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Problem Chosen

A

2009 Mathematical Contest in Modeling (MCM) Summary Sheet

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Pseudo-Finite Jackson Networks and Simulation: A Roundabout Approach To Traffic Control

Roundabouts, a foreign concept a generation ago, are an increasingly common sight in the United States. In principle, they are simple and effective methods of traffic control that reduce accidents and delays. A natural question for today's traffic engineer is "What is the best method to control traffic flow within a roundabout?" Using mathematics, it is possible to distill the essential features of traffic entering and exiting a roundabout into a system which can be analyzed, manipulated, and optimized for a wide variety of situations. As the metric of effective flow control, we choose time spent in the system.

We use the concept of Jackson networks to create an analytic model. A roundabout can be thought of as a network of queues, where the entry queues receive external arrivals, which move into the roundabout queue before exiting the system. To form this model, we must assume that an equilibrium state may exist and that arrival rates are constant. The Jackson network is useful because, if certain conditions are met, a closed form stationary distribution may be found. Furthermore, the parameters for this system may be obtained empirically: how often cars arrive at an entrance (external arrival rate), how quickly they may enter the roundabout (internal arrival rate), and how quickly they exit (departure rate). We account for the traffic control method by thinning the internal arrival process with a "signal" parameter that represents the fraction of time that a signal light is green.

One pitfall of this formulation is that restricting the capacity of the roundabout queue to a finite limit will destroy the useful analytic properties of the system. We utilize a "pseudo-finite" capacity formulation, where we allow the roundabout queue to receive a theoretically infinite number of cars, but optimize over the signal parameter to create a steady state in which a minimal number of waiting cars is overwhelmingly likely. Using lower bound calculations, we prove that the yield sign produces the optimal behavior for all sets of allowed parameters. The analytic solution, however, sacrifices important aspects of a real roundabout, such as time-dependent flow.

To test the theoretical conclusions, we develop a computer simulation which incorporates more parameters: roundabout radius, car length, car spacing, car velocity inside the roundabout, periodicity of traffic signaling, and time-dependent input flow rates. The simulation uses these parameters to stochastically model individual vehicles as they move through the system, resulting in more realistic output. In addition to comparing yield and traffic signal control, we also examined variable distributions of input rates, non-standard roundabout constructions, and the relationship between traffic flow volume, radius size, and average total time. Our simulation is, however, limited to a single-lane roundabout. This model is also compromised by the very stochasticity that enhances its realism. Since it is non-deterministic, there is a good deal of randomness which may mask the true behavior. Another drawback of the stochastic formulation is that, as we show, the computational cost of mathematical minimization is enormous. We use our background research to set up the model as an empirical experiment to verify the hypothesis that a yield sign is almost always the best form of flow control.

PSUEDO-FINITE JACKSON NETWORKS AND SIMULATION: A ROUNDABOUT APPROACH TO TRAFFIC CONTROL

Team # 4806

February 9, 2009

1 Introduction

A report from the Wisconsin Department of Transportation noted that "to many, the idea of replacing four way signaling with a roundabout seems like replacing hot dogs with crepes at the ballpark"[3]. For many Americans, the roundabout[1] is a foreign idea- even though the first unidirectional circular traffic installation was actually built in New York in 1903. Roundabouts fell out of favor in the U.S. by mid century, as recent studies show how much safer and more efficient they can be, there has been a resurgence in roundabout construction [7]. Half the states in the U.S. now have roundabouts, for a total over 1000 installations. In many types of intersections, roundabouts improve traffic flow, decrease the occurrence of accidents, and reduce the amount of gasoline wasted during idling [6]. In fact, a U.S. study indicated that, on average, fatal crashes decreased 90% after traditional traffic lights were replaced by roundabouts [5].

A crucial aspect of efficiency and safety is the method of entry. A modern roundabout (Figure 1) is distinguished by two key characteristics: incoming traffic yields to traffic within the circle, and incoming traffic changes direction to some extent. Initially, roundabout entry rules were inconsistent and varied from place to place. Until the 1920s, "yield-to-right" regulations gave the right-of-way to incoming cars rather than to those within the circle. This tended to cause "locking" and delays in traffic circles at high traffic volumes. British studies indicated that adopting "priority-to-the-circle" rules allowed more cars to to move through the circle more quickly and diminished accident rates. The deflection of entering traffic serves to prevent excessive speed within the roundabout and to further reduce incidence of accidents [7].

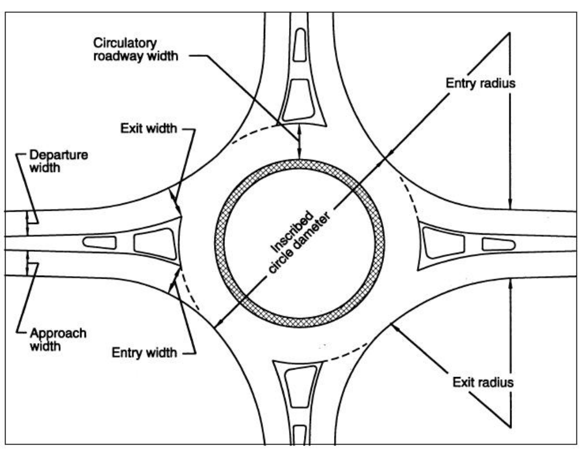


Figure 1: Roundabout and geometric parameters [8]

[1] Although formal definitions make a distinction between the terms "roundabout" and "traffic circle", much of the literature uses the terms interchangeably and we will do the same.

Within the framework of the "priority-to-the-circle" rule, roundabout entry may be governed in different ways. The simplest and most common method is a "Yield" sign placed at each entry point to indicate that incoming cars must relinquish the right-of-way to cars already in the circle. The U.S. Department of Transportation advises that roundabouts "should never be planned for metering or signalization" [8]. Nonetheless, although roundabouts are in principle very simple systems, they are often besieged with complications. These intricacies include geometric irregularities, discrepancies between inflow volume at different entry points, and temporal traffic flow fluctuation.

In order to evaluate the effectiveness of different methods of input control we develop a mathematical model for traffic flow within roundabouts. We first introduce the assumptions utilized in determining the key parameter inputs and developing a metric for "effectiveness." We subsequently formulate and solve a simple analytic model of networked queues in an equilibrium state. After discussing the limitations of the analytic model, we adapt this model into a computer simulation which allows greatly enhanced flexibility and complexity. This computer simulation allows for detailed analysis and may be used by traffic engineers to optimize the flow-control method.

1.1 Assumptions

The following assumptions were used in both the analytic and computer models:

Exponential Arrivals/Departures: This model is based on arrivals and departures which follow a Poisson process with exponentially distributed interarrival times. The Poisson model is advantageous because, not only is it mathematically convenient, it is widely accepted as a realistic model for many situations involving random arrivals [4]. Rates for the Poisson process may be empirically obtained by taking the average number of arrivals or departures in a certain period of time.

Local Variable Selection: In our model, we use only variables which contribute significantly to traffic flow patterns on a regular basis. External forces such as weather, special events, or acts of God may alter the system; however, we do not address these factors.

Unbounded Output: When embedded in actual road systems, blockages in nearby areas may affect the output rate from the roundabout under study, causing output from one or more exits to slow down or cease. Although this phenomenon significantly alters flow in some locations, regulation of input rates in one roundabout would be unable to correct for blockages in another part of the system. Hence, this factor unnecessarily increases the complexity of the model, and we assume that cars are always able to leave the roundabout once they reach their exit.

Yield and Stop Sign Equivalence: In our model, we will only examine the effect of a yield sign versus a traffic light. This is due to the fact that a stop sign will only perform as well as (or worse) than a yield sign in terms of efficiency. Therefore, we assume that if efficiency is the only consideration, a yield sign would be preferable to a stop sign. Stop signs may, however, be appropriate to facilitate a safer roundabout (such as in situtations where there is high pedestrian traffic).

1.2 Defining Effectiveness

Both personal experience and literature searches led us to define "effectiveness" in the following way: an *effective* roundabout is one through which traffic may flow freely and efficiently without delay. The most effective roundabout design minimizes delay. Each model will quantify delay in a different manner.

2 Analytic Formulation

Our analytic model consists of a Jackson network of M/M/s queues (indicating queues with Markovian arrivals, Markovian departures, and s servers). A diagram of the queue is presented in Figure 2. In this model (which we infer to have a steady-state and no explicit time dependence), effectiveness is quantified by the probability that there will be few cars waiting within the system. This may be calculated once we find a stationary distribution for the Jackson network. For the most effective roundabout, the most likely stationary states will be those with the fewest total cars, implying that delay is minimized.

2.1 Additional Assumptions

In order to formulate a tractable analytic model, the following assumptions were also made:

Constant Arrival Rates: Although there is a very low likelihood that any real traffic system would be time-independent, utilizing constant arrival rates allowed us to set up a system in which it was possible to analytically derive a stationary distribution and understand the asymptotic behavior. It is valuable to have a detailed understanding of how the system behaves in equilibrium, even though the system may never stay in equilibrium for a very long period of time. The equilibrium behavior acts as a snapshot of a particular moment in time, and serves as a basis to build up a more complex and realistic simulation.

Perfect Driver Behavior: In the analytic model, we consider only the asymptotic behavior of a probabilistic system. We do not allow for people to miss their exits or break the rules imposed by the system. In reality, there will be people who swerve wildly while talking on cell phones, scrape their tires on the curb, mow down pedestrians, or cut in front of other drivers. These behaviors lead to accidents and major traffic delays. However, they are infrequent enough that we assume them to have little bearing on how the system looks when averaged over a long period of time.

2.2 Description of Simple Queuing Network

The basic idea behind our Jackson network is to break up the system into compartments modeled by queues. We assume that the roundabout is located at the intersection of N streets, which yields a network of $N+1$ queues. Each street contributes an input stream of cars, as well as an opportunity to leave the roundabout. Each input stream of cars is modeled as an M/M/1 queue with its own unique external arrival rate, λ_i. All input queues are assumed to release cars at rate σ_i which represents the rate at which cars transition from the incoming street into the roundabout. The presence of traffic lights or yield signs is represented by a thinning parameter, g, which represents the percentage of time when a traffic light located at the intersection is green. Setting $g = 1$ corresponds to a yield sign. Thus, cars enter the roundabout at a thinned rate $g\lambda_i$. They "return" to the same queue (i.e. remain there) with probability $(1-g)$. The queue representing the roundabout itself is an M/M/N queue, where the N servers represent the N exits.

