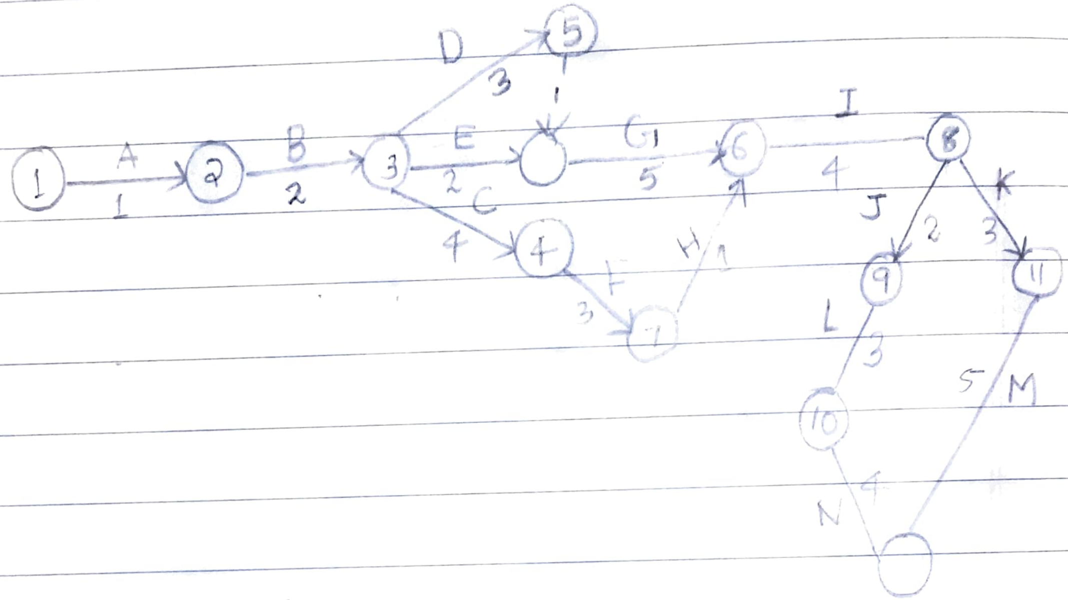
Queuing theory & Project Management

1.



