Dynamic model of autonomous ground vehicle for the path planning module

Conference Paper · December 2011

DOI: 10.1109/ICARA.2011.6144859 · Source: DBLP

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***Abstract*—This paper discusses the importance of kinematic and dynamic constraints in a path planning module. The kinematic and dynamic models are taken into account to model the actual mobile robot including the torques provided by the motors and the restriction to the robot mobility induced by the constraints. The dynamic model of the mobile robot is simulated and compared with the behaviour of a real Pioneer 3DX mobile robot.**

***Index Terms*—Dynamic modelling, Pioneer 3DX mobile robot, Path Planning**

I. Introduction

Autonomous Ground Vehicles (AGVs) have been the topical area in the recent past in which the researchers are working to enhance the intelligence of the vehicles for different applications. Path planning has been considered as the main challenge in the field of autonomous mobile systems. Several researchers are currently looking at this problem in the presence of uncertainties like dynamic obstacles, partially known environments, etc. A number of algorithms for path planning have been developed over the years. One of the reasons for the continued work in this field can be summarised in the words of David P Miller [1], as "*the simplifying assumptions made about the modules associated with the module actually being designed are too simple. Implementing a module in a real robot system makes it difficult to make oversimplifications.*"

In general, it is difficult to come up with a very good model for a mobile robot without any assumptions. In order to develop an effective path planning technique, knowledge about the robot dynamics is very important. Mathematical path planning techniques assume that the robot is a point mass and can move in any direction and able to reach the goal flawlessly [2]. Most researchers use a simplified kinematic constraint to plan a feasible path. For real world scenarios, these assumptions are often invalid.

Many path planning techniques have been discussed in the literature. However, the resulting paths by the search algorithms do not tend to be smooth and hence do not include kinematic constraints. Some well-known methods [3] have been used to generate the smooth path but they have a drawback of discontinuity of curvature at the joint nodes connecting the lines and arc [4]. Clothoid curve [5], Bezier curve [6] are used used by the researchers to model the smooth curves to get the continuous curve, which meet the curvature constraint. The planned path for a real robot should satisfy the practical conditions imposed by: (a) Speed and acceleration limitation which is caused by the actuator (DC motor), (b) Dynamics of the vehicle in reacting to the actuator forces and (c) Position & velocity [7]. The kinematic model of a mobile robot is essentially the description of the admissible instantaneous motions in respect of the constraints. On the other hand, the dynamic model accounts for the reaction forces and describes the relationship between the motions and the generalized forces acting on the robot. The trajectories have to be generated in respect of the kinematic constraints imposed by the wheels.For the control of rotary motion, a common actuator is the DC motor [8]. Such a device is constrained in input voltage and currents, which, without load, is equivalent to velocity and acceleration.

This paper proposes a model which replicates the actual pioneer robot's behaviour. First the collision free path consists of a series of coordinates, generated for the path planning module. These sequences of points are generated by suitable path planning technique from the literature. These set of points are formed as a continuous curve. To form a smooth movement, the curve fitting is used as a part of path planning module. The path planning algorithm takes into account the model of the robot. Results are discussed to show the variations in the paths due to the incorporation of the dynamics.

The rest of the paper is organised as follows: Section II describes the grid based path planning technique and smoothing by cubic spline, Section III shows the modelling of Pioneer mobile robots, and Section IV gives an overview on models implementation in MATLAB. Paper ends with conclusion and future work on this topic.

II. Path Planning

A. Robot Position

The vector of joint variables, q provides a representation of a configuration space $\mathcal{Q} = \Re^2$. The robot is denoted by the notation $\wp$ and $\wp(q)$ represents the subset of the workspace that is occupied by the robot in the Cartesian space. The posture coordinates of the robot is completely specified by the

3 variables $q = x, y, \theta$, where x, y represent the position in the 2D space and θ represents the orientation with respect to the OX axis. The path planning problem is to find a path from an initial posture q_{start} to a particular target posture q_{goal} in the configuration space, such that the robot does not collide with any obstacle as it traverses [9]. In the path planning literature, a collision free path from start to final location is represented as a continuous map, ($\mathcal{O}$).

$$\mathcal{O} : [start, goal] \Rightarrow \mathcal{Q}_{collisionfreepath}$$
$$\mathcal{O}(start) = q_{start} : \mathcal{O}(goal) = q_{goal}$$

The collision free path is determined by the D^* lite search algorithm [10]. After the path is calculated, cubic spline interpolations are used to create a smooth path using kinematic constraints.