The stationary distribution for this system of queues, if it exists, denoted $\pi(n_1, \ldots n_{N+1})$, indicates the asymptotic fraction of time that the system spends in the state where there are n_1 cars in queue 1, n_2 cars in queue 2, etc. We are interested in creating a network of queues such that the stationary distribution may be found. Then we may choose g such that the system spends a larger fraction of time in a state where the total number of cars within the system, $n_1 + \ldots n_{N+1}$, is low. The most intuitive way to do this is to have a queue representing each input street, and a limited-capacity queue representing the roundabout. The input queues would only put cars into the system if there is space in the roundabout queue. However, finite-capacity queuing networks do not generally yield closed form solutions for stationary distributions [1].

Therefore, to ensure an analytic solution, we allow every car which leaves an input queue to "enter" the roundabout queue; i.e., this queue has infinite capacity. Although this does not mirror what is physically happening, it allows us to construct a stationary distribution. This stationary distribution will be able to give us the same information: the probability that a certain *total* number of cars is "stuck" within the system. In the original model, cars wait in the street they are coming in on; in our model, they wait inside of the (infinitely large) roundabout. We are not actually concerned with where they wait but rather how many wait, and how likely it is that that many cars will be waiting.

In the case where the roundabout is not full, the finite- and infinite-capacity roundabout queue cases are clearly equivalent because an incoming car can always enter the system. Now suppose the roundabout is full to capacity, and observe that:

(i) If a car waits in the street outside the roundabout, three events must occur before it exits the system: another car inside the circle must leave the roundabout (with some departure rate μ), the car in question must enter the roundabout (with rate σ) and exit with rate μ.

(ii) If a car "waits" in the roundabout, and then exits, three events will also have occurred: the car will have entered the roundabout with rate σ, and joined the interior queue. Since there are only N servers, another car must exit the queue before the car in question is served; each of these processes happens with rate μ. Thus, either treatment is simply the superposition of three Poisson processes, and order is unimportant. Thus the queue exhibits "pseudo-finite capacity" as it mimics the qualitative behavior of a network where one queue size is bounded and the others are infinite.

2.3 Formulation of Stationary Distribution

For an equilibrium state, the input process for each queue in the network equals the output process [2]. This motivates us to define r_i, the asymptotic departure rate from queue i. Each r_i will be equivalent to the sum of the arrival rates to queue i. Defining $p(i,j)$ as the probability that a car leaving queue i enters queue j, we can write an expression for the asymptotic departure rates:

$$r_j = \lambda_j + \sum_{i=1}^{N+1} r_i p(i,j)$$

Or, in matrix form, we have:

$$\mathbf{r} = \mathbf{\Lambda} + \mathbf{rp}$$

Where $\mathbf{r}$ is a row vector of departure rates, $\mathbf{\Lambda}$ is a row vector of arrival rates, and $\mathbf{p}$ is simply the matrix with elements $p(i,j)$ as defined above. Define two conditions on the system:

(A) There exists, for each queue i, a path of positive probability along which it is possible to exit the system.

(B) Defining $\varphi_i(n)$ as the departure rate from queue i when that queue contains n people, and letting

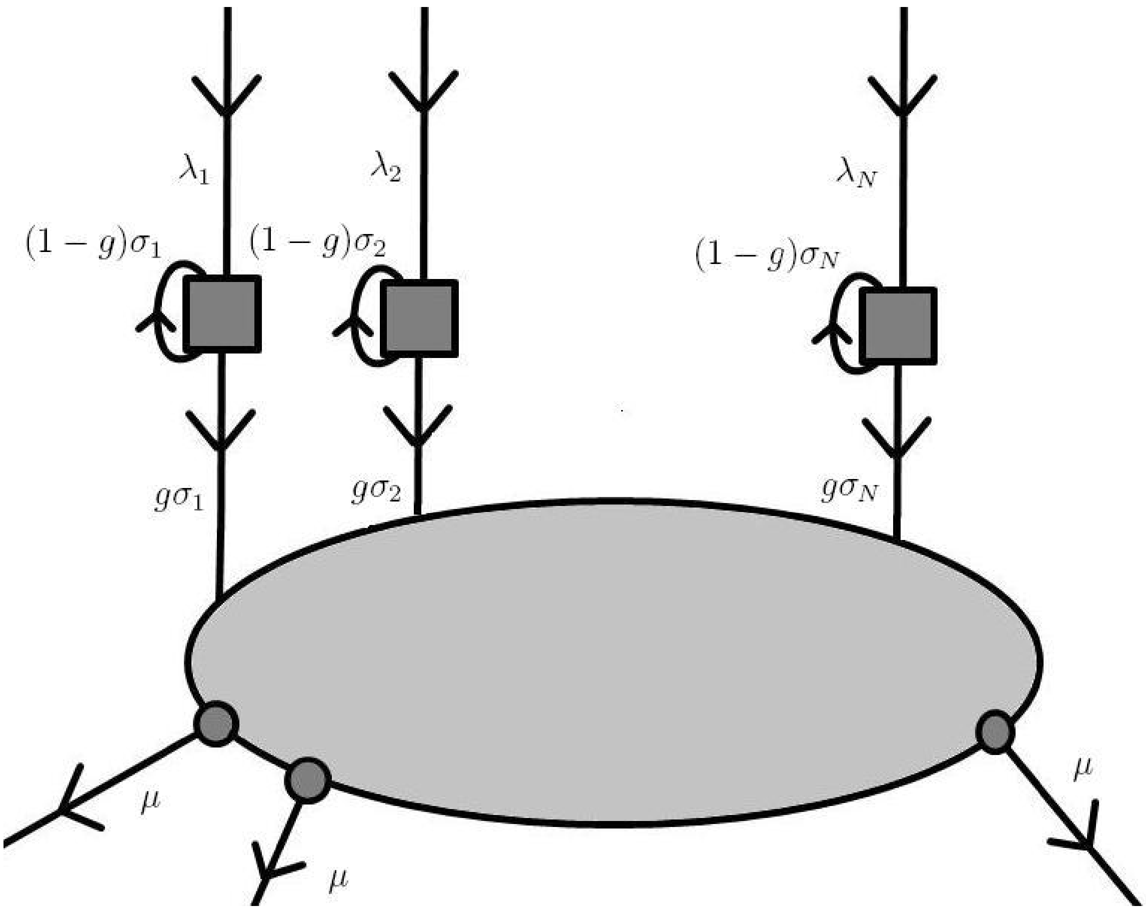


Figure 2: Visual Schematic of Queuing Network

Summary of Analytic Model Parameters

N	Number of streets which connect to roundabout
λ_i	External arrival rate of cars to entrance i
σ_i	Rate at which cars may enter roundabout from entrance i
μ	Rate at which cars may exit roundabout
$\pi(n_1, \ldots n_{N+1})$	Stationary Distribution for the network

$$\psi_i(n) = \begin{cases} \prod_{m=1}^{n} \varphi_i(m) & n \geq 1 \\ 1 & n = 0 \end{cases}$$

There exists some positive constant c_j such that

$$\sum_{n=0}^{\infty} \frac{c_j r_j^n}{\psi_j(n)} < \infty$$

It can be shown that, if condition (**A**) is met, then the matrix $(\mathbf{I} - \mathbf{p})$ is invertible. If condition (**B**) is also met, then a stationary distribution π exists [2] with the form:

$$\pi(n_1, \ldots, n_{N+1}) = \prod_{j=1}^{N+1} \frac{c_j r_j^{n_j}}{\psi_j(n_j)}$$

For our system, we define the vector $\mathbf{\Lambda}$ as:

$$(\lambda_1 \ \ldots \lambda_N \ 0)$$

Each λ_i corresponds to the external arrival rate for queue i; the $N+1$ queue has a zero external arrival rate because this is the queue corresponding to the roundabout. The $(N+1) \times (N+1)$ **p** matrix has the form:

$$\begin{pmatrix} 1-g & 0 & \ldots & g \\ 0 & \ddots & \ldots & \vdots \\ 0 & \ddots & 1-g & g \\ 0 & 0 & \ldots & 0 \end{pmatrix}$$

From any location in this particular network, there is a nonzero probability of exiting the system. Thus, Condition **(A)** is satisfied, the matrix $\mathbf{I} - \mathbf{p}$ is invertible, and we may write the vector of asymptotic release rates as:

$$\mathbf{r} = \mathbf{\Lambda}(\mathbf{I} - \mathbf{p})^{-1}$$

The simplicity of our system allows us to directly solve for the inverse matrix, $(\mathbf{I} - \mathbf{p})^{-1}$, via Gauss-Jordan elimination:

$$\begin{pmatrix} \frac{1}{g} & 0 & \ldots & 1 \\ 0 & \ddots & \ldots & \vdots \\ 0 & \ddots & \frac{1}{g} & 1 \\ 0 & 0 & \ldots & 1 \end{pmatrix}$$

Thus, the asymptotic departure rates are found to have the following form:

$$\mathbf{r_j} = \begin{cases} \frac{\lambda_j}{g} & 1 \leq j \leq N \\ \sum_{i=1}^{N} \lambda_i & j = N+1 \end{cases}$$

Now, we formulate the parameters necessary to solve for a stationary state. We observe that for the entry queues:

$$\varphi_j(n) = \sigma_j \quad 1 \leq j \leq N$$

and that for the roundabout queue:

$$\varphi_{N+1}(n) = \begin{cases} n\mu & 1 \leq n \leq N \\ N\mu & n > N \end{cases}$$

Also, we formulate for the entry queue:

$$\psi_j(n) = \sigma_j^n \quad 1 \leq j \leq N$$

and for the roundabout queue:

$$\psi_{N+1}(n) = \begin{cases} (n!)(\mu)^n & 1 \leq n \leq N \\ (N!)(N\mu)^n & n > N \end{cases}$$

