EE 303 Assignment

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Generating PDFs of six different distributions using Gaussian distribution in MATLAB

A report by:

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Objective

We aim to generate and plot probability distribution function (PDF) of the following six distributions using the normal (Gaussian) distribution as the base distribution :

- Chi distribution with degrees of freedom k for 3 different values of k
- Chi-Squared distribution with degrees of freedom k for 3 different values of k
- Rice distribution for 3 different values of shape factor
- Non-Central Chi-Squared distribution with degrees of freedom k for 3 different values of k
- Gamma distribution for 3 different values of shape factor
- Nakagami-m distribution for 3 different values of shape factor

Methodology

<u>Chi Distribution</u>

It is the distribution of the positive square root of the sum of squares of a set of independent random variables each following a standard normal distribution. Thus, If a square root operation is applied on the Chi-Squared random variable, it gives the Chi random variable.

If Z1,Z2,...,Zn are independent, normally distributed random variables with mean 0 and standard deviation 1, then mathematically chi-distribution is given as follows

$$Y = \sqrt{\sum_{i=1}^{k} Z_i^2}$$ [1]

The same is implemented in the code by summing up the squared normally distributed random variables in y and then taking the square root of the same.

Chi-squared Distribution

The chi-squared distribution (χ^2-distribution) with k degrees of freedom is the distribution of a sum of the squares of k independent standard normal random variables.

If Z1,Z2,...,Zn are independent, normally distributed random variables with mean 0 and standard deviation 1, then mathematically chi-distribution is given as follows

$$Q = \sum_{i=1}^{k} Z_i^2,$$

[2]

The same is implemented in the code by summing up the squared normally distributed random variables in y.

Rice Distribution

A simple Rician fading channel can be simulated by setting the variables *g*1 and *g*2, as follows .

$$g_1 = \sqrt{\frac{\kappa}{2(1+\kappa)}}, \qquad g_2 = \frac{1}{\sqrt{2(1+\kappa)}}$$

For this, the total power of the line-of-sight path and the scattered path is assumed to be unity

$$\xi(t) = |\mu_\rho(t)| = |\mu(t) + m(t)|$$

Normally distributed variables are hence taken so and the value is computed with mean and variance as given above and implemented in the code.

Non-Central chi squared

If squares of k independent standard normal random variables are added, it gives rise to central Chi-squared distribution with k degrees of freedom. Instead, if squares of k independent normal random variables with non-zero means are added, it gives rise to non-central Chi-squared distribution. Hence a non centrality parameter is included, hence 1 variable with different mean than the normally distributed other variables. The the number of independent variables is defined by the degree of freedom. The same is implemented in the MATLAB code.

Gamma Distribution

The gamma distribution is given by the following formula as given below where Beta is the scale factor and alpha is the shape factor . Alpha is a positive half integer and Beta > 0. Zi are the normally distributed random variables.

$$\frac{1}{2\beta} \sum_{1}^{2\alpha} (Z_i)^2$$

The following is implemented in the MATLAB code as well.

