DESIGN OF LOCAL FILTERS FOR SPARSE SPIKE TRAINS DECONVOLUTION

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Abstract: This paper deals with a new method of deconvolution of sparse spike signals: local filtering. A local filter is a filter whose characteristics evolve, generally with time. It is based on a Bayesian interpretation of the algorithm ARTUR presented by Charbonnier *et al.* (Charbonnier *et al.*, 1997). We compare this new method with the one of the reference: the SMLR algorithm.

Keywords: sparse spike trains deconvolution - regularisation - local filter.

1. INTRODUCTION

The aim of deconvolution is to estimate an unknown signal $\boldsymbol{x}$, input of a system characterized by its impulse response $\boldsymbol{H}$, and distorted by an additive noise $\boldsymbol{n}$, supposed to be a white Gaussian zero-mean noise with variance $\sigma_n^2\boldsymbol{I}$:

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}. \tag{1}$$

The impulse response $\boldsymbol{H}$ and the system output $\boldsymbol{y}$ are known. We note N the length of signals $\boldsymbol{x}$ and $\boldsymbol{y}$ and k refers to time.

Among the many applications of this problem, we would like to mention those where $\boldsymbol{x}$ is considered as a sparse spike train: geophysics, medicine, spectroscopy or non-destructive evaluation, including partial discharges analysis which was the primer goal of this work.

$\boldsymbol{x}$ being a sparse spike train, it is modelled as a BG process given by:

$$\forall\, k,\ x(k) = q(k)x(k),$$

where $q(k) = 0$ if $x(k) = 0$ and $q(k) = 1$ otherwise. $\boldsymbol{q}$ follows a Bernoulli law:

$$p(q(k) = 1) = 1 - p(q(k) = 0) = \lambda$$

and, when $q(k) = 1$, $p(x(k))$ is a zero-mean Gaussian with variance σ_x^2, otherwise $x(k) = 0$. Then, $\forall\, k$:

$$p(x(k)) = \lambda\mathcal{N}(0, \sigma_x^2) + (1 - \lambda)\delta[x(k)].$$

In this paper, the signal $\boldsymbol{x}$ is estimated by deconvolution, using a *local filter*, or *linear time varying filter*, whose characteristics evolve in function of the signal to restore.

The deconvolution consists in restoring the input $\boldsymbol{x}$, knowing $\boldsymbol{H}$ and $\boldsymbol{y}$. It is an ill-posed problem, in the sense of Hadamard, that is there is no unique and stable solution. Regularisation aims at finding an unique and stable solution to the problem. There are many regularisation approaches that can be classified into two broad categories: dimension control and minimisation of a compound criterion. The later will be considered here. So, the signal estimation will be achieved by minimising a criterion $\mathcal{J} = \mathcal{J}_1 + \mathcal{J}_2$ where:

- $\mathcal{J}_1 = ||\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x}||^2$ is the least squares criterion which is a data fitting measure;
- $\mathcal{J}_2$ is the *a priori* criterion, coding the desirable information about the solution. $\mathcal{J}_2$ can be written as:

$$\mathcal{J}_2 = \sum_k \varphi[Dx(k)]$$

where φ is a function and D the differential operator (see (Idier, 2001) for a discussion about the choice of φ and D).

The Bayesian interpretation allows to link energetic regularisation (the criterion) with the probabilistic model of the data (the *a posteriori* probability density):

$$\mathcal{J} = -\ln p(\boldsymbol{x}|\boldsymbol{y}, \boldsymbol{\theta})$$

which, due to Bayes theorem and omitting the term $p(\boldsymbol{y}|\boldsymbol{\theta})$ which is a constant for $\boldsymbol{x}$, becomes:

$$\mathcal{J} = \underbrace{-\ln p(\boldsymbol{y}|\boldsymbol{x}, \boldsymbol{\theta})}_{\mathcal{J}_1} \underbrace{-\ln p(\boldsymbol{x}|\boldsymbol{\theta})}_{\mathcal{J}_2}$$

where $\boldsymbol{\theta}$ is the vector of hyperparameters of the problem.

