A realistic two-lane traffic model for highway traffic

W Knospe†, L Santen‡, A Schadschneider§ and M Schreckenberg†

† Theoretische Physik FB 10, Gerhard-Mercator-Universität Duisburg, Lotharstr. 1, D-47048 Duisburg, Germany
‡Fachrichtung Theoretische Physik, Universität des Saarlandes, Postfach 151150, 66041 Saarbrücken, Germany
§ Institut für Theoretische Physik, Universität zu Köln, Zülpicher Str. 77, D-50937 Köln, Germany

PACS numbers: 45.70, 05.60, 02.50

Abstract. A two-lane extension of a recently proposed cellular automaton model for traffic flow is discussed. The analysis focuses on the reproduction of the lane usage inversion and the density dependence of the number of lane changes. It is shown that the single-lane dynamics can be extended to the two-lane case without changing the basic properties of the model which are known to be in good agreement with empirical single-vehicle data. Therefore it is possible to reproduce various empirically observed two-lane phenomena, like the synchronization of the lanes, without fine-tuning of the model parameters.

1. Introduction

In [1] a cellular automaton model for traffic flow is proposed that is based on the Nagel and Schreckenberg (NaSch) model [2] and that is capable to reproduce all of the empirically observed traffic states [3, 4], i.e., free flow, wide moving jams and especially synchronized traffic (see [5, 6] for recent reviews on the modeling of traffic flow). Moreover, the accordance of the model with the empirics can be found even on a microscopic level of description. In [7] this degree of realism is underscored by considering a more realistic simulation setup, i.e., a simulation of a two-lane highway segment with an onramp. Empirical observations suggest [4, 8] that lane changes are responsible for the occurrence of strong correlations of the velocity and flow measurements between neighboring lanes in the synchronized states and in wide moving jams. However, it was concluded [7] that lane changes do only select rather than generate traffic states.

Although the results of [7] do not depend on the special choice of the lane change update, the application of the model in more sophisticated topologies should be provided with realistic lane changing rules.

Much progress has been made in the simulation of large traffic networks like the inner-city of Duisburg [9], the Dallas/Forth-Worth area [10] and the highway network of North-Rhine-Westfalia [11]. These simulations were based on the NaSch model and use a detailed representation of the systems infrastructure like multi-lane traffic, on- and offramps and highway intersections. However, a realistic microscopic description of the dynamics of each individual vehicle in a network requires to take into account the peculiarities of the complex road structures. It is therefore necessary to formulate simple rules in the language of cellular automata which mimic the behavior of the drivers as simple as possible and yield reasonable results compared with empirical findings. Unfortunately it turned out that the incorporation of two-lane traffic even in the NaSch model is a formidable task and the formulation of realistic lane changing rules is very difficult.

Only a few empirical results for two-lane traffic exist that help to specify lane changing rules [12, 13, 14, 15]. The ability to change lanes should increase with the density, shows a maximum in the vicinity of the flow maximum and then decreases with increasing density [12]. Nevertheless, lane changes are still possible at large densities. A special feature of German highway traffic is the empirically observed lane usage inversion. Although there is a right lane preference, the distribution of the flow becomes asymmetric and the flow is larger on the left than on the right lane [12, 16]. In contrast, in urban traffic or on highways without the right lane constraints the traffic flow is evenly distributed on both lanes [13, 14, 15].

Several two-lane extensions [17, 18, 19, 20, 21, 22] of the NaSch model were proposed which are able to reproduce the empirically observed lane usage inversion. However, they fail to model a realistic density dependence of the number of lane changes. Moreover, it was shown that some problems of the cellular automaton approach exist if one introduces different kinds of vehicles, but these shortcomings can be minimized by the consideration of anticipation effects [23].

Here, we extend the recently introduced traffic model [1] to two-lane traffic and answer the question which mechanism gives rise to the empirically observed lane usage inversion. It is clarified that this extension can be made without changing the basic realistic properties of the single-lane model. Moreover, it is demonstrated that lane changes do lead to strong correlations between neighboring lanes.

The simulations are (if not stated otherwise) performed on a highway of length 50000 cells with periodic boundary conditions. In addition to the local measurements of [1] traffic data are also recorded by averaging over the whole highway segment. This allows, in particular, to observe lane changes which cannot be measured by local detectors. The averaging process, however, does not distinguish between the various traffic states on different parts of the highway [8].

In the next section we briefly summarize the definition of the single-lane model and present a simple two-lane extension. Because it is not possible to generate a lane usage inversion by the introduction of different kinds of cars, we present in section 3 an asymmetric lane changing rule set. In section 4 we show that the vehicle dynamics of the single-lane model is not influenced by the lane-changes, but the car dynamics is determined by the single-lane update rules only. The advantages of two-lane traffic in comparison to single-lane traffic become obvious in section 5. Finally, we conclude with section 6.

2. Symmetric model

Before we will present our lane changing rule set, we briefly define the single-lane model (see [1] for a detailed explanation). The model used here is based on the Nagel-Schreckenberg cellular automaton model for traffic flow [2]. The road is divided into cells and a vehicle has a length of five cells. The movement of the vehicles is determined by a set of simple update rules and the system update is performed in parallel. The model of [1] is based on the desire of the drivers for smooth and comfortable driving

variable		parameter	
x	position	gap_{safety}	controls the effectiveness of the anticipation
v	velocity	$v_{\max}$	maximum velocity
v_{anti}	anticipated velocity	p_b	deceleration probability
d	distance-headway	p_0	deceleration probability
d^{eff}	effective distance-headway	p_d	deceleration probability
b	status of the brake light	t_h	time-headway
p	deceleration probability	t_s	safety time-headway

Table 1. Summary of the variables and of the parameters of the model used in the model definition.

whereas the dynamics of the NaSch model only assures the avoidance of crashes. This more sophisticated behavior is incorporated in a driving strategy which comprises three aspects: (i) At large distances the movement of a vehicle is determined by the maximum possible velocity. Vehicles at rest have a smaller acceleration capability than moving vehicles. Additionally, the velocity of the leading car is anticipated which allows for smaller gaps and larger velocities. (ii) At intermediate distances vehicles anticipate last minute braking by means of brake lights. (iii) At small distances the drivers adjust their velocity such that safe driving is possible.

