SLS Unit: Rate and Speed

Level: P6

Learning objectives

(a) Apply the definition of rate to problems involving direct and indirect proportion (all pupils)

(b) Solve word problems involving the following scenarios (all pupils)

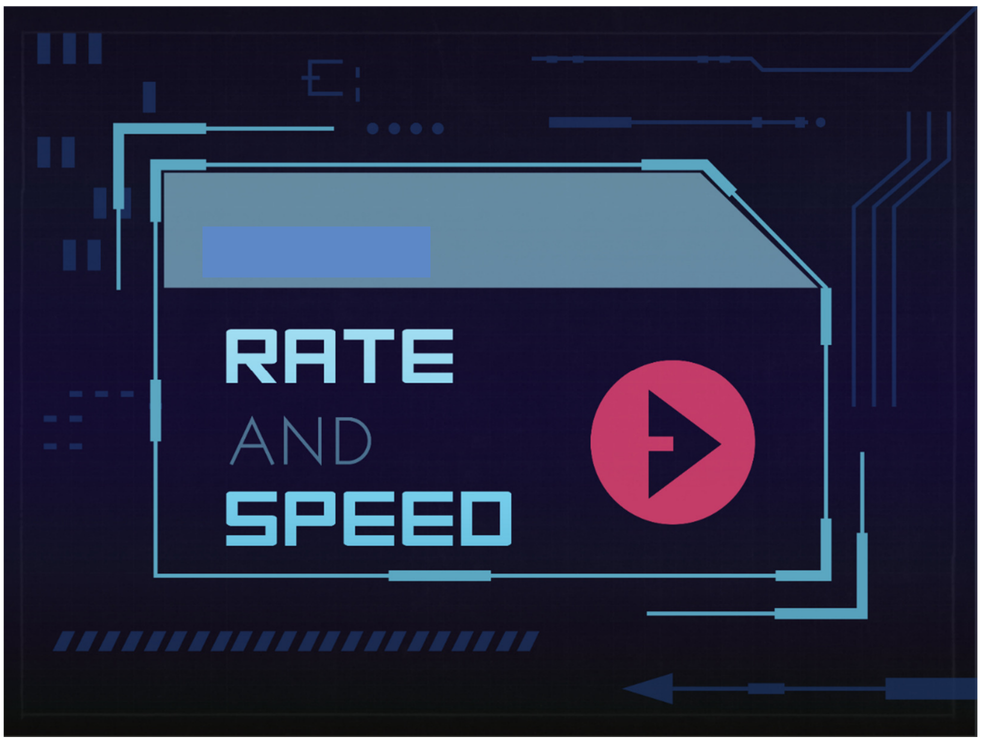
  i. Two objects moving in opposite direction
  ii. Two objects moving in same direction
  iii. Closing the gap
  iv. Circular journey

(c) Interpret distance-time and speed-time graphs (all pupils)

(d) Relate the shape of graph to the rate of change (advanced pupils)

Scene 1 – Introduction to Rate and Speed

<u>1.1 Title slide</u>



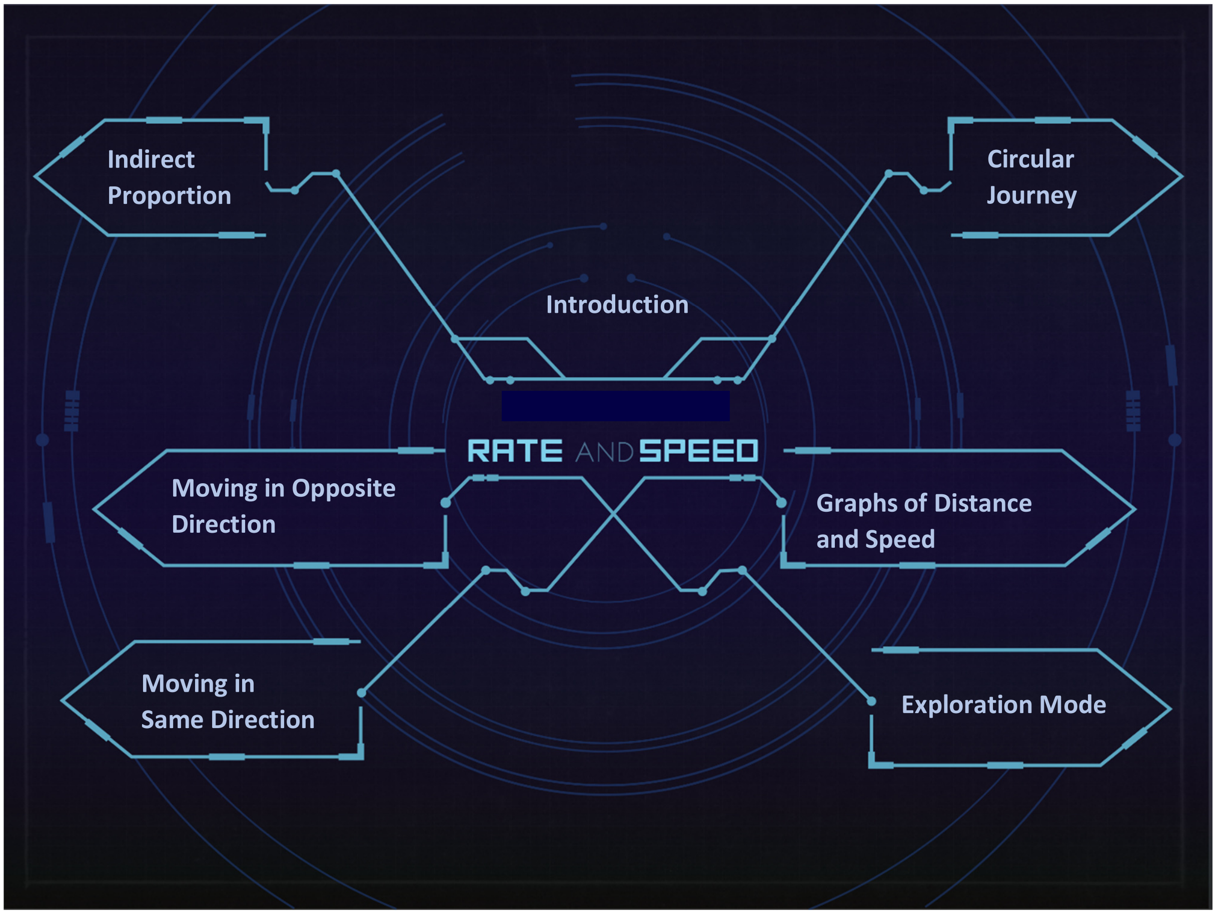
Start button to link to **1.2 Home**.

<u>1.2 Home</u>

Present the Home page which consists a main menu with options:

- Introduction (link to **1.3**)
- Indirect Proportion (link to **2.1**)
- Two objects moving in opposite direction (link to **3.1**)
- Two objects moving in same direction (link to **4.1**)
- Circular journey (link to **5.1**)
- Graphs of distance and speed (link to **6.1**)
- Exploration Mode (link to **7.1**)

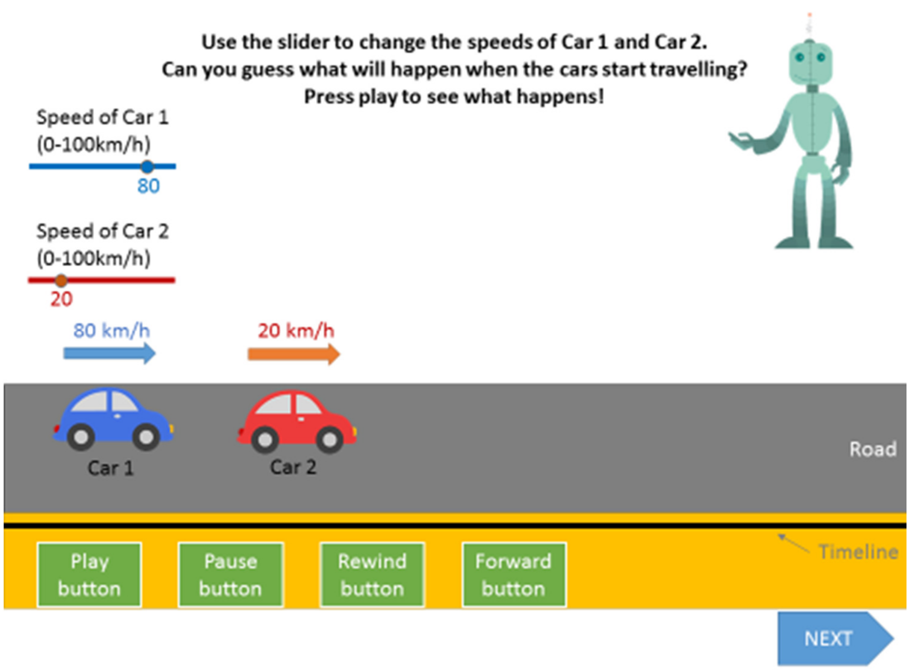
As the user mouses over the option, it should glow red.



1.3 Introduction

Robot will be introduced as the guide for the topic. Users can click on the robot whenever they need instructions to be repeated.

Introductory Exploration appears.



Variables (sliders or text entry): • Speed of Car 1 • Speed of Car 2 Text entry is preferred as it is easier to use than sliders. Choice of sliders or text entry depends on whether the programme is able to accept and work with **any** number keyed in. If it can only work with a range of numbers, text entry may not be feasible. For advice. For simulation, require navigational buttons and timeline to indicate time elapsed.

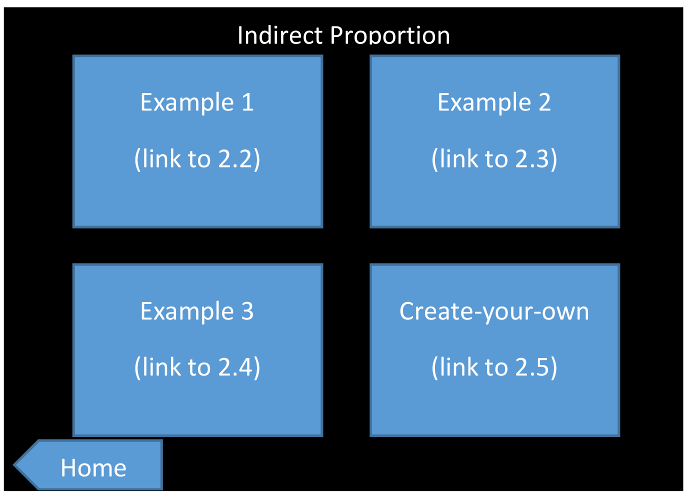
Users can use slider to adjust the speeds of the cars (any number from 0 to 100 km/h). The speed indicated above the arrows for each car will change accordingly to reflect the specified speed.

Upon pressing play, the 2 cars will move at the specified speed until a car reaches the end of the road, which prompts the simulation to stop. Simulation can be paused at any point. The simulation will play again when the user press on the Play button again.

Next button to link to **1.2 Home**.

Scene 2 – Indirect Proportion

2.1 Menu 1



Robot: "For a given speed, a car will take a longer time to travel a longer distance. Hmmm, does it always mean that whenever one quantity increases, another related quantity will increase as well? (Pause) Not necessarily.

For example, the more people you have to complete a piece of work, the lesser the time taken to complete the work. Check out this menu for more of such examples."

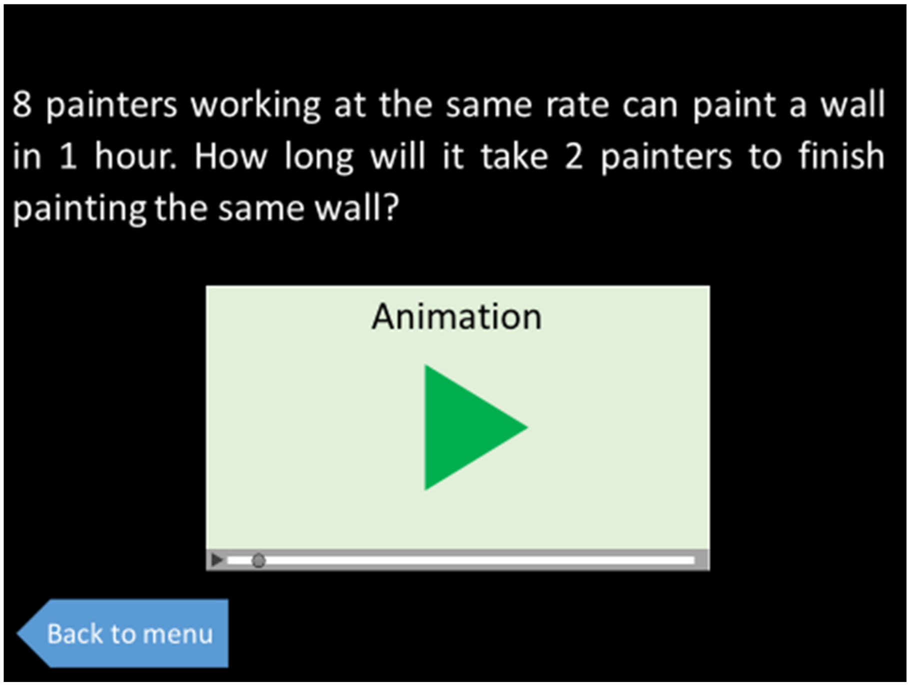
Menu has 4 options:

1. Example (2.2)
2. Question (2.3)
3. Question (2.4)
4. Create-your-own (2.5)

Options will glow when user mouses over. Users can access any option in any order. No need to access all options.

Include button for returning to **1.2 Home**.

2.2 Example 1



Animation should include timeline and function for pausing, forward, rewind.
Include a button which returns to **2.1 Menu 1**.

Animation details and Script

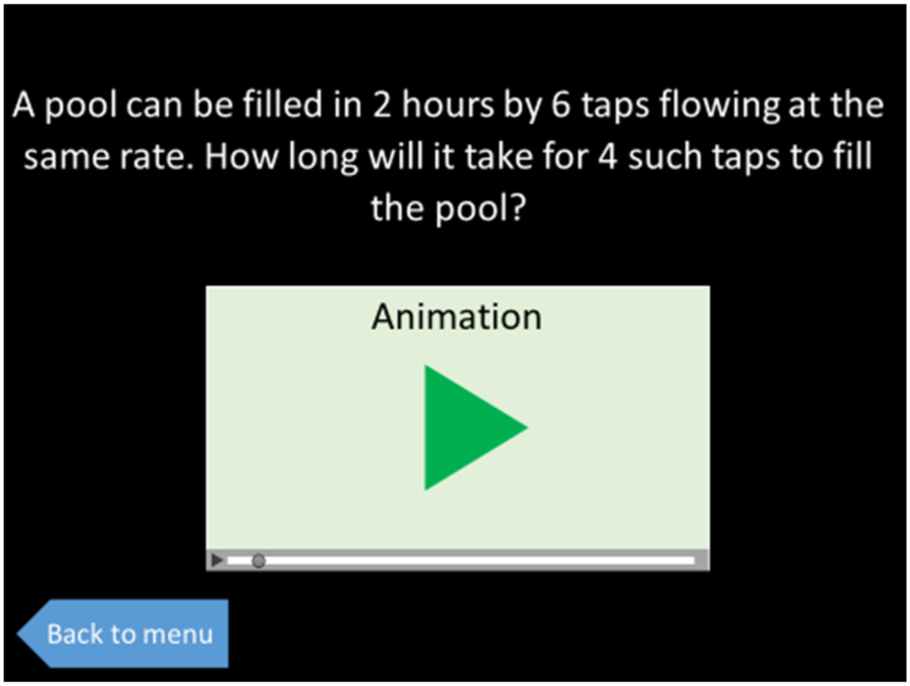
Robot: "Here we have 8 painters painting a wall. They can finish the job in 1 hour."
[Animation shows the wall being divided into 8 equal sections. Each of the 8 painters then concurrently paints each section of the wall, as the clock moves 1 hour.]

Robot: "What if we only have 1 painter? How much time would 1 painter take to finish painting the wall?"
[Animation shows 1 painter painting all 8 sections of the wall one by one, clock moves 8 hours (an hour at a time as the worker completes painting each section).]

Robot: "Now, we expect that the time taken by 2 painters to do the same amount of work as 1 painter will be less because there are more people working at the same time. How long will 2 painters take to finish painting the wall together?
[Animation shows the 8 sections of the wall being distributed such that 4 sections are assigned each painter. Each of the 2 painters then concurrently paints each section of the wall assigned to them, as the clock moves 1 hour for every section completed. Clock moves 4 hours in total.]

Robot: "We have seen that 8 painters take 1 hour; 1 painter takes 8 hours; and 2 painters take 4 hours. What do you notice? (pause)
With *fewer* workers, *more* time would be needed to complete the same job. With *more* workers, *less* time would be needed. We say there is an *inverse relationship* between the *number of workers* and the *amount of time taken*. When the number of workers is multiplied by a number, the amount of time taken will be divided by the same number. We call this **indirect proportion**.
[Illustrate this and animate the key words on the screen]

2.3 Example 2



Animation should include timeline and function for pausing, forward, rewind.
Include a button which returns to **2.1 Menu 1**.

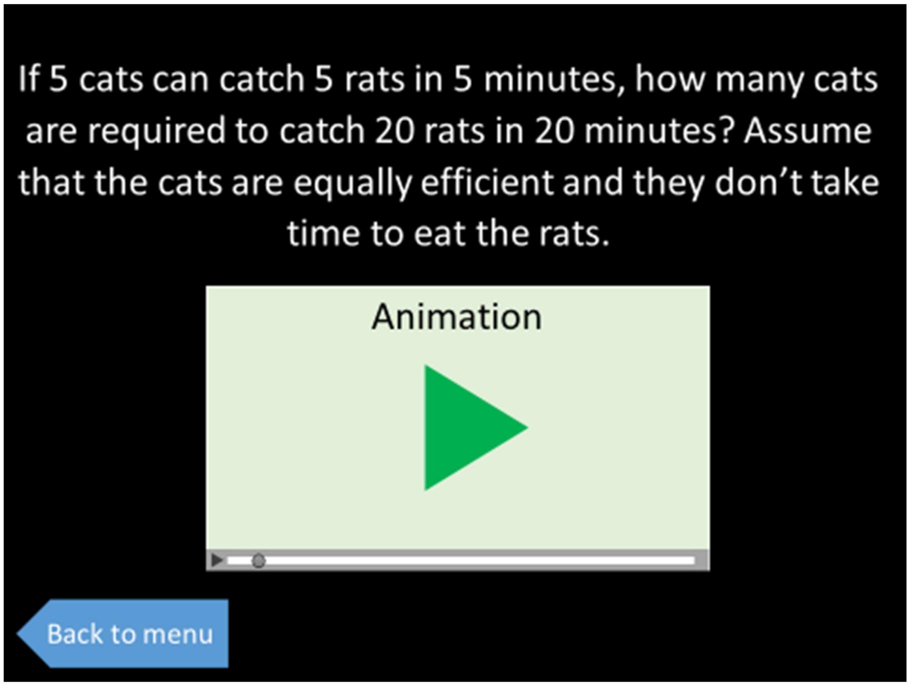
Animation details and Script

Robot: "Here we have 6 taps filling a pool. The pool is filled in 2 hours."
[Animation shows the pool being divided into 6 equal sections. Each of the 6 taps then concurrently fills each section of the pool, as the clock moves 2 hours.]