A-B-E-G-I-K-M

A-B-C-F-H-I-J-L-N

A-B-D-G-I-K-M

A-B-D-G-I-J-L-N

A-B-C-F-H-I-K-M

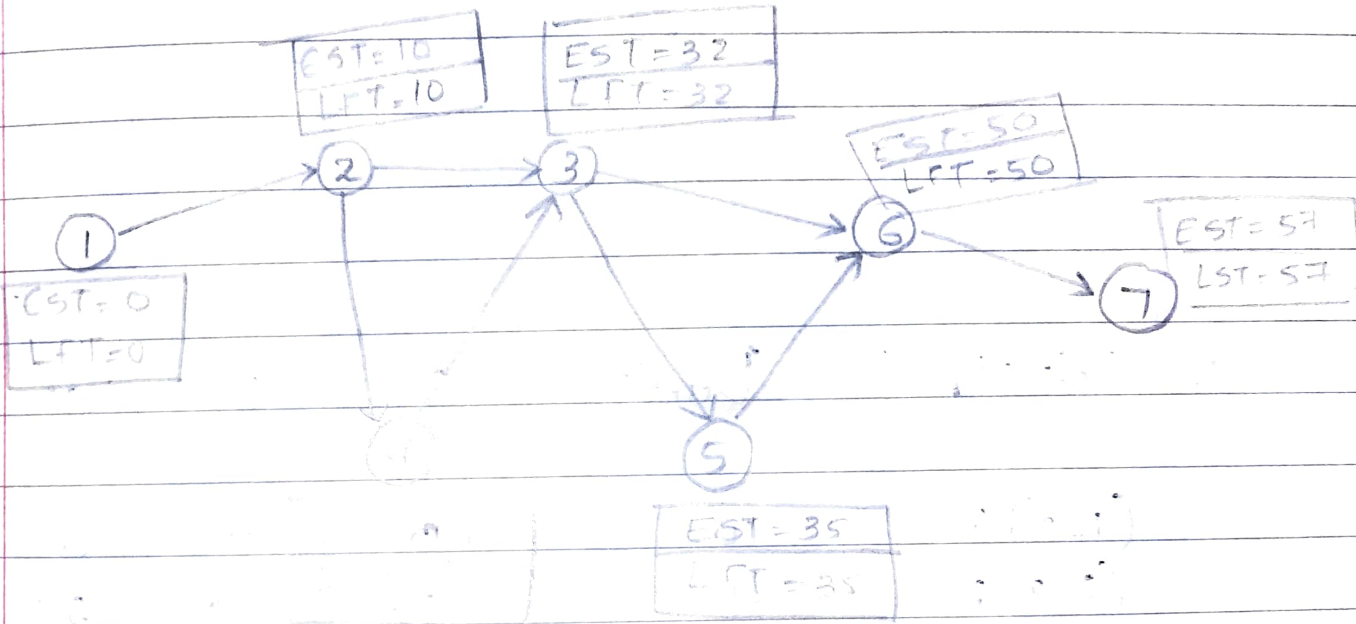
A-B-E-G-I-K-M

A-B-E-G-I-J-L-N

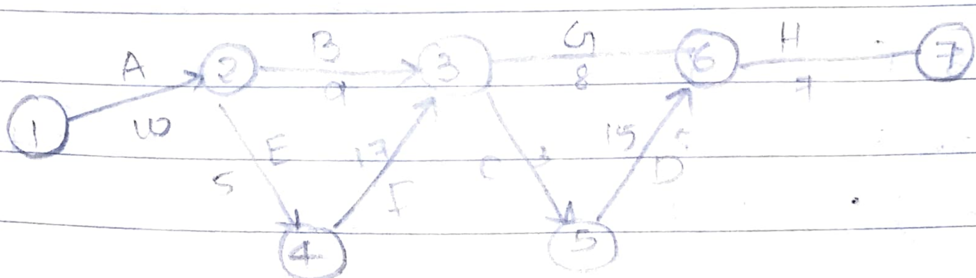
Path	Duration
A-B-E-G-I-K-M	20
A-B-C-F-H-I-J-L-N	24

A - B - D - G - I - K - M 21
A - B - D - G - I - J - L - N 24
A - B - C - F - H - I - K - M 21
A - B - E - G - I - K - M 20
A - B - E - G - I - J - L - N 23

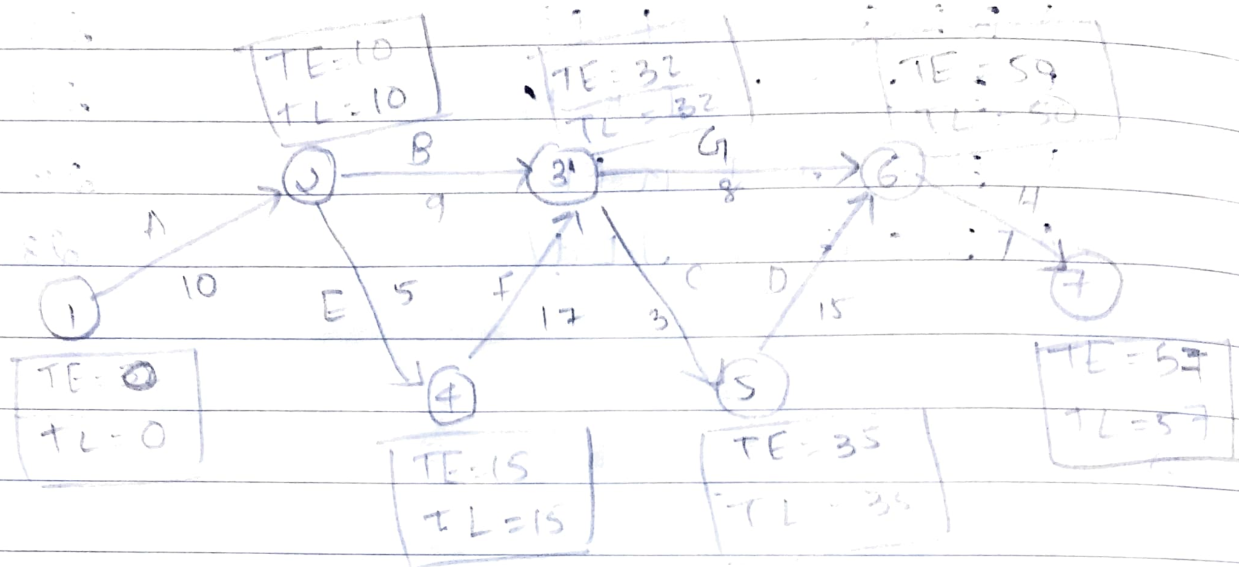
2.



a



b



c.

