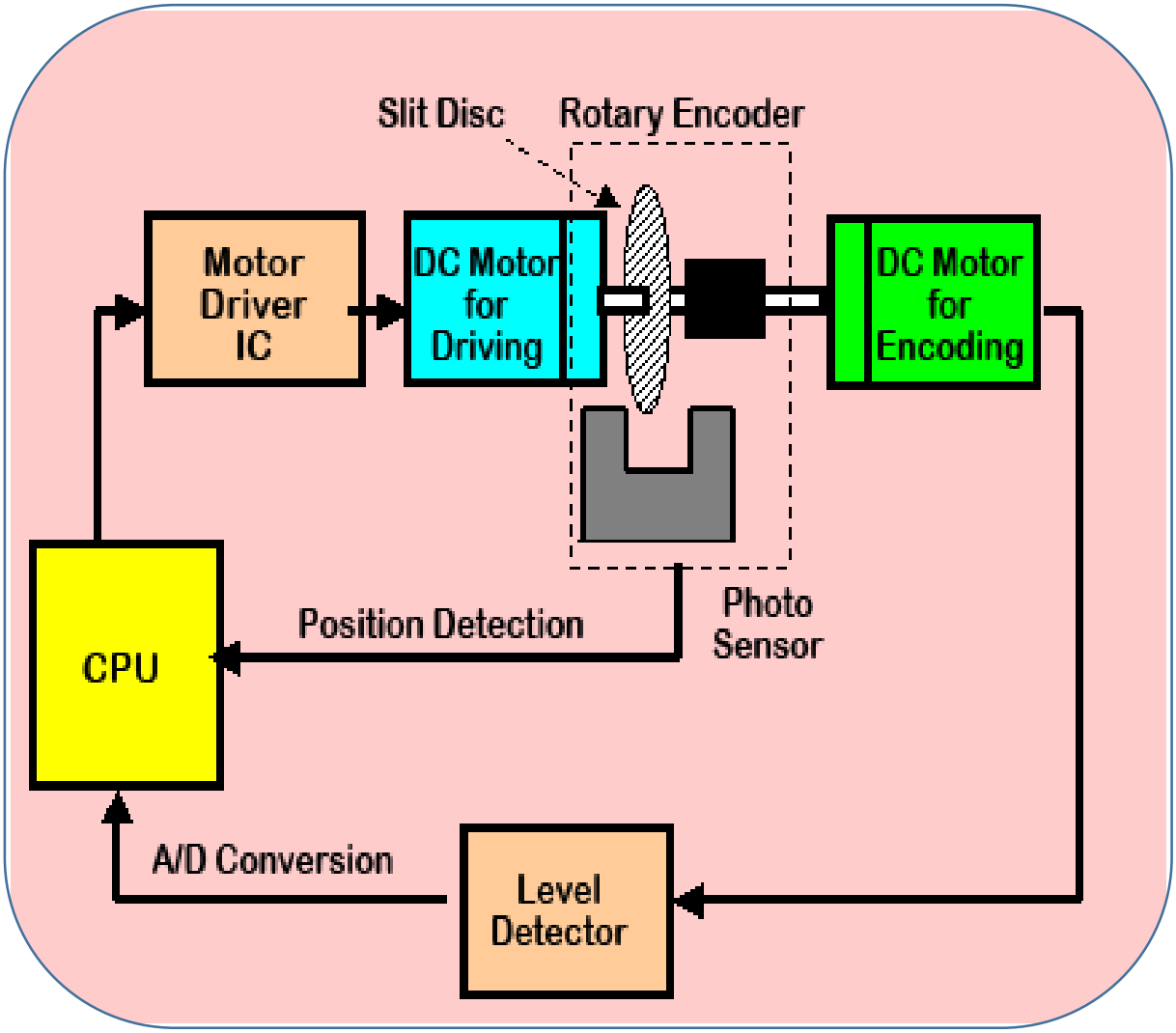
DC MOTOR CONTROLLER DESIGN



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DC MOTOR STATESPACE CONTROL

1. Full-state Feedback Controller DC motor Speed.

A. Mathematical Model of DC motor

A common actuator in control systems is the DC motor. It directly provides rotary motion and, coupled with wheels or drums and cables, can provide translational motion. The electric circuit of the armature and the free-body diagram of the rotor are shown in the following figure 1.1:

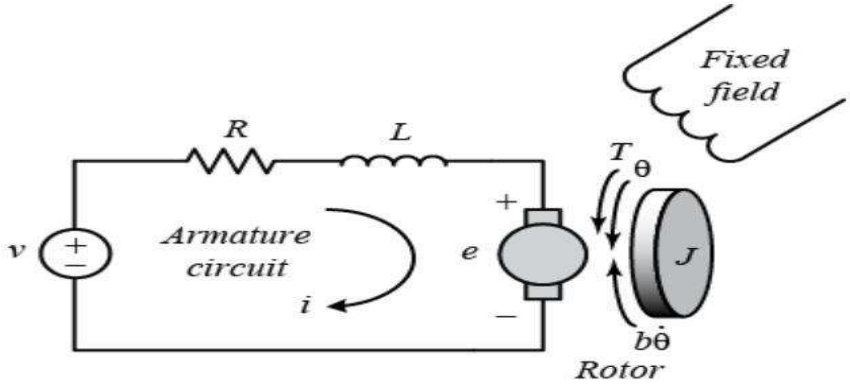


Figure 1.1 Mathematical Model Dc motor.

(http://ctms.engin.umich.edu)

For this project the following values for the physical parameters already knowns and listed in tabel 1.1.

Tabel 1.1 Physical Dc motor parameters.

No	**Parameters**	**Unit**
1	Moment of Inertia of the rotor (J)	0.01 kg.m^2
2	Motor viscous fricttion constant (b)	0.1 N.m.s
3	Electromotive force constant (Ke)	0.01 V/rad/sec
4	Motor torque constant (Kt)	0.01 N.m/Amp
5	Electrical resistance (R)	1 Ohm
6	Electircal Inductance (L)	0.5 H

B. Drive The Equation motion of the system.

B.1 FBD Electrical System.

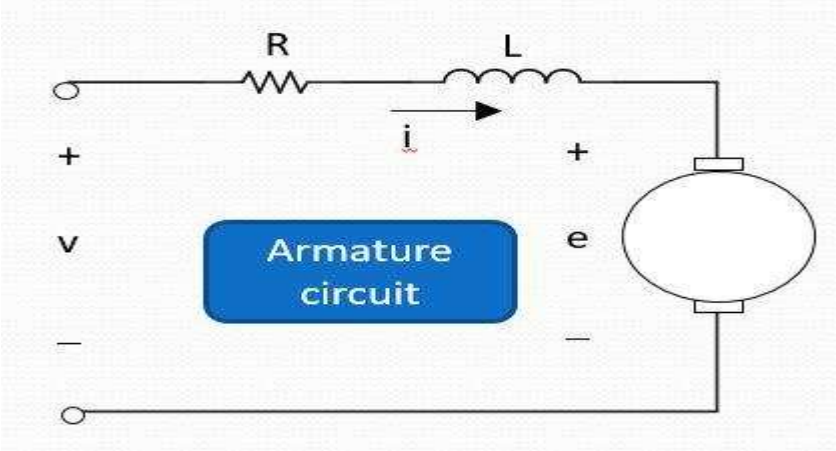


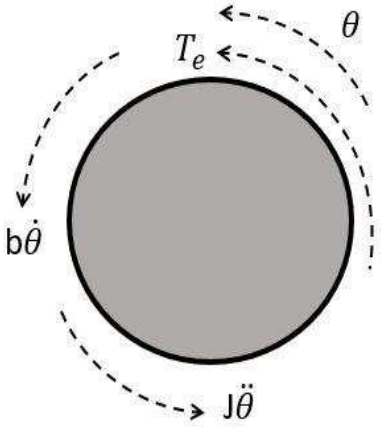
Figure 1.2 FBD Electrical System

$T_e = K\emptyset i = K_t i$ Electromagnectic Torque

$e = K\emptyset\omega = K_e\dot{\theta}$ Armature back e.m.f

$$L_a \frac{di}{dt} = -Ri + V - e$$

B.2 FBD Mechanical System



$$\sum M = 0$$

$$J\frac{d^2\theta}{dt^2} = T_e - b\frac{d\theta}{dt}$$

C. Drive State-space model of the system

Total Equation Motion of the system :