Robot: "What if we only have 1 tap? How much time would 1 tap take to finish filling the pool alone?"
[Animation shows 1 tap filling all 6 sections of the pool one by one, clock moves 12 hours (2 hours at a time as the tap fills each section).]

Robot: "Now, we expect that the time taken by 4 taps to fill the same pool will be shorter because there are more taps filling at the same time. How long will 4 taps take to fill the pool together?
[Animation shows the 6 sections of the pool being combined and re-divided into 4 sections. Each of the 4 taps then concurrently fills each new section of the pool, and the clock moves 3 hours. To illustrate the inverse relationship, it is shown clearly that the time taken is a quarter of the 12 hours previously taken by 1 tap.]

2.4 Example 3



Animation should include timeline and function for pausing, forward, rewind.
Include a button which returns to **2.1 Menu 1**.

Animation details and Script

Robot: "Here we have 5 cats catching 5 rats. They can do it in 5 minutes."
[Animation shows the 5 cats and 5 rats. Each of the 5 cats is matched to a rat. The cats then concurrently chase and catch their assigned rat, as the clock moves 5 minutes.]

Robot: "It looks like a cat takes 5 minutes to catch a rat. There are three variables here: number of cats, number of rats and the time taken. If we keep to the same amount of time, what happens when we have more cats? (pause) With more cats, more rats can be caught at the same time. The number of cats and number of rats are directly proportionate.
[Animation shows that as the number of cats increases, the number of rats also increases. For every new cat, a new rat is added.]

Robot: "If we keep to the same number of cats, for more rats to be caught, would we need more or less time? (pause) With more rats, more time would be needed, so the number of rats and the time taken are directly proportionate.
[Animation shows a fixed number of cats. As the number of rats increases, the cats take a longer time to catch them all.]

Robot: "Now, the number of rats is fixed. If we have more cats, would we need more or less time? (pause) With more cats, less time would be needed. On the other hand, with fewer cats, more time would be needed. We can say that the number of cats and the time taken are inversely proportionate.
[Animation shows a fixed number of rats to be caught. With more cats, the time taken is shorter. With fewer cats, the time taken is longer.]

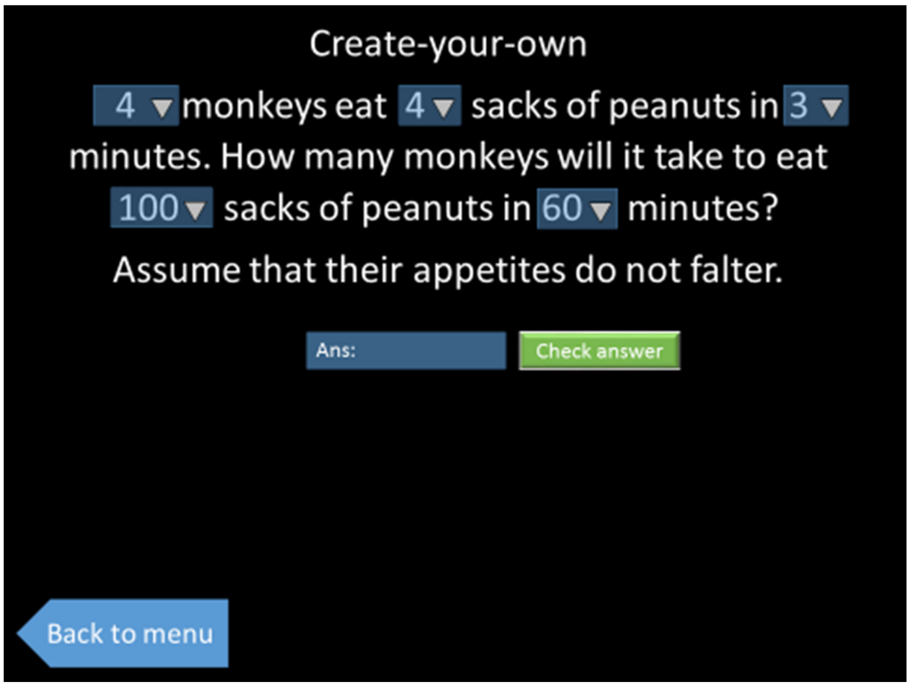
Robot: "What if we have 20 rats? How much time would 5 cats take to catch them all?"
[Animation shows the 5 cats and 20 rats. Each of the 5 cats is matched to 4 rats. The cats then concurrently chase and catch their assigned rats, as the clock moves 20 minutes (5 minutes at a time).]

Robot: "What if we have 30 rats to be caught in 10 minutes? How many cats would we need?

[Animation shows the 5 cats and 30 rats. Each of the 5 cats is matched to 6 rats. The cats then concurrently chase and catch their assigned rats, as the clock moves 30 minutes (5 minutes at a time). Clock shakes to show that it has exceeded the 10 minutes. It then splits the 30 minutes into 3 separate screens showing 5 cats each catching concurrently in 10 mins i.e. 15 cats in total.]

2.5 Create-your-own

Robot: "Here, you are free to create your own question by changing some parts of the original question. To change a number, click on the arrow next to the number and click on the number you would like to use in your own question. After that, try solving your very own question and key in your answer to check if you got it right! Have fun!"



Include drop-down which allows user to change the value of the input variable. Input for Ans should be marked according to the correct answer calculated from the input variables.

Formula

$\boxed{m_1}$ monkeys eat $\boxed{p_1}$ sacks of peanuts in $\boxed{t_1}$ minutes. How many monkeys will it take to eat $\boxed{p_2}$ sacks of peanuts in $\boxed{t_2}$ minutes? Assume that their appetites do not falter.

$$\text{Ans} = m_1 \times \frac{p_2 t_1}{p_1 t_2}$$

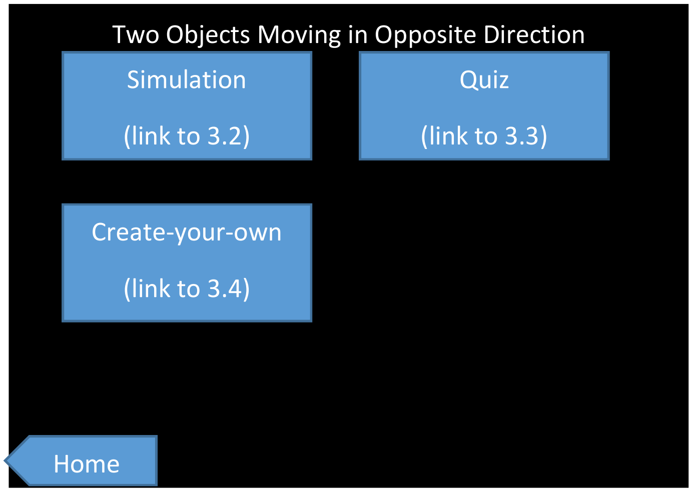
Include feedback layers:

> If answer is correct, provide feedback: "Good job!" Then return to the question for user to view the question.
>
> If answer is incorrect, provide feedback: "Among the monkeys, peanut sacks, and time taken, can you guess which of these are directly proportionate, and which are inversely proportionate? Try again."

Include a button which returns to **2.1 Menu 1**.

Scene 3 – Two Objects Moving in Opposite Direction

3.1 Menu 2



Robot: "When we have 2 cars on a road moving in the opposite direction, what can possibly happen? Will they ever meet? Check out this menu for more of such scenarios."

Menu has 4 options:

1. Simulation (3.2)
2. Quiz (3.3)
3. Create-your-own (3.4)

Options should glow when user mouses over. Users can access any option in any order. No need to access all options.

Include button for returning to **1.2 Home**.

3.2 Simulation

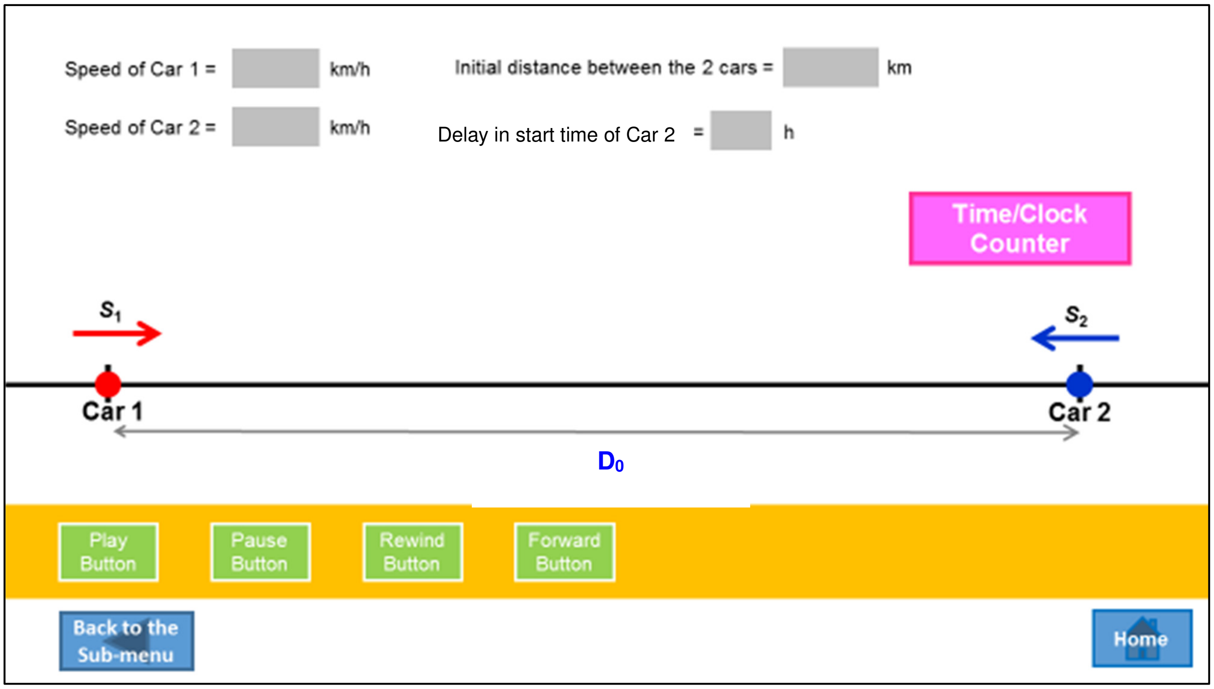
Prior to simulation, there will be an animation (with voice over) of the Robot giving brief introduction and instructions to enter the variables.

Robot: "Let's see what happens when we have two cars which are moving towards each other in the opposite direction. What happens when they start moving at different times?"

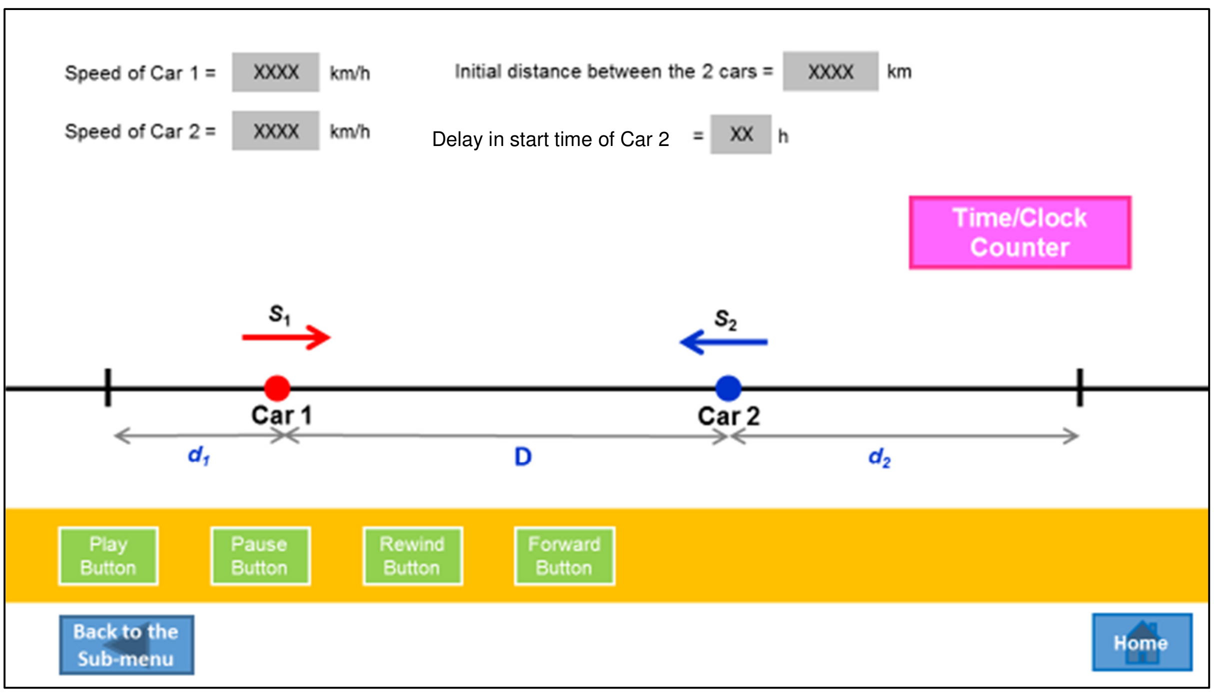
"First key in the speeds of the 2 cars. Next, key in the initial distance between the 2 cars. Lastly, key in the number of hours of delay for Car 2. When you are ready, press play to see the cars go!"

"You may click on the pre-set time buttons to see where they are at each hour and when they meet."

The diagram below shows how the simulation looks like before any of the variables are entered.



The diagram below shows how it looks like at time, *t*, when the simulation is played.



Features of Simulation for Opposite Direction

Users can change the following independent variables in the simulation before playing:

- Speed of Car 1 (any number between 0 to 100 km/h)
- Speed of Car 2 (any number between 0 to 100 km/h)
- Initial distance between the two cars, D_0 (in km). Once the value is being entered by the user, the distance value will appear between the 2 cars on the screen. This value will **vary** accordingly as the 2 cars travel towards each other.
- Starting time of Car 2, t_2, after Car 1 has started.

There will be 3 other dependent variables that will be reflected on the screen as the simulation is played.

- Time elapsed – There should be a clock/time counter (in hours; values may be in decimals) to show the **time elapsed** (that changes **continuously**) as Car 1 starts moving. Car 2 will start moving after time t_2, determined by the user at the beginning of the simulation. **There should be preset buttons for users to click on to jump to specific times during the simulation** e.g.
    - 1 hour later,
    - 2 hours later,
    - 3 hours later,
    - meeting point (when distance between 2 cars is 0 i.e. at time $t = \frac{D_0 + s_2 t_2}{s_1 + s_2}$)
- Distance travelled by Car 1 (measuring from its starting point)
- Distance travelled by Car 2 (measuring from its starting point)
- Distance between the 2 cars at any time *t*.

After entering the 4 independent variables, speed of Car 1, speed of Car 2 and Distance between the 2 cars and time at which Car 2 starts, user may click on play to start the simulation. The 2 cars will move at the specified speed until they meet, which prompts the animation to **indicate that the cars have met**. The animation continues to play as both cars pass each other. **When both cars have reached the end of the road, this prompts the animation to stop**. Animation can be paused at any point. The animation will play again when the user presses on the Play button again.

Formula

The following are formulas that should be embedded in the simulation to help compute all the dependent variables, so that they can be accurately reflected on the screen as the simulation plays:

At any time t (in hours) [may be in decimals],

$$d_1 = s_1\, t$$

$$d_2 = s_2\,(t - t_2)$$

$$D = D_0 - s_1\, t - s_2\,(t - t_2)$$

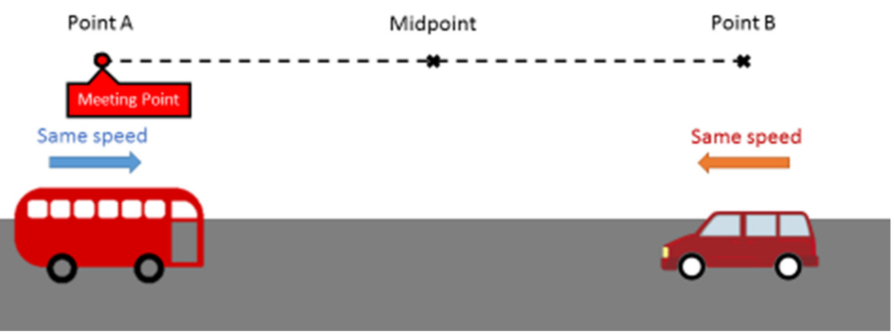
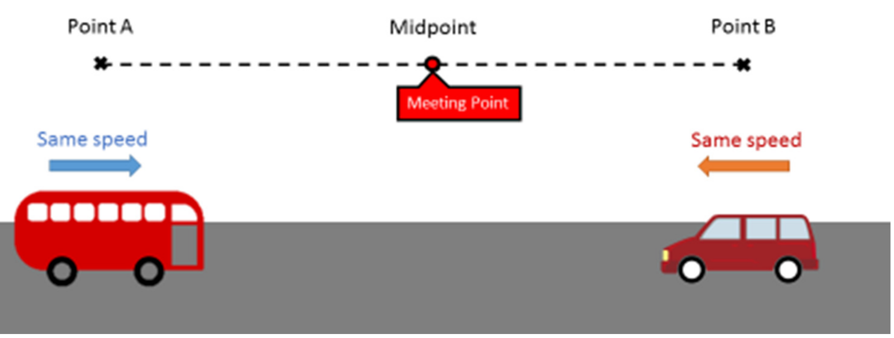
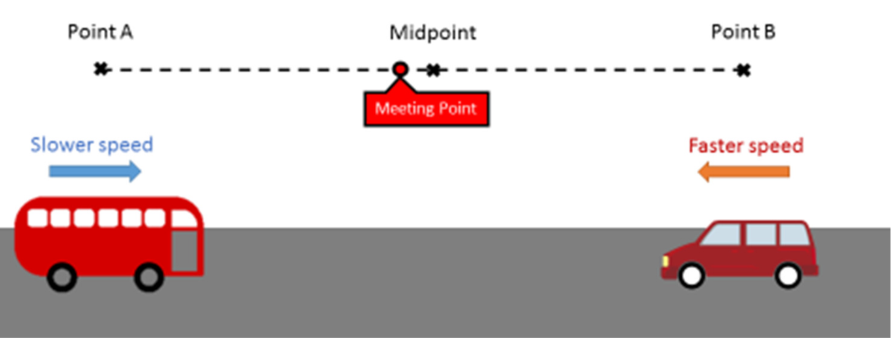
where $\boxed{d_1}$ and $\boxed{d_2}$ are the distance travelled by Car 1 and Car 2 respectively. $\boxed{D_0}$ is the initial distance between the 2 cars entered by the user. $\boxed{t_2}$ is the starting time of Car 2 after Car 1 has started.

