Debt or No Debt?

Team number 1901

Contents

1 The Problem

Given the problem of national debt, we define **national debt** as the "public debt securities issued by the U.S. Treasury, consisting of marketable Treasury securities (bills, notes, bonds), savings bonds, and special securities issued by state and local governments." [6] The first step in modeling national debt lies in finding a general formula for calculating national debt. In the most general sense,

$$\text{National debt} = \text{Spending - Income}$$

where **spending** and **income** are furher divided.

The general idea for modeling the national debt is to determine a model for various aspects of spending and income as a function of year (based on previous data) and sum the predicted values to determine a function to forecast national debt as a function of year.

2 Assumptions

To assume the success of our model, we do not assume causation between year and any variable we model using nonlinear regressions. The regressions we calculate based on trends from year and various factors only serve to predict the factor from the future year. We do assume that any cyclic trends (usually modeled by sine functions) occuring within each set of data has a negligible effect on the predicted value. So unless the coeffients of the predicted regression line were greater than .001 (a tolerance level of .001), the sine term was dropped.

We also assume that national debt is only affected by the factors that we discuss. Any other influences of national debt are part of the natural variability of our proposed model.

3 Terminology

An **inelastic good** is a good or service that has a demand that is not affected dramatically by price. An **elastic good** is a good that has a demand that is affected dramatically by a change in price. A **normal good** is a good whose consumption increases as income increases while the price remains constant. An **inferior good** is a good whose consumption decreases as income increases while the price remains constant.

4 Model Development

To narrow down factors affecting national debt, we have two major categories: **spending** and **income**.

The factors that are grouped under **spending** are: pensions, healthcare, education, defense spending, welfare spending, research subsidies, and interest payments. Factors grouped under **income** are: tax revenue (including sales tax, social insurance tax, and income tax) and bond sales[1].

For each factor (whether under **income** or **spending**), we found published online data from the U.S. Government for the years 1970 to 2008, or the most available data up to date. Using each set of data, we determined a regression line that modeled the data roughly, either with a quintic or a polynomial plus a sine term.

4.1 Income: Income Tax

One aspect of national debt is the amount of income tax the government collects. Using data from the United States Government Revenue, we constructed a quartic regression that modeled the data from 1970 to 2008. The equation

$$\hat{y} = 1.77791 \times 10^{12} - 3.55901 \times 10^{9}x + 2.67165 \times 10^{6}x^2 - 891.346x^3 + 0.111517x^4$$

models the data for revenue from income tax based on year, where $\hat{y}$ is income tax revenue in billions of dollars and x is the year.

The graph depicting the data and the regression of best fit is shown below (Figure 1), where the x-axis is year and the y-axis is income tax revenue in billions of dollars:

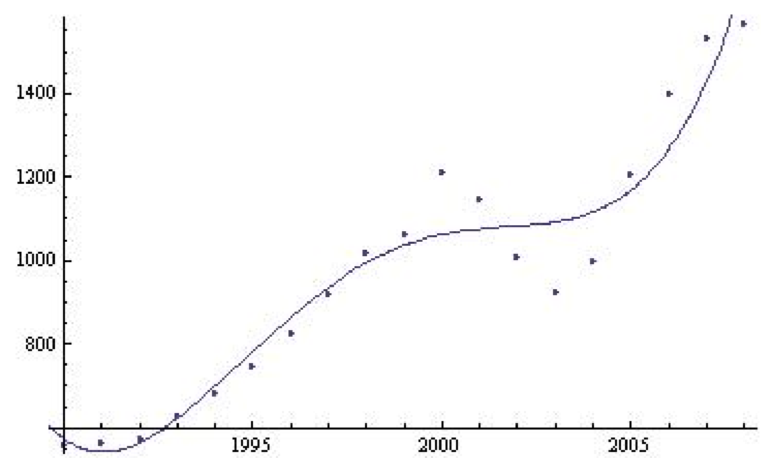

Figure 1: Income tax versus year

[1]In some years, bond sales may be negative because the government bought more bonds than it sold, so in some cases the predicted bond sales are negative even though they are listed as part of **income**.

4.2 Income: Social Insurance Tax

Another factor influencing government income is social insurance tax. From the same data source, we calculated a quartic regression for the social insurance tax revenue, modeled by this equation:

$$\hat{y} = 5.62999 \times 10^9 - 1.13198 \times 10^7 x + 8535.07x^2 - 2.86025x^3 + 0.000359455x^4$$

where $\hat{y}$ is the social insurance tax revenue in billions of dollars and x is the year. The graph of the data (Figure 2) and the regression line is depicted below, where the x-axis is year and the y-axis is social insurance tax in billions of dollars:

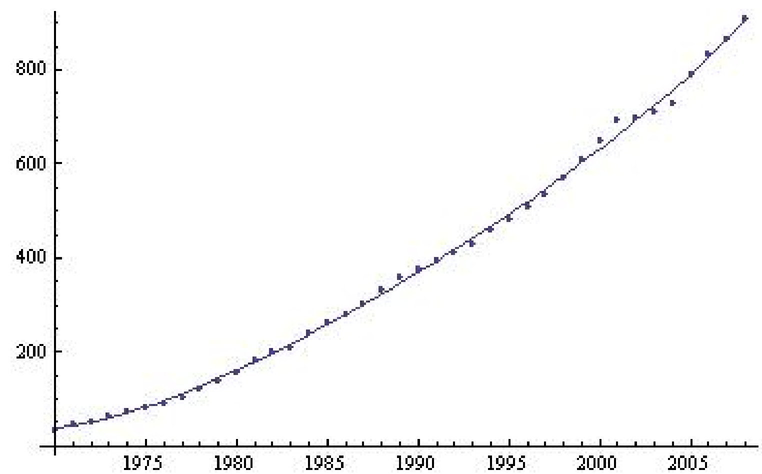

Figure 2: Social insurance tax versus year

4.3 Income: Sales Tax

Sales tax also is a component of government income. The model for the data taken from [1] predicts the sales tax revenue in billions of dollars ($\hat{y}$) from the year (x) as modeled by the equation

$$\hat{y} = 1.27974 \times 10^9 - 2.56699 \times 10^6 x + 1930.86x^2 - 0.645489x^3 + 0.0000809191x^4 + 3.82922 \sin\left(\frac{x}{\pi}\right)$$

and the graph (Figure 3) of the regression line and data is shown below where the x-axis is year and the y-axis is predicted sales tax revenue in billions of dollars:

