# The logarithmic negativity in random spin chains

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## I. INTRODUCTION

## II. THE DISORDERED $XX$ CHAIN

The disordered $XX$ chain is defined by the Hamiltonian

$$\mathcal{H}_{XX}=\sum_{i=1}^{L-1}J_i(S_i^xS_{i+1}^x+S_i^yS_{i+1}^y)+h\sum_{i=1}^{L}S_i^z, \quad (1)$$

with $S_i^{x,y,z}\equiv\sigma_i^{x,y,z}/2$, $\sigma_i^\alpha$ being the Pauli matrices acting on site $i$. For periodic boundary conditions one has an extra term in Eq. (1) connecting site $L$ with site 1. Hereafter we fix $h=0$ and choose $J_i$ uniformily distributed in $[0,1]$. After the Jordan-Wigner transformation

$$c_i=\Big(\prod_{m=1}^{i-1}\sigma_m^z\Big)\frac{\sigma_i^x-i\sigma_i^y}{2}, \quad (2)$$

(1) is recast in the free-fermionic form

$$\mathcal{H}_{XX}=\frac{1}{2}\sum_{i=1}^{L-1}J_i(c_i^\dagger c_{i+1}+c_{i+1}^\dagger c_i)+\frac{h}{2}\sum_{i=1}^{L-1}c_i^\dagger c_i, \quad (3)$$

with $c_i$ spinless fermionic operators satisfying the canonical anticommutation relations $\{c_m,c_n^\dagger\}=\delta_{m,n}$. The mapping between Eq. (1) and Eq. (3) is exact apart from boundary terms (that we neglect here) giving a vanishing contribution (as $1/L$) to physical quantities in the large chain limit.

By imposing that the single-particle eigenstates $|\Psi_q\rangle$ (with $q$ an integer labelling the different eigenstates) of (3) are of the form

$$\eta_q^\dagger|0\rangle\equiv|\Psi_q\rangle=\sum_i\Phi_q(i)c_i^\dagger|0\rangle, \quad (4)$$

with $|0\rangle$ the vacuum and $\Phi_q(i)$ the eigenstate amplitudes. The Schrödinger equation gives the equation for $\Phi_q(i)$ as

$$(J_i\Phi_q(i+1)+J_{i-1}\Phi_q(i-1))/2=\epsilon_q\Phi_q(i), \quad i=1,\dots,L, \quad (5)$$

and $J_L=0$. Eq. (5) corresponds to finding the eigenvectors of the banded matrix $T=(J_j\delta_{i,j+1}+J_{j-1}\delta_{i,j-1})/2$. The eigenvalues corresponds to the single-particle eigenergies $\epsilon_q$.

The ground state $|GS\rangle$ of (1) is obtained by filling all the negative modes $\epsilon_q$ as

$$|GS\rangle=\eta_{q_M}^\dagger\eta_{q_{M-1}}^\dagger\cdots\eta_{q_1}^\dagger|0\rangle. \quad (6)$$

One has the anticommutation relations

$$\{\eta_q^\dagger,c_j^\dagger\}=\{\eta_q,c_j\}=0 \quad (7)$$

and

$$\{\eta_q^\dagger,c_j\}=\Phi_q(j)\delta_{k,j}, \quad \{\eta_q,c_j^\dagger\}=\Phi^*(j)\delta_{k,j} \quad (8)$$

These imply that the expectation value of the two-point function in a generic eigenstate of (1) is given as

$$\langle c_i^\dagger c_j\rangle=\sum_q\Phi_q^*(i)\Phi_q(j), \quad (9)$$

where the sum if over the $q$ single-particle excitations forming the eigenstate.

One can show that the eigenvalus of the matrix $T$ are organized in pairs with opposite sign. Given the components $\Phi_1(i)$ of the eigenvector with eigenvalue $\epsilon_q$, the components of the eigenvector with $-\epsilon_q$ are given as $(-1)^{i+1}\Phi_q(i)$. This also implies that the ground state of (1) is in the sector with $M=L/2$ fermions.

[1] F. Iglói, R. Juhász, and H. Rieger, Phys. Rev. B **61**, 11552 (2000).