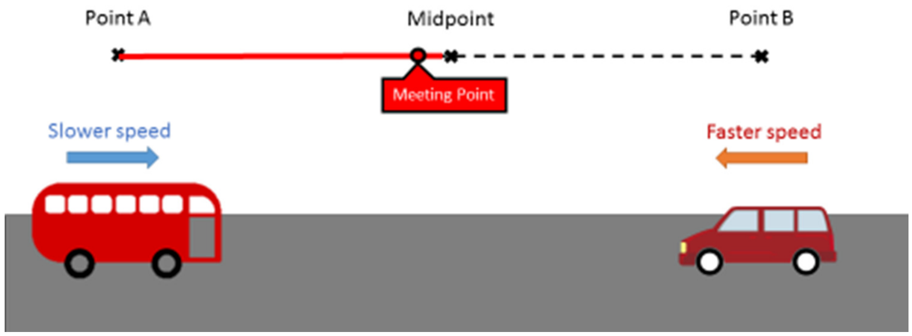
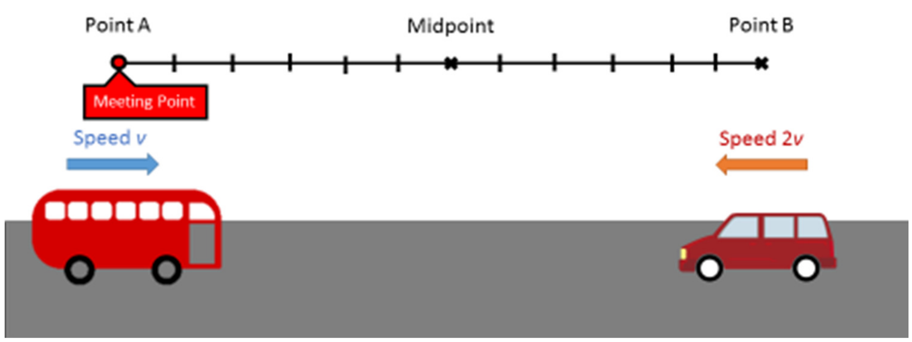
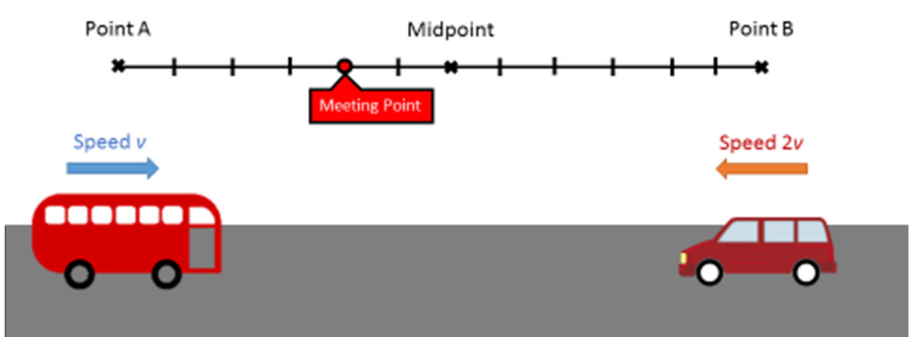
Include button for returning to **1.2 Home,** and a back button for returning to **3.1 Menu 3.**

3.3 Quiz

There are 4 questions to be attempted in sequence and scored.

Question 1	When 2 objects move in the opposite direction, will they ever meet? (show the diagram below)  ☐ Yes ☐ No

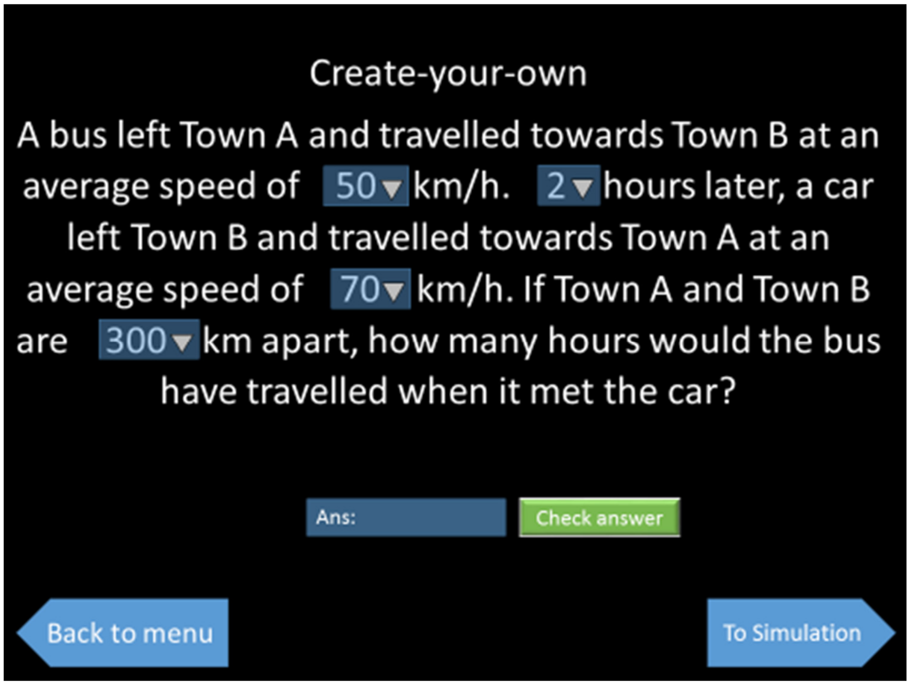
Answer 1	☒ Yes ☐ No
Feedback 1	If answer is correct, provide feedback: “Good job!” Then move to Question 2 for user to attempt. If answer is incorrect, provide feedback: “Try again.” Then move to Question 1 for user to attempt again.
Question 2	2 objects start moving in the opposite direction at the same time. If they move at the same speed, where will they meet? Drag the “Meeting Point” to where you think they will meet. (show the diagram below with slider for “Meeting Point”) 
Answer 2	The Meeting Point will be exactly at the midpoint indicated in red in diagram below. 
Feedback 2	If answer is correct, provide feedback: “Good job!” Then move to Question 3 for user to attempt. If answer is incorrect, provide feedback: “If the objects are travelling at the same speed, it means they cover the same distance at any time. How would this affect their meeting point? Try again.” Then move to Question 2 for user to attempt again.
Question 3	2 objects start moving in the opposite direction at the same time. If they move at **different** speeds, where will they meet? Drag the “Meeting Point” to where you think they will meet. (show the diagram below with slider for “Meeting Point”) 

Answer 3	The Meeting Point can be anywhere on the red line indicated in diagram below, <u>excluding</u> Point A and midpoint. 
Feedback 3	If answer is correct, provide feedback: “Good job!” Then move to Question 4 for user to attempt. If answer is incorrect, provide feedback: “If the object on the right is travelling at a faster speed, it means it covers more distance compared to the object on the left. How would this affect their meeting point? Try again.” Then move to Question 3 for user to attempt again.
Question 4	2 objects start moving in the opposite direction at the same time. If the object on the right moves at twice the speed of the object on the left, where will they meet? Drag the “Meeting Point” to where you think they will meet. (show the diagram below with slider for “Meeting Point”. Include tick marks which divide the line into 12 equal segments.) 
Answer 4	The Meeting Point will be exactly at the point indicated in diagram below. 
Feedback 4	If answer is correct, provide feedback: “Good job!” Then move to **3.1 Menu 2**. If answer is incorrect, provide feedback: “If the object on the right is travelling at double the speed, it means it covers twice the distance covered by the object on the

	left. How would this affect their meeting point? Try again." Then move to Question 4 for user to attempt again.

3.4 Create-your-own

Robot: "Here, you are free to create your own question by changing some parts of the original question. To change a number, click on the arrow next to the number and click on the number you would like to use in your own question. After that, try solving your very own question and key in your answer to check if you got it right! Have fun!"



Include drop-down which allows user to change the value of the input variable. Input for Ans should be marked according to the correct answer calculated from the input variables.

Formula A bus left Town A and travelled towards Town B at an average speed of $\boxed{s_1}$ km/h. $\boxed{t_2}$ hours later, a car left Town B and travelled towards Town A at an average speed of $\boxed{s_2}$ km/h. If Town A and Town B are $\boxed{D_0}$ km apart, how many hours would the bus have travelled when it met the car? Ans: $t = \dfrac{D_0 + s_2 t_2}{s_1 + s_2}$ hours

Include feedback layers:

If answer is correct, provide feedback: "Good job!" Then return to the question for user to view the question.

If answer is incorrect, provide feedback: "When two objects moving in the opposite direction meet/pass one another, what can we say about their distances travelled? Try again."

Include a button which returns to **3.1 Menu 2**.

Include a button which moves to **3.2 Simulation** for users to solve the question with the help of the simulation.

Scene 4 – Two Objects Moving in the Same Direction

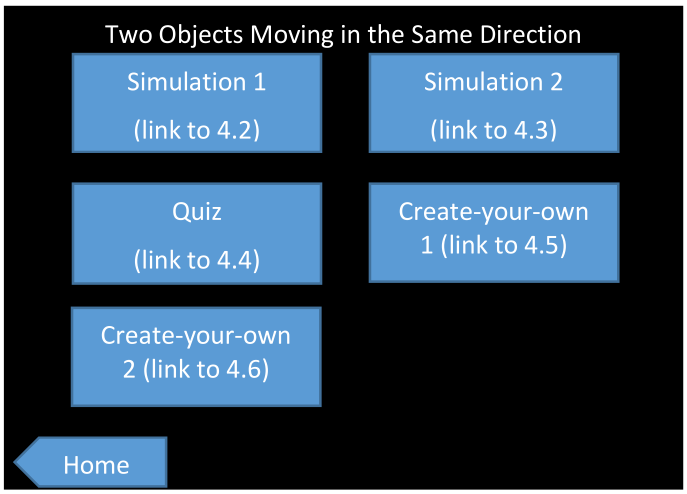
4.1 Menu 3

Robot: “When we have 2 cars on a road moving in the same direction, what can possibly happen? Will they ever meet? Check out this menu for more of such scenarios.”

Menu has 5 options:

1. Simulation 1 (4.2)
2. Simulation 2 (4.3)
3. Quiz (4.4)
4. Create-your-own 1 (4.5)
5. Create-your-own 2 (4.6)

Options should glow when user mouses over. Users can access any option in any order. No need to access all options.



Include button for returning to **1.2 Home**.

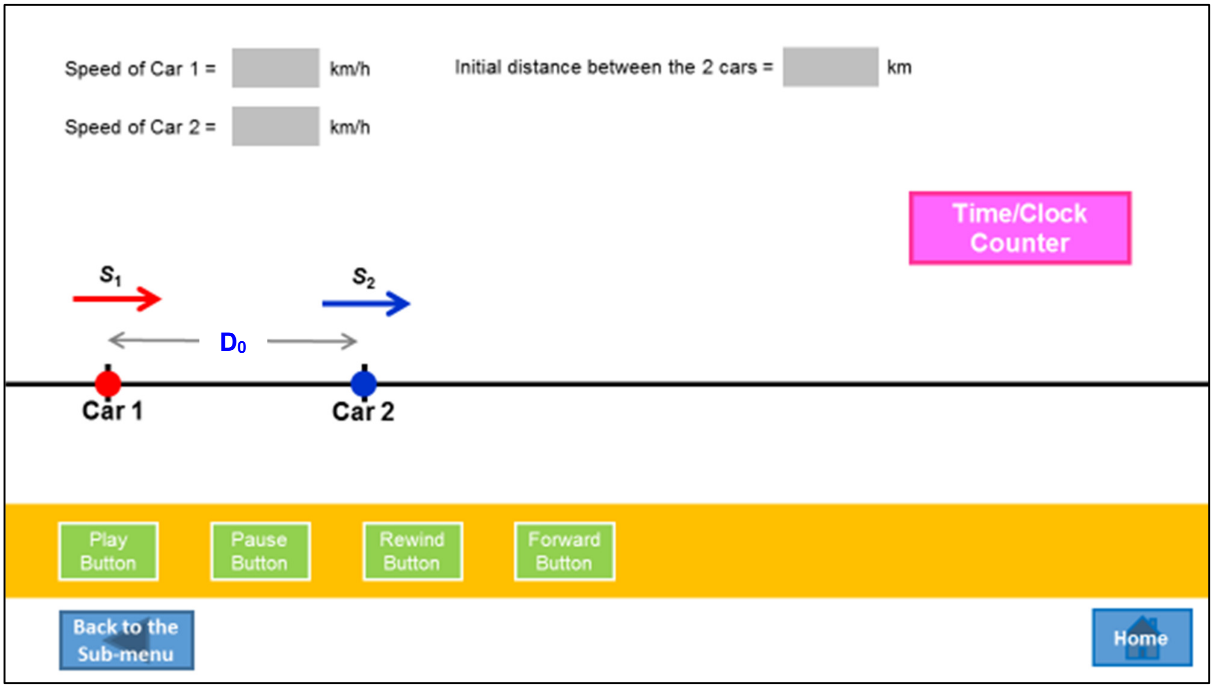
4.2 Simulation 1 (2 Objects Moving in the Same Direction at Same Time Different Place)

Prior to simulation, there will be an animation (with voice over) of the Robot giving brief introduction and instructions to enter the variables.

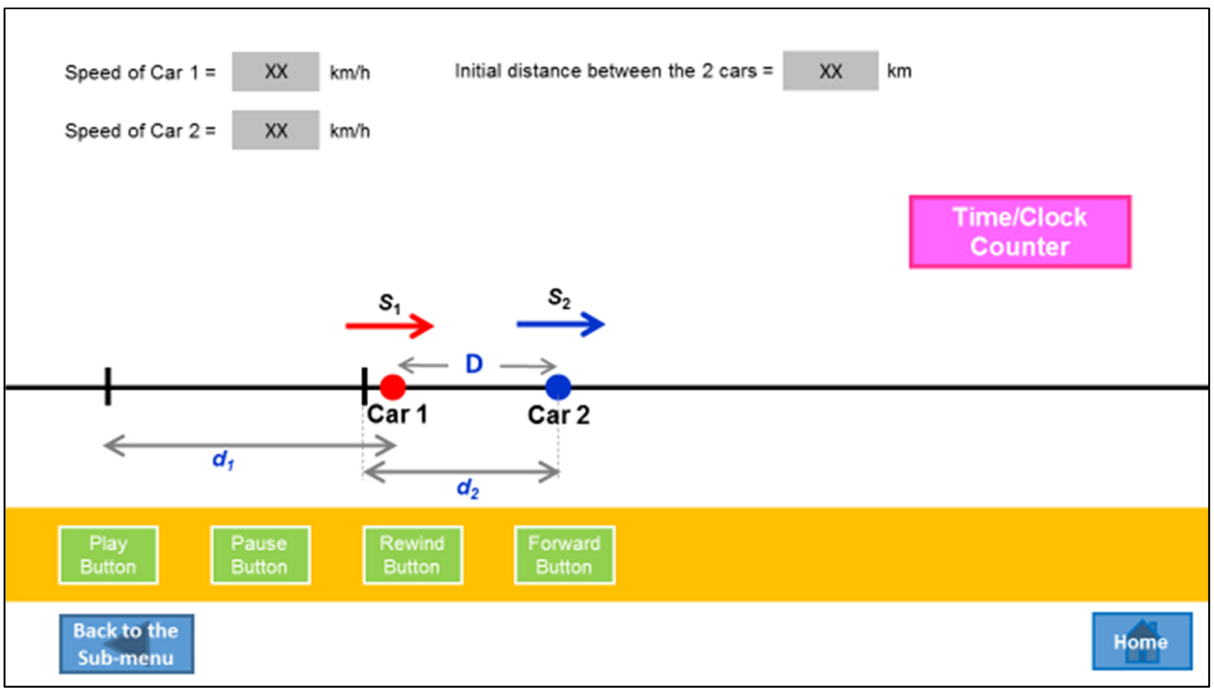
Robot: “Let’s see what happens when we have two cars which are a distance apart. What happens when they start to move at the same time in the same direction?”

“First key in the speeds of the 2 cars. Next, key in the initial distance between the 2 cars. When you are ready, press play to see the cars go! Did you predict it correctly?”

“You may click on the pre-set time buttons to see where they are at each hour and when they meet.”



The diagram below shows how it may appear at time *t*, when the simulation is played.



Features of the simulation
Users can change the following independent variables in the simulation before playing:

- Speed of Car 1 (any number between 0 to 100 km/h)
- Speed of Car 2 (any number between 0 to 100 km/h)
- Initial distance between the two cars, D_0 (in km). Once the value is being entered by the user, the distance value will appear between the 2 cars on the screen. This value will **vary** accordingly as the 2 cars travel towards each other.

There will be 3 other dependent variables that will be reflected on the screen as the simulation is played.

- Time elapsed – There should be a clock/time counter (in hours; values may be in decimals) to show the **time elapsed** (that changes continuously) as the Car 1 and Car 2 start moving simultaneously. At the end of each simulation, the 2 cars will meet or will not meet depending on the speeds of Car 1 and Car 2. **There should be preset buttons for users to click on to jump to specific times during the simulation** e.g.
    - 1 hour later,
    - 2 hours later,
    - 3 hours later,
    - meeting point (when distance between 2 cars is 0 i.e. at time $t = \frac{D_0}{s_1 - s_2}$)
- Distance travelled by Car 1 (measuring from its starting point)
- Distance travelled by Car 2 (measuring from its starting point)
- Distance between the 2 cars at time *t*.

After entering the 3 variables, speed of Car 1, speed of Car 2 and Distance between the 2 cars, user may click on play to start the simulation. The 2 cars will move at the specified speed. If they meet, it will prompt the animation to **indicate that the cars have met**. The animation continues to play as both cars move apart after meeting. Regardless of whether the cars meet or not meet, **when both cars have reached the end of the road, this prompts the animation to stop**. Animation can be paused at any point. The animation will play again when the user press on the Play button again.