There are many deconvolution methods. The Hunt filter (see section 3.1) results from the Tikhonov criterion minimisation, where the *a priori* is quadratic (Demoment, 1985): that's why it does not result a BG signal. A Bayesian interpretation gives the optimal value of the regularisation coefficient.

A better algorithm to restore a sparse spike signal is to use a L_p norm with p between 1 and 2 rather than a quadratic one: we talk about L_p deconvolution. There is no explicit solution, but, because of the criterion convexity, one can use minimum search algorithms (Idier, 2001).

The SMLR (Champagnat *et al.*, 1996; Kormylo and Mendel, 1982), IWM (Kaaresen, 1997), MAP, MPM or ICM (Lavielle, 1993) algorithms are well-adapted to the problem of BG deconvolution. These algorithms use a two-step approach where detection of pikes and estimation of their amplitude are performed successively. Note that, except for the amplitude estimation, the SMLR involves combinatory optimisation problems.

At last, local filters have been already used in speech processing or vibratory modelling. Nevertheless, their use for solving inverse problems seems to have never been broached except by us. Brie *et al.* (Brie *et al.*, 2001*c*) have used a local low-pass filter with a variable bandwidth. This approach has given good results for the deconvolution of electron energy loss spectrum. Local filters have also been applied on partial discharge signals for denoising (Caironi, 2002) and deconvolution (Mazet, 2002).

Hyperparameters may be estimated using a MCMC (Monte Carlo Markov Chain) method, for example the SEM algorithm (Doucet and Duvaut, 1997; Idier, 2001).

In section 2, we recall the SMLR method which will serve as a reference method to assess the performances of the proposed approach. Section 3 gives the main contribution of this paper: firstly we present the Hunt filter and the principle of the deconvolution by local filtering; then we present the proposed method which is based on an interpretation of the algorithm ARTUR (Charbonnier *et al.*, 1997) in terms of local Hunt filtering. To approximate the BG distribution, the signal to restore is modelled as a mixture of two Gaussian distributions, making possible to use ARTUR for the criterion minimisation. The hyperparameters are estimated using a SEM algorithm. Section 4 presents a comparison of the results achieved by both the SMLR and the local filter methods. Finally, we conclude this paper by giving some perspectives.

2. PRINCIPLE OF SMLR ALGORITHM

The SMLR algorithm (Champagnat *et al.*, 1996; Kormylo and Mendel, 1982) is an iterative method of sparse spike trains deconvolution. It separates the detection of pikes and their amplitude estimation in order to prevent false detections (Champagnat *et al.*, 1996):

$$\widehat{\boldsymbol{q}} = \arg\max_{\boldsymbol{q}} p(\boldsymbol{q}|\boldsymbol{y}, \boldsymbol{\theta}),$$
$$\widehat{\boldsymbol{x}} = \arg\max_{\boldsymbol{x}} p(\boldsymbol{x}|\boldsymbol{y}, \widehat{\boldsymbol{q}}, \boldsymbol{\theta})$$

where $q(k) = 1$ if there is a pulse at time k, $q(k) = 0$ otherwise.

Firstly, we deal with supervised deconvolution which considers that the hyperparameters of the problem are known (sections 2.1 and 2.2), secondly we estimate these hyperparameters (section 2.3).

