

Programming Languages Design Workshop

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May 02, 2025

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Chapter 1: Introduction to Programming Language Design

Section 1: What is a Programming Language?

A **programming language** is a formal language comprising a set of strings (instructions) that produce various kinds of machine output. Programming languages are used in computer programming to implement algorithms.

Key Characteristics of Programming Languages

- **Syntax:** The form or structure of the expressions, statements, and program units
- **Semantics:** The meaning of the expressions, statements, and program units
- **Type System:** The set of types and rules for how types are assigned to various constructs in the language
- **Runtime Model:** How the language executes on a computer, including memory management
- **Standard Library:** Common functionality provided out of the box

Programming Languages vs. Natural Languages

Programming languages differ from natural languages in several important ways:

1. **Precision:** Programming languages are designed to be precise and unambiguous
2. **Vocabulary:** Programming languages have a limited vocabulary defined by the language specification
3. **Grammar:** Programming languages have a strict, formal grammar with precise rules
4. **Evolution:** Programming languages evolve through explicit design decisions, not organic usage
5. **Purpose:** Programming languages are designed to instruct machines, not primarily for human communication

Section 2: The Importance of Programming Language Design

Why should we care about programming language design?

Programming Languages Shape How We Think

Programming languages are not just tools for instructing computers; they are frameworks for human thinking. Different languages emphasize different concepts and approaches:

- **Imperative languages** (C, Pascal) focus on step-by-step instructions
- **Functional languages** (Haskell, Lisp) emphasize expressions and function composition
- **Object-oriented languages** (Java, C++, Python) organize code around objects and their interactions
- **Logic languages** (Prolog) express programs as logical relations

Language Design Affects Software Quality

The design of a programming language can significantly impact:

- **Reliability:** How easy is it to write correct code?
- **Maintainability:** How easy is it to understand and modify existing code?
- **Performance:** How efficiently can the code be executed?
- **Security:** How easily can programmers avoid security vulnerabilities?
- **Developer Productivity:** How quickly can developers write and debug code?

The Evolution of Programming Languages

Programming languages have evolved dramatically over time, reflecting changes in hardware, software engineering practices, and problem domains:

- **1950s:** Assembly languages and early high-level languages (FORTRAN, LISP)
- **1960s:** ALGOL, COBOL, and structured programming concepts
- **1970s:** C, Pascal, and the rise of procedural programming
- **1980s:** C++, Ada, and the adoption of object-oriented programming

- **1990s:** Java, Python, Ruby, and the focus on portability and productivity
- **2000s:** C#, JavaScript frameworks, and web-centric languages
- **2010s:** Go, Rust, Swift, and the focus on safety and concurrency
- **2020s:** Continued evolution with AI assistance, type inference improvements, and more

Section 3: Modern Language Design Principles

What principles guide the design of modern programming languages?

Abstraction

Abstraction is the process of removing details to focus on the essential features of a concept or object.

Examples in programming languages: - Functions abstract away implementation details - Classes abstract data and behavior - Interfaces abstract expected behaviors - Modules abstract related functionality

Expressiveness

Expressiveness refers to how easily and concisely a language can express computational ideas.

Factors that contribute to expressiveness: - Rich set of operators and built-in functions - Support for higher-order functions - Pattern matching - Concise syntax for common operations

Safety

Safety features help prevent programmers from making mistakes or make it easier to find and fix errors.

Safety mechanisms in modern languages: - Static type checking - Bounds checking - Memory safety guarantees - Exception handling systems - Null safety features

Performance

Performance considerations affect how efficiently a language can be implemented and executed.

Performance factors: - Compilation vs. interpretation - Memory management approach - Static vs. dynamic typing - Optimization opportunities - Support for concurrency and parallelism

Consistency

Consistency in language design makes languages easier to learn and use correctly.

Consistency principles: - Similar concepts should have similar syntax - Minimal special cases - Orthogonal features (features that can be used in any combination) - Principle of least surprise (intuitive behavior)

Section 4: Using Python to Explore Programming Language Concepts

Why use Python for studying programming language design?

Python's Suitability for Language Implementation

Python is well-suited for implementing language interpreters and exploring language concepts:

- **Readability:** Python's clean syntax makes interpreter code easier to understand
- **High-level constructs:** Python provides lists, dictionaries, and other structures useful for language implementation
- **Dynamic typing:** Simplifies working with diverse language constructs
- **Rich standard library:** Includes parsing tools, regular expressions, and other useful utilities
- **Interactive development:** Makes experimenting with language features easier

Python 3.10+ Features Relevant to Language Design

Recent Python versions have introduced features that make it particularly interesting for PL experiments:

- **Type hints:** Allows for static type checking while maintaining dynamic execution
- **Pattern matching:** Provides elegant structural decomposition similar to functional languages
- **Dataclasses:** Simplifies creating data-carrying classes with minimal boilerplate
- **Functional programming tools:** Map, filter, reduce, lambdas, and comprehensions
- **AST module:** Allows inspection and manipulation of Python's abstract syntax tree

Section 5: Implementing Language Features in Python

Let's explore how we can implement core language components in Python.

Representing Syntax: Abstract Syntax Trees (ASTs)

An abstract syntax tree (AST) is a tree representation of the abstract syntactic structure of source code. Here's a simple example of representing expressions:

```
from dataclasses import dataclass
from typing import Union, List

# Define the node types
@dataclass
class Number:
    value: float

@dataclass
class Variable:
    name: str

@dataclass
class BinaryOp:
    left: 'Expr'
    operator: str
    right: 'Expr'

# Define the expression type
Expr = Union[Number, Variable, BinaryOp]

# Example: 2 + (x * 3)
expr = BinaryOp(
    left=Number(2.0),
    operator='+',
    right=BinaryOp(
        left=Variable('x'),
        operator='*',
        right=Number(3.0)
    )
)
```

Implementing an Evaluator

The evaluator traverses the AST and computes the result. For example:

```
def evaluate(expr: Expr, environment: dict = None) -> float:
    """Evaluate an expression in the given environment."""
    if environment is None:
        environment = {}

    if isinstance(expr, Number):
        return expr.value
    elif isinstance(expr, Variable):
        if expr.name not in environment:
            raise NameError(f"Variable '{expr.name}' not defined")
        return environment[expr.name]
    elif isinstance(expr, BinaryOp):
        left_val = evaluate(expr.left, environment)
        right_val = evaluate(expr.right, environment)

        if expr.operator == '+':
            return left_val + right_val
        elif expr.operator == '-':
            return left_val - right_val
        elif expr.operator == '*':
            return left_val * right_val
        elif expr.operator == '/':
            return left_val / right_val
        else:
            raise ValueError(f"Unknown operator: {expr.operator}")
```

Pattern Matching for AST Processing

Python 3.10's pattern matching provides a more elegant way to implement evaluators:

```
def evaluate_with_match(expr: Expr, environment: dict = None) -> float:
    """Evaluate an expression using pattern matching."""
    if environment is None:
        environment = {}

    match expr:
        case Number(value):
            return value
        case Variable(name):
            if name not in environment:
                raise NameError(f"Variable '{name}' not defined")
            return environment[name]
        case BinaryOp(left, operator, right):
            left_val = evaluate_with_match(left, environment)
            right_val = evaluate_with_match(right, environment)

            match operator:
                case '+':
                    return left_val + right_val
                case '-':
                    return left_val - right_val
                case '*':
                    return left_val * right_val
                case '/':
```



```

        return left_val / right_val
    case _:
        raise ValueError(f"Unknown operator: {operator}")

```

Simple Type Checking

We can implement basic type checking for our language:

```

from enum import Enum, auto
from dataclasses import dataclass
from typing import Dict

class Type(Enum):
    NUMBER = auto()
    BOOLEAN = auto()
    STRING = auto()

def type_check(expr: Expr, type_env: Dict[str, Type]) -> Type:
    """Determine the type of an expression."""
    match expr:
        case Number(_):
            return Type.NUMBER
        case Variable(name):
            if name not in type_env:
                raise TypeError(f"Variable '{name}' not defined")
            return type_env[name]
        case BinaryOp(left, operator, right):
            left_type = type_check(left, type_env)
            right_type = type_check(right, type_env)

            # Type checking rules for operators
            if operator in ['+', '-', '*', '/']:
                if left_type != Type.NUMBER or right_type != Type.NUMBER:
                    raise TypeError(f"Operator '{operator}' requires number operands")
                return Type.NUMBER
            elif operator in ['==', '!=', '<', '>', '<=', '>=']:
                if left_type != right_type:
                    raise TypeError("Comparison operators require operands of the same type")
                return Type.BOOLEAN
            else:
                raise ValueError(f"Unknown operator: {operator}")

```

Section 6: Course Structure

This course will introduce you to programming language design concepts through hands-on implementation in Python.