Introducing the randomization function

$$p(v_n(t), b_{n+1}(t), t_h, t_s) = \begin{cases} p_b & \text{if } b_{n+1} = 1 \text{ and } t_h < t_s \\ p_0 & \text{if } v_n = 0 \text{ and not } (b_{n+1} = 1 \text{ and } t_h < t_s) \\ p_d & \text{in all other cases.} \end{cases}$$

and the effective distance

$$d_n^{\mathrm{eff}} = d_n + \max(v_{\mathrm{anti}} - gap_{\mathrm{safety}}, 0),$$

where d_n is the gap of vehicle n, $v_{\mathrm{anti}} = \min(d_{n+1}, v_{n+1})$ the expected velocity of the preceding vehicle in the next time step and gap_{safety} controls the effectiveness of the anticipation, the update rules consist of the following 5 steps (see table 1 for a summary of the parameters and variables of the model):

0. Determination of the randomization parameter p:
   $p = p(v_n(t), b_{n+1}(t), t_h, t_s)$
   $b_n(t+1) = 0$
1. Acceleration:
   if $((b_{n+1}(t) = 0)$ and $(b_n(t) = 0))$ or $(t_h \geq t_s)$ then:
   $v_n(t+1) = \min(v_n(t) + 1, v_{\max})$
2. Braking rule:
   $v_n(t+1) = \min(d_n^{(\mathrm{eff})}, v_n(t))$
   if $(v_n(t+1) < v_n(t))$ then: $b_n(t+1) = 1$
3. Randomization and braking:
   if $(rand() < p)$ then:
   $v_n(t+1) = \max(v_n(t+1) - 1, 0)$
   if $(p = p_b)$ then: $b_n(t+1) = 1$
4. Car motion:
   $x_n(t+1) = x_n(t) + v_n(t+1)$

Step 0 determines the value of the randomization parameter p which depends on the status of the "brake light" b_n (on/off) and the velocity v_n of the n-th car (Cars are numbered in the driving direction, i.e., vehicle $n+1$ precedes vehicle n. In case of two-lane traffic this numbering is applied to each single lane separately. Thus, cars are relabeled after lane changes.). In the next three steps the velocity v_n of vehicle n is calculated. A car accelerates if its own brake light and that of the leading car is switched off. If the time-headway t_h to the next car is large enough the car tries to approach the desired velocity in any case. The braking rule ensures safe driving – the velocity is adapted to the effective distance $d_n^{(\mathrm{eff})}$ to the leading vehicle. The effective distance is used instead of the real spatial distance d_n to the leading vehicle. If the velocity is reduced the brake light is switched on. The next step introduces the stochastic element into the model, i.e., the velocity is reduced with probability $p(v_n(t), b_{n+1}, t_h, t_s)$ as determined in step 0. Please note that the implementation also allows for the propagation of the brake lights. In the final step the positions of the vehicles are updated and the status of the brake light is reseted. The two times $t_h = d_n/v_n(t)$ and $t_s = \min(v_n(t), h)$, where h determines the range of interaction with the brake light, are introduced to compare the time t_h needed to reach the position of the leading vehicle with a velocity-dependent (temporal) interaction horizon t_s. Realistic behavior is observed for the model parameters $v_{\max} = 20$, $p_d = 0.1$, $p_b = 0.94$, $p_0 = 0.5$, $h = 6$, $gap_{\mathrm{safety}} = 7$ which have been used in the simulations. The cell length is 1.5 m, a car has a length of 5 cells [1]. A time-step corresponds to 1 s in reality. Note that no fine-tuning of the parameters is necessary and the model is quite robust against changes of the parameter values.

In order to extend the model to two-lane traffic one has to introduce lane-changing rules. A lane changing rule set should (i) consist of simple, local rules, (ii) be robust regarding slow vehicles, that is the flow should not be dominated by the introduction of a small fraction of slow vehicles, (iii) reproduce empirical lane changing curves, (iv) show the empirically observed lane usage inversion (in the asymmetric case) and (v) should not change the dynamical behavior of the single-lane system.

In general, lane changing rules can be symmetric or asymmetric with respect to the lanes or to the cars. While symmetric rules treat both lanes equally (see, e.g., urban traffic or highway traffic in the USA), asymmetric rule sets especially have to be applied for the simulation of German highways, where lane changes are dominated by a right lane preference and a right lane overtaking ban. Moreover, an asymmetry between, e.g., cars and trucks is provided on a two-lane highway since trucks are not allowed to change to the left lane.

In principle, all lane changing rule sets of cellular automaton models for traffic flow are formulated analogously [18, 5]. First, a vehicle needs an incentive to change a lane. Second, a lane change is only possible if some safety constraints are fulfilled. Asymmetry is introduced if one applies different criteria for the change from left to right and right to left.

A system update is performed in two sub-steps: In the first step the cars change the lanes according to the lane changing rules and do not move. In the second step, the cars move according to the calculated velocity. Both sub-steps are performed in parallel for all vehicles.

As a first step towards a realistic two-lane model for highway traffic we studied symmetric lane changing rules with identical rule sets for the change from the left to the right and the right to the left lane. The lane changing strategy comprises two aspects: First, the interaction of a vehicle with its predecessor on its lane should be minimized for safety or just for comfort reasons. Thus, a vehicle can optimize its travel time by driving as fast as possible due to an optimal gap usage. In order to keep the model as simple as possible we restrict the lane interaction to vehicles which have to brake in the next time step due to an insufficient gap in front. Vehicles which had to brake (i.e., their brake light is activated) are not allowed to change the lane. Second, safety reasons or constraints by law require the minimization of the interaction of a changing vehicle with its predecessor as well as with its successor on the destination lane. Thus, a vehicle is allowed to change the lane only if the gap between successor and predecessor on the destination lane is sufficient. In order to increase the efficiency of the lane changes, the movement of the preceding vehicle on the destination lane is anticipated.

The lane changing rules then are as follows:

(i) Incentive criterion:

$$(b_n = 0) \ \textit{and} \ (v(t) > d)$$

(ii) Safety criterion:

$$(d_{\text{pred}}^{(\text{eff})} \geq v(t)) \ \textit{and} \ (d_{\text{succ}} \geq v_{\text{succ}})$$

$v(t)$ and d are the velocity and the gap of the vehicle, d_{succ} and v_{succ} are the gap to the succeeding vehicle on the destination lane and its velocity, respectively. d_{pred} denotes the gap and $d_{\text{pred}}^{(\text{eff})} = d_{\text{pred}} + \max(v^{(\text{anti})} - gap_{\text{safety}}, 0)$ denotes the effective gap to the preceding vehicle on the destination lane (where $v^{(\text{anti})} = \min(gap, v_{\text{pred}})$ is the expected velocity of the leading vehicle on the destination lane in the next time step and gap and v_{pred} is the gap and the velocity of the predecessor on the destination lane) (see Fig. 1). The effectiveness of the anticipation is controlled by the parameter gap_{safety}. Accidents are avoided only if the constraint $gap_{\text{safety}} \geq 1$ is fulfilled. The same value for gap_{safety} as in the single-lane model is applied.

