Problem A Solution

Team Control Number 2108

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1 Summary

Rather than to explicitly construct a model for the national debt, this project first built an equation for the change in national debt and used differential equation techniques to solve for an expression which predicts future values of the national debt. The factors that this project incorporated were interest on the national debt, the inflation rate of the United States economy, the rate at which the debt is being paid off by the government per year, and the amount of deficit spending by the government per year.

The differential equation which was constructed is as follows:

$$D'(t) = rD(t) + ID(t) - \alpha y(t) + \mu s(t)$$

This equation is used to model the national debt per year. The r value signifies the interest on the national debt. The I value is the inflation rate of the economy. The α constant represents the percent of the government's profit used to pay off the national debt, and the μ constant indicates what percent of the planned deficit spending is actually performed. The μ constant will be left at $\mu = 1$ until future plans regarding the national debt are discussed. The $y(t)$ function is the amount money that the government has as pure profit, and the $s(t)$ function is the amount of deficit spending performed by the government.

Obtaining $y(t)$ and $s(t)$ involved performing regressions on GDP and deficit spending data, respsectively, in order to project deficit in the equation above. The resulting equation and constants projecting $D(t)$, the national debt with respect to year, and used for proposed solutions is displayed within the paper.

In order to use this model to arrive at solutions to the United States' national debt problem, the constants representing the percentage of government income devoted towards paying off the debt and the percent of current deficit spending rates were manipulated. The percentage of government income from the GDP

was not manipulated, as this ratio has remained essentially constant since 1790. The manipulation of these constants would require significant changes in national economic policy, which need to be carefully considered when evaluating the solutions produced from the model. In order to achieve a change in α, the government would need to divert funds from other investments, such as infrastructure and social security, to increase the amount paid against the debt. For most of the 21st century, the government has been employing a large amount of deficit spending to accommodate the expenses associated with the War on Terror. It follows that manipulating the μ constant would most likely require a reduction in defense spending and an appropriate cutback in many War on Terror and homeland security issues.

Our model produced an optimal solution to counter rising national debt. This solution proposed to increase α to .096 and decrease μ to .525. This hybrid solution calls for only a modest increase in payments against the national debt and a decrease in deficit spending that could feasibly be accomplished by a decrease in defense spending. Making these adjustments would arrest the growth of the national debt by 2017.

2 Restatement of Problem

National debt is one of the most prevalent issues in American politics today. National debt, also referred to as Government Debt, is any currency or credit owed from a given government to holders of that government's debt instruments. Based on reports from the United States Treasury Department, from 2000 to 2008 the national debt has increased from $5,000,000,000,000 to $10,000,000,000,000, and this trend is continuing. This becomes a major concern for the citizens of America because the government is forced to allocate more money from federal taxes to pay off the debt. The government goes about solving this problem in two ways: by increasing taxes and by cutting money spent on governmental services for the citizens of the United States. Action needs to be taken to help resolve national debt crisis before it reaches an even higher and un-payable rate due to interest, inflation, and deficit spending.

National debt is generated when the government encounters a budget deficit, meaning that more money is spent than is received in taxes. This causes the government to have to borrow money from the public and increases the amount of national debt that exists. Borrowing from the public is implemented through the selling of government bonds. Reasons for encountering such deficits come from government overspending, tax cuts, and war. Another reason for constant accumulation of the national debt is the interest on its debts. Due to the size of the national debt, raw interest adds a significant amount every year. Thus, a large portion of the money the government pays against the national debt must be allocated to simply paying off the interest.

In order to handle these situations, the government needs to produce several budget surpluses to repay the national debt. By using differential equations to model the United States' economy, the projected national debt for future years can be determined. By casting a function of the adjusted gross domestic product (essentially the income on the federal taxes) per capita, inflation and population do not have to be variables in the equation because it is implicitly predicted using the adjusted gross domestic per capita. With these equations, optimal plans for decreasing the amount of national debit can be formulated.

3 Assumptions

- The interest rate on the national debt and the current inflation rate will not deviate significantly from their current values in the interval from 2009 to 2017.
- Data gathered from government and other sources was accurately and correctly reported by those sources.

4 Model Design

There are many factors which play a part in the national debt. Rather than to explicitly construct a model for the national debt, this project first built an equation for the change in national debt and used differential equation techniques to solve for an expression which predicts future values of the national debt. The factors that this project incorporated were interest on the national debt, the inflation rate of the United States economy, the rate at which the debt is being payed off by the government per year, and the amount of deficit spending by the government per year.