Nakagami-m distribution

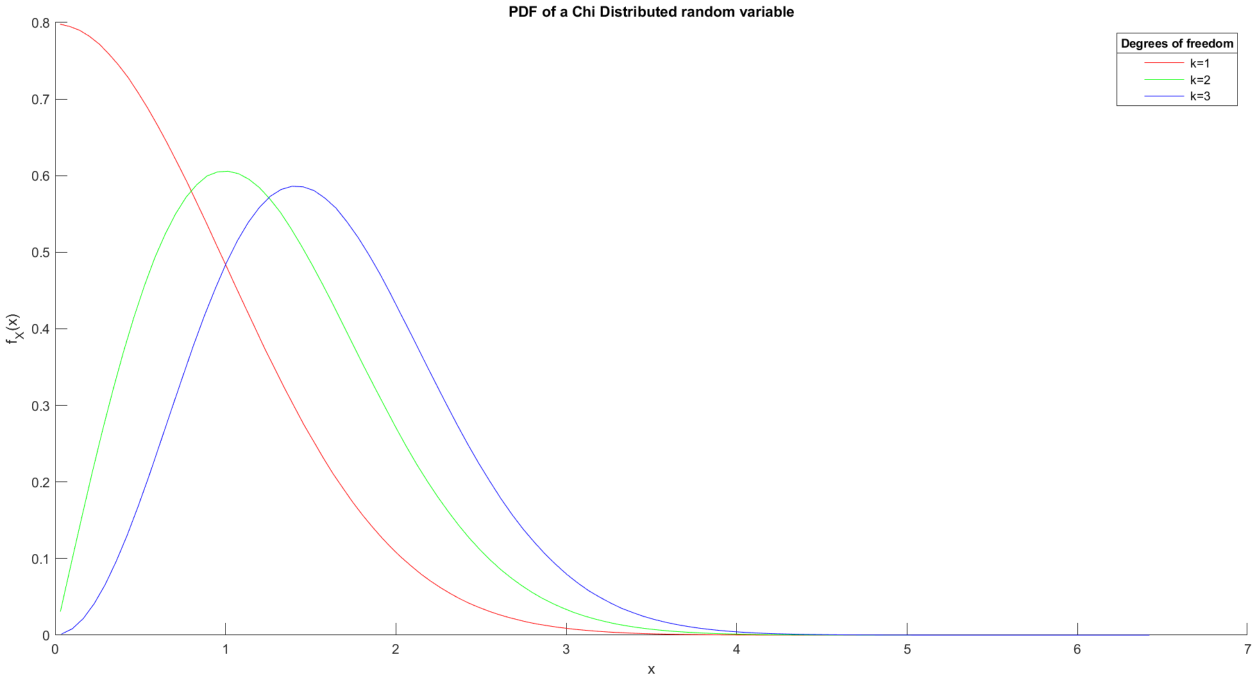
$$Y = \sqrt{\frac{\omega}{2m}}\chi(2m)$$

Thus we can see that Nakagami-m distribution can be generated by by producing a chi-distribution with degrees of freedom 2m and multiply the result with the given function consisting of shape factor **m** and scale factor **omega .** The same is implemented in the code by first finding the chi-distribution and then multiplying with the desired quantity.