Course Topics

Throughout this course, we will cover:

1. **Language Syntax and Semantics**
 - Parsing and lexical analysis
 - Abstract syntax trees
 - Operational semantics
2. **Type Systems**

- Static vs. dynamic typing
 - Type inference
 - Polymorphism
 - Advanced type features (generics, algebraic data types)
3. **Language Features**
 - Functions and closures
 - Pattern matching
 - Object-oriented programming
 - Concurrency models
 - Memory management approaches
 4. **Interpreter and Compiler Implementation**
 - Building a simple interpreter
 - Environment and scope
 - Evaluation strategies
 - Introduction to compilation concepts

Projects and Exercises

The course will include:

- Regular programming exercises to reinforce concepts
- Progressive development of a language interpreter
- Exploration of existing language implementations
- Analysis of language design trade-offs

Section 7: Prerequisites and Setup

Knowledge Prerequisites

To get the most out of this course, you should have:

- Basic Python programming experience
- Understanding of fundamental programming concepts (variables, functions, control flow)
- Familiarity with basic data structures (lists, dictionaries, trees)
- Interest in how programming languages work “under the hood”

No prior experience with compiler or interpreter development is required.

Python Environment Setup

To follow along with the course examples and exercises:

1. **Install Python 3.10 or later**
 - Required for pattern matching and other modern features
2. **Recommended development tools**
 - Visual Studio Code with Python extension
 - PyCharm
 - Jupyter Notebook/Lab for interactive exploration
3. **Useful libraries**
 - mypy for static type checking
 - pytest for testing your implementations

Section 8: Additional Resources

Books on Programming Language Design

- “Crafting Interpreters” by Robert Nystrom

- **“Programming Language Pragmatics”** by Michael Scott
- **“Types and Programming Languages”** by Benjamin Pierce
- **“Concepts of Programming Languages”** by Robert Sebesta
- **“Structure and Interpretation of Computer Programs”** by Abelson and Sussman

Online Resources

- [Python Documentation](#)
- [Python Type Hints](#)
- [Pattern Matching in Python 3.10](#)
- [The AST Module](#)
- [Building a Simple Interpreter](#)

Chapter 2: Types and Pattern Matching in Python

Section 1: Python's Type System

What are Type Annotations?

Python's type system allows developers to add optional type hints to variables, function parameters, and return values. These annotations help catch errors early, improve code documentation, and enhance IDE support without changing the runtime behavior of the code.

```
def greet(name: str) -> str:
    return f"Hello, {name}!"
```

Type annotations are part of the Python Enhancement Proposal (PEP) 484 and have been continuously improved in subsequent PEPs. They provide a way to make Python code more robust through static type checking, although Python remains dynamically typed at runtime.

Basic Types in Python

Python provides several built-in types for annotations:

- **Primitive types:** `int`, `float`, `bool`, `str`
- **Collection types:** `list`, `tuple`, `dict`, `set`

Example from `types_and_matching.py`:

```
my_tuple: tuple[int, str, float] = (1, "hello", 1.0)
my_list: list[int] = [1, 2, 3, 4, 5]
```

These annotations tell the type checker that `my_tuple` is a 3-element tuple containing an integer, a string, and a float, while `my_list` is a list containing only integers.

Generic Types

Generic types allow you to create reusable, type-safe components. In Python 3.12+, the syntax for generic classes uses square brackets:

```
class Stack[T]:
    def __init__(self) -> None:
        self.items: list[T] = []

    def push(self, item: T) -> None:
        self.items.append(item)

    def pop(self) -> T | None:
        return self.items.pop() if self.items else None
```

In this example from our code: - `T` is a type parameter representing any type - `Stack[T]` is a generic class that can be specialized for specific types - `list[T]` indicates a list containing elements of type `T`

To use this generic class:

```
stack_1 = Stack[int]() # A stack of integers
stack_1.push(1) # Valid
stack_1.push("hello") # Type error: expected int, got str
```

Union Types and Optional Values

Union types allow a variable to have multiple possible types, expressed using the `|` operator (introduced in Python 3.10):

```
def process_data(data: int | str) -> None:
    # Can handle both integers and strings
    pass
```

From our example code:

```
def pop(self) -> T | None: # Use | for union types
    return self.items.pop() if self.items else None
```

This indicates that the `pop` method returns either a value of type `T` or `None` if the stack is empty.

Type Aliases

Type aliases help simplify complex type annotations:

```
type JsonData = dict[str, int | str | list | dict]
type A = int | dict[str, A] # Recursive type
```

In `types_and_matching.py`, we use type aliases for recursive types:

```
type MyBaseList[T] = None | MyList[T]
type Expr = int | Sum
```

These aliases make the code more readable and allow for recursive type definitions.

Literal Types

Literal types restrict values to specific constants:

```
y: Literal["hello", "world"] = "hello" # Valid
z: Literal[1, 2, 3] = 9 # Type error
```

From our example code:

```
@dataclass
class Human:
    name: str
    drivingLicense: Literal[True, False]
```

This constrains the `drivingLicense` attribute to be either `True` or `False` only.

Section 2: Structural Pattern Matching

Introduction to Pattern Matching

Introduced in Python 3.10, the `match` statement provides powerful pattern matching capabilities, similar to switch statements in other languages but with more expressive power:

```
def describe(value):
    match value:
        case 0:
            return "Zero"
        case int(x) if x > 0:
            return "Positive integer"
        case str():
            return "String"
        case _:
            return "Something else"
```

Pattern matching allows for more concise and readable code, especially when dealing with complex data structures and multiple conditions.

Basic Patterns

Basic patterns match against simple values and types:

```
match x:
    case 0:
        print("Zero")
    case int():
        print("Integer")
    case str():
        print("String")
    case _: # Wildcard pattern
        print("Something else")
```

The `_` pattern is a wildcard that matches anything and is often used as a catch-all case.

Sequence Patterns

Sequence patterns match against sequence types like lists and tuples:

```
def process_lst(lst: list[int]) -> None:
    match lst:
        case []:
            print("List: empty")
        case [head, *tail]:
            print(f"List: head {head}, tail {tail}")
```

This example shows destructuring a list into its head (first element) and tail (remaining elements), demonstrating how pattern matching facilitates recursive list processing.

Class Patterns and Attribute Matching

Pattern matching works particularly well with dataclasses:

```
def greet(person: Person | str) -> None:
    match person:
        case Person(name="Alice", age=25):
            print("Hello Alice!")
        case Person(name=x, age=y):
            print(f"Hello {x} of age {y}!")
        case str():
            print(f"Hello {person}!")
```

This example shows matching against specific attribute values and binding attributes to variables.

Complex Pattern Matching Examples

Recursive Pattern Matching Pattern matching excels at handling recursive data structures:

```
def sum_list_2(lst: MyBaseList[int]) -> int:
    match lst:
        case None:
            return 0
        case MyList(head=x, tail=y):
            return x + sum_list_2(y)
```

This function processes a custom linked list structure using pattern matching to handle the base case (`None`) and recursive case elegantly.

Parsing Expressions Pattern matching can implement simple interpreters:

```
def eval_expr(expr: Expr) -> int:
    match expr:
        case int(x):
            return x
        case Sum(left=x, right=y):
            return eval_expr(x) + eval_expr(y)
```

This code evaluates a simple arithmetic expression tree, showing how pattern matching simplifies traversal of complex structures.

Section 3: Combining Types and Pattern Matching

Algebraic Data Types in Python

Python can implement algebraic data types (ADTs) using classes, dataclasses, and union types:

Sum Types (Tagged Unions) Sum types represent values that could be one of several alternatives:

```
@dataclass
class Human:
    name: str
    drivingLicense: Literal[True, False]

@dataclass
class Dog:
    name: str
    kind: DogKind
    colour: Literal["brown", "black", "white"]

type Record = Human | Dog
```

This example defines two distinct record types (`Human` and `Dog`) and a union type `Record` that can be either of them.

Pattern Matching with Sum Types Pattern matching works seamlessly with sum types:

```
def describe_person(person: Human | Dog) -> str:
    match person:
        case Human(name=name, drivingLicense=True):
            return f"Person {name} has a driving license"
        case Human(name=name, drivingLicense=False):
            return f"Person {name} does not have a driving license"
        case Dog(name=name, kind=kind, colour=colour):
            return f"Dog {name} is a {kind} and has a {colour} colour"
```

This function handles different record types with specific patterns for each case.

Product Types Product types represent combinations of values:

```
@dataclass
class Person:
    name: str
    age: int
```

A dataclass like `Person` is a product type, representing a combination of a string and an integer.

Type Checking and Pattern Matching

Type checkers like mypy can catch errors in pattern matching code:

```
def describe_person_2(person: Record) -> str:
    if isinstance(person, Human):
        return person.name + person.colour # Type error: Human has no attribute 'colour'
    else:
        return person.name + person.kind
```

Type checking helps identify incorrect attribute access, which might otherwise lead to runtime errors.

