

The topological approach to **Spatial Model Checking**

with applications to imaging and 3D meshes.

GRETA Seminar

2024-12-06

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Formal Methods and Tools Laboratory @ ISTI-CNR Pisa

Joint work(s) with Gina Belmonte, Giovanna Broccia, Laura Bussi, Fabio Gadducci, Diego Latella, Mieke Massink, & many more coauthors

Goal

Spatial logic +

Model checker =

Declarative Domain-Specific Executable Language

for graphs, images, 3d meshes, ...

Who we are

CNR-ISTI FMT

G. Broccia, L. Bussi, V. Ciancia, D. Latella, M. Massink, S. Colantonio, M. ter Beek

AUSL TOSCANA NORDOVEST

G. Belmonte, A. Tofani, M. Imbriani

UNIPi

F. Gadducci, M. Cosottini, I. Pesaresi, ...

UNIUD

M. Miculan, V. della Mea, ...

ILLC Amsterdam

N. Bezhanisvili, ...

Univ. Tbilisi

D. Gabelaia, M. Jibladze, ...

Univ. Barcellona

D. Fernandez-Duque, ...

TU Eindhoven

(E. de Vink, J. F. Groote, ...)

Formal methods & AI

L. Bussi, V. Ciancia, D. Latella, M. Massink, M. ter Beek, F. Gadducci, M. Miculan, E. de Vink, J. F. Groote, S. Colantonio, ...

Logics

- N. Bezhanisvili, D. Gabelaia, M. Jibladze, D. Fernandez-Duque, ...

Neuroradiology

M. Cosottini, I. Pesaresi, ...

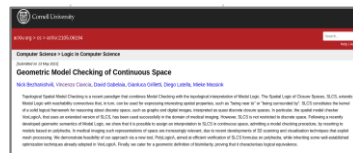
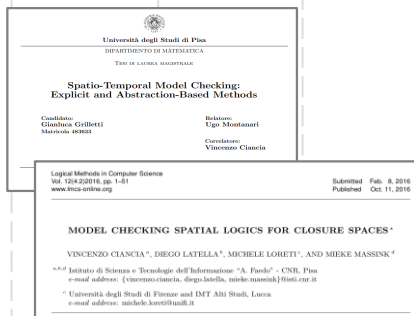
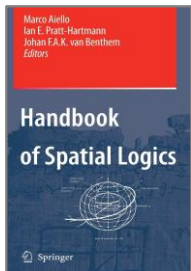
Multi-disciplinary collaboration network

Human-Computer Interaction

G. Broccia, A. Strippoli, ...

Medical Physics

G. Belmonte, M. Imbriani,
A. Tofani, ...



Nick Bezhanishvili, Laura Bussi, Vincenzo Ciancia, David Fernández-Duque, David Gabelaia:
Logics of Polyhedral Reachability. [AIML 2024](#): 187-204



2014

2015

2016

2017

2018

2019

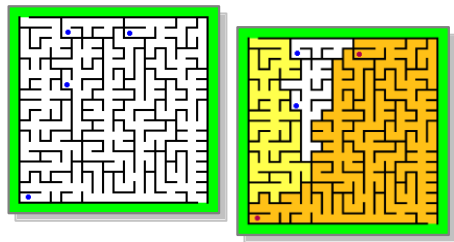
2020

2021

2022

2023

2024

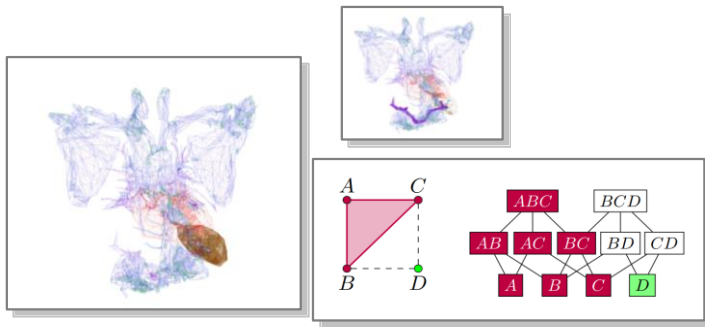


Reachability

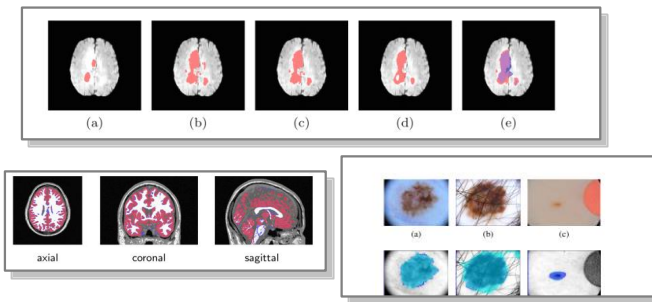
One goal:
make
spatial logics
practical



Map annotation



3D meshes (simplicial complexes)



Medical Imaging

Advantages

Logical language

concise, unambiguous, collaborative,
knowledge-based, **Human-Centric**, explainable, accountable...

Familiar application domain

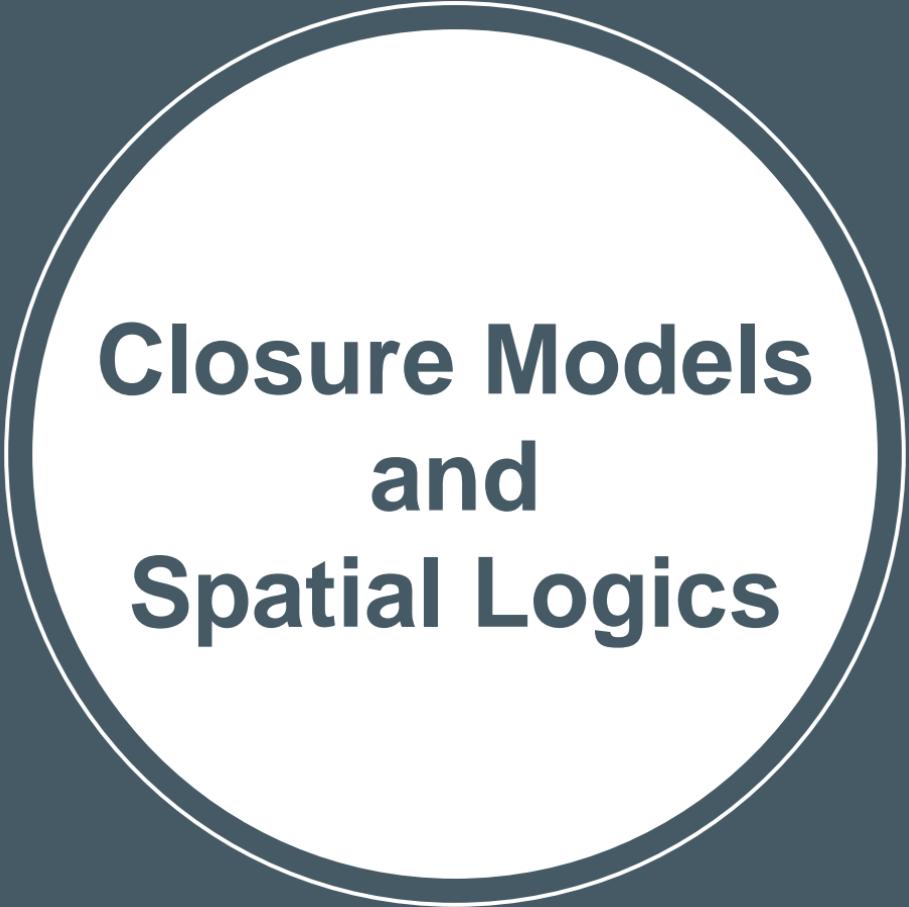
using spatial structures, logical specifications are easier to grasp for non-specialists

No side effects

syntactic manipulation, memoization, query optimization

Global model checking

parallel execution, GPU computation, caching, distributed execution, minimization, ...



**Closure Models
and
Spatial Logics**

Modal logic is Topological



[McKinsey-Tarski 1944]:

“Modal logic is **topological**”

$$\Phi ::= \top \mid p \mid \Phi \wedge \Phi \mid \neg \Phi \mid \boxed{\Diamond \Phi}$$

What does that mean?

Kuratowski definition of Topological Spaces


A **topological** space is a pair (X, \mathcal{C})

- X is a set
- $\mathcal{C} : \mathcal{P}(X) \rightarrow \mathcal{P}(X)$

Continuous function

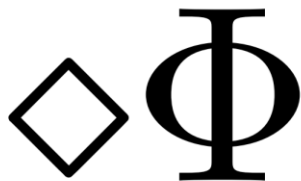
$$f : X_1 \rightarrow X_2 \text{ with } f(\mathcal{C}_1(A)) \subseteq \mathcal{C}_2(f(A))$$

- $\mathcal{C}(\emptyset) = \emptyset$
- $A \subseteq \mathcal{C}(A)$
- $\mathcal{C}(A \cup B) = \mathcal{C}(A) \cup \mathcal{C}(B)$
- $\mathcal{C}(\mathcal{C}(A)) = \mathcal{C}(A)$



Idempotency axiom

McKinsey & Tarski's proposal:



Interpret **diamond** as closure

(and box as interior)

Topological modal logic

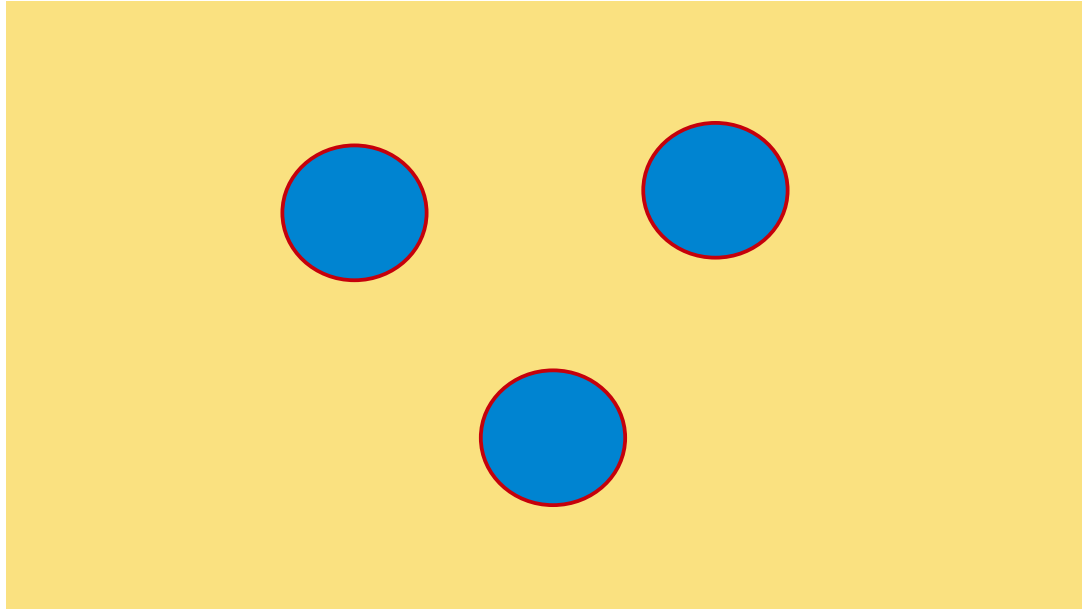
$$\mathcal{M} = ((X, \mathcal{C}), \mathcal{V})$$

Topological space
(Kuratowski)

Valuation of atomic
propositions

$$\mathcal{V}(p) \subseteq X$$

Topological model: a “coloured” topological space



Satisfaction relation

$$\mathcal{M}, x \in X \models \phi$$

point of the space

denotes a set of points
("region"?)