Formula

The following are formulas that should be embedded in the simulation to help compute all the dependent variables, so that they can be accurately reflected on the screen as the simulation plays:

At any time t (in hours) [may be in decimals],

$$d_1 = s_1 t$$

$$d_2 = s_2 t$$

$$D = (D_0 + s_2 t) - s_1 t$$

where $\boxed{d_1}$ and $\boxed{d_2}$ are the distance travelled by Car 1 and Car 2 respectively. $\boxed{D_0}$ is the initial distance between the 2 cars entered by the user.

Include button for returning to **1.2 Home,** and a back button for returning to the **4.1 Menu 3.**

4.3 Simulation 2 (2 Objects Moving in the Same Direction from Same Place Different Time)

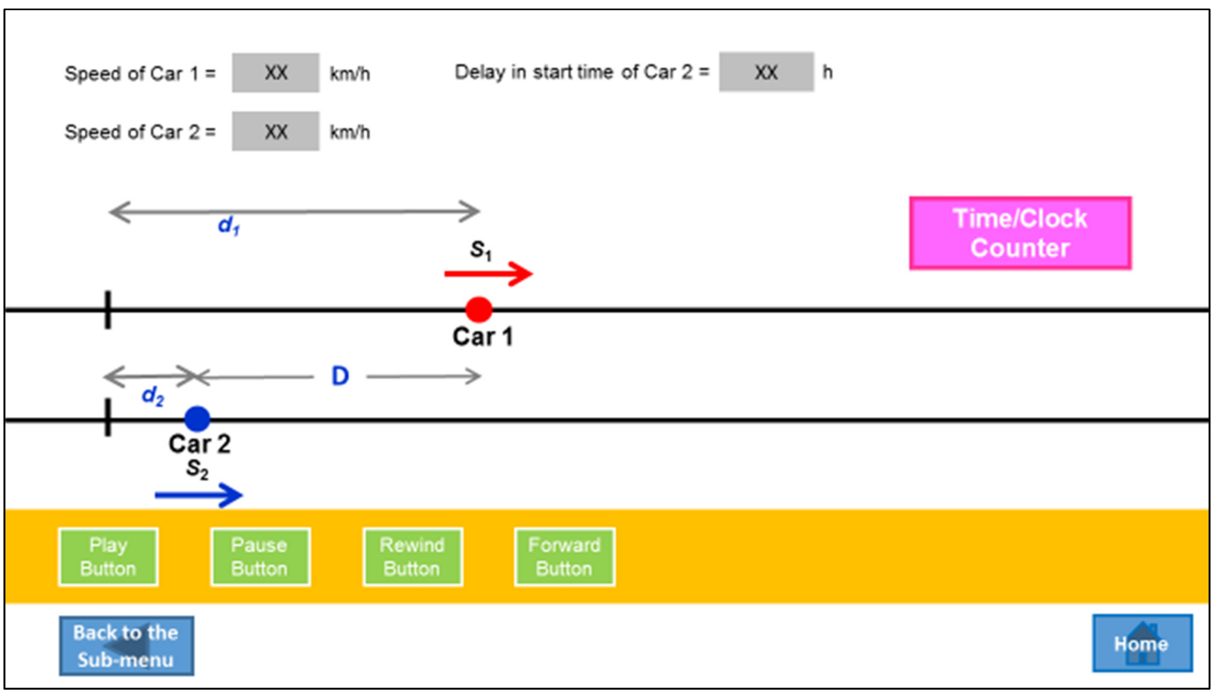
Prior to simulation, there will be an animation (with voice over) of the Robot giving brief introduction and instructions to enter the variables.

Robot: "Let's see what happens when we have two cars which are at the same place. What happens when they start to move in the same direction but at different times?"

"First key in the speeds of the 2 cars. Next, key in the number of hours of delay for Car 2. When you are ready, press play to see the cars go! Did you predict it correctly?"

"You may click on the pre-set time buttons to see where they are at each hour and when they meet."

The diagram below shows how it may appear at time *t*, when the simulation is played.



Features of the simulation

Users can change the following independent variables in the simulation before playing:

- Speed of Car 1 (any number between 0 to 100 km/h)
- Speed of Car 2 (any number between 0 to 100 km/h)
- Starting time of Car 2, t_2, after Car 1 has started.

There will be 3 other dependent variables that will be reflected on the screen as the simulation is played.

- Time elapsed – There should be a clock/time counter (in hours; values may be in decimals) to show the **time elapsed** (that changes continuously) as the Car 1 and Car 2 start moving simultaneously. At the end of each simulation, the 2 cars will meet or will not meet depending on the speeds of Car 1 and Car 2. **There should be preset buttons for users to click on to jump to specific times during the simulation** e.g.
    - 1 hour later,
    - 2 hours later,
    - 3 hours later,
    - meeting point (when distance between 2 cars is 0 i.e. at time $t = \dfrac{s_2 t_2}{s_2 - s_1}$)
- Distance travelled by Car 1 (measuring from its starting point)
- Distance travelled by Car 2 (measuring from its starting point)
- Distance between the 2 cars at time *t*.

After entering the 3 variables, speed of Car 1, speed of Car 2 and Distance between the 2 cars, user may click on play to start the simulation. The 2 cars will move at the specified speed. If they meet, it will prompt the animation to **indicate that the cars have met**. The animation continues to play as both cars move apart after meeting. Regardless of whether the cars meet or not meet, **when both cars have reached the end of the road, this prompts the animation to stop**. Animation can be paused at any point. The animation will play again when the user presses on the Play button again.

Formula

The following are formulas that should be embedded in the simulation to help compute all the dependent variables, so that they can be accurately reflected on the screen as the simulation plays:

At any time t (in hours) [may be in decimals],

$$d_1 = s_1\, t$$

$$d_2 = s_2\left(t - t_2\right)$$

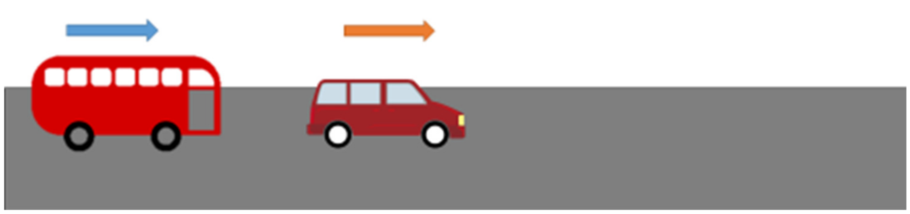
$$D = s_1 t - s_2\left(t - t_2\right)$$

where $\boxed{d_1}$ and $\boxed{d_2}$ are the distance travelled by Car 1 and Car 2 respectively. $\boxed{D_0}$ is the initial distance between the 2 cars entered by the user. $\boxed{t_2}$ is the starting time of Car 2 after Car 1 has started.

Include button for returning to **1.2 Home,** and a back button for returning to the **4.1 Menu 3 for Two Objects Moving in the Same Direction.**

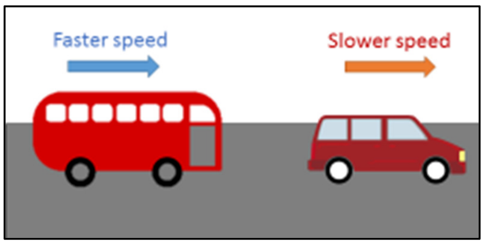
4.4 Quiz

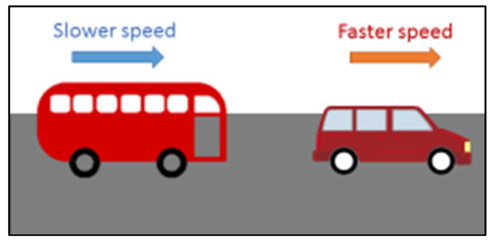
There are 2 questions to be attempted in sequence and scored.

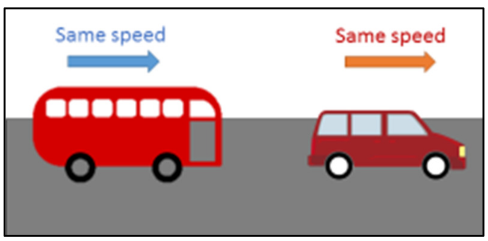
Question 1	When 2 objects move in the same direction, will they always meet? (show the diagram below)  ☐ Yes ☐ No

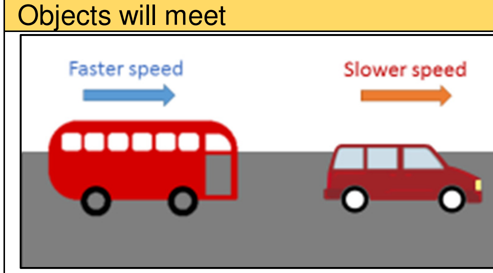
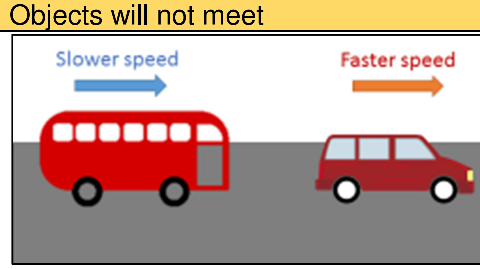
<table>
<tr><td>Answer 1</td><td>☐ Yes
☒ No</td></tr>
<tr><td>Feedback 1</td><td>If answer is correct, provide feedback: “Good job!” Then move to Question 2 for user to attempt.

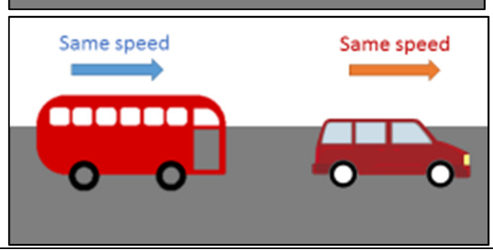
If answer is incorrect, provide feedback: “Try again.” Then move to Question 1 for user to attempt again.</td></tr>
<tr><td>Question 2</td><td>2 objects start moving in the same direction at the same time. Drag and drop the 3 pictures into the correct box. (Drag and drop is just a suggestion. The question can also be presented in other forms e.g. checkbox)

<table>
<tr><th>Objects will meet</th><th>Objects will not meet</th></tr>
<tr><td></td><td></td></tr>
</table>






</td></tr>
<tr><td>Answer 2</td><td>
<table>
<tr><th>Objects will meet</th><th>Objects will not meet</th></tr>
<tr><td>
</td><td>


</td></tr>
</table>
</td></tr>
<tr><td>Feedback 2</td><td>If answer is correct, provide feedback: “Good job!” Then move to 4.1 Menu 3.

If answer is incorrect, provide feedback: “Try again.” Then move to Question 2 for user to attempt again.</td></tr>
</table>

4.5 Create-your-own 1

Interactive and visual requirements are similar to what was shown in **2.5** and **3.4**. The question is as follows:

Towns A, B and C are sited along the same road (refer to diagram). A car and a bus were both travelling to Town C using that road. The car departed from Town A at an average speed of $\boxed{s_1}$ km/h. At the same time, the bus departed from Town B at an average speed of $\boxed{s_2}$ km/h. If Town A and Town B are $\boxed{D_0}$ km apart, how many hours later would the car catch up with the bus? (Show diagram below)

Include drop-down for the variables (indicated with the box drawn around it) which allows user to change the value of the input variable. Input for Ans should be marked according to the correct answer calculated from the input variables.

Formula
Towns A, B and C are sited along the same road. A car and a bus were both travelling to Town C using that road. The car departed from Town A at an average speed of $\boxed{s_1}$ km/h. At the same time, the bus departed from Town B at an average speed of $\boxed{s_2}$ km/h. If Town A and Town B are $\boxed{D_0}$ km apart, how many hours later would the car catch up with the bus?
Ans: $t = \dfrac{D_0}{s_1 - s_2}$ hours
If $s_1 \leq s_2$, there is no valid answer.

Include feedback layers:

> If answer is correct, provide feedback: "Good job!" Then return to the question for user to view the question.
>
> If answer is incorrect, provide feedback: "When two objects are moving in the same direction, what conditions must be fulfilled for them to catch up? Try again."

Include a button which returns to **4.1 Menu 3**.

Include a button which moves to **4.2 Simulation 1** for users to solve the question with the help of the simulation.

4.6 Create-your-own 2

Interactive and visual requirements are similar to what was shown in **2.5** and **3.4**. The question is as follows:

A bus left Town A for Town B at an average speed of $\boxed{s_1}$ km/h. $\boxed{t_2}$ hours later, a car left Town A for Town B at an average speed of $\boxed{s_2}$ km/h. How many hours would the bus have travelled when the car caught up with it?

Include drop-down for the variables (indicated with the box around it) which allows user to change the value of the input variable. Input for Ans should be marked according to the correct answer calculated from the input variables.

Formula
A bus left Town A for Town B at an average speed of $\boxed{s_1}$ km/h. $\boxed{t_2}$ hours later, a car left Town A for Town B at an average speed of $\boxed{s_2}$ km/h. How many hours would the bus have travelled when the car caught up with it?

Ans: $t = \dfrac{s_2 t_2}{s_2 - s_1}$ hours

If $s_1 > s_2$, there is no valid answer.

Include feedback layers:

> If answer is correct, provide feedback: “Good job!” Then return to the question for user to view the question.
>
> If answer is incorrect, provide feedback: “When two objects are moving in the same direction, what conditions must be fulfilled for them to catch up? Try again.”

Include a button which returns to **4.1 Menu 3**.

Include a button which moves to **4.3 Simulation 2** for users to solve the question with the help of the simulation.

Scene 5 – Circular Journey

5.1 Menu 4

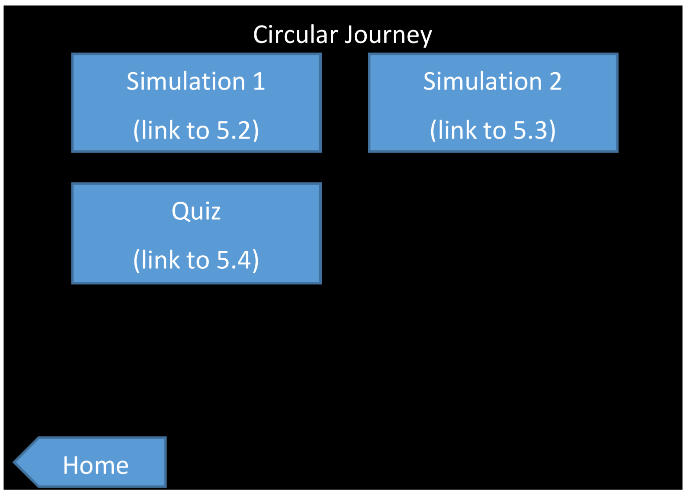
Robot: “When 2 people jog at different speed on a circular running track in the same direction, what can possibly happen? Likewise, what can possibly happen when 2 people jog in the opposite direction? Check out this menu for more of such scenarios.”

Menu has 7 options:

1. Simulation 1 (5.2)
2. Simulation 2 (5.3)
3. Quiz (5.4)

Options should glow when user mouses over. Users can access any option in any order. No need to access all options.

Include button for returning to **1.3 Home**.



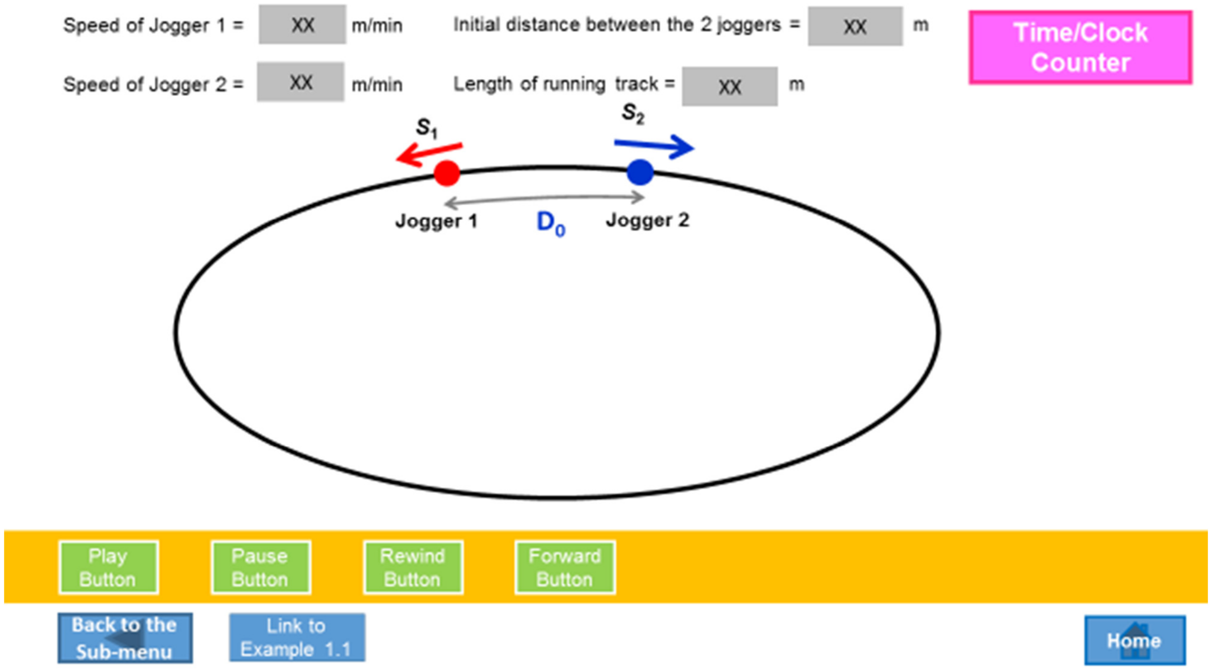
5.2 Simulation 1 (2 Objects Moving in Opposite Direction in a Circular Route)

Prior to simulation, there will be an animation (with voice over) of the Robot giving brief introduction and instructions to enter the variables.

Robot: “Let’s see what happens when we have two joggers who are running away from each other in a circular running track. What happens when they start moving? Will they meet?”

“First key in the speeds of the 2 joggers. Next, key in the length of the running track. Lastly, key in the initial distance between the 2 joggers. When you are ready, press play to see them go!”

“You may click on the pre-set time buttons to see where they are at each minute and when they meet.”



Features of the simulation

Users can change the following independent variables in the simulation before playing:

- Speed of Jogger 1 (any number between 0 to 200 m/min)
- Speed of Jogger 2 (any number between 0 to 200 m/min)
- Distance between the two joggers (any whole number between 0 to L m). Once the value is being entered by the user, the distance value will appear between the 2 joggers on the screen. This value will **change** accordingly as the 2 persons jog.
- Length of the running track, L m (any number between 100 to 10 000 m)

There will be 5 other dependent variables that will be reflected on the screen as the simulation is played.

