

Implications of Convolution Theorem

Method 1: convolution $f * g$ runs in N^2.
Method 2: FFT and IFFT run in $N \log N$ and mult in N.

Separable Filters

[TODO]