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String content of the Néel state

Abstract. Bla bla bla ...

1. Reduced overlaps with the Neel state

Let us consider a generic n-string state

$$\lambda_\alpha^{n,a} = \lambda_\alpha^n + \frac{i}{2}(n+1-2a) + i\delta_\alpha^{n,a} \quad \text{with } a = 1,\ldots,n. \tag{1}$$

The overlap with the Neel state reads

$$\frac{\langle\Psi_0|\{\pm\lambda_j\}_{j=1}^m, n_\infty\rangle}{|||\{\lambda_j\}_{j=1}^m, n_\infty\rangle||} = \frac{\sqrt{2}N_\infty!}{\sqrt{(2N_\infty)!}}\left[\prod_{j=1}^m \frac{\sqrt{\lambda_j^2+1/4}}{4\lambda_j}\right]\sqrt{\frac{\det_m(G^+)}{\det_m(G^-)}} \tag{2}$$

where

$$G_{jk}^{\pm} = \delta_{jk}\left(NK_{1/2}(\lambda_j) - \sum_{l=1}^m K_1^+(\lambda_j,\lambda_l)\right) + K_1^{\pm}(\lambda_j,\lambda_k), \quad j,k = 1,\ldots,m \tag{3}$$

and

$$K_1^{\pm}(\lambda,\mu) = K_1(\lambda-\mu) \pm K_1(\lambda+\mu) \tag{4}$$

and

$$K_\alpha(\lambda) \equiv \frac{2\alpha}{\lambda^2+\alpha^2} \tag{5}$$

We start focusing on G^+. The term $K_1^+(\lambda,\mu)$ diverges when $|\lambda-\mu| = i$, which happens if λ and μ are successive members of the same string.

The number of eigenstates excluding the zero-momentum strings is given by the sequence A014495. It is given as a function of the chain length as

$$C\left(\frac{L}{2}, \left\lfloor\frac{L}{4}\right\rfloor\right) - 1 \tag{6}$$

with $C(x,y)$ denoting the binomial coefficient. Notice that the number of states, which have non-zero overlap with the Neel state, is in principle

$$2^{\frac{L}{2}-1} + \frac{(1+(-1)^{\frac{L}{2}})}{4} C\left(\frac{L}{2}, \frac{L}{4}\right) \tag{7}$$

Bethe states with nonzero Néel overlap ($N = 12$)

String content	$2I_n^+$	E	$\vert\langle\{\lambda\}\vert\Psi_0\rangle\vert^2$	here
6 inf	-	0	0.002164502165	0.002164502165
2 one, 4 inf	1_1	-3.918985947229	0.096183409244	0.096183409244237
	3_1	-3.309721467891	0.011288497947	0.0112884979464673
	5_1	-2.284629676547	0.004542580506	0.0045425805061850
	7_1	-1.169169973996	0.002752622983	0.0027526229835876
	9_1	-0.317492934338	0.002116006203	0.0021160062026402
4 one, 2 inf	$1_1 3_1$	-7.070529325964	0.310133033838	0.554809782804
	$1_1 5_1$	-5.847128730477	0.129277023687	
	$1_1 7_1$	-4.570746557876	0.085992436024	
	$3_1 5_1$	-5.153853093221	0.015256395523	
	$3_1 7_1$	-3.916336243695	0.010091113504	
	$5_1 7_1$	-2.817696043731	0.004059780228	
2 two, 2 inf	1_2	-1.905667167442	0.001207238321	0.005468702625
	3_2	-1.368837200825	0.002340453815	
	5_2	-0.681173793635	0.001921010489	
1 one, 1 three, 2 inf	$0_1 0_3$	-2.668031843135	0.034959609810	0.034959609810
6 one	$1_1 3_1 5_1$	-8.387390917445	0.153412152966	0.153412152966
2 two, 2 one	$1_1 1_2$	-5.401838225870	0.040162686361	0.046134750850
	$3_1 1_2$	-4.613929948329	0.004636541934	
	$5_1 1_2$	-3.147465758841	0.001335522556	
1 three, 3 one	$0_1 2_1 0_3$	-6.340207488736	0.052743525774	0.078910020729
	$0_1 4_1 0_3$	-5.203653009936	0.015022005621	
	$0_1 6_1 0_3$	-3.788693957250	0.011144489334	
1 five, 1 one	$0_1 0_5$	-2.444293750583	0.005887902992	0.005887902992
2 three	1_3	-1.111855930538	0.001342476001	0.001342476001
1 two, 1 four	$0_2 0_4$	-1.560671012472	0.000026982174	0.000026982174

Table 1. All Bethe states for $N = 12$ with nonzero overlap with the zero-momentum Néel state. The overlap squares add up to 1 up to the precision in which the Bethe equations were solved. The $2I_n^+$ in the second column give the positive n-string quantum numbers of the parity-invariant Bethe states.