$$\frac{d^2\theta}{dt^2} = \frac{1}{J}(K_t i - b\frac{d\theta}{dt})$$

$$\frac{di}{dt} = \frac{1}{L}(-Ri + V - K_e\frac{d\theta}{dt})$$

The state-space model have the standard form shown below where the state vector x = $\dot{\theta}\imath$ and the input for the system is u = V, and the output velocity which is thetha_dot will be my output desire.

$$x = A\dot{x} + Bu$$
$$y = Cx + du$$

$$x = \frac{d}{dt}\begin{bmatrix}\dot{\theta}\\ i\end{bmatrix} = \begin{bmatrix}-\frac{b}{J} & \frac{K}{J}\\ -\frac{K}{L} & -\frac{R}{L}\end{bmatrix}\begin{bmatrix}\dot{\theta}\\ i\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{L}\end{bmatrix}V$$

$$y = [1 \quad 0]\begin{bmatrix}\dot{\theta}\\ i\end{bmatrix}$$

D. Simulink Model without control

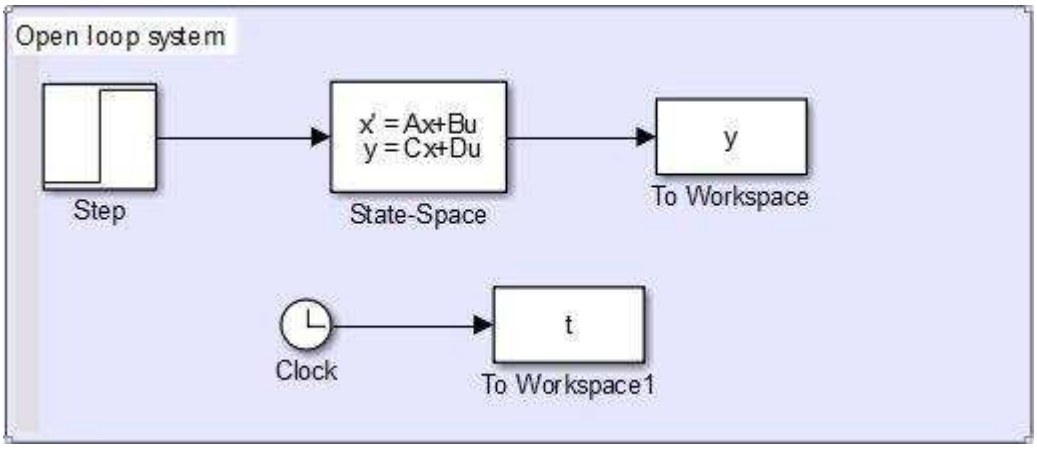


Figure 1.3 Open Loop system Simulink diagram block

E. Matlab Commands.

```
% Define motor Parameters
J = 0.01; % Moment Inertia of the rotor
b = 0.1;  % Motor viscous friction constant
K = 0.01; % Electromotive force constant,motor torque
constant
R  = 1;   % Electrical Resistance
L  = 0.5; % Electrical Inductance

% Define motor state variable model
A = [-b/J K/J;-K/L -R/L];
B = [ 0;1/L ];
C = [ 1 0 ];
D = 0;

plot(t,y,'g','linewidth',2);
xlabel('time in second');
ylabel('Angular Velocity (rad/s)');
title('Step respon for Open Loop System');
grid on
```

F. Plotted result of open loop system

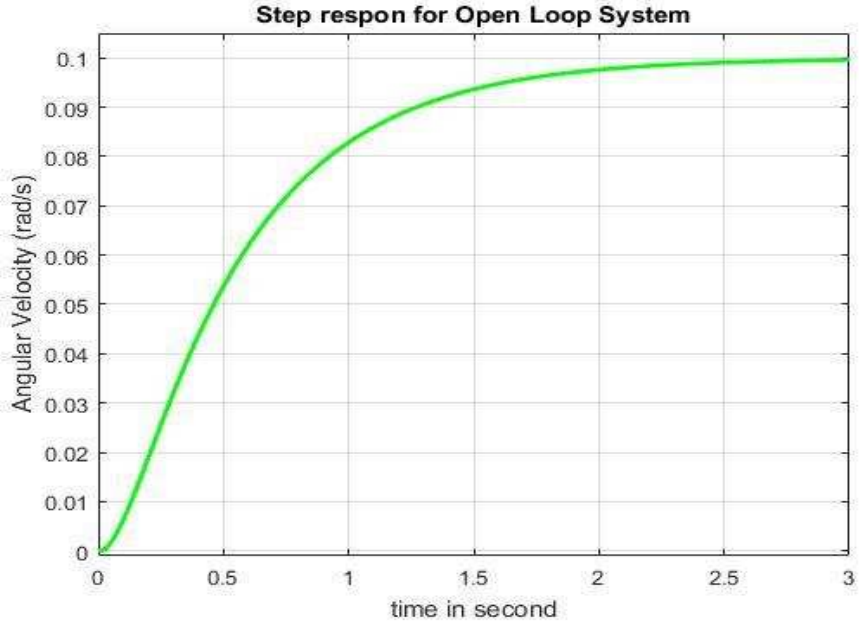


Figure 1.4 Step respon plot for Open loop system

By using **stepinfo** command below, we obtain the performances of the openloop system.

```
%Obtain Open-loop Performance
S = stepinfo(sys)

And the result are:

                    S =

              RiseTime: 1.1351
          SettlingTime: 2.0652
           SettlingMin: 0.0899
           SettlingMax: 0.0998
              Overshoot: 0
             Undershoot: 0
                   Peak: 0.0998
              PeakTime: 3.6758
```

As you can see , when the system was given an input voltage about 1 Volt, the motor can only achieved the maximums angular speed about 0.1 rad/sec and become stable at 3 second, the system has a large steady state error when the final value of the step input reference was given by default 1 rad/s.

G. Designing the full-stateback controller

Full state feedback (FSF), or pole placement, is a method employed in feedback control system theory to place the closed-loop poles of a plant in pre-determined locations in the s-plane. Placing poles is desirable because the location of the poles corresponds directly to the eigenvalues of the system, which control the characteristics of the response of the system.