- Time elapsed – There should be a clock/time counter (in minutes and hours) to show the **time elapsed** (that changes continuously) as the 2 joggers jog. **There should be preset buttons for users to click on to jump to specific times during the simulation** e.g.
  - 1 min later,
  - 2 mins later,
  - 3 mins later,
  - 1st meeting point (when distance between 2 joggers is L i.e. at time $t = \dfrac{L - D_0}{s_1 + s_2}$)
  - 2nd meeting point (when distance between 2 joggers is $2L$ i.e. at time $t = \dfrac{2L - D_0}{s_1 + s_2}$)
  - 3rd meeting point (when distance between 2 joggers is $3L$ i.e. at time $t = \dfrac{3L - D_0}{s_1 + s_2}$)
  - 4th meeting point (when distance between 2 joggers is $4L$ i.e. at time $t = \dfrac{4L - D_0}{s_1 + s_2}$)
  - 5th meeting point (when distance between 2 joggers is $5L$ i.e. at time $t = \dfrac{5L - D_0}{s_1 + s_2}$)

- Distance travelled by Jogger 1, d_1 (measuring from its starting point)
- Distance travelled by Jogger 2, d_2 (measuring from its starting point)
- Distance between the two joggers, D
- Number of times joggers met, n

After entering the 4 variables, speed of Jogger 1, speed of Jogger 2, Initial distance between the 2 joggers and Length of running track, user may click on play to start the simulation. The 2 joggers will move at the specified speed. Every time they meet, it prompts the animation to **indicate that the joggers have met**. The animation continues to play as both joggers pass each other. **The animation continues in an infinite loop**. Animation can be paused at any point. The animation will play again when the user presses the Play button again.

Formula

The following are formulas that should be embedded in the simulation to help compute all the dependent variables, so that they can be accurately reflected on the screen as the simulation plays:

At any time t (in hours) [may be in decimals],

$$d_1 = s_1 t$$

$$d_2 = s_2 t$$

Distance between joggers, $D = \min\left(\text{MOD}(D_0 + d_1 + d_2, L), L - \text{MOD}(D_0 + d_1 + d_2, L)\right)$

Number of times joggers met, $n = \dfrac{D_0 + s_1 t + s_2 t}{L}$

where $\boxed{d_1}$ and $\boxed{d_2}$ are the distance travelled by Jogger 1 and Jogger 2 respectively. $\boxed{D_0}$ is the initial distance between the 2 joggers entered by the user. $\boxed{L}$ is the length of the running track entered by the user.

Include button for returning to **1.3 Home,** and a back button for returning to the **5.1 Menu 4**.

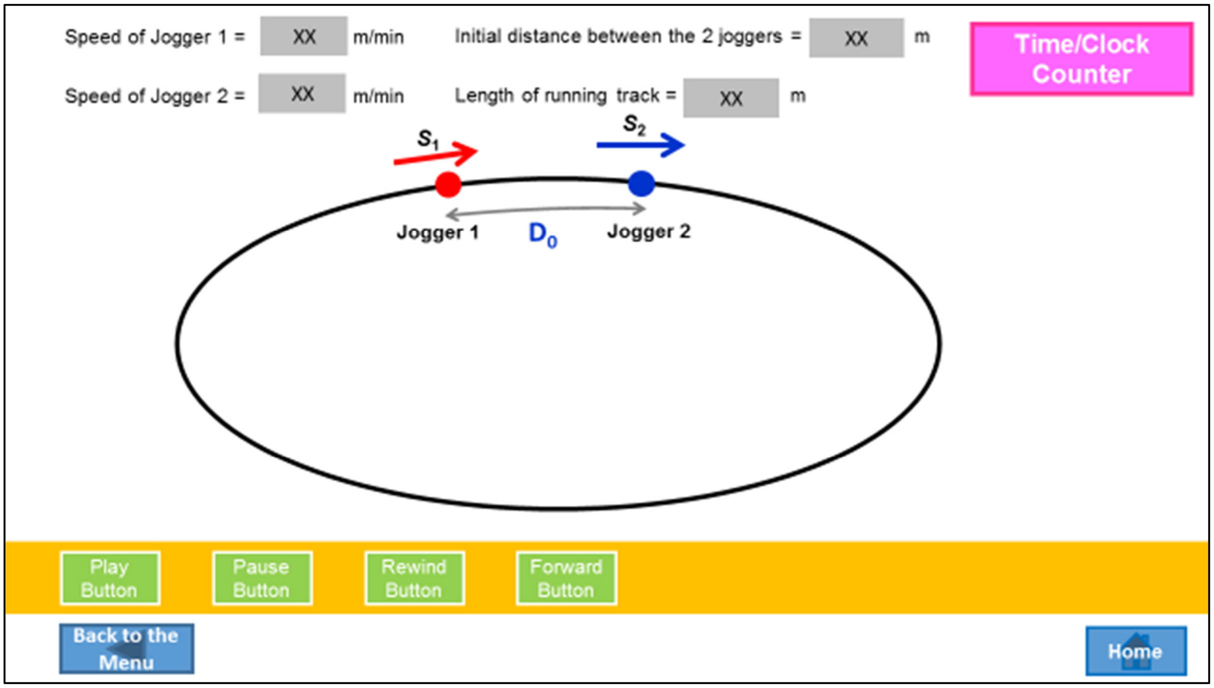
5.3 Simulation 2 (2 Objects Moving in the Same Direction in a Circular Route)

Prior to simulation, there will be an animation (with voice over) of the Robot giving brief introduction and instructions to enter the variables.

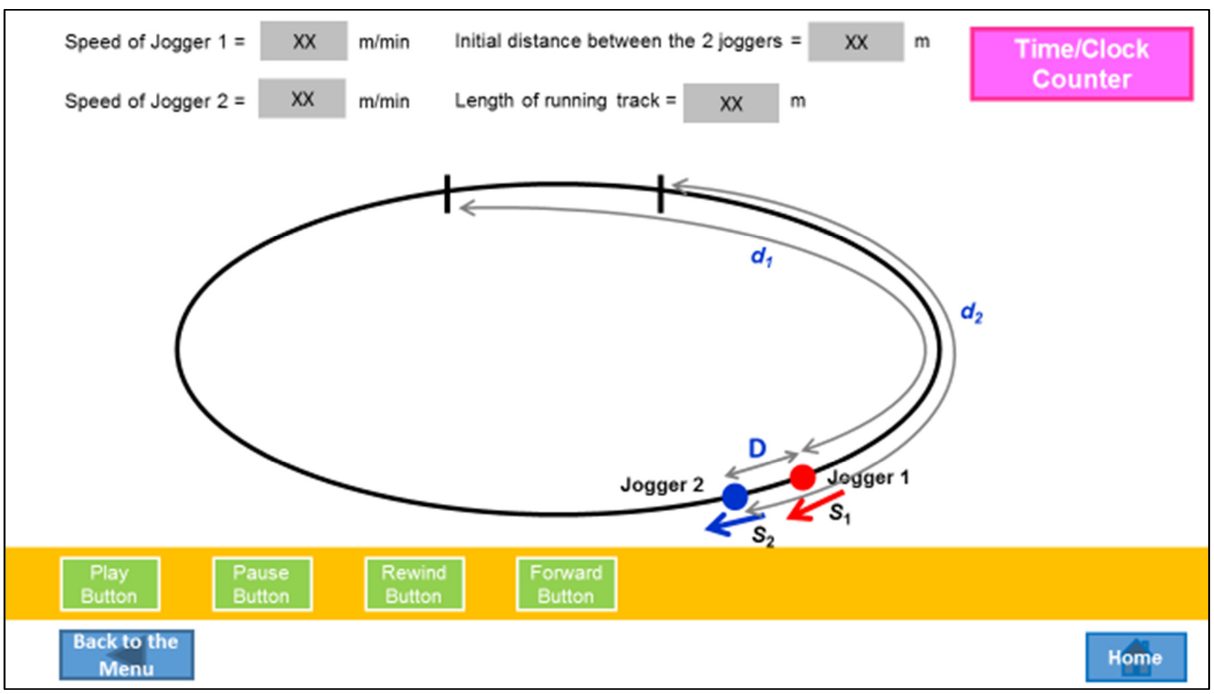
Robot: "Let's see what happens when we have two joggers who are running in the same direction on a circular running track. What happens when they start moving? Will they ever meet?"

"First key in the speeds of the 2 joggers. Next, key in the length of the running track. Lastly, key in the initial distance between the 2 joggers. When you are ready, press play to see them go!"

"You may click on the pre-set time buttons to see where they are at each minute and when they meet."



The diagram below shows how it may appear at time, t, when the simulation is played.



Features of the simulation

Users can change the following independent variables in the simulation before playing:

- Speed of Jogger 1 (any number between 0 to 200 m/min)
- Speed of Jogger 2 (any number between 0 to 200 m/min)
- Distance between the two joggers (any whole number between 0 to L m). Once the value is being entered by the user, the distance value will appear between the 2 joggers on the screen. This value will **change** accordingly as the 2 persons jog.
- Length of the running track, L m (any number between 100 to 10 000 m)

There will be 5 other dependent variables that will be reflected on the screen as the simulation is played.

- Time elapsed – There should be a clock/time counter (in minutes and hours) to show the **time elapsed** (that changes continuously) as the 2 joggers jog in the same direction. **There should be preset buttons for users to click on to jump to specific times during the simulation** e.g.
    - 1 min later,
    - 2 mins later,
    - 3 mins later,
    - 1st meeting point (when distance between 2 joggers is 0 i.e. at time $t = \dfrac{D_0}{s_1 - s_2}$)
    - 2nd meeting point (when distance between 2 joggers is L i.e. at time $t = \dfrac{L + D_0}{s_1 - s_2}$)
    - 3rd meeting point (when distance between 2 joggers is $2L$ i.e. at time $t = \dfrac{2L + D_0}{s_1 - s_2}$)
    - 4th meeting point (when distance between 2 joggers is $3L$ i.e. at time $t = \dfrac{3L + D_0}{s_1 - s_2}$)
    - 5th meeting point (when distance between 2 joggers is $4L$ i.e. at time $t = \dfrac{4L + D_0}{s_1 - s_2}$)
- Distance travelled by Jogger 1, d_1 (measuring from its starting point)
- Distance travelled by Jogger 2, d_2 (measuring from its starting point)
- Distance between the two joggers, D
- Number of times joggers met, n

After entering the 4 variables, speed of Jogger 1, speed of Jogger 2, Initial distance between the 2 joggers and Length of running track, user may click on play to start the simulation. The 2 joggers will move at the specified speed. Depending on the speed of the joggers, the following will happen:

- If $s_1 > s_2$, Jogger 1 will catch up with Jogger 2
- If $s_1 < s_2$, Jogger 2 will catch up with Jogger 1
- If $s_1 = s_2$, the joggers will never meet

Every time they meet, it prompts the animation to **indicate that the joggers have met**. The animation continues to play as both joggers pass each other. **The animation continues in an infinite loop**. Animation can be paused at any point. The animation will play again when the user press on the Play button again.

Formula

The following are formulas that should be embedded in the simulation to help compute all the dependent variables, so that they can be accurately reflected on the screen as the simulation plays:

At any time t (in hours) [may be in decimals],

$$d_1 = s_1 t$$

$$d_2 = s_2 t$$

Distance between joggers,

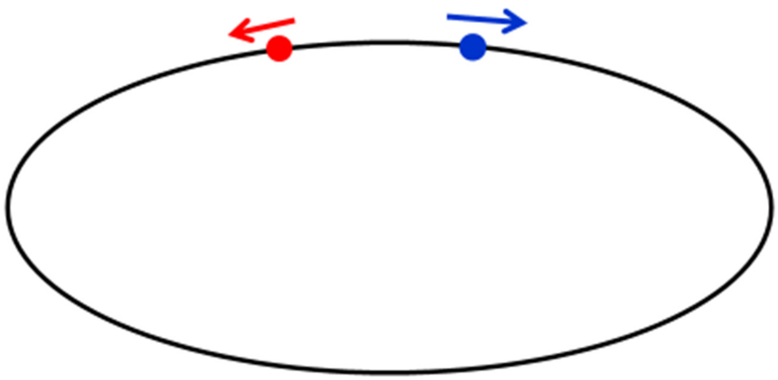
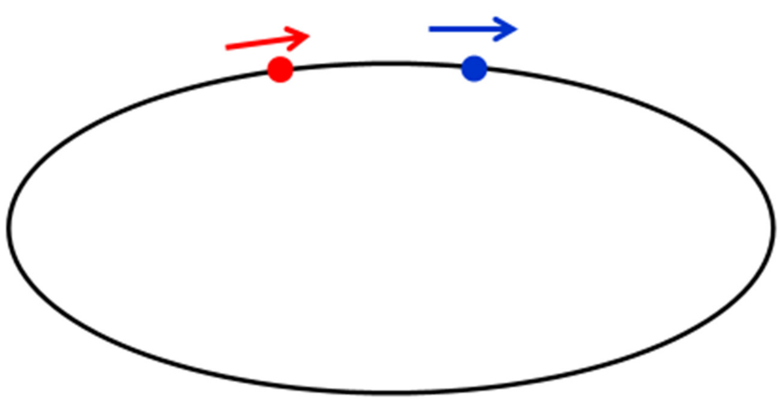
$$D = \min\left(\mathrm{MOD}\left(\left|(D_0 + s_2 t) - s_1 t\right|, L\right), L - \mathrm{MOD}\left(\left|(D_0 + s_2 t) - s_1 t\right|, L\right)\right)$$

Number of times joggers met, $n = \frac{\lvert D_0 + s_2 t - s_1 t \rvert}{L}$ where $\boxed{d_1}$ and $\boxed{d_2}$ are the distance travelled by Jogger 1 and Jogger 2 respectively. $\boxed{D_0}$ is the initial distance between the 2 joggers entered by the user. $\boxed{L}$ is the length of the running track entered by the user.

Include button for returning to **1.3 Home,** and a back button for returning to the **5.1 Menu 4.**

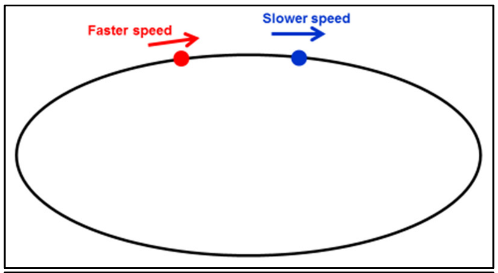
5.4 Quiz

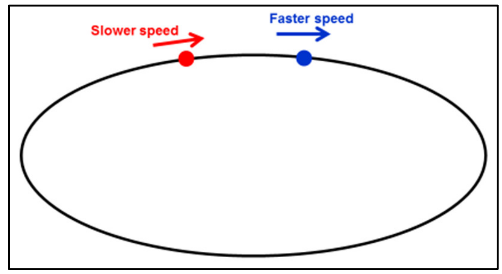
There are 3 questions to be attempted in sequence and scored.

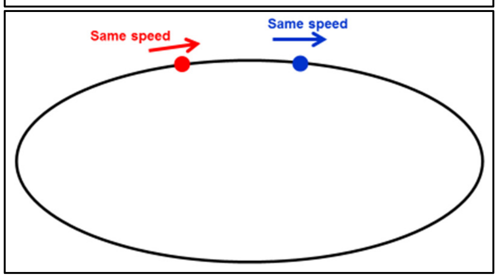
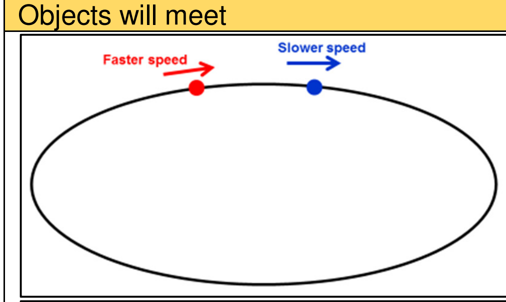
Question 1	When 2 objects move in the opposite direction in a circular route, will they ever meet? (show the diagram below)  ☐ Yes ☐ No
Answer 1	☒ Yes ☐ No
Feedback 1	If answer is correct, provide feedback: “Good job!” Then move to Question 2 for user to attempt. If answer is incorrect, provide feedback: “Try again.” Then move to Question 1 for user to attempt again.
Question 2	When 2 objects move in the same direction, will they **always** meet? (show the diagram below)  ☐ Yes ☐ No

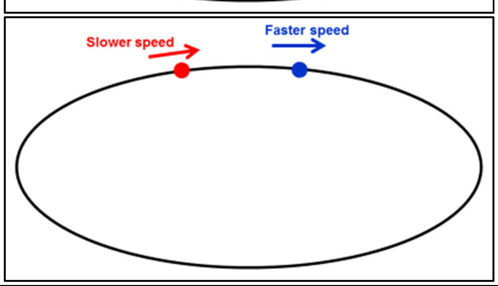
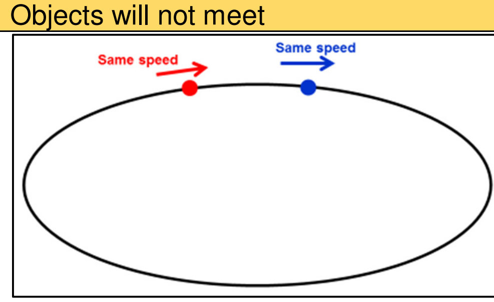
<table>
<tr><td>Answer 2</td><td>☐ Yes
☒ No</td></tr>
<tr><td>Feedback 2</td><td>If answer is correct, provide feedback: “Good job!” Then move to Question 3 for user to attempt.

If answer is incorrect, provide feedback: “Try again.” Then move to Question 2 for user to attempt again.</td></tr>
<tr><td>Question 3</td><td>2 objects start moving in the same direction at the same time. Drag and drop the 3 pictures into the correct box. (Drag and drop is just a suggestion. The question can also be presented in other forms e.g. checkbox)

<table><tr><th>Objects will meet</th><th>Objects will not meet</th></tr><tr><td></td><td></td></tr></table>





</td></tr>
<tr><td>Answer 3</td><td><table><tr><th>Objects will meet</th><th>Objects will not meet</th></tr><tr><td>


</td><td>
</td></tr></table></td></tr>
<tr><td>Feedback 3</td><td>If answer is correct, provide feedback: “Good job!” Then move to 5.1 Menu 4.

If answer is incorrect, provide feedback: “Try again.” Then move to Question 3 for user to attempt again.</td></tr>
</table>

Scene 6 – Graphs of Distance and Speed

6.1 Travel Graphs

Prior to simulation, there will be an animation (with voice over) of the Robot giving brief introduction and instructions to enter the variables.

Robot: “How can we represent the motion of a car? Here we will find out how!”

“First, select which type of motion you would like to explore – constant speed, increasing speed, or decreasing speed. Next, select which graphs you would like to view.”