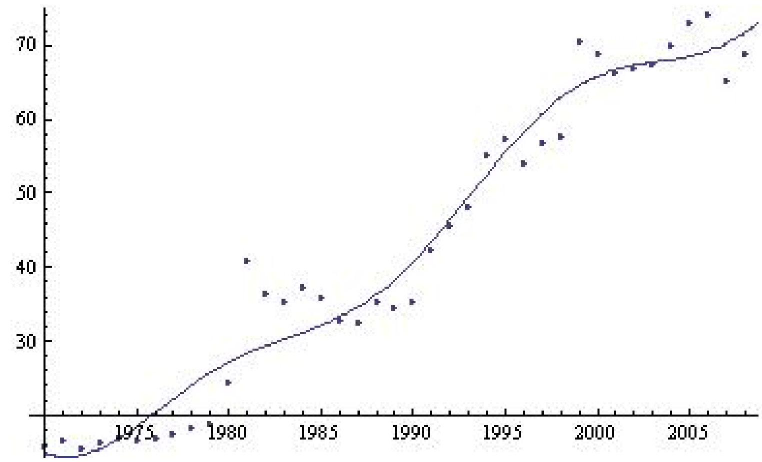

Figure 3: Sales tax versus year

4.4 Income: Bonds

The final source of income for the government is through the selling of bonds, both to corporations and to individuals. Assuming that there is no significant difference between the price of bonds sold to corporations or individuals, the quintic regression for predicting bond revenue in billions of dollars ($\hat{y}$) from year (x) is given by:

$$\hat{y} = 2.04442 \times 10^{11} - 3.08163 \times 10^{8} x + 154827.x^2 - 25.9279x^3$$

Because some years may yield more bond buys than sells, this model may predict a negative bond income for a given year. The graph (Figure 4) showing the data and the quintic curve is given below, where the x-axis is year and the y-axis is predicted bond revenue in billions of dollars:

4.5 Spending: Pensions

The first component of government spending is pensions, which the government pays to elderly citizens. From the US Government Revenue [1] the quintic curve predicting pension expenses in billions of dollars ($\hat{y}$) from year (x) is:

$$\hat{y} = -3.45058 \times 10^{11} + 8.70202 \times 10^{8} x - 877816.x^2 + 442.742x^3 - 0.111651x^4 + 0.0000112623x^5$$

The graph (Figure 5) showing the data and the regression line is given below, where the x-axis is year and the y-axis is predicted pension expenses in billions of dollars for that given year.

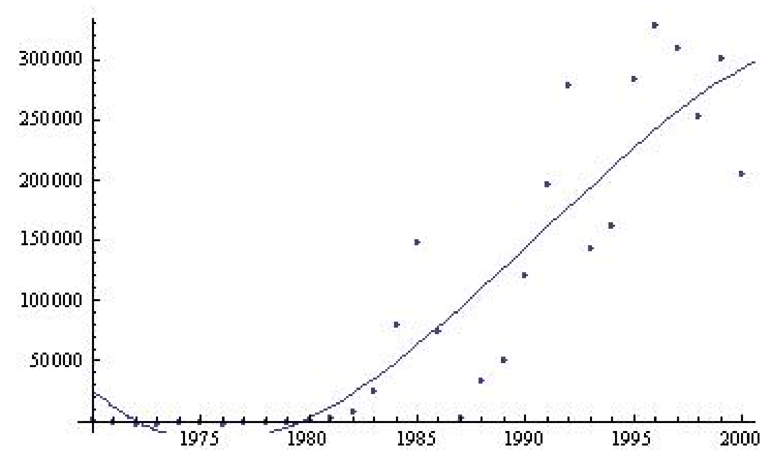

Figure 4: Bond sales versus year

4.6 Spending: Healthcare

Another component of government spending is healthcare, in the form of Medicaid, Medicare, National Institute of Health employee salaries, National Institute of Health research subsidies, Center for Disease control subsidies and salaries, etc. Using the data in [1] we constructed a quintic curve predicting healthcare expenditure in billions of dollars ($\hat{y}$) from year (x). The equation is given below:

$$\hat{y} = -1.13558\times10^{12}+2.85746\times10^{9}x-2.87608\times10^{6}x^{2}+1447.4x^{3}-0.364205x^{4}+0.0000366572x^{5}$$

The graph (Figure 6) for the data and the regression line is given below, where the x-axis is given in years and the y-axis is healthcare expenditure in billions of dollars.

4.7 Spending: Education

Another component of government spending is money directed towards education in all forms: pre-primary, primary, secondary, and university. As new reforms and ideas come into play with differing leaders in Congress and the presidency, educational reforms impact the amount of government spending directed towards education. We can model this trend using data from [1] to find a logarithmic cubic curve predicting government costs to education in billions of dollars ($\hat{y}$) from year (x). The equation is given below[2]:

$$\ln(\hat{y}) = -1.33846 \times 10^{6} + 2017.01x - 1.01319x^{2} + 0.000169651x^{3}$$

[2] We used a logarithmic model to predict eduation expenditure because the curves we received from a logarithmic model was more accurate than a polynomial model

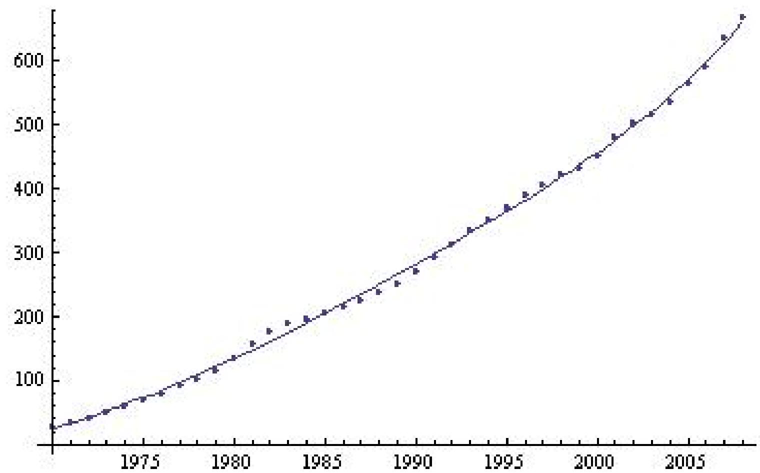

Figure 5: Pension expenditure versus year

$$\hat{y} = e^{-1.33846\times 10^{6}+2017.01x-1.01319x^{2}+0.000169651x^{3}}$$

and the graph (Figure 7) of the data and the curve is given, with the x-axis given in year and the y-axis given in education expenditure in billions of dollars.

4.8 Spending: Defense

A part of government spending is always allocated to defense, whether that be in the form of civil defense, military defense, veteran pensions and aid, foreign military aid, or foreign economic acid. Some of these costs are reflectant upon disasters happening around the world, but the plotted data follows a distinct quintic trend for the last 38 years. The equation

$$\hat{y} = 2.37973\times 10^{12} - 5.95595\times 10^{9}x + 5.96233\times 10^{6}x^{2}$$
$$-2984.24x^{3} + 0.746799x^{4} - 0.0000747507x^{5}$$

predicts the expected defense expenditure ($\hat{y}$) from year (x) based on the graph (Figure 8) of the data, where the x-axis is given in year and the y-axis is given in defense cost in billions of dollars:

4.9 Spending: Welfare

Government welfare spending varies depending on the current administration in power and the mixture of policymakers in Congress. But the aspects of government aid listed under welfare include for example housing aid, social security benefits, childcare, unemployment benefits, foodstamps, and WIC, among others. The graph of this data has a more pronounced cyclic trend, so the inclusion

