String content of the Néel state

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Bla bla bla ...

I. REDUCED OVERLAPS WITH THE NEEL STATE

Let us consider a generic string state

$$\lambda_\alpha^{n,a} = \lambda_\alpha^n + \frac{i}{2}(n+1-2a) + i\delta_\alpha^{n,a} \quad \text{with } a = 1,\dots,n. \tag{1}$$

The overlap with the Neel state reads

$$\frac{\langle \Psi_0 | \{\pm\lambda_j\}_{j=1}^m, n_\infty\rangle}{||\,|\{\lambda_j\}_{j=1}^m, n_\infty\rangle||} = \frac{\sqrt{2}N_\infty!}{\sqrt{(2N_\infty)!}}\left[\prod_{j=1}^m \frac{\sqrt{\lambda_j^2+1/4}}{4\lambda_j}\right]\sqrt{\frac{\det_m(G^+)}{\det_m(G^-)}} \tag{2}$$

where

$$G_{jk}^\pm = \delta_{jk}\left(NK_{1/2}(\lambda_j) - \sum_{l=1}^m K_1^+(\lambda_j,\lambda_l)\right) + K_1^\pm(\lambda_j,\lambda_k), \quad j,k = 1,\dots,m \tag{3}$$

and

$$K_1^\pm(\lambda,\mu) = K_1(\lambda-\mu) \pm K_1(\lambda+\mu) \tag{4}$$

and

$$K_\alpha(\lambda) \equiv \frac{2\alpha}{\lambda^2+\alpha^2} \tag{5}$$

We start focusing on G^+. The term $K_1^+(\lambda,\mu)$ diverges when $|\lambda-\mu| = i$, which happens if λ and μ are successive members of the same string.

Let us first discuss the situation with two 2-strings. The matrix G^+ has the structure

$$\begin{pmatrix} D_1 - F_{12}^- - F_{12}^+ & D_1 \\ D_1 & D_1 + D_2 \end{pmatrix} \tag{6}$$

where we defined $D_j \equiv LK_{1/2}(\lambda_j)$ and $F_{ij}^\pm \equiv K_1(\lambda_i \pm \lambda_j)$. Since $F_{12}^- \sim -1/\delta^2$, with δ the string deviation, one can consider the matrix

$$\begin{pmatrix} -1/\delta^2 & 0 \\ 0 & D_1 + D_2 \end{pmatrix} \tag{7}$$

For G^- one can write

$$\begin{pmatrix} \tilde{D}_1 - F_{12}^- - F_{12}^+ & \tilde{D}_1 - 2F_{12}^+ \\ \tilde{D}_1 - 2F_{12}^+ & \tilde{D}_1 + \tilde{D}_2 - 4F_{12}^+ \end{pmatrix} \tag{8}$$

where $\tilde{D}_j \equiv (L-1)K_{1/2}(\lambda_j)$ which at the leading order in $1/\delta$ can be written as

$$\begin{pmatrix} -1/\delta & 0 \\ 0 & \tilde{D}_1 + \tilde{D}_2 - 4F_{12}^+ \end{pmatrix} \tag{9}$$

For a 3-string by simple row and column manipulations we can reduce G^+ to

$$\begin{pmatrix} D_1 - F_{12}^+ - F_{12}^- - F_{13}^- - F_{13}^+ & D_1 - F_{13}^- - F_{13}^+ & D_1 \\ D_1 - F_{13}^- - F_{13}^+ & D_1 + D_2 - F_{23}^+ - F_{23}^- - F_{13}^- - F_{13}^+ & D_1 + D_2 \\ D_1 & D_1 + D_2 & D_1 + D_2 + D_3 \end{pmatrix} \tag{10}$$

The leading order of $\det(G^+)$ is obtained from the reduced matrix

$$\begin{pmatrix} -1/\delta^2 & 0 & 0 \\ 0 & -1/\delta^2 & 0 \\ 0 & 0 & D_1+D_2+D_3 \end{pmatrix} \tag{11}$$

Similarly, for the matrix G^- one has

$$\begin{pmatrix} \tilde D_1 - F^+_{12} - F^-_{12} - F^-_{13} - F^+_{13} & \tilde D_1 - 2F^+_{12} - F^-_{13} - F^+_{13} & \tilde D_1 - 2F^+_{12} - 2F^+_{13} \\ \tilde D_1 - 2F^+_{12} - F^-_{13} - F^+_{13} & \tilde D_1 + \tilde D_2 - 4F^+_{12} - F^+_{23} - F^-_{23} - F^-_{13} - F^+_{13} & \tilde D_1 + \tilde D_2 - 4F^+_{12} - 2F^+_{23} - 2F^+_{13} \\ \tilde D_1 - 2F^+_{12} - 2F^+_{13} & \tilde D_1 + \tilde D_2 - 4F^+_{12} - 2F^+_{13} - 2F^+_{23} & \tilde D_1 + \tilde D_2 + \tilde D_3 - 4F^+_{12} - 4F^+_{13} - 4F^+_{23} \end{pmatrix} \tag{12}$$

while the leading order of $\det G^-$ is obtained from the matrix

$$\begin{pmatrix} -1/\delta^2 & 0 & 0 \\ 0 & -1/\delta^2 & 0 \\ 0 & 0 & \tilde D_1 + \tilde D_2 + \tilde D_3 - 4(F^+_{12} + F^+_{13} + F^+_{23}) \end{pmatrix} \tag{13}$$