Section 4: Applications and Best Practices

When to Use Type Annotations

Type annotations are particularly valuable in: - Large codebases with multiple developers - APIs and libraries that will be used by others - Performance-critical code where type-specific optimizations matter - Complex data processing pipelines

Domain-specific languages, interpreters, ADTs!!

When to Use Pattern Matching

Pattern matching excels at: - Processing complex recursive data structures - Implementing interpreters and compilers - Handling case-based logic with destructuring - Processing structured data like JSON or ASTs

Best Practices for Type Annotations

1. Be consistent with type annotations across your codebase
2. Use type aliases for complex or repetitive type expressions
3. Leverage tools like mypy, pyright, or pylance for static type checking
4. Balance between type precision and code readability
5. Document non-obvious type constraints with comments

Best Practices for Pattern Matching

1. Order cases from most specific to most general
2. Use the wildcard pattern (`_`) as the last case
3. Consider using guard clauses for complex conditions
4. Break complex pattern matching into smaller functions
5. Leverage destructuring to avoid redundant variable assignments

Exercises

- Consider the expression evaluator written in Lecture 01. Turn it into a parser that converts a string of the form “x op y op z ...” separated by spaces into an AST in the sense of Lecture 02. Concatenate the new parser and the evaluation function that takes AST as input to define a mini-interpreter.
- Add the “==” boolean operator to the AST and the evaluator.

Additional Resources

- [PEP 484 – Type Hints](#)
- [PEP 585 – Type Hinting Generics In Standard Collections](#)
- [PEP 604 – Allow writing union types as X | Y](#)
- [PEP 634 – Structural Pattern Matching: Specification](#)
- [PEP 636 – Structural Pattern Matching: Tutorial](#)

- [Mypy Type Checker Documentation](#)
- [Real Python: Python Type Checking](#)
- [Real Python: Structural Pattern Matching in Python](#)

Chapter 3: Building a Mini Interpreter

Section 1: Introduction to Interpreters

An interpreter is a program that executes source code directly, without requiring compilation to machine code. Let's explore how interpreters work and how to build a simple one in Python.

The Role of Interpreters

Interpreters serve several important purposes in programming language implementation:

1. **Direct Execution:** Execute source code without a separate compilation step
2. **Immediate Feedback:** Provide instant results for interactive programming
3. **Portability:** Run on any platform that supports the interpreter
4. **Simplicity:** Often easier to implement than full compilers
5. **Debugging:** Allow for interactive debugging and inspection

Languages like Python, JavaScript, and Ruby primarily use interpreters, while others like Java use a hybrid approach with compilation to bytecode followed by interpretation.

Components of an Interpreter

A typical interpreter includes the following components:

1. **Lexer (Tokenizer):** Converts source code text into tokens
2. **Parser:** Transforms tokens into an abstract syntax tree (AST)
3. **Evaluator:** Executes the AST to produce results
4. **Environment:** Stores variables and their values
5. **Error Handler:** Manages and reports errors

We'll implement each of these components in our mini interpreter.

Section 2: Lexical Analysis

The first step in interpreting code is breaking it down into tokens - the smallest meaningful units in the language.

Tokens and Lexical Structure

Tokens are the building blocks of a language, similar to words in natural language. Common token types include:

- **Keywords:** Reserved words with special meaning (e.g., `if`, `while`)
- **Identifiers:** Names given to variables, functions, etc.
- **Literals:** Constant values (numbers, strings, booleans)
- **Operators:** Symbols that perform operations (`+`, `-`, `*`, `/`)
- **Punctuation:** Symbols that structure the code (`;`, `,`, `{}`, `()`)

Implementing a Lexer

Our lexer will convert a string of source code into a list of tokens:

```
from dataclasses import dataclass
from enum import Enum, auto
from typing import List, Optional

# Define token types
class TokenType(Enum):
    NUMBER = auto()
    PLUS = auto()
```

```

MINUS = auto()
MULTIPLY = auto()
DIVIDE = auto()
LPAREN = auto()
RPAREN = auto()
EOF = auto() # End of file

@dataclass
class Token:
    type: TokenType
    value: Optional[str] = None

def tokenize(text: str) -> List[Token]:
    tokens = []
    i = 0

    while i < len(text):
        char = text[i]

        # Skip whitespace
        if char.isspace():
            i += 1
            continue

        # Process numbers
        if char.isdigit():
            num = ""
            while i < len(text) and text[i].isdigit():
                num += text[i]
                i += 1
            tokens.append(Token(TokenType.NUMBER, num))
            continue

        # Process operators
        if char == '+':
            tokens.append(Token(TokenType.PLUS))
        elif char == '-':
            tokens.append(Token(TokenType.MINUS))
        elif char == '*':
            tokens.append(Token(TokenType.MULTIPLY))
        elif char == '/':
            tokens.append(Token(TokenType.DIVIDE))
        elif char == '(':
            tokens.append(Token(TokenType.LPAREN))
        elif char == ')':
            tokens.append(Token(TokenType.RPAREN))
        else:
            raise ValueError(f"Unexpected character: {char}")

        i += 1

    tokens.append(Token(TokenType.EOF))
    return tokens

```

Testing the Lexer

Let's test our lexer with a simple arithmetic expression:

```
def test_lexer():
    source = "3 + 4 * (2 - 1)"
    tokens = tokenize(source)

    expected = [
        Token(TokenType.NUMBER, "3"),
        Token(TokenType.PLUS),
        Token(TokenType.NUMBER, "4"),
        Token(TokenType.MULTIPLY),
        Token(TokenType.LPAREN),
        Token(TokenType.NUMBER, "2"),
        Token(TokenType.MINUS),
        Token(TokenType.NUMBER, "1"),
        Token(TokenType.RPAREN),
        Token(TokenType.EOF)
    ]

    assert tokens == expected, f"Expected {expected}, got {tokens}"
    print("Lexer test passed!")

test_lexer()
```

Section 3: Parsing and Abstract Syntax Trees

Once we have tokens, we need to organize them into a structured representation of the program - an Abstract Syntax Tree (AST).

Understanding Abstract Syntax Trees

An AST is a tree representation of the abstract syntactic structure of source code. Each node in the tree represents a construct in the source code.

For example, the expression $3 + 4 * 2$ would be represented as:

```

      (+)
     /  \
    3    (*)
       /  \
      4    2
```

The tree captures the structure and precedence of operations.

Defining AST Nodes

Let's define classes for our AST nodes:

```
from dataclasses import dataclass
from typing import Union, Optional

@dataclass
class Number:
    value: float

@dataclass
```

```

class BinaryOp:
    left: 'Expression'
    operator: str
    right: 'Expression'

# Define our Expression type
Expression = Union[Number, BinaryOp]

```

Implementing a Recursive Descent Parser

A recursive descent parser is a top-down parser that uses a set of recursive procedures to process the input:

```

class Parser:
    def __init__(self, tokens: List[Token]):
        self.tokens = tokens
        self.current = 0

    def peek(self) -> Token:
        return self.tokens[self.current]

    def previous(self) -> Token:
        return self.tokens[self.current - 1]

    def advance(self) -> Token:
        if not self.is_at_end():
            self.current += 1
        return self.previous()

    def is_at_end(self) -> bool:
        return self.peek().type == TokenType.EOF

    def check(self, type: TokenType) -> bool:
        if self.is_at_end():
            return False
        return self.peek().type == type

    def match(self, *types: TokenType) -> bool:
        for type in types:
            if self.check(type):
                self.advance()
                return True
        return False

    def consume(self, type: TokenType, message: str) -> Token:
        if self.check(type):
            return self.advance()
        raise Exception(message)

    def parse(self) -> Expression:
        return self.expression()

    def expression(self) -> Expression:
        return self.term()

```

```

def term(self) -> Expression:
    expr = self.factor()

    while self.match(TokenType.PLUS, TokenType.MINUS):
        operator = self.previous().type
        right = self.factor()
        expr = BinaryOp(
            expr,
            "+" if operator == TokenType.PLUS else "-",
            right
        )

    return expr

def factor(self) -> Expression:
    expr = self.primary()

    while self.match(TokenType.MULTIPLY, TokenType.DIVIDE):
        operator = self.previous().type
        right = self.primary()
        expr = BinaryOp(
            expr,
            "*" if operator == TokenType.MULTIPLY else "/",
            right
        )

    return expr

def primary(self) -> Expression:
    if self.match(TokenType.NUMBER):
        return Number(float(self.previous().value))

    if self.match(TokenType.LPAREN):
        expr = self.expression()
        self.consume(TokenType.RPAREN, "Expect ')' after expression.")
        return expr

    raise Exception(f"Unexpected token: {self.peek()}")

```

This parser implements the following grammar:

```

expression → term
term       → factor (( "+" | "-" ) factor)*
factor     → primary (( "*" | "/" ) primary)*
primary    → NUMBER | "(" expression ")"

```

Testing the Parser

Let's test our parser with the same expression:

```

def test_parser():
    tokens = tokenize("3 + 4 * 2")
    parser = Parser(tokens)
    ast = parser.parse()

```

```

# Expected: BinaryOp(Number(3), "+", BinaryOp(Number(4), "*", Number(2)))
expected = BinaryOp(
    Number(3),
    "+",
    BinaryOp(
        Number(4),
        "*",
        Number(2)
    )
)

assert ast == expected, f"Expected {expected}, got {ast}"
print("Parser test passed!")

test_parser()

```

Section 4: Evaluating Expressions

Now that we have an AST, we can evaluate it to produce a result.