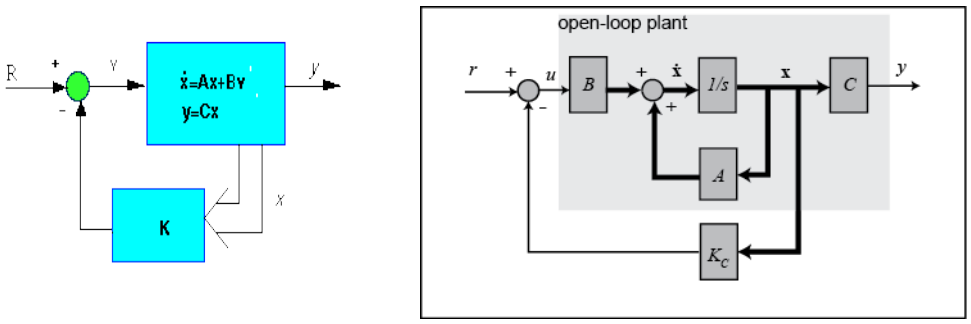


Figure 1.5 Closed Loop Full state feedback controller in state-space model

G.1 Design Criteria

For a 1-rad/sec step reference, the design criteria are shown in table 1.2

Table.1.2 Design criteria

Settling time	**<**	**2 second**
Overshoot	<	5%
Steady-state error	<	1%

G.2 Simulink Block Diagram

The Simulink block diagram are used to get the model and design the pole placement controller with goal to achieved the design criteria.

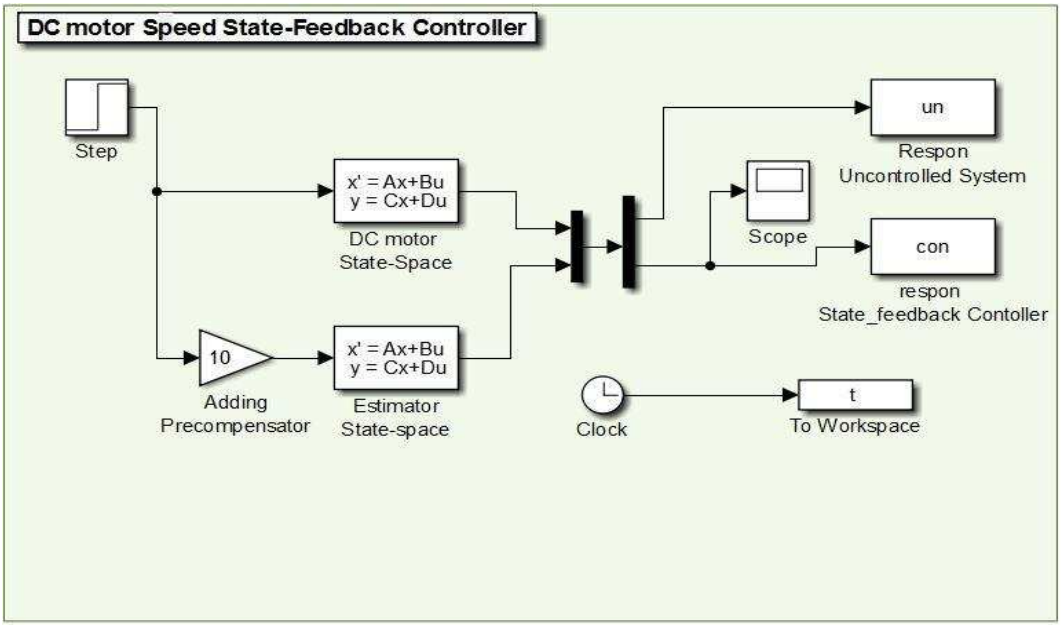


Figure 1.6 Simulink Block Diagram State-feedback Controller DC motor.

First Statespace block contains the initial state and output matrix of Dc motor and the second statespace block contains the estimator matrix where the feedback matrix came in the form look like this $sI - (A - B*Kc)$ here s is the Laplace variable. Since the matrices A and $B*Kc$ are both 2x2 matrices, there should be 2 poles for the system. To eliminate the steady state error we need to ***adding precompensator*** . It could be done by simply using ***rscale*** function in matlab command, but in this project we've using gain by the value of ten, so that the output in turn is scaled to the desired level.

G.3 Matlab Source-Code

Before running the simulink file, the matlab command to define all the parameters must be done.

```
% Define motor Parameters
J = 0.01; % Moment Inertia of the rotor
b = 0.1;  % Motor viscous friction contant
K = 0.01; % Electromotive force constant, motor torque constant
R  = 1;   % Electrical Resistance
L  = 0.5; % Electrical Inductance
% Define motor state variable model
A = [-b/J K/J;-K/L -R/L]
B = [ 0;1/L ]
C = [ 1 0 ]
D = [ 0]

% check observability
O = obsv(A,C)
rank(O)

% Obtain feedback gain by placing 2 poles with vary value
p1 = -7;      % Change based on table 1.3
p2 = -7;      % Change based on table 1.3
K = acker(A,B,[p1 p2])

% Define the Estimator statespace
Aes= A-B*K   % Change matrix A , the rest still same

sys=ss(Aes,B,C,D);

% plot the root locus
subplot(2,1,1);
pzmap(sys)

% Plotting State Respon with State-Feedback controller
Uncontrolled_system = un;
With_state_feedback = con;
subplot(2,1,2);
plot
(t,Uncontrolled_system,'r',t,With_state_feedback,'b','linewidth',2
);
xlabel('Time in second');
ylabel('Amplitude (rad/s)');
title ('State Respon Uncontrolled VS State-feedback controller')
legend ('Uncontrolled system','With state feedback');
grid on
hold off

% Obtain the system performances
S= stepinfo(con,t)
```

G.4 Vary pole location

To know how the system behave cause of changing of the poles value, we need to try several poles value. Table 1.3 shown vary poles location.

Table 1.3 Vary poles location

Type of poles	Poles 1	Poles 2
Real number same value	-7	-7
Real number different value	-6.5	-4.35
Complex number	-5+1.5i	-5-1.5i
Complex number	-3+3.5i	-3-3.5i

G.4.1 System respon.

1. The root locus and respon for twin pole (±7) in the real part

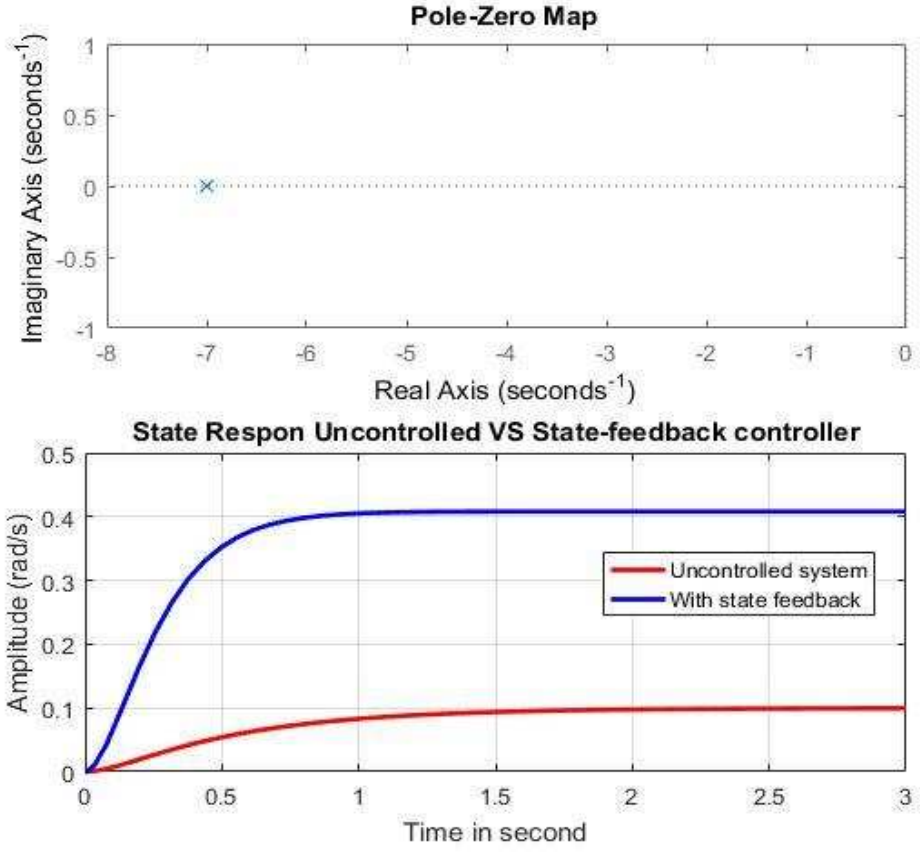


Figure 1.7 Root locus and respon with poles place at (±7)