Interpretation

See J. van Benthem, G. Bezhanishvili, Modal Logics of Space, Handbook of Spatial Logics Chapter 5

$$\llbracket p \rrbracket = v(p)$$

$$\llbracket \top \rrbracket = X$$

$$\llbracket \neg \phi \rrbracket = \overline{\llbracket \phi \rrbracket}$$

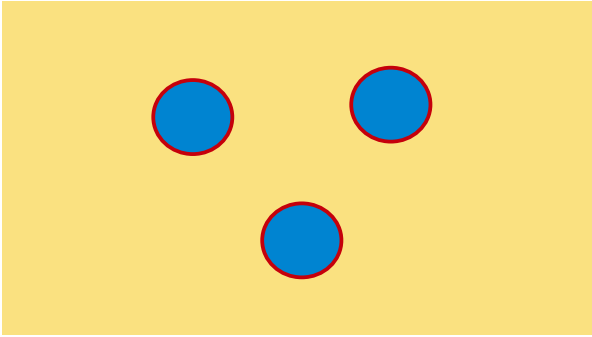
$$\llbracket \phi \wedge \psi \rrbracket = \llbracket \phi \rrbracket \cap \llbracket \psi \rrbracket$$

$$\llbracket \diamond \phi \rrbracket = \mathcal{C}(\llbracket \phi \rrbracket) \quad \leftarrow \text{Closure}$$

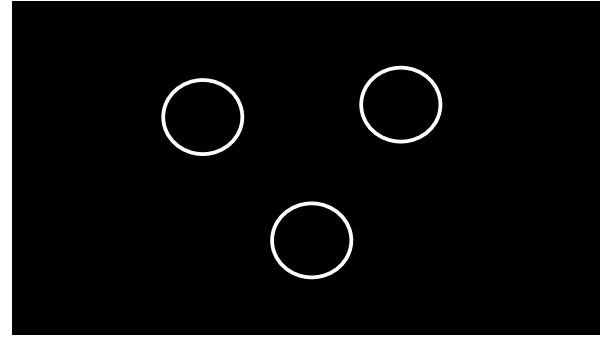
$$\llbracket \Box \phi \rrbracket = \mathcal{I}(\llbracket \phi \rrbracket) \quad \leftarrow \text{Interior (dual)}$$

$\mathcal{I}(A) = \overline{\mathcal{C}(\overline{A})}$

Topological model: a “coloured” topological space



Model



$$\Diamond blue \wedge \neg blue$$

(the points satisfying the formula are painted in white here)

Example (Ch. 5, Handbook of Spatial Logics)



red



$\Box red$



$\Diamond red$



$\neg \Box red \wedge \Diamond red$

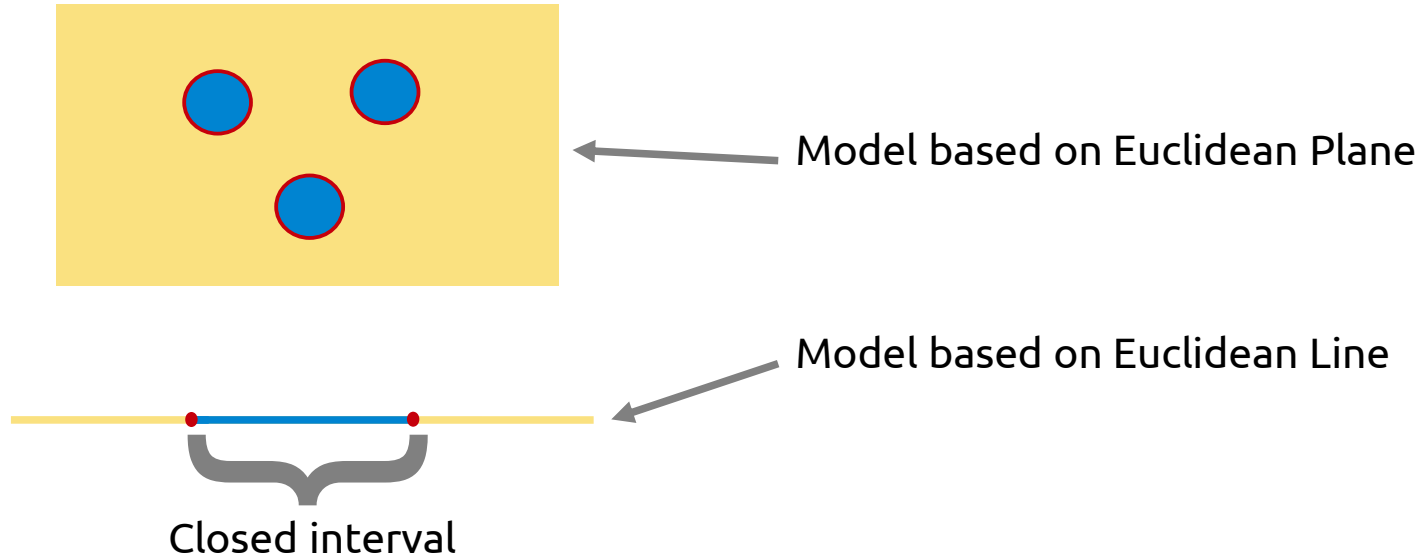


$\Diamond \Box red$



$red \wedge \neg \Diamond \Box red$

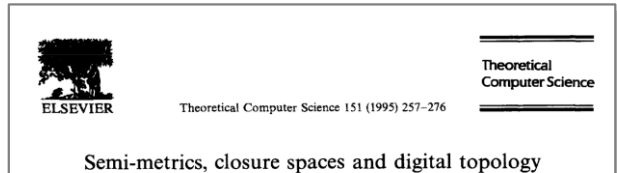
Logical equivalence, bisimilarity*



* Hennessy-Milner theorem! (See Handbook of Spatial Logics, Chapter 5).

Next please!

Generalise topology, for practical purposes



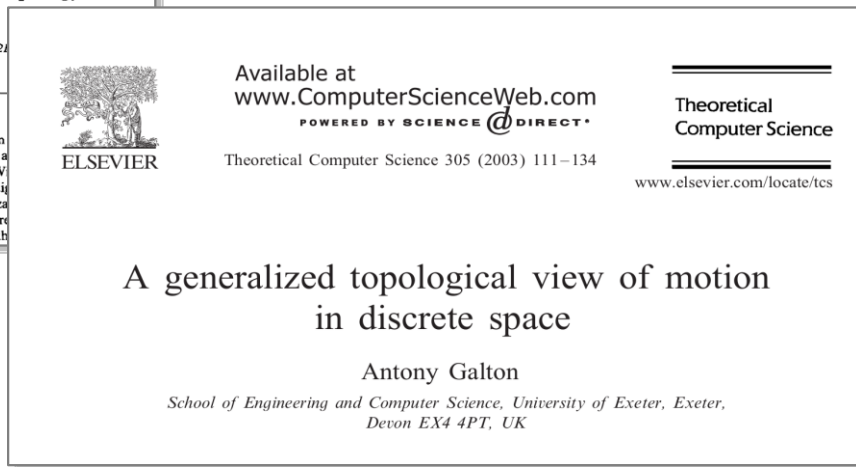
M.B. Smyth

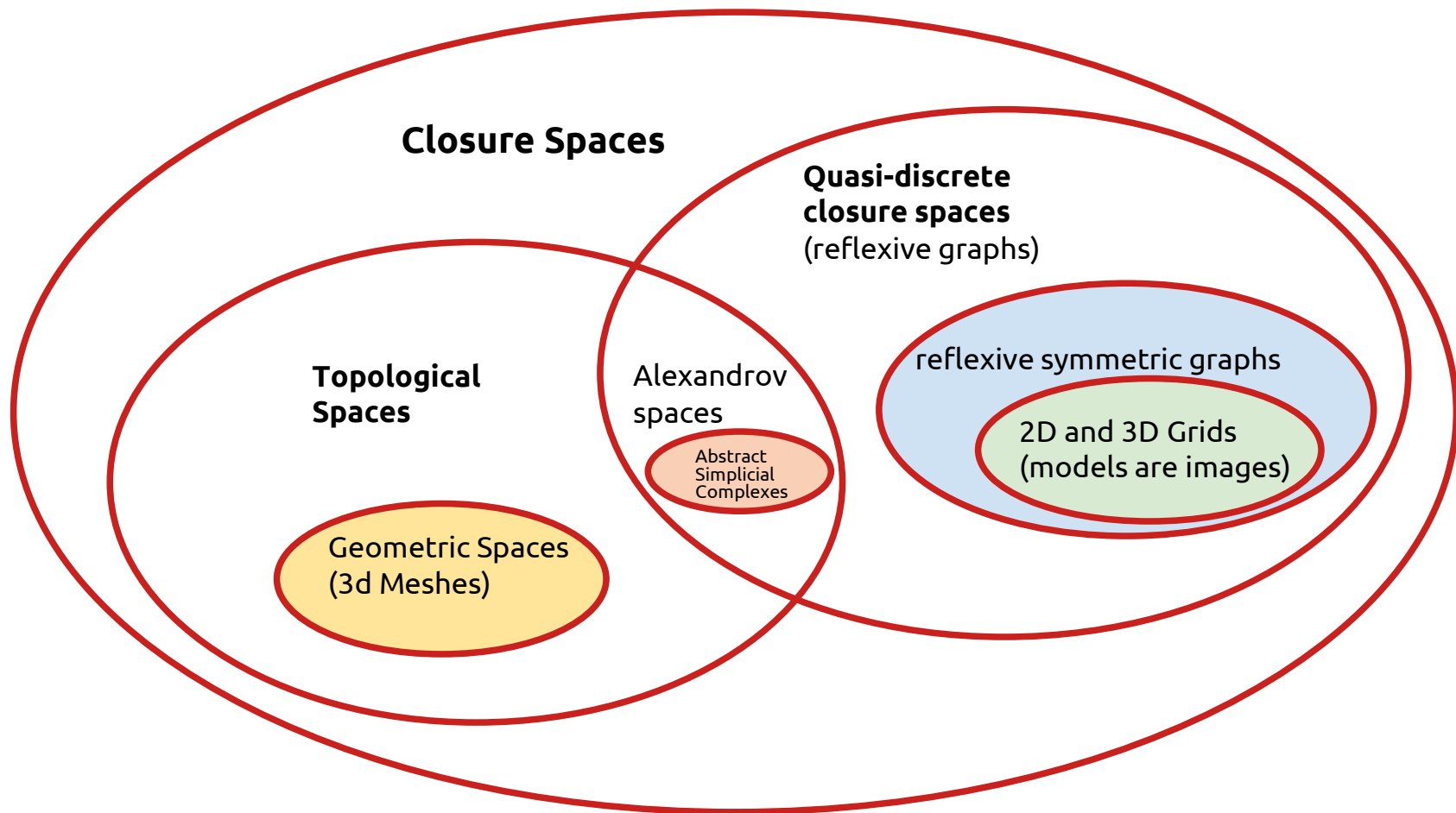
Department of Computing, Imperial College, London SW7 2BZ