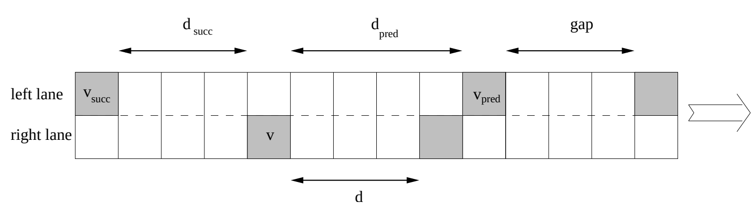

Figure 1. Sketch of a road segment and illustration of the quantities relevant for the lane changing rules. The hatched cells are occupied by a vehicle.

The symmetry of the lane changing update rules is reflected in the fundamental diagram‡ (Fig. 2): both lanes show the same flow density relationship, especially they have the same maximum flow $J_{\max}$ at the same density $\rho_{\max}$. Nevertheless, there is a strong exchange of vehicles between the lanes. Although the empirical lane change curve [12] was taken from measurements on German highways where asymmetric rules

‡ Note, that the kink in the fundamental diagram is an artifact of the global measurement process: since it is not distinguished between the various traffic states on different highway sections, one simply averages over free flow and synchronized states.

have to be applied, it is quite well reproduced on a qualitative level by simulations of the symmetric model while on a quantitative level the number of lane changes is too large (Fig. 3). A detailed analysis reveals that at small densities about 50% of the lane changes are ping-pong changes [17]. Since anticipation is only applied in step 2 of the lane changing update, once a vehicle with an effective gap changed the lane the condition $v > d$ of step 1 is always fulfilled. As a consequence, the vehicle changes the lane again.

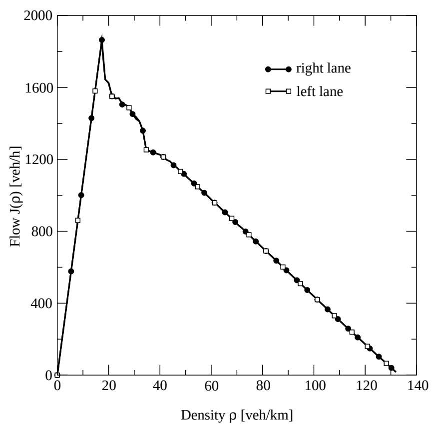

Figure 2. Fundamental diagram of the individual lanes in the symmetric model.

If we drop the single-lane model extensions (slow-to-start rule, anticipation, brake lights) successively, the lane change curve changes drastically (Fig. 4). The original NaSch model as well as the NaSch model with velocity-dependent randomization where a slow-to-start rule is applied (VDR-model [24]) show a maximum of the lane change number at large densities, but not at $\rho_{\max}$ [12]. Additionally, the number of lane changes is reduced considerably compared to the full model (Fig. 3). However, the introduction of brake lights or anticipation shifts the maximum of the curve to $\rho_{\max}$. On one hand, anticipation increases the effectiveness of the lane changes because the gap acceptance is increased. Moreover, since platoons of vehicles with a gap smaller than the velocity exist, the first lane change rule is often fulfilled leading to an artificial enhancement of the lane changing probability. On the other hand, brake lights visualize breakdowns of the downstream traffic timely, so that vehicles are able to swerve to the other lane in order to avoid abrupt brakings. Thus, from a microscopic point of view only the introduction of brake lights is responsible for the realistic lane changing behavior.

The symmetry of the lane changing rules is responsible for the even distribution of the density on both lanes. In contrast, lane usage inversion was observed on German highways where the left lane is designated as the passing lane and the lane changing behavior is determined by the right lane preference and the right lane overtaking ban.

In order to implement an artificial asymmetry we introduced disorder by

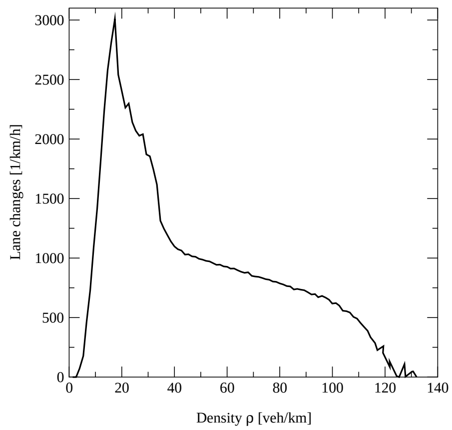

Figure 3. Number of lane changes per lane in the symmetric model.

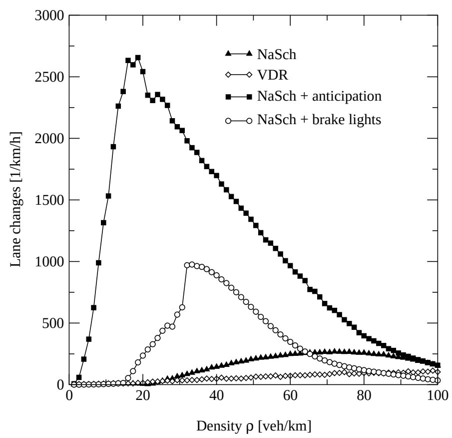

Figure 4. Lane changes of successive extensions of the symmetric two-lane NaSch model.

considering different types of vehicles like cars and trucks. Unfortunately, the introduction of disorder in cellular automaton models for two-lane traffic has some shortcomings: it is possible that two slow vehicles driving side by side on different lanes can form a plug which blocks the succeeding traffic [23, 25]. These plugs are very stable in the free flow regime and their dissolution is determined by velocity fluctuations. As a result, the lifetime of the plugs is very large and the flow is dominated by the slowest vehicle. In the NaSch model without anticipation, even one slow car can dominate the flow. Anticipation reduces the formation of plugs considerably [23], but they still seem to be slightly overestimated by the model. To avoid the formation of plugs all trucks are therefore initialized on the right lane only, and are not allowed to change the lane.

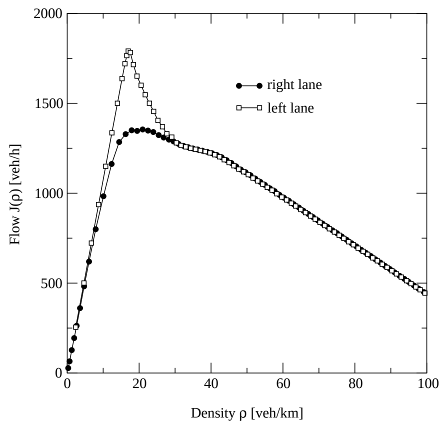

Figure 5. Fundamental diagram of the individual lanes of the symmetric model with 10% trucks. Note, that the trucks are not allowed to change to the left lane.