The system has performance shown below:

Settling time	**0.8359s**
Overshoot	0
Steady-state error	<1%
Rise time	0.4804
Peak time	3

2. The root locus and respon of differents poles (-6.5 and -4.35) in the real part.

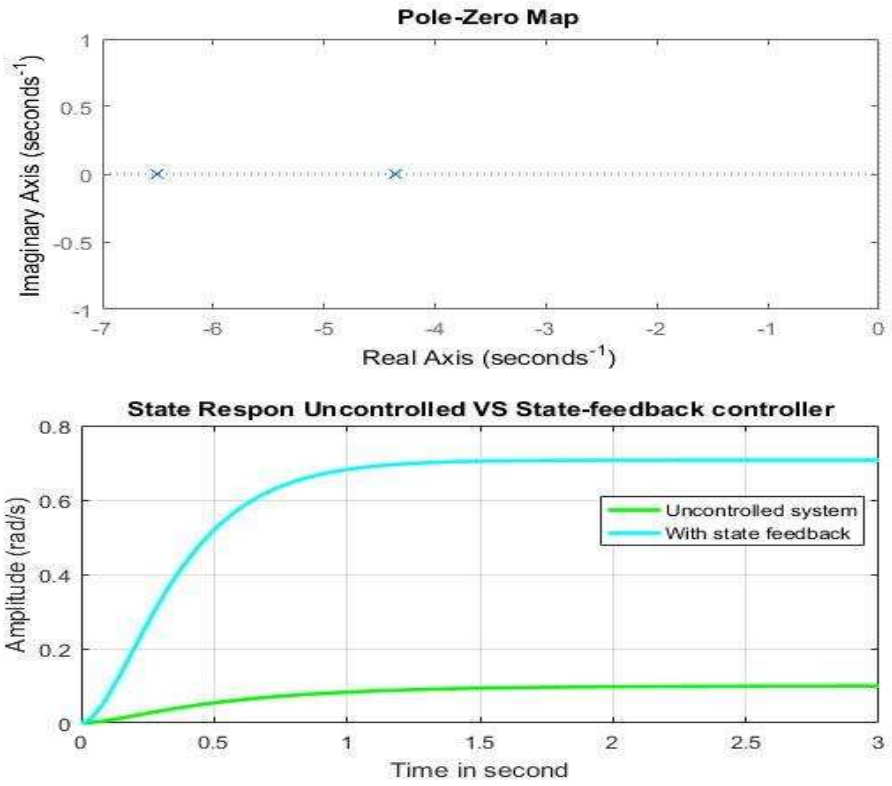


Figure 1.8 Root locus and respon with poles place at (-6.5 and -4.35)

The system has performance shown below:

Settling time	**1.1417**
Overshoot	0
Steady-state error	<1%
Rise time	0.6523
Peak time	3

3. The root locus and respon of complex poles (-5±1.5i)

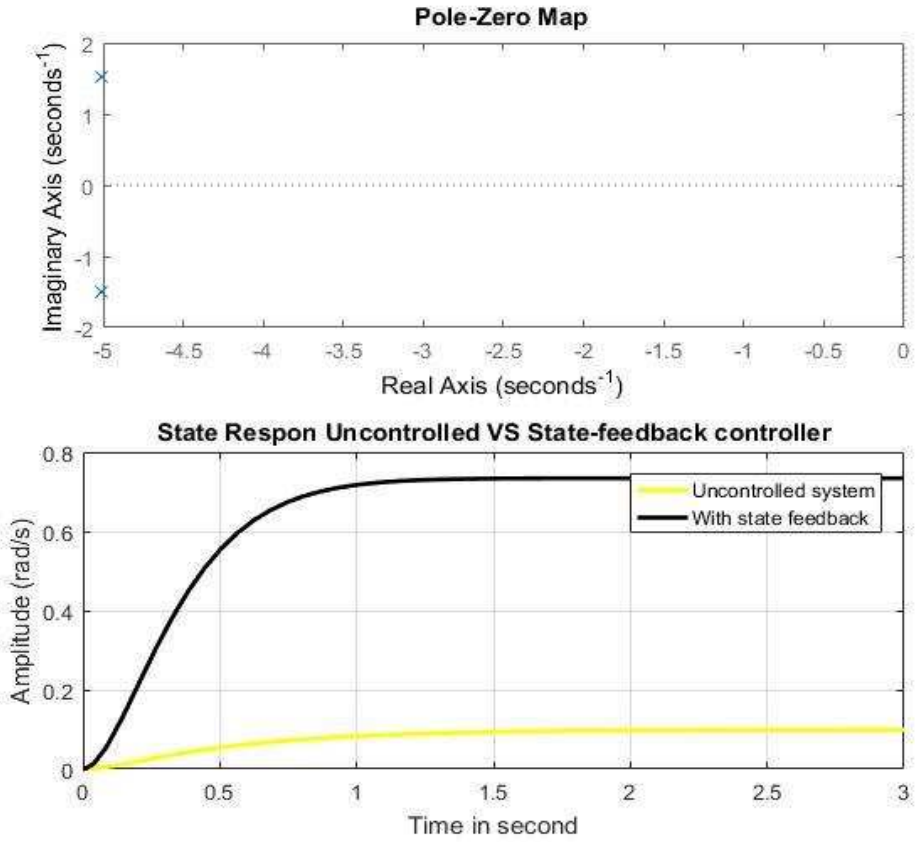


Figure 1.9 Root locus and respon with poles place at (-5±1.5i)

The system has performance shown below:

Settling time	**1.0270**
Overshoot	0.0027
Steady-state error	<1%
Rise time	0.6069
Peak Time	2.1252

The root locus and respon of complex poles (-3±3.5i)