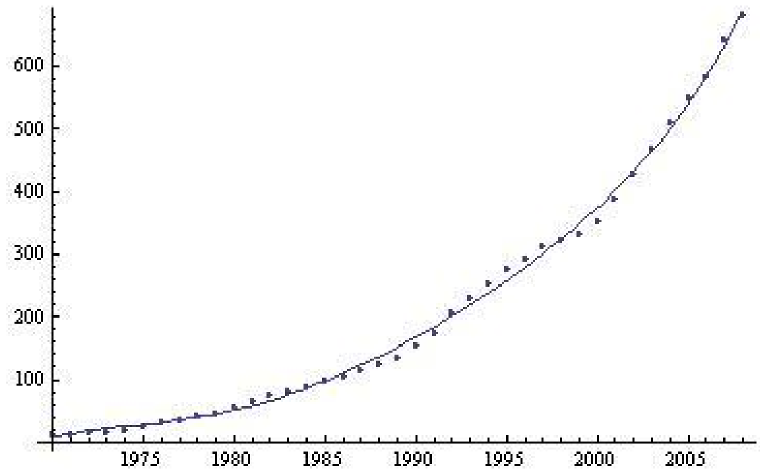

Figure 6: Healthcare spending versus year

of a sine term in the function was necessary to account for this. The quartic and the sine are given here:

$$\hat{y} = -6.41924\times10^9 + 1.29055\times10^7 x - 9729.5x^2 + 3.25995x^3 - 0.000409593x^4 - 8.20864\sin\left(\frac{x}{2}\right)$$

predicting government expenditure on welfare in billions of dollars ($\hat{y}$) from year (x). The graph (Figure 9) of the data and the function are given below, with the x-axis given in year and the y-axis given in welfare spending in billions of dollars.

4.10 Spending: Research

Government subsidized research spending stems primarily from subsidies given to NASA. This spending is extremely variable, and it depends on current world events and the current administration's policies towards these events. Because of the high variability of this data, we included a cosine term to account for some of the variability. The quartic and the cosine function predicting government spending on research ($\hat{y}$) from year (x) is given by:

$$\hat{y} = 7.58778\times10^9 - 1.52778\times10^7 x + 11535.3x^2 - 3.87088x^3 + 0.000487094x^4 - 12.7976\cos\left(\frac{x}{2}\right)$$

and the graph (Figure 10) of the data and the function is given below, with the x-axis given in year and the y-axis given in research spending in billions of dollars.

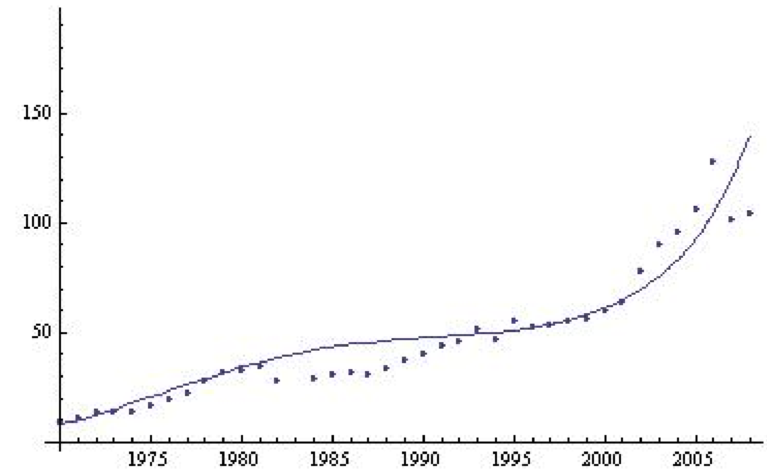

Figure 7: Education spending versus year

4.11 Spending: Interest

An often forgotten component of government spending is the paying off of interest on bank loans. This depends on interest rates set by the federal government, which in turn influence inflation and money circulation. The data can be modeled by the quintic curve

$$\hat{y} = -3.80651 \times 10^{12} + 9.58018 \times 10^{9}x - 9.64437 \times 10^{6}x^2 + 4854.43x^3$$
$$-1.2217x^4 + 0.000122984x^5$$

where $\hat{y}$ is predicted government interest payment adn x is year. The graph (Figure 11) of the data and the curve is given below, with the x-axis given in year and the y-axis given in interest payment in billions of dollars.

5 The Model Itself

Once all contributors of national debt have been analyzed and regression curves calculated for each one based on year, we return to the general formula for national debt as stated before:

$$\text{National debt} = \text{Spending - Income}$$

Using this formula we can combine all regression curves to get one formula predicting total national debt ($\hat{y}$) from year (x) as follows:

$$\hat{y} = [(\text{Income Tax}) + (\text{Social Insurance Tax}) + (\text{Sales Tax}) + (\text{Bond revenue})] -$$

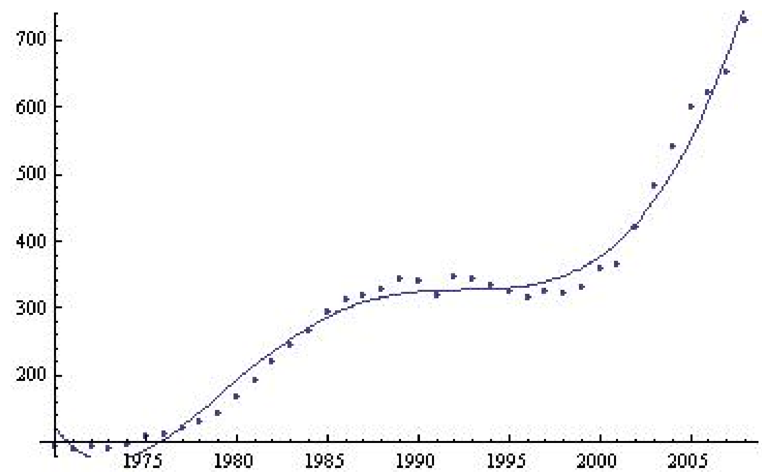

Figure 8: Defense spending versus year

$$[(\text{Pensions}) + (\text{Healthcare}) + (\text{Education}) + (\text{Defense}) +$$
$$(\text{Welfare}) + (\text{Research}) + (\text{Interest})] =$$
$$[(1.77791\times10^{12} - 3.55901\times10^{9}x + 2.67165\times10^{6}x^2 - 891.346x^3 + 0.111517x^4)+$$
$$(5.62999\times10^{9} - 1.13198\times10^{7}x + 8535.07x^2 - 2.86025x^3 + 0.000359455x^4)+$$
$$(1.27974\times10^{9} - 2.56699\times10^{6}x + 1930.86x^2 - 0.645489x^3 + 0.0000809191x^4 + 3.82922\sin(\frac{x}{\pi}))+$$
$$(2.04442\times10^{11} - 3.08163\times10^{8}x + 154827.x^2 - 25.9279x^3)]-$$
$$[(-3.45058\times10^{11} + 8.70202\times10^{8}x - 877816.x^2 + 442.742x^3 - 0.111651x^4 + 0.0000112623x^5)+$$
$$(-1.13558\times10^{12} + 2.85746\times10^{9}x - 2.87608\times10^{6}x^2 + 1447.4x^3 -$$
$$0.364205x^4 + 0.0000366572x^5) + (e^{-1.33846\times10^{6} + 2017.01x - 1.01319x^2 + 0.000169651x^3})+$$
$$(2.37973\times10^{12} - 5.95595\times10^{9}x + 5.96233\times10^{6}x^2 - 2984.24x^3 + 0.746799x^4 - 0.0000747507x^5)+$$
$$(-6.41924\times10^{9} + 1.29055\times10^{7}x - 9729.5x^2 + 3.25995x^3 - 0.000409593x^4 - 8.20864\sin(\frac{x}{2}))+$$
$$(7.58778\times10^{9} - 1.52778\times10^{7}x + 11535.3x^2 - 3.87088x^3 + 0.000487094x^4 - 12.7976\cos(\frac{x}{2}))+$$
$$(-3.80651\times10^{12} + 9.58018\times10^{9}x - 9.64437\times10^{6}x^2 + 4854.43x^3 - 1.2217x^4 + 0.000122984x^5)]$$

and combining everything into one expression,

$$\hat{y} = 4.8955\times10^{12} - e^{-1.33846\times10^{6} + 2017.01x - 1.01319x^2 + 0.000169651x^3} -$$
$$1.12306\times10^{10}x + 1.02711\times10^{7}x^2 - 4680.5x^3 + 1.06264x^4 - 0.0000961523x^5 +$$