Suppose that 10 % of the vehicles are trucks with $v_{\max}^{\text{trucks}} = 15 = 81\ km/h$. As a result of the asymmetric vehicle distribution the flow on the right lane is dominated by trucks, while the flow on the left lane is comparable to the flow of the homogeneous model (Fig. 5). Obviously, the introduction of trucks does not allow the reproduction of the lane usage inversion (Fig. 6): The flow on the left lane is larger than on the right lane for *all* densities since fast vehicles avoid to drive on the right lane.

However, since one value of the total flow can be related to either free flow or congested traffic, the lane-usage is not a unique function of the total flow. In contrast, the lane usage depending on the density gives unique results in the whole density regime. Therefore, the lane usage curve described by the density is used in the further analysis since, as an advantage, density can be calculated explicitly in the simulations. Moreover, a comparison of both methods shows that the lane usage inversion point has the same flow-density value.

Due to the trucks on the right lane, at small densities nearly all fast cars are on the left lane. With increasing density the velocity differences between the cars and the trucks decrease so that both lanes are evenly occupied (Fig. 6). In contrast to the empirically observed lane usage inversion, neither most of the cars are on the right lane at small densities, nor most of the cars are on the left lane at large densities. In order to obtain the lane usage inversion, it is therefore necessary to incorporate an asymmetry into the lane changing rules §.

§ Note, that the lane usage inversion can be traced back to an asymmetric distribution of the density

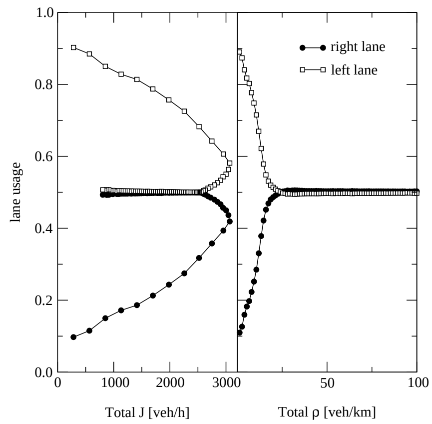

Figure 6. Lane usage of the symmetric model with 10% trucks as a function of the flow (left) and the density (right). Note, that the trucks are not allowed to change to the left lane.

On German highways two mechanisms lead to an asymmetry between the lanes:

(i) The *right lane preference* is enforced by the legal regulation to use the right lane as often as possible.

(ii) The *right lane overtaking ban* prohibits a car driving on the right lane to overtake a car which is driving on the left lane.

In order to clarify which of these two mechanisms is responsible for the lane usage inversion, we first focus on the right lane overtaking ban. Therefore, lane changes to the left lane are forced in the sense that a car on the right lane has to change to the left lane in case of a velocity larger than its gap to the predecessor on the destination lane. In addition to that, the safety criterion is weakened, so that cars even with $v > d^{(\mathrm{eff})}_{\mathrm{pred}}$ are allowed to change the lane. However, the overtaking ban is not formulated in a strict sense since vehicles are allowed to overtake a car driving on the left lane if a lane change is not possible. As a result, most of the cars are on the left lane for all densities. With increasing density this difference of the lane usage decreases, but nevertheless it is not possible to observe a lane usage inversion.

Second, we focus on the right lane preference and force changes from left to right. Obviously, the right lane preference alone is not sufficient for the correct description of the lane usage inversion, because cars now are predominantly on the right rather than on the left lane.

To summarize, neither it is possible to get a lane usage inversion by implementing disorder in the symmetric model, like different types of cars, nor by introducing either

on the lanes but not to a negative velocity gradient from the left to the right lane that leads to larger flows on the left lane. Different speed limits for the lanes increase the inversion but are not its reason, since lane usage inversion can also be observed on highways with speed limit.

the right lane preference or the right lane overtaking ban alone. One therefore has to introduce the right lane preference *and* the right lane overtaking ban simultaneously.

3. Asymmetric model

The straightforward implementation of the right lane overtaking ban of the preceding section leads to large lane changing frequencies. Furthermore, a stricter formulation of the right lane overtaking ban may lead to difficulties with regard to velocity anticipation or to the update procedure. For the sake of simplicity, however, the consequences of a right lane overtaking ban rather than the basic mechanisms are modeled. Vehicles are still allowed to overtake their predecessor on the left lane but the left lane should be preferred. Therefore, we reduced the ability to change from the left to the right lane [18, 19] so that vehicles on the left lane change back to the right lane only if there is a sufficient gap on both, the right and the left lane. The lane change rules are then as follows:

right → left

Incentive criterion: $(b_n = 0)$ *and*
$(v(t) > d)$

Safety criterion: $(d_{\text{pred}}^{(\text{eff})} \geq v(t))$ *and* $(d_{\text{succ}} \geq v_{\text{succ}})$

left → right

Incentive criterion: $(b_n = 0)$ *and*
$(t_{\text{pred}}^h > 3.0)$ *and* $((t^h > 6.0)$ *or* $(v > gap))$

Safety criterion: $(d_{\text{pred}}^{(\text{eff})} \geq v(t))$ *and* $(d_{\text{succ}} \geq v_{\text{succ}})$

The two times $t^h = d/v$ and $t_{\text{pred}}^h = d_{\text{pred}}/v$ give the time a vehicle needs to reach the position of its predecessor and its predecessor on the destination lane. Since the time-headways take the velocity of the vehicles into account, slow vehicles are allowed to change the lane even at small distances ‖.

With the two parameters t_{pred}^h and t^h it is possible to trigger the lane usage inversion. If the movement of a vehicle is undisturbed (at small densities) it changes the lane if its time-headway to the preceding vehicle on its own and on the destination lane is larger than 6 s and 3 s, respectively. Otherwise, at large densities the condition $v > d$ ensures an incentive to change to the right lane.

Figure 7 illustrates the asymmetry of the lane changing rules with the fundamental diagram: at small densities, the flow on the left lane is larger than on the right lane. As a consequence, the flow breakdown occurs first on the left lane. Once the maximum flow on the left lane is reached more and more cars swerve on the right lane until the flow is distributed evenly on both lanes. Therefore, the system breakdown is mainly

‖ Instead of comparing time-headways it is possible to use the vehicles gap reduced by a constant offset in the incentive criterion (e.g, $v < gap - offset$) in order to retard the change from the left to the right lane. Unfortunately, this has major drawbacks since for larger densities this condition is never be fulfilled [19].