2.1 *Pulse Detection ($\widehat{\boldsymbol{q}}$)*

One can prove that maximising the *a posteriori* probability $p(\boldsymbol{q}|\boldsymbol{y}, \boldsymbol{\theta})$ returns to maximise the following criterion (Champagnat *et al.*, 1996):

$$\begin{aligned}\mathcal{J}(\boldsymbol{q}, \boldsymbol{\theta}) &\triangleq 2\ln p(\boldsymbol{q}|\boldsymbol{y}, \boldsymbol{\theta}) + N \ln 2\pi \\ &= -\ln|\boldsymbol{R}_{\boldsymbol{y}|\boldsymbol{q}}| - \boldsymbol{y}^T {\boldsymbol{R}_{\boldsymbol{y}|\boldsymbol{q}}}^{-1} \boldsymbol{y} + 2N_q \ln\lambda \\ &\quad + 2(N - N_q)\ln(1-\lambda).\end{aligned}$$

As there is no explicit expression of $\widehat{\boldsymbol{q}}$, and as we cannot test the 2^N combinations, we use the SMLR algorithm which allows to find a local minimum of $\mathcal{J}$ (Champagnat *et al.*, 1996; Idier, 2001; Kormylo and Mendel, 1982). Thus, a sequence $\boldsymbol{q}_0$ is created randomly. N other sequences $\boldsymbol{q}_k$ are also created, equal to $\boldsymbol{q}_0$ except in sample k where $\boldsymbol{q}_k(k) = 1 - \boldsymbol{q}_0(k)$. Then the two criteria $\mathcal{J}(\boldsymbol{q}_0)$ and $\mathcal{J}(\boldsymbol{q}_k)$ are compared and the most likely sequence becomes the new $\boldsymbol{q}_0$ for the next iteration.

2.2 *Amplitude Estimation* ($\widehat{\boldsymbol{x}}$)

Similarly to pulse detection, one can prove that:

$$-\ln p(\boldsymbol{x}|\boldsymbol{y},\widehat{\boldsymbol{q}},\boldsymbol{\theta}) = \frac{N}{2}\ln 2\pi + \frac{1}{2\sigma_n^2}(\boldsymbol{y}-\boldsymbol{H}\boldsymbol{x})^T(\boldsymbol{y}-\boldsymbol{H}\boldsymbol{x}) + \sum_{\substack{k=1\\ q(k)=1}}^{N}\left[\frac{1}{2}\ln 2\pi + \frac{1}{2}\ln\sigma_x^2 + \frac{1}{2\sigma_x^2}x^2(k)\right]$$

which gives an explicit expression of $\boldsymbol{x}$:

$$\widehat{\boldsymbol{x}} = \sigma_x^2 \widehat{\boldsymbol{Q}}\boldsymbol{H}^T \boldsymbol{R}_{\boldsymbol{y}|\widehat{\boldsymbol{q}}}^{-1}\boldsymbol{y} \tag{2}$$

with $\boldsymbol{R}_{\boldsymbol{y}|\widehat{\boldsymbol{q}}} = \sigma_x^2\boldsymbol{H}\widehat{\boldsymbol{Q}}\boldsymbol{H}^T + \sigma_n^2\boldsymbol{I}$.

2.3 *Hyperparameter Estimation* ($\widehat{\boldsymbol{\theta}}$)

Hyperparameters are: λ, $r_x = \sigma_x^2$ and $r_n = \sigma_n^2$. In non-supervised deconvolution, they are estimated during pulse detection with Gibbs algorithm, which is defined by the following steps (Champagnat *et al.*, 1996):

- *Sampling* (S):
  sample $\boldsymbol{q}$ from $p(\boldsymbol{q}|\boldsymbol{y},\boldsymbol{\theta}_i)$ (SMLR algorithm);
- *Maximization* (M):
  $\widehat{\boldsymbol{\theta}}_{i+1} = \arg\max_{\boldsymbol{\theta}} \ln p(\boldsymbol{y},\boldsymbol{q}|\boldsymbol{\theta}_i)$.

So, introducing the variable $\mu = r_x/r_n$, one can prove that:

$$\widehat{\lambda} = N_q/N, \tag{3}$$

$$\widehat{r_n} = \boldsymbol{y}^T\widetilde{\boldsymbol{R}}_{\boldsymbol{y}|\boldsymbol{q}}^{-1}\boldsymbol{y}/N, \tag{4}$$

$$\widehat{r_x} = \widehat{\mu}\widehat{r_n} \tag{5}$$

and $\widehat{\mu}$ is estimated by a minimisation search algorithm (a golden section search in our programs).

