Title. Formulation of a Mathematical Equation for a Precise and Accurate Prediction of the Relative Location of a Self-Driving Robot.

Research question. How can the location of a robot be determined based purely on mathematical manipulation of its wheel rotations?

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SECTION 1 – INTRODUCTION

My curiosity, piqued by mathematics and robotics from a young age, started my investigation of precisely tracking a robot's position using mathematics. As I gained more knowledge and earned more opportunities, especially after learning vectors in IB HL Mathematics and earning the title of robotics team captain at my school, I became more passionate about this investigation.

Most robotics competitions have an autonomous driving event. My early exposure to robotics familiarized me with this concept. Our robotics team had difficulties perfecting our autonomous driving, especially in autonomous steering. I investigated the problem, realizing that wheel slippage caused inaccurate encoder readings in the sanctioned motors' built-in encoders. After research and rumination, I solved the slippage problem, which led me to develop my mathematical equations to track a robot's location based on wheel rotations.

SECTION 2 – CORE

2.1 Background

From a third person's perspective of a volume in space in an instance of time, selecting an arbitrary point in a 3-dimensional (3D) space allows the position representation of any object in this space with a single vector relative to that point. Similarly, a bird's-eye-view of 3D space (i.e., 2D) enables the position representation of any object by a single vector. The vector's initial point is any coordinate such as the object's initial position. As a result, one way to track an object's motion is to monitor the change in its 2D position vector.

Globally, there are many existing real-world applications of this concept of position tracking in space. Global Navigation Satellite System (GNSS) is a constellation of satellites from different countries that provides worldwide coverage of the location of objects on Earth within a few meters of accuracy (*What is GNSS?*) (*GNSS / GPS Technology Differences*). One subset of GNSS is the Global Positioning System (GPS), operated by the United States Space Force. The GPS tracks the location of objects on Earth through satellite communication by devices such as smartphones (*What is GPS?*) (*What is a GPS? How does it work?*). The accuracy of GPS-enabled smartphones is within 4.9 meters (*GPS Accuracy*).

Internationally, self-driving cars are a type of vehicle that can move accurately without direct human control by using internal and external sensors. One of the biggest challenges associated with autonomous vehicles is their collision risk (Zein et al.), giving rise to ethical concerns. A reduction in the risk of collision directly relates to the amelioration of ethical problems. So, knowledge of a car's accurate location is crucial to ensure the safety of life inside and outside the vehicle. Typical position tracking methods include using Light Detection and Ranging (LIDAR) and RAdio Detection And Ranging (RADAR), cameras, deep-learning algorithms, and computer vision with proportional-integral-derivative (PID) controllers (Prabhakar et al.) (Iqbal) (*What is LIDAR?*) (*Radar (radio detection and ranging)*).

Transnationally, virtual reality equipment closely monitors the motion of the eyes, hands, head, and body to ensure a precise and accurate representation of real-world movements in virtual space, maximizing a user's enjoyment (National Research Council). This application of position tracking involves using external sensors, such as cameras (*Positional tracking*). Programming virtual reality entails complex computer image and video analysis, which are high-tech concepts of machine learning. So, position tracking in virtual reality position tracking is difficult because of its use of cameras to track motion.

Locally, incremental encoders track rotary electromechanical signals, giving the direction and speed of rotation. Examples of such tracking include physical volume control knobs, trackballs in traditional mice, and conveyor belts for rotation speed tracking. Rotary quadrature encoder sensing mechanisms such as optical encoders (see fig. 1) allow for this tracking (*Introduction to Incremental Encoders*) (Collins).

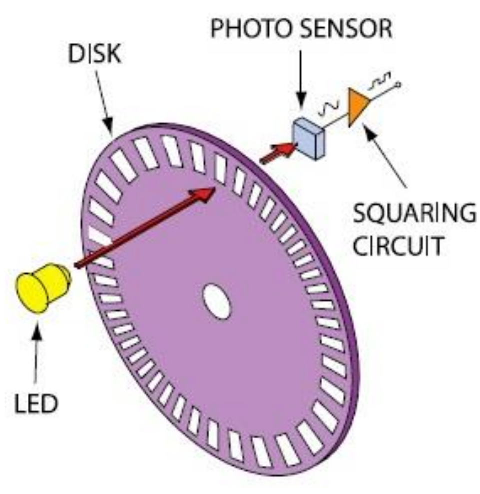

Figure 1. Optical rotary encoder used in rotational position tracking (Schweber).

2.2 Wheelbase Model

A standard competition robot wheelbase is the four-wheeled skid-steer model. A motor (blue rectangles, see fig. 2) powers each wheel (black rectangles) through an axle (horizontal gray line). Chains (vertical gray line) connect the two wheels on the left side, forming a wheelset. The chained two wheels on the right side is another wheelset.

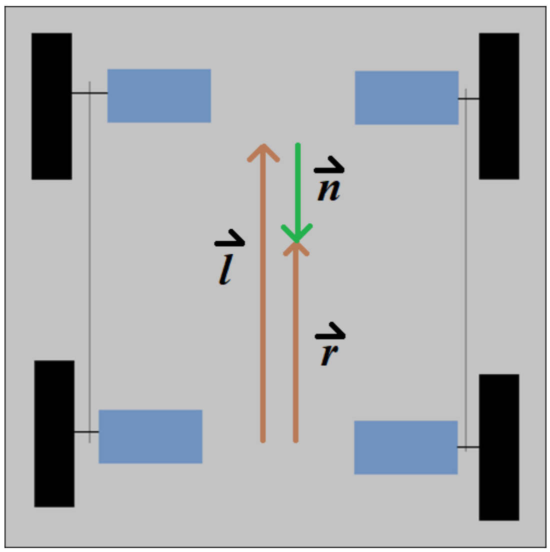

Figure 2. A chained four-wheeled skid-steer wheelbase turns right.