B. Cubic Splines

Considering the points generated by the path planning algorithms in x-direction, q_x, the ith segment can be constructed [11] as in (1).

$$X_i(t) = a_i(t-1)^2(2t+1) + b_i t^{-2}\,(3-2t) \\ +c_i t(t-1)^2 - d_i t^2(1-t) \quad (1)$$

where X_i is the parameter connecting the points q_i and q_{i+1} in the x-direction, $0 \leq t \leq 1$ is the relative time of motion in each segment i = 1,2,...,N-1 and N is the total number of points. The smoothen continuous curves are generated through a series of unique cubic polynomials which are fitted between each of these points, with the stipulation that the curve obtained be continuous and appear smooth.

III. Dynamic Modelling

For a mobile robot, the wheel configuration may restrict the controllable degree of freedom (DOF) to two: typically the robot can turn on the spot or move forward but it cannot directly move side-wards. The vehicle is said to be under actuated or non-holonomic. In all physical systems actuators are subject to saturation as they can only deliver a certain amount of force. The inputs of the kinematic models are analogous to velocities while the inputs of the dynamic models are either torques or acceleration [7]. The structure of the robot model is given in Fig. 1.

Here, r - radius of the wheel, d - distance between the centre of mass and the centre of wheel axis, b - distance from the centre of the robot frame to driving wheel. The x_c, y_c are the Cartesian coordinates of the point C, which is the origin of mobile robot body axis and located at the intersection of the X_R-axis and Y_R-axis. The point L is the look-ahead point located on the X_R-axis of the mobile robot.

A. Kinematic model

Let x_g, y_g be the coordinates of the centre of the mass of the robot platform and θ be the angle between the heading direction and the OX-axis. The wheel angular velocities are dependant on linear and angular velocities of the robot as in (2). The notation ν represents the linear velocity and $\omega = \dot{\theta}$ denotes the angular velocity of the robot [12].

$$\begin{bmatrix} \nu \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{2b} & \frac{r}{-2b} \end{bmatrix} \begin{bmatrix} \omega_r \\ \omega_l \end{bmatrix} \quad (2)$$

The kinematic model of the mobile robot [13] with two conventional fixed wheels on the same axis and one rear conventional off-centered orientable wheel (caster wheel) can be written as in (3).

$$\begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = \begin{bmatrix} \frac{r(b\cos\theta - d\sin\theta)}{2b} & \frac{r(b\cos\theta + d\sin\theta)}{2b} \\ \frac{r(d\cos\theta + b\sin\theta)}{2b} & \frac{r(-d\cos\theta + b\sin\theta)}{2b} \\ \frac{r}{2b} & \frac{r}{-2b} \end{bmatrix} \begin{bmatrix} \omega_r \\ \omega_l \end{bmatrix} \quad (3)$$

Due to the nonholonomic nature, the mobile robot cannot move sidewise, the knife-edge constraint is described as (4).

$$\dot{y}_g \cos\theta - \dot{x}_g \sin\theta - d\dot{\theta} = 0 \quad (4)$$

B. Dynamic Model

The dynamic equations of the motion of a mobile robot can be deduced by Lagrangian formulation [11] as in (5).

$$M(q)\ddot{q} + C(q,\dot{q}) + F(q,\dot{q}) = B(q)\tau + A^T(q)\lambda \quad (5)$$

where,

$M(q)$ is a positive symmetric inertia matrix.
$C(q,\dot{q})$ represents the Centrifugal & Coriolis force and torques.
$B(q)$ is the matrix of motor torques applied to the robot.
$A(q)$ is the constraints matrix associated with the non-holonomic system.
λ is associated with the independent kinematic constraints.