Abstract

We investigate certain generalized topological structures, with the aim of providing a suitable framework within which to compare various types of topology (including tolerance geometry) with “ordinary” topology. Within this category of these structures, ordinary spaces arise as inverse limits of digital spaces.

In the first instance, the structures can be taken to be (a slight generalization of) digital spaces. The structures of this type which seem to be the most useful are realized as *topological graphs* (see Section 3). Domain equations for these





Čech Closure Spaces

A **closure** space is a pair (X, \mathcal{C})

- X is a set
- $\mathcal{C} : \mathcal{P}(X) \rightarrow \mathcal{P}(X)$

- $\mathcal{C}(\emptyset) = \emptyset$
- $A \subseteq \mathcal{C}(A)$
- $\mathcal{C}(A \cup B) = \mathcal{C}(A) \cup \mathcal{C}(B)$
- ~~$\mathcal{C}(\mathcal{C}(A)) = \mathcal{C}(A)$~~

Idempotency axiom

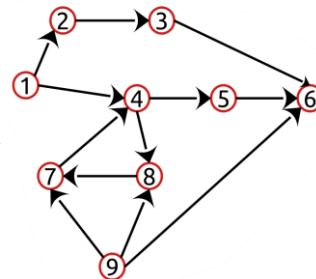
$$f : X_1 \rightarrow X_2 \text{ with } f(\mathcal{C}_1(A)) \subseteq \mathcal{C}_2(f(A))$$

Quasi-discrete closure spaces

Closure is **generated by a relation**

$$R \subseteq X \times X$$

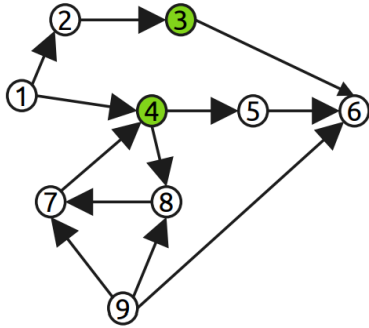
$$\mathcal{C}_R(A) = A \cup \{x \in X \mid \exists a \in A. (a, x) \in R\}$$



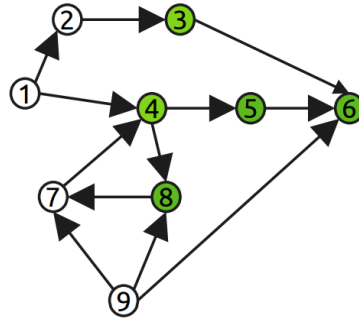
EQUIVALENT: closure of a set **determined by the singletons**: $\mathcal{C}(A) = \bigcup_{a \in A} \mathcal{C}(\{a\})$

[EQUIVALENT: **minimal neighbourhoods exist**], cf. *Alexandrov spaces*

$$\mathcal{C}_R(A) = A \cup \{x \in X \mid \exists a \in A. (a, x) \in R\}$$



A



$\mathcal{C}_R(A)$

*Clearly non
idempotent*

Example: graph paths as a continuous function from a Q.D.C.S.

$$[0, 1) \rightarrow (X, \mathcal{C}_R)$$

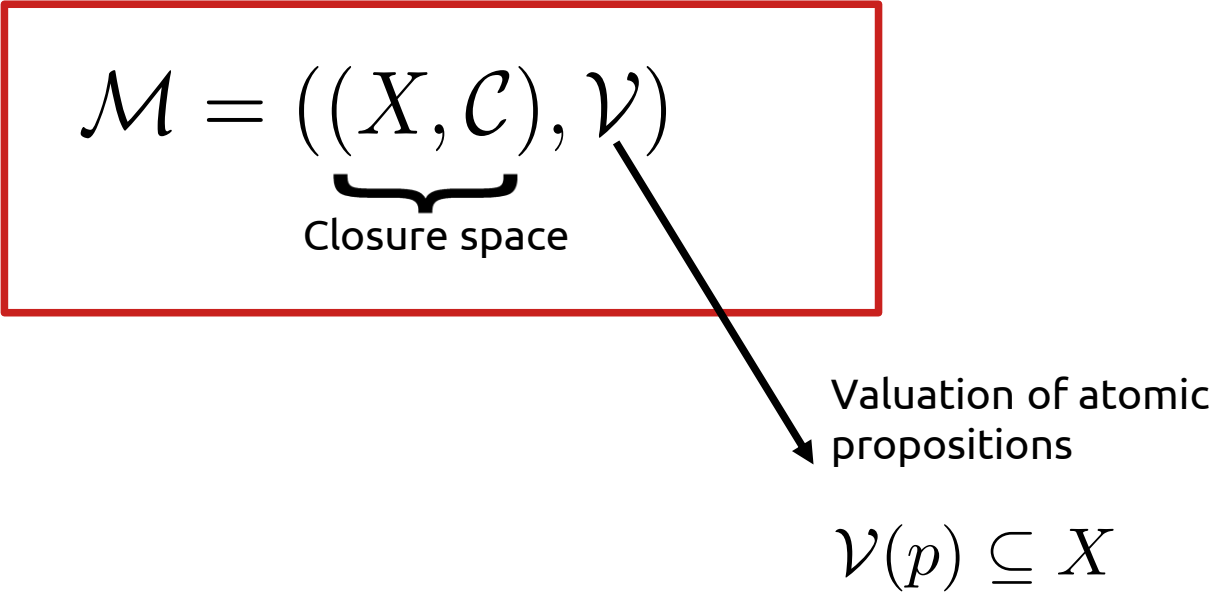
Continuous path

$$(\mathbb{N}, \mathcal{C}_{Succ}) \rightarrow (X, \mathcal{C}_R)$$

Graph theoretical path

Closure model

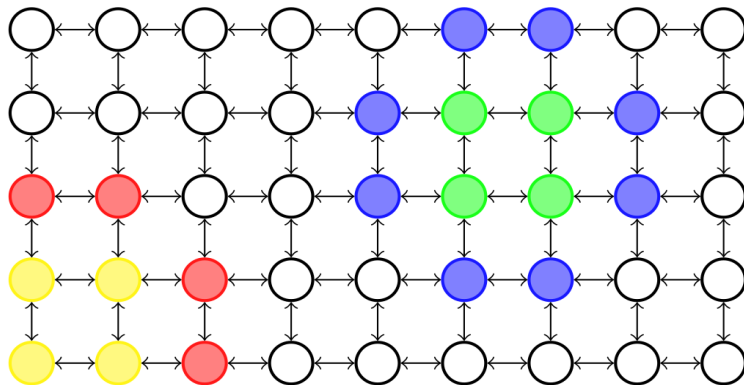
$$\mathcal{M} = ((X, \mathcal{C}), \mathcal{V})$$


Closure space

Valuation of atomic
propositions

$$\mathcal{V}(p) \subseteq X$$

Images are quasi-discrete closure models



images are graphs
regular grids with chosen adjacency



Spatial
Logic
for
Closure
Spaces

Syntax of the **Spatial Logic of Closure Spaces SLCS**

See V. Ciancia, D. Latella, M. Loretì, M. Massink, Model Checking Spatial Logics for Closure Spaces, LMCS 2016

$\Phi ::= p$ atomic proposition

| \top true

| $\neg\Phi$ negation

| $\Phi \wedge \Phi$ conjunction

| $\mathcal{N}\Phi$ closure

| $\Phi\mathcal{S}\Phi$ surrounded

$$\llbracket p \rrbracket = v(p)$$

$$\llbracket \top \rrbracket = X$$

$$\llbracket \neg \phi \rrbracket = \overline{\llbracket \phi \rrbracket}$$

$$\llbracket \phi \wedge \psi \rrbracket = \llbracket \phi \rrbracket \cap \llbracket \psi \rrbracket$$

$$\llbracket \mathcal{N}\phi \rrbracket = \mathcal{C}(\llbracket \phi \rrbracket)$$

*A spatial
“Until”*

$$\mathcal{M}, x \models \phi \mathcal{S} \psi$$

“To get out of ϕ , one must first pass by ψ ”

