A Geometric System with Infinity at the Origin: A Hyperreal Framework for Scale and Dynamics

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Abstract

This article introduces a novel geometric framework where infinity is positioned at the origin, an infinitesimal scale defines the boundary, and zero is excluded from the system. Constructed using hyperreal numbers, this framework reimagines the traditional plane and extends to configuration spaces for multi-point systems with generalized properties. It leverages scale-shape decomposition, flexible time transformations, and compatibility with curved spaces to unify individual points, configurations, and dynamics. A quantum adaptation enhances the system by incorporating quantum states, operators, and time transformations, optimizing its utility for quantum computing algorithms. Key properties—such as spatial compactification, metric inversion, and asymptotic duality—support applications in asymptotic analysis, numerical computation, differential equations, multi-body dynamics, gravitational systems, numerical simulation, economic modeling, and quantum computing. This versatile system equips mathematicians and scientists with tools to explore phenomena across vast, minute, and quantum scales with precision and clarity.

1 Introduction

The Cartesian plane, with zero at its center and infinity at its extremities, has been a foundational tool in mathematics. However, it struggles at extreme scales: infinities complicate limit evaluations, singularities disrupt behavior near zero, and unbounded domains challenge computational methods. This article proposes a geometric system that inverts this paradigm by placing infinity at the origin and excluding zero beyond an infinitesimal boundary. Built on the hyperreal numbers, the τ-plane tames infinite scales and encapsulates all phenomena within an infinitesimal frontier.

For multi-point systems, the framework employs logarithmic mappings and dynamic time adjustments to separate scale from shape, facilitating analysis of overall size and relative configuration. Grounded in a rigorous axiomatic foundation, it transcends conventional geometry, offering theoretical depth and practical applicability. A quantum adaptation further extends its scope, integrating quantum states and operators to address quantum computing challenges. This system finds applications in diverse fields—from asymptotic function behavior to celestial mechanics, economic dynamics, and quantum algorithms—providing a unified lens to examine phenomena across scales and fulfilling a perennial mathematical aspiration: *to measure the immeasurable.*

2 Axiomatic Foundation

The framework is built upon the hyperreal numbers $*\mathbb{R}$, encompassing finite reals, infinitesimals (positive numbers smaller than any real), and infinite numbers (larger than any real). This foundation ensures robust handling of extreme scales. The axioms are categorized into three classical domains—the τ-plane for individual points, configuration spaces for multiple points with generalized properties, and dynamics—augmented by a quantum adaptation for quantum computing.

2.1 Axioms for the τ-Plane (Individual Points)

1. **The τ-Plane** The τ-plane is defined as $(*\mathbb{R})^d$, the space of d-tuples $\boldsymbol{\tau} = (\tau_1, \tau_2, \ldots, \tau_d)$, where each $\tau_i \in *\mathbb{R}$, generalizing the system to d dimensions. It is equipped with the Euclidean metric $d_\tau = \sqrt{\tau_1^2 + \cdots + \tau_d^2}$.

2. **Mapping to the r-Plane** For $\boldsymbol{\tau}$ with $\tau_i \neq 0$ for all i, the corresponding r-plane position is $\mathbf{r} = \left(\frac{1}{\tau_1}, \frac{1}{\tau_2}, \ldots, \frac{1}{\tau_d}\right)$, defined over $(*\mathbb{R} \setminus \{0\})^d$. The r-plane inherits the Euclidean metric $d_r = \sqrt{r_1^2 + \cdots + r_d^2}$.

3. **Origin as Infinity** The origin $\boldsymbol{\tau} = \mathbf{0}$ maps to an infinite scale in the r-plane. Infinitesimal $\boldsymbol{\tau}$ corresponds to infinite $\mathbf{r}$, preserving the signs of components.