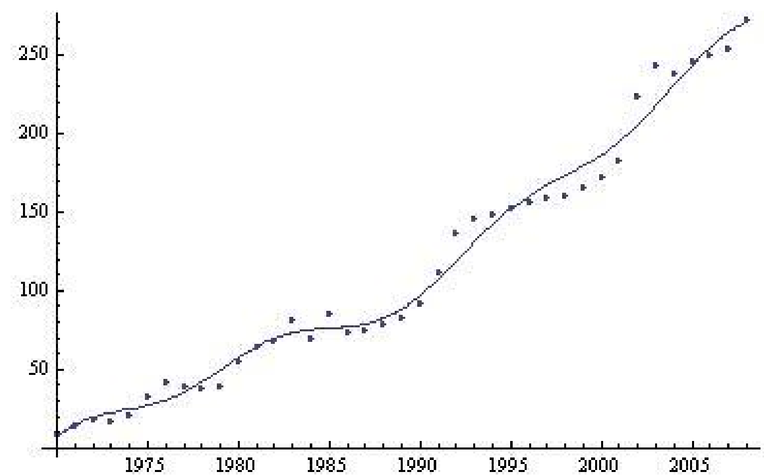

Figure 9: Welfare spending versus year

$$12.7976\cos(\frac{x}{2}) + 8.20864\sin(\frac{x}{2}) + 3.82922\sin(\frac{x}{\pi})$$

where x is year and $\hat{y}$ is predicted national debt.

The final step in our calculation of national debt is to include GDP and population in our calculation of national debt. There is no feasible way to incorporate both GDP and population into the calculation, we determined methods of calculating two different statistics, one for the ratio of national debt to GDP and one for national debt per capita.

To calculate the ratio of national debt to GDP, we need to predict the GDP based on the GDP from 1970 onward. Using GDP values in 2000-dollars [5] converted to 2008-dollars using the ratio of GDP from 2000 to 2008, we calculated the regression line estimating GDP (in 2008 dollars) from year. Because we don't know what the inflation rate will be in a later year, we constructed our GDP graph using the GDP for a year in that year's currency. Although this is economically incorrect, for a mathematical model it is more useful because it incorporates two aspects of the economy, inflation (interest rates and money circulation) and GDP (economic strength), into one equation that models both factors. Therefore, the equation for predicting GDP for a given year (in that year's dollars) is given by:

$$\hat{y} = -6.26059 \times 10^8 + 955979.x - 486.672x^2 + 0.0825999x^3$$

where $\hat{y}$ is predicted GDP in that year's dollars and x is year. The graph of GDP versus year is given (Figure 12) where the x-axis is given in year and the y-axis is given in billions of that year's dollars.

To get the statistic for the ratio of national debt to GDP, combine the two equations into one statistic:

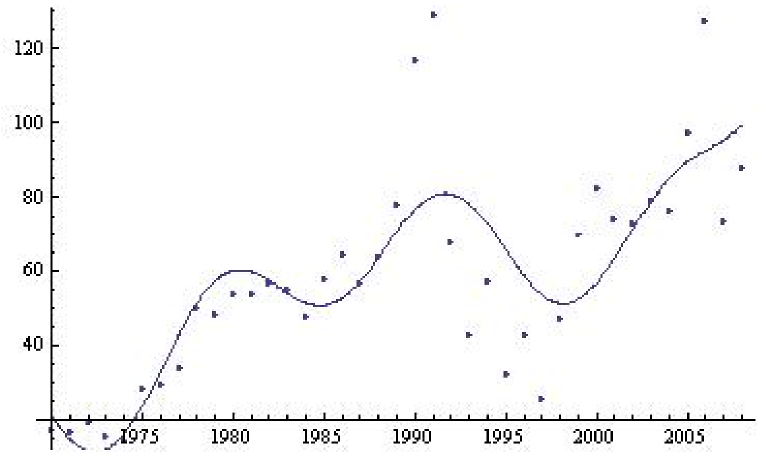

Figure 10: Research spending versus year

$$\text{National debt ratio to GDP} = \frac{\text{National debt}}{\text{GDP in that year's dollars}} =$$

$$\frac{1}{-6.26059 \times 10^8 + 955979.x - 486.672x^2 + 0.0825999x^3} \times (4.8955 \times 10^{12} -$$

$$e^{-1.33846\times10^6+2017.01x-1.01319x^2+0.000169651x^3} -$$

$$1.12306 \times 10^{10}x + 1.02711 \times 10^7x^2 - 4680.5x^3 + 1.06264x^4 - 0.0000961523x^5 +$$

$$12.7976\cos(\frac{x}{2}) + 8.20864\sin(\frac{x}{2}) + 3.82922\sin(\frac{x}{\pi}))$$

where you plug in the year for x and the answer becomes the national debt as a percentage of GDP.

Another statistic useful in analyzing national debt is national debt per capita, because if the national debt for a certain country is high and the population is low, then the country is more in debt than a country with the same amount of national debt with a high population. So it is useful to calculate a statistic named national debt per capita. However, we need to estimate the population for a given year based on the population of America in the years 1970 to 2008 based on U.S. Census data [4].

The equation for estimating population from year is given by:

$$\hat{y} = -4.56159 \times 10^{12} + 6.92705 \times 10^9x - 3.50728 \times 10^6x^2 + 592.112x^3$$

where $\hat{y}$ is predicted population and x is year. The graph of the population based on year is given (Figure 13), where x-axis is given in year and y-axis is given in population (number of people).