3. LOCAL FILTER

A *local filter* (Brie *et al.*, 2001*a*; Brie *et al.*, 2001*b*; Brie *et al.*, 2001*c*) is a filter whose characteristics evolve in function of an evolution variable, generally time. Because we will use Hunt filter as local filter, let us first introduce it.

3.1 *Hunt Filter*

The Hunt filter results from the Tikhonov criterion minimisation (Demoment, 1985; Idier, 2001):

$$\mathcal{J} = \int |y(t) - [h*x](t)|^2 dt + \alpha\int |[d*x](t)|^2 dt \tag{6}$$

where $d(t) = \delta(t)$ to favour zero values. So, using Parseval theorem and minimizing this criterion gives the expression of the Hunt filter:

$$\widehat{X}(\omega) = \frac{H^*(\omega)Y(\omega)}{H^*(\omega)H(\omega)+\alpha}. \tag{7}$$

$\widehat{x}(t)$ is obtained by the inverse Fourier transform.

The optimal value of α is determined by a Bayesian interpretation on the criterion $\mathcal{J}$.

Since $\boldsymbol{n}$ is a white Gaussian zero-mean noise with variance $\sigma_n^2\boldsymbol{I}$, the likelihood function is:

$$p(\boldsymbol{y}|\boldsymbol{x},\boldsymbol{\theta}) = \mathcal{N}(\boldsymbol{H}\boldsymbol{x},\sigma_n^2\boldsymbol{I}).$$

$\boldsymbol{x}$ is considered zero-mean Gaussian:

$$p(\boldsymbol{x}|\boldsymbol{\theta}) = \mathcal{N}(\boldsymbol{0},\sigma_x^2\boldsymbol{I}).$$

Using Bayes theorem, and setting $\mathcal{J}' = -\ln p(\boldsymbol{x}|\boldsymbol{y},\boldsymbol{\theta})$ (Bayesian interpretation), the *a posteriori* criterion is:

$$\mathcal{J}' = \frac{1}{2\sigma_n^2}(\boldsymbol{y}-\boldsymbol{H}\boldsymbol{x})^T(\boldsymbol{y}-\boldsymbol{H}\boldsymbol{x}) + \frac{1}{2\sigma_x^2}\boldsymbol{x}^T\boldsymbol{x}$$

which is equal, by multiplying by $2\sigma_n^2$, to $\mathcal{J}$:

$$\mathcal{J} = (\boldsymbol{y}-\boldsymbol{H}\boldsymbol{x})^T(\boldsymbol{y}-\boldsymbol{H}\boldsymbol{x}) + \frac{\sigma_n^2}{\sigma_x^2}\boldsymbol{x}^T\boldsymbol{x} \tag{8}$$

with $\alpha_{opt} = \sigma_n^2/\sigma_x^2$ (Demoment, 1985; Idier, 2001). Here, α is a constant (then a scalar), but in a local filter it will be the variable parameter (then considered as a vector).

Minimising $\mathcal{J}$ gives the discrete expression of the Hunt filter:

$$\widehat{\boldsymbol{x}} = (\boldsymbol{H}^T\boldsymbol{H} + \alpha\boldsymbol{I})^{-1}\boldsymbol{H}^T\boldsymbol{y}. \tag{9}$$

Note that the continuous-time solution (7) and the discrete-time one (9) are equivalent under the assumption that the signals are periodical and the matrix $\boldsymbol{H}$ is circulant (Demoment, 1985).

3.2 *Generality about Deconvolution by Local Filtering*

A local filter is completely defined by its bidimensional impulse response $g(t,t')$ which is the filter response at time t' to a pulse applied at time t.