Now, we investigate under what conditions the Condition **(B)** is met. For the entry queues ($1 \leq j \leq N$), we need to find a positive constant c_j such that

$$\sum_{n=0}^{\infty} c_j \left(\frac{\lambda_j}{g\sigma_j}\right)^n < \infty$$

We may only choose a non-zero c_j if this geometric series converges, which occurs when

$$\left(\frac{\lambda_j}{g\sigma_j}\right) < 1$$

For the roundabout queue, we examine the convergence of

$$c_j \left(\sum_{n=0}^{N} \left(\frac{r_{N+1}}{\mu}\right)^n \frac{1}{n!} + \sum_{n=N+1}^{\infty} \frac{1}{N!} \left(\frac{r_{N+1}}{N\mu}\right)^n \right)$$

The first term of the sum is finite for fixed N and does not affect convergence. The second sum can be re-written as:

$$\sum_{n=0}^{\infty} \frac{1}{N!} \left(\frac{r_{N+1}}{N\mu}\right)^n - \sum_{n=0}^{\infty} \frac{1}{N!} \left(\frac{r_{N+1}}{N\mu}\right)^n$$

The second term of *this* sum is also finite for fixed N, so we are only concerned with the first series. This geometric series converges if

$$\left(\frac{r_{N+1}}{N\mu}\right) < 1$$

. Thus, we state the two conditions necessary for the existence of equilibrium and a stationary distribution in our queuing network:

$$\begin{array}{ll} (\mathbf{i}) & \lambda_j < g\sigma_j \\ (\mathbf{ii}) & \displaystyle\sum_{i=1}^{N} \lambda_i < N\mu \end{array}$$

If these conditions are met, we may solve for the stationary distribution. First, choose the constant c_j such that

$$\sum_{n=0}^{\infty} \frac{c_j r_j^n}{\psi_j(n)} = 1$$

Solving for c_j, we find:

$$\frac{1}{c_j} = \frac{1}{1 - \frac{\lambda_j}{g\sigma_j}} \quad q \leq j \leq N$$

$$\frac{1}{c_{N+1}} = \sum_{n=0}^{N} \left(\frac{r_{N+1}}{\mu}\right)^n \frac{1}{n!} + \frac{1}{N!}\left(\frac{1}{1 - \frac{r_{N+1}}{N\mu}} - \sum_{n=0}^{N} \left(\frac{r_{N+1}}{N\mu}\right)^n\right)$$

Now, we formulate the closed form of the stationary distribution [2]:

$$\pi(n_1, \ldots, n_{N+1}) = \left(1 - \frac{\lambda_1}{g\sigma_1}\right)\left(\frac{\lambda_1}{g\sigma_1}\right)^{n_1} \cdots \left(1 - \frac{\lambda_N}{g\sigma_N}\right)\left(\frac{\lambda_N}{g\sigma_N}\right)^{n_N} \left(\frac{c_{N+1}}{N!}\right)\left(\frac{r_{N+1}}{\mu N}\right)^{n_{N+1}}$$

2.4 Optimization of Stationary State

The parameters μ, λ_j, and σ_j are presumably fixed by the physical location of the roundabout and the number of people who use it. Hence, the stationary state π is a function of g, and may be optimized over g. The idea is to maximize the amount of time spent in a state in which the total number of cars in the system is less than or equal to the capacity of the roundabout. Define $\mathcal{K} \equiv \{\text{all } \{n_i\}_{i=1}^{N+1} \text{ such that } n_1 + \ldots + n_{N+1} = k\}$. Now we can define:

$$\pi(k) = \underbrace{\sum}_{\text{all } \{n_i\} \in \, \mathcal{K}} \pi(n_1, \ldots, n_{N+1}).$$

It would be useful to analyze how $\pi(k)$ depends on g for small k. Notice that for any given k, the number of terms in the sum is equivalent to the number of non-negative integer solutions to the equation

$$n_1 + \ldots + n_{N+1} = k$$

which is given by the well-established formula [9]

$$\frac{(N+k)!}{N!k!}.$$

Clearly, the number of terms in the sum will grow exceptionally quickly, and directly examining g dependence will become impossible. Instead of direct analysis, we establish a lower bound for $\pi(k)$ in terms of $\pi(0, 0, \ldots, 0)$, the fraction of time in which no cars remain in the system. For this case, denoted $\pi(0)$, we have:

$$\pi(0) = \prod_{i=1}^{N} \left(1 - \frac{\lambda_i}{g\sigma_i}\right) \left(\frac{c_{N+1}}{N!}\right).$$

Notice that neither c_{N+1} nor $N!$ depend on our choice of g. Therefore, $\pi(0)$ will be maximized over g if the product

$$\prod_{i=1}^{N} \left(1 - \frac{\lambda_i}{g\sigma_i}\right)$$

is maximized over g. The conditions under which this stationary distribution was constructed assert that

$$\frac{\lambda_i}{g\sigma_i} < 1,$$

ensuring that all terms of the product are between 0 and 1. Therefore, for a fixed set of $\{\lambda_i/\sigma_i\}$ (the constraints of the system), the optimal choice of g would minimize each $\lambda_i/g\sigma_i$ so as to maximize the quantity $1 - (\lambda_i/g\sigma_i)$. Therefore, the largest choice of g will maximize $\pi(0)$. Given the constraint $0 < g \leq 1$, the optimal choice is $g = 1$.// Every other stationary state may be written in terms of $\pi(0)$:

$$\pi(n_1, \ldots, n_{N+1}) = \pi(0) \left(\frac{c_{N+1}}{N!}\right) \left(\frac{\lambda_1}{g\sigma_1}\right)^{n_1} \cdots \left(\frac{r_{N+1}}{N\mu}\right)^{n_{N+1}}$$

We establish a lower bound for $\pi(k)$ by defining:

$$\frac{\epsilon}{g} \equiv \min\left\{\frac{\lambda_i}{g\sigma_i}, \frac{r_{N+1}}{N\mu}\right\}$$
$$C \equiv \frac{c_{N+1}}{N!}$$

Now, we assert that, since each term in the product is less than or equal to m, the sum of the powers on these terms is k, and there are $\frac{(N+k)!}{N!k!}$ distinct elements of $\mathcal{K}$:

$$\pi(k) \geq \frac{(N+k)!}{N!k!} C \left(\frac{\epsilon}{g}\right)^K \pi(0)$$

In the event that

$$\min\left\{\frac{\lambda_i}{g\sigma_i}, \frac{r_{N+1}}{N\mu}\right\} = \frac{r_{N+1}}{N\mu}$$

all g dependence comes from $\pi(0)$, which is maximized for $g = 1$. In the event that, for some index j,

$$\min\left\{\frac{\lambda_i}{g\sigma_i}, \frac{r_{N+1}}{N\mu}\right\} = \frac{\lambda_j}{g\sigma_j}$$

we first define:

$$\max\left\{\frac{\lambda_i}{g\sigma_i}\right\} = \frac{\delta}{g}$$

which allows us to assert

$$\pi(0) \geq \left(1 - \frac{\delta}{g}\right)^N$$

which implies that

$$\pi(k) \geq \frac{(N+k)!}{N!k!} C \left(\frac{\epsilon}{g}\right)^k \left(1 - \frac{\delta}{g}\right)^N$$

We turn our attention to the behavior of the function which governs the g dependence of the lower bound of $\pi(k)$:

$$f(g) = \left(\frac{\epsilon}{g}\right)^k \left(1 - \frac{\delta}{g}\right)^N$$

We differentiate with respect to g and find that

$$\frac{\partial f}{\partial g} = \frac{\epsilon^k (g-\delta)^{N-1}((N-k)(g-\delta) + Ng)}{g^{2(k-N)}}.$$

Since $\epsilon > 0$, and $g - \delta > 0$ according to the assumptions with which we set up the system, the sign of $\frac{\partial f}{\partial g}$ is determined by the expression