The displacement of the left and right wheelsets is maintained even if one motor of a wheelset breaks because chains connect the motors. Thus, simplifying the wheelbase to the tracked (e.g., tanks) robot model. Differing velocities of the wheelsets control steering (i.e., the faster wheelset determines the steering direction). Let the left wheelset have velocity $\vec{l}$, and the right wheelset velocity $\vec{r}$. Vector subtraction of $\vec{r}$ - $\vec{l}$ indicates the net velocity $\vec{n}$ (left, right, straight) of the robot (see fig. 2).

Since $\vec{l}$ and $\vec{r}$ are fixed to the left and right side, respectively, $\vec{n}$ on the right side represents a right turn, and $\vec{n}$ on the left side represents a left turn. Algebraically, there is no left and right side as the vectors overlap, but for clarity, assume there are two sides. If $\vec{l} = \vec{r}$, then $\vec{n} = \vec{0}$, there is no rotation but either linear motion or no motion. If $\vec{l} = -\vec{r} \neq 0$, $\vec{n} = \vec{l} + \vec{r}$, indicating rotation on the spot.

A simplification of the chained four-wheel skid-steer wheelbase is the tracked wheelbase. A further simplification of the tracked wheelbase is the differential drive wheelbase, where a wheel replaces each chained wheelset (Northwestern Robotics). This simplification can be applied since three models move in the same manner. The subsequent analysis will use the differential drive model (see fig. 3).

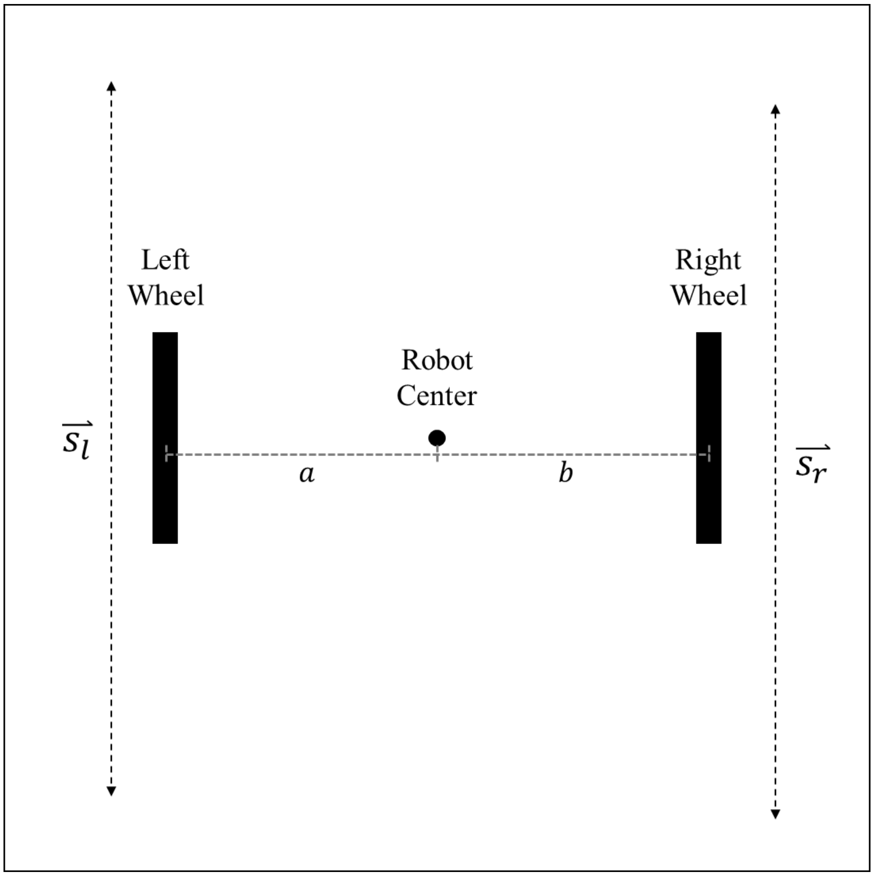

Figure 3. Showing the differential drive wheelbase model of a robot.

Table 1. Notation in Figure 3.

Symbol	**Representation**
Rectangle	Wheel.
Circle	Robot (wheelbase) center.
a	Distance from left wheel to robot center.
b	Distance from right wheel to robot center. $a = b$.
$\overrightarrow{s_l}$	Left wheel displacement vector. Dashed double-ended arrows represent that the wheel can move both forwards and backwards.
$\overrightarrow{s_r}$	Right wheel displacement vector.

2.3 Arc Model for Motion

Given both wheels spin forward at a constant non-zero rate in a differential drive wheelbase, when the right wheel spins faster than the left wheel, the robot will drive in a circular path with a constant radius. Suppose a robot in this circular motion stops after completing 45° (see fig. 4).