4. **Infinitesimal Boundary** Points where $|\boldsymbol{\tau}| = \sqrt{\tau_1^2 + \cdots + \tau_d^2}$ is infinite correspond to infinitesimal $\mathbf{r}$, delineating the system's smallest scales.

5. **Scale Spheres** For a hyperreal $\rho > 0$, the sphere $|\boldsymbol{\tau}| = \rho$ implies $|\mathbf{r}| \approx \frac{1}{\rho}$. Infinitesimal ρ yields infinite $|\mathbf{r}|$, while infinite ρ yields infinitesimal $|\mathbf{r}|$.

6. **Scaling Identity** For an infinitesimal $\delta > 0$ and infinite $H = \frac{1}{\delta}$, the identity $H \cdot \delta = 1$ links reciprocal scales.

7. **Directional Continuity** As τ_i transitions through infinitesimals around zero, $r_i = \frac{1}{\tau_i}$ shifts from positive to negative infinity, maintaining directional consistency.

8. **Extension to Curved Spaces** For curved r-spaces modeled as Riemannian manifolds (N, g_r), the τ-plane generalizes via a diffeomorphism $\phi : M \to N$, where M is a manifold with metric g_τ. Here, ϕ maps a designated point $p_0 \in M$ to infinity in N, and points distant from p_0 to small scales in N. The flat τ-plane serves as a local approximation for computational simplicity.

2.2 Axioms for Configuration Spaces (Multiple Points)

9. **Configuration Space for Multiple Points** For n points, each point i has a weight $w_i > 0$ and a property vector $\mathbf{p}_i = (p_{i1}, p_{i2}, \ldots, p_{ik})$, where w_i weights positional contributions, and $\mathbf{p}_i$ denotes system-specific attributes (e.g., mass, charge, capital). Positions are $\mathbf{r}_1, \ldots, \mathbf{r}_n \in \mathbb{R}^d$, potentially constrained by $\sum_{i=1}^n w_i \mathbf{r}_i = \mathbf{0}$. Weights and properties are system-dependent, e.g., $w_i = ||\mathbf{p}_i||$ or optimized values.

10. **Scale Factor** The scale factor is defined as $s = \sqrt{\frac{\sum_{i=1}^n w_i |\mathbf{r}_i|^2}{W}}$, where $W = \sum_{i=1}^n w_i$, quantifying the configuration's overall size.

11. **Flexible Scale Coordinate** Define $\sigma = \log s$, where $\sigma \in (-\infty, \infty)$, mapping $s \to \infty$ to $\sigma \to \infty$ and $s \to 0$ to $\sigma \to -\infty$.

12. **Shape and Orientation Coordinates** Shape coordinates θ (scale-invariant) describe relative positions, while orientation coordinates ϕ (in the rotation group) account for rotation—e.g., in 2D, θ spans a shape sphere, and ϕ is an angle.

13. **Full Parameterization** The configuration space is parameterized as (σ, ϕ, θ), with σ on the real line, ϕ in the rotation group, and θ in the shape space.

14. **Configuration Extremes** As $\sigma \to \infty$, the system expands infinitely; as $\sigma \to -\infty$, it collapses, with θ preserving shape.

15. **Interaction with τ-Mappings** Each $\mathbf{r}_i$ maps to $\boldsymbol{\tau}_i = \left(\frac{1}{r_{i1}}, \ldots, \frac{1}{r_{id}}\right)$, connecting local and global coordinates.

2.3 Axioms for Dynamics

16. **Flexible Time Transformation** Define time τ such that $\frac{dt}{d\tau} = e^{f(\sigma)}$, where $f(\sigma)$ regularizes the system's driving function F (e.g., kinetic energy, cost functions), ensuring F in τ-time lacks exponential dependence on σ. For systems where $F \sim e^{k\sigma} G(\theta, \phi)$, set $f(\sigma) = \frac{k}{m}\sigma$, with m as the homogeneity degree in derivatives.