$$((N-k)(g-\delta) + Ng),$$

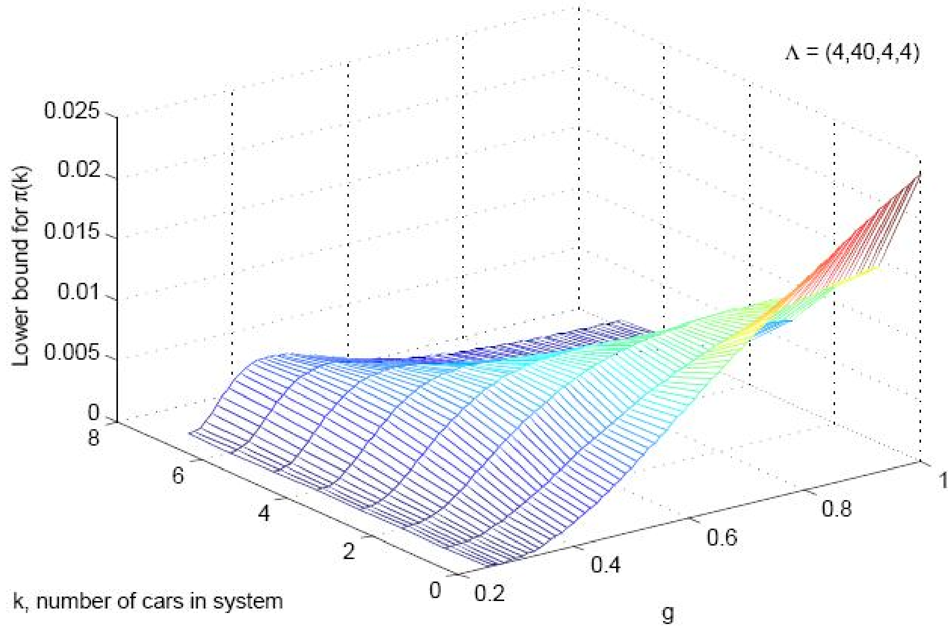
which is guaranteed positive for

$$k < N + Ng/(g-\delta).$$

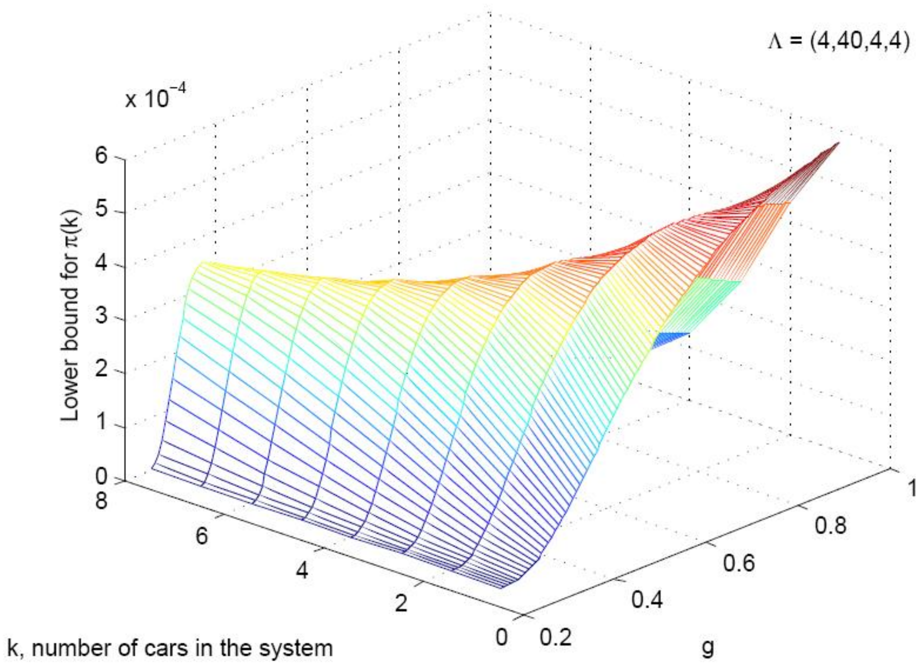
Therefore, for small k, the slope is positive for all g in our domain, implying that increasing g increases the lower bound on $\pi(k)$. Increasing the lower bound ensures the stationary distribution is larger. For our analytic model, the value of g which guarantees the largest lower bound on $\pi(k)$ for small k is $g = 1$, regardless of other parameters. Therefore, our analytic model will always recommend a yield sign.

To examine the actual stationary state behavior, we implemented a computer program which calculates $\pi(k)$ for each fixed value of k, summed up over all the stationary states for which the total number of cars in the system is equal to k. We examined this for a wide range of λ, σ, and μ values. In all cases, the stationary distribution for lower k values is highest for $g = 1$. In Figures 3 and 4, we compare the lower bound behavior and the actual behavior for a 4-entrance roundabout. We examine both the case where all input rates are equal, and the case where they are not. Our lower bound estimate curves and calculated curves have very similar shapes. Thus, a choice of g which maximizes the area under the lower bound curve for small k also maximizes the area under the actual curve. This validates our use of the lower bound estimate as a basis for the optimal choice of g.

Our analytic formulation always finds the optimal entrance rule to be a yield sign at every intersection. Although this is in part a result of the limitations of the model, such as lack of time dependence, it is mostly consistent with both the results of our computer simulation and our research into real-world practices. As noted in the introduction, roundabouts are generally neither planned nor implemented with traffic lights. Our computer simulation will attempt to find some of the situations in which traffic lights are more efficient than yield signs.

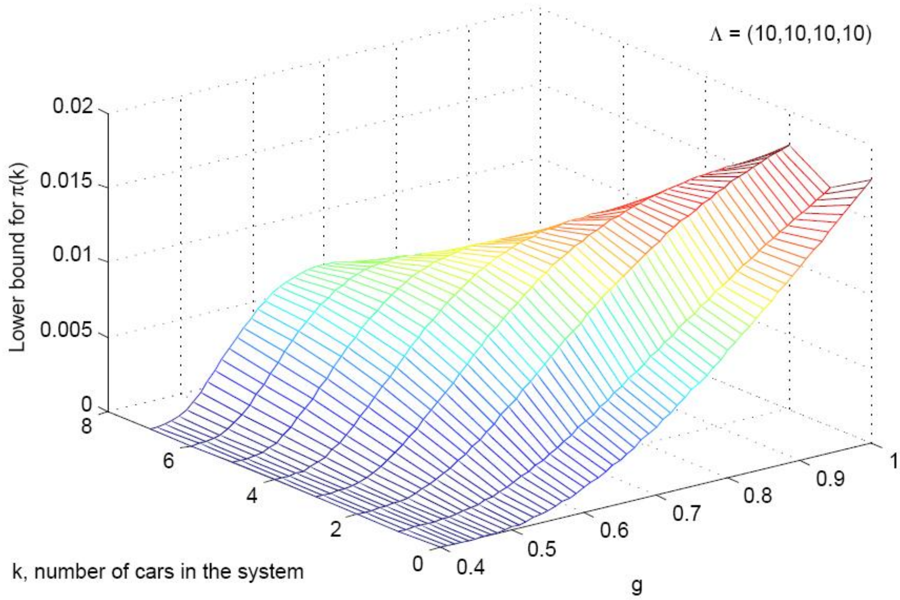


(a) Actual Value

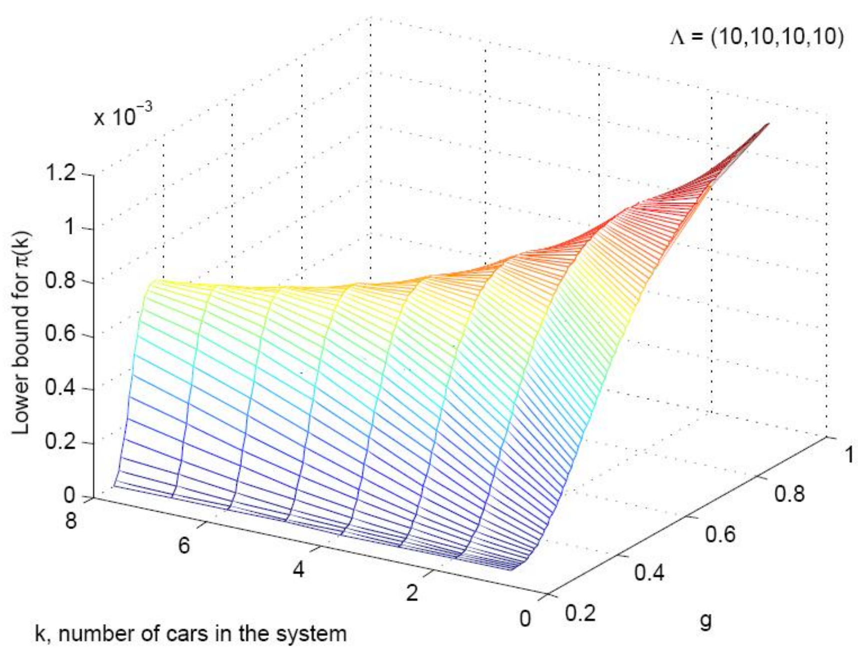


(b) Lower Bound Estimate

Figure 3: Comparison of Actual Stationary Distribution and Lower Bound Estimate for Unequal Input Rates



(a) Actual Value



(b) Lower Bound Estimate

Figure 4: Comparison of Actual Stationary Distribution and Lower Bound Estimate for Equal Input Rates

3 Computer Simulation

Given the weaknesses of the analytic model, we adapted it to create a computer simulation with the freedom to change some assumptions in order to create a more realistic model.

3.1 Assumption Modifications

Independent Arrival Processes: We assume that whatever process determining the arrival of cars through a given entrance street remains independent of the processes determining the behavior of other streets and the circle itself. Thus, the probability of a car approaching the circle from one street does not depend on the probability of a car approaching the circle from a different street, nor does it depend on the probability distribution of how cars enter or leave the traffic circle. This assumption is reasonable, since we would not expect a driver to have prior knowledge of what is happening in the traffic circle at any given moment.

Drivers' Intentions: We assume that every driver in the computer-simulated model wants to leave the traffic circle through a specific exit and do so in the least amount of time possible. However, since it is quite possible for a driver to be confused or unaware of his or her surroundings, we define a probabilistic constant for each car successfully leaving the circle. While this allows for the possibility of getting stuck in the circle (reminiscent of Chevy Chase in National Lampoon's *European Vacation*), the probability of continually missing the exit is vanishingly low. Also, we assume that the driver never takes the wrong exit or alters his or her destination once inside the circle, not only because this makes the model simpler but also because no traffic engineer could possibly gather such information.

Constant Car Length and Speed: We assume that the vehicles going through the traffic circle have fixed length and speed. While it is possible for a car to be very long and occupy more space in the circle, we also reason that another car could be very short and occupy less space, potentially nullifying the effects of the longer vehicle. Also, the actual vehicle speed varies between drivers, but adding variation would introduce unnecessary complexity into the model.

Yield Sign is Optimal for Low Traffic Volume: According to both literature and common sense, a traffic light in a roundabout with few cars will only hamper traffic flow. This is clear when one considers that if the roundabout is less than full to capacity, incoming traffic will have little trouble entering if allowed to do so by traffic signals. If the signals periodically prevent them from entering, this only serves to decrease the efficient flow of traffic.

3.2 Computer Simulation of One-Lane Roundabout

Now that we have obtained an analytical model for the expected behavior of the system using Jackson networks, we want to compare these results to a more realistic simulation of traffic flow. We will simulate cars actually arriving to a theoretical traffic circle, entering the circle according to some heuristic process, moving through the traffic circle toward a specific exit, and finally leaving the circle at the desired exit (granted that the driver does not miss his or her turn - which is also possible).

During the simulation, we fix the length of the car at 5 meters. The speed of the cars inside the circle is varied between 8 and 13 m/s, based on the ranges presented in [8]. The capacity of the roundabout, or the number of cars which can be inside at any one time, is determined by vehicle length, vehicle velocity, and roundabout radius. At full capacity, cars inside the roundabout are spaced by one second of driving, ensuring sufficient space to maneuver.

3.3 Description of Simulation Process

Before we simulate the actual flow of traffic through the circle, our simulation determines the exact times that cars arrive to the circle from each entrance street. Because we assumed the arrival processes to be independent, we can simulate the arrival times for each street individually. Using the principle that, for any random variable $U \sim Uniform[0,1]$, the variable $(-\frac{1}{\lambda}\ln U)$ is exponentially distributed with parameter λ, we can easily determine the inter-arrival times for each entrance road for the entire time period of inquiry, which will be one 24-hour day in this simulation.