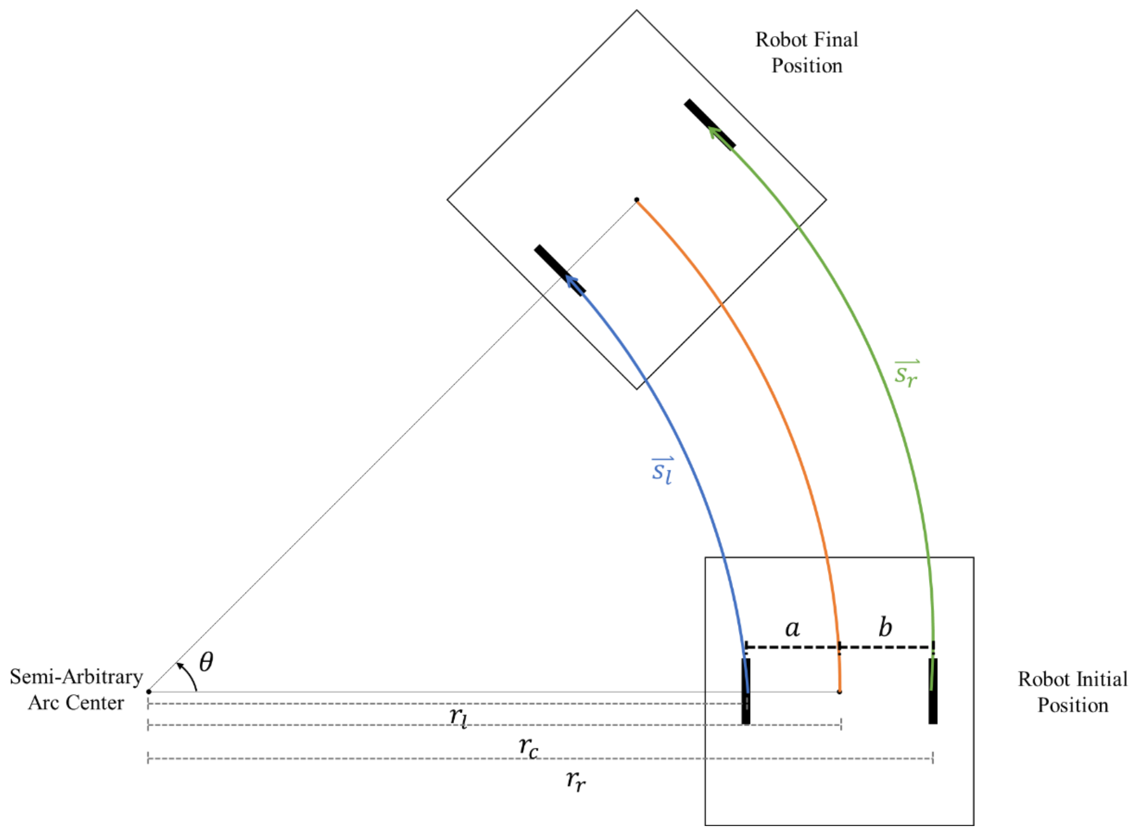

Figure 4. Arc model to calculate robot motion by modelling the pathway of a robot in this arc motion.

Table 2. Notation in Figure 4.

Symbol	**Representation**
Square outline	Initial and final robot position.
Bottom left dot	Semi-arbitrary arc center. Semi-arbitrary because net subtraction vector determines the radius of the arc.
θ	Robot rotation angle in standard position.
$\overrightarrow{s_l}$	Left wheel displacement.
$\overrightarrow{s_r}$	Right wheel displacement.
r_l	Radius of $\overrightarrow{s_l}$ (blue arc).
r_r	Radius of $\overrightarrow{s_r}$ (green arc).
r_c	Radius of orange arc (distance from arc center to robot center).

If a robot only performs arc motion (a portion of circular motion), measuring how much the left and right wheels rotate leads to finding its location on the circular pathway circumference. Furthermore, the vector representing the direction the robot faces is tangent to the circular path. One perspective of motion in a coordinate plane is a composition of infinitely many circular pathways. Specifically, the robot center represents the position of a robot at an instance in time. For example, if the robot drives straight, its path composes of infinitely many arc motions. Each arc motion comprises an arc center and displacement of the robot center.

Calculating the direction the robot faces and how much it moved relative to a starting position requires the distance from the robot center to wheels and the radius of the wheel. Measuring with a ruler accomplishes both requirements. One remaining piece of information is the displacement of the left and right wheels. Thus, to calculate a robot's location and orientation relative to a point, find the constant specification values and measure the wheel displacements.

2.4 Wheel Displacement

The first step in calculating the orientation and location of a robot is measuring constant specification values. Afterwards, find $\overrightarrow{s_l}$ and $\overrightarrow{s_r}$. Calculating $\overrightarrow{s_l}$ and $\overrightarrow{s_r}$ requires measuring wheel radii and tracking wheel rotations in degrees. Equations 1 and 2 shows wheel displacement calculations.

$$Equation\ 1. \overrightarrow{s_l} = 2\pi r(\frac{R_l}{360°})$$

$$Equation\ 2. \overrightarrow{s_r} = 2\pi r(\frac{R_r}{360°})$$

Table 3. Additional notation in Equations 1 and 2.

Symbol	**Representation**
r	Wheel radius.
R_l	Left wheel rotation in degrees. $R_l \in \mathbb{R}$.
R_r	Right wheel rotation in degrees. $R_r \in \mathbb{R}$.