The Evaluation Process

Evaluation is the process of computing the result of an expression. It typically involves:

1. Walking the AST recursively
2. Computing the value of each node based on its type and children
3. Combining results according to the language semantics

Implementing an Evaluator

For our mini interpreter, we'll implement a simple evaluator that computes arithmetic expressions:

```

def evaluate(expr: Expression) -> float:
    """Evaluate an expression and return its value."""
    match expr:
        case Number(value):
            return value
        case BinaryOp(left, operator, right):
            left_value = evaluate(left)
            right_value = evaluate(right)

            match operator:
                case "+":
                    return left_value + right_value
                case "-":
                    return left_value - right_value
                case "*":
                    return left_value * right_value
                case "/":
                    if right_value == 0:
                        raise ZeroDivisionError("Division by zero")
                    return left_value / right_value
                case _:
                    raise ValueError(f"Unknown operator: {operator}")

```

Testing the Evaluator

Let's test our evaluator with a few expressions:

```
def test_evaluator():
    expressions = [
        ("3 + 4", 7),
        ("3 * 4", 12),
        ("10 - 2", 8),
        ("20 / 5", 4),
        ("3 + 4 * 2", 11), # Tests operator precedence
        ("(3 + 4) * 2", 14), # Tests parentheses
    ]

    for source, expected in expressions:
        tokens = tokenize(source)
        parser = Parser(tokens)
        ast = parser.parse()
        result = evaluate(ast)

        assert result == expected, f"For '{source}', expected {expected}, got {result}"

    print("Evaluator tests passed!")

test_evaluator()
```

Section 5: Putting It All Together

Now we'll combine our lexer, parser, and evaluator into a complete mini interpreter.

The Full Interpreter

```
def interpret(source: str) -> float:
    """Interpret a source string and return the result."""
    try:
        tokens = tokenize(source)
        parser = Parser(tokens)
        ast = parser.parse()
        return evaluate(ast)
    except Exception as e:
        print(f"Error: {e}")
        return float('nan') # Return NaN for errors

def run_repl():
    """Run a simple Read-Eval-Print Loop."""
    print("Mini Interpreter REPL")
    print("Enter expressions to evaluate, or 'exit' to quit")

    while True:
        try:
            source = input("> ")
            if source.lower() in ("exit", "quit"):
                break

            result = interpret(source)
```



```

        print(f"= {result}")
    except KeyboardInterrupt:
        break
    except Exception as e:
        print(f"Error: {e}")

    print("Goodbye!")

if __name__ == "__main__":
    run_repl()

```

Example Usage

Using our interpreter:

```

Mini Interpreter REPL
Enter expressions to evaluate, or 'exit' to quit
> 3 + 4
= 7.0
> 3 * (4 + 2)
= 18.0
> 10 / (2 - 2)
Error: Division by zero
> (3 + 4) * (5 - 2)
= 21.0
> exit
Goodbye!

```

Section 6: Extending the Interpreter

Our mini interpreter is very basic, but we can extend it with more features.

Adding Variables

To add variable support, we need:

1. An environment to store variable bindings
2. New AST nodes for variable references and assignments
3. Updated parsing rules
4. Updated evaluation logic

Let's implement a simple environment first:

```

@dataclass
class Environment:
    values: dict[str, float] = None

    def __post_init__(self):
        if self.values is None:
            self.values = {}

    def define(self, name: str, value: float) -> None:
        """Define a variable with the given name and value."""
        self.values[name] = value

    def get(self, name: str) -> float:

```

```

"""Get the value of a variable."""
    if name in self.values:
        return self.values[name]
    raise ValueError(f"Undefined variable: {name}")

```

New AST Nodes for Variables

```

@dataclass
class Variable:
    name: str

@dataclass
class Assignment:
    name: str
    value: 'Expression'

# Update Expression type
Expression = Union[Number, BinaryOp, Variable, Assignment]

```

Updating the Parser

```

class Parser:
    # ... existing code ...

    def parse(self) -> Expression:
        return self.assignment()

    def assignment(self) -> Expression:
        expr = self.expression()

        if self.match(TokenType.EQUAL):
            if isinstance(expr, Variable):
                value = self.assignment()
                return Assignment(expr.name, value)
            raise Exception("Invalid assignment target")

        return expr

    # ... update tokenize() to handle identifiers and '=' ...

```

Updating the Evaluator

```

def evaluate(expr: Expression, env: Environment) -> float:
    """Evaluate an expression in the given environment."""
    match expr:
        case Number(value):
            return value
        case Variable(name):
            return env.get(name)
        case Assignment(name, value):
            result = evaluate(value, env)
            env.define(name, result)
            return result

```

```

    case BinaryOp(left, operator, right):
        # ... existing code ...

```

Adding Control Flow

We can extend our language with if-expressions:

```

@dataclass
class If:
    condition: Expression
    then_branch: Expression
    else_branch: Expression

# Update Expression type
Expression = Union[Number, BinaryOp, Variable, Assignment, If]

# Update evaluator
def evaluate(expr: Expression, env: Environment) -> float:
    match expr:
        # ... existing cases ...
        case If(condition, then_branch, else_branch):
            if evaluate(condition, env) != 0: # Non-zero is true
                return evaluate(then_branch, env)
            else:
                return evaluate(else_branch, env)

```

Section 7: Further Exploration

Here are some ways you could extend our mini interpreter further:

Additional Features to Implement

1. **Functions:** Add function declarations and calls
2. **Loops:** Implement while or for loops
3. **More Operators:** Add comparison, logical, and bitwise operators
4. **Error Handling:** Improve error messages and recovery
5. **Type System:** Add a simple type system
6. **Standard Library:** Implement built-in functions for common operations

Learning Resources

To learn more about interpreters and language implementation:

- “**Crafting Interpreters**” by Robert Nystrom: A comprehensive guide to implementing interpreters
- “**Programming Language Pragmatics**” by Michael Scott: Covers theoretical aspects of language design
- “**Structure and Interpretation of Computer Programs**” by Abelson and Sussman: A classic text on programming language concepts

Conclusion

Building a mini interpreter helps understand how programming languages work under the hood. We’ve seen how to:

1. Tokenize source code into lexical tokens
2. Parse tokens into an abstract syntax tree
3. Evaluate the AST to produce results
4. Extend the interpreter with new features

This foundation can be expanded to build more complex languages and tools.

Chapter 4: Semantic Domains and Environment-Based Interpreters

Section 1: Introduction to Semantic Domains

In programming language semantics, **semantic domains** are mathematical structures used to give meaning to syntactic constructs. They provide the foundation for defining the behavior of programs in a precise, mathematical way.

Key Semantic Domains in Programming Languages

- **Syntactic Domains:** Represent the structure of programs (tokens, parse trees, syntax trees).
- **Semantic Domains:** Represent the meaning of programs (values, environments, states).

The relationship between these domains is at the heart of language semantics:

```
Syntax -> Semantics
Program -> Meaning
```

Core Semantic Domains

In our mini-interpreter, we'll work with several fundamental semantic domains:

- **Expressible Values:** Values that can be produced by evaluating expressions
- **Denotable Values (DVal):** Values that can be bound to identifiers in the environment
- **Memorable Values (MVal):** Values that can be stored in memory/state
- **Identifiers:** Names of constants, variables, functions, modules, etc.
- **Locations:** Memory addresses of variables
- **Environment:** Maps identifiers to denotable values, representing the binding context
- **State:** Maps memory locations to memorable values, representing the program's memory

While these domains often overlap, they aren't necessarily identical. Understanding the differences is crucial for language design.

Side Effects and Pure Functions

A fundamental distinction in programming language semantics is between **pure** computations and those that produce **side effects**.