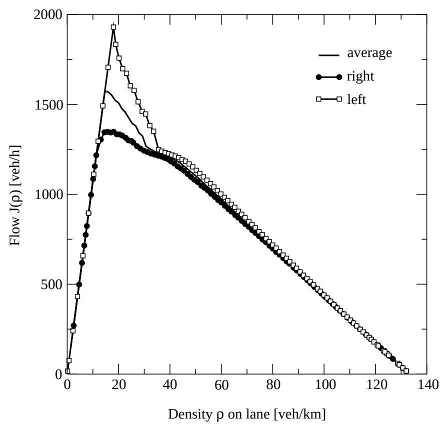

Figure 7. Fundamental diagram of the individual lanes in the asymmetric model.

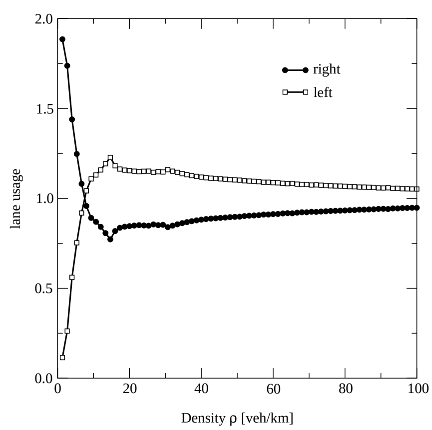

Figure 8. Lane usage in the asymmetric model.

triggered by a single-lane breakdown on the left lane [18]. However, the mean velocity on the right lane starts to decrease before the breakdown on the left lane occurs, so that the velocity is always larger on the left rather than on the right lane. Thus, the system separates into a fast and a slow lane.

The two competing mechanisms of the lane changing rules, namely the right lane overtaking ban and the right lane preference, lead to a lane usage inversion (Fig. 8).

At small densities the right lane preference dominates so that most of the vehicles are driving on the right lane. However, as a consequence of the right lane overtaking ban, with increasing density more vehicles avoid driving on the right lane. The position of the inversion point is mainly controlled by t^h while the extent of the lane usage inversion is controlled by t^h_{pred}. With increasing total density the lane usage inversion decreases (Fig. 8).

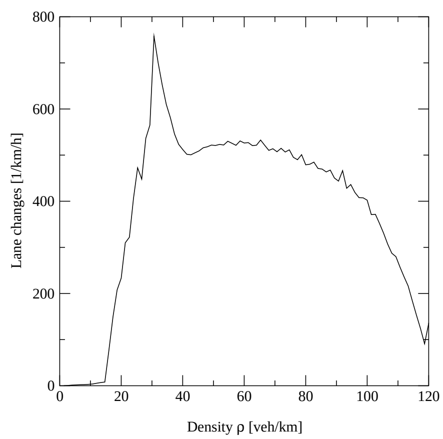

Figure 9. Number of lane changes in the asymmetric model.

Due to the retarded lane change from left to right the number of lane changes is decreased significantly compared to the symmetric model. Like in the symmetric model, the maximum of lane changes is reached in the vicinity of the maximum flow. Further increasing the density the lane change number drops to a plateau value until lane changes are no longer possible at very large densities (Fig. 9). In contrast to the symmetric model, the ping-pong changes are strongly suppressed due to the asymmetric rule set.

Next, we consider an inhomogeneous model with 10% trucks. In contrast to the symmetric model trucks are now allowed to change from the right to the left lane. In Fig. 10 the distribution of the trucks on both lanes is depicted. Although at small densities considerably more trucks are on the right than on the left lane, the trucks do not condensate completely on the right lane. Therefore, the flow is dominated by trucks on both lanes even for small densities and a small fractions of trucks. As a result of the lane usage inversion and a small number of trucks on the left lane, vehicles can go faster on the right rather than on the left lane. However, at densities larger than $\rho_{\max}$ the maximum velocity of the vehicles are smaller than the mean velocity so that the trucks are evenly distributed on both lanes.

The stability of the plugs is mainly determined by velocity fluctuations since the distance between two trucks driving side by side in free flow can only be increased by a repeated deceleration of one truck in the randomization step of the velocity update. We therefore distributed the maximum velocity of the cars $v_{\max}$ according to a Gaussian

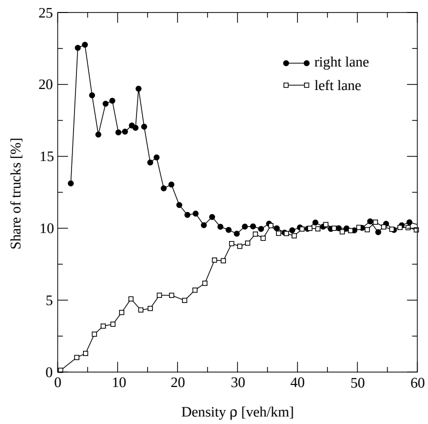

Figure 10. Distribution of the trucks on both lanes in the inhomogeneous asymmetric model. Note, that trucks are allowed to change the lanes.

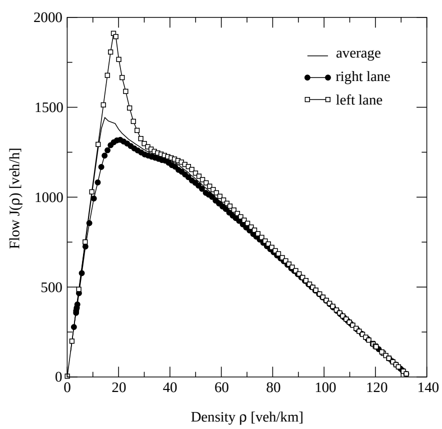

Figure 11. Fundamental diagram of the individual lanes in the inhomogeneous asymmetric model. Note, that trucks are not allowed to change the lane.

profile with different variances and a mean $\bar{v}_{\max} = 20$. Despite the possibility that now small differences of the maximum velocity of the trucks are allowed, the plugs remain still very robust. Even different Gaussian distributions of the maximum velocities of cars and trucks or special lane changing rules for trucks do not decrease the stability of the plugs. Plugs are therefore no artifact of the discretization but are formed by the

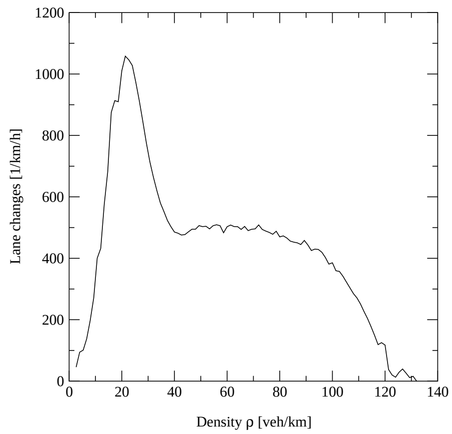

Figure 12. Lane changes in the inhomogeneous asymmetric model. Note, that trucks are not allowed to change the lane.