Features of the simulation

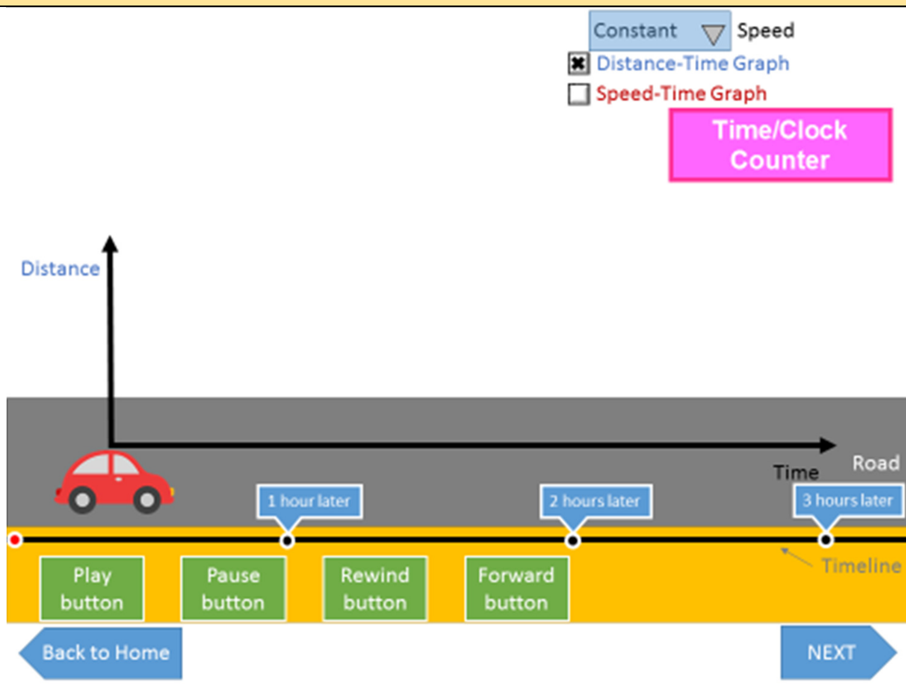
Users can make choices on the following:

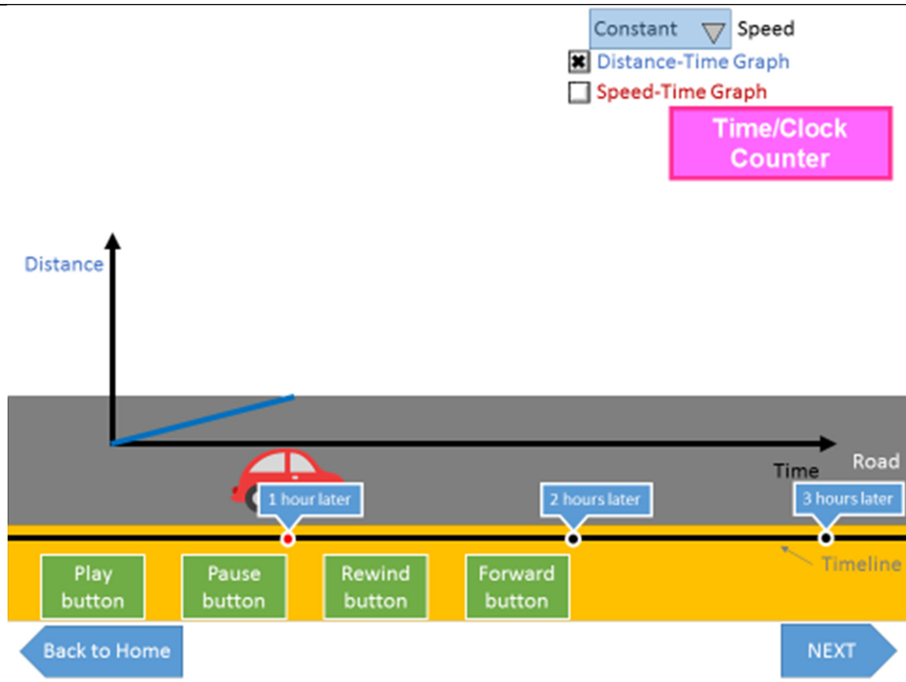
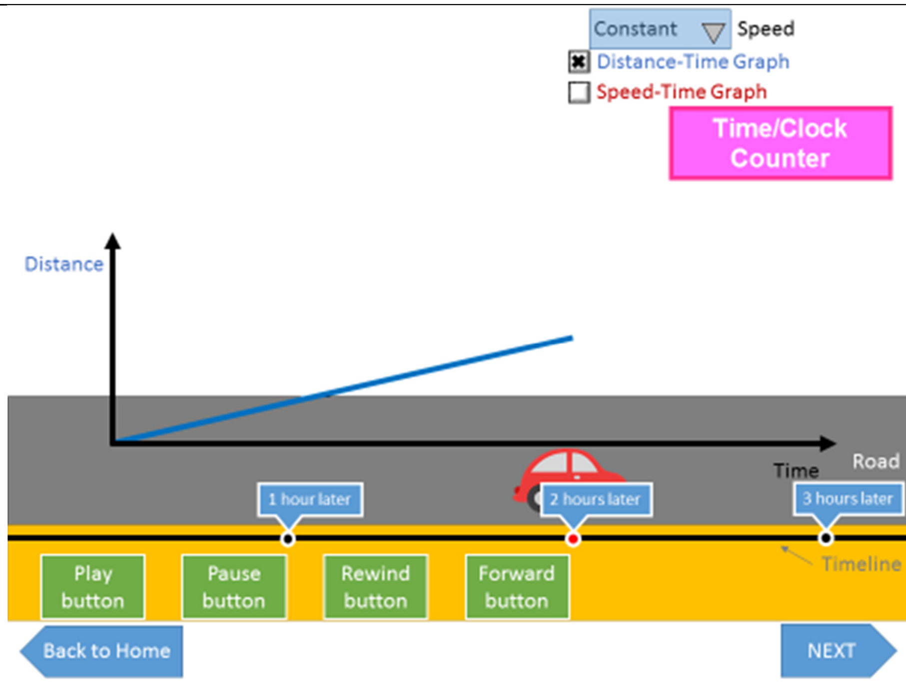
- Checkbox to select either constant speed, increasing speed, or decreasing speed. Can only select one at a time.
- checkbox to select distance-time graph or speed-time graph or both. Once checked, the selected graph(s) would be displayed and drawn simultaneously as the car moves. If speed-time graph is selected, a speedometer appears next to the car and displays current speed of car.

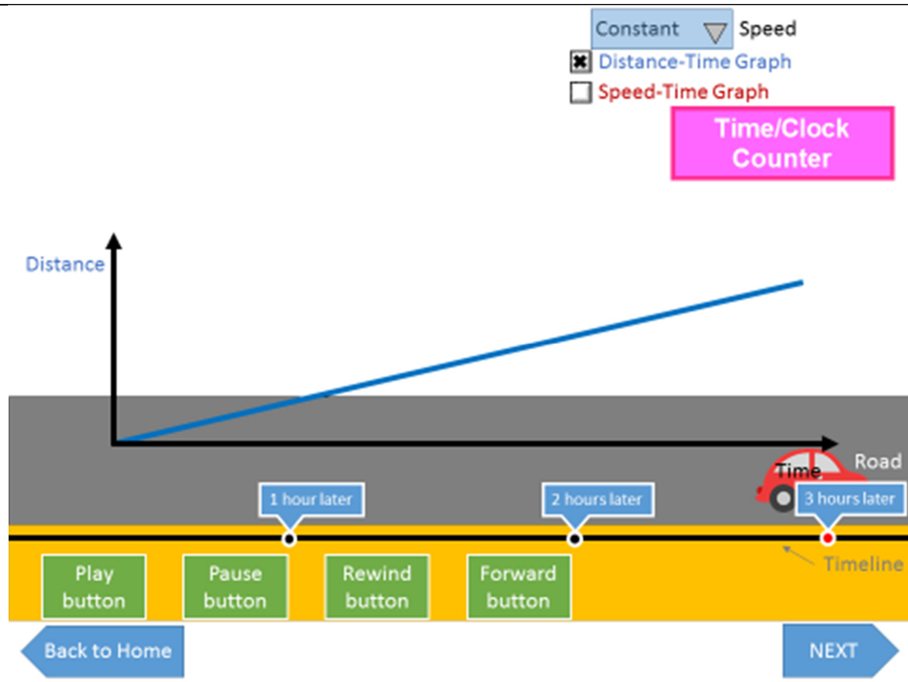
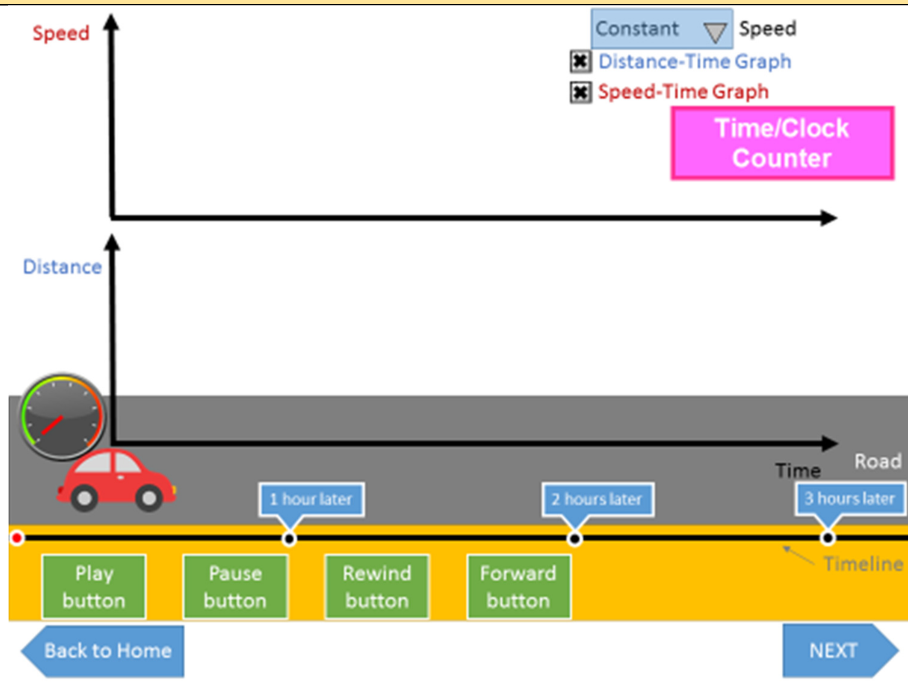
Once the two choices are made, users are able to press play to view the motion of the car and the selected graph(s) should be drawn simultaneously as the car moves. Motion should be **continuous** (note that diagrams below only show hour by hour as an example).

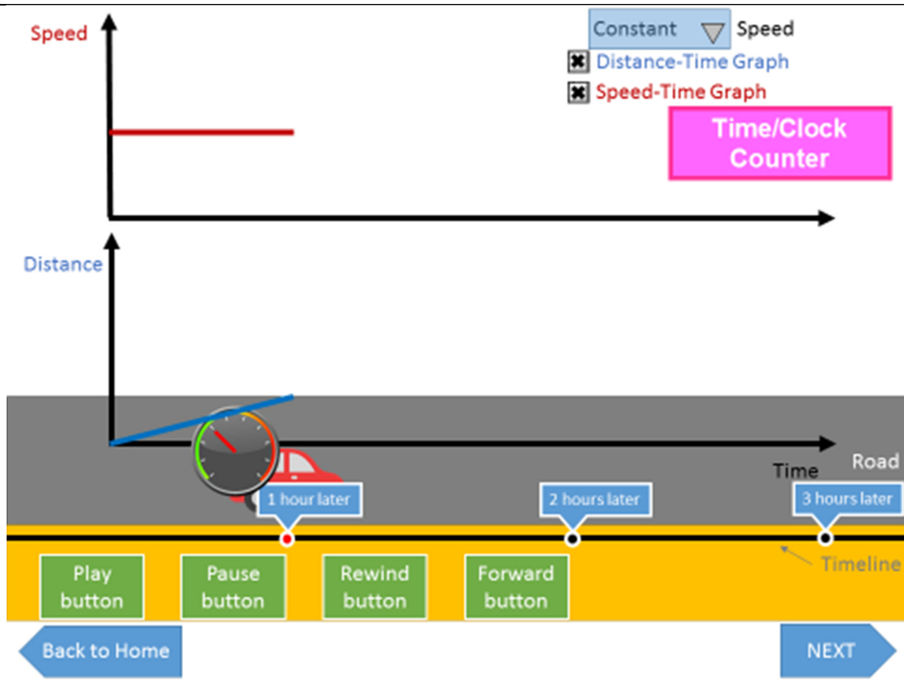
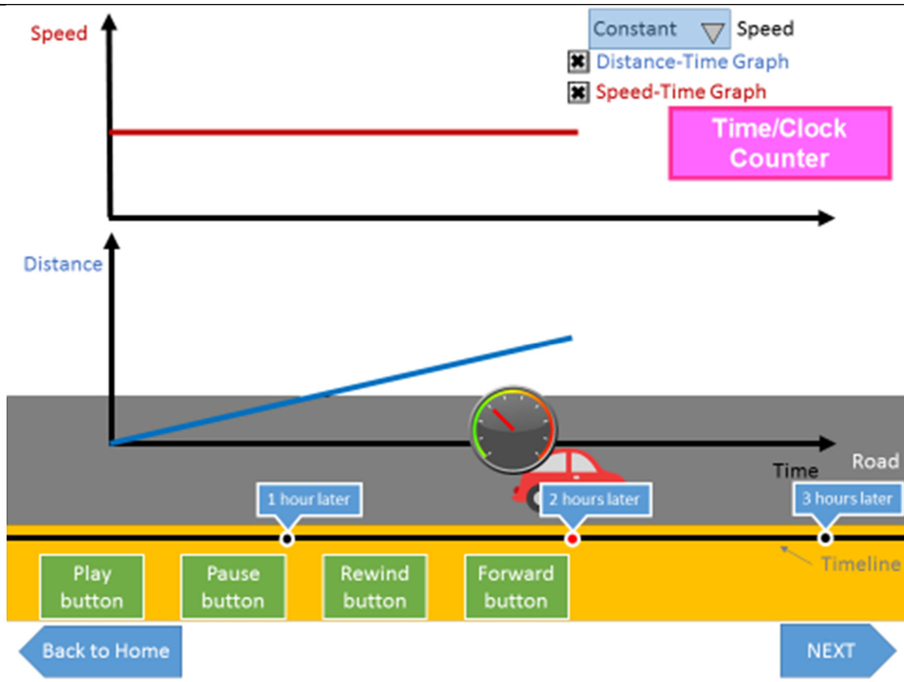
It should include timeline and buttons for play, pause, rewind and forward. Include button for returning to **1.3 Home**.

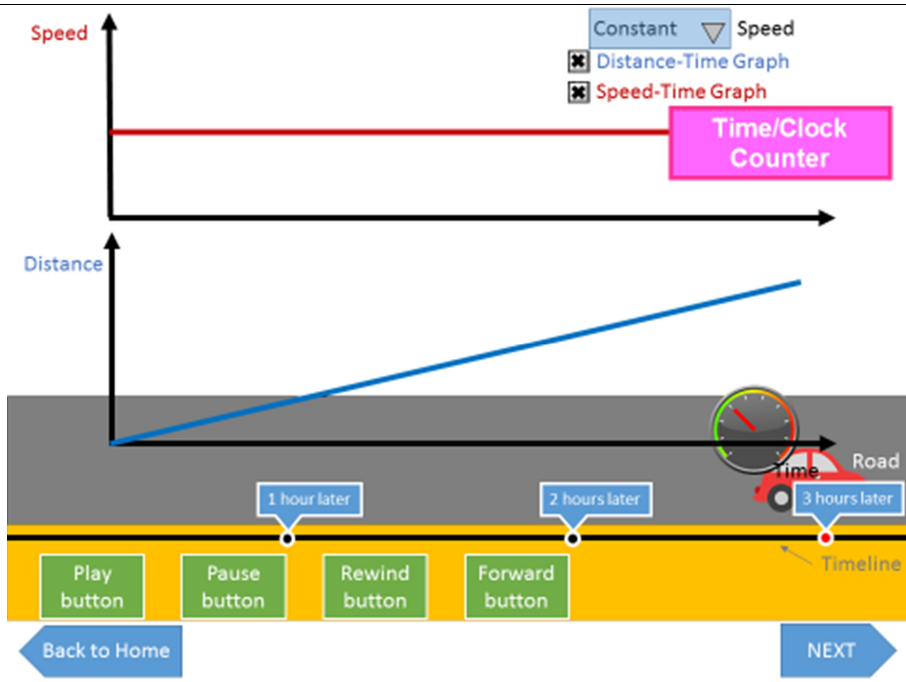
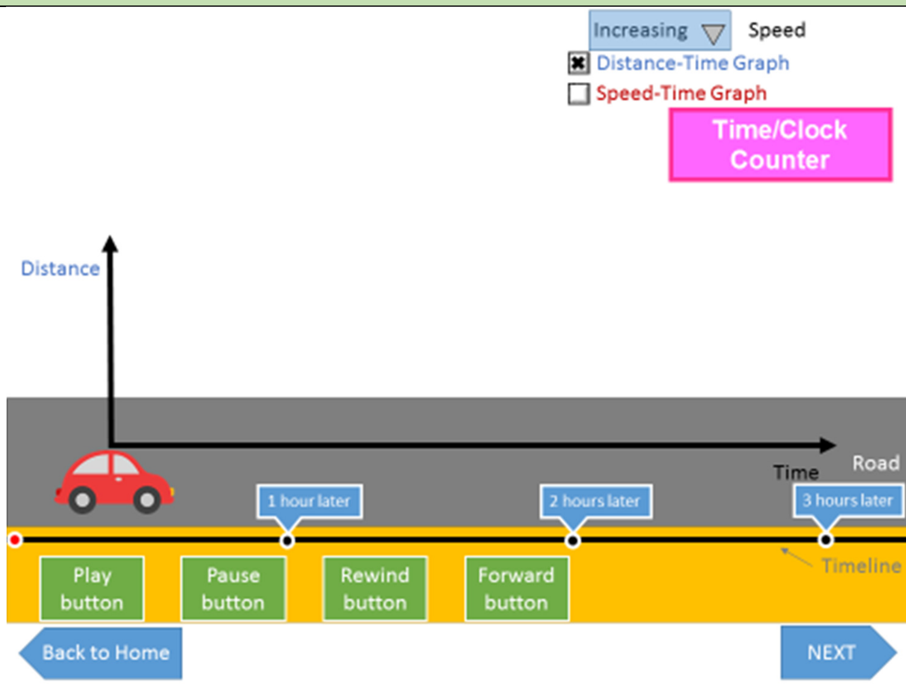
The following diagrams illustrate the graphs for constant speed, increasing speed and decreasing speed.