2.5 Robot Orientation

The next crucial variable to calculate is the arc angle. Ultimately, the robot's rotation relative to the arc center defines its orientation.

$$Equation\ 3.\ r_c = r_l + a$$

$$Equation\ 4.\ r_c = r_r - b$$

Express displacement with radius using $s = r\theta$.

$$Equation\ 5.\ \overrightarrow{s_l} = r_l\theta$$

$$Equation\ 6.\ \overrightarrow{s_r} = r_r\theta$$

Ultimately, θ needs to be expressed in terms of "a", "b", $\overrightarrow{s_l}$, and $\overrightarrow{s_r}$ because these are known values. First substitute r_l and r_r in Equations 5 and 6 with r_l and r_r in Equations 3 and 4.

$$Equation\ 7.\ \overrightarrow{s_l} = (r_c - a)\theta$$

$$Equation\ 8.\ \overrightarrow{s_r} = (r_c + b)\theta$$

Now, r_c is the only unknown. Rearrange and equate Equations 7 and 8.

$$r_c = \frac{\overrightarrow{s_l}}{\theta} + a$$

$$r_c = \frac{\overrightarrow{s_r}}{\theta} - b$$

$$Equation\ 9.\ \frac{\overrightarrow{s_l}}{\theta} + a = \frac{\overrightarrow{s_r}}{\theta} - b$$

Simplify and rearrange Equation 9 for θ.

$$\overrightarrow{s_l} + a\theta = \overrightarrow{s_r} - b\theta$$

$$Equation\ 10.\ \theta = \frac{\overrightarrow{s_r} - \overrightarrow{s_l}}{a + b}$$

Replace unknown variables in Equation 10 with known variables in Equations 1 and 2.

$$\theta = \frac{2\pi r(\frac{R_r}{360°}) - 2\pi r(\frac{R_l}{360°})}{a+b}$$

$$\theta = \frac{\frac{2\pi r}{360°}(R_r - R_l)}{a+b}$$

$$\theta = \frac{\pi r(R_r - R_l)}{180°(a+b)}$$

Since $a = b$ (distance from robot center to each wheel is equal), eliminate “b”.

$$Equation\ 11. \theta = \frac{\pi r(R_r - R_l)}{360°(a)}$$

Equation 11 shows the relationship between robot orientation, distance from the wheels to the robot center and the amount the wheels displaced. For example, when $R_l = R_r$, $\theta = 0$. That is, when the robot drives straight, its orientation does not change. Equation 11 successfully calculates the orientation of the robot.

It is unlikely for the motion of a robot to remain in a circular motion, so accuracy increases as θ approaches 0. Thus, place a multivariable limit on the wheel rotations.

$$\lim_{(R_r,\ R_l)\to(0,0)} \frac{\pi r(R_r - R_l)}{360°(a)}$$

To ensure θ is minimum in a programming context, calculate θ as frequently as possible.

2.6 Robot Displacement

The next step is to calculate the robot center's change in the coordinate plane (see Fig. 5).

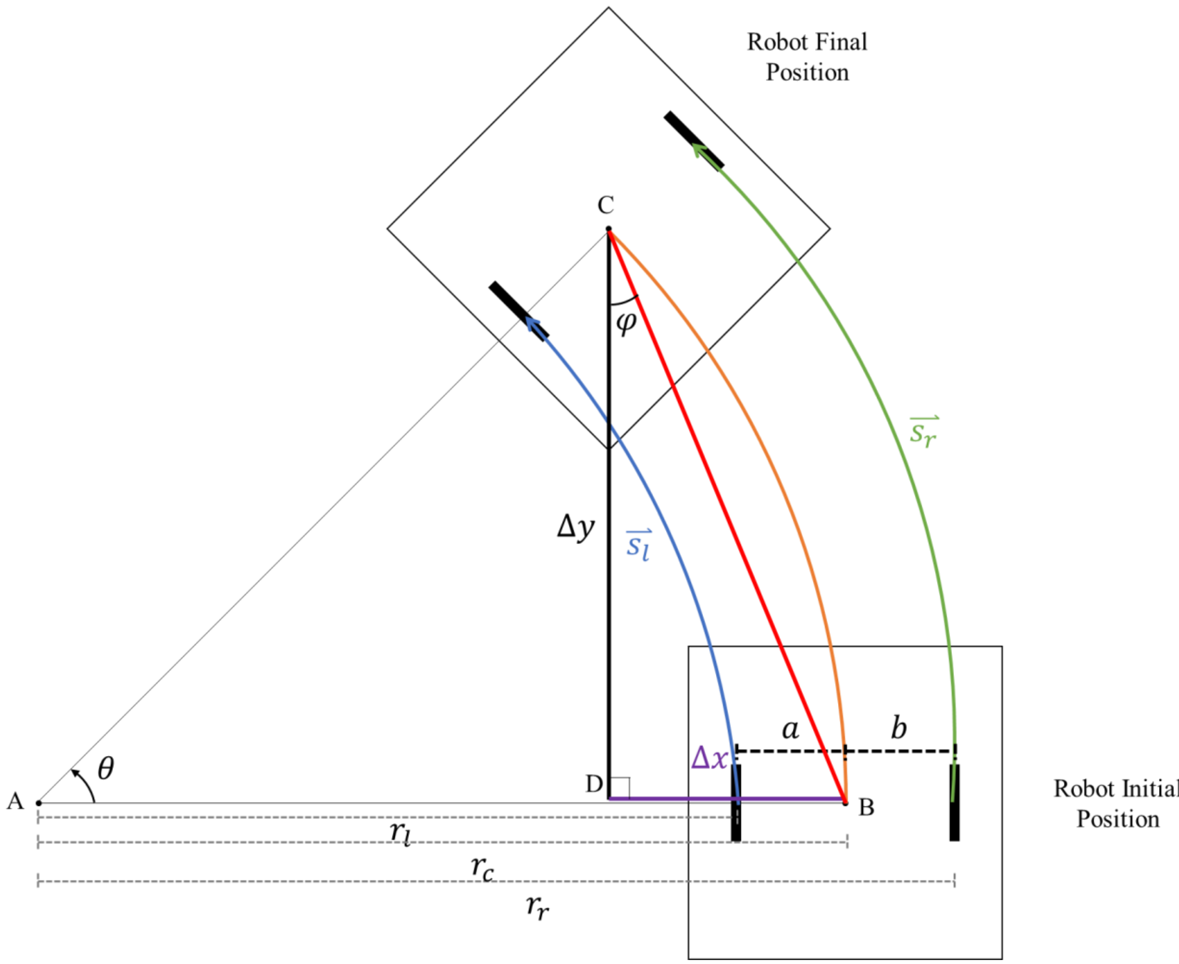

Figure 5. Arc model showing a change in the position of the robot center, where displacement in x- and y-position is in thick purple and black lines, respectively.