In regards to the inter-arrival times, we wish to vary traffic arrival rates depending on the time of day. To emulate real traffic flow, fewer cars should arrive at night than the middle of the day, and even more cars should appear during the morning and evening rush hours. To account for this behavior, we scale the peak arrival rates for the street. The scaling function $f(t)$ consists of one relatively narrow Gaussian centered at each rush hour time, and one smaller amplitude, slowly varying Gaussian centered at midday. This function is plotted in Figure 5 for rush periods of 1 hour each at 8:00 AM and at 5:00 PM. At the time t that each new arrival time is calculated, the rate parameter λ is scaled by $f(t)$. This creates the desired time variation in arrival rates.

The arrival times for each entrance queue are recorded and computed prior to simulation. During the simulation, at each simulated arrival time, we add a vector to the right end of a

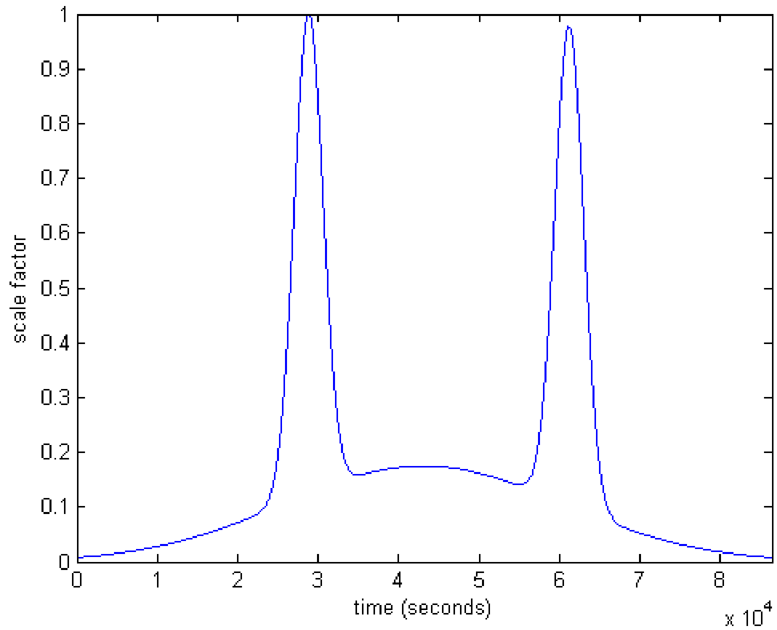


Figure 5: Time Dependent Arrival Rate Multiplier

dynamic matrix representing the entry queue. Each vector corresponds to a car, and contains the parameters that govern each individual car's behavior: arrival time, destination, and probability of missing the exit. The matrix columns represent the order of cars waiting to enter traffic circle, and because we treat the entrance queue as a "First In, First Out" buffer, only the car represented in the leftmost column of the matrix is allowed to enter the traffic circle.

The destination of the car is determined using a given parameter that represents relative exit popularity. We reason that cars will be slightly less likely to use the circle to make a U-turn (i.e. exit adjacent to the position that it entered), so we determine probabilistically if the car should be able to make a U-turn (return to the street they entered on) before selecting its destination. For all cars, the probability of a U-turn is 0.05. The distribution is translated into a partition of the interval [0,1]. When a car arrives at the queue, a random variable is simulated; the interval into which it falls determines where the car will exit. When the car arrives at its exit, a random variable $U \sim Uniform[0, 1]$ is simulated, and if this number is less than 0.05, the car misses its exit and stays in the traffic circle.

To simulate traffic moving through the circle, we divide the traffic circle into discrete positions based on the circumference of the circle and the length of the typical car that drives through it. We then number these positions in the same direction of the flow of traffic, as in a car in position 1 would go into position 2, position 2 would go into position 3, and so on until it reaches the last numbered position, at which point the car returns to position 1. Vectors from the leftmost position in an entry queue matrix are placed into the traffic circle if the entry position and the position immediately behind the entrance are both vacant. Thus, we obtain a "circle matrix" where each column pertains to a position. Moving an entire column of the matrix simulates an individual's movement through the circle.

At regular time intervals based on the speed of the cars and the size of the circle, we rotate the columns of the matrix according to the method described above (1 to 2, 2 to 3, etc.). After each rotation interval, cars in the circle check to see if they have reached their destination and, consequently, determine if they successfully exit the circle. Once a car exits, it calculates time spent in the circle by subtracting the arrival time from the exit time. The simulation then erases the values of the vector representing its current position from the circle matrix to indicate that that car has left the circle. After exiting cars leave, cars waiting to enter the circle make the following two checks, both of which must be satisfied in order to enter the circle:

Check traffic signal: The car checks a "signal matrix" whose rows are indexed by the entrance locations and whose columns represent a fraction of time in a traffic light cycle. Thus, each entry (i, j) of the signal matrix indicates whether the i^{th} light is red or green during the j^{th} signal interval, where each signal interval is $20/\text{round}(\frac{\text{carlength+speed}}{\text{speed}})$ seconds long. At the start of the simulation, $j = 1$; once one signal interval has elapsed, $j = j + 1$. This continues until we reach the end of our signal matrix, signifying the end of the traffic light cycle. At that point, j is set to 1. The time t of the simulation step determines which value of j is used. If the entry of the matrix is a zero, the light is red, and the car may not enter the circle. If the entry is an one, the light is green, and the car may enter the circle if there is space. For each run of the simulation, three signal matrices are used: one for late night/early morning, one for rush hours, and one for midday. A signal matrix whose entries are all identically 1 is referred to as a "yield matrix" because it acts like a yield sign. It should be noted that the late night/early morning signal matrix is always a yield matrix due to the diminished traffic flow.

Check for cars in the circle: Cars that are permitted to enter the circle by the signal matrix must nonetheless yield to traffic already in the circle. As a result, entering cars must check the circle matrix to see if both the entrance position on the circle and the position before it are unoccupied so that it does not hit a car in the circle nor cut one off.

Assuming that both conditions are satisfied, the simulation puts the car into the circle by removing the left-most column of its entry matrix and copying it into the entrance position on the circle matrix. The process of adding vectors to the dynamic matrices at the arrival times, rotating the columns in the circle matrix, removing cars that successfully exit, and adding cars that successfully enter continues until the end of the total interval, taken in this simulation to be a day.

3.4 A Metric to Measure Effective Traffic Flow

Now that we have a simulation-based model, we discuss metrics to quantify the effectiveness of a given flow-control method . Since we record the times that a car finally leaves the traffic circle, we can use this data to determine certain statistics that describe the process.

Because of the large number of cars that enter and leave our simulation, we can use the average time spent in the system per car for that day as a good estimator of how the simulation behaved. However, cars driving during rush hour should be waiting longer than cars driving during midday or at night. As a result, the maximum time spent should give us a sense of the worst-case scenario for the flow-control model. After all, a driver waiting for almost an hour to go through one traffic circle probably will not care that the average driver waits for only twenty seconds.

Another metric that could be considered is the minimum time spent, but this minimum will very likely measure a car arriving at an empty traffic circle at night, possibly because the driver is a student staying up late to work on a modeling contest. With no one in the traffic circle, the driver receives immediate service, and if his or her exit is adjacent to the entrance, there exists a good chance that the driver only spends a few computation cycles in the simulated traffic circle before successfully exiting on the first attempt. Thus, the minimum will not be a very descriptive statistic. The only real conclusion to be drawn from the minimum time is that either the situation described above occurred occurred at some time or, due to random chance alone, no car in the empty system was lucky enough to enter the circle adjacent to his or her exit or was able to successfully leave on the first attempt.

In short, we will qualify our simulated traffic circles mainly due to the average and maximum time spent in the traffic circle. A “good” flow-control system (or signal matrix) will produce lower values for these two statistics, while a “poor” flow-control system will produce higher values.

3.5 Justification of Experimental Methodology

Now that we have created a simulation for a traffic circle that uses signal matrices as input, we theoretically could fix all of the other parameters such as radius, speed, etc. and minimize the average time spent in the system over the three signal matrices. However, there are many reasons why this method would not work. First we have a discrete input space that is all possible matrices in $\mathbb{R}^{N\times\alpha}$ $\alpha \in \mathbb{Z}_+$ with entries either 0 or 1. We typically used $\alpha = N$ as we felt that having N intervals gave us enough ability to vary the signal matrix. Because of the nature of the input space traditional minimization methods would not work.

Furthermore, due to the fact that our simulation is a random process, the same set of input simulated twice would produce 2 distinct results. While this is representative of how a real traffic circle would behave on various days, it proves problematic in minimization methods. One possible solution to this would be to minimize the average of a large number of simulation runs on each set of input. However, the computational cost of such a method is very large; there are $2^{\alpha N}$ possibilities for each of the three inputs. This strategy could potentially be refined by eliminating impractical signal matrices, but the computational cost to minimize the average of a large number of simulation runs would still be very high.

Due to all the aforementioned difficulties in the use of any traditional minimization method, we took a different approach. Our literature searches and analytic model revealed that yield control is by far the most common and effective form of roundabout traffic flow control. We decided to use our simulation to test the effectiveness of a yield sign versus a traffic light in a large number of experiments. First, we assumed that late at night and early in the morning when traffic flow is minimal a yield sign, or perpetually green traffic light, would be the optimal choice. Then, to account for the randomness of our simulation, we decided to run three simulations on each of 100 combinations of matrices, 98 of which are randomly generated. We always compared the random signal matrix results to the yield signal matrix and a fixed non-yield signal matrix, and every matrix set was run on the same roundabout.

We want to eliminate matrices that represent periods of red light that are unrealistic. Thus we force that our midday signal matrix satisfies the following (where $\mathbf{g_{yield}}$ represents the matrix of all ones and $\mathbf{g_{mid}}$ is our midday matrix):

$$\|\mathbf{g_{yield}} - \mathbf{g_{mid}}\|_\infty <= 2 \tag{1}$$

For our rush hour signal matrix we enforce the following condition (where $\mathbf{g_{rush}}$ is our rush hour signal matrix):

$$\|\mathbf{g_{yield}} - \mathbf{g_{rush}}\|_\infty <= 3 \tag{2}$$

These conditions force that there is a sufficient amount of ones in any given row of the matrix. We enforce slightly different conditions during rush hour and midday because of the decreased traffic volume during midday. We felt that as traffic volume decreased, the necessity of control decreased. If the traffic circle was more likely to be empty, we did not want a car to have to wait at a red light.