The differential equation which was constructed is as follows:

$$D'(t) = rD(t) + ID(t) - \alpha y(t) + \mu s(t)$$

This equation reflects in change in the national debt per year. The r value signifies the interest on the national debt. The I value is the inflation rate of the economy. The α constant represents what percent of the government's profit it uses to pay off the national debt, and the μ constant indicates what percent of the planned deficit spending is actually performed. The μ constant will be left at $\mu = 1$ until future plans regarding the national debt are discussed. The $y(t)$ function is the amount money that the government has as pure profit, and the $s(t)$ function is the amount of deficit spending performed by the government.

Inflation is handled by adding the percent of the debt that is increased by the inflation rate every year. For example, if the national debt is $100 million, and the inflation rate is 3%, our model will add $3 million the first year and 3% of the existing national debt every subsequent year.

Running a linear regression on the values for nominal gross domestic product (GDP) per capita from 2000 to 2006 given by the Bureau of Economic Analysis yields the equation

$$G(t) = e^{\frac{t-a}{b}} = e^{\frac{t+249.058}{23.970}}$$

Where $t = 0$ corresponds to 1999. The values from 2000-2006 were chosen due to the relevance of the Iraq War and the War on Terrorism to our economy. Also, the nominal GDP per capita was used so that the value of the money for the time period would be preserved and also to ensure that the changing values of the population would not skew the predictions of the model.

Naturally, this project is not interested in GDP, but more in the actual amount of money that the government has to spend. The relationship to the amount of available money the government has to the GDP is

$$y(t) = pG(t) = pe^{\frac{t-a}{b}}$$

Where p is a constant, approximately $p = .18$ according to the Tax Policy Center. This equation can be adjusted for any time interval, assuming similar growth to the period 2000-2006:

$$y(t) = e^{\frac{-a}{b}} pe^{\frac{t}{b}} = pg_0 e^{\frac{t}{b}}$$

Where g_0 is the GDP for $t = 0$.

Using similar data from the Office of Management and Budget for the United States deficit spending from 2000-2006 the following equation was obtained:

$$s(t) = e^{\frac{t-A}{B}} = e^{\frac{t+144.136}{25.047}} = s_0 e^{\frac{t}{25.047}}$$

Substituting the aforementioned equations into the original differential equation yields

$$D'(t) = (r + I)D(t) - \alpha p g_0 e^{\frac{t}{b}} + \mu s_0 e^{\frac{t}{B}}$$

This equation can be solved by simply multiplying through by an integrating factor and integrating both sides. Solving yields the solution equation

$$D(t) = Ce^{(r+I)t} - \left(\frac{\mu s_0 B}{(r+I)B-1}\right) e^{\frac{t}{B}} + \left(\frac{b\alpha p g_0}{b(r+I)-1}\right) e^{\frac{t}{b}}$$

Where C is an arbitrary constant. Using $D(0) = D_0$, the solution becomes

$$D(t) = \left[D_0 + \left(\frac{\mu s_0 B}{(r+I)B-1}\right) - \left(\frac{b\alpha p g_0}{b(r+I)-1}\right)\right] e^{(r+I)t} - \left(\frac{\mu s_0 B}{(r+I)B-1}\right) e^{\frac{t}{B}} + \left(\frac{b\alpha p g_0}{b(r+I)-1}\right) e^{\frac{t}{b}}$$

Thus, given the constants D_0, s_0, r, I, b, B, p, and g_0 the equation will model the national debt as a function of time. The constants α and μ are in the equation so that the amount of money used to attempt to pay off the debt can be altered and also so that the effects of cutting back spending can be viewed quantitatively. This model was chosen for its versatility as well as its accuracy, which will be discussed later in the paper. Differential equations are routinely used as models for economic events, especially economic growth. Thus, a differential equation model was the most appropriate for the given problem.

5 Model Testing and Analysis

Constant values were set to reflect the reality of the United States economy. The initial values were set relative to 2000. During testing, constants were set as $\mu = 1$, $\alpha = .07$, and $p = .18$ in accordance with data from Bureau of Economic analysis. In billions of dollars, these initial values were $g_0 = 34785$, $s_0 = 330.765$, and $D_0 = 5674$. Further, the r and I values were chosen by taking the average of the interest rate on the national debt and the inflation rate, respectively, between 2000 and 2006.