Then the statistic for calculating national debt per capita is

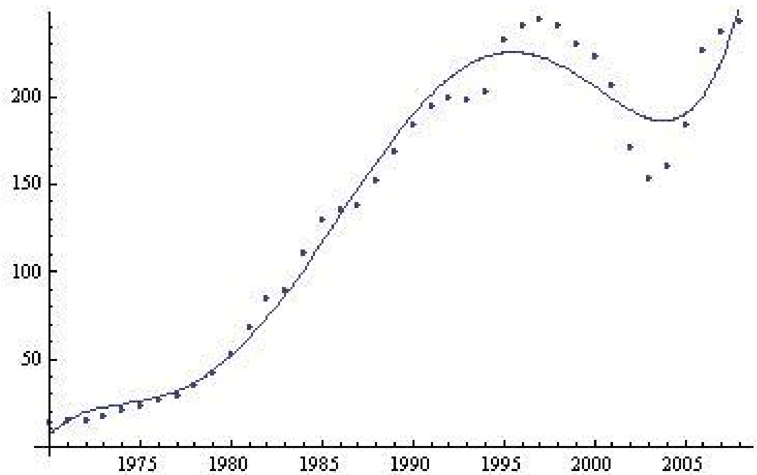

Figure 11: Interest spending versus year

$$\text{National debt per capita} = \frac{\text{National debt}}{\text{Population}} =$$

$$\frac{1}{-4.56159 \times 10^{12} + 6.92705 \times 10^{9}x - 3.50728 \times 10^{6}x^{2} + 592.112x^{3}} \times (4.8955 \times 10^{12} -$$

$$e^{-1.33846\times 10^{6}+2017.01x-1.01319x^{2}+0.000169651x^{3}} - 1.12306 \times 10^{10}x +$$

$$1.02711 \times 10^{7}x^{2} - 4680.5x^{3} + 1.06264x^{4} - 0.0000961523x^{5} +$$

$$12.7976\cos(\frac{x}{2}) + 8.20864\sin(\frac{x}{2}) + 3.82922\sin(\frac{x}{\pi}))$$

where x is year.

Figure 14 gives the graph of predicting national debt from year.

6 Predictions for 2009-2017

Using our model, we predict the national debt and the two statistics we developed (ratio of national debt to GDP and national debt per capita) for each of the years 2009 through 2017.

7 Sensitivity Testing

The model is only sensitive in terms of the bond expenditure parameter. If changes are made to the income tax, social insurance tax, sales tax, pensions expenditure, healthcare expenditure, education expenditure, defense spending, welfare spending, research expenditure, and interest spending, then the model will not change dramatically in terms of predicting the national debt.

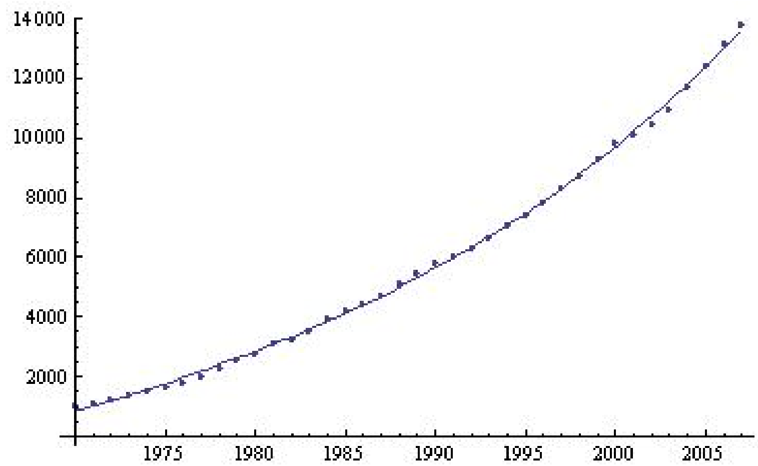

Figure 12: GDP in that year's dollars versus year

Table 1: Year, National Debt, Ratio, Per Capita

Year	National debt (billions $	Ratio of national debt to GDP	National debt per capita
2009	303954	20.4123	.000974037
2010	293128	18.8246	.000926617
2011	279378	17.1659	.000871002
2012	262569	15.4433	.000807168
2013	242568	13.6636	.000735116
2014	219240	11.833	.000654865
2015	192444	9.9569	.000566445
2016	162032	8.04016	.00046988
2017	127837	6.0864	.000365163

An interesting result about national debt as described by our model is that bond revenue (or expenditure, depending on the context) is the major force in describing or eliminating national debt. Through the buying or selling of these bonds, the government is able to maintain a relatively constant level of national debt with slow changes. Any possible improvements needed to eradicate national debt must be centered around the changes in bonds, because changes in the other factors affecting national debt do not have as significant an impact.

Figure 15 depicts the national debt equation predicted from every factor described above without bonds, and Figure 16 depicts the national debt equation including bonds. The drastic difference between the two shows that bonds are a major factor in influencing the model for national debt.

Our model is not sensitive to small changes in income tax, social insurance tax, sales tax, pensions expenditure, healthcare expenditure, education expenditure, defense spending, welfare spending, research expenditure, and interest

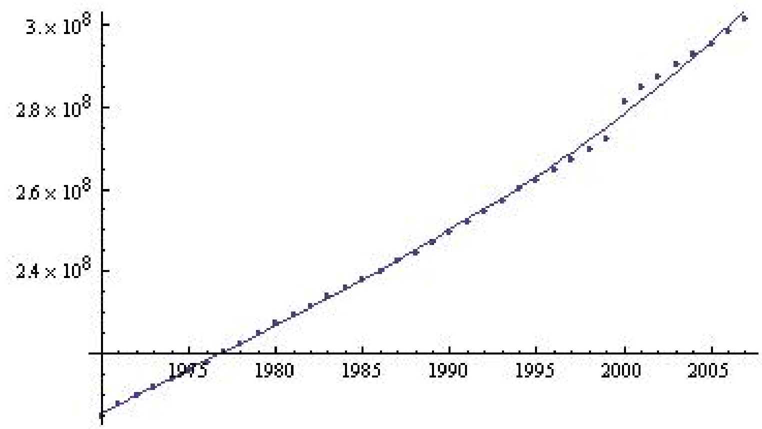

Figure 13: Population versus year

spending, but is extremely sensitive to changes in bond revenue.

8 Two Alternative Plans for 2009 - 2017

8.1 Two plans: Democratic and Republican

To test our model, we tested two different scenarios of government policies and changed our national debt formula based on these two alternatives. Based on different political affiliations, we developed two plans for possible future scenarios.

Alternative 1 is based on the Democratic platform:

- Cut income tax by 0.3%
- Increase sales tax by 0.2%
- Assume universal healthcare, which in our model means increase healthcare by 100%
- Pull out of the war in Iraq, so defense spending is decreased by 150 billion dollars[2] [3]

Alternative 2 is based on the Republican platform:

- Cut income tax by 2.0%
- Cut sales tax by 5.0%

[3] There is possible bias from the source of the projected cost of the Iraq war because it was taken from an anti-war website.

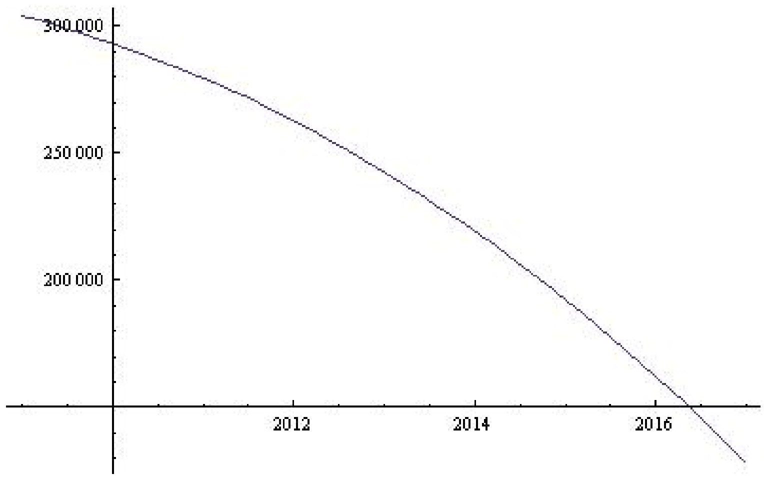

Figure 14: The model predicting national debt

- No change in defense spending

Comparing the graphs of the two alternatives, we see the differences in terms of income tax revenue (Figure 17), sales tax revenue (Figure 18), healthcare expenditure (Figure 19), and defense spending (Figure 20).