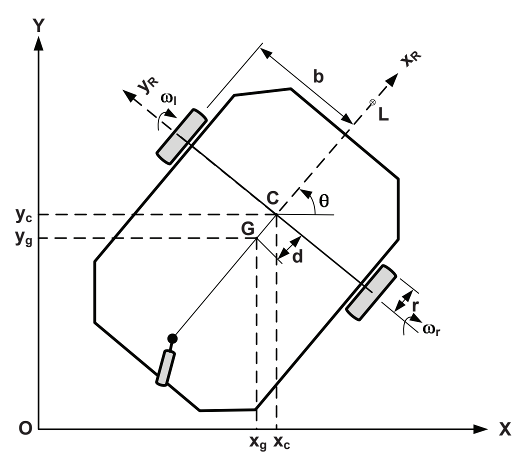

Fig. 1. Structure of mobile robot model

Due to the constraints, there exists a jacobian matrix $S(q)$ that satisfies the following relation (6).

$$\dot{q} = S(q)\eta \tag{6}$$

where, $\eta = [\nu, \omega]^T$.
The constraint can be written in the Pfaffian form [14] as in (7).

$$S(q)^T A(q)^T = 0 \tag{7}$$

Practically, the mobile robot is restricted to its maximum steering angle and maximum speed. The translation and rotational speeds must be within their limits. The turning rate is limited by the speed of the mobile robot. Due to the constraints imposed by the dynamics of the vehicle, it is necessary to pre-analyse the path that is planned, to ensure it is traversable by the mobile robot dynamics.

C. Actuator Model

The mobile robot is driven by two differential DC motors. The mathematical model characterises the behaviour of DC motor controlled by the armature voltage. The dynamic model relating the voltage v to the torque τ is obtained as in (8). The load torque is obtained after the gear box which has a gears reduction ratio. The speed of the wheel $\dot{\Phi}_i$ is related with the rotational force, torque applied to the wheels.

$$\tau = g_r \mu \{K_T(\frac{V_a - K_E g_r \dot{\Phi}_i}{R_a}) - \tau_{fr}\} \tag{8}$$

The motor parameters are provided by the manufacturer (PITTMAN) [15] as given in Table I.

TABLE I
PITTMAN DC MOTOR PARAMETERS

Symbol	Parameter	Value	Units
g_r	Gear ratio	38.3:1	
K_T	Torque-constant	0.023	Nm/A
K_E	Back-EMF constant	0.023	$V/rad/s$
μ	Gearbox efficiency	73	%
τ_{fr}	Friction force	0.0056	Nm
R_a	Resistance	0.71	Ω
L_a	Inductance	0.66	mH
V	Voltage	12.0	v

Most mobile robots have low level PID velocity controllers to track the input reference velocities and do not allow the motor voltage to be driven directly [8]. The PD velocity controller is used in this model to track the reference velocities, which is described by the following equations (9).

$$\begin{bmatrix} v_\nu \\ v_\omega \end{bmatrix} = \begin{bmatrix} K_p(\nu_{ref} - \nu) - K_D(\dot{\nu}_{ref} - \dot{\nu}) \\ K_p(\omega_{ref} - \omega) - K_D(\dot{\omega}_{ref} - \dot{\omega}) \end{bmatrix} \tag{9}$$

Fig. 2. Pioneer 3DX mobile robot

D. Pioneer mobile robot

The Pioneer $3DX$, shown in Fig.2, has high-speed, high-torque, reversible-DC motors, each equipped with encoders for position and speed and advanced dead-reckoning. The mobile robot parameters [16] can be obtained from the mobile robot manufacturer data as shown in Table II. Some of the parameters are estimated.

The pioneer servers use a common PID system with the wheel-encoder feedback to adjust a pulse-width-modulated (PWM) signal at the motor drives to control the power to the motors. The motor-duty cycle is $50\ ms$; pulse-width is proportional $0-500$ for $0-100$ % of the duty cycle. Number of encoder ticks per revolution equals 500. The swing radius of the mobile robot is $0.26\ m$.