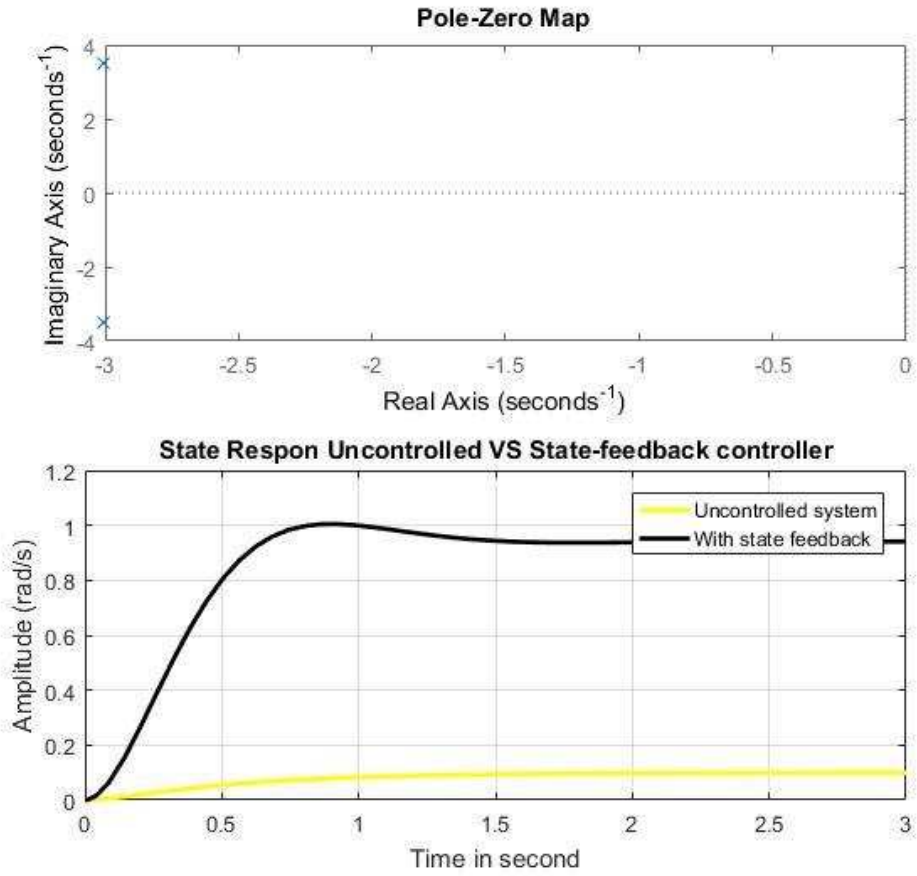


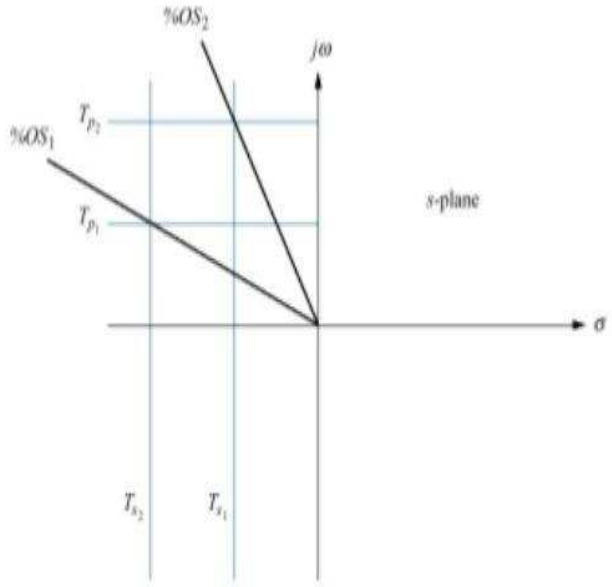
Figure 1.9 Root locus and respon with poles place at (-3±3.5i)

The system has performance shown below:

Settling time	**1.3029**
Overshoot	6.711
Steady-state error	<1%
Rise time	0.4347
Peak Time	0.9252

H. Analysis of poles location.

We've been simulate and plotted all the poles location in table 1.3 and the result has shown above. The relationship between the pole location and the specification of the system are demostrate by figure 1.20 below.



$T_{s2} < T_{s1}$

$T_{p2} < T_{p1}$

$\%OS_1 < \%OS_2$

Figure 1.20 pole location in the s-plane

Since horizontal lines on the s-plane are lines of constant imaginary value, they're also lines of constant peak time, and the vertical lines on the s-plane are lines of constant real value, they're also the lines of constant settling time. As the poles move in a horizontal direction either right or left, the real part has change and keeping the imaginary part remain constant. This condition will change the setlling time performance, as we can see from figure 1.7-1.9, we were changing the poles location move to the right it caused the settling time performance increased. But at figure 1.6-1.7 has the same peak time, it maked sense because the system weren't having any imajinary part, so the imajinary part remain the same.

Let us now figure out what happens at figure 1.8-19, obviously the system now has imajinary part, since $\zeta = \cos\theta$ represent overshoot and as we move the poles in vertical direction its increasing the overshoots because its change the radial position of θ.

2. Full-state Feedback+ Integrall Controller DC motor Angular Position.

We've been created speed controller for Dc motor where the output comes out from the statespace matrix is velocity. Now we'll considering controller for angular position of the rotor. To do that, we take the same model as shown above but we need some modification through the statespace matrix and the parameters listed in table 1.4 below to get poper model.

Table 2.1 Phsiycal parameters for Dc motor

No	**Parameters**	**Unit**
1	Moment of Inertia of the rotor (J)	3.2284E-6 kg.m^2
2	Motor viscous fricttion constant (b)	3.5077E-6 N.m.s
3	Electromotive force constant (Ke)	0.0274 V/rad/sec
4	Motor torque constant (Kt)	0.0274 N.m/Amp
5	Electrical resistance (R)	4 Ohm
6	Electircal Inductance (L)	2.75E-6 H

A. Drive the state space model for the system

From the main problem, the dynamic equations in state-space form are given below.

$$\frac{d}{dt}\begin{bmatrix}\theta\\ \dot{\theta}\\ i\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & -\frac{b}{J} & \frac{K}{J}\\ 0 & -\frac{K}{L} & -\frac{R}{L}\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\\ i\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \frac{1}{L}\end{bmatrix}V$$

$$y = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\\ i\end{bmatrix}$$

For the initial respon from the system is pretty close the same with Dc motor speed shown in sub section D until F with all we need just change the matrices composition into 3x3 matrices and we good to go. The respon of uncotrolled system shown in figure 1.21 below :

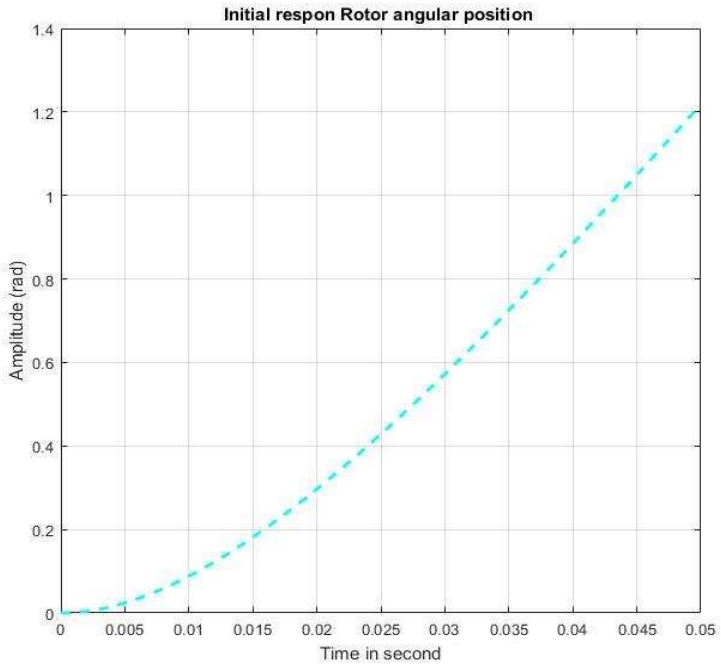


Figure 2.1 Respon the angular position of rotor without controller

As we can see, its seem like the amplitude of the angular position goes to the infinity . The system performance listed below :

```
S =

Rise time : 0.0344      Settling time: 0.0493

Overshoot : 0             Peak time : 0.05
```

B. Designing the full-stateback controller

The control law for a full-state feedback system has the form u =r –K*x , the characteristic polynomial for this closed-loop system os the derminant of sI-(A-B*K) where s is the laplace variable, the associated block diagram is given by figure1.5. Since the matrices of A and B*K are both 3x3 matices, it should be 3 poles exist in the system. Before we create the controller we need to verify our system is controllable or not, by checking through the system order and its deteminant. For this case the design criteria and the vary poles location has given by table 2.1. and table 2.2

Table 2.1 Given Design Criteria with a 1-radian step reference

Settling time	**<**	**2 second**
Overshoot	<	5%
Steady-state error	<	1%

Table 2.2 Vary poles location

Type of poles	Poles 1	Poles 2	Pole 3
Real number same value	-450	-450	-450
Real number different value	-735	-567	-325
Complex number	-255+50i	-255-50i	-100
Complex number	-100+100i	-100-100i	-200

C. Disturbance Respon

In order to observe the system's disturbance response, we must provide the proper input to the system. In this case, a disturbance will fit as a physically a load torque that acts on the inertia of the motor. This load torque acts as an additive term in the second state equation. We can simulate this by modifying our close-loop input matrix, to have a 1/J in the second row what will act like our current input is only the disturbance. We can perform this by define the B matrices in matlab command below, so it can be use to simulate the whole system through our simulink file.