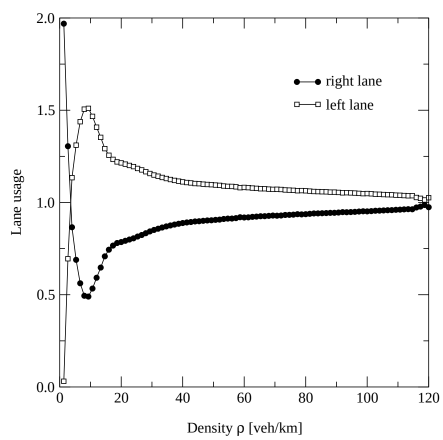

Figure 13. Lane usage in the inhomogeneous asymmetric model. Note, that trucks are not allowed to change the lane.

dynamics. Because of the asymmetry of the lane changing rules, once a truck changed to the left lane, there has to be a large gap in front on the left and on the right lane to change back. This increases the time trucks are on the left lane and supports the formation of plugs. In contrast to the symmetric model, even one truck on the left lane is able to dominate the flow of the left lane completely. As long as the truck stays on the left lane passing is not possible: due to the right lane overtaking ban, like in reality, the fast cars pile up behind the truck waiting for a possibility to pass.

In order to avoid the formation of plugs we reintroduce the lane changing ban for the trucks, so that the trucks now are lane stationary on the right lane. Like in the symmetric model, the flow of the right lane is dominated by trucks only in the vicinity of $\rho_{\max}$ (Fig. 11) which is a consequence of the increased robustness of the model regarding slow vehicles. As a result of the moving hindrances on the right lane, the number of lane changes is increased compared to the homogeneous model (Fig. 12). Now, lane changes are necessary even at very small densities to avoid being trapped behind a truck. Therefore, the lane usage inversion is increased significantly in the vicinity of $\rho_{\max}$ (Fig. 13).

Like in the homogeneous model the traffic breakdown is mainly controlled by the capacity of the left lane (Fig. 14). At small densities the flow on the left lane is larger than on the right lane since the density is distributed asymmetrically. Further increasing the density shifts the excess flow on the right lane up to its capacity. At large densities more vehicles are on the left than on the right lane because of the lane usage inversion, which results in a larger flow on the right lane.

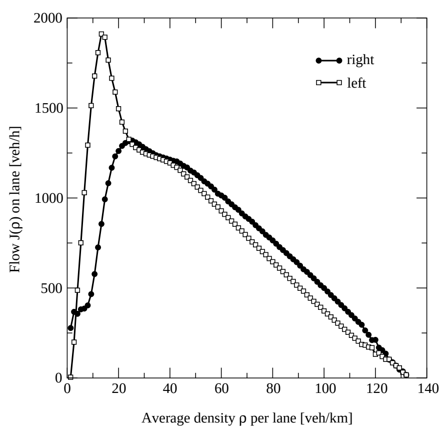

Figure 14. Fundamental diagram of the individual lanes in the asymmetric model. The density is given as the average density of the individual lanes. Note, that trucks are not allowed to change the lane.

4. Local measurements

In the last sections we focused our analysis on two-lane properties induced by lane changing maneuvers like the number of lane changes, the flow enhancement and the lane usage inversion. It turned out that it is possible to produce a realistic lane changing behavior which can reproduce empirical data already by means of a simple asymmetric rule set. Nevertheless, it is an open question whether the vehicle dynamics of the single-lane model is modified by the introduction of lane changing rules.

In order to compare the two-lane model with the corresponding single-lane model, we evaluated the simulation data by a virtual inductive loop, i.e., we measured the speed and the time-headway of the vehicles at a given link of the lattice. The density is calculated via the relation $\rho = J/v$ where J and v are the mean flow and the mean velocity of cars passing the detector in a time interval of 1 min. In addition to the aggregated data, the single-vehicle data of each car passing the detector are also analyzed.

A detailed analysis of the locally measured data of the homogeneous model shows no difference of the two-lane model to the single-lane model on a macroscopic (i.e., data averaged over an interval of 1 min) as well as on a microscopic (i.e., single-vehicle data) level in the congested regime. Moreover, there is no difference of the locally measured data between the two-lanes in the congested regime. In contrast, in the free flow regime the time-headway distribution of the left lane shows an increased number of small headways compared to the right lane. The lane usage inversion at small densities increases the car-car interactions on the left lane which leads to clusters of cars with small gaps and large velocities. In the inhomogeneous model the inclusion of disorder dissolves these clusters so that the difference between the time-headway distributions to the single-lane model vanishes again. Moreover, trucks on the right lane reduce the mean velocity in the free flow regime and shift the velocity-headway curve to smaller velocities at small distances compared to the left lane. Nevertheless, these results show, that the vehicle dynamics is not influenced by the lane changing rules but by the traffic state ¶.

Empirical observations suggest a strong coupling of the lanes in the synchronized state [8]. In order to quantify the interaction of the lanes, we calculated the crosscorrelation $cc(\tau)$ of the time-series of measurements of the density, the flow and the velocity on different lanes:

$$cc(x_i, x_j) = \frac{\langle x_i(t) x_j(t+\tau)\rangle - \langle x_i(t)\rangle\langle x_j(t+\tau)\rangle}{\sqrt{\langle x_i^2(t)\rangle - \langle x_i(t)\rangle^2}\sqrt{\langle x_j^2(t+\tau)\rangle - \langle x_j(t+\tau)\rangle^2}}. \tag{1}$$

Here x_i denotes either the flow, the density or the velocity in lane i.

In the free flow state the weak interaction of the lanes results in a small correlation of all quantities (Fig. 15). Since the flow is mainly controlled by density fluctuations the density and the flow measurements show the same correlations. In contrast, at larger densities the velocity and the flow on both lanes is strongly correlated ($cc(x_i, x_j) \leq 0.6$ at $\tau = 0$), i.e., indicating that the system is in the synchronized regime. However, due to the large variance of the density measurements in the synchronized regime the density on both lanes is not correlated (Fig. 16).