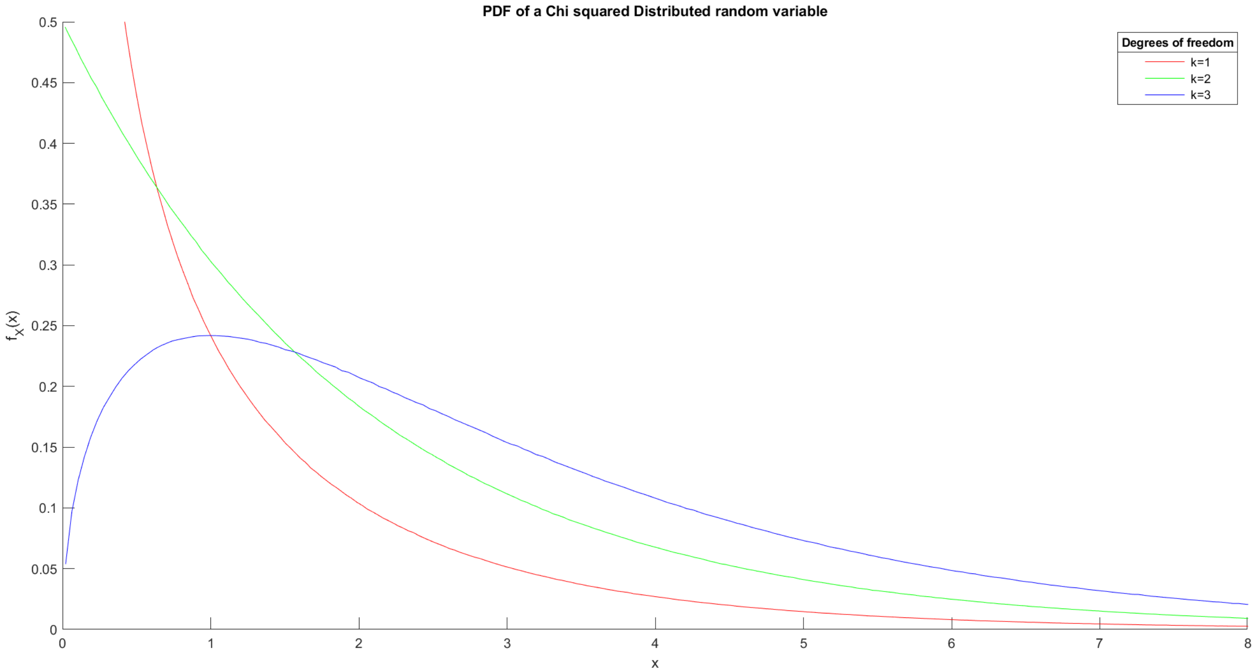
Results

The following Probability distribution functions were found for the given distributions :