Table 4. Additional notation in Figure 5.

Symbol	Representation
Δx	Horizontal displacement.
Δy	Vertical displacement.
A	Arc center.
B	Initial robot center.
C	Final robot center.
D	Intersection of Δy and line AB.
△BCD	Right-angle triangle.
△ABC	Isosceles triangle.
BC	Hypotenuse of triangle BCD.

The first step to calculating Δx and Δy is finding φ, which requires the length of line AB. Knowing θ allows the expression of φ in terms of θ.

Since $\triangle ABC$ is isosceles,

$$Equation\ 14.\ \angle ABC = \angle ACB = \frac{180° - \theta}{2}$$

Looking at $\triangle BCD$,

$$\varphi = 180° - 90° - \angle ABC$$

$$Equation\ 15.\ \varphi = 90° - \angle ABC$$

Substitute $\angle ABC$ from Equation 14 to Equation 15.

$$\varphi = 90° - \frac{180° - \theta}{2}$$

$$Equation\ 16.\ \varphi = \frac{\theta}{2}$$

After finding φ, BC needs to be found to reach the goal of finding Δx and Δy. Since θ is known, if AB and AC are known, the cosine law can be applied to find BC. Equation 5 will be used to aid in the process.

$$AB = AC = r_l + a$$

$$Equation\ 5. \vec{s_l} = r_l \theta$$

$$Equation\ 17. AB = AC = \frac{\vec{s_l}}{\theta} + a$$

Applying cosine law,

$$c = \sqrt{a^2 + b^2 - 2ab\cos C}$$

$$BC = \sqrt{AB^2 + AC^2 - 2(AB)(AC)\cos\theta}$$

AB = AC since both are radii of the arc.

$$BC = \sqrt{2AB^2 - 2(AB^2)\cos\theta}$$

$$BC = AB\sqrt{2 - 2\cos\theta}$$

Using Equation 17,

$$Equation\ 18. BC = (\frac{\vec{s_l}}{\theta} + a)\sqrt{2 - 2\cos\theta}$$

Change in x

And so, by trigonometric ratios, Δx can be found.

$$\Delta x = BC\sin\varphi$$

Using Equation 16 and 18,

$$Equation\ 16. \varphi = \frac{\theta}{2}$$

$$\Delta x = (\frac{\vec{s_l}}{\theta} + a)\sqrt{2 - 2\cos\theta}\sin\frac{\theta}{2}$$

$$Equation\ 19. \Delta x = (\sin\frac{\theta}{2})(\frac{\overrightarrow{s_l}}{\theta} + a)\sqrt{2 - 2\cos\theta}$$

To express everything as a single vector in polar form later, AD can be expressed in terms of AB and Δx from Equation 17 and 19.

$$AD = AB - \Delta x$$

$$AD = (\frac{\overrightarrow{s_l}}{\theta} + a) - (\sin\frac{\theta}{2})(\frac{\overrightarrow{s_l}}{\theta} + a)\sqrt{2 - 2\cos\theta}$$

$$Equation\ 20. AD = (\frac{\overrightarrow{s_l}}{\theta} + a)\left(1 - \sin\frac{\theta}{2}\sqrt{2 - 2\cos\theta}\right)$$

Change in y

By trigonometric ratios, Δy can be found.

$$\Delta y = BC\cos\varphi$$

Using Equation 16 and Equation 18,

$$Equation\ 16. \varphi = \frac{\theta}{2}$$

$$Equation\ 18. BC = (\frac{\overrightarrow{s_l}}{\theta} + a)\sqrt{2 - 2\cos\theta}$$

$$\Delta y = (\frac{\overrightarrow{s_l}}{\theta} + a)\sqrt{2 - 2\cos\theta}\cos\frac{\theta}{2}$$

$$Equation\ 21. \Delta y = (\cos\frac{\theta}{2})(\frac{\overrightarrow{s_l}}{\theta} + a)\sqrt{2 - 2\cos\theta}$$

2.7 Single Displacement Vector

Overall, Equation 1, 11, 20, and 21 can be combined to form one single displacement vector in polar form.