The red line depicts the predicted Republican line, the blue line depicts the predicted Democratic line, and the yellow line depicts the line predicted by our model without any changes. In Figure 17, the yellow line blends very closely with the blue and red lines, so it is very difficult to distinguish between the different policies.

As predicted, Figure 18 depicts the sales tax revenue increasing through the Democratic policy.

Figure 16 predicts a drastic increase in healthcare if a Democratic policy (Alternative 1) is implemented, resulting in a general increase in national debt if the policy of universal healthcare is put forth. The Republican policy does not change the current healthcare policy, so the red and yellow lines are exactly overlapping.

Because the Republican policy is not geared towards pulling out of Iraq, the Republican model for predicted defense spending is overlapping with our original model prediction for defense spending. Democratic policy promises pulling out of Iraq, so the Democratic model predicts a decrease in defense spending drastically over time.

It is useful to compare the three different equations for modeling national debt: Democratic, Republican, and the original model. The calculated equation for predicting national debt based on the Democratic model is

$$\hat{y} = \text{model} - \text{income tax revenue} \times .003 - \text{sales tax revenue} \times .02 -$$

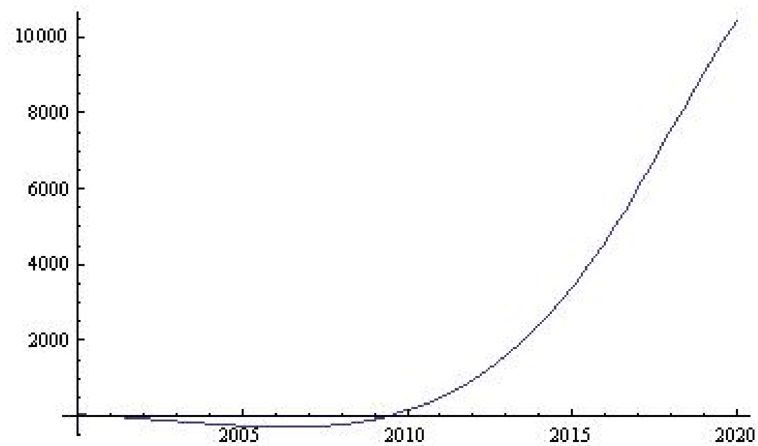

Figure 15: The model predicting national debt not including bonds

$$\text{healthcare cost in the national debt} + \text{healthcare} \times \frac{1}{.14} - 150 =$$
$$-2.0855\times10^{12} - 1.154316273796274\times10^{-581284} e^{x\times(2017.01-1.01319x+0.000169651x^2)} +$$
$$6.33303\times10^{9}x - 7.40426\times10^{6}x^2 + 4213.35x^3 - 1.17495x^4 + 0.000129028x^5 +$$
$$12.7976\cos(\frac{x}{2}) + 8.20864\sin(\frac{x}{2}) + 3.90581\sin(\frac{x}{\pi})$$

Adding universal healthcare into the Democratic budget includes adding every person into the government healthcare plan. Because currently only 14% of Americans are insured by Medicaid.[3] To change this to universal healthcare, 100% of Americans must be covered by government healthcare. The subtracted 150 in the formula represents the 150 billion dollars projected to be the cost of Iraq for another year, and because the Democrats promise to remove all troops from Iraq, those 150 billion dollars will be decreased from national debt. Figure 21 depicts the graph of the Democratic model of national debt.

Under the Republican policy (Alternative 2), the equation for calcuating national debt comes out to be:

$$\hat{y} = \text{model} - \text{income tax revenue} \times .02 - \text{sales tax revenue} \times .05 =$$
$$4.85988\times10^{12} - 1.154316274\times10^{-581284} e^{x\times(2017.01-1.01319x+0.000169651x^2)} -$$
$$1.11593\times10^{10}x + 1.02175\times10^{7}x^2 - 4662.64x^3 + 1.06041x^4 - 0.0000961523x^5 +$$
$$12.7976\cos(\frac{x}{2}) + 8.20864\sin(\frac{x}{2}) + 3.63776\sin(\frac{x}{\pi})$$

The equation differs from the equation of the model and the equation of Alternative 1 (Democratic model) only slightly. Figure 22 depicts the graph of the Republican model for calculating national debt.

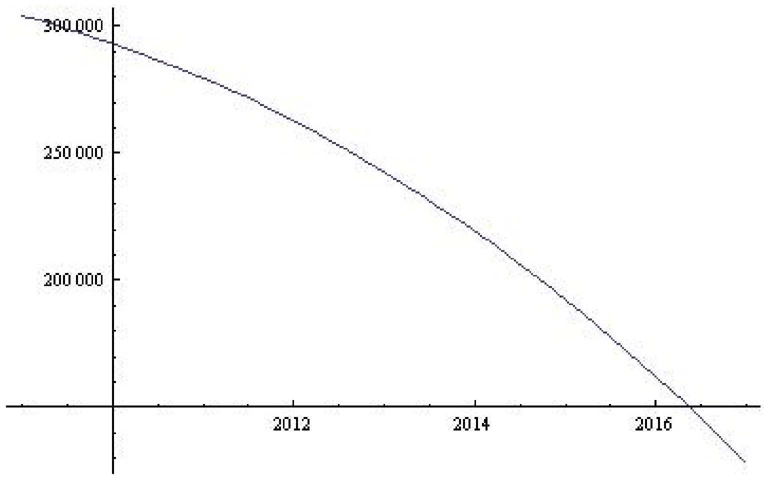

Figure 16: The model predicting national debt including bonds

Comparing all three graphs side by side, it is evident that any significant change in policy does not drastically change the model used to predict the national debt.

Using the two models (Democratic and Republican) it is easy to predict the national debt will not be significantly different. Table 2 gives values for predicted national debt by both the Democratic and Republican models.

Table 2: National debt from Democratic and Republican models

Year	National debt from Democratic model (billions of dollars)	National debt from Republican model (billions of dollars)
2009	308408	303910
2010	298006	293074
2011	284733	279310
2012	268461	262484
2013	249065	242461
2014	226416	219104
2015	200386	192274
2016	170835	161820
2017	137606	127576

From Table 2, we can see that the Republican policy (Alternative 2) makes the national debt decrease more dramatically than the Democratic policy (Alternative 1). However, the national debt is still very high, and the difference between the Republican and Democratic policies is relatively small. National debt, according to our model (and according to both the Democratic and Republican models) do not completely eradicate national debt by 2017.