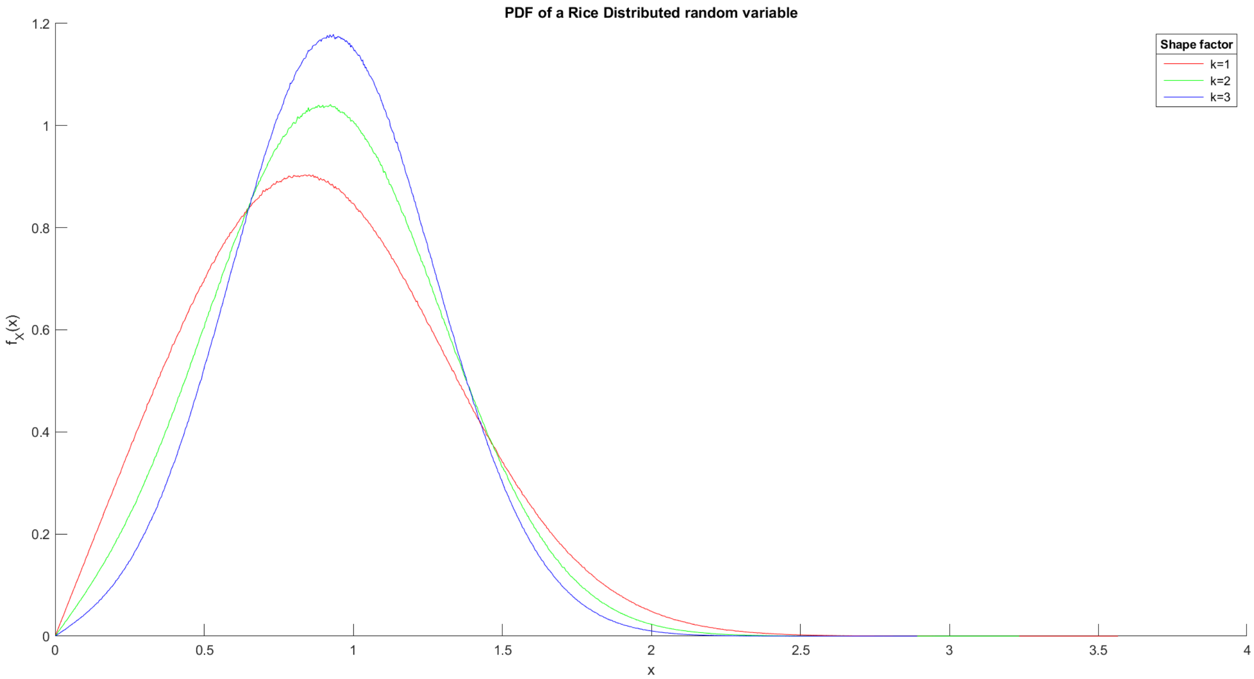
Chi Distribution



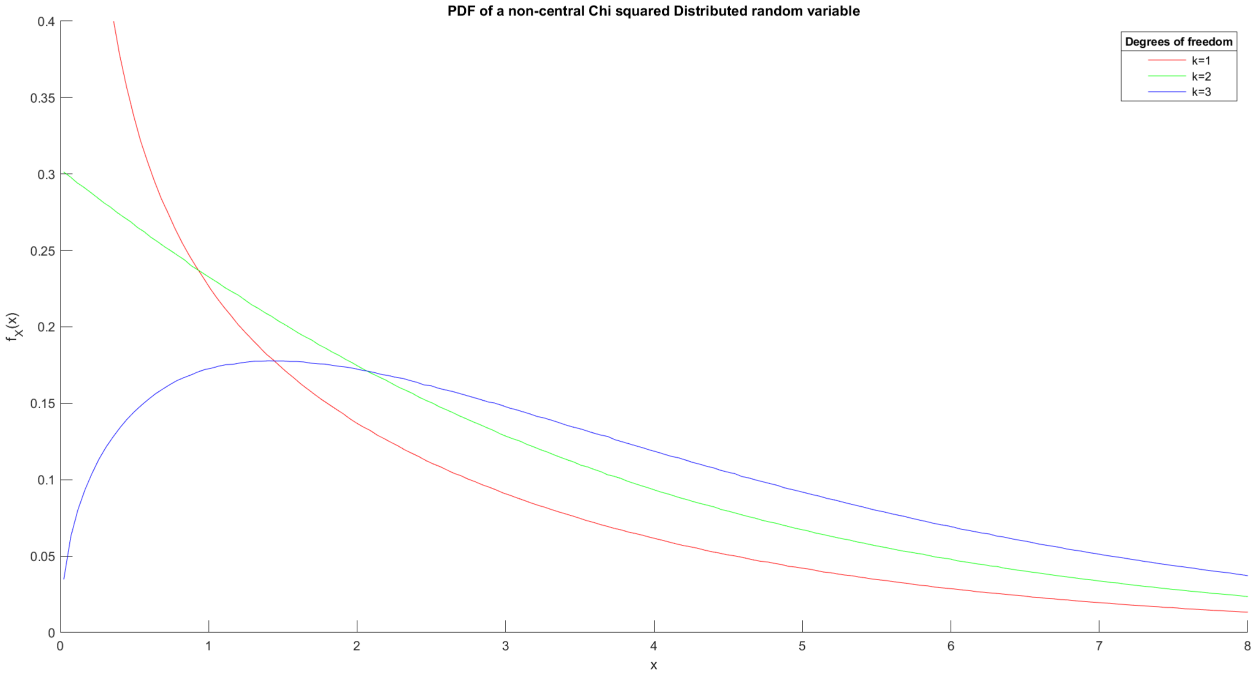
Chi-squared



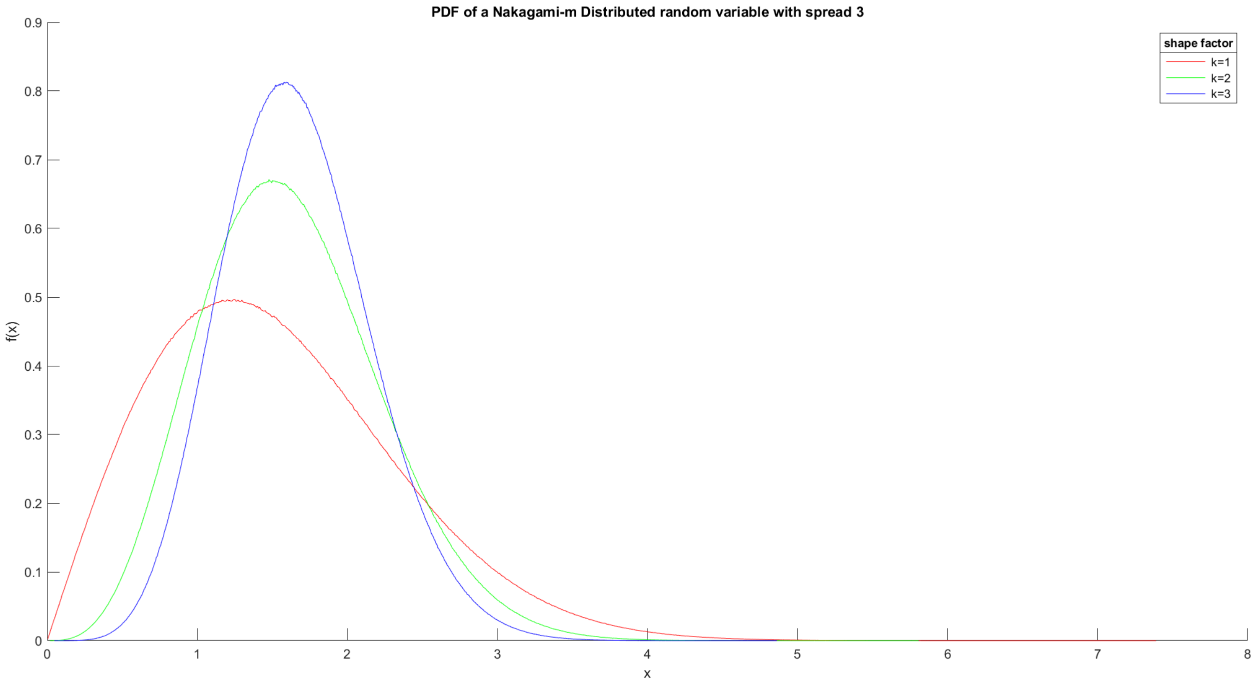
Rice Distribution



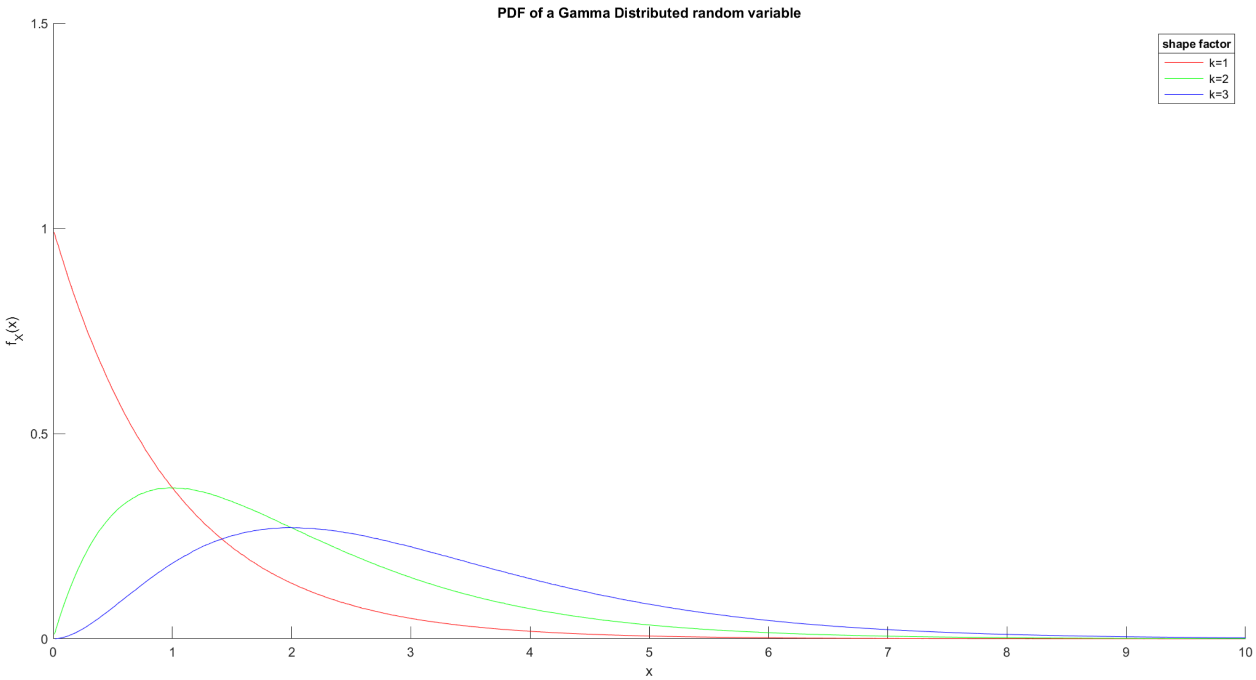
Non-central Chi Squared



Nakagami-m with spread 3



Gamma distribution



Conclusion

We have successfully implemented the probability distribution functions of the following distributions :

- Chi distribution with degrees of freedom k for 3 different values of k
- Chi-Squared distribution with degrees of freedom k for 3 different values of k
- Rice distribution for 3 different values of shape factor
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Using normal distribution variables calculated from the box-muller method and plotted the distributions for various degrees of freedom and shape factors.