Lane changes are responsible for the synchronization of the velocities of different lanes [4]. In case of a velocity difference between the lanes, vehicles are changing to the faster lanes, thus decreasing the velocity difference. This leads to a strong coupling of the lanes in the congested regime. In particular, in the vicinity of on- and offramps cars enter or leave only the right lane, but both lanes are disturbed at large in- or outflows. In order to verify this, we simulated a system with open boundary conditions and generated a jam on the right lane by a large input of an onramp. As a result, the jam spreads to the left lane, so that the time-series of the velocity measurements of

¶ A detailed analysis of the single-lane model reveals the independence of the vehicle dynamics in the congested regime even on the maximum velocity.

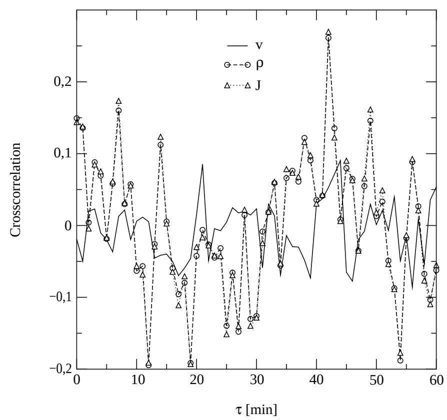

Figure 15. Crosscorrelation of the locally measured flow, density and velocity of neighboring lanes in the free flow regime ($\rho = 6$ veh/km) in the asymmetric model.

both, the left and the right lane are highly correlated. Moreover, the traffic breakdown occurs at the same time (Fig. 17) leading to a synchronization of the lanes.

5. Single-lane vs. two-lane traffic

A detailed comparison of the single-lane model with the corresponding two-lane models shows that it is not possible to increase the flow of a two-lane model to more than twice the flow of a single-lane model by means of an asymmetric lane interaction. In contrast to symmetric models, the flow of the asymmetric model on the right lane is decreased whereas the flow on the left lane is increased compared with the single-lane model (Fig. 18). The right lane overtaking ban and the right lane preference lead to a suboptimal gap usage and, therefore, to a reduction of the flow on the right lane. As a result, the mean flow per lane is smaller than the flow of a homogeneous single-lane model.

However, the benefits of the two-lane model become visible if different types of cars with different maximum velocities are considered. (Note, that the trucks are not allowed to change the lane.) Due to the possibility of overtaking, the system is more robust regarding disorder. While the flow of a single-lane model is dominated already by just one slow car [23, 25] (since passing is not possible), the flow on the right lane of the two-lane model is only decreased in the vicinity of $\rho_{\max}$ compared with the homogeneous model.

Furthermore, the travel-time of a vehicle is reduced considerably by the lane interaction. The travel-time is measured as the time a vehicle needs to pass a given fixed link of the lattice twice. Thus the travel-time on the right lane is given by vehicles that passed the measurement section on the right lane. Here it is not distinguished whether the car started on the left or the right lane and lane changes during the trip

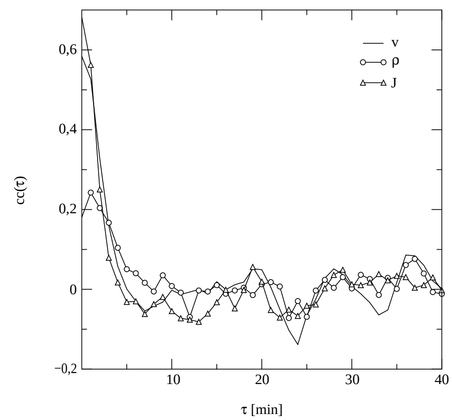

Figure 16. Crosscorrelation of the local measured flow, density and velocity of neighboring lanes in the synchronized state ($\rho = 50\ veh/km$) in the asymmetric model.

through the system are allowed.

Compared to the single-lane model, the width of the travel-time distribution in the congested regime of the two-lane model is larger (Fig. 19) but the maximum has moved to smaller times. On one hand, vehicles which cannot change the lane due to an insufficient safety gap lead to larger travel-times. On the other hand, the possibility to change the lane reduces the travel-time. Moreover, the asymmetric distribution of the density leads to a smaller travel-time on the right lane than on the left lane Again, the results in the congested regime are independent on the fraction of trucks since driving with maximum velocity is no longer possible.

In the free flow regime, the inclusion of trucks on the right lane is responsible for a larger mean travel-time compared with the homogeneous two-lane model (Fig. 20). The trucks on the right lane enforce clustering so that the travel-time of cars trapped behind a truck increases which leads to a second small peak at large times in the travel-time distribution. Therefore, the more trucks are in the system, the larger is the variance of the travel-time distribution.

6. Conclusion

We have presented an asymmetric two-lane model with a simple and local lane changing rule set that shows the empirically observed lane usage inversion and reproduces the density dependence of the number of lane changes. In order to observe lane usage inversion it is not sufficient to implement asymmetry into the model by means of disorder, i.e., different types of cars. Moreover, the introduction of the right lane overtaking ban or the right lane preference alone fails to reproduce the lane usage inversion, too. Thus, realistic behavior is only obtained by incorporating both a right lane preference and a right lane overtaking ban simultaneously into the model's rules.

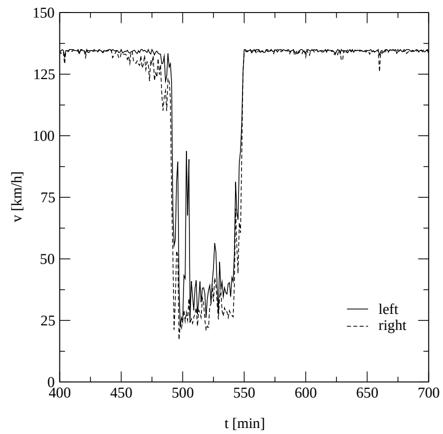

Figure 17. Asymmetric model with open boundary conditions. The large input rate of 0.35 of the onramp induces jams on both lanes.

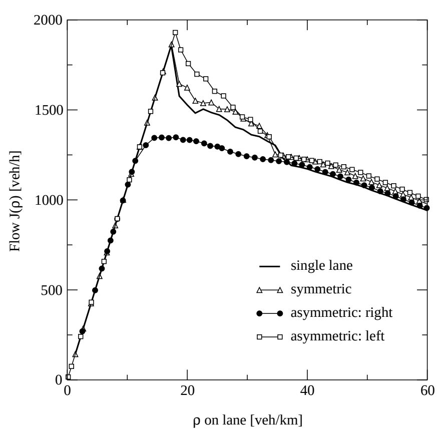

Figure 18. Comparison of the fundamental diagram of the single-lane model with the homogeneous two-lane models.