Pure Functions A **pure function** is a computation that: 1. Always produces the same output for the same input 2. Has no observable effects beyond computing its result

In mathematical terms, a pure function is simply a mapping from inputs to outputs, like mathematical functions (e.g., $\sin(x)$, $\log(x)$).

```
def add(x: int, y: int) -> int:
    return x + y # Pure: same inputs always give same output
```

Side Effects A **side effect** is any observable change to the system state that occurs during computation, beyond returning a value. Common side effects include:

1. **Memory updates:** Modifying variables or data structures

```
x = 10 # Changes the program's state
```

Q: in this context, what is a function that does **not** return always the same value for the same inputs?

2. **Input/Output operations:**

```
print("Hello") # Affects the external world (terminal)
```

3. **File operations:**

```
with open("data.txt", "w") as f:
    f.write("data") # Changes the file system
```

4. Network operations:

```
requests.get("https://example.com") # Interacts with external systems
```

Memory Updates as Side Effects In our interpreter design, memory (state) updates are a primary form of side effect. When we update state:

```
def state_update(state: State, location: int, value: MVal) -> State:
    new_state = state.copy()
    new_state[location] = value
    return new_state
```

We're representing a change to the program's memory. In a real computer, this would modify memory cells directly. Our functional implementation returns a new state rather than modifying the existing one, but conceptually it represents the same side effect.

Side Effects in Programming Languages Languages differ in how they handle side effects:

- **Purely functional languages** (like Haskell) isolate side effects using type systems and monads
- **Imperative languages** (like C, Python) embrace side effects as their primary mechanism for computation
- **Hybrid languages** (like Scala, OCaml) support both styles

Understanding side effects is crucial for language design because they impact: - Program correctness (pure functions are easier to reason about) - Parallelization (side effects complicate parallel execution) - Optimization (pure functions allow more aggressive optimizations)

In our interpreter, we'll model side effects using explicit state passing, maintaining the mathematical clarity of our semantics while accurately representing the behavior of stateful programs.

Section 2: Denotable vs. Memorizable Values

Denotable Values (DVal)

Denotable values are those that can be bound to identifiers in an environment. In our implementation, they include:

```
type Num = int # A type alias for integers
type DenOperator = Callable[[int, int], int]
type DVal = int | DenOperator # Denotable values
```

DVal includes: - **Numbers**: Simple integer values - **Operators**: Functions that take two integers and return an integer

Memorizable Values (MVal)

Memorizable values are those that can be stored in memory (the state). In our implementation:

```
type MVal = int # Memorizable values
```

MVal only includes integers, not functions. This highlights an important distinction:

Not everything that can be bound to a name can be stored in memory.

This distinction is crucial for understanding: - Why some languages don't support first-class functions - Why some types require special treatment in memory management - How languages with different type systems handle values differently

Section 3: Environment and State as Functions

In our treatment of semantic domains, we adopt a purely functional approach where environments are represented as functions, and states are represented as dataclasses containing store functions, rather than mutable data structures. This approach aligns with the mathematical view of semantic domains and provides a clean conceptual model for understanding program behavior.

Functional Programming: A Brief Digression

Before diving into our function-based implementation of environments and state, it's worth taking a brief detour to discuss functional programming concepts, as they form the foundation of our approach.

Functional programming is a paradigm where computations are treated as evaluations of mathematical functions, emphasizing immutable data and avoiding side effects. Python, while not a pure functional language, supports many functional programming techniques.

Functions as First-Class Citizens In functional programming, functions are “first-class citizens” — they can be: - Assigned to variables - Passed as arguments to other functions - Returned from functions - Stored in data structures

For example:

```
# Function assigned to a variable
increment = lambda x: x + 1

# Function passed as an argument
def apply_twice(f, x):
    return f(f(x))

result = apply_twice(increment, 3) # Returns 5
```

Higher-Order Functions: Map A common pattern in functional programming is applying a function to each element in a collection. Python's `map` function does exactly this:

```
numbers = [1, 2, 3, 4, 5]

# Apply a function to each element
squared = list(map(lambda x: x**2, numbers)) # [1, 4, 9, 16, 25]

# Equivalent to a list comprehension
squared_alt = [x**2 for x in numbers] # [1, 4, 9, 16, 25]
```

Function Composition Functional programming emphasizes building complex behaviors by composing simpler functions:

```
def compose(f, g):
    return lambda x: f(g(x))

# Compose two functions
negate_and_square = compose(lambda x: -x, lambda x: x**2)
result = negate_and_square(5) # -25
```

Pure Functions and Immutability Pure functions always produce the same output for the same input and have no side effects. This property makes them predictable and easier to reason about:

```
# Pure function
def add(a, b):
    return a + b
```

```
# Impure function (has side effects)
def add_and_print(a, b):
    result = a + b
    print(f"The result is {result}") # Side effect: printing
    return result
```

Relevance to Semantic Domains These functional programming concepts directly inform our approach to implementing semantic domains:

1. We represent environments and stores as functions, not data structures
2. We use higher-order functions to create updated environments and stores
3. We maintain immutability through functional updates rather than mutations
4. We compose simple operations to build complex behaviors

With this foundation in mind, let's explore how we represent environments and states as functions.

Environment as a Function

An environment is mathematically a function that maps identifiers to denotable values:

```
type Environment = Callable[[str], DVal]
```

This means an environment is a function that: - Takes an identifier (string) as input - Returns a denotable value (DVal) - Raises an error if the identifier is not defined

While many practical implementations use dictionaries or hash tables for efficiency, conceptually an environment is simply a function:

```
Environment: Identifier → DVal
```

State as a Dataclass

Unlike environment, which is purely a function, a state encapsulates both a store function and allocation information:

```
@dataclass
class State:
    store: Callable[[Location], MVal] # The store function
    next_loc: int # Next available location
```

This represents a state as: - A dataclass containing a store function and next location counter - The store function takes a location as input and returns the value at that location - The next_loc field tracks the next available memory location for allocation

Conceptually, the store component maintains the mapping:

```
Store: Location → MVal
```

While the next_loc component tracks allocation state.

Section 4: Functional Updates

In a purely functional approach, we don't mutate existing environments or states. Instead, we create new functions or dataclasses that encapsulate the updated behavior.

Environment Updates

Instead of modifying a dictionary, we define a new function that returns the new value for the updated identifier and delegates to the original environment for all other identifiers:


```
def bind(env: Environment, name: str, value: DVal) -> Environment:
    """Create new environment with an added binding"""
    def new_env(n: str) -> DVal:
        if n == name:
            return value
        return env(n)
    return new_env
```

This function returns a new environment that: - Returns `value` when asked for `name` - Delegates to the original environment for all other identifiers

This approach: - Preserves referential transparency - Enables easy implementation of lexical scoping - Facilitates reasoning about program behavior - Models the mathematical concept of function extension

State Updates

Similarly, state updates create new State objects with updated store functions:

```
def state_update(state: State, location: Location, value: MVal) -> State:
    """Create new state with an updated value at given location"""
    def new_store(loc: Location) -> MVal:
        if loc == location:
            return value
        return state.store(loc)
    return State(store=new_store, next_loc=state.next_loc)
```

This function creates a new State object that: - Contains a new store function that returns the new value when asked for the specified location - Delegates to the original state's store function for all other locations - Preserves the `next_loc` value from the original state

Empty Environment and State

The primitives for creating empty environments and states define initial values:

```
def empty_environment() -> Environment:
    """Create an empty environment function"""
    def env(name: str) -> DVal:
        raise ValueError(f"Undefined identifier: {name}")
    return env

def empty_memory() -> State:
    """Create an empty memory state"""
    def store(location: Location) -> MVal:
        raise ValueError(f"Undefined memory location: {location}")
    return State(store=store, next_loc=0)
```

Note that `empty_memory` returns a State dataclass initialized with an empty store function and `next_loc` set to 0.

Memory Allocation

In a complete interpreter, we use the State dataclass to implement memory allocation elegantly:

```
def allocate(state: State, value: MVal) -> tuple[State, Location]:
    """Allocate a new memory location and store a value there.
    Returns the updated state and the new location."""
    location = state.next_loc
    new_state = state_update(state, location, value)
```

```
# Return state with incremented next_loc and the allocated location
return State(store=new_state.store, next_loc=location + 1), location
```

This function: 1. Gets the next available location from the state 2. Updates the store function to map this location to the provided value 3. Returns a new state with the incremented next_loc and the allocated location

By bundling the store function with the next_loc counter in our State dataclass, we maintain a purely functional approach while elegantly handling the allocation challenge. This design demonstrates how functional programming can manage state without side effects by making state changes explicit in the return values of functions.

Initial Environment Setup

In our functional implementation, the initial environment is built by starting with an empty environment and extending it with each operator:

```
def create_initial_env() -> Environment:
    """Create an environment populated with standard operators"""
    env = empty_environment()
    env = bind(env, "+", add)
    env = bind(env, "-", subtract)
    env = bind(env, "*", multiply)
    env = bind(env, "/", divide)
    env = bind(env, "%", modulo)
    return env
```

This builds up the environment incrementally, adding each binding through functional extension.