TABLE II
PIONEER ROBOT PARAMETERS

Parameter	Value	Units
M_c	15.764	kg
M_w	0.35	kg
r	0.09765	m
b	0.1689	m
d	0.07	m
I_c	15.625	kgm^2
I_w	0.00477	kgm^2
I_m	0.4879	kgm^2

Here, M_c - mass of the robot chassis, M_w - mass of a single wheel & its rotor, I_c - moment of inertia of the robot chassis about the vertical axis, I_w - moment of inertia of the wheel about the wheel diameter. I_m - moment of inertia of the wheel about the wheel axis.

IV. EXPERIMENT

The physical model of the Pioneer mobile robot is incorporated in the simulation model as seen in Fig. 3, which is modelled from the actual motor parameters and mobile robot parameters. The inputs of the model are the desired position (q).

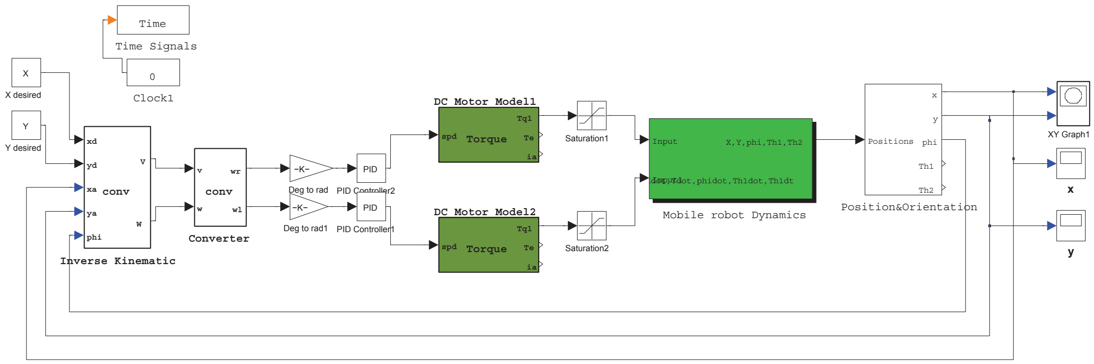

Fig. 3. Robot dynamic model in Simulink

The model is assigned to the following constraints similar to the Pioneer $3DX$ mobile robot.

Max. Linear velocity	$400\ mm/sec$
Max. Angular velocity	$300\ mm/sec$
Heading angle, θ	0
Max. Acceleration	$300\ mm/sec/sec$
Max. Deceleration	$300\ mm/sec/sec$

The PD controller which is used in the model to track the velocity, has the gain value of

$$k_{p_1} = k_{p_2} = 10; k_{D_1} = k_{D_2} = 10$$

For the experiment, the heuristic based search algorithm, D^* lite algorithm is used to get the collision free path from the start to goal position. These set of points which represents the x, y position in the continuous map ($\mathcal{O}$), are smoothed to create a continuous path using cubic spline interpolation.

$$\mathcal{O} : [start, goal] \Rightarrow \mathcal{Q}_{collisionfreepath}$$

The model is tested in two different scenarios as shown in Fig 4 and Fig. 5. In the Figures, the blue circle represents the start position q_{start}, the green rhombus represents the target goal position q_{goal}, the black square blocks and circles denote the obstacles where the robot can't traverse, the purple circles are the set of points which is obtained from the D^* lite algorithm. Cubic splines are used to create a smooth path from the points obtained above.

During path planning, generally, the robot is considered to be a point mass as shown in Fig. 4 and Fig. 5. Due to the physical dimensions of the robot, the posture and the limitations on the achievable velocities/accelerations, the trajectory will get modified as shown in Fig. 6 and Fig. 7. Since the mobile robot has its own dynamic constraints, the path which is traced by the robot, is shown in Fig. 6 and Fig. 7. This indicates the changes in the allowable path by taking the dynamic constraints into account. The simulated model behaviour is tested with the actual pioneer robot for different conditions. The model exactly replicates the mobile robot path with the same time.