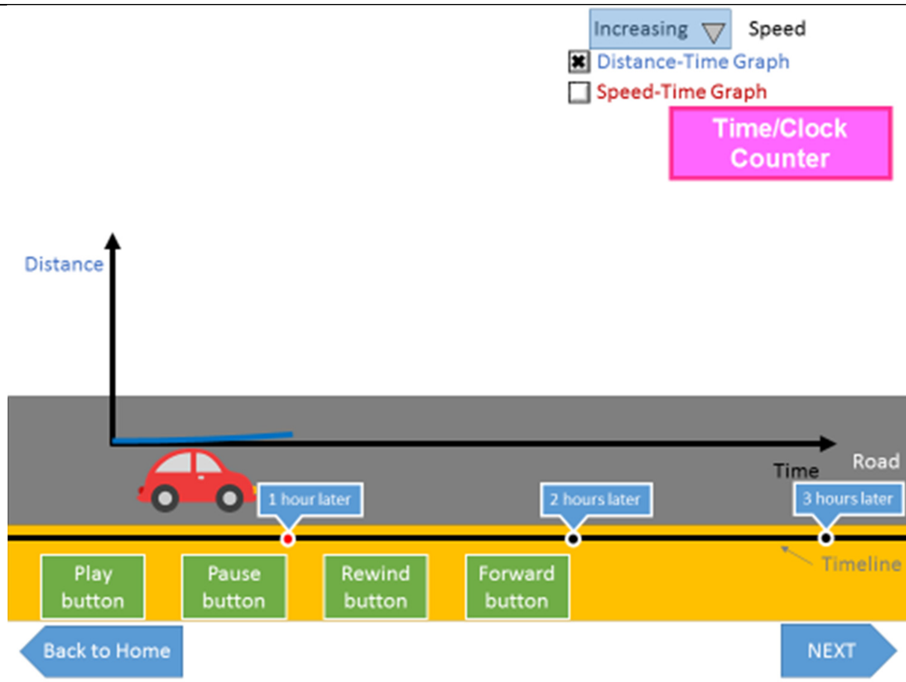
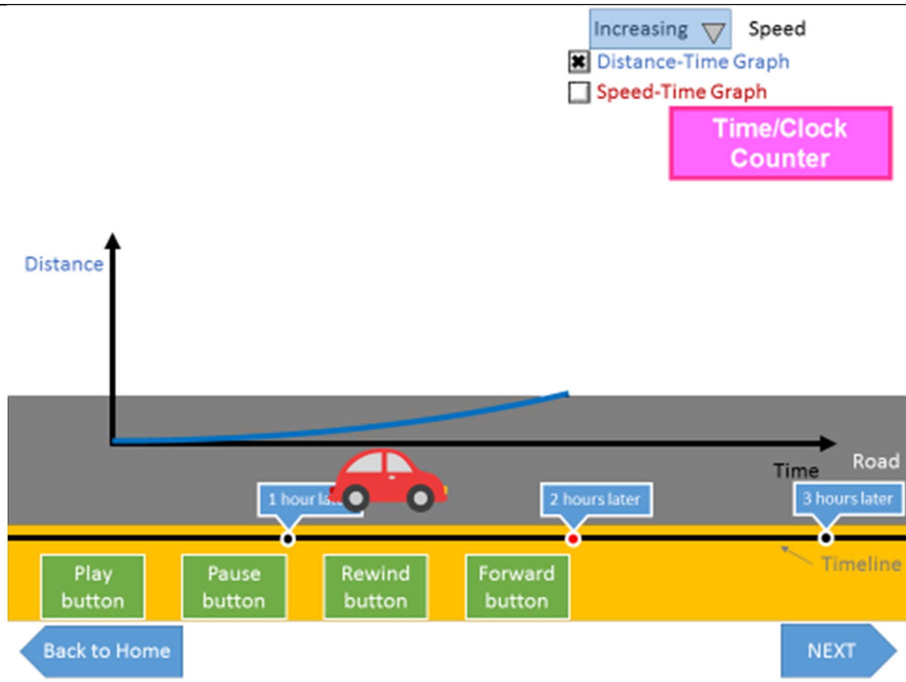
1. **Constant speed**: If user selects distance-time graph only. Car must move at the same speed throughout the motion.	
1a	

1b	Constant Speed Distance-Time Graph Speed-Time Graph Time/Clock Counter Distance Time Road 1 hour later 2 hours later 3 hours later Timeline Play button Pause button Rewind button Forward button Back to Home NEXT
1c	Constant Speed Distance-Time Graph Speed-Time Graph Time/Clock Counter Distance Time Road 1 hour later 2 hours later 3 hours later Timeline Play button Pause button Rewind button Forward button Back to Home NEXT

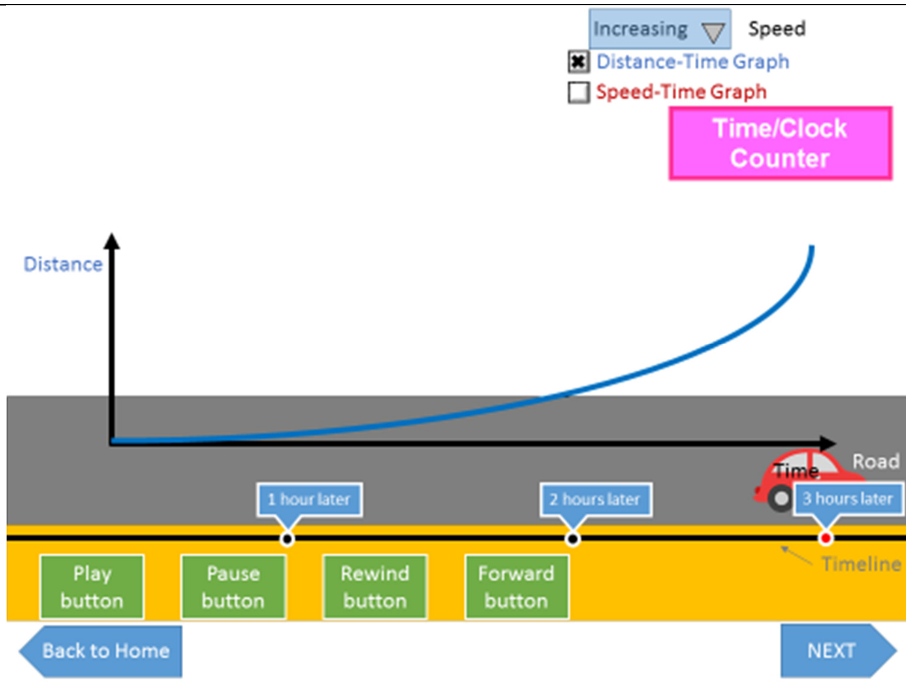
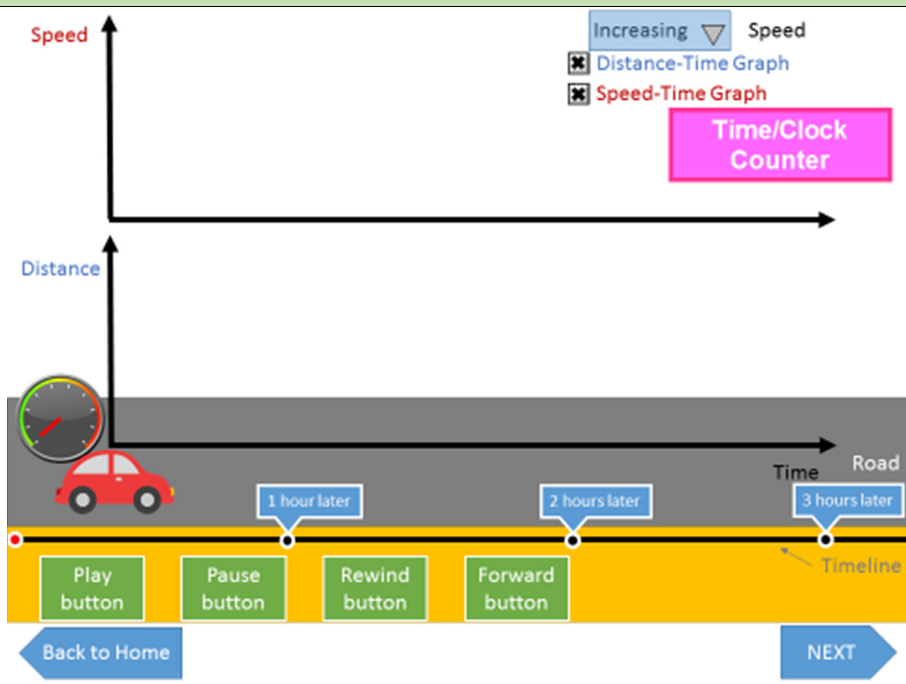
1d	
2. **Constant speed**: If user selects both distance-time and speed-time graphs Note that speedometer must show the speed remaining constant.	
2a	

<table>
<tr><td>2b</td><td>
</td></tr>
<tr><td>2c</td><td>
</td></tr>
</table>

2d	
3. **Increasing speed**: If user selects distance-time graph only. Car must accelerate throughout the motion.	
3a	

3b	
3c	

Increasing
Speed
Distance-Time Graph
Speed-Time Graph
Time/Clock Counter
Distance
Time
Road
1 hour later
2 hours later
3 hours later
Timeline
Play button
Pause button
Rewind button
Forward button
Back to Home
NEXT

<table>
<tr><td>3d</td><td>
</td></tr>
<tr><td colspan="2">4. Increasing speed: If user selects both distance-time and speed-time graphs
Note that speedometer must show the speed increasing at a constant rate.</td></tr>
<tr><td>4a</td><td>
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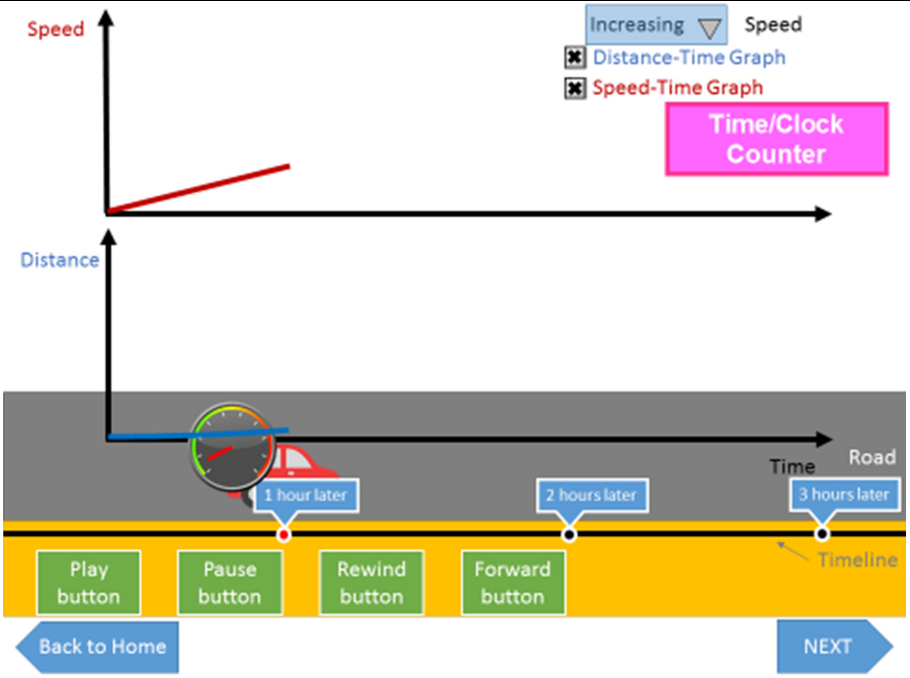
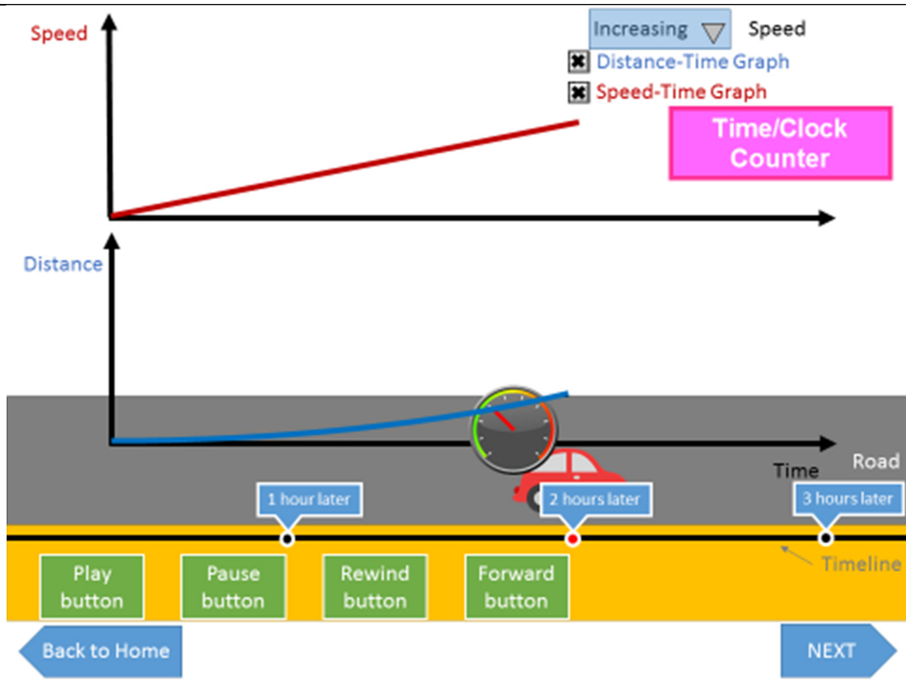
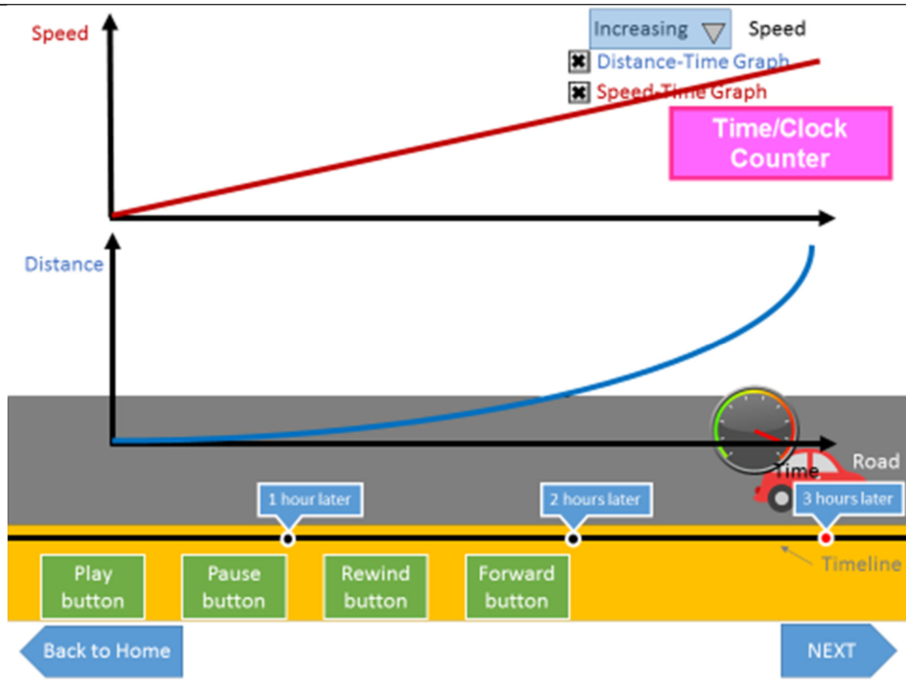
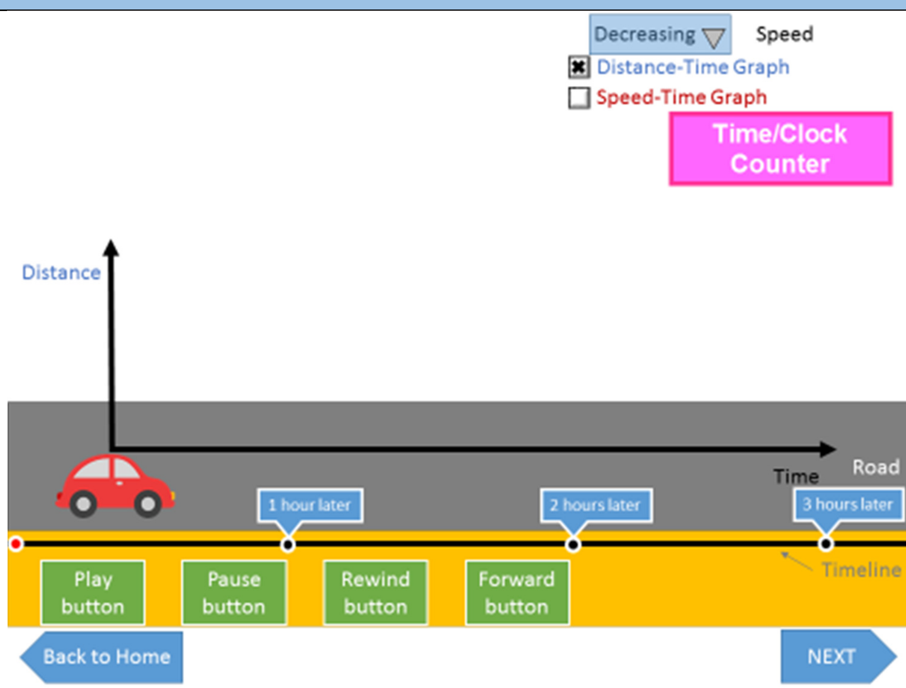
4b	Speed
Increasing
Speed
Distance-Time Graph
Speed-Time Graph
Time/Clock Counter
Distance
Time
Road
1 hour later
2 hours later
3 hours later
Timeline
Play button
Pause button
Rewind button
Forward button
Back to Home
NEXT
4c	Speed
Increasing
Speed
Distance-Time Graph
Speed-Time Graph
Time/Clock Counter
Distance
Time
Road
1 hour later
2 hours later
3 hours later
Timeline
Play button
Pause button
Rewind button
Forward button
Back to Home
NEXT