Satisfaction relation for surrounded

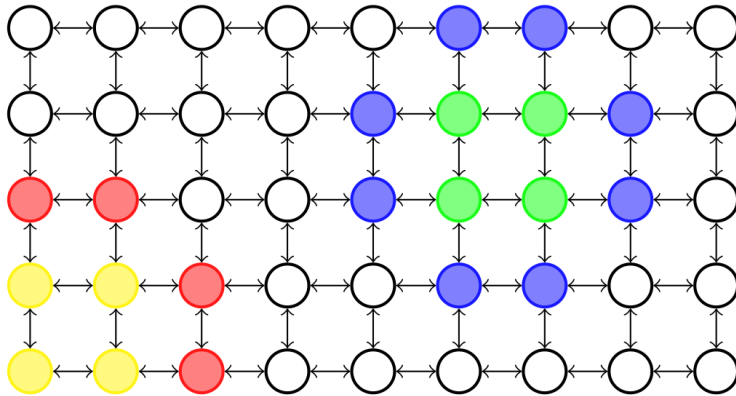
$$\mathcal{M}, x \models \phi \mathcal{S} \psi$$

if and only if

$$\forall p : \text{path}(x). \forall \ell.$$

$$\mathcal{M}, p_\ell \models \neg \phi \implies \exists k \in (0, \ell]. \mathcal{M}, p_k \models \psi$$

Example



yellow \mathcal{S} red

yellow $\wedge \mathcal{N}$ red

green \mathcal{S} violet

white \mathcal{S} red

Derived operators

$$\perp \triangleq \neg \top$$

$$\mathcal{I}\phi \triangleq \neg(\mathcal{N}\neg\phi)$$

$$\delta^-\phi \triangleq \phi \wedge (\neg\mathcal{I}\phi)$$

$$\phi_1 \mathcal{R} \phi_2 \triangleq \neg((\neg\phi_2) \mathcal{S}(\neg\phi_1))$$

$$\phi_1 \vee \phi_2 \triangleq \neg(\neg\phi_1 \wedge \neg\phi_2)$$

$$\delta\phi \triangleq (\mathcal{N}\phi) \wedge (\neg\mathcal{I}\phi)$$

$$\delta^+\phi \triangleq (\mathcal{N}\phi) \wedge (\neg\phi)$$

$$\mathcal{E}\phi \triangleq \phi \mathcal{S} \perp$$

Duality between “Reachable” and “Surrounded”

$$\phi_1 \mathcal{R} \phi_2 \triangleq \neg((\neg \phi_2) \mathcal{S}(\neg \phi_1))$$

Theorem:

$$\mathcal{M}, x \models \phi_1 \mathcal{R} \phi_2 \iff$$

$$\exists p : path(x). \exists \ell. \mathcal{M}, p_\ell \models \phi_2$$

$$\wedge \forall j. 0 < j \leq \ell \implies \mathcal{M}, p_j \models \phi_1$$

A “Modernised” Syntax*

$$\Phi ::= p \mid \top \mid \neg\Phi \mid \Phi \wedge \Phi \mid \mathcal{N}\Phi \mid \overrightarrow{\rho}\Phi[\Phi] \mid \overleftarrow{\rho}\Phi[\Phi] \mid \mathcal{D}_c\Phi$$



Two reachability operators
(forward & backward)



Distance modality

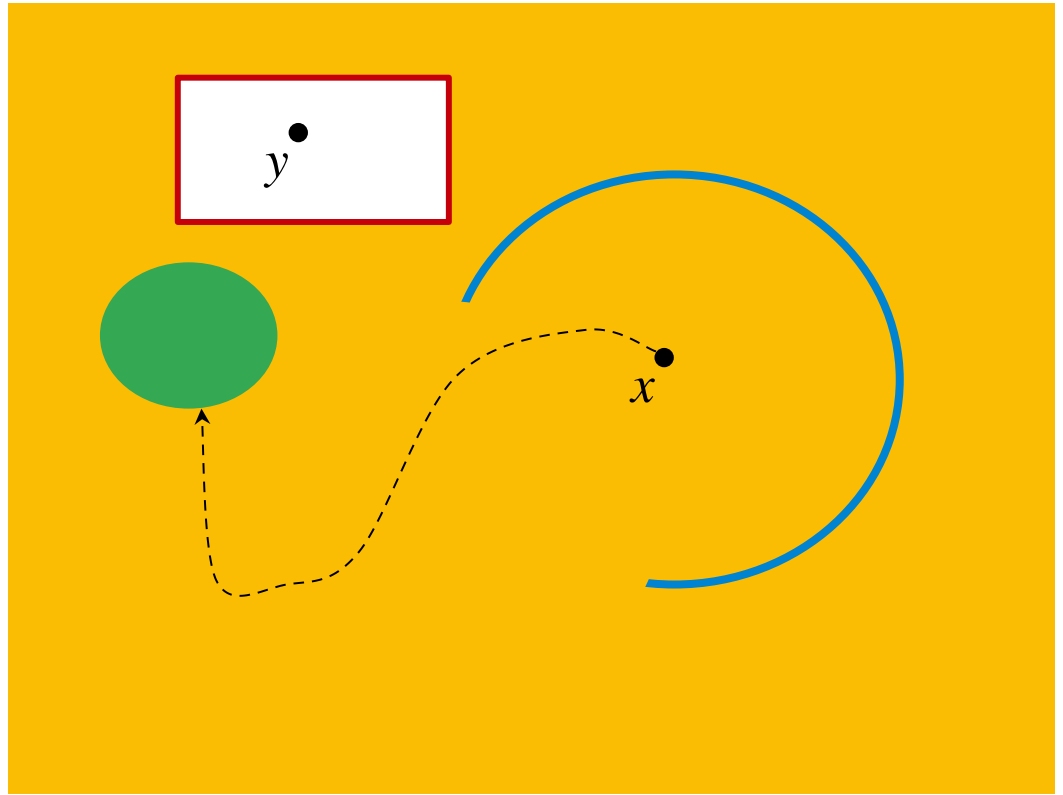
Interpretation of reachability

$$\begin{aligned}\mathcal{M}, x \models \vec{\rho} \Phi_1[\Phi_2] &\Leftrightarrow \text{there exist path } \pi \text{ and index } \ell \text{ such that} \\ &\quad \pi(0) = x \text{ and} \\ &\quad \pi(\ell) \models \Phi_1 \text{ and} \\ &\quad \text{for all } j \text{ such that } 0 < j < \ell : \\ &\quad \pi(j) \models \Phi_2; \\ \mathcal{M}, x \models \overleftarrow{\rho} \Phi_1[\Phi_2] &\Leftrightarrow \text{there exist path } \pi \text{ and index } \ell \text{ such that} \\ &\quad \pi(\ell) = x \text{ and} \\ &\quad \pi(0) \models \Phi_1 \text{ and} \\ &\quad \text{for all } j \text{ such that } 0 < j < \ell \\ &\quad \pi(j) \models \Phi_2.\end{aligned}$$

All our definitions depend upon the notion of path!

$M, x \models \vec{\rho} \text{ green}[\neg \text{red}]$

$M, y \not\models \vec{\rho} \text{ yellow}[\neg \text{red}]$



Open problem: a uniform definition of “path”

Logical Methods in Computer Science
Vol. 12(4:2)2016, pp. 1–51
www.lmcs-online.org

Submitted Feb. 8, 2016
Published Oct. 11, 2016

MODEL CHECKING SPATIAL LOGICS FOR CLOSURE SPACES*

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e-mail address: {vincenzo.ciancia, diego.latella, mieke.massink}@isti.cnr.it

^c Università degli Studi di Firenze and IMT Alti Studi, Lucca
e-mail address: michele.loreti@unifi.it

Definition 2.31. For each closure space (X, \mathcal{C}) , assume a chosen closure space \mathfrak{I} , equipped with a linear order \leq with bottom 0, and call path a continuous function $p : \mathfrak{I} \rightarrow (X, \mathcal{C})$. In particular, call *Euclidean path* any continuous function whose domain is the half-line $\mathbb{R}_{\geq 0} = \{x \in \mathbb{R} \mid 0 \leq x\}$, equipped with the Euclidean (topological) closure operator. Call *quasi-discrete path* any continuous function whose domain is the quasi-discrete closure space $(\mathbb{N}, \mathcal{C}_{Succ})$ where $(n, m) \in Succ \iff m = n + 1$. Whenever (X, \mathcal{C}) is an Euclidean topological space (resp. a quasi-discrete closure space), call *path* an Euclidean (resp. quasi-discrete) path whose codomain is (X, \mathcal{C}) .

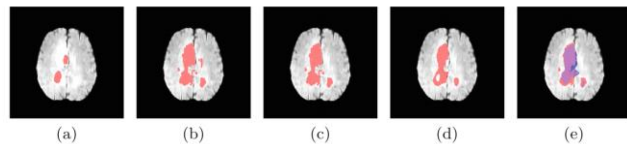
The logo consists of a large white circle with a dark blue border, centered on a dark blue background. Inside the circle, the text "VoxLogicA" is written in a bold, dark blue, sans-serif font.

VoxLogicA

VoxLogicA: some applications

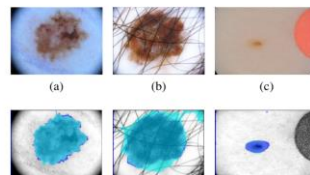
Segmentation of brain tumours

TACAS 2019 -- STTT 2020 -- ESMRMB 2017, 2019



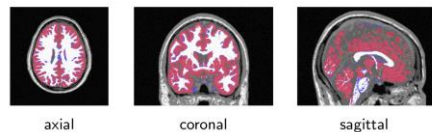
Segmentation of nevi - FORMALISE@ ICSE 2021