C.1 Simulink block Diagram

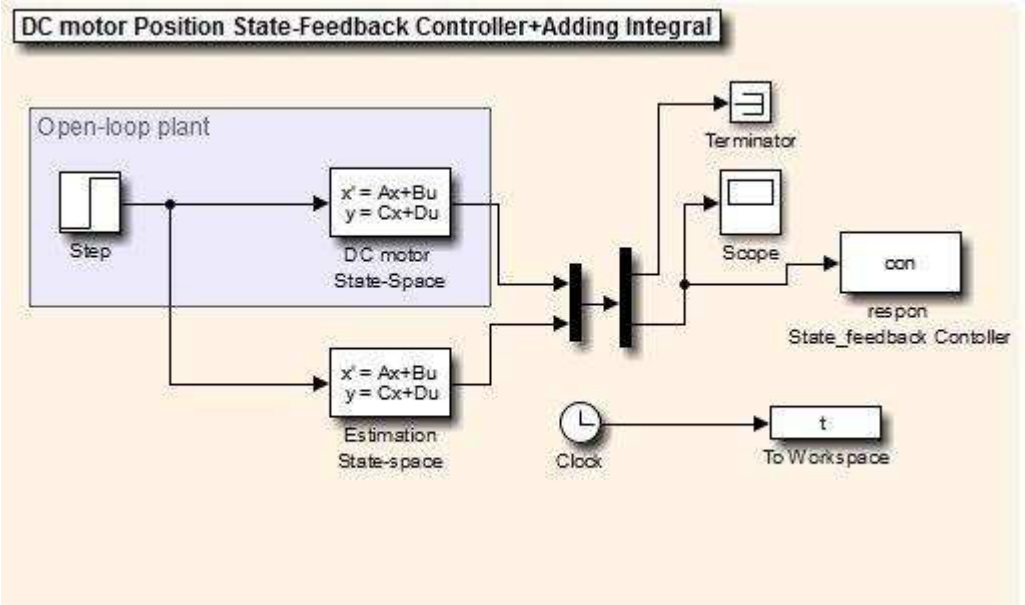


Figure 2.2 Dc motor position block diagram with feedback and disturbance

C.2 Matlab Source-Code

```
% Define motor Parameters
J = 3.2284E-6; % Moment Inertia of the rotor
b = 3.5077E-6; % Motor viscous friction contant
K = 0.0274;    % Electromotive force constant, motor torque
R  = 4;        % Electrical Resistance
L = 2.75E-6; % Electrical Inductance
```

```
% Define motor state variable model
A = [0 1 0; 0 -b/J K/J; 0 -K/L -R/L]
B = [0 ; 0 ; 1/L]
C = [1  0  0]
D = [0]

% check observability and controllable
determinant = det(ctrb(A,B))
O = obsv(A,C);
rank(O)


% Obtain feedback gain by placing 3 poles with vary value
p1 = -450; % Change based on table 2.2
p2 = -450; % Change based on table 2.2
p3 = -450; % Change based on table 2.2
K = place(A,B,[p1, p2, p3])

% Modifying the close-loop input matrix B
F = A-B*K   % state feed back  A matrices
Bd = [0; 1/J ; 0] % Disturbance input matrices, the rest are still
same.

% Plotting State Respon with State-Feedback controller
Disturbance = con;
plot (t,Disturbance,'linewidth',2 );
xlabel('Time in second');
ylabel('Amplitude (rad)');
title ('Disturbance respon of rotor angular position')
grid on

% Obtain the system performances
S= stepinfo(con,t)
```

D. Plotted result for disturbance respon and vary poles location

1. The Disturbance and feedback respon when same poles place at (-450)

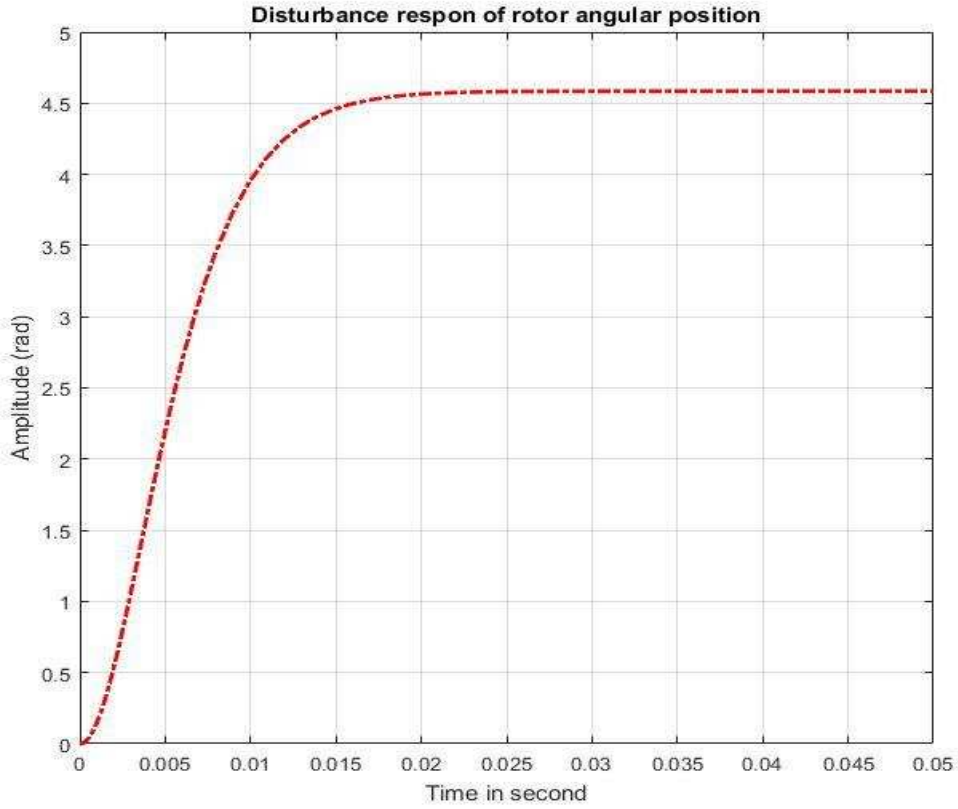


Figure 2.3 Disturbance respon of rotor angular position at poles location (-450)

The system performance listed below :

```
S =

Rise time : 0.0092      Settling time: 0.0159

Overshoot : 0           Peak time : 0.05
```