$$\vec{z} = |\vec{z}|(\cos\theta + i\sin\theta)$$

$$\vec{z} = \sqrt{x^2 + y^2}(\cos\theta + i\sin\theta)$$

$$\vec{z} = \sqrt{x^2 + y^2}(\operatorname{cis}\theta)$$

$$\vec{z} = (\operatorname{cis}\theta)\sqrt{x^2 + y^2}$$

$$\vec{z} = (\operatorname{cis}\theta)\sqrt{AD^2 + \Delta y^2}$$

$$Equation\ 20.\, AD = (\frac{\vec{s_l}}{\theta} + a)\left(1 - \sin\frac{\theta}{2}\sqrt{2 - 2\cos\theta}\right)$$

$$Equation\ 21.\, \Delta y = (\cos\frac{\theta}{2})(\frac{\vec{s_l}}{\theta} + a)\sqrt{2 - 2\cos\theta}$$

$$\vec{z} = \operatorname{cis}\theta\sqrt{\left(\left(\frac{\vec{s_l}}{\theta} + a\right)\left(1 - \sin\frac{\theta}{2}\sqrt{2 - 2\cos\theta}\right)\right)^2 + \left(\left(\cos\frac{\theta}{2}\right)\left(\frac{\vec{s_l}}{\theta} + a\right)\sqrt{2 - 2\cos\theta}\right)^2}$$

$$= \operatorname{cis}\theta\sqrt{\left(\frac{\vec{s_l}}{\theta} + a\right)^2\left(1 - \sin\frac{\theta}{2}\sqrt{2 - 2\cos\theta}\right)^2 + \left(\cos\frac{\theta}{2}\right)^2\left(\frac{\vec{s_l}}{\theta} + a\right)^2\left(\sqrt{2 - 2\cos\theta}\right)^2}$$

$$= (\operatorname{cis}\theta)\left(\frac{\vec{s_l}}{\theta} + a\right)\sqrt{\left(1 - \sin\frac{\theta}{2}\sqrt{2 - 2\cos\theta}\right)^2 + \cos^2\frac{\theta}{2}(2 - 2\cos\theta)}$$

$$= (\operatorname{cis}\theta)$$

$$\left(\frac{\vec{s_l}}{\theta} + a\right)\sqrt{1 - 2\sin\frac{\theta}{2}\sqrt{2 - 2\cos\theta} + \sin^2\frac{\theta}{2}(2 - 2\cos\theta) + \left(1 - \sin^2\frac{\theta}{2}\right)(2 - 2\cos\theta)}$$

$$= (\operatorname{cis}\theta)\left(\frac{\vec{s_l}}{\theta} + a\right)\sqrt{1 - 2\sin\frac{\theta}{2}\sqrt{2 - 2\cos\theta} + (2 - 2\cos\theta)}$$

$$= (\operatorname{cis}\theta)\left(\frac{\overrightarrow{s_l}}{\theta} + a\right)\sqrt{3 - 2\cos\theta - 2\sin\frac{\theta}{2}\sqrt{2 - 2\cos\theta}}$$

$$= (\operatorname{cis}\theta)\left(\frac{\overrightarrow{s_l}}{\theta} + a\right)\sqrt{3 - 2\cos\theta - 2\sin\frac{\theta}{2}\sqrt{2 - 2\cos\theta}}$$

Using Equation 1 and Equation 11,

$$Equation\ 1. \overrightarrow{s_l} = 2\pi r\left(\frac{R_l}{360°}\right)$$

$$Equation\ 11. \theta = \frac{\pi r(R_r - R_l)}{360°(a)}$$

$$\vec{z} = (\operatorname{cis}\theta)\left(\frac{\frac{2\pi r R_l}{360°}}{\frac{\pi r(R_r - R_l)}{360°(a)}} + a\right)\sqrt{3 - 2\cos\theta - 2\sin\frac{\theta}{2}\sqrt{2 - 2\cos\theta}}$$

$$= (\operatorname{cis}\theta)\left(\frac{2aR_l}{R_r - R_l} + a\right)\sqrt{3 - 2\cos\theta - 2\sin\frac{\theta}{2}\sqrt{2 - 2\cos\theta}}$$

$$= (\operatorname{cis}\theta)\left(\frac{a(2R_l + R_r - R_l)}{R_r - R_l}\right)\sqrt{3 - 2\cos\theta - 2\sin\frac{\theta}{2}\sqrt{2 - 2\cos\theta}}$$

$$Equation\ 22. \vec{z} = a\frac{R_r + R_l}{R_r - R_l}\left(\operatorname{cis}\frac{\pi r(R_r - R_l)}{360°(a)}\right)$$

$$\sqrt{3 - 2\cos\frac{\pi r(R_r - R_l)}{360°(a)} - 2\sin\frac{\pi r(R_r - R_l)}{720°(a)}\sqrt{2 - 2\cos\frac{\pi r(R_r - R_l)}{360°(a)}}}$$

Where a is the distance from the robot center to either wheel, r is the radius of the wheel, R_l is the number of rotations of the left wheel, and R_r is the number of rotations of the right wheel.

2.8 Conclusion

This section calculated a robot's location and orientation relative to a starting point based on mathematical manipulation of the wheel rotations, starting with the data collection of known constants and variables. The constants are the wheel radius and the distance from the wheels to the robot center. Variables are the left and right wheel rotations. The values allow expressing the wheel displacement with measured data. Then the angle of the arc was found, which tells the orientation of the robot. Next, the finding of the robot location gave the vertical and horizontal displacement vectors. Finally, combining the direction the robot faces, the horizontal component, and the vertical component of the displacement vector into a single vector equation represents the robot's orientation and location relative to a point.

SECTION 3 – EVALUATION

3.1 Application

The overarching force that drove the development of this essay was to help solve problems in robotics. The vector equation in Equation 22 keeps track of the position and orientation of a robot. Recall that wheel slippage caused the inaccuracy in autonomous driving. One method to overcome this challenge is adding an independent set of tracking wheels (see fig. 7).