joint work with G. Belmonte, G. Broccia, D. Latella, M. Massink



Segmentation of white and grey matter

G. Belmonte, V. Ciania, D. Latella, M. Massink LNCS 11865



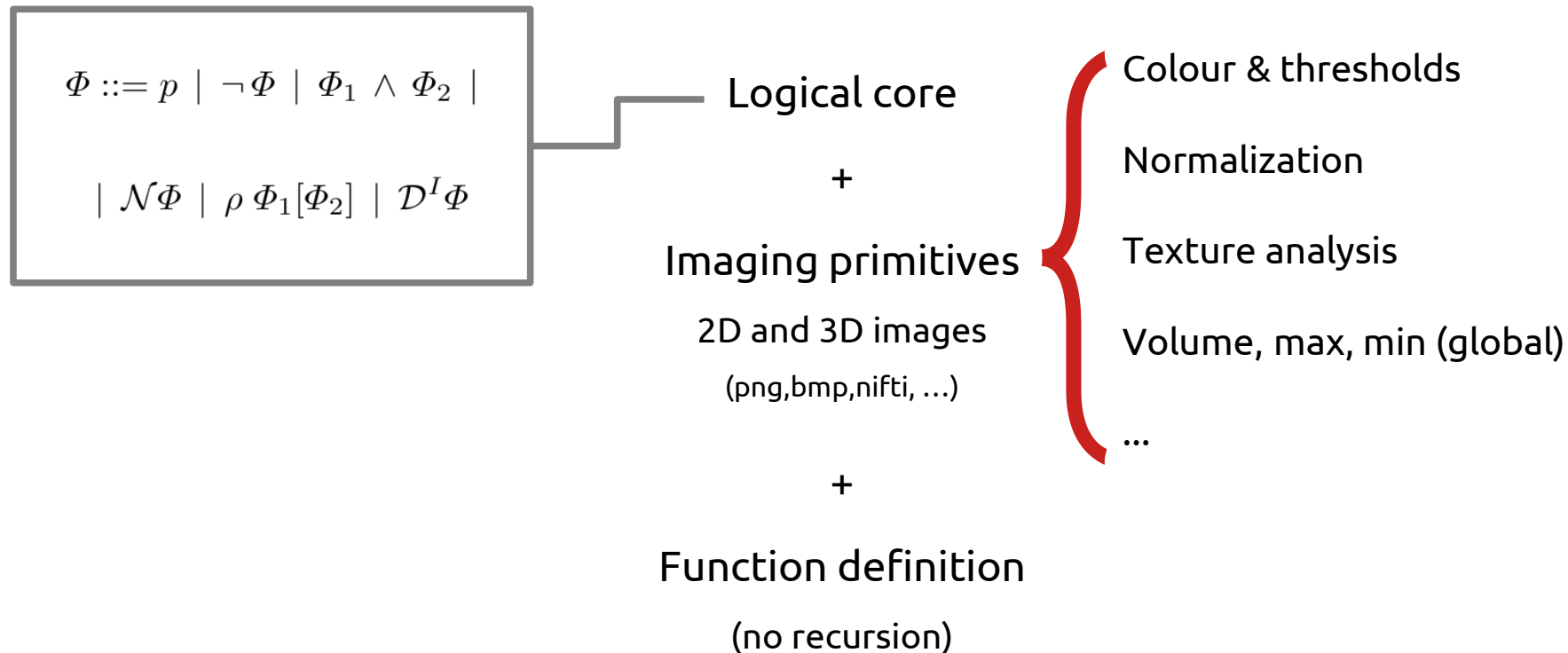
Analysis of video streams in real-time

L. Bussi, V. Ciania, F. Gadducci, D. Latella, M. Massink
DataMod 2021



Domain-specific language **ImgQL**

joint work with Gina Belmonte, Diego Latella, Mieke Massink (TACAS 2019)

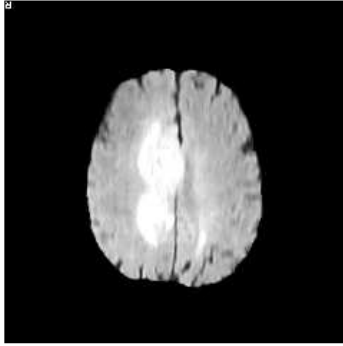




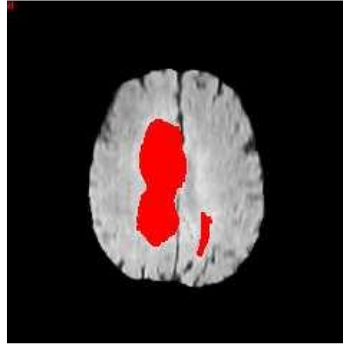
Dr. Maxime Menard's radiology
department, Hôpital Cochin, Paris 1914

Formal methods **In Medical Imaging**

Example from Radiotherapy: **Brain tumour segmentation**



Input



Output

*Usually 3D images
(multiple slices)*

- Highly specialised working time (hours, per case)
- Bottleneck in radiotherapy planning
- Precise guidelines / protocols to follow

Can we automate contouring?

Yes! State of the art is promising!

(hint: uses machine learning)

But rarely used in clinical & research workflows

Why?

State of the art

Menze, B.H.e.a.: The multimodal brain tumor image segmentation benchmark (brats). IEEE Transactions on Medical Imaging 2015

MICCAI-BraTS challenge since 2013

2017: circa 50 papers,
all machine learning

Some excellent results,

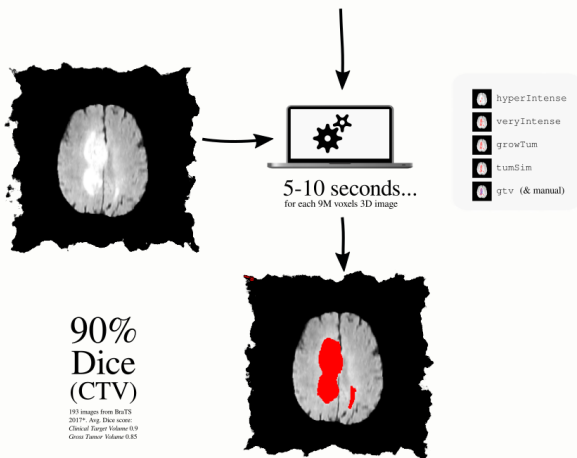
... but also **bad**
results!

Not so
easy

Tumour delineation for radiotherapy using VoxLogicA

Example: contouring glioblastoma in 10 lines

```
background removal {  
  load flair = "flair.nii.gz"  
  let background = touch(intensity(flair) <. 0.1, border)  
  let brain = complement(background)  
  
  thresholding {  
    let normFlair = percentiles(intensity(flair), brain)  
    let hyperIntense = filter(5.0, normFlair >. 0.95)  
    let veryIntense = filter(2.0, normFlair >. 0.86)  
  
    semantic noise removal {  
      let growTum = grow(hyperIntense, veryIntense)  
  
      texture similarity {  
        let tumSim = similarTo(growTum)  
        let tumStatCC = filter(2.0, tumSim >. 0.6)  
        let gtv = grow(growTum, tumStatCC)  
      }  
    }  
  }  
}
```



Accuracy:

in par with humans
and best-in-class machine learning

Validation:

circa 200 cases, ground truth available
MICCAI-BraTS Challenge 2017 dataset

Speed:

<5 seconds for each 3d image (9 milion
voxels)
on a intel Core-i7 desktop computer

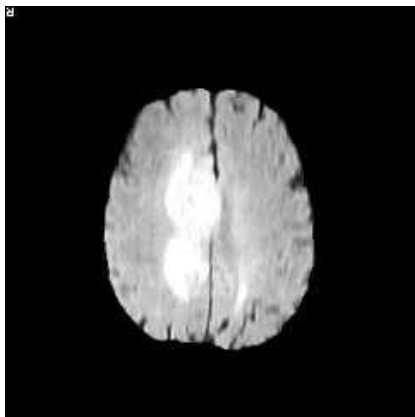
A few reasons why

- Accountability.
- Compliance to protocols and guidelines (a case for Formal Methods!).
- Quality assurance.
- Innovative ideas do not have training data.

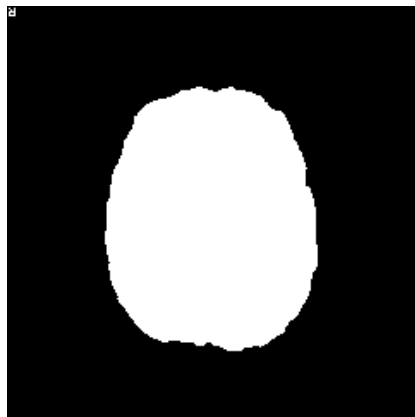
We can do it in three slides...

Step 1: Background removal

```
let background = touch(intensity <. 0.1, border)  
let brain = !background
```



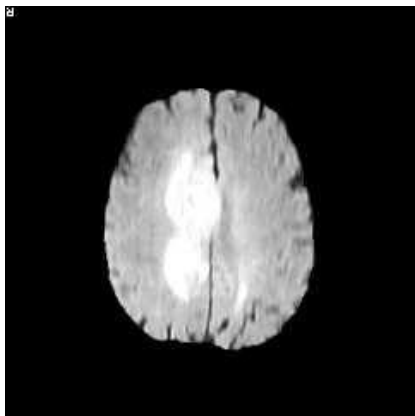
intensity



brain

Step 2: Thresholding

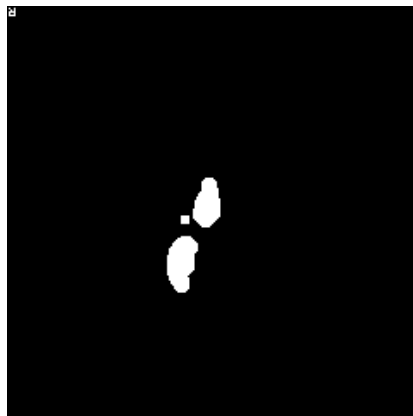
```
let normIntensity = percentiles(intensity,brain)
let hyperIntense = filter(5.0,intensity >. 0.95)
let veryIntense = filter(2.0,intensity >. 0.85)
```



intensity



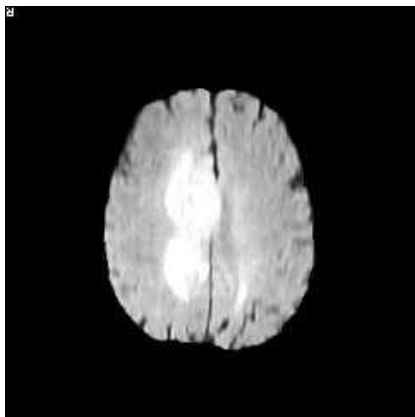
veryIntense



hyperIntense

Step 3: Noise removal, semantically

```
let growTum = grow(hyperIntense,veryIntense)
let tumSim = filter(2.0,similarTo(growTum) >. 0.6)
let gtv = grow(growTum,tumSim)
```



intensity



growTum



tumSim

Results

VoxLogicA vs manual segmentation

BRATS17 dataset

Type	Dice 193 cases	Dice 210 cases
Gross Tumour Volume	0.85(0.10)	0.81(0.18)
Clinical Target Volume	0.90(0.09)	0.87(0.15)

Higher is not better!