2. The Disturbance and feedback respon when differents poles place at (-735 -567 -325).

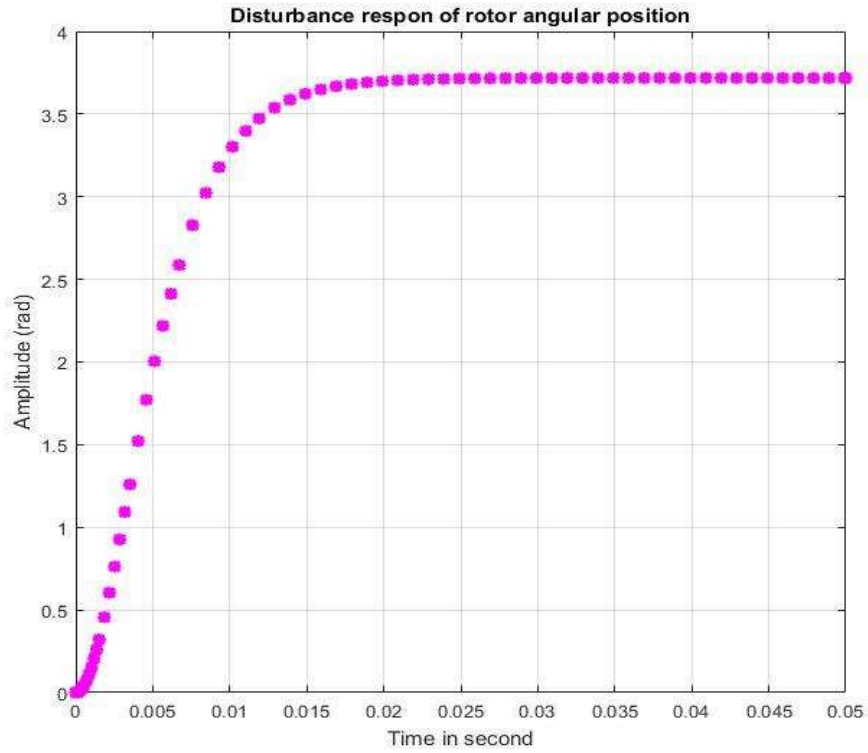


Figure 2.3 Disturbance respon of rotor angular position at poles location (-735 ,-567, -325).

The system performance listed below :

```
S =

Rise time : 0.0089      Settling time: 3.3984

 Overshoot : 0            Peak time : 0.05
```

3. The Disturbance and feedback respon when complex poles place at (-255+50i, -255-50i, -100).

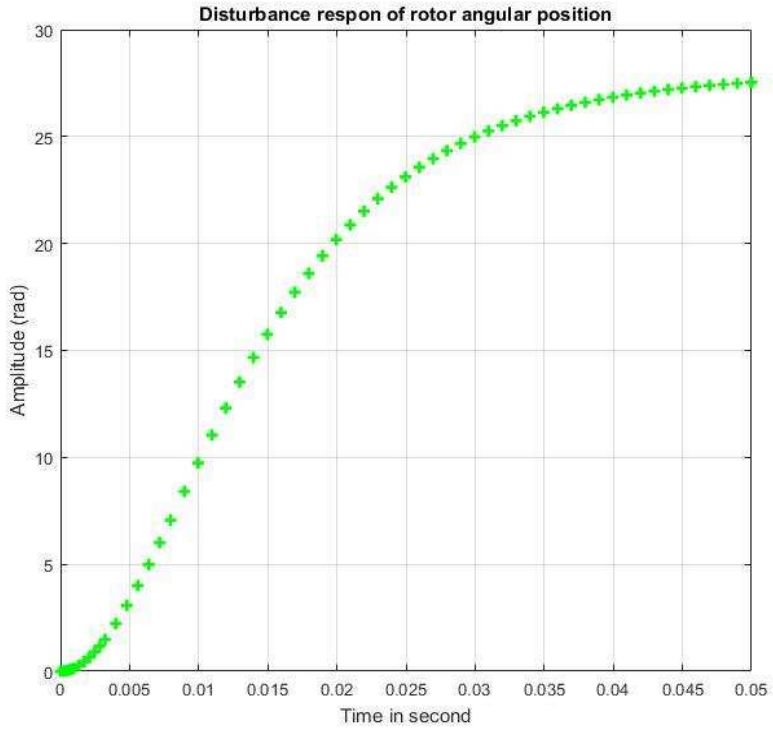


Figure 2.4 Disturbance respon of rotor angular position at poles location (-255+50i, -255-50i, -100).

The system performance listed below :

```
S =

Rise time : 0.0248      Settling time: 0.0414

 Overshoot : 0            Peak time : 0.05
```

4. The Disturbance and feedback respon when complex poles place at (-100+100i, -100-100i -200).

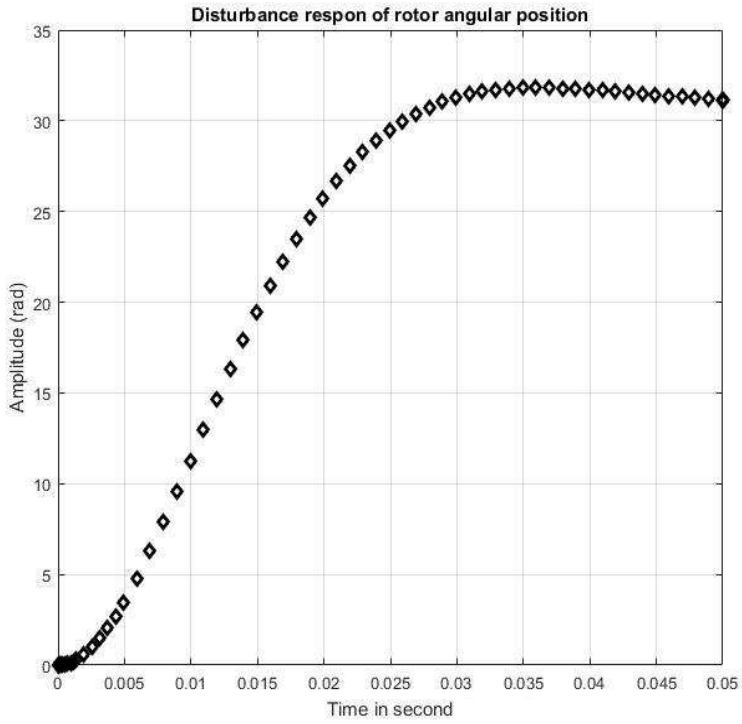


Figure 2.5 Disturbance respon of rotor angular position

at poles location (-100+100i, -100-100i, -200).

The system performance listed below :

```
S =

Rise time : 0.018        Settling time: 0.0382

 Overshoot : 0 2.1132        Peak time : 0.00359
```

E. Adding integral action

Putting an extra integrator in series with the plant it can remove the steady-state error due to a step reference. If the integrator comes before the injection of the disturbance, it will also cancel a step disturbance input in steady state. the associated block diagram is given by figure 2.4.