The relation between an input and output system is given by:

$$y(t) = \int g(t, t-t')x(t')dt'.$$

Using the local frequency response of $g(t,t')$ defined as $G(t,\omega') = \mathrm{FT}_{t'\to\omega'}[g(t,t')]$, $y(t)$ may be written also as:

$$y(t) = \int G(t,\omega')X(\omega')\mathrm{e}^{j\omega't'}d\omega'$$

where $X(\omega')$ is Fourier transform or $x(t)$.

Consider now the signal $y(t) = [h*x](t)+n(t)$. Thus, deconvolution by local filtering may be achieved by the procedure described on figure 1.

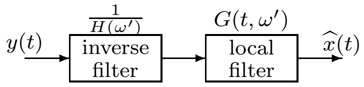

Figure 1. Deconvolution by local filtering

The restored signal $\widehat{x}(t)$ is given by:

$$\widehat{x}(t) = \int G(t,\omega') \frac{1}{H(\omega')} Y(\omega') \mathrm{e}^{j\omega' t} d\omega'.$$

The problem is to design $G(t,\omega')$ to obtain the best $\widehat{x}(t)$. To do that, we give an interpretation of the algorithm ARTUR (Charbonnier *et al.*, 1997) in terms of local filtering.

3.3 *Interpretation of ARTUR in terms of Local Filtering*

Charbonnier *et al.* (Charbonnier *et al.*, 1997) have proposed a deconvolution method, called ARTUR, for edge-preserving image restoration.

The searched solution is given as the minimum of a compound criterion $\mathcal{J}$:

$$\begin{aligned} \widehat{\boldsymbol{x}} &= \arg\min_{\boldsymbol{x}} \mathcal{J}(\boldsymbol{x}) \\ \text{where} \quad \mathcal{J}(\boldsymbol{x}) &= \mathcal{J}_1(\boldsymbol{x}) + \mathcal{J}_2(\boldsymbol{x}) \\ \text{and} \quad \mathcal{J}_2(\boldsymbol{x}) &= \sum_k \varphi\left(x(k)\right). \end{aligned}$$

As for the Hunt method, the differential operator is set to $\delta(t)$ to favour zero value signal. The function φ has to be adapted to the pulse restoration. This point will be addressed in section 3.4.

The half-quadratic regularisation (Charbonnier *et al.*, 1997; Ciuciu and Idier, 2002; Geman and Yang, 1995) is intended to make the optimisation easier. Its principle is to introduce an auxiliary variable $\alpha(\boldsymbol{x})$ (thus becoming a vector noted $\boldsymbol{\alpha}$) and the criteria $\mathcal{J}^*$ and $\mathcal{J}_2^*$ such as:

$$\mathcal{J}^*(\boldsymbol{x},\boldsymbol{\alpha}) = \mathcal{J}_1(\boldsymbol{x}) + \mathcal{J}_2^*(\boldsymbol{x},\boldsymbol{\alpha})$$

where $\mathcal{J}^*$ has the same global minimum than $\mathcal{J}$. The term "half-quadratic" means that, $\boldsymbol{\alpha}$ being fixed, $\mathcal{J}_2^*$, and consequently $\mathcal{J}^*$, are quadratic w.r.t. $\boldsymbol{x}$. Then an explicit solution of $\boldsymbol{x}$ does exist. Moreover, when $\boldsymbol{x}$ is fixed, $\mathcal{J}^*$ is convex and an explicit expression of $\boldsymbol{\alpha}$ does exist too.

Provided that φ satisfies certain conditions (see appendix A), the iterative algorithm ARTUR allows to obtain the solution according to:

Initialisation of $\widehat{\boldsymbol{x}}_0$.
Repeat until convergence :

$$\boldsymbol{\alpha}_{i+1} = \arg\min_{\boldsymbol{\alpha}} \mathcal{J}^*(\boldsymbol{x}_i,\boldsymbol{\alpha})$$

$$\boldsymbol{x}_{i+1} = \arg\min_{\boldsymbol{x}} \mathcal{J}^*(\boldsymbol{x},\boldsymbol{\alpha}_{i+1})$$