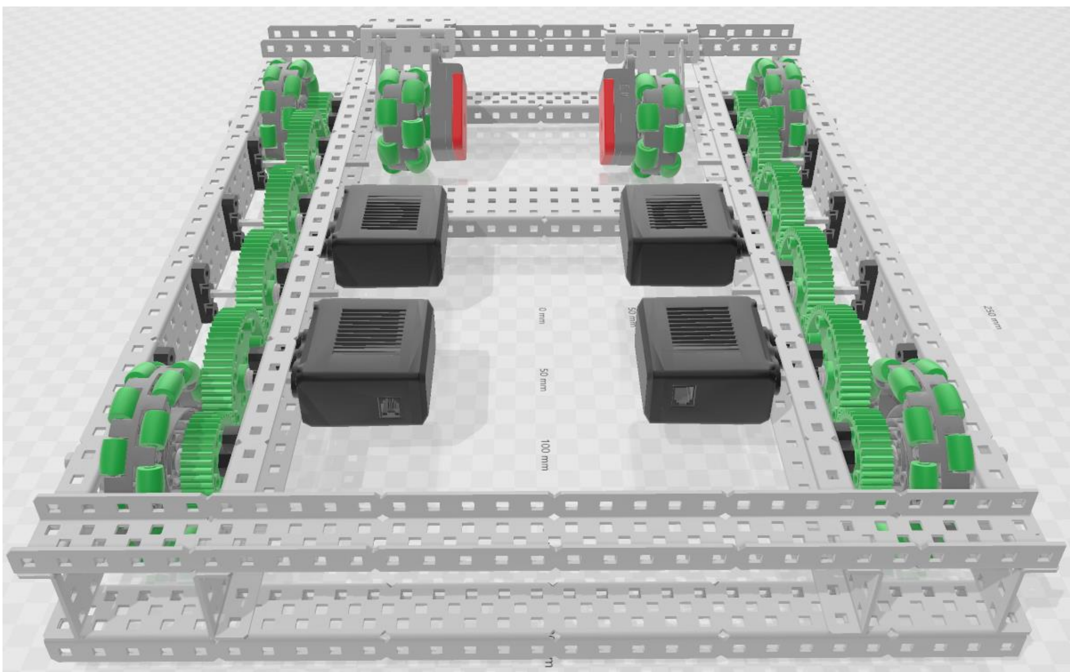

Figure 7. Robot drivetrain with four black motors rotating green gears connected to four green and gray wheels. Hinges secure two smaller sized tracking wheels to the chassis. Tracking wheels and the red and grey optical rotary encoders are connected. Model created using 3D Builder.

In this wheelbase, the four motors that power the four driving wheels control the robot's motion. These wheels are subject to wheel slippage, which occurs when a robot wheel temporarily leaves the ground. One common cause is a sudden change in the direction of motion, which is common in this robotics competition. These robots have relatively low mass hence less moving inertia. Its centripetal force in a circular motion in a horizontal plane would be smaller, leading to a smaller radius of rotation, during which time the wheels may slip. Connecting the driving wheels to the optical rotary encoder would result in inaccuracy in measuring wheel rotation. Thus, there is a need to isolate the driving system and the tracking system such that the tracking system is free from wheel slippage.

Rubber bands physically press down two tracking wheels via the elastic force exerted (gravitational force would not suffice to ensure the wheels always remain on the ground due to their small mass) (see fig. 8, 9, 10).

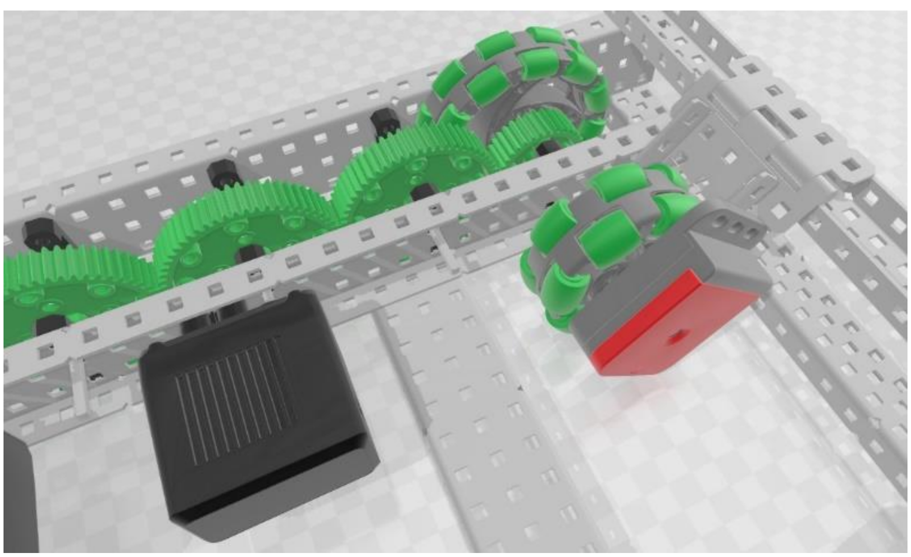

Figure 8. Showing the right-back side of the robot. The driving wheel and tracking wheel are not connected.

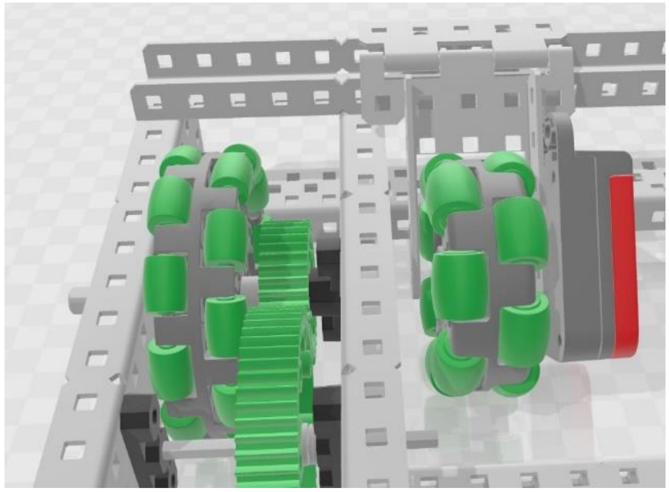

Figure 9. Showing a closer view of the right-back side.