Human factors, decisions, uncertainty. 0.90 is actually “perfect”.

$$\text{Dice}(X, Y) = \frac{2 \cdot |X \cap Y|}{|X| + |Y|} = \frac{2 \cdot \text{TruePos}}{2 \cdot \text{TruePos} + \text{FalsePos} + \text{FalseNeg}}$$

18 selected techniques of BraTS17

(of 50 papers, those submitting > 100 cases)

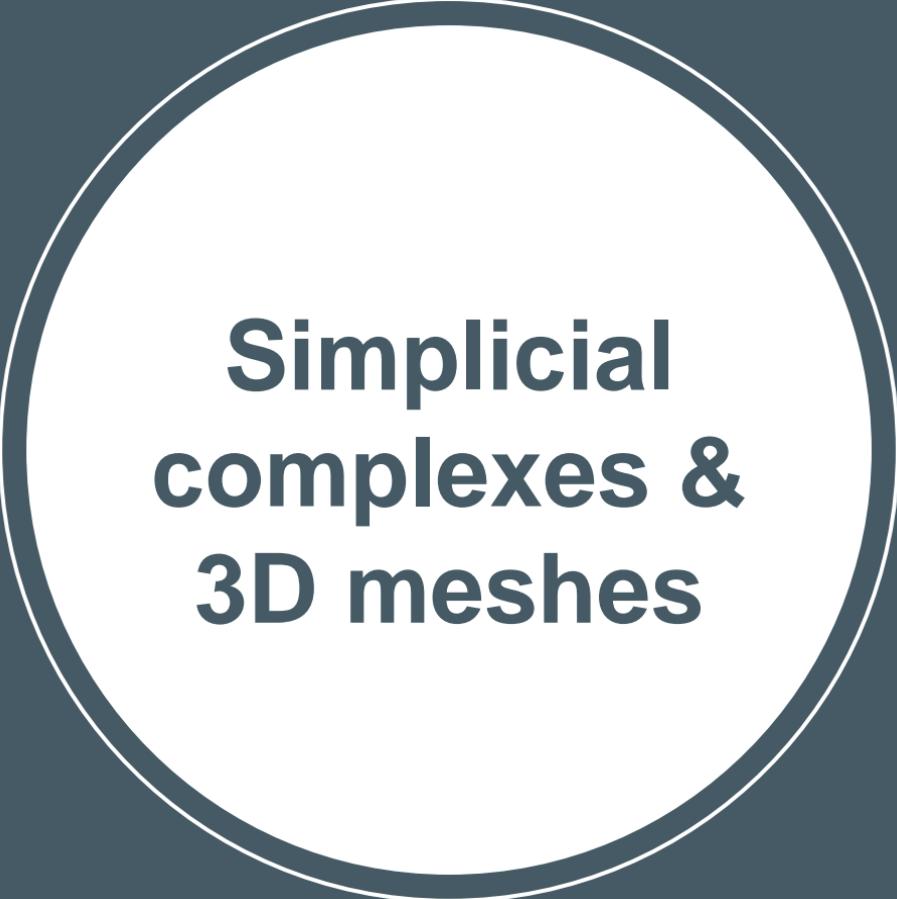
Dice GTV avg & range: **0.88 (0.64 to 0.96)**

Work in progress

Hybrid AI via model checking






Use the state-of-the-art **nnUNet** together with logical specifications

Combine black-box and explainable procedures to get to human-centric AI in medical imaging



**Simplicial
complexes &
3D meshes**

GEOMETRIC MODEL CHECKING OF CONTINUOUS SPACE

NICK BEZHANISHVILI ^a, VINCENZO CIANCIA ^b, DAVID GABELAIA ^c,
GIANLUCA GRILLETTI ^d, DIEGO LATELLA ^b, AND MIEKE MASSINK ^b

^a Institute for Logic, Language and Computation, University of Amsterdam, The Netherlands
e-mail address: n.bezhanishvili@uva.nl

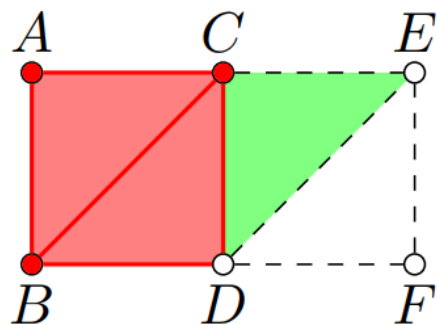
^b Istituto di Scienza e Tecnologie dell'Informazione “A. Faedo”, Consiglio Nazionale delle Ricerche,
Pisa, Italy
e-mail address: {vincenzo.ciancia,diego.latella,mieke.massink}@cnr.it

^c TSU Razmadze Mathematical Institute, Tbilisi, Georgia
e-mail address: gabelaia@gmail.com

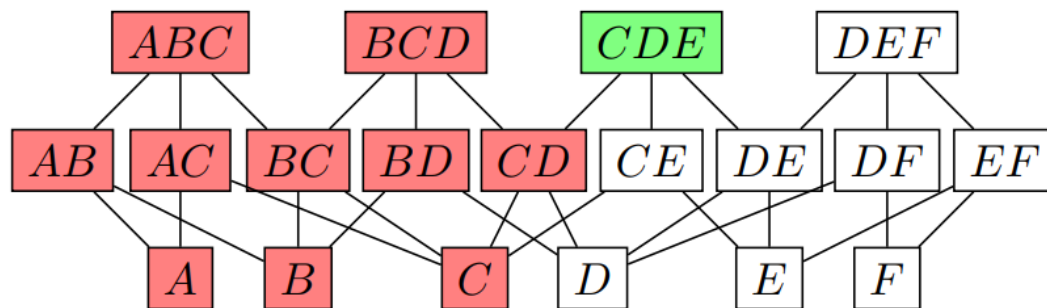
Model checking polyhedra (and 3D meshes)*

joint work with N. Bezhanisvili, D. Gabelaia, G. Grilletti, D. Latella, M. Massink
“Geometric Model Checking of Continuous Space”, <https://arxiv.org/abs/2105.06194>

Idea: the face graph of a **simplicial complex** is sufficient to compute the topological semantics of SLCS



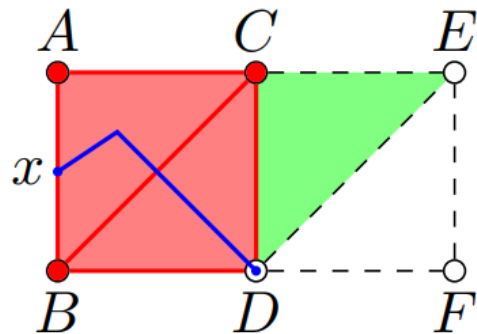
(A)
Polyhedral model \mathcal{X}



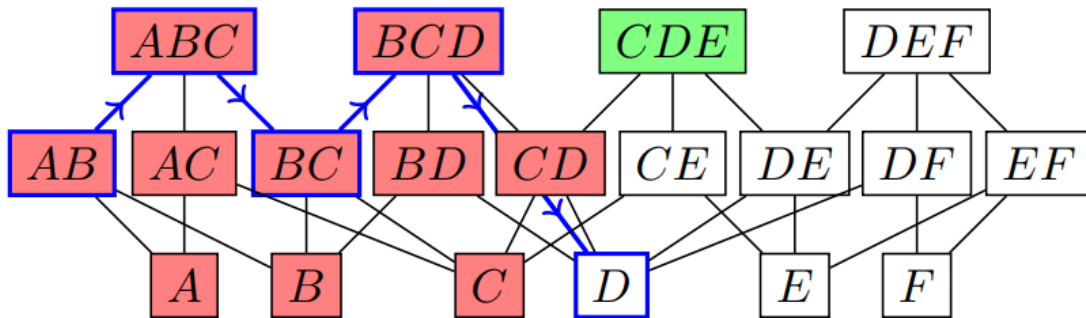
(B)
Kripke model $\mathcal{M}(\mathcal{X})$

Model checking reachability on the face graph

joint work with N. Bezhanisvili, D. Gabelaia, G. Grilletti, D. Latella, M. Massink
“Geometric Model Checking of Continuous Space”, <https://arxiv.org/abs/2105.06194>



(A)
PL-path π'



(B)
Kripke Model $\mathcal{M}(\mathcal{X})$

- **Prototype**
 - still not optimized
 - custom .json format
 - simple visualizer
- Up to 400k polygons “.obj” mesh (tested so far)
- 20k polygons: 5 seconds
400k polygons: 2 minutes

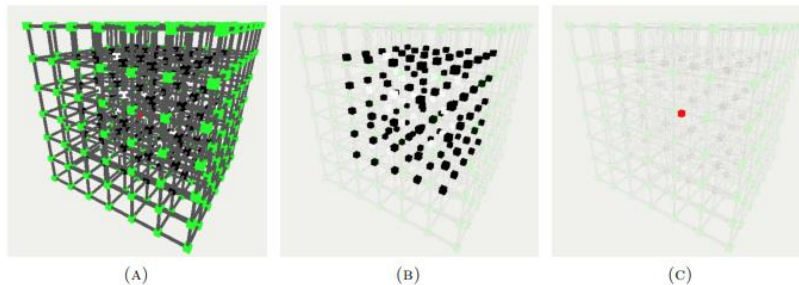
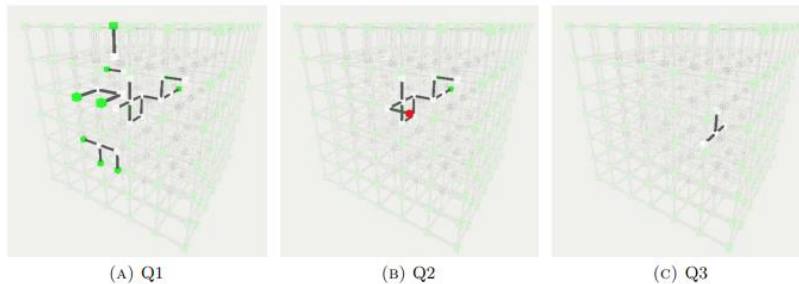


Figure 10: 3Dmaze (10a), black and white (10b) and red rooms (10c) in the 3D maze.