The benefits of the two-lane model compared with the single-lane model become visible by introducing disorder: there is no flow enhancement of the two-lane model with one type of cars, but the robustness of the flow regarding slow vehicles can be increased significantly by means of lane interaction. This lane interaction leads to a strong coupling of both lanes in the synchronized regime, which results in strong

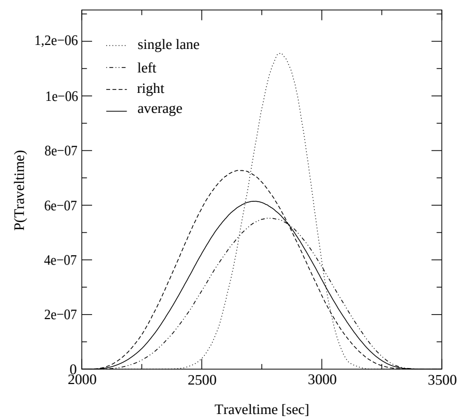

Figure 19. Travel-time distribution of the asymmetric two-lane model compared with the single-lane model at a density of 54 veh/km. The data points were smoothed by a moving average. The system length is 15 km.

correlations of the time-series of the velocity.

As a consequence, a jam generated on the right lane by an onramp can branch to the left lane leading to a congested region on both lanes. In order to improve and optimize the throughput of the highway it is possible to use ramp metering systems [26, 27, 28]. Here, such systems will help to reduce perturbations of the right lane and, thus, stabilize the flow even on the left lane.

Finally, a detailed analysis of local measurements shows that the car dynamics remains unchanged by the introduction of the lane changing rules. Therefore, the lane change extension of the single-lane model can be used for the propagation of traffic state information to all lanes, e.g., in order to synchronize the lanes, or to trigger distinct traffic states (e.g., in the vicinity of on- and offramps). Moreover, the independence of the velocity update and the lane change update allows for further improvements of the car dynamics and/or developments of more sophisticated lane changing rules (e.g., for bottlenecks).

Acknowledgments: The authors are grateful to the Ministry of Economics and Midsize Businesses, Technology and Transport for the financial support. L. S. acknowledges support from the Deutsche Forschungsgemeinschaft under Grant No. SA864/1-2.

References

[1] W. Knospe, L. Santen, A. Schadschneider and M. Schreckenberg, J. Phys. A **33**, 477 (2000).
[2] K. Nagel and M. Schreckenberg, J. Physique I **2**, 2221 (1992).
[3] B.S. Kerner, in *Traffic and Granular Flow'99*, p. 253, D. Helbing, H.J. Herrmann, M. Schreckenberg and D. Wolf (eds.) (Springer, 2000).
[4] B. S. Kerner, Networks and Spatial Economics, 1, 35 (2001).
[5] D. Chowdhury, L. Santen, and A. Schadschneider, Phys. Rep. **329**, 199 (2000).

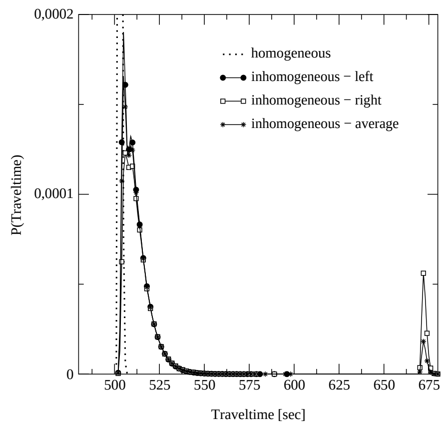

Figure 20. Comparison of the travel-time distributions of the homogeneous and the inhomogeneous asymmetric model at a density of 14 veh/km. The system length is 15 km.

[6] D. Helbing, to appear in Rev. Mod. Phys. [cond-mat/0012229].
[7] W. Knospe, L. Santen, A. Schadschneider, and M. Schreckenberg, Phys. Rev. E, (2001), in press.
[8] L. Neubert, L. Santen, A. Schadschneider, and M. Schreckenberg, Phys. Rev. E **60**, 6480 (1999).
[9] J. Esser and M. Schreckenberg, Int. J. Mod. Phys. C 8, 1025 (1997).
[10] M. Rickert and K. Nagel, Int. J. Mod. Phys. C 8, 483 (1997).
[11] O. Kaumann, K. Froese, R. Chrobok, J. Wahle, L. Neubert, and M. Schreckenberg, in *Traffic and Granular Flow '99*, p. 351, D. Helbing, H. J. Herrmann, M. Schreckenberg, D. E. Wolf (eds.) (Springer, 2000).
[12] U. Sparmann, *Spurwechselvorgänge auf zweispurigen BAB-Richtungsfahrbahnen*, Forschung Straßenbau und Straßenverkehrstechnik, Heft 263, 1978 (in German).
[13] Gang-Len Chang and Yang-Ming Kao, Trans. Res. A **25**, 375 (1991).
[14] F. L. Hall and T. N. Lam, Trans. Res. A **22**, 45 (1988).
[15] M. Brackstone, and M. McDonald, in *Traffic and Granular Flow*, p. 151 , D. E. Wolf, M. Schreckenberg and A. Bachem (eds.), (World Scientific, 1996)
[16] W. Leutzbach and F. Busch, *Spurwechselvorgänge auf dreispurigen BAB-Richtungsfahrbahnen*, Institut für Verkehrswesen, Universität Karlsruhe (1984) (in German).
[17] M Rickert, K. Nagel, M. Schreckenberg, and A. Latour, Physica A **231**, 534 (1996).
[18] K. Nagel, D.E. Wolf, P. Wagner, and P. Simon, Phys. Rev. E **58**, 1425 (1998).
[19] P. Wagner, K. Nagel, and D.E. Wolf, Physica A **234**, 687 (1997).
[20] P. Wagner, in *Traffic and Granular Flow*, D.E. Wolf, M. Schreckenberg and A. Bachem (eds.) (World Scientific, 1996).
[21] T. Nagatani, in *Traffic and Granular Flow*, p. 57, D. E. Wolf, M. Schreckenberg and A. Bachem (eds.) (World Scientific, 1996).
[22] T. Nagatani, J. Phys. A **29**, 6531 (1996)
[23] W. Knospe, L. Santen, A. Schadschneider, and M. Schreckenberg, Physica A **265**, 614 (1999).
[24] R. Barlovic, L. Santen, A. Schadschneider, and M. Schreckenberg, Eur. Phys. J **5**, 793 (1998).
[25] D. Chowdhury, D.E. Wolf, and M. Schreckenberg, Physica A **235**, 417 (1997).
[26] M. Papageorgiou, Transpn. Res. C **3**, 19 (1995).
[27] H. H. Salem, M. Papageorgiou, Transpn. Res. A **29**, 303 (1995).
[28] B. A. Huberman, D. Helbing, Europhys. Lett. **47**, 196 (1999).