<table>
<tr><td>4d</td><td>
</td></tr>
<tr><td colspan="2">5. Decreasing speed: If user selects distance-time graph only.
Car must decelerate throughout the motion.</td></tr>
<tr><td>5a</td><td>
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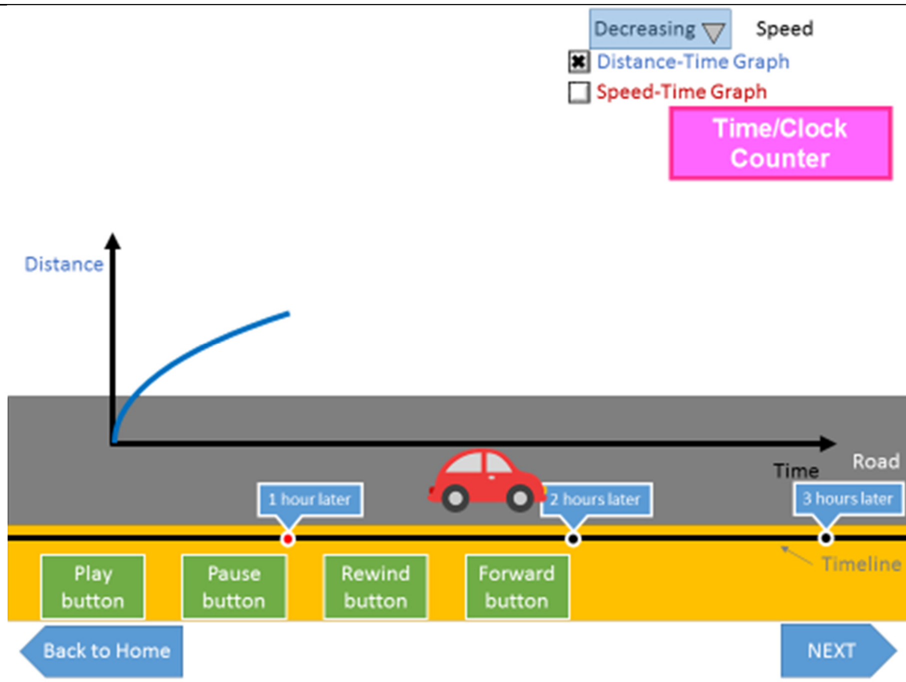
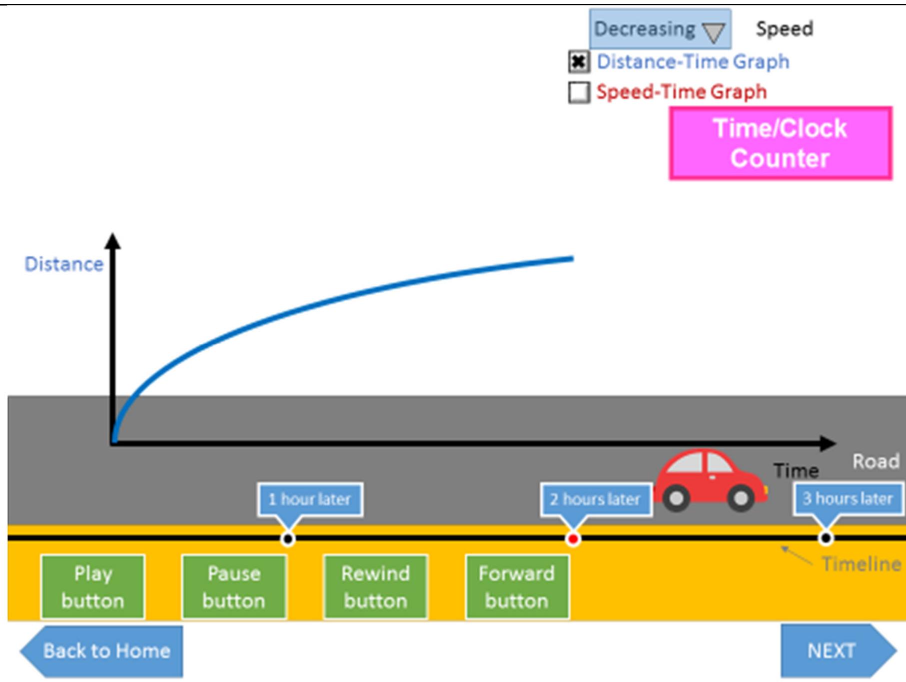
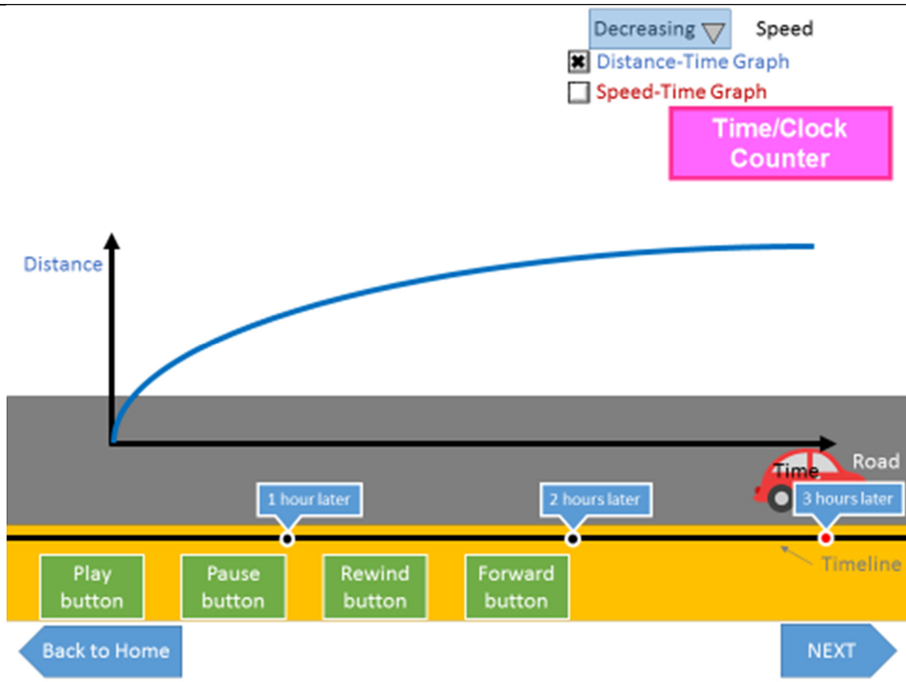
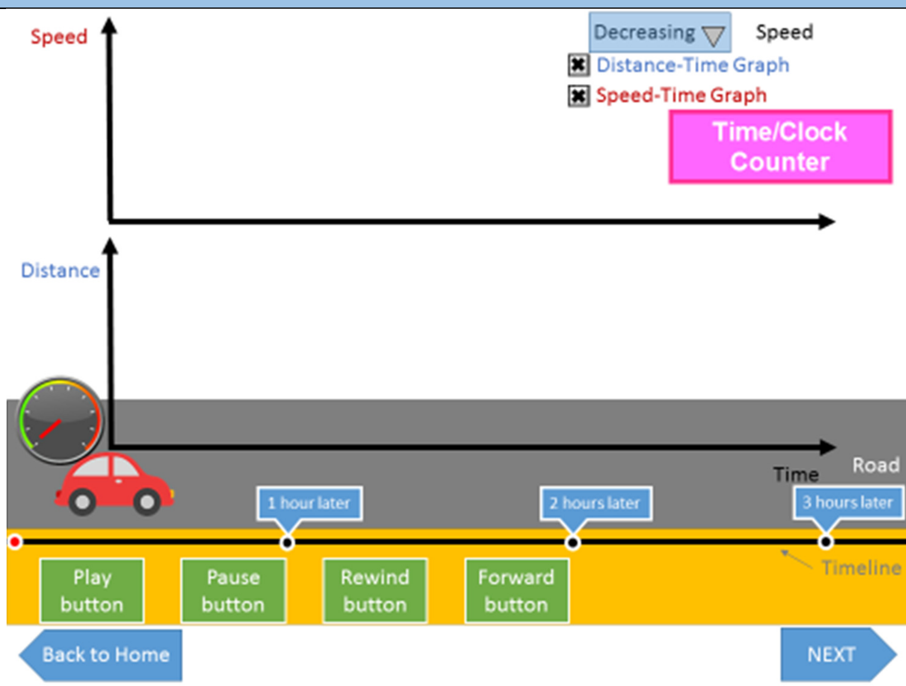
5b	Decreasing Speed Distance-Time Graph Speed-Time Graph Time/Clock Counter Distance Time Road 1 hour later 2 hours later 3 hours later Timeline Play button Pause button Rewind button Forward button Back to Home NEXT
5c	Decreasing Speed Distance-Time Graph Speed-Time Graph Time/Clock Counter Distance Time Road 1 hour later 2 hours later 3 hours later Timeline Play button Pause button Rewind button Forward button Back to Home NEXT

<table>
<tr><td>5d</td><td>
</td></tr>
<tr><td colspan="2">6. Decreasing speed: If user selects both distance-time and speed-time graphs
Note that speedometer must show the speed decreasing at a constant rate.</td></tr>
<tr><td>6a</td><td>
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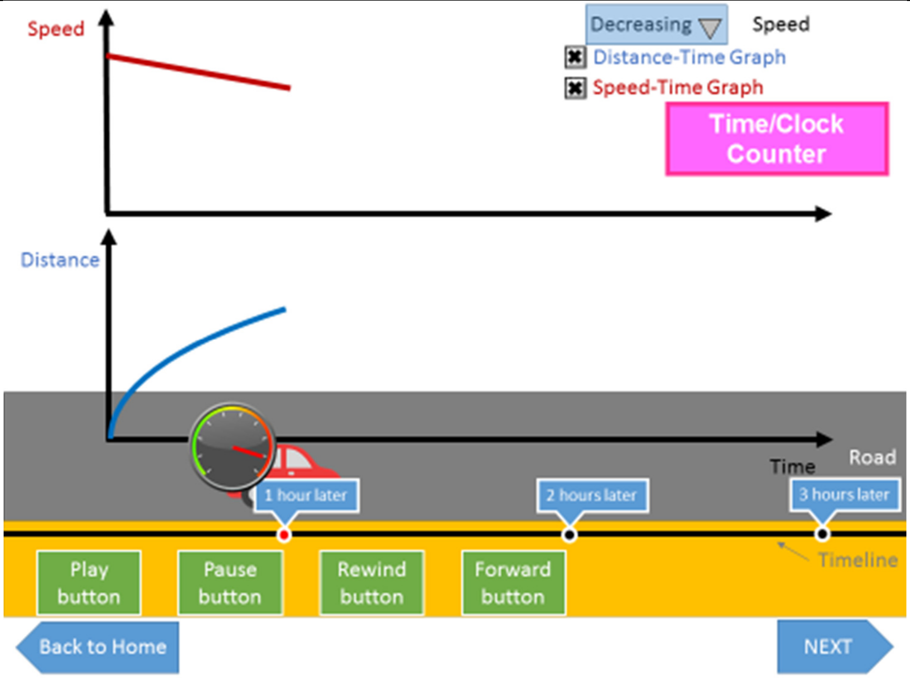
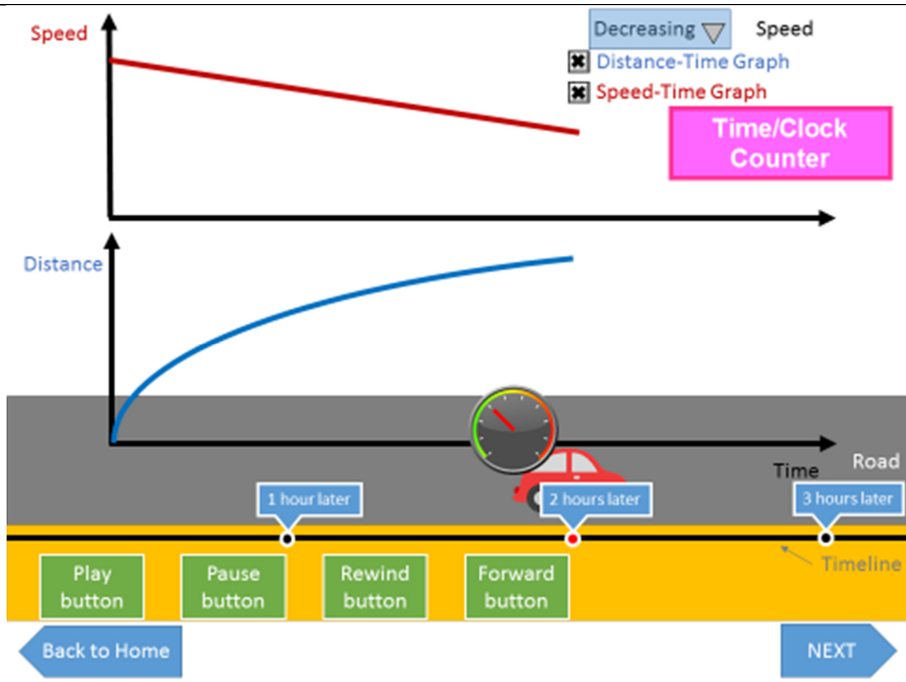
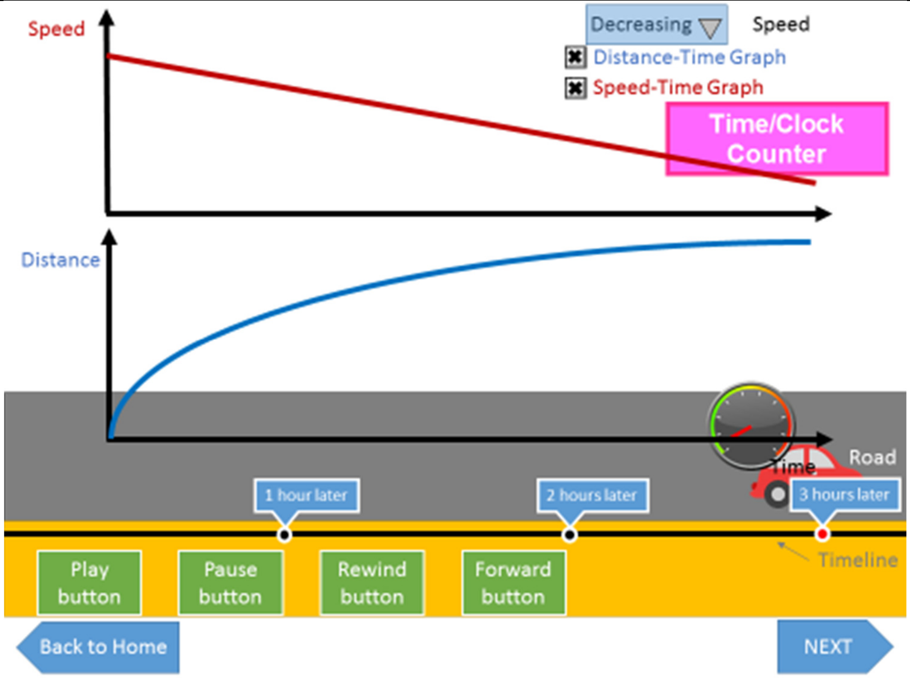
6b	 Speed Decreasing Speed Distance-Time Graph Speed-Time Graph Time/Clock Counter Distance Time Road 1 hour later 2 hours later 3 hours later Timeline Play button Pause button Rewind button Forward button Back to Home NEXT
6c	 Speed Decreasing Speed Distance-Time Graph Speed-Time Graph Time/Clock Counter Distance Time Road 1 hour later 2 hours later 3 hours later Timeline Play button Pause button Rewind button Forward button Back to Home NEXT

6d	

Speed
Decreasing
Speed
Distance-Time Graph
Speed-Time Graph
Time/Clock Counter
Distance
Time
Road
1 hour later
2 hours later
3 hours later
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Play button
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Rewind button
Forward button
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NEXT

Scene 7 – Exploration Mode

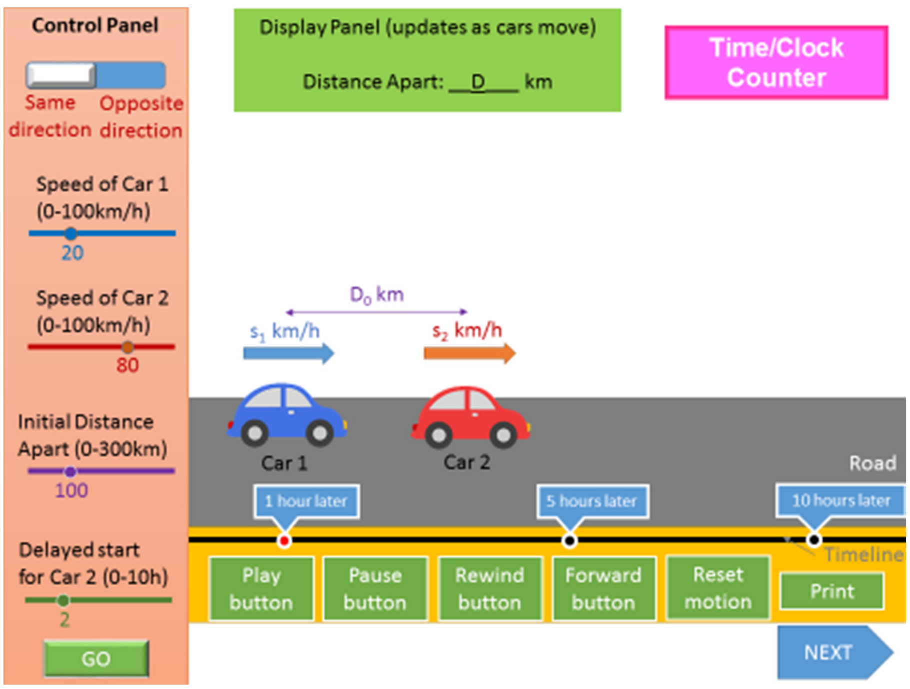
7.1 Exploration Mode

Robot introduces the exploration mode. Where animations and simulations were previously specific to a particular scenario with limited variables to focus pupils' exploration, the exploration mode would be a more comprehensive environment where users can vary the following factors more freely and generate the simulation for two cars:

- Whether cars are moving in opposite or same direction
- Speed of Car 1
- Speed of Car 2
- Initial distance apart
- Delay in start time of Car 2

After specifying the variables in the control panel, users click on "Go". The specified values for speeds and distance apart will appear on the screen. When users click on "Play", the simulation begins. As the simulation progresses, the display panel shows the distance apart and the clock reflects the time elapsed.

Include an option to print the scenario created by pupils as a record of their exploration.



Formula

For opposite direction,

$$D = D_0 - s_1 t - s_2 (t - t_2)$$

where s_1 and s_2 are the speed of Car 1 and Car 2 respectively. D_0 is the initial distance between the 2 cars entered by the user. t_2 is the starting time of Car 2 after Car 1 has started.

For same direction (Car 2 in front of Car 1),

$$D = \left| s_1 t - D_0 - s_2 (t - t_2) \right|$$

where d_1 and d_2 are the distance travelled by Car 1 and Car 2 respectively. D_0 is the initial distance between the 2 cars entered by the user. t_2 is the starting time of Car 2 after Car 1 has started.