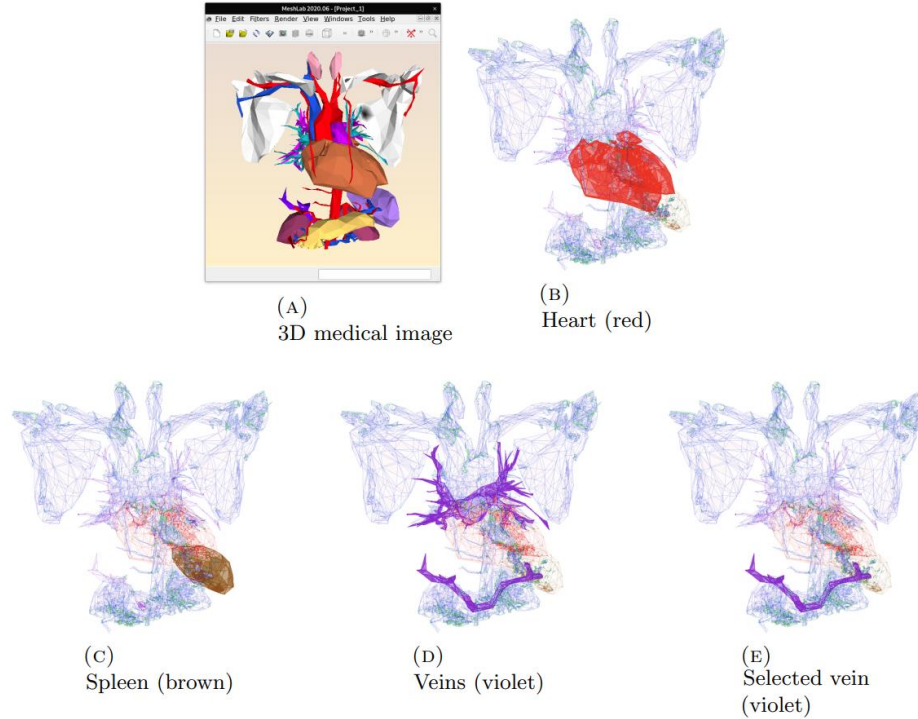


Figure 13: (13a) A 3D medical illustration, courtesy of www.sketchfab.com (copyright: COEUR et vaissaux by Chair_Digital_Anatomy – The Unesco Chair of digital anatomy (Paris University) – is licensed under Creative Commons Attribution, see <https://creativecommons.org/licenses/by/4.0/legalcode>)



Bisimilarity

Weak Simplicial Bisimilarity for Polyhedral Models and SLCS η

Conference paper | First Online: 13 June 2024

pp 20–38 | [Cite this conference paper](#)

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**Formal Techniques for
Distributed Objects,
Components, and...**

(FORTE 2024)

[Nick Bezhanishvili](#), [Vincenzo Ciancia](#), [David Gabelaia](#), [Mamuka Jibladze](#), [Diego Latella](#), [Mieke Massink](#)  & [Erik P. de Vink](#)


 Part of the book series: [Lecture Notes in Computer Science](#) ((LNCS, volume 14678))


Minimisation of Spatial Models Using Branching Bisimilarity

Conference paper | First Online: 03 March 2023

pp 263–281 | [Cite this conference paper](#)

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Formal Methods

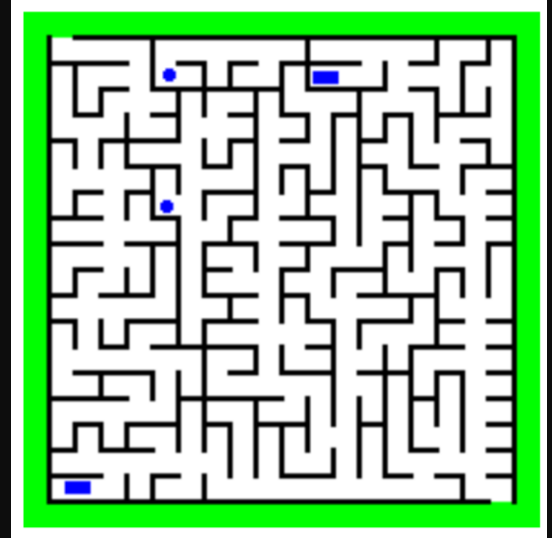
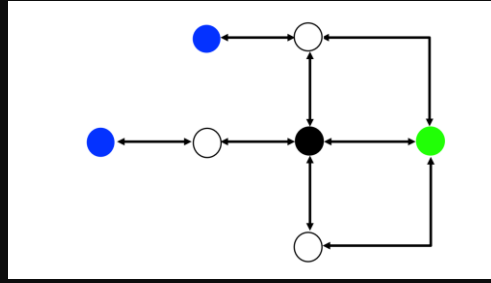
(FM 2023)

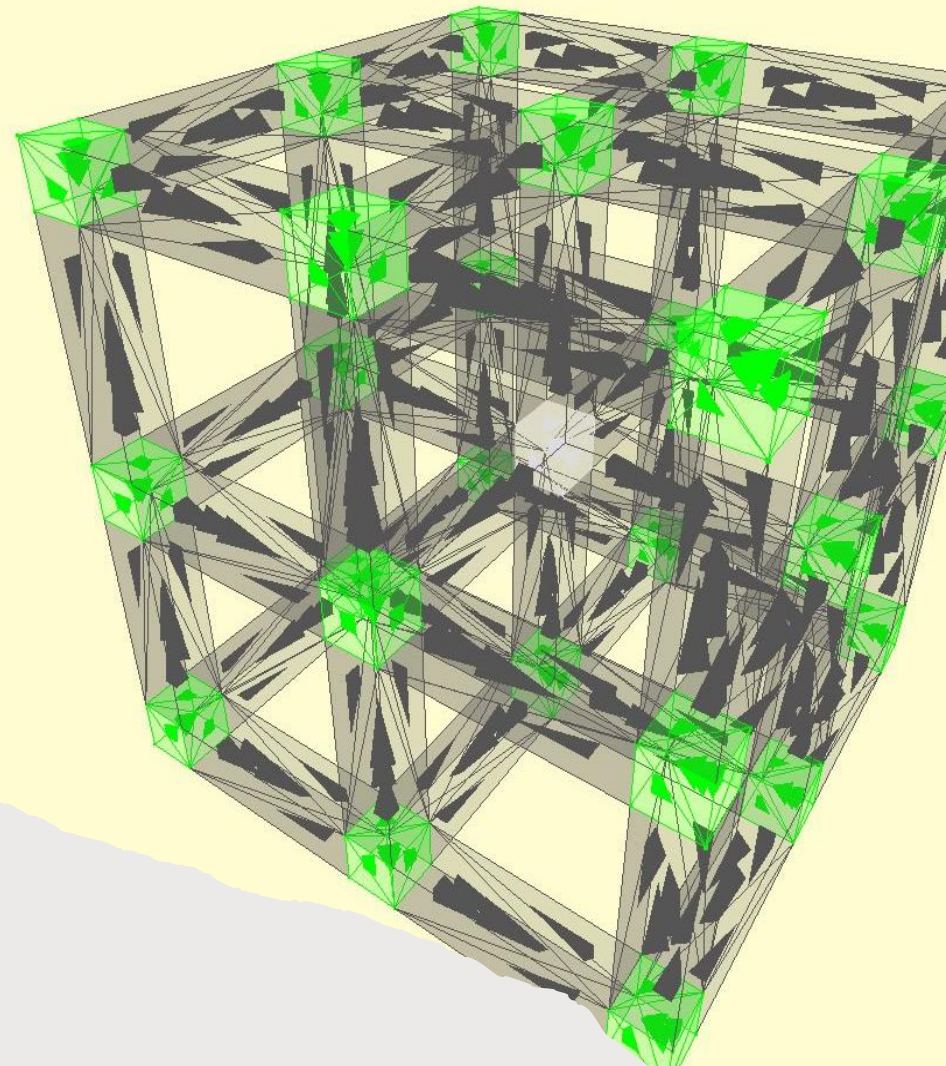
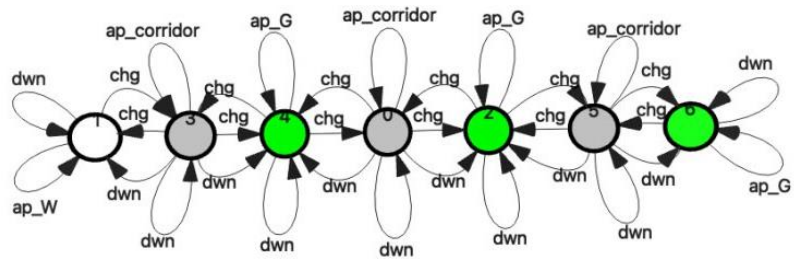
[Vincenzo Ciancia](#), [Jan Friso Groote](#), [Diego Latella](#), [Mieke Massink](#)  & [Erik P. de Vink](#)

Sections

Fig

Bisimilarity for images



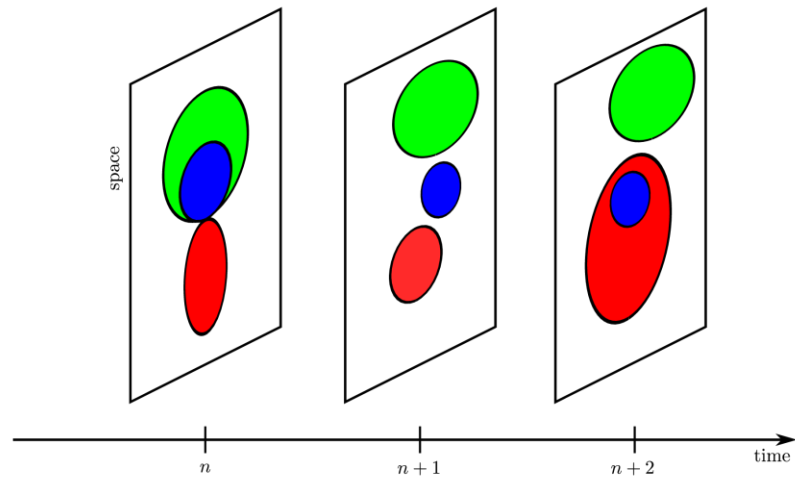


Bisimilarity for polyhedra

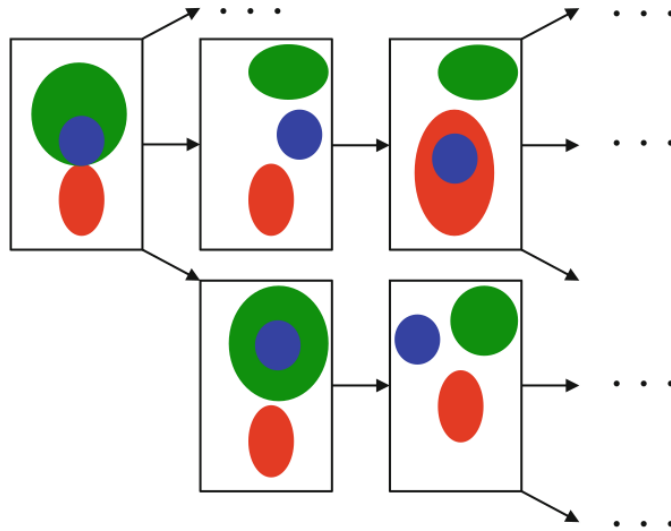


**Spatio-Temporal
Logic & model
checking**

Snapshot traces



Snapshot models



STLCS
=
SLCS x CTL

Φ	::=	\top	[TRUE]
		p	[ATOMIC PREDICATE]
		$\neg\Phi$	[NOT]
		$\Phi \wedge \Phi$	[AND]
		$\mathcal{N}\Phi$	[NEAR]
		$\Phi\mathcal{S}\Phi$	[SURROUNDED]
		$\mathbf{A}\varphi$	[ALL FUTURES]
		$\mathbf{E}\varphi$	[SOME FUTURE]