Activity	Duration	EST	EFT	LST	LFT
(1-2) A	10	0	10	0	10
(2-3) B	9	10	19	23	32
(2-4) E	5	15	15	10	15
(3-5) C	3	32	35	32	35
(5-6) D	15	35	50	35	50
(4-3) F	17	15	32	15	32
(3-6) G	8	32	40	42	50
(6-7) H	7	50	57	50	57

EFT = EST + te

LST = LFT - te

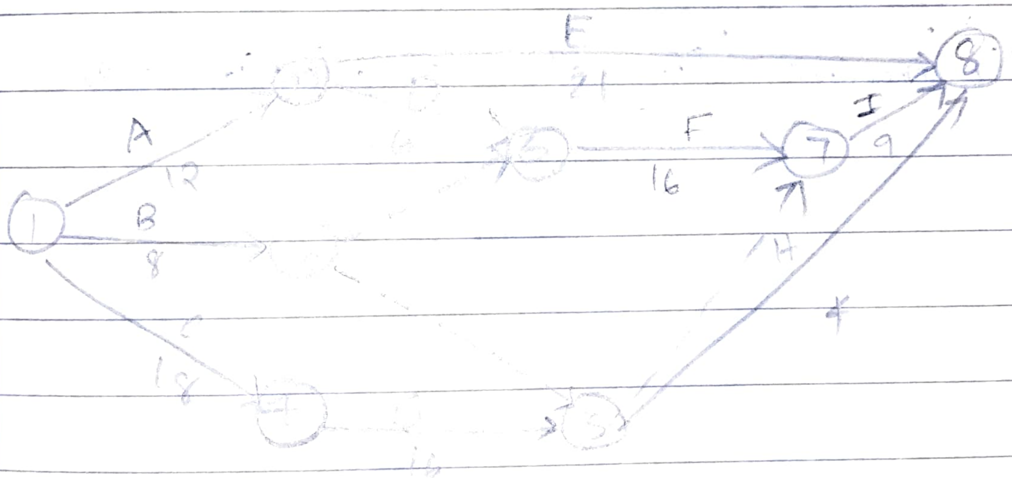
d Critical Path

1 - 2 - 4 - 3 - 5 - 6 - 7

A - E - F - C - D - H

10 + 5 + 17 + 3 + 15 + 7 = 57

3. Network diagram.



Activity	Predece activi
A	–
D	A
E	A
F	BD
G	C
H	B, G
I	F, G
B	–
C	–

To find the minimum time of completion, we need to calculate the critical path of the network diagram.

The paths are:

~~AE~~ ~~= 33~~

A - E = 33

A - D - F - I = 53

B - F - I = 33

B - H = 12

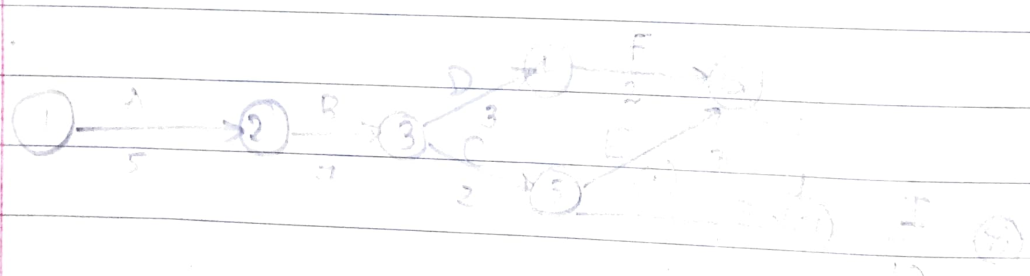
C - G - H = 38

C - G - I = 43

Hence the critical path = A - D - F - I.

Minimum time of completion = 53

4



The paths are :

A - B - D - F - H - I = 30

A - B - C - E - H - I = 28

A - B - C - G - I = 25

(i) Critical path is A - B - D - F - H - I

(ii) Minimum time of completion is 30 days

5.