17. **Dynamic Simplification** This transformation regularizes F, simplifying analysis at extreme scales. In mechanics, kinetic energy T becomes polynomial in $\frac{d\sigma}{d\tau}$, stabilizing large-scale dynamics.

2.4 Quantum Adaptation

These axioms extend the framework to quantum computing, enhancing its relevance to quantum algorithms.

18. **Quantum τ-Plane** The τ-plane extends to a *quantum τ-plane* over a Hilbert space $\mathcal{H}$. Each point $\boldsymbol{\tau}$ corresponds to a quantum state $|\psi_{\boldsymbol{\tau}}\rangle \in \mathcal{H}$,

with the origin $\boldsymbol{\tau} = \mathbf{0}$ representing maximal entanglement or infinite computational complexity (e.g., encoding solutions to unbounded problems).

19. **Quantum Configuration Space** For n quantum entities (e.g., qubits), the configuration space assigns each entity i a quantum state $|\psi_i\rangle$, a weight $w_i > 0$, and a property operator $\hat{P}_i$ acting on $\mathcal{H}$. Classical positions $\mathbf{r}_i$ are replaced by quantum position operators $\hat{\mathbf{r}}_i$, and the scale factor s becomes a quantum scale operator $\hat{s}$, measuring complexity or entanglement entropy.

20. **Quantum Scale and Shape** Define the quantum scale coordinate as $\sigma = \log \hat{s}$, where $\hat{s}$ is the quantum scale operator. Shape coordinates θ become quantum shape operators $\hat{\theta}$, describing quantum circuit topology or entangled states. Orientation coordinates ϕ transform into unitary transformations or quantum gates adjusting orientation in Hilbert space.

21. **Quantum Time Transformation** Introduce a quantum time parameter τ, with $\frac{dt}{d\tau} = e^{f(\sigma)}$, where $f(\sigma)$ regularizes the quantum driving function $\hat{F}$ (e.g., a Hamiltonian). Quantum state evolution is governed by:

$$i\hbar \frac{d}{d\tau}|\psi\rangle = \hat{H}_\tau |\psi\rangle,$$

where $\hat{H}_\tau$ is the effective Hamiltonian, optimized for scale-independent efficiency.

22. **Quantum Dynamic Simplification** The quantum driving function $\hat{F}$ in τ-time is regularized to eliminate exponential dependence on σ, ensuring quantum computations (e.g., circuit depth) remain polynomial as problem size scales exponentially.

3 Derived Properties

The axioms give rise to properties that enhance the system's utility across classical and quantum domains:

1. **Compactification of Space** The r-plane's infinite expanse maps to the τ-plane's origin $\boldsymbol{\tau} = \mathbf{0}$, while infinitesimal $\mathbf{r}$ extends to infinite $|\boldsymbol{\tau}|$. In the quantum τ-plane, this corresponds to maximal entanglement or complexity at $\boldsymbol{\tau} = \mathbf{0}$, compactifying quantum information.

2. **Metric Inversion** The Euclidean distance $d_\tau = |\boldsymbol{\tau}|$ inversely relates to $d_r = |\mathbf{r}|$: small d_τ near the origin corresponds to large d_r, and large d_τ maps to small d_r. In quantum terms, this inversion relates to entanglement measures or computational complexity.

3. **Asymptotic Duality** As $|\mathbf{r}| \to \infty$, $\boldsymbol{\tau} \to \mathbf{0}$; as $|\mathbf{r}| \to 0$, $|\boldsymbol{\tau}| \to \infty$. In quantum contexts, this duality facilitates analysis of algorithm behavior for large inputs near $\boldsymbol{\tau} = \mathbf{0}$.

4. **Series Transformation** Laurent series at $\mathbf{r} = \infty$ become Taylor series at $\boldsymbol{\tau} = \mathbf{0}$, simplifying asymptotic analysis. This extends to quantum operator expansions near the origin.