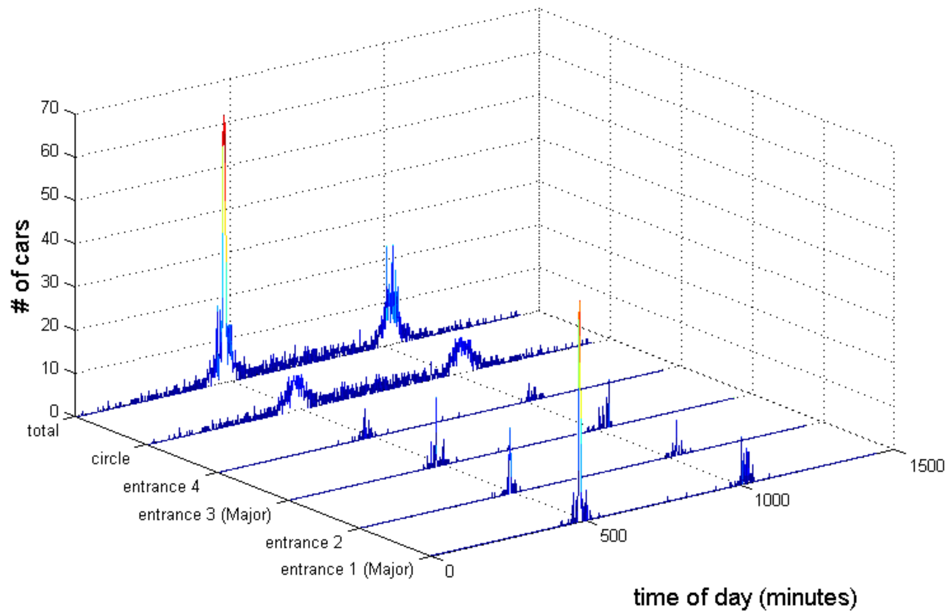
3.6 Simulation Results, Part 1: Flow Control Considerations

From the analytic model, we concluded that the most effective flow-control model involved all incoming cars to yield to cars inside the circle, so we wish to see if this result holds when we use the more complicated simulation. In terms of the simulation variables, we want to know if the yield matrix is the optimal choice of signal matrix.

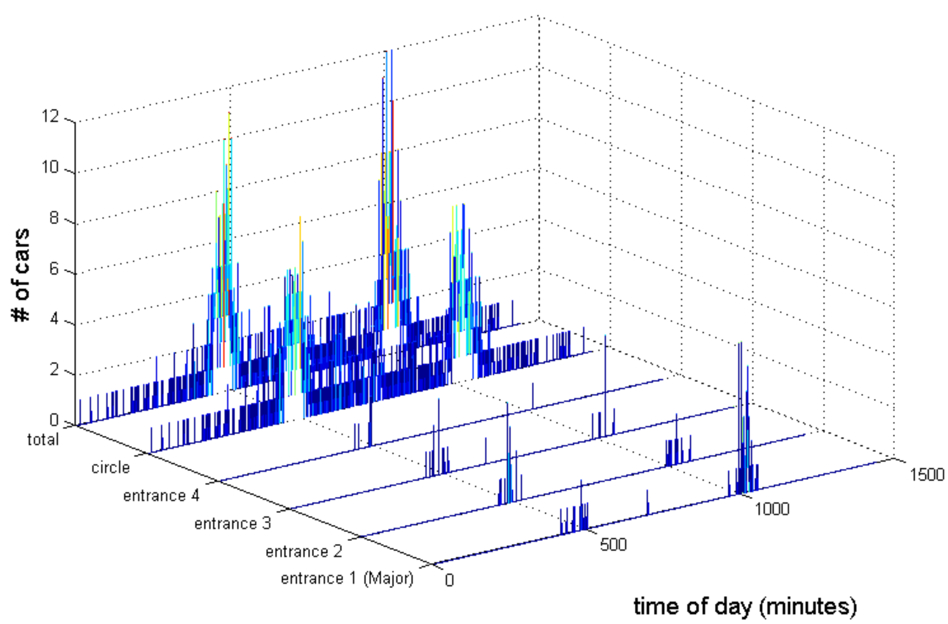
Using the yield matrix, we ran several simulations using different relative distributions for the input rates from four entrance streets, which is the standard construction of an intersection. The ratio of smallest input rate to largest input rate ranged from 1:1 to 1:8. To gauge their effectiveness in controlling traffic, we plot the number of cars in each part of the system against time. In Figures 7(a), these plots appear for the yield flow-control method using the same entrance rate for all of the side streets. As one would expect, the peak number of cars in the system occurs during the morning rush hour, when the input rates are highest. Traffic congestion appears in the plots as extreme peaks in the density, and as one can see in the second row from the back, the majority of cars are entering the circle almost immediately.

Similar behaviors were observed in plots of the other systems where some streets have higher input rates then the others. In these systems, we consider a street to have a "major" input rate if its input rate is higher than the input rates of other streets. In Figure 6(a), two streets have major inputs, but the plots appear almost exactly the same as in Figure 7(a), with only a discrepancy in the peak total density. Furthermore, when we have only one major street, we saw even better performance, as demonstrated by the plots in Figure 6(b). The peaks are significantly decreased, mostly due to the fact that the sum of the input rates is much lower than in the other systems, but this shows that yield signs are self-regulating enough to be fairly well-behaved in high input and low input systems. We will explore this concept more in a later section.

We now turn our attention to systems without yield signs at all streets. Instead, we install theoretical traffic lights at each entrance that prevent cars from entering on red lights. Essentially, this means that the traffic signal matrices contain both ones and zeros, although we enforce the condition that no row contains all zeros (stopping all traffic). Also, we only use these non-yield flow-control methods during the rush hour and midday periods and use the standard yield matrix during night hours. We make this adjustment because yield matrices are extremely efficient for low input rates.