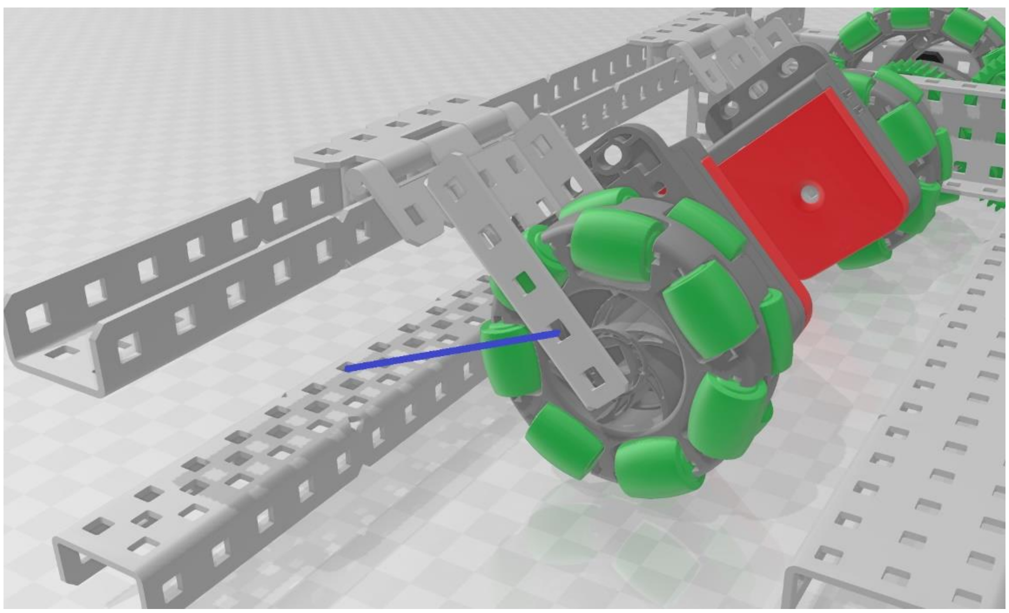

Figure 10. the blue rubber band presses the tracking wheel to the ground. Note: the removal of the drivetrain's right side shows the tracking wheel system clearer.

When the robot's backside lifts slightly above the ground due to a sudden rotation, this tracking wheel will remain in contact with the ground surface. Thus, the vector equation would be accurate because the wheel slippage probability drastically decreases, providing more precise wheel rotation measurements.

3.2 Extension

In the robotics competition, all robots on the field move during the autonomous period. The initial positions of obstacles do not change for any match, allowing for easy implementation of the mathematical equation developed in this essay by pre-programming it using C++ (a computer programming language that this robotics competition software uses) into the robot.

There is a rule in this competition that robots of one alliance may not cross a line during the autonomous period. The penalty for violating this rule is an immediate win for the opposing teams. An extension of the position tracking equations is implementing the machine learning algorithms (which rely heavily on statistical mathematics).

For example, this machine learning algorithm may function to prevent the robot from crossing the autonomous line. One method is the logistic regression algorithm, which calculates the probability an event occurs based on a neural network with an activation function (commonly sigmoid function – resembles the graph of arctangent – or ReLU – Rectified Linear Unit – function) generated from "experience." The experience can be from practice runs where every successful practice routine is stored and used as training data for the logistic regression algorithm. During the competition, if the logistics regression algorithm decides that the probability the robot crosses the line is greater than 50% based on too much deviation from its experience, then artificial intelligence will terminate the program. Implementing this artificial intelligence to make a decision would require plenty of statistical analysis, matrix operations, and calculus (e.g., partial derivatives used to minimize the cost function in a process called gradient descent).

A key concept in machine learning is minimizing the cost function, and this is similar for proportional-integral-derivative (PID) controllers. The difference is minimizing an error term, rather than a cost function, in PID controllers. Thus, using PID controllers to control robot movement is another extension.

A final extension of this essay in a bigger context is its application potential to real-world autonomous vehicles. As mentioned in the Background section, artificial intelligence must face the ethical dilemma of safety prioritization in car accidents, raising concerns about discriminatory prejudice (TEDEducation). Preventing accidents altogether would mitigate such problems. Hence, it is a hope that this essay can be foundational for more sophisticated research into this booming autonomous vehicle industry.

SECTION 4 – CONCLUSION

This essay formulated a mathematical equation to predict the instantaneous location of an autonomous robot precisely and accurately. Given the left and right tracking wheels rotations, the distance from the robot center to either tracking wheel, and wheel radius, this equation yields the instantaneous location and orientation of a robot relative to its initial position. The summation of instantaneous location and orientation calculations over infinitely short time intervals define a robot's overall location and orientation.

This mathematical equation is easily implementable in code, which allows for its application in competitive robotics to improve performance in autonomous events. An extension of this equation includes adaptations for control theory and artificial intelligence development. Finally, this equation and its extension may create impacts of global significance for its usefulness for the prospectively thriving autonomous vehicle industry.

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