Section 5: Environment-Based Interpretation

Traditional interpreters often use pattern matching on operators directly in the evaluation function. An environment-based approach takes a more abstract view, treating operators as first-class values in the environment.

Traditional Approach (from Chapter 3)

```
def evaluate(ast: Expression) -> int:
    match ast:
        case Number(value):
            return value
        case BinaryExpression(op, left, right):
            left_value = evaluate(left)
            right_value = evaluate(right)
            match op:
                case "+":
                    return left_value + right_value
                case "-":
                    return left_value - right_value
            # ... other operations
```

Environment-Based Approach

```
def evaluate(ast: Expression, env: Environment) -> MVal:
    match ast:
        case Number(value):
            return value
        case BinaryExpression(op, left, right):
```

```

try:
    # Get operator from environment
    operator = lookup(env, op)

    # Ensure it's a DenOperator
    if not isinstance(operator, Callable):
        raise ValueError(f"{op} is not a function")

    # Evaluate operands and apply operator
    left_value = evaluate(left, env)
    right_value = evaluate(right, env)

    # Apply the operator to the evaluated operands
    return operator(left_value, right_value)
except ValueError as e:
    raise ValueError(f"Evaluation error: {e}")

```

Benefits of the Environment-Based Approach

- **Extensibility:** New operators can be added to the environment without modifying the evaluator
- **First-class operations:** Operators are values that can be passed, returned, and manipulated
- **Consistent treatment:** Operators and other identifiers are handled uniformly
- **Semantic clarity:** The environment explicitly represents the mapping from names to meanings

Section 6: Implementing an Environment-Based Interpreter

Our `domains.py` file implements a complete environment-based interpreter:

1. **Define semantic domains:** Denotable and memorizable values
2. **Implement operators:** Define functions for arithmetic operations
3. **Create the environment:** Populate with standard operators
4. **Evaluate expressions:** Using the environment to look up operators
5. **REPL:** Interactive environment for testing the interpreter

Section 7: Extending the Interpreter

This approach makes it easy to extend the language with new features:

Adding New Operators

To add a new operator, simply define its function and add it to the environment:

```

def power(x: int, y: int) -> int:
    return x ** y

# Extend environment
env = bind(create_initial_env(), "**", power)

```

Adding Variables

To support variables, extend the AST with a variable node and update the evaluator:

```

```python
@dataclass
class Variable:
 name: str

```

```
Update Expression type
type Expression = Number | BinaryExpression | Variable
```

## Update evaluate function

```
def evaluate(ast: Expression, env: Environment) -> MVal:
 match ast:
 case Variable(name):
 value = lookup(env, name)
 # Additional check might be needed if variables can only be Numbers
 if not isinstance(value, int):
 raise ValueError(f"{name} is not a number")
 return value
 # ... existing cases
```

## Additional Resources

- [Denotational Semantics \(Wikipedia\)](#)
- [Programming Language Semantics \(Stanford\)](#)
- [Functional Programming and Lambda Calculus](#)
- [Environment and Store in Programming Languages](#)

## Section 9: Practical Implementation and Testing

Our environment-based interpreter includes practical components for execution and testing, demonstrating how theoretical concepts translate to code.

### Parsing and AST Construction

In our implementation, we use the Lark parser to convert text expressions into parse trees:

```
def parse_ast(expression: str) -> Expression:
 """Parse a string expression into an AST"""
 parse_tree = parser.parse(expression)
 return transform_parse_tree(parse_tree)
```

The `transform_parse_tree` function then converts these parse trees into our AST representation, ready for evaluation.

### Interactive REPL

A Read-Eval-Print Loop allows interactive testing of the interpreter:

```
def REPL():
 """Read-Evaluate-Print Loop with environment"""
 env = create_initial_env()

 print("Mini-interpreter with environment (type 'exit' to quit)")
 print("Available operators: +, -, *, /, %")

 while True:
 expression = input("Enter an expression (exit to quit): ")
 if expression == "exit":
 break
```

```

try:
 ast = parse_ast(expression)
 result = evaluate(ast, env)
 print(result)
except Exception as e:
 print(f"Error: {e}")

```

The REPL shows how our environment-based interpreter integrates with user interaction.

## Automated Tests

Testing ensures the interpreter behaves as expected across various expressions:

```

def run_tests():
 """Run test expressions to verify the parser and evaluator"""
 test_expressions = [
 "1+2",
 "3*4",
 "5-3",
 "10/2",
 "10%3",
 "(1+2)*3",
 "1+(2*3)",
 "10/(2+3)",
 "10%(2+3)",
]

 env = create_initial_env()

 for expr in test_expressions:
 try:
 ast = parse_ast(expr)
 result = evaluate(ast, env)
 print(f"{expr} = {result}")
 except Exception as e:
 print(f"{expr} -> Error: {e}")

```

These tests validate core functionality while providing examples of valid expressions.

## Section 10: Conclusion and Next Steps

### Summary of Key Concepts

In this chapter, we've explored:

1. **Semantic domains** as mathematical structures that give meaning to programs
2. **Environment-based interpretation** as a flexible approach to language implementation
3. **Functional representations** of environments and state
4. **Primitive operations** for manipulating environments and memory
5. **The distinction** between denotable and memorizable values

These concepts form the foundation for understanding more complex language features in subsequent chapters.

### Looking Forward

The environment-based approach introduced here will be extended in future chapters to support:

1. **Variables and assignment:** Using environments to bind identifiers to memory locations

2. **Scoping mechanisms:** Creating nested environments for block-structured code
3. **Functions and closures:** Capturing environments for later execution
4. **Typing systems:** Adding constraints on what values expressions can produce

By building on these semantic foundations, we can construct a rich understanding of programming language design and implementation.

### Exercises for the Reader

1. Extend the interpreter to support variables using the Variable AST node described in Section 7
2. Add support for multi-character operators (e.g., “\*\*” for exponentiation)
3. Implement a memory system with the primitives described in Section 8
4. Add support for conditional expressions (if-then-else)
5. Implement a simple block structure with local variables

These exercises will deepen your understanding of language semantics and interpreter design while preparing you for more advanced topics in the next chapters.

## Chapter 5: Variable Binding and Let Expressions

### Section 1: Introduction to Variable Binding

In this chapter, we extend our mini-interpreter to support **variable binding** through `let` expressions. This is a fundamental capability that allows programs to:

1. Name and reuse values
2. Create modular components
3. Build abstractions that hide implementation details

The extension represents our first step toward a more practical and powerful language.

#### Variable Binding vs. Assignment

It's important to distinguish between binding and assignment:

- **Variable Binding:** Associates a name with a value in a specific scope or environment

```
let x = 10 in ...
```

- **Assignment:** Changes the value associated with an existing name

```
x = 10
```

In our mini-language, we implement variable binding but not assignment. This makes our language more functional in nature and simplifies reasoning about program behavior.

### Section 2: Let Expressions

#### Syntax and Semantics of Let Expressions

We extend our grammar with `let` expressions:

```
let: "let" IDENTIFIER "=" expr "in" expr
```

A `let` expression has three components: 1. The name (identifier) being bound 2. The expression that provides the value to bind 3. The body expression where the binding is in scope

For example, in `let x = 10 in x + 5`, we: - Bind `x` to the value `10` - Evaluate the body `x + 5` in this extended environment, resulting in `15`

#### AST Representation

To support `let` expressions in our abstract syntax tree, we add a new class:

```
@dataclass
class Let:
 name: str
 expr: Expression
 body: Expression
```

And we extend our `Expression` type to include `Let` and `Var` (variable reference) nodes:

```
type Expression = Number | BinaryExpression | Let | Var
```

#### Variable References

To use bound variables, we need to support variable references in our language:

```
@dataclass
class Var:
 name: str
```

When evaluating a variable reference, we look up its value in the current environment:

```
case Var(name):
 x = lookup(env, name)
 match x:
 case int():
 return x
 case _:
 raise ValueError(f"Unexpected value type: {type(x)}")
```

## Section 3: The Environment

### Environment as a First-Class Citizen

In our mini-interpreter, the environment is a first-class entity that's explicitly passed around during evaluation. This makes the binding context explicit in our semantics.

The key operations on environments are: - **Creation**: Building an initial environment with primitive operations - **Lookup**: Finding the value associated with a name - **Extension**: Adding new bindings to create a derived environment

### Environment Implementation

Our environment is implemented as a function that maps names to values:

```
type Environment = Callable[[str], DVal]
```

We extend the environment when evaluating a `let` expression:

```
case Let(name, expr, body):
 value = evaluate(expr, env)
 extended_env = bind(env, name, value)
 return evaluate(body, extended_env)
```

This creates a new environment that includes the original bindings plus the new binding.

### Static vs. Dynamic Scoping

Our implementation uses **lexical (static) scoping**, where variable references are resolved in the environment where they are defined, not where they are used.

This is in contrast to **dynamic scoping**, where variable references are resolved in the current execution environment.

Lexical scoping is more predictable and easier to reason about, which is why it's the dominant approach in modern programming languages.

**Static vs. Dynamic Scoping Examples** Consider this expression:

```
let x = 5 in
 let f = (let y = 10 in y + x) in
 let x = 42 in
 f
```

With **static scoping** (our implementation), this evaluates to: - `f` is bound to the result of `(let y = 10 in y + x)` where `x` is 5, so `f` is 15 - Even though we later rebind `x` to 42, the reference to `x` inside `f` still refers to the original binding - Final result: 15

With **dynamic scoping**, this would evaluate to: - `f` is bound to the expression `(let y = 10 in y + x)` (not its value yet) - When `f` is evaluated, it looks for the current binding of `x`, which is 42 - Final result: 52

Another example:



```
let a = 1 in
 let f = (a + 10) in
 let a = 100 in
 f
```

With **static scoping**: - `f` evaluates to 11 using the binding `a = 1` - The later binding of `a = 100` has no effect on `f`

With **dynamic scoping**: - `f` would be 110 since it would use the latest binding of `a` when evaluated

## Examples

Let expressions dramatically increase the expressiveness of our language. Consider a few examples:

```
let x = 10 in x + 5
```

Evaluates to 15 by binding `x` to 10 and adding 5.

```
let x = 1 in let y = 2 in x + y
```

Nested binding.

## Section 5: Conclusion

Adding variable binding through `let` expressions marks a significant step in the evolution of our mini-language. In future chapters, we'll build on this foundation to add more capabilities:

- Conditionals and booleans
- Iteration (while and for loops)
- Functions and application
- Recursion

## Parser and Grammar Extensions

Compared to Lecture 4, we've made the following extensions to the parser and grammar:

1. Added a new production rule for `let` expressions:

```
let: "let" IDENTIFIER "=" expr "in" expr
```

2. Extended the expression rule to include both variables and `let` expressions:

```
?expr: bin | mono | let
mono: ground | paren | var
var: IDENTIFIER
```

3. Added new pattern matching cases in the parse tree transformation function:

```
case Tree(data="var", children=[Token(type="IDENTIFIER", value=name)]):
 return Var(name=name)

case Tree(
 data="let",
 children=[
 Token(type="IDENTIFIER", value=name),
 expr,
 body,
],
):
 return Let(
 name=name,
 expr=transform_parse_tree(expr),
```

```
 body=transform_parse_tree(body),
)
```

#### 4. Added evaluation rules for the new AST node types:

```
case Let(name, expr, body):
 value = evaluate(expr, env)
 extended_env = bind(env, name, value)
 return evaluate(body, extended_env)
case Var(name):
 x = lookup(env, name)
 match x:
 case int():
 return x
 case _:
 raise ValueError(f"Unexpected value type: {type(x)}")
```

These extensions allow our language to support both variable definitions and references while maintaining the core evaluation semantics from Lecture 4 that will be useful in the second part of the course.

## Chapter 6: State

### Section 1: Introduction to State

In this chapter, we extend our mini-language with the concept of **state**. Until now, our language has been purely functional, where expressions are evaluated to produce values without side effects. By adding state, we can:

1. Store and update values in memory
2. Observe changes to data over time
3. Model real-world systems that have changing state

#### Expressions vs. Commands

Our language extension introduces a new distinction:

- **Expressions:** Compute values (e.g., `x + 1`, `let x = 42 in x * 2`)
- **Commands:** Perform actions with side effects (e.g., `var x = 42`, `x <- 42`, `print x`)

This distinction is common in many programming languages:

```
Expression (computes a value)
x + 1

Command (performs an action)
var x = 42
x <- 42
```

### Section 2: Commands and Command Sequences

To implement state, we introduce two new syntactic categories:

1. **Commands:** Individual actions that can modify state
2. **Command Sequences:** Ordered lists of commands to be executed in sequence

#### Commands in Our Language

We implement three basic command types:

1. **Variable Declaration:** Declares a new variable and initializes it

```
var x = 42
```

2. **Assignment:** Updates a variable with a new value (only if already declared)

```
x <- 10
```

3. **Print:** Outputs the value of an expression

```
print x + 1
```

#### Command Sequences

Command sequences represent multiple commands separated by semicolons:

TODO: also use an assignment in the example

```
var x = 10;
var y = x + 5;
print y
```

This sequence declares `x`, then declares `y` as `x + 5` (which is 15), and finally prints the value of `y`.

## Grammar Extensions

We extend our language grammar to support commands and sequences:

```
?program: command_seq

?command_seq: command
 | command ";" command_seq

?command: vardecl
 | assign
 | print

vardecl: "var" IDENTIFIER "=" expr
assign: IDENTIFIER "<-" expr
print: "print" expr
```

## AST Representation

We represent commands and command sequences with these AST node types:

```
@dataclass
class VarDecl:
 name: str
 expr: Expression

@dataclass
class Assign:
 name: str
 expr: Expression

@dataclass
class Print:
 expr: Expression

@dataclass
class CommandSequence:
 first: Command
 rest: Optional[CommandSequence] = None

type Command = VarDecl | Assign | Print
```

## Section 3: The Store Model of State

To represent state, we use the “store model”:

1. **Environment:** Maps variable names to locations (memory addresses)
2. **Store:** Maps locations to values

This two-level indirection allows multiple variables to refer to the same location or for a variable’s value to change without changing its location.

### The State as a Functional Dataclass

Our implementation uses a `State` dataclass to manage the store in a functional style:

- The store is **not** a dictionary, but a function from locations to values (or raises an error if not found), just like the environment is a function from names to locations. This ensures a fully functional approach.

```

@dataclass(frozen=True)
class State:
 store: Callable[[int], Any]
 next_loc: int

def empty_store() -> Callable[[int], Any]:
 def store_fn(loc: int) -> Any:
 raise ValueError(f"Location {loc} not allocated")
 return store_fn

def empty_state() -> State:
 return State(store=empty_store(), next_loc=0)

def alloc(state: State, value: Any) -> tuple[int, State]:
 loc = state.next_loc
 prev_store = state.store
 def new_store(l: int) -> Any:
 if l == loc:
 return value
 return prev_store(l)
 return loc, State(store=new_store, next_loc=loc + 1)

def update(state: State, loc: int, value: Any) -> State:
 state.store(loc) # will raise if not found
 prev_store = state.store
 def new_store(l: int) -> Any:
 if l == loc:
 return value
 return prev_store(l)
 return State(store=new_store, next_loc=state.next_loc)

def lookup(state: State, loc: int) -> Any:
 return state.store(loc)

```

## Denotable Values (DVal)

Denotable values are those that can be bound to identifiers in an environment. In our implementation, they include:

DVal includes:

- **Numbers:** Simple integer values
- **Operators:** Functions that take two integers and return an integer
- **Locations:** Memory addresses (as int, but semantically distinct)
- **Booleans:** For forward compatibility with later chapters

```

Python 3.13 union syntax

type Num = int # A type alias for integers
type DenOperator = Callable[[int, int], int]
type Location = int

type DVal = int | DenOperator | bool | Location # Denotable values: everything that could be bound in Lecture 5, plus locations

```

**Note:** DVal is broader than what can be stored in memory (MVal). Not everything that can be bound to a name can be stored in memory.

## Section 4: Evaluating Expressions with State

When evaluating expressions in a stateful language, we need to pass both the environment and the state:

```
def evaluate_expr(expr: Expression, env: Environment, state: State) -> int:
 # ...
```

Variable lookup now has two steps:

1. Use the environment to find the location
2. Use the state to look up the value at that location

```
case Var(name):
 # Look up variable's location and then its value in the state
 try:
 loc = lookup_env(env, name)
 return lookup(state, loc)
 except ValueError as e:
 raise ValueError(f"Variable error: {e}")
```

## Section 5: Executing Commands

Commands modify the state or produce output. The `execute_command` function returns both the updated environment and state as a tuple:

```
env, state = execute_command(cmd, env, state)
```

- `env` is the updated environment (with new variable bindings, if any)
- `state` is the updated state (with new or updated values)

You can access the first and second components of the tuple as `env` and `state` respectively.

**Note:** The `print` command is not part of the mathematical semantics; it is just an aid for using the interpreter.

### The Variable Declaration Command

The variable declaration command (`var x = expr`) creates a new variable and initializes it. If the variable already exists, it is an error.

### The Assignment Command

The assignment command (`<-`) only updates an existing variable. If the variable does not exist, it is an error.

### The Print Command

The print command evaluates the expression and prints its value. It does not affect the environment or state.

### Command Sequences

Executing a command sequence involves:

1. Executing the first command
2. Executing the rest of the sequence with the updated environment and state