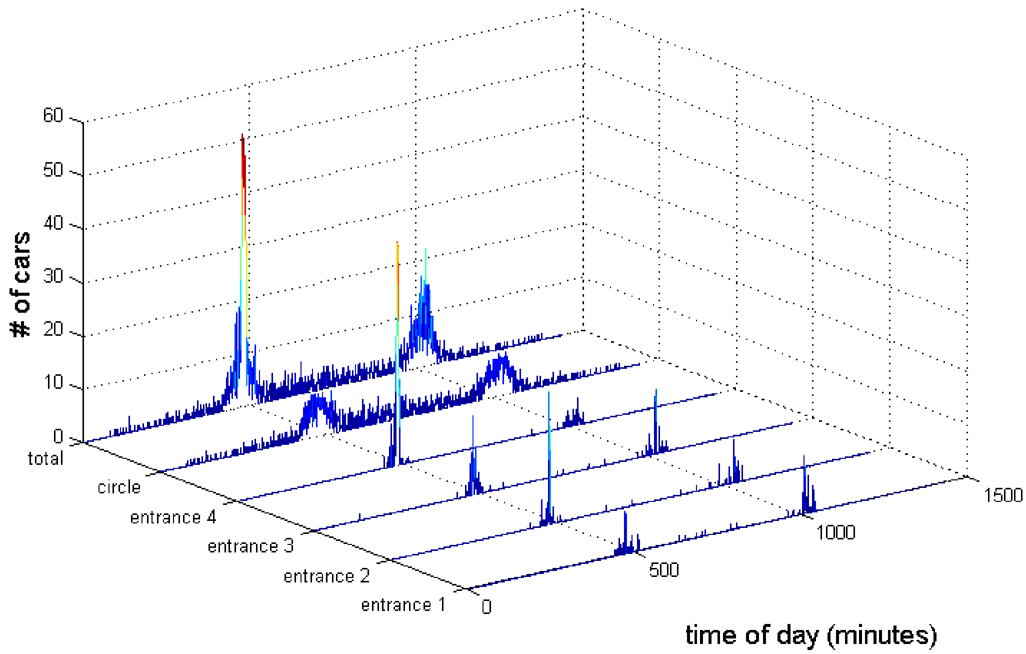


(a) Two Major Input Rates

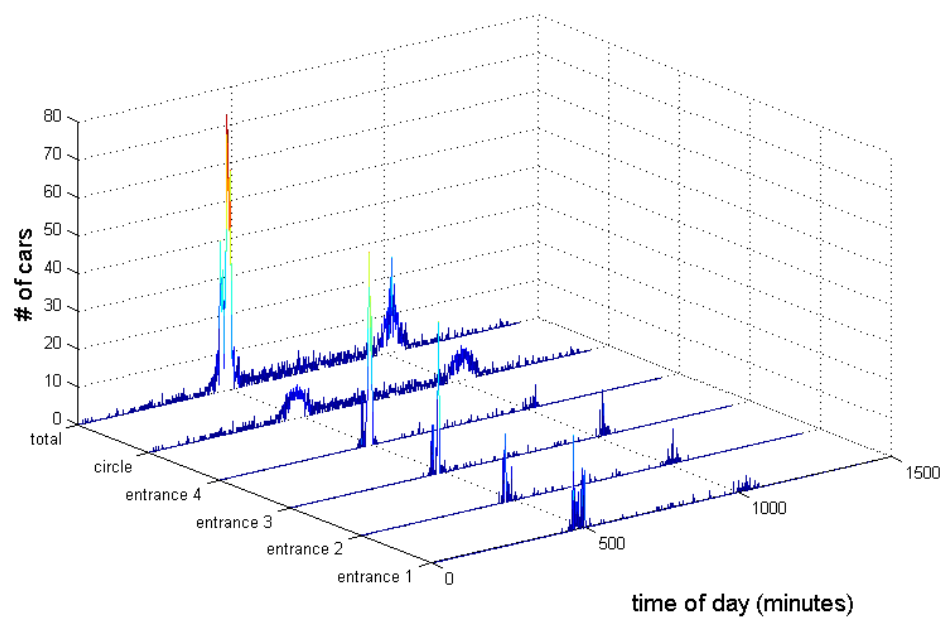


(b) One Major Input Rate

Figure 6: Car Density for Yield Flow-Control



(a) Yield Flow-Control With Similar Input Rates



(b) Non-Yield Flow-Control with Similar Input Rates

Figure 7: Car Density Comparison of Yield versus Non-Yield

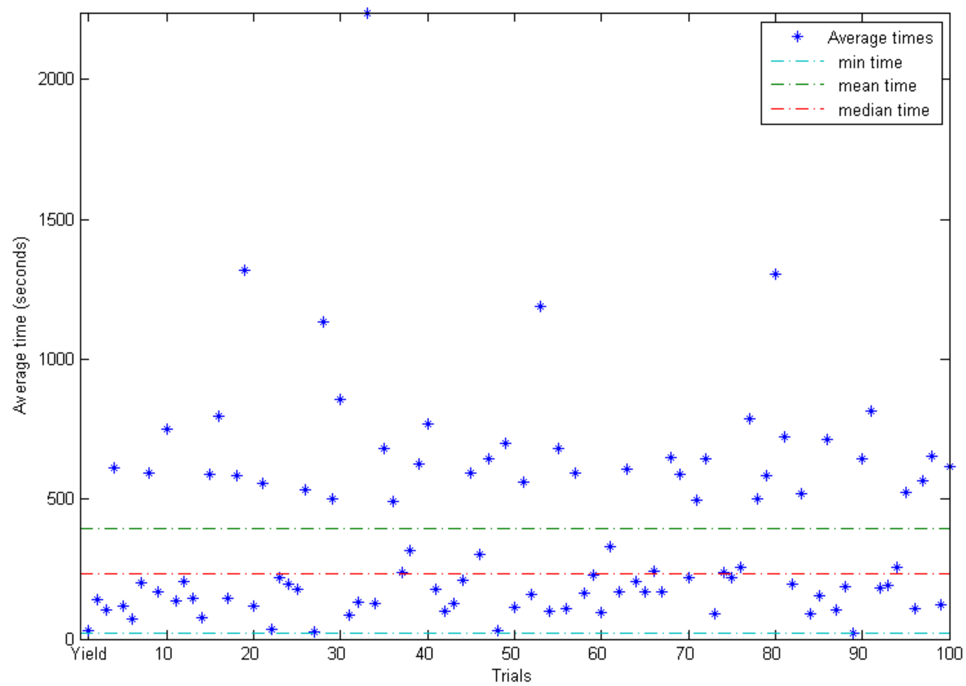
Using the same input rates as the system from Figure 7(a), we obtain the density plots found in Figure 7(b) for an arbitrary non-yield traffic matrix. The shape of the plots appear quite similar to those of the previous systems, but the scaling is quite different. The maximum peak of the total cars in Figure 7(a) barely reaches over 50 cars, but in Figure 7(b), the peak reaches over 70 cars. This result infers that non-yield flow-control may not be optimal.

However, no conclusions can be drawn from only one trial, so we ran 100 trials of our simulation with different random traffic signal matrices and compared them with another trial using the yield matrix. The results are plotted in Figure 8(a). The horizontal lines indicate the values of the mean (395.8 seconds), median (232.3 seconds), and minimum (22.55 seconds) values of the data. Although the granularity of the plot partially hides this fact, we observed that the yield matrix, with an average of 31.83 seconds, was NOT the best trial during the simulation. In fact, four trials with random, non-yield traffic matrices beat the yield matrix by a margin of about 9 seconds.

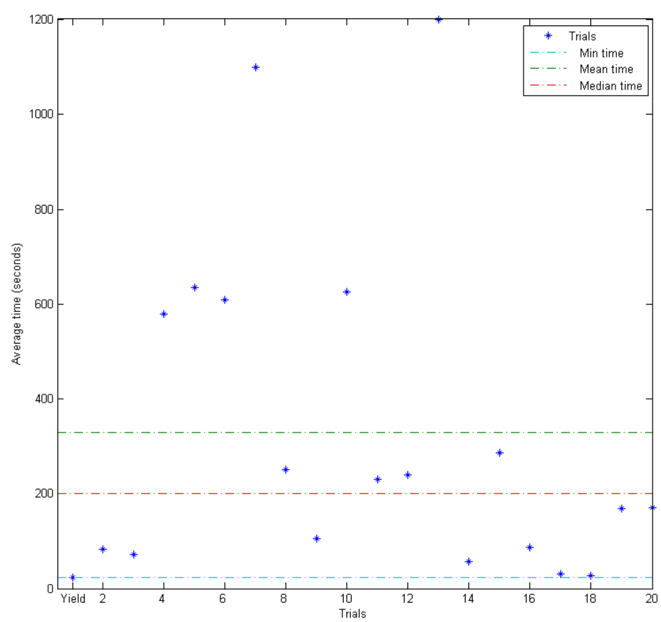
Nonetheless, these results do not shatter the conclusions we drew from our analytic model. Upon further inspection of the matrices that seemed to improve the traffic flow, we noticed that these matrices were extremely similar to the yield matrix, with only one or two zeroes in the entire matrix and no row containing more than one zero. In physical terms, these roundabouts would only have one or two entrances having a red light one-fourth of the time during a few hours of the day. Also, we only used these matrices during peak traffic hours (representing less than 1/12 of the entire time interval), so the fact that these matrices showed better performance than the yield matrix can be attributed simply to random chance. The overal experimental result is telling: 96% of the trials were significantly worse than the yield matrix. It should also be noted that the "better" matrices improved the process by only a small margin - too small to warrant spending money on installing expensive traffic lights rather than simple yield signs.

We also tested our simulation on various different roundabouts. For five other parameter sets, varying size, speed, and input flow, we ran the same type of experiment with 19 random matrices per parameter set. In each of these trials the yield matrix performed as wells as, or better than, any of the signal matrices. One such example is seen in Figure 8(b), where the roundabout contains one large street and four small streets for a total of five inputs and exits. Each point on the figure represents the average time over three trials for a given set of signal matrices. In this example, as in all our trials, using a yield sign all of the time provided the lowest average time. Thus, we base our recommendation on the consistent results over 200 trials across various roundabout designs.

In short, while some non-yield traffic signs may improve performance on any given day, our simulations show that the yield sign system is statistically better than the majority of other flow-control methods, even when given different variations of input rates.



(a) 100 Trials



(b) 20 Trials

Figure 8: Average Time Spent in System with Random Traffic Signal Matrices

3.7 Simulation Results, Part 2: Size Considerations

Because larger traffic circles have higher capacities, we also investigate the effects of the traffic circle radius on traffic flow. As the radius of the circle increases, the number of cars that can fit in the circle also increases, so less cars should be waiting in the entrance queues as there will be more gaps between cars in the circle. Thus, if the total input of cars is large, a larger traffic circle should perform better than a smaller one. Of course, if we increase the radius infinitely, the traffic circle becomes less of a circle and more like a very large one-way street that curves. Cars in that roundabout would take a long time to pass through the system simply because they have to drive farther. Also, larger circles would cost more money and demand more space, so we wish to find an optimum radius for any given situation.

However, we cannot use our simulation to find an exact relation between total input rate and optimum radius. Simulation results may demonstrate typical behaviors, yet the very nature of random simulation simulation prevents the creation of an exact function, say $r(\lambda_T)$, of optimum radius in terms of total input radius.

Using the yield matrix and a ratio of two major streets with two minor streets, we ran a set of trials while varying the total input rates and the traffic circle radius and recorded the average time spent in the system. In Figure 9, the flat plane represents well-behaved systems

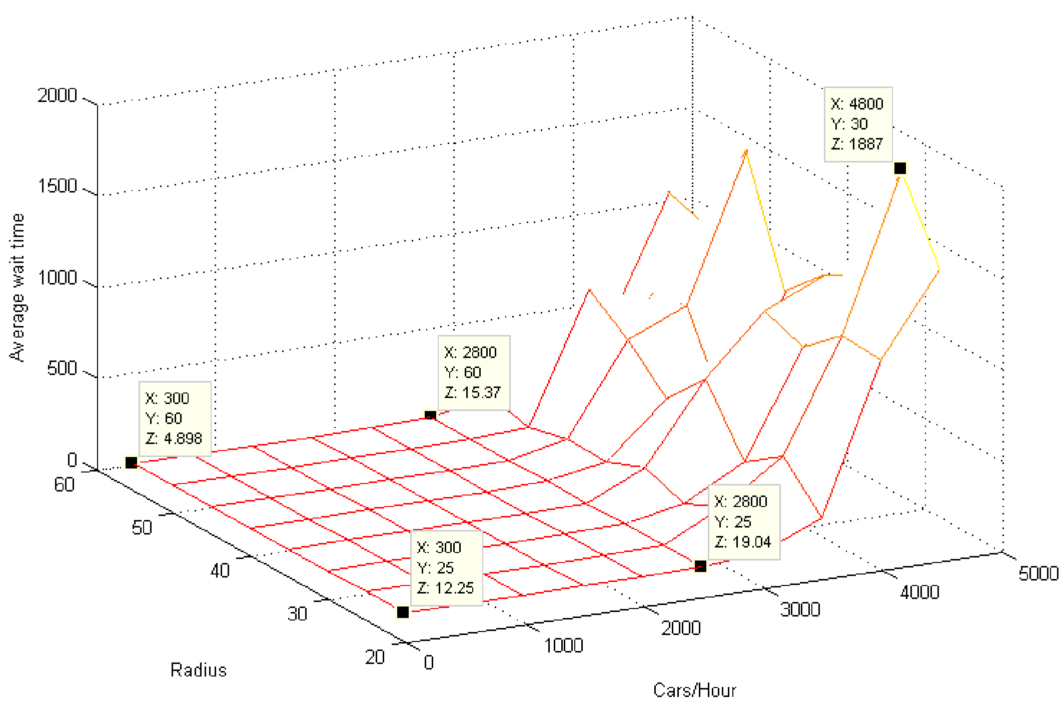


Figure 9: Average Time Spent in System for Various Input Rates and Circle Radii

where the single-lane roundabouts with larger radii exhibit lower average time spent inside the roundabout. Also, when we fix the radius to be a certain value, we see that the average time spent in the system increases as the total input rates increase.