Because GDP and deficit spending values were collected in the interval from 2000 to 2006, it was in these years that the actual national debt and the model's projected national debt were compared (Figure 1). A Linear Regression T-Test was performed on these two debts with alternative hypothesis that they were not equal, and an r^2 value of 0.9904 was obtained. This strongly indicated that the two data sets were equivalent, and that our projections of deficit are very accurate.

The model is resilient against changes in the present state of the United States. Data was observed beginning in 2000 primarily because the Afghanistan and Iraq wars which fought since then have changed the workings of the US economy, especially by creating substantial deficit spending. However, a conversion to peacetime economy can be incorporated into the model by setting the percent of projected deficit spending

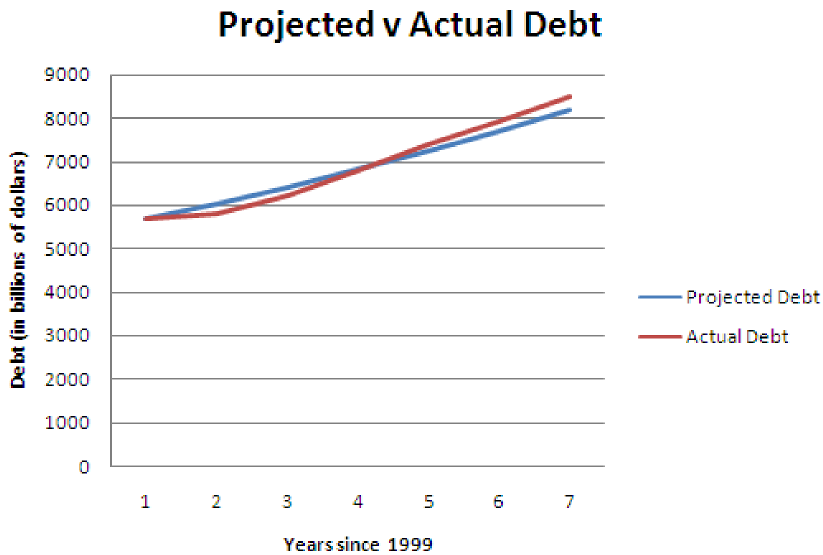

Figure 1: Debt Values.

to zero. Similarly, α values can be decreased when more domestic spending is necessary. The only clear weakness in this model lies in the slight inaccuracy of projected GDP and deficit spending values which inevitably occurred when modeling these with regressions.

The model is sensitive to modifications to the constants α, μ, and p. Leaving α as its base of .07 allows the US debt to rise indefinitely, whereas, as discussed in solutions below, changing it to .01829 will cause the debt to be entirely removed by 2017. Similarly, reducing μ to .075 changes the debt from increasing to decreasing by 2017. Changes in p, though politically unlikely, have similarly large effects. By adjusting these constants, ideal solutions to the rising national debt can be found.

6 Proposed Solutions

In order to use this model to arrive at solutions to the United State's national debt problem, the constants representing the percentage of government income devoted towards paying off the debt and the percent of current deficit spending rates were manipulated. The percentage of government income from the GDP was not manipulated, as this ratio has remained essentially constant since 1790. The manipulation of these constants would require significant changes in national economic policy, which need to be carefully considered when evaluating the solutions produced from the model. In order to achieve a change in α, the government would need to divert funds from other investments, such as infrastructure and social security, to increase

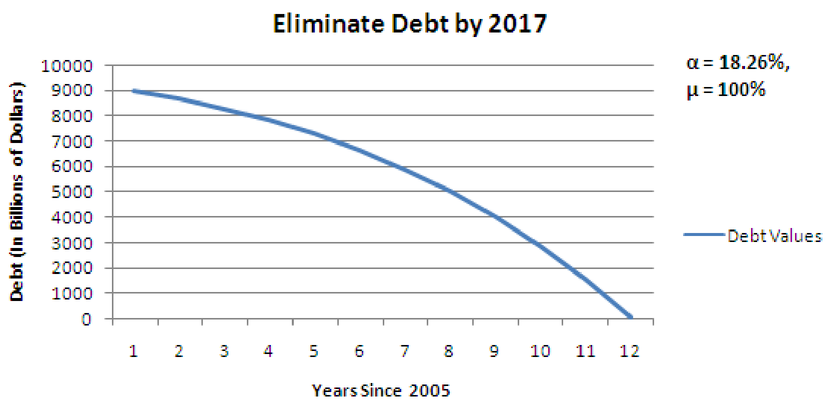

Figure 2: Solution 1.

the amount paid against the debt. For most of the 21st century, the government has been employing a large amount of deficit spending to accommodate the expenses associated with the War on Terror. It follows that manipulating the μ constant would most likely require a reduction in defense spending and an appropriate cutback in many War on Terror and homeland security issues.