5. **Scale Compactification in Configurations** Infinite separation ($s \to \infty$) maps to finite σ, with time transformation bounding dynamics. In quantum configuration spaces, this ensures computational efficiency as problem size increases.

6. **Shape Invariance** Shape coordinates θ remain finite under scale changes, isolating geometric structure. Quantum shape operators $\hat{\theta}$ preserve circuit topology or state entanglement across scales.

7. **Energy Decomposition** Potentials often factor as $V = -e^{-\sigma}U(\theta)$, separating scale ($\sigma$) from shape ($\theta$). In quantum systems, this applies to potentials or optimization cost functions.

8. **Dynamic Simplification in Mechanics and Quantum Systems** Classically, kinetic energy T becomes polynomial in $\frac{d\sigma}{d\tau}$, stabilizing dynamics. In quantum systems, $\hat{F}$ is regularized, ensuring efficient scaling of computations.

4 Practical Applications

The framework's versatility is demonstrated through applications enriched by the quantum adaptation:

1. **Asymptotic Analysis**
   - *Classical Example*: For $f(\mathbf{r}) = |\mathbf{r}|^2$ as $|\mathbf{r}| \to \infty$, $f\left(\frac{1}{\boldsymbol{\tau}}\right) = \frac{1}{|\boldsymbol{\tau}|^2}$ enables Taylor series analysis near $\boldsymbol{\tau} = \mathbf{0}$.
   - *Quantum Example*: Study quantum algorithm asymptotics for large inputs near $\boldsymbol{\tau} = \mathbf{0}$.

2. **Numerical Computation**
   - *Classical Example*: The integral $\int_{|\mathbf{r}|>1} e^{-|\mathbf{r}|^2}\, dV$ becomes bounded in the τ-plane.
   - *Quantum Example*: Compute quantum integrals or sums over unbounded domains efficiently in the quantum τ-plane.

3. **Differential Equations**
   - *Classical Example*: The singular equation $|\mathbf{r}|^2 u'' + u = 0$ transforms to a regular problem in the τ-plane.
   - *Quantum Example*: Regularize singular quantum differential equations for solution.

4. **Multi-Body Dynamics**
   - *Classical Example*: In the three-body problem, (σ, ϕ, θ) and τ-time stabilize simulations across scales.
   - *Quantum Example*: Manage complexity in quantum many-body systems using (σ, ϕ, θ).
5. **Gravitational Systems**
   - *Classical Example*: The potential $V = -e^{-\sigma} U(\theta)$ separates scale and shape.
   - *Quantum Example*: Apply to quantum gravitational systems or field theories.
6. **Numerical Simulation**
   - *Classical Example*: Multi-body motion with $\sigma \in (-\infty, \infty)$ and τ-time remains feasible.
   - *Quantum Example*: Simulate quantum systems with extreme scales, e.g., quantum gravity.
7. **Economic Market Analysis**
   - *Classical Example*: Firms with positions $\mathbf{r}_i$, weights w_i, and properties $\mathbf{p}_i$ use σ for market size and θ for structure.
   - *Quantum Example*: Optimize quantum financial algorithms using the quantum configuration space.
8. **Quantum Computing Algorithms**
   - *Example*: Design algorithms leveraging compactification of infinite scales in the quantum τ-plane for large problem sizes.
   - *Example*: Optimize entanglement in quantum circuits using $\hat{s}$, enhancing efficiency and depth.

5 Conclusion

This geometric system, positioning infinity at the origin and excluding zero, redefines the analysis of scale and dynamics. Its hyperreal foundation, flexible transformations, support for curved spaces, and quantum adaptation unify points, configurations, motion, and quantum states, overcoming traditional constraints. Applicable to celestial orbits, market dynamics, and quantum algorithms, it provides tools to explore extremes with precision. As a foundation for mathematical and quantum inquiry, it invites further development while already offering profound insights into the vast, the minute, and the quantum.