What is most interesting in the plot is the rapid change in behavior after the total input rate goes above 3000 cars per hour, or more than one car every second, and radius is allowed to vary. We expect more delays as more cars try to enter the system, but we also expect larger radii to decrease the delays with some kind of proportionality. For example, when we fix the total input to be 4000 cars per hour, we see that a circle with a radius of 35 meters performed better than a circle with a radius of 30 meters, which is expected. However, the circle with a radius of 40 meters performed worse than the both the 35 meter circle and the 30 meter circle, which is entirely unexpected. Thus, we conclude that for total peak flow of less than 3000 cars per hour, increasing radius is directly correlated with decreasing average total time, but at higher flow rates, the correspondence between radius and flow rate becomes erratic.

This unexpected behavior reveals the limitations of our model. A single-lane roundabout with four entrances cannot handle grossly inflated input rates, regardless of size, so areas with extremely high traffic densities need to use other constructions, such as multiple lanes to increase traffic capacity or express lanes to thin out cars who need to only drive to the next street over. However, our computer simulation model cannot handle these extra cases.

4 Strengths and Weaknesses

4.1 Analytic Model

The analytic model, although satisfyingly simple to write down and perform calculus upon, is limited in many ways. We compromised many kinds of complexity in order to formulate a closed-form stationary distribution, but in the end, the sheer variety of equivalent states which our system could take thwarted our analysis. Our lower bound calculations for the stationary distribution are pretty but provide a bound which is, according to several numerical trials, an order of magnitude less than the function itself. We could show that the lower bound grows with g for small k, but we did not prove that the overall shape of the lower bound always emulates the actual function. We did show that the two functions are behaviorally similar in two specific cases, lending credence to our lower-bound based optimization.

This model was useful in forming a basis for our computer simulation and narrowing our search for effective flow control systems. There is an alarmingly large number of ways to create a signal matrix; our analytic calculations led us to search around those similar to the yield matrix rather than perform tens of thousands of costly and unproductive calculations.

4.2 Computer Simulation

The computer simulation was able to cope with many of the limitations of the analytic model. It introduced time dependent flow, limited the capacity of the roundabout, and more directly simulated the action of a traffic light as a discrete system rather than a time-averaged parameter. This formulation allowed us to explore a wide range of parameters beyond the convergence constraints of the analytic model. In doing so, it gave us useful insight about the relationship between parameters.

The computer simulation is limited by the vastness of the parameter space. Implementing an optimal signal matrix search could not be done by any function-based search algorithm because the functional value of a given signal matrix must be determined by how it performs in a computationally intensive random simulation, and because the dimensionality of the variable space is so large. The independent variable space for one signal matrix for a 4-entrance roundabout has 16 dimensions, and the simulation utilizes three different signal matrices during every run. Directly performing a search algorithm on random calculations in this variable space requires computing power far beyond our means. The analytic model was useful, therefore, in restricting our search to signal matrices close to the yield matrix. We ran hundreds of trials with randomly generated signal matrices containing no more than three zeros per row. Within this search space, the yield matrix performed better in the vast majority of cases. Thus, our simulation successfully confirmed that, compared to yield signs, traffic signals have at best comprable efficacy.

The simulation is limited in scope, however. It does not account for pedestrian traffic, for driver mistakes and accidents, or for effects of weather conditions or other factors. It is also limited to roundabouts with only one lane. As Figure 9 shows, for our model, flow rates in excess of 2500 cars per hour will clog the roundabout for any input control. To some extent, the effect of increasing flow can be mitigated by increasing roundabout radius; however, for flow rates in excess of 3000 cars per hour, we believe a two lane roundabout would be necessary. A simple case of this would be a roundabout with outer "express" lanes from which a vehicle may travel only from one entrance to the next exit, such as in Figure 10. In this case, traffic signaling would always impair flow, because the "express" lanes are always vacant for an entering vehicle. More complex scenarios would require a more sophisticated model. Such cases include multi-lane roundabouts in which vehicles are free to merge from lane to lane throughout, or large roundabout systmes comprised of multiple smaller roundabouts.

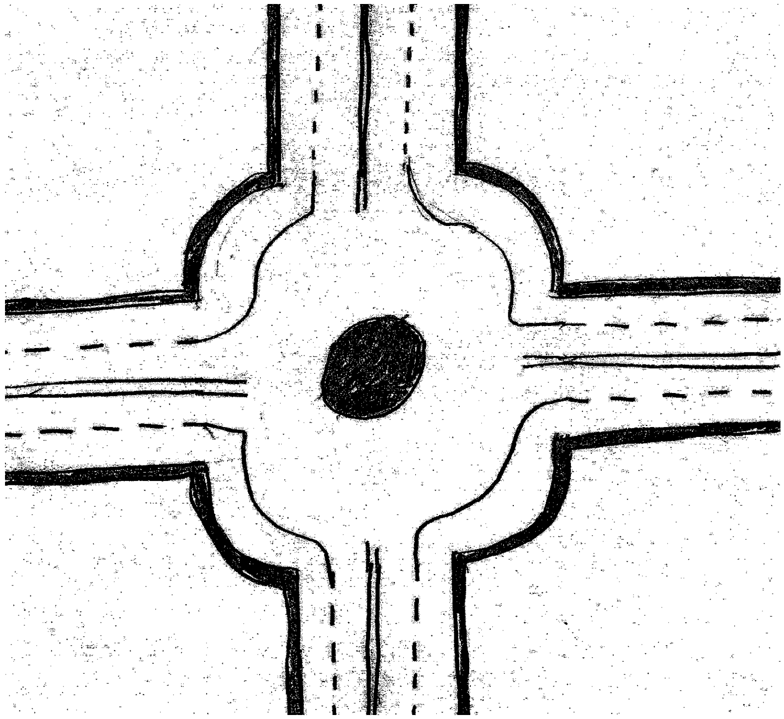


Figure 10: An “Express” Roundabout

5 Conclusion

Our search through literature, parameter space, and computer-generated experimental results brought us to a conclusion validated in intersections across the United States: yield sign control is nearly always the best way to regulate roundabout entry. Our analytic formulation of a Jackson network led us to calculate that the optimal choice of thinning parameter for the system corresponds to a yield sign at each entrance. Our computer simulation concluded that single-lane roundabouts with yield signs at all entrances are extremely well-behaved with input rates below 3000 cars per hour, although higher flow rates may necessitate additional lanes. Although we concede that extenuating circumstances may require the use of a traffic signal, in the vast majority of cases a yield sign is the most effective method of flow control in a roundabout.

Technical Summary

This analysis is best suited for single-lane roundabouts which are not unduly effected by neighboring traffic installations. In order to analyze roundabout efficiency and recommend an appropriate flow control method, the following information must be obtained:

Number of Entrances and Exits: Include relative popularity of all exits.

Separation Distance Between Entrances and Exits: This can be reported as an anglular or a circumferential distance.

Peak Input Flow from Each Entrance: These numbers should be reported as vehicles per hour, sampled during "rush hour".

Roundabout Radius: This is the distance from the center of the roundabout to the center point of the vehicle lane.

Peak Flow Times: This model requires the times of peak flow and average duration. In this model, traffic flow will follow a basic "rush hour" pattern; that is, there will be two periods during the day for which peak flow rates occur; at other times, flow is scaled down. Flow is scaled down most during the late night and early morning hours. For situations with less noticeable rush-hour patterns, a large "duration" parameter will smooth out flow variation over the course of the day.

This model will be able to output the average time spent in the roundabout or waiting to enter the roundabout. Optimal efficiency is defined as minimizing this time. Our models of roundabout flow indicate that, in the vast majority of cases, the optimal mechanism for flow control is a yield sign at each entry point. This is largely because the traffic signal acts as a restricted yield sign, tending to impair traffic flow whenever volume dips below peak flow. We recommend a yield sign if your parameters fit into the ranges below:

Any Number of Input Lanes: This is 2 or more. Variation of this parameter does not alter flow control recommendations.

Adequate Separation Between Entrances and Exits: As long as there is enough space to fit all entrances and exits on the roundabout circumference, this parameter does not effect flow control recommendations.

Low to Medium Total Peak Input Flow: The sum of the inputs is in the range of 200 and 3000 cars per hour.

Relative Intensity of Input Flow: The maximum ratio of peak input flow from any two entrances is less than 8:1. For larger discrepancies, more analysis may be necessary.

Slow to Medium Roundabout Travel Velocity: Vehicles in the roundabout are moving at speeds less than 30 mph, slower for smaller roundabouts.

Roundabout Radius: Roundabout radius is between 15 m and 45 m.

Peak Flow Times: This parameter does not in general effect efficiency.

For all different combinations of size, velocity, and flow within these ranges, yield signs inevitably proved to be the best solution. Furthermore, these parameter ranges represent a full spectrum of radii, input flows, and velocities commonly found in U.S. roundabouts [7].

There are a few situations where signaled entry may be appropriate. If a certain entry point is subject to highly concentrated flow during very short times (such as a sports venue emptying out after a game), a traffic signal during those times may permit efficient release of cars. We recommend, however, that the light be left entirely green during all other times. If the roundabout experiences a large volume of pedestrian traffic, signals (or stop signs) may be a reasonable way to decrease accident likelihood, but this determination requires analysis of pedestrian habits and attitudes.

If location permits, other design elements may be modified to improve roundabout efficiency. For traffic flows of up to 3000 vehicles per hour, single lane yield-control roundabouts are very efficient. For peak flow up to 2000 vehicles per hour, a roundabout of 20-40 m radius will be be sufficient to ensure average total roundabout wait and travel time below 20 seconds. For peak flows between 2000 and 3000 vehicles per hour, we recommend planning for a 40-60 m roundabout radius, which will keep average wait a travel time below 25 seconds. As flow increases beyond 3000 vehicles per hour, average wait and travel time will increase very quickly, and we recommend designing a multiple-lane roundabout if possible.

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