```
def execute_command_seq(seq: CommandSequence, env: Environment, state: State) -> tuple[Environment, State]:
 env, state = execute_command(seq.first, env, state)
 if seq.rest:
 return execute_command_seq(seq.rest, env, state)
 return env, state
```

## Section 6: Examples of State in Action

Let's look at some examples that demonstrate the power of state:

### Example 1: Basic Declaration, Assignment, and Printing

```
var x = 42;
print x
```

This simple example shows creating a variable and reading its value. The output is 42.

### Example 2: Updating Variables

```
var x = 10;
var y = x + 5;
print y;
x <- 20;
print x;
print y
```

This example demonstrates that changing `x` doesn't automatically change `y`, even though `y` was defined in terms of `x`. The output is:

```
15
20
15
```

The value of `y` remains 15 because the expression `x + 5` was evaluated at the time of declaration.

### Example 3: Let Expressions in Commands

```
var x = let y = 5 in y * 2;
print x
```

This example shows how we can use let expressions within commands. The output is 10.

## Section 7: Differences from Previous Chapters

Adding state represents a significant shift in our language:

Purely Functional (Ch. 5)	Stateful (Ch. 6)
Values are immutable	Values can change over time
No side effects	Commands have side effects
Referential transparency	Expressions may evaluate differently
Environment maps names to values	Environment maps names to locations
Easier to reason about	More expressive

## Section 8: Conclusion and Next Steps

Adding state to our mini-language significantly increases its expressiveness, making it capable of modeling real-world problems that involve change over time. In the next chapter, we'll build upon this foundation by adding control flow structures like loops and conditionals.

With state, environment, and control flow, our language will have all the essential ingredients of a complete programming language.

## Exercises: Extending State

### 1. Alias Command

**Exercise:** Implement an `alias x = y` command that makes `x` and `y` point to the same location in the environment (i.e., after `alias x = y`, both names refer to the same memory cell).

#### Example 1:

```
var a = 10;
alias b = a;
b <- 20;
print a # Output: 20
```

#### Example 2:

```
var x = 5;
alias y = x;
print y # Output: 5
x <- 42;
print y # Output: 42
```

#### Example 3:

```
var m = 1;
alias n = m;
m <- 7;
print n # Output: 7
```

---

### 2. Conditional Command

**Exercise:** Implement an `if condition then command1 else command2` construct, where `condition` is an expression and `command1/command2` are single commands (not sequences).

#### Prompt:

- What if either `command1` or `command2` is a `var` declaration? Can this be used to conditionally declare variables?
- What should happen if the same variable is declared in both branches? Should the environment be merged, or should this be an error?
- How would you design the semantics to handle these cases?

Reflect on these questions and try to implement a solution that is both safe and intuitive.



## Chapter 7: Control Flow

WORK IN PROGRESS.

### Section 1: Introduction to Booleans and Conditionals

In this chapter, we extend our mini-interpreter to support **boolean expressions** and **conditional (if-then-else) expressions**. These features are essential for making decisions in programs and are foundational to all programming languages.

#### Why Booleans and Conditionals?

- **Booleans** allow us to represent truth values (`True` and `False`).
- **Conditionals** enable programs to choose between different actions based on boolean conditions.
- Together, they make our language expressive enough to encode logic and control flow.

### Section 2: Boolean Expressions

#### Syntax and Semantics

We extend our language with boolean literals and boolean operations:

- Boolean literals: `True`, `False`
- Comparison operators: `==`, `!=`, `<`, `>`, `<=`, `>=`
- Logical operators: `and`, `or`

#### AST Representation

We add new AST node types for booleans and binary operations:

```
@dataclass
class Boolean:
 value: bool

@dataclass
class BinOp:
 op: str # '+', '-', '*', '/', '==', '!=', '<', '>', '<=', '>=', 'and', 'or'
 left: Expression
 right: Expression
```

### Section 3: Conditional Expressions

#### Syntax and Semantics

We add the `if` expression to our language:

```
if: "if" expr "then" expr "else" expr
```

This allows us to write expressions like:

```
if x > 0 then x else -x
```

#### AST Representation

```
@dataclass
class If:
 cond: Expression
```

```
then_branch: Expression
else_branch: Expression
```

## Section 4: Parser and Evaluation Changes

### Parser Extensions

The parser is extended to recognize boolean literals, comparison and logical operators, and conditional expressions. For demonstration, we use a simple tuple-based parser:

```
def parse(expr):
 if isinstance(expr, int):
 return Number(expr)
 if isinstance(expr, bool):
 return Boolean(expr)
 if isinstance(expr, str):
 return Var(expr)
 if isinstance(expr, tuple):
 tag = expr[0]
 if tag == 'let':
 _, name, value_expr, body_expr = expr
 return Let(name, parse(value_expr), parse(body_expr))
 if tag == 'if':
 _, cond, then_branch, else_branch = expr
 return If(parse(cond), parse(then_branch), parse(else_branch))
 if tag in {'+', '-', '*', '/', '==', '!=', '<', '>', '<=', '>=', 'and', 'or'}:
 _, left, right = expr
 return BinOp(tag, parse(left), parse(right))
 raise ValueError(f"Cannot parse: {expr}")
```

### Evaluation Extensions

The evaluation function is extended to handle booleans, binary operations, and conditionals:

```
def eval_expr(expr: Expression, env: Environment) -> DVal:
 if isinstance(expr, Number):
 return expr.value
 if isinstance(expr, Boolean):
 return expr.value
 if isinstance(expr, Var):
 return env(expr.name)
 if isinstance(expr, Let):
 value = eval_expr(expr.expr, env)
 extended_env = bind(env, expr.name, value)
 return eval_expr(expr.body, extended_env)
 if isinstance(expr, BinOp):
 # Short-circuit for 'and' and 'or'
 if expr.op == 'and':
 left = eval_expr(expr.left, env)
 if not left:
 return False # short-circuit
 return bool(eval_expr(expr.right, env))
 if expr.op == 'or':
 left = eval_expr(expr.left, env)
 if left:
```

```

 return True # short-circuit
 return bool(eval_expr(expr.right, env))
 left = eval_expr(expr.left, env)
 right = eval_expr(expr.right, env)
 # ... handle other operators ...
 if isinstance(expr, If):
 cond = eval_expr(expr.cond, env)
 if cond:
 return eval_expr(expr.then_branch, env)
 else:
 return eval_expr(expr.else_branch, env)
 raise ValueError(f"Unknown expression: {expr}")

```

## Section 5: Short-Circuit Evaluation

Short-circuit evaluation means that in logical expressions like `and` and `or`, the second operand is only evaluated if necessary:

- For `and`, if the first operand is `False`, the result is `False` (second operand not evaluated).
- For `or`, if the first operand is `True`, the result is `True` (second operand not evaluated).

This is important for avoiding errors and improving efficiency.

### Example: Avoiding Division by Zero

```

let x = 0 in if (x != 0) and (10 // x > 1) then 1 else 2
ast = parse(('let', 'x', 0, ('if', ('and', ('!=', 'x', 0), ('>', ('/', 10, 'x'), 1)), 1, 2)))
print(eval_expr(ast, empty_env)) # Output: 2 (no error)

```

## Section 6: Non-trivial Examples

### Example 1: Simple Conditional with Binding

```

let x = 10 in if x > 5 then x else 0
ast1 = parse(('let', 'x', 10, ('if', ('>', 'x', 5), 'x', 0)))
print(eval_expr(ast1, empty_env)) # Output: 10

```

### Example 2: Nested Binding and Conditionals

```

let x = 3 in let y = 4 in if (x < y) and (y < 10) then x + y else 0
ast2 = parse(('let', 'x', 3, ('let', 'y', 4, ('if', ('and', ('<', 'x', 'y'), ('<', 'y', 10)), ('+', 'x', 'y'), 0))))
print(eval_expr(ast2, empty_env)) # Output: 7

```

### Example 3: Short-circuiting to Avoid Errors

```

let x = 0 in if (x != 0) and (10 // x > 1) then 1 else 2
ast3 = parse(('let', 'x', 0, ('if', ('and', ('!=', 'x', 0), ('>', ('/', 10, 'x'), 1)), 1, 2)))
print(eval_expr(ast3, empty_env)) # Output: 2

```

## Section 7: Conclusion and Next Steps

Adding booleans and conditionals makes our mini-language much more expressive, enabling it to represent logic and control flow. In future chapters, we will build on this by adding:

- Iteration (while and for loops)
- Functions and application
- Recursion

These features will allow us to write even more powerful and interesting programs in our mini-language.