Several plans have been created based on the model in order to eliminate the national debt by 2017 or at least arrest its growth.

To get the debt to zero by 2017 there are two options. The first option is to increase the percent of the projected gross domestic product used to pay the national debt to 18.29%. As seen in Figure 2, making this adjustment would cause the national debt to reach zero by 2017. Unfortunately, the solution requires that the amount being paid against the national debt be increased by a factor of roughly 2.6. In order to accomplish such a dramatic increase in spending in this area, the government would need to cut spending significantly in other areas, though it would not be required to adjust its level of deficit spending.

The second option is to decrease the amount of predicted deficit spending by 50% and increase the amount of projected gross domestic produced used to pay the national debt to 15.46%. As seen in Figure 3, making this adjustment would cause the national debt to reach zero by 2017. While this solution requires a smaller increase in the amount to be paid against the national debt each year, the amount required is still burdeningly significant. Furthermore, this increase is more difficult to pay for because the amount of allowed deficit spending is also decreased.

Overall, it may not be realistic to enact a policy that would completely eliminate the national debt by 2017. However, it may be feasible to halt the increase in government debt by this date and begin decreasing it.

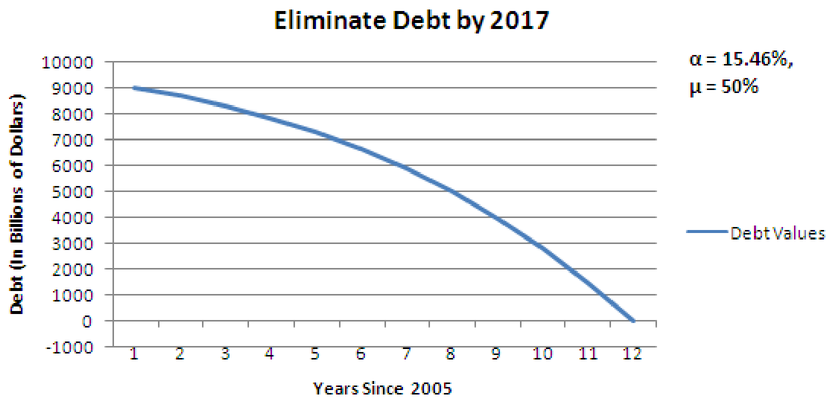

Figure 3: Solution 2.

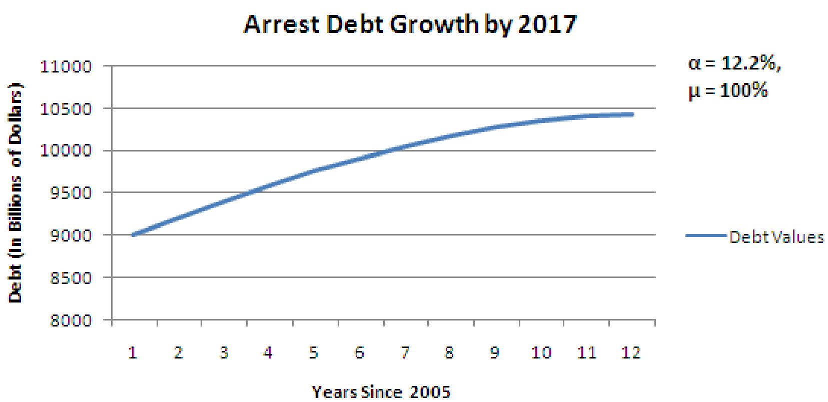

Figure 4: Solution 3.

The first method by which this may be achieved is to leave the deficit spending where it is now, but increase the percent of gross domestic product paying the national debt to 12.2%.As seen in Figure 4, making this adjustment would terminate the increase in the national debt by 2017. While this requires a significantly less drastic increase in the percent of the GDP being paid against the national debt than the previous two solutions, the increase is still significant enough to adversely affect many other elements of the government budget.

The second way is to leave the payment of domestic product where it is now, but decrease the projected deficit spending to 7.5% of its current rate. As seen in Figure 5, making this adjustment would cause the national debt to level out by 2017. Naturally, it would not be feasible under the current political setting to almost completely eliminate deficit spending, but this solution still proposes a useful starting point for a third method of arresting the growth of the national debt.