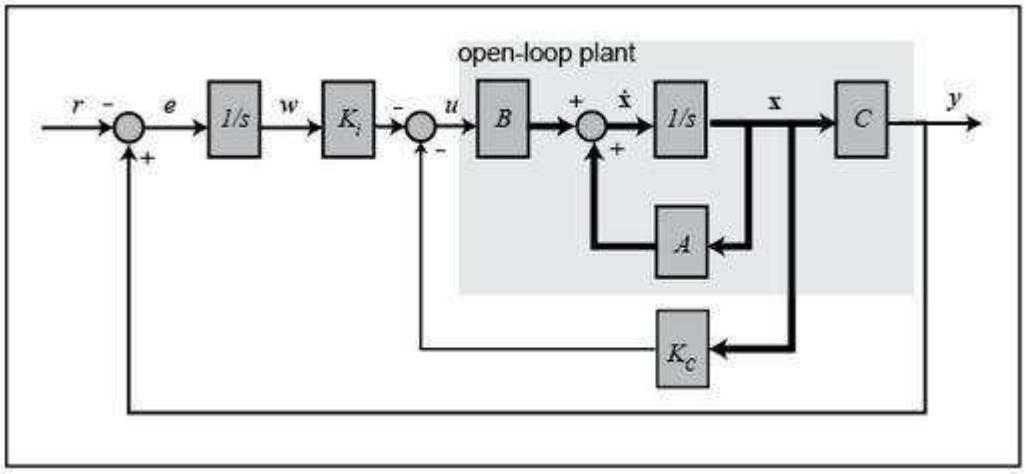


Figure 2.6 Associated block diagram for series adding integral with the plant

We can model the addition of this integrator by augmenting our state equations with an extra state for the integral of the error which we will identify with the variable *w*. This adds an extra state equation, where the derivative of this state is then just the error, $e = y - r$ where $y = theta$. This equation will be placed at the bottom of our matrices. The reference *r*, therefore, now appears as an additional input to our system. We're gonna need to rebuilt the estimator block in figure 2.1 into augmented block by implemented the new 4x4 matrices shown below.

$$\frac{d}{dt}\begin{bmatrix} \theta \\ \dot{\theta} \\ i \\ w \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{b}{J} & \frac{K}{J} & 0 \\ 0 & -\frac{K}{L} & -\frac{R}{L} & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \theta \\ \dot{\theta} \\ i \\ w \end{bmatrix}$$

$$+\begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \\ 0 \end{bmatrix} V + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \theta \\ \dot{\theta} \\ i \\ w \end{bmatrix}$$

The augmented additional integrator state are represent by matrices Aa, Ba, Ca and Da. In order to find the closed-loop equations, all we need to do is to look at how the step input, affects the plant. In this case, it affects the system in exactly the same

manner as in the unaugmented equations except now u = -*Kc* **x** - *Ki w*. We can also rewrite this in terms of our augmented state as u = -*Ka* **xa** where *Ka* = [*Kc Ki*]. Substituting this u into the equations above provides the following closed-loop equations.

$$\dot{X}_a = (A_a - B_a K_a)x_a + B_r r$$

$$y = C_a x_a$$

As we can see the integral of the error will fed back again, then it would eliminate the steady state error goes to zero. Since the augmented matrices has 4x4 form,it should have 4 poles inside, moreover we need to adding fourth pole into the system we will use -600.

E.2 Matlab Source-Code

We used the same simulink block in figure 2.2 then we change all the parameters based on the matlab command below.

```
% Obtain augmented 4x4 state matrices Aa, Ba, Ca, Da
Aa = [0 1 0 0; 0 -b/J K/J 0 ; 0 -K/L -R/L 0; 1 0 0 0];
Ba = [0 ; 0 ; 1/L ; 0 ];
Br = [0 ; 0 ; 0; -1];
Ca = [1 0 0 0];
Da = [0];

% Obtain feedback gain by placing 4 poles and other poles same as
before
p1 = -100+100i;
p2 = -100-100i;
p3 = -200;
p4 = -600;
Ka = acker(Aa,Ba,[p1, p2, p3,p4])

% Define the augmanted statespace
G = Aa-Ba*Ka
H = Br
I = Ca
J = Da

% Plotting State Respon with State-Feedback controller
integral = con;
plot (t,integral,'','linewidth',2 );
xlabel('Time in second');
ylabel('Amplitude (rad)');
ylim([0 1.05]);
title ('Rotor respon with Adding integral action')
grid on

% Obtain the system performances
S= stepinfo(con,t)
```

F. Plotted result for adding integral action

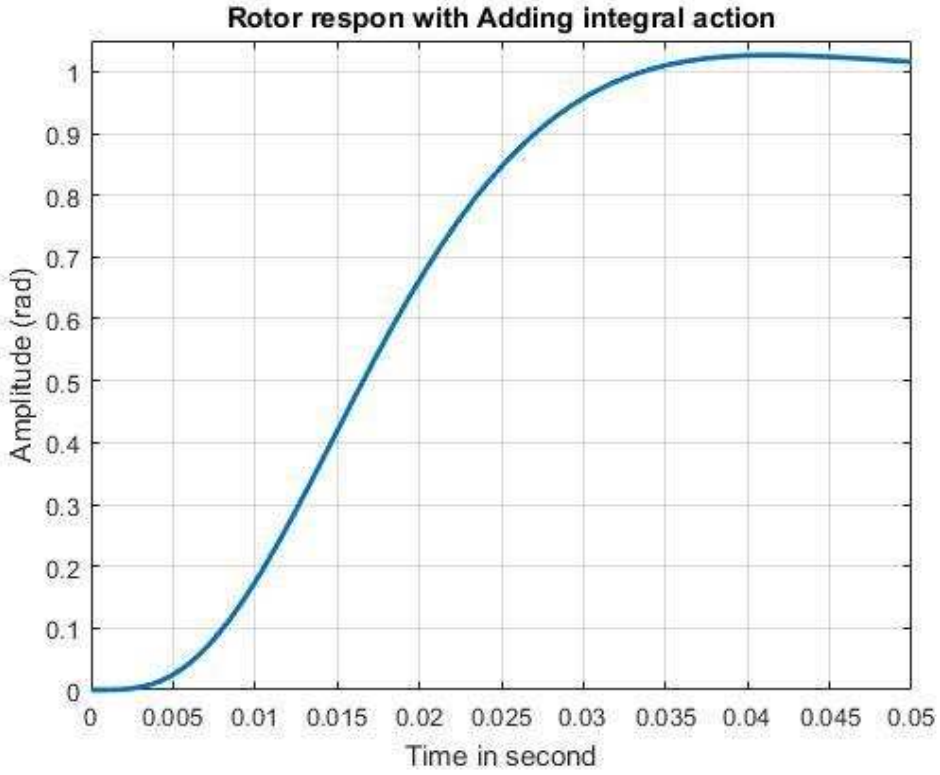


Figure 2.6 Disturbance respon of rotor angular position

at poles location (-100+100i, -100-100i, -200, -600).

The system performance listed below :

```
S =

Rise time : 0.0196      Settling time: 0.0331

Overshoot : 1.0634      Peak time : 0.0413
```

G. Analysis of poles location.

The effect of varying poles location it seem pretty close the same with case of created controller for Dc motor speed. When we gave the system with real poles move further to the left it will change the setlling time of the system, since we only had poles in real part, no matter how far we drag it to the left the peak time of the system will be the same due to constant imajinary part of the system. The inversly for peak time, peak time only change when we drag the poles vertically since now the imajinary axis take a part. But when we move the poles diagonaly since the ωn is fuction of radial distance from the origin, it will increase the system frequency while the envelope of the system remains the same. The number of imajinary part

in poles also effect the overshoot of the system since the overshoot is a fuction of zheta. Adding extra integral will remove the steady state error of the system, obviously this could happens because when we add integral in series with the plant, it change the whole system type. Since steady state-state error are depend upon the number ontegration in the forward path. We already know, since we define the system type to be the value of n in the denominator or eqivalently, the number of pure integration in the forward path. Therefore,a system with n = 0 is a Type 0 system. If n = 1 or n = 2, the coressponding system is a type 1 or type 2 system. Thats why when we adding integral the type of system change into type 1 or 2 when we know the value of steady state error with step input u(t) would be zero.