The explicit solutions $\boldsymbol{x}_{i+1}$ and $\boldsymbol{\alpha}_{i+1}$ are given by:

$$\boldsymbol{\alpha}_{i+1} = \varphi'(\boldsymbol{x}_i)/2\boldsymbol{x}_i, \tag{10}$$

$$\boldsymbol{x}_{i+1} = (\boldsymbol{H}^T\boldsymbol{H} + \mathrm{diag}\{\boldsymbol{\alpha}_{i+1}\})^{-1}\boldsymbol{H}^T\boldsymbol{y}. \tag{11}$$

Equation (11) may be interpreted in terms of local filtering since $\boldsymbol{x}$ depends on $\boldsymbol{\alpha}$, depending itself on the evolving variable k. The continuous-time expression of equation (11) is:

$$\widehat{x}_{i+1}(t) = \int \frac{H^*(\omega')Y(\omega')}{H^*(\omega')H(\omega') + \alpha_{i+1}(t)} \mathrm{e}^{j\omega' t} d\omega'. \tag{12}$$

At iteration $i+1$, the local frequency response is:

$$G_{i+1}(t,\omega') = \frac{H^*(\omega')H(\omega')}{H^*(\omega')H(\omega') + \alpha_{i+1}(t)}. \tag{13}$$

On the one hand, it should be noted that the local filter (13) may be interpreted as a local Hunt filter. On the other hand, the conditions for a strict equivalence between equations (11) and (12) (which refer to equations (9) and (7) respectively for the non-local Hunt filter) has not been studied yet and remains an open problem.

To conclude this section, we would like to stress up that the proposed interpretation of ARTUR shows that the minimisation of $\mathcal{J}$ may be performed by a local filter. In addition, the implementation of (12) yields a lower computational burden than (11).

3.4 *Expression of Criterion $\mathcal{J}$*

The problem is now to design the criterion $\mathcal{J}$ that should be minimised. As before, the criterion is composed of two parts: the first ($\mathcal{J}_1$) is the least squares criterion, the second ($\mathcal{J}_2$) is the *a priori* criterion which has to be defined. Thus we have to find a function φ that fulfils the conditions of appendix A and favors the restoration of sparse spike trains. To do that, a probabilistic approach will be adopted. Our goal being to restore sparse spike trains, the best criterion we should use is the BG criterion, as for the SMLR algorithm (see section 2). But we cannot use the BG criterion for two reasons: firstly, we cannot calculate a Dirac pulse logarithm; secondly, the corresponding function φ would not satisfy the necessary conditions. So, we propose to use a probabilistic model where the Dirac pulse is replaced by a zero-mean Gaussian with a very small variance σ_ε^2 (Idier, 2001; Santamaría-Caballero *et al.*, 1996). Then we obtain a probability distribution which is a mixture of two zero-mean Gaussians.

The searched solution is defined as the maximum *a posteriori* (MAP):

$$\mathcal{J} = -\ln p(\boldsymbol{x}|\boldsymbol{y},\boldsymbol{\theta})$$

with $p(\boldsymbol{x}|\boldsymbol{y},\boldsymbol{\theta}) \propto p(\boldsymbol{y}|\boldsymbol{x},\boldsymbol{\theta})p(\boldsymbol{x}|\boldsymbol{\theta})$ because of the Bayes theorem.

Since $\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}$, the likelihood is written as:

$$p(\boldsymbol{y}|\boldsymbol{x}, \boldsymbol{\theta}) = \mathcal{N}(\boldsymbol{H}\boldsymbol{x}, \sigma_n^2 \boldsymbol{I}).$$

Then :

$$\mathcal{J}_1 = \frac{N}{2} \ln 2\pi + \frac{N}{2} \ln \sigma_n^2 + \frac{1}{2\sigma_n^2} (\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x})^2. \quad (14)$$

The *a priori* is the Gaussian mixture:

$$p(x(k)|\boldsymbol{\theta}) = \lambda \mathcal{N}(0, \sigma_x^2) + (1 - \lambda) \mathcal{N}(0, \sigma_\varepsilon^2).$$

