

# **The Generalized Gibbs Ensemble in the Heisenberg spin chain: A Hilbert space Monte Carlo approach**

Vincenzo Alba<sup>1</sup> and Maurizio Fagotti<sup>2</sup>

<sup>1</sup>*International School for Advanced Studies (SISSA), Via Bonomea 265, 34136, Trieste, Italy, INFN, Sezione di Trieste*

<sup>2</sup>*Département de Physique, Ecole normale supérieure, CNRS, 24 rue Lhomond, 75005 Paris, France*

(Dated: June 18, 2015)

## **I. INTRODUCTION**

---

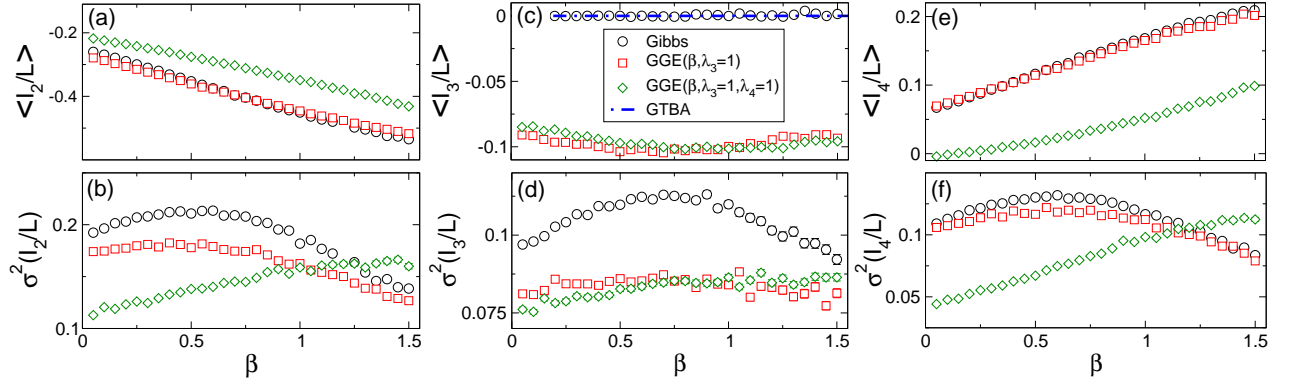


FIG. 1. The Generalized Gibbs Ensemble (GGE) for the finite-size Heisenberg spin chain with  $L = 16$  sites. The GGE is constructed including the conserved charges  $I_2, I_3, I_4$ . The corresponding Lagrange multipliers are denoted as  $\lambda_2, \lambda_3, \lambda_4$ . Here  $I_2$  is the Hamiltonian and  $\lambda_2 \equiv \beta$  the inverse temperature. (a) The GGE average  $\langle I_2/L \rangle$  of  $I_2/L$  plotted as a function of  $\beta$ . The data are obtained using the Hilbert space Monte Carlo approach described in the manuscript. The different symbols correspond to GGEs with different fixed values of  $\lambda_3$  and  $\lambda_4$ . The circles correspond to the Gibbs ensemble. (b) The fluctuations  $\sigma^2(I_2)/L \equiv \langle (I_2/L)^2 \rangle - \langle I_2/L \rangle^2$  as function of  $0 \leq \beta \leq 1.5$ . (c)(d) and (e)(f): Same as in (a)(b) for  $I_3$  and  $I_4$ , respectively. In all panels the dash-dotted lines are the analytical results obtained using the Generalized Thermodynamic Bethe Ansatz (GTBA) approach.

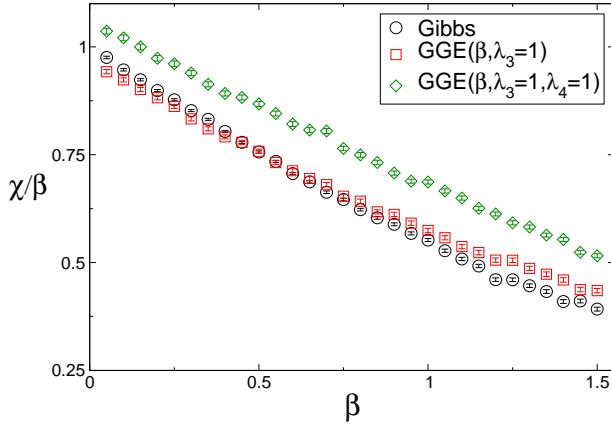


FIG. 2. The GGE average of the spin susceptibility  $\chi$  in the Heisenberg chain with  $L = 16$  sites:  $\chi/\beta$  plotted versus the inverse temperature  $\beta$ . The GGE is constructed including the conserved charges  $I_2, I_3, I_4$ . The corresponding Lagrange multipliers are denoted as  $\lambda_2, \lambda_3$ , and  $\lambda_4$ . Here  $I_2$  is the Hamiltonian and  $\lambda_2 = \beta$ . The different symbols are Monte Carlo data for GGEs with different values of  $\lambda_3, \lambda_4$ . Notice that the circles correspond to the Gibbs ensemble.

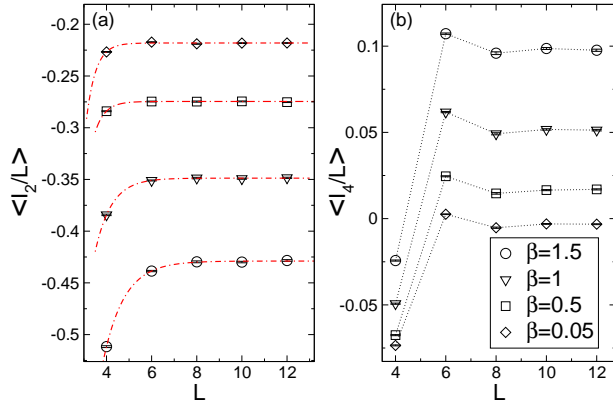


FIG. 3. Finite-size scaling of the GGE averages in the Heisenberg chain. Here the GGE is constructed using the conserved charges  $I_2, I_3, I_4$ , with associated Lagrange multipliers  $\lambda_2, \lambda_3, \lambda_4$ .  $I_2$  is the Hamiltonian and  $\lambda_2 \equiv \beta$  the inverse temperature. Here we fix  $\lambda_3 = \lambda_4 = 1$ . (a) The GGE average  $\langle I_2/L \rangle$  (Monte Carlo data) plotted versus the chain size  $L$ . Different symbols correspond to different values of  $\beta$ . The dash-dotted lines are fits to an exponential behavior. (b) Same as in (a) for  $I_4$ .