Task	to	tp	tm	te
A	5	10	8	7.83 = 8
B	18	22	20	20
C	26	40	33	33
D	16	20	18	18
E	15	25	20	20
F	6	12	9	9
G	7	12	10	9.8 = 10
H	7	9	8	8
I	3	5	4	4

(i) Calculate the expected task time

$$t_e = \frac{t_o + t_p + 4t_e}{6}$$

$\lambda = 5 \quad \mu_1 = 10 \quad \mu_2 = 6$

$\rho = \frac{\lambda}{\mu}$

$\rho_1 = 1/2 \quad \rho_2 = 5/6$

$Lq = \rho^2 / 1-\rho$

$Lq_1 = (1/2)^2 / 1 - 1/2 = 2/4 = 1/2$

$Lq_2 = (5/6)^2 / 1 - 5/6 = \frac{25/36}{1/6} = 25/6$

$Wq = \frac{Lq_1}{\lambda} = \frac{1}{2(5)} = \frac{1}{10}$

$Wq_2 = Lq_2/\lambda = 25/6(5) = 25/30$

$W_1 = Wq_1 + 1/\mu_1 = 1/10 + 1/10 = 1/10$

$W_2 = Wq_2 + 1/\mu_2 = 25/30 + 1/6 = 30/30 = 1$

$L_1 = \lambda W_1 = 5(1/10) = \frac{1}{2}$

$L_2 = \lambda W_2 = 5 \times 1 = 5$

- The avg queue length is $1/2$ during morning and 5 during afternoon pack time
  The expected waiting time in the que is $1/10$ min during early morning and $25/30$ min during afternoon pack period.

Blue bird

Date
Page

7. $\lambda = 20$ cus/per

$\mu = 30$ Cus/per

$\therefore P = \frac{\lambda}{\mu} = \frac{20}{30} = \frac{2}{3}$

(i) The system will be idle $P = 1 - P$

$= 1 - 2/3$

$= \frac{3-2}{3} = \frac{1}{3} = 0.33$

33% of the time

(ii) Time the customer has to wait before reaching the server $\rho/_{\mu-\lambda}$

$= \frac{2/3}{30-20}$ $= \cancel{0.0}0.67$ hours

$= 4.02$ minutes

$W = Wq + \frac{1}{\mu} = 4.02 + 0.033$

$= 4.0533.$

(iii) The fraction of customers who will have to wait

$P = \rho/_{(1-\rho)} = (2/3)/_{(1-2/3)}$

$= 2/_1 = 2$

All the customer has to wait.

8. Avg no of customers

$$= \frac{\text{Avg arrival rate } \lambda}{\text{Avg waiting time}}$$

$$\text{Avg waiting time} = \frac{\text{Avg no of customer}}{\text{Avg arrival rate}}$$

$$\Rightarrow (\text{Avg. arrival rate})^2 \times \frac{\text{Service time}}{1 - \text{utilization}}$$

$$\Rightarrow \frac{(2.833)^2 \times 2.8}{1-2.33} \Rightarrow 0.83 \text{ time.}$$

10. This is a M/M/1 system with infinite population with $\lambda = 20$ customers per hour and $\mu = 30$ customer per hour.

(a) P(idle bank teller) $= P_0$

$= 1 - \rho = 1/3$

33.33 % of the time is idle

(b) $W_Q = 1/15 = 4$ minutes

(c) $L = 2$, $L_Q = 4/3$

Fraction in queue $= 2/3$

66.67 % of the system customers are waiting in the queue.

11. This is M/M/2 system with population with $\lambda = 10$ people per hour

$\mu = 15$ people per hour.

(a) P(waiting time) $= 1 - P_0 - P_1 = 4/9$

(b) $= L_Q = 4/3$ people in the queue

(c) $W_Q = 2/15$ hours

8 minutes on average in the queue.

12. This is a M/M/1 system with infinite population, with $\lambda = 8$ customer per hours and $\mu = 12$ Customers per hour

(a) $W_q = 0.16$ hours = 10 minute

(b) $L_q = 1.3$ customers

(c) The average queue length when the queue is not empty is calculated by Lq / prob.(queue not empty).

The probability = $1 - P_0 - P_1$

Finally, the queue length when it is not empty) in 3 people. The probability that there are at least 2 people in the system is 0.4.

13.

(a) $L = 39/760$ patient

(b) $W = 15$ min 4 23 5

(c) $W_q = 233$

(d) $L_q = 1/760$ Patients