So the posterior criterion is written as:

$$\mathcal{J}' = \underbrace{\frac{N}{2} \ln 2\pi + \frac{N}{2} \ln \sigma_n^2 + \frac{1}{2\sigma_n^2} (\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x})^2}_{\mathcal{J}_1} + \underbrace{\sum_{k=1}^{N} \frac{1}{2\sigma_n^2} \varphi(x(k))}_{\mathcal{J}_2}. \quad (15)$$

To be coherent with equation (8), this equation can be written as:

$$\mathcal{J} = \sigma_n^2 N \ln 2\pi + \sigma_n^2 N \ln \sigma_n^2 + (\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x})^2 + \sum_{k=1}^{N} \varphi(x(k)) \quad (16)$$

where the function φ is defined by:

$$\varphi(\tau) = -2\sigma_n^2 \ln \left[\frac{\lambda}{\sqrt{2\pi\sigma_x^2}} \exp\left(-\frac{\tau^2}{2\sigma_x^2} \right) + \frac{1-\lambda}{\sqrt{2\pi\sigma_\varepsilon^2}} \exp\left(-\frac{\tau^2}{2\sigma_\varepsilon^2} \right) \right]. \quad (17)$$

Conditions that φ satisfies are presented in appendix A.

3.5 *Estimation of the Signal*

$\widehat{\boldsymbol{x}}$ is estimated using a discretised version of equation 12. But the restored signal is not strictly BG because the *a priori* is the compound probability of two Gaussians. To obtain a BG signal, only the N_q greatest values of the signal are kept, the others being set to zero; N_q is determined during the hyperparameter estimation phase (see next section).

3.6 *Hyperparameter Estimation*

To dispose of a non-supervised algorithm, an hyperparameter estimation procedure is now presented. It is based on a SEM method as presented in section 2.3.

λ is estimated by calculating the ratio of the pulse number N_q on the sample number N. N_q is determined by classifying samples into two classes: those with a small amplitude, whose class variance is noted σ^2; and those whose amplitude is greater than $5\sigma^2$ (the coefficient 5 was determined empirically to achieved the best result).

The variance σ_n^2 appears only in the criterion $\mathcal{J}_1$ whose minimisation gives the explicit solution:

$$\sigma_n^2 = \frac{1}{N} (\boldsymbol{y} - \boldsymbol{H}\widehat{\boldsymbol{x}})^T (\boldsymbol{y} - \boldsymbol{H}\widehat{\boldsymbol{x}}).$$

This estimation supposes that the signal is BG, so $\widehat{x}(k)$ is put to zero if and only if $\widehat{x}(k) < 5\sigma^2$.