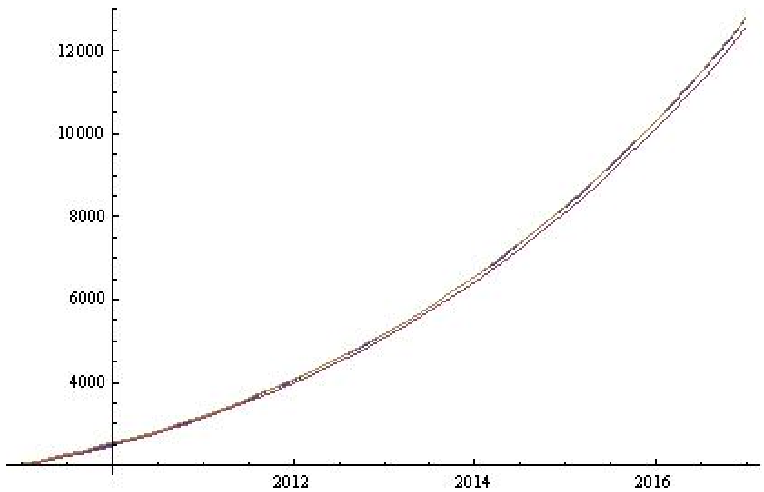

Figure 17: Income tax revenue for Democrats, Republicans, and Model

8.2 Impact of Policies

Impact of Alternative 1 (Democratic policy): All troops are removed from Iraq, so national moral increases because there are no American troops dying in a foreign country. The idea of universal healthcare is politically feasible, because it was proposed by the Democrats for some time. Universal healthcare will impact all Americans because they all will have access to healthcare. This is economically feasible because as the model predicts, universal healthcare does not create a huge deficit, contrary to common belief. The impact of increasing sales tax impacts consumers in two ways: (1) for inelastic goods, the demand is not greatly affected by the price of the product, so the sales tax burden will be mainly paid for by the consumers since the demand does not change greatly, (2) for elastic goods, the demand for goods is affected by the price of the product, so the sales tax burden will be paid for mainly by the producers. Because income tax is decreased, the consumers will have more money to spend, so normal goods (like for example cars, houses, computers) are purchased more often and inferior goods (for example used cars, used clothes) are not purchased as often. In this scenario, the demand for all normal products will increase because the price of the normal goods will make up a smaller proportion of people's income.

Impact of Alternative 2 (Republican policy): Because the sales tax is decreased, consumers pay less on inelastic goods because the demand is not affected by the price of the product. For elastic goods, producers save money because the sales tax is decreased and the burden of payment of the good is placed on the consumer, since demand is influenced by the price of the product. Because income tax is decreased, the consumers will have more money to spend, so normal goods (like for example cars, houses, computers) are purchased more often and inferior goods (for example used cars, used clothes) are not purchased

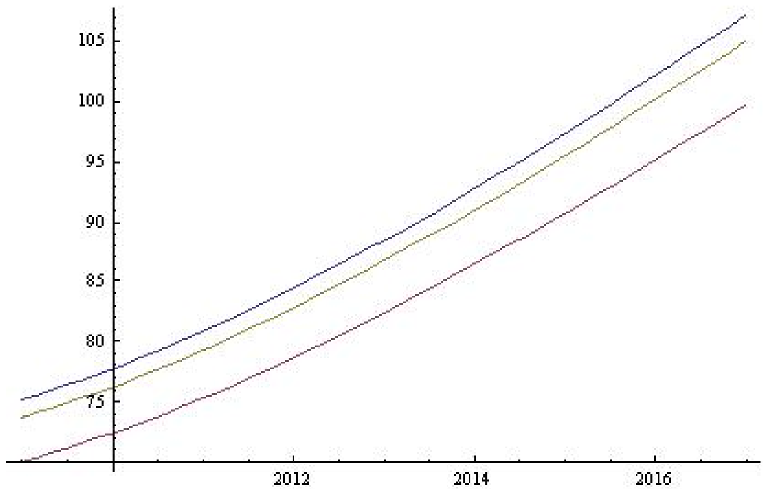

Figure 18: Sales tax revenue for Democrats, Republicans, and Model

as often. In this scenario, the demand for all normal products will increase because the price of the normal goods will make up a smaller proportion of people's income.

9 Strengths and Weaknesses

Our model is an indicator for predicting national debt from year. We also developed two statistics for better understanding the nature of national debt (as opposed to one raw number for how much money the government is in debt by. One major flaw in the model is that any unforseen circumstances are not accounted for in terms of national debt. For example, government costs for natural disaster relief is not accounted for in the formula. When Hurricane Katrina hit in 2005, the government spent more money on disaster relief than they originally allocated. Defense disasters like 9/11 also increased the allocated defense spending. So the sensitivity of our model is not perfect because it does not account completely for all the spending of the government.

Another weakness of our model is that it does not incorporate the idea of paying back bonds that are bought from the government. If more bonds are bought in the present day, then in approximately 20 years the government will have to pay back the bonds to the consumers. Our model does not incorporate this cyclic pattern in bonds, so the effects of selling too many bonds to consumers in the present day is not noticed by our predicting function.

A strength of the model developed is that it is dependent (or completely based on) data from 1970 to 2008, so it takes into account changes in the recent past as an indicator of future changes. This coincides with the prediction that changes in major policies except for bonds have no dramatic effect on the

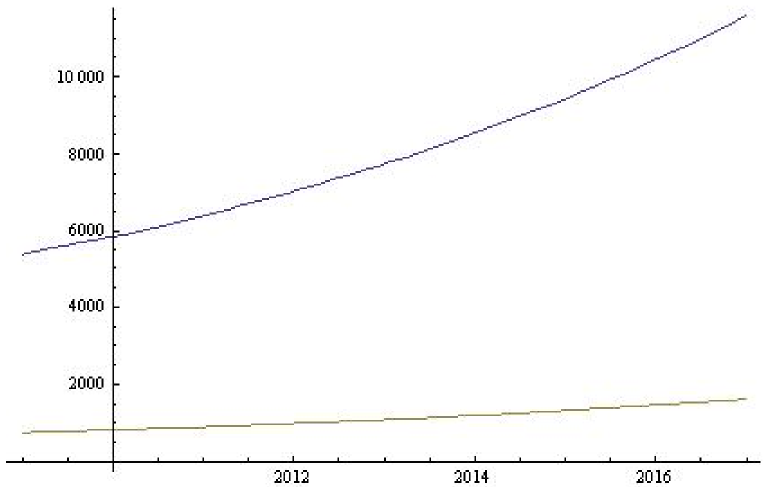

Figure 19: Health care expenditure for Democrats, Republicans, and Model

eradication or exacerbation of national debt.

10 Conclusion

The model we developed to predict national debt incorporated predictions of income tax, social insurance tax, sales tax, bond expenditure, pensions expenditure, healthcare expenditure, education expenditure, defense spending, welfare spending, research expenditure, and interest spending into the prediction for national debt. Using this model, the main component of predicting national debt is the influence of bonds, either in the form of expenditures or revenue. Therefore, if a government wants to eliminate its national debt, then its main focus should be on maximizing bond revenue.

Our model is sensitive to changes in bond revenue, but not sensitive to any other changes in income tax, social insurance tax, sales tax, pensions expenditure, healthcare expenditure, education expenditure, defense spending, welfare spending, research expenditure, and interest spending.