φ	::=	$\mathcal{X}\Phi$	[NEXT]
		$\mathbf{F}\Phi$	[EVENTUALLY]
		$\mathbf{G}\Phi$	[GLOBALLY]
		$\Phi \mathcal{U} \Phi$	[UNTIL]

Spatio-temporal model checking of vehicular movement in public transport systems

[Vincenzo Cincia](#) , [Stephen Gilmore](#), [Gianluca Grilletti](#), [Diego Latella](#), [Michele Loreti](#) & [Mieke Massink](#)

International Journal on Software Tools for Technology Transfer 20: 289–311 (2018) | [Cite this article](#)



Università degli Studi di Pisa

DIPARTIMENTO DI MATEMATICA

TESI DI LAUREA MAGISTRALE

Spatio-Temporal Model Checking: Explicit and Abstraction-Based Methods

Candidato:
Gianluca Grilletti
Matricola 483633

Relatore:
Ugo Montanari

Correlatore:
Vincenzo Cincia

Anno Accademico 2015–2016



[International Symposium on Leveraging Applications of Formal Methods](#)

ISOFA 2016: [Leveraging Applications of Formal Methods, Verification and Validation: Foundational Techniques](#) pp 657–673 | [Cite as](#)

A Tool-Chain for Statistical Spatio-Temporal Model Checking of Bike Sharing Systems

Authors [Authors and affiliations](#)

Vincenzo Cincia , [Diego Latella](#), [Mieke Massink](#), [Rytis Paškuskas](#), [Andrea Vandin](#)

[Home](#) > [Dynamic Logic. New Trends and Applications](#) > Conference paper

A Spatial Logic with Time and Quantifiers

Conference paper | First Online: 13 January 2024

pp 1–19 | [Cite this conference paper](#)

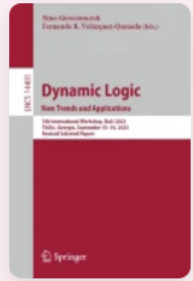
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[Dynamic Logic. New Trends and Applications](#)

(DaLí 2023)

Sections	Figures	References
Abstract		
Introduction		

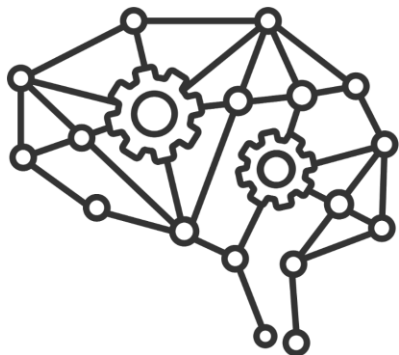
Spatio-temporal models coming from Graph Rewriting?



**Ongoing
work**

ISOLA 2024

Training as a primitive



Training can be **freely mixed with logical primitives**

Trained models are cached

the result can be used anywhere in the analysis specification
(«source code»)

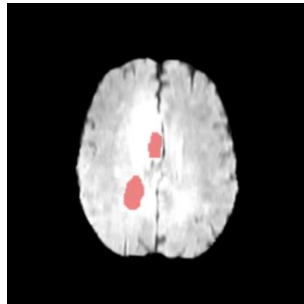
Distributed execution with **trusted** parties

Provenance tracking!

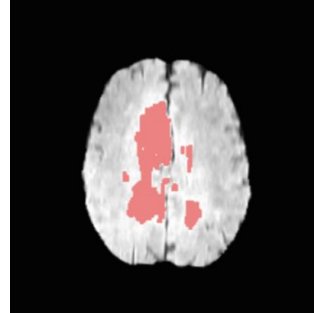
```
let tumour_model = train(tumour_dataset_augmented)
let oedema_model = train(oedema_dataset_augmented)
```

Prediction as a primitive

```
let ML_tumour_core = predict(tumour_model,validation_dataset)  
let ML_oedema = predict(oedema_model,validation_dataset)
```



Tumour core



Oedema, noisy

Nick Bezhanishvili, Laura Bussi, Vincenzo Ciancia, David Fernández-Duque, David Gabelaia:

Logics of Polyhedral Reachability.

AiML 2024: 187-204

- Complete axiomatisation of polyhedral logic
- **Task: use graph rewriting** techniques to minimize formulas
- **Bouns points: minimize cost**
predict the cost of each operator, **in-context**

VoxLogicA-GPU:

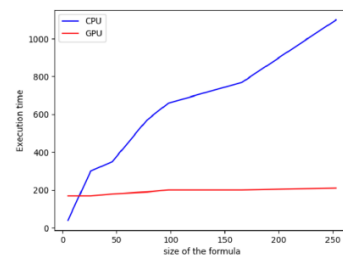
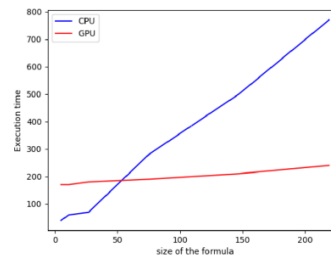
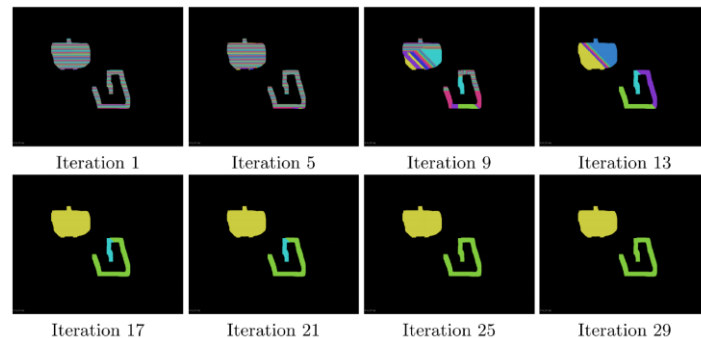
joint work with Laura Bussi, Fabio Gadducci (FORTE 2021)

Massively parallel implementation.

Reachability via Connected Components.

Consistent speed up.

Garbage collection.



Performance, garbage collection

No. of Tasks	CPU	GPU		GPU-GC	
	Time	Time	Speed-up	Time	Speed-up
11	410ms	190ms	2.15	200ms	2.05
35	1470ms	190ms	7.73	230ms	6.39
67	1800ms	190ms	9.47	230ms	7.82
195	8200ms	200ms	41.00	320ms	25.62
259	10900ms	210ms	51.90	360ms	30.27
1027	43600ms	350ms	124.57	980ms	44.48
4099	174600ms	Out of memory	-	4100ms	42.58
8195	479000ms	Out of memory	-	12000ms	39.91

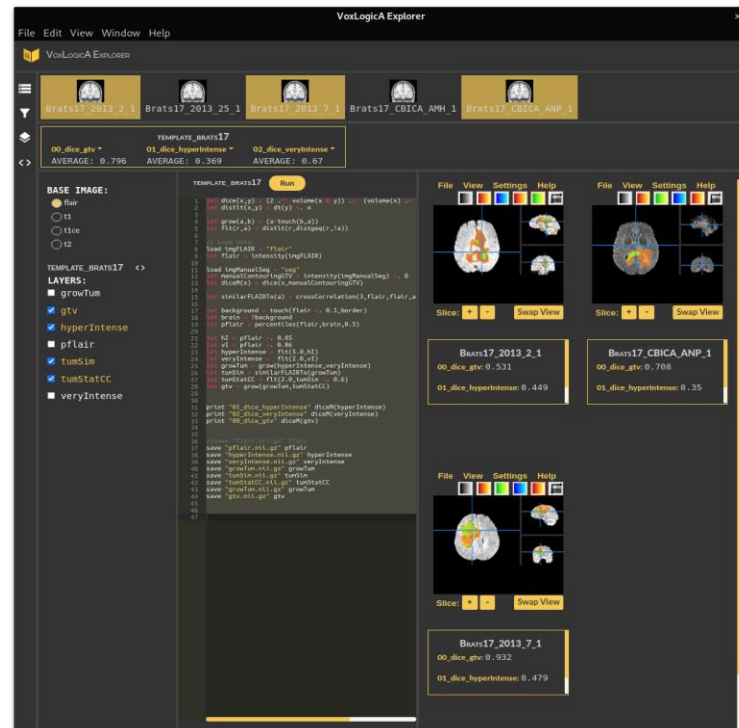
HCI: Dataset-oriented User interface (work in progress)

joint work (in progress) with G. Broccia, D. Latella, M. Massink

- **HCI methods**

to improve understanding
of logic formulas by end users.

- Reduce the **cognitive load**
of imaging tasks!



Some References

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<https://www.springer.com/la/book/9781402055867>

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<https://lmcs.episciences.org/2067> (definition of SLCS, LMCS 2016)

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<http://www.voxlogica.org> Official web site.

<https://github.com/vincenzoml/VoxLogicA> Free and Open Source, Apache2-Licensed.

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Thanks* for **Listening!**

*Also from the coauthors...

F. Banci Buonamici, D. Basile, G. Belmonte, N. Bezhanisvili, G. Broccia, L. Bortolussi, L. Bussi, E. de Vink, D. Gabelaia, F. Gadducci, S. Gilmore (RIP), S. Gnesi, G. Grilletti, M. Jibladze, D. Latella, M. Loreti, M. Massink, L. Nenzi, R. Paskauskas, G. Spagnolo, M. ter Beek, A. Vandin.