The generalizion to the n-string for G^+ should be

$$G^+_{an} = G^+_{na} = \begin{cases} D_a - \sum\limits_{k=n+1}^{M} (F^+_{ka} + F^-_{ka}) & \text{for } a < n \\ \sum\limits_{k=1}^{n} (D_k - \sum\limits_{l=n+1}^{M} (F^+_{kl} + F^-_{kl})) & \text{for } a = n \\ -\sum\limits_{k=1}^{n} (F^+_{ka} + F^-_{ka}) & \text{for } a > n \end{cases} \tag{14}$$

For the generic case one has that

$$G^+_{n,\alpha,m,\beta} = \begin{cases} 2L\frac{d}{d\lambda^n_\alpha}\theta(\lambda^n_\alpha/n) - 2\sum\limits_{(l,\gamma)\neq(n,\alpha)} \frac{d}{d\lambda^n_\alpha}(\Theta_{n,l}(\lambda^n_\alpha - \lambda^l_\gamma) + \Theta_{n,l}(\lambda^n_\alpha + \lambda^l_\gamma)) & \text{if } (n,\alpha) = (m,\beta) \\ 2\frac{d}{d\lambda^n_\alpha}(\Theta_{n,m}(\lambda^n_\alpha - \lambda^m_\beta) + \Theta_{n,m}(\lambda^n_\alpha + \lambda^m_\beta)) & \text{if } (n,\alpha) \neq (m,\beta) \end{cases} \tag{15}$$

where

$$\Theta_{nm}(x) \equiv \theta(x/(|n-m|)) + 2\theta(x/(|n-m|+2)) + \cdots + 2\theta(x/(n+m-2)) + \theta(x/(n+m)) \tag{16}$$

and $\theta(x) \equiv 2\arctan(x)$. For G^- one obtains

$$G^-_{n,\alpha,m,\beta} = \begin{cases} 2(L-1)\frac{d}{d\lambda^n_\alpha}\theta(\lambda^n_\alpha/n) - 4\sum\limits_{k=1}^{n-1}\frac{d}{d\lambda^n_\alpha}\theta(\lambda^n_\alpha/k) - 2\sum\limits_{(l,\gamma)\neq(n,\alpha)} \frac{d}{d\lambda^n_\alpha}(\Theta_{n,l}(\lambda^n_\alpha - \lambda^l_\gamma) + \Theta_{n,l}(\lambda^n_\alpha + \lambda^l_\gamma)) & \text{if } (n,\alpha) = (m,\beta) \\ 2\frac{d}{d\lambda^n_\alpha}(\Theta_{n,m}(\lambda^n_\alpha - \lambda^m_\beta) - \Theta_{n,m}(\lambda^n_\alpha + \lambda^m_\beta)) & \text{if } (n,\alpha) \neq (m,\beta) \end{cases} \tag{17}$$

with

$$\Theta_{nn}(x) \equiv 2\theta(x/2) + 2\theta(x/4) + \cdots + 2\theta(x/(2n-2)) + \theta(x/(2n)) \tag{18}$$

Moreover for a n-string one has for odd n

$$\prod_{a=1}^{n} \frac{\sqrt{(\lambda^{n,a}_\alpha)^2 + 1/4}}{4\lambda^{n,a}_\alpha} = \frac{1}{4^n}\left(\frac{\lambda^n_\alpha}{\sqrt{n^2 + (\lambda^n_\alpha)^2}} \prod_{k=0}^{\lfloor n/2\rfloor} \frac{(2k+1)^2 + (\lambda^n_\alpha)^2}{(2k)^2 + (\lambda^n_\alpha)^2}\right) \tag{19}$$

while for even n one has

$$\prod_{a=1}^{n} \frac{\sqrt{(\lambda^{n,a}_\alpha)^2 + 1/4}}{4\lambda^{n,a}_\alpha} = \frac{1}{4^n}\left(\frac{\sqrt{n^2 + (\lambda^n_\alpha)^2}}{\lambda^n_\alpha} \prod_{k=0}^{\lfloor n/2\rfloor - 1} \frac{(2k)^2 + (\lambda^n_\alpha)^2}{(2k+1)^2 + (\lambda^n_\alpha)^2}\right) \tag{20}$$

A. Zero-momentum strings

Extra divergencies arise when there are strings with zero center. We analyze first the case with one 1-string and a 3-string. It is very interesting to notice that one obtains