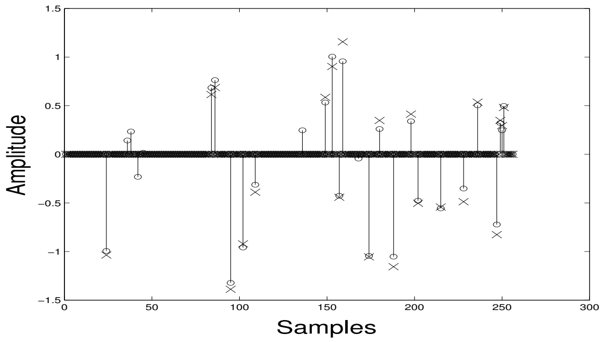

Figure 2. SMLR estimate

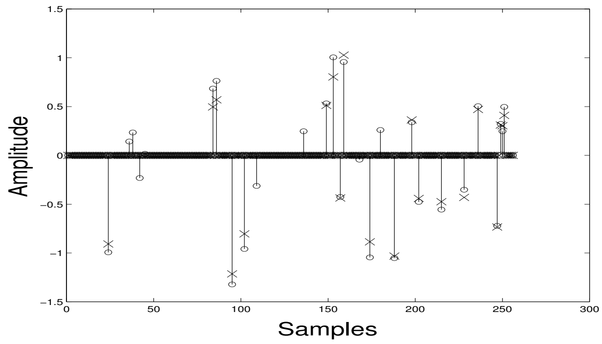

Figure 3. local filtering estimate

σ_x^2 and σ_ε^2 only appear in the criterion $\mathcal{J}_2$, but they do not admit explicit expressions. So, we use a minimum search algorithm (in our programs, we successively apply the golden section search algorithm to estimate σ_x^2 and σ_ε^2). Here, signal $\widehat{\boldsymbol{x}}$ follows the Gaussian mixture, so we use the signal given by the local filter.

4. EXPERIMENTAL RESULTS

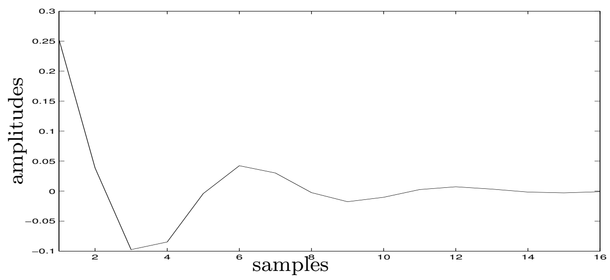

Figure 4. impulse response

A simulation has been made with a 256-sample input, a 16-sample impulse response (figure 4) and the following real parameters:

$$\lambda^* = 0.1 \quad {\sigma_x^*}^2 = 0.5 \quad {\sigma_n^*}^2 = 4.0145.10^{-4}.$$

Results are presented in figures 2 and 3. $\circ$ represents real pulses, and $\times$ represents estimated pulses. The following table presents statistics on the results:

	time	D.P.	F.D.	N.D.
SMLR	25.42 s	22	0	6
L.F.	6.49 s	20	0	8

D.P.: detected pulses; F.D.: false detections; N.D.: Non detections.

Simulations show that the SMLR method is very well-adapted to this problem: it finds most of the pulses. But it's a very slow method, because of matrix inversions. Local filtering is quicker than the SMLR method, and gives equivalent results. But we can notice that amplitudes are under-estimated.

5. CONCLUSION

In conclusion, we have presented a new method of deconvolution, called local filtering. We have applied this method to the deconvolution of BG processes and have compared it with the SMLR method, which is the reference. The results of the two methods are closely equivalent, but local filtering is about 4 times quicker than SMLR algorithm.

The local filtering is not the best method yet and have to be improved for BG signals. But this approach is able to deconvolve other signals than BG signals contrary to the SMLR method, and it should be better with signals following a mixture of laws.

Appendix A. NECESSARY CONDITIONS ON φ

Conditions imposed on φ are:

Basic assumptions:
(1) $\forall\, \tau, \varphi(\tau) \geq 0$ and $\varphi(0) = 0$;
(2) $\varphi(-\tau) = \varphi(\tau)$;
(3) φ continuously differentiable;
(4) $\forall\, \tau \geq 0,\ \varphi'(\tau) \geq 0$;
Edge preservation:
(5) $\varphi'(\tau)/2\tau$ continuous and strictly decreasing on $[0, +\infty[$;
(6) $\lim_{\tau\to+\infty} \varphi'(\tau)/2\tau = 0$;
(7) $\lim_{\tau\to 0^+} \varphi'(\tau)/2\tau = c$ with $0 < c < +\infty$;
Convergence of the algorithm:
(8) $\varphi^{(3)}(0) = 0$;
(9) $\varphi^{(4)}(0)$ exists.

The function φ of the equation (17) satisfies all the conditions, except No. (6).

But conditions (5), (6) and (7) are open to criticism. Indeed, Condition (6) signifies that α must be equal to zero when pulse amplitude tends to infinity: that means that the bandwidth (proportional to the inverse of α) must be infinite, what is not logically obligatory. The very important conditions for the convergence of the algorithm are conditions (8) and (9), and they are respected.

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