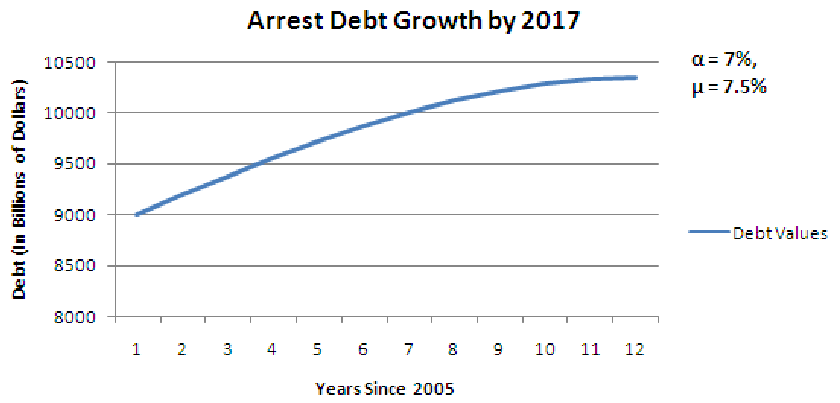

Figure 5: Solution 4.

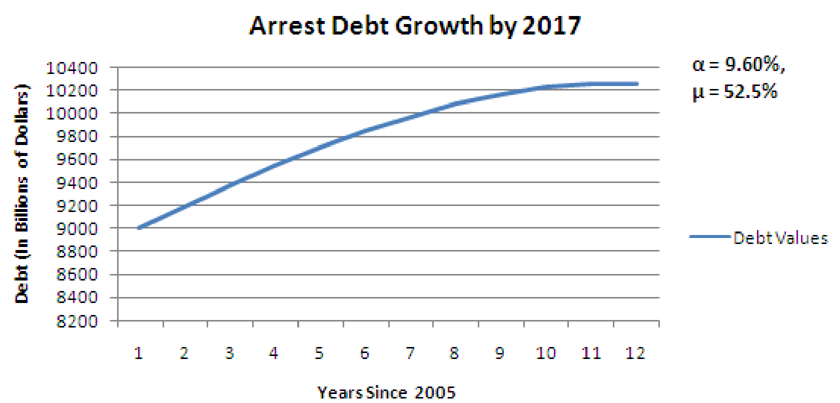

Figure 6: Solution 5.

The final solution is to increase α to .097 and decrease the μ to .525 of their current values. This hybrid solution calls for only a modest increase in payments against the national debt and a decrease in deficit spending that could feasibly be accomplished by a decrease in defense spending. Figure 6 shows how making these adjustments would arrest the growth of the national debt by 2017.

7 Letter to the President

Mr. President,

In order to handle rapidly increasing national debt, the government needs to produce several budget surpluses to repay the national debt. By using differential equations to model the United States' economy, the projected national debt for future years can be determined. By casting a function of the adjusted gross domestic product (essentially the income on the federal taxes) per capita, inflation and population do not have to be variables in the equation because it is implicitly predicted using the adjusted gross domestic product per capita. With these equations, optimal plans for decreasing the amount of national debt can be formulated. Our team formed such an equation. Factors incorporated were interest on the national debt, the inflation rate of the United States economy, the rate at which the debt is being paid off by the government per year, and the amount of deficit spending by the government per year.

Once the equation was formed, it was determined that in order to use this model to arrive at solutions to the United States' national debt problem, the constants representing the percentage of government income devoted towards paying off the debt and the percent of current deficit spending rates were manipulated. The percentage of government income from the GDP was not manipulated, as this ratio has remained essentially constant since 1790. The manipulation of these constants would require significant changes in national economic policy, which need to be carefully considered when evaluating the solutions produced from the model. In order to achieve a change in the percent of government revenue used yearly to pay off the national debt, the government would need to divert funds from other investments, such as infrastructure and social security, to increase the amount paid against the debt. For most of the 21st century, the government has been employing a large amount of deficit spending to accommodate the expenses associated with the War on Terror. It follows that manipulating the percent of projected deficit spending allowed to be spent would most likely require a reduction in defense spending and an appropriate cutback in many War on Terror and homeland security issues.

Our model predicted an optimal solution to counter increasing national debt. This solution is to increase the aforementioned percent of government revenue used yearly to pay off the national debt to 9.60% and decrease percent of the projected deficit spending actually allowed to be spent to 52.5%. This hybrid solution calls for only a modest increase in payments against the national debt and a decrease in deficit spending that could feasibly be accomplished by a decrease in defense spending. Making these adjustments would arrest the growth of the national debt by 2017. We strongly advise you to read our full report on this model and solution.

8 References

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