Comparing two alternative policies for the next eight years (modeled after Democratic and Republican policies), we predict that the national debt will remain fairly constant regardless of which policies are implemented.

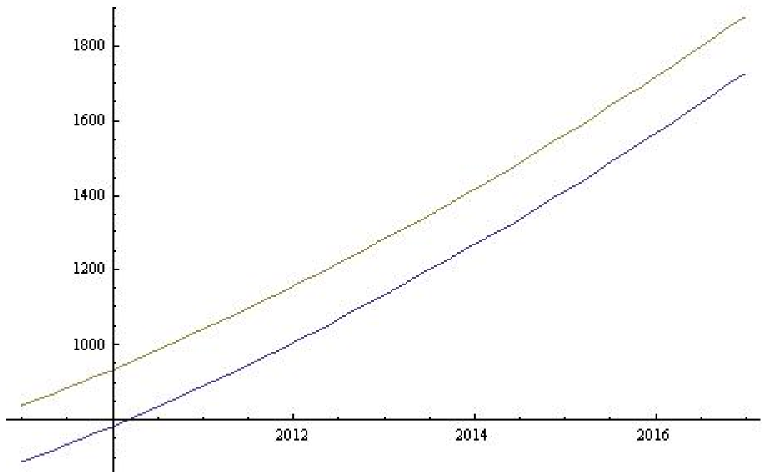

Figure 20: Defense spending for Democrats, Republicans, and Model

11 Letter to the President

Dear Mr. President,

An essential function of a new administration is to study national debt and predict future national debt using a mathematical model. We have developed a mathematical model that incorporates elements of income tax, social insurance tax, sales tax, bonds revenue, pensions expenditure, healthcare expenditure, education expenditure, defense spending, welfare spending, research expenditure, and interest spending in an attempt to synthesize the major components of national debt.

The general formula we used to develop a model is the fact that

$$\text{National debt} = \text{Spending} - \text{Income}$$

where components of spending is broken down into income tax, social insurance tax, sales tax, and bond revenue and components of income are broken down into pensions expenditure, healthcare expenditure, education expenditure, defense spending, welfare spending, research expenditure, and interest spending. Using methods of nonlinear regression, we predict various components of spending and income based on data given from 1970 onwards and combine the various regressions into one formula for predicting national debt from year. The final formula calculated is:

$$\hat{y} = 4.8955 \times 10^{12} - e^{-1.33846\times 10^{6}+2017.01x-1.01319x^2+0.000169651x^3} -$$
$$1.12306 \times 10^{10}x + 1.02711 \times 10^{7}x^2 - 4680.5x^3 + 1.06264x^4 - 0.0000961523x^5 +$$
$$12.7976\cos(\frac{x}{2}) + 8.20864\sin(\frac{x}{2}) + 3.82922\sin(\frac{x}{\pi})$$

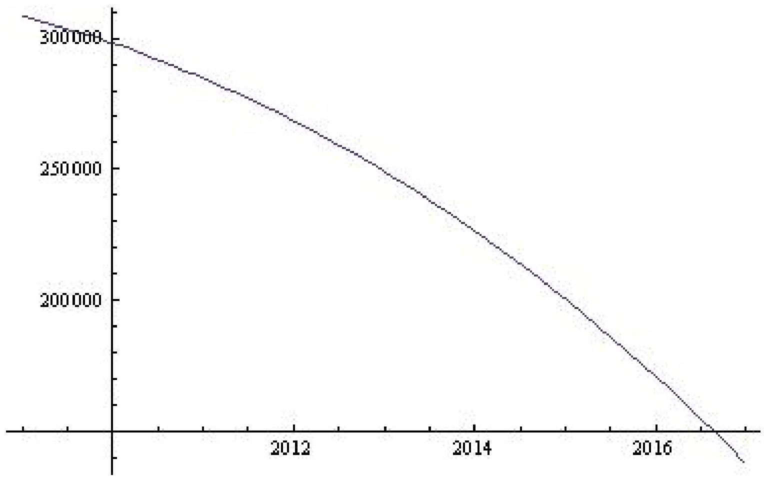

Figure 21: National debt model under the Democrats

Two additional statistics were developed to understand national debt: the ratio of national debt to GDP and national debt per capita.

The formulas for each of these components is given as follows:

$$\text{Ratio of national debt to GDP} = \frac{1}{-6.26059 \times 10^8 + 955979.x - 486.672x^2 + 0.0825999x^3} \times (4.8955 \times 10^{12} - e^{-1.33846\times10^6+2017.01x-1.01319x^2+0.000169651x^3} - 1.12306 \times 10^{10}x + 1.02711 \times 10^7x^2 - 4680.5x^3 + 1.06264x^4 - 0.0000961523x^5 + 12.7976\cos(\frac{x}{2}) + 8.20864\sin(\frac{x}{2}) + 3.82922\sin(\frac{x}{\pi}))$$

and

$$\text{National debt per capita} = \frac{1}{-4.56159 \times 10^{12} + 6.92705 \times 10^9x - 3.50728 \times 10^6x^2 + 592.112x^3} \times (4.8955 \times 10^{12} - e^{-1.33846\times10^6+2017.01x-1.01319x^2+0.000169651x^3} - 1.12306 \times 10^{10}x + 1.02711 \times 10^7x^2 - 4680.5x^3 + 1.06264x^4 - 0.0000961523x^5 + 12.7976\cos(\frac{x}{2}) + 8.20864\sin(\frac{x}{2}) + 3.82922\sin(\frac{x}{\pi}))$$

Using a combination of these three measures, national debt can be studied and understood to an even greater extent.

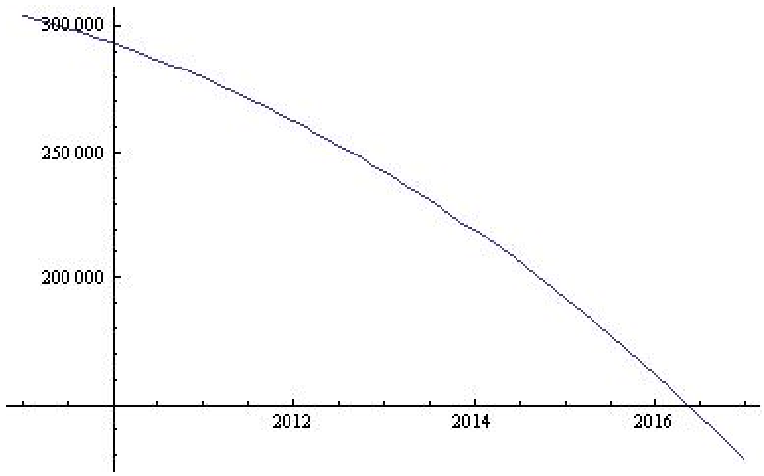

Figure 22: National debt model under the Republicans

Some limitations of this model include the fact that it is extremely sensitive to policies regarding bond sales. Accordingly, the predictions we calculated based on Republican and Democratic policies showed not very significant changes in national debt over the next 8 years.

Using our calculated model, any policies that you choose to put forth will have no effect on the national debt in the next 8 years. So, from our model we predict that if all of your promised policies are implemented, the effect on national debt will still be relatively minor. It is feasible to take all troops out of Iraq and put forth universal healthcare with minimal effect on America's debt situation.

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