Initially the mobile robot is located along the x-axis at $(0, 0)$ in the Scenario 1 as in Fig. 4 and Fig. 5. Due to the dynamics

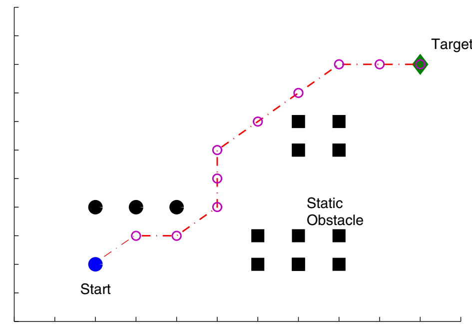

Fig. 4. Scenario 1

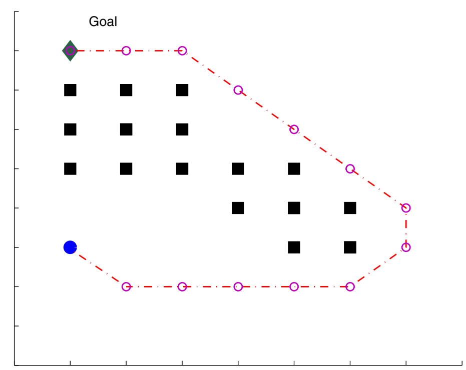

Fig. 5. Scenario 2

of the AGV, it takes a curvy path to reach its next point. The time taken by the Pioneer $3DX$ mobile robot from start location $(0, 0)$ to reach the goal location $(2m, 1m)$ is $7.0sec$ and is the same as the simulation time to reach the target. The model has also been verified for other target conditions as well. The Fig. 8 and Fig. 9 show the path traced by the point L, look-ahead point of the mobile robot for the generated smooth curves.

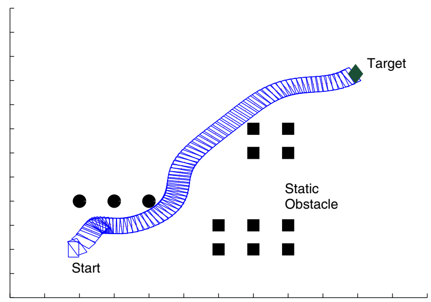

Fig. 6. Scenario 1: Path traced by the robot model

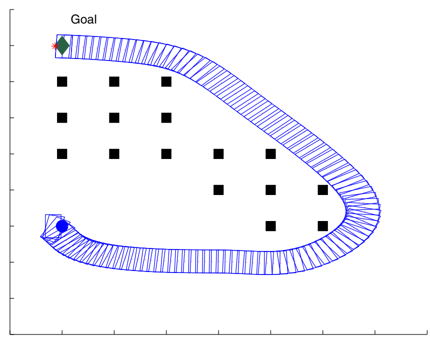

Fig. 7. Scenario 2: Path traced by the robot model

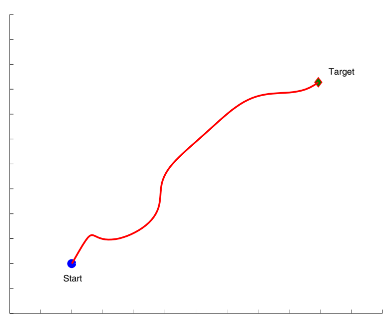

Fig. 8. Scenario 1: Path traced by the look-ahead point, L

V. CONCLUSION & FUTURE WORK

In this paper, the dynamic model of the mobile robot has been simulated and the behaviour is compared with the Pioneer 3DX robot. Then the proposed model has been used for the path planning module. The heuristic based search algorithm, D^* lite algorithm is accomplished by using a cubic spline interpolation to create a smooth path using the kinematic constraints. Our future work will focus on the real time dynamic path planning algorithm in cluttered environment.

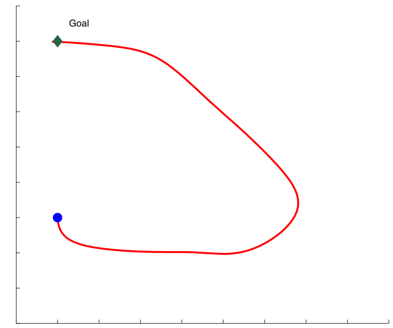

Fig. 9. Scenario 2: Path traced by the look-ahead point, L

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