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# Basic Motion Model of Autonomous Ground Vehicle

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**Abstract:** This paper deals with the basic design and implementation of the mathematical motion model of an autonomous unmanned ground vehicle. This model comes from the specific construction of the vehicle, which was designed in order to verify and demonstrate its autonomous motion possibilities indoors and outdoors. This article presents two different issues of the basic model: a) computing vehicle position depending on values of parameters of its motion system and b) deriving values of these parameters according to current demands for movement of the vehicle from a current position to a new position.

**Key-Words:** - model, unmanned vehicle, autonomous motion, manual control, autonomous control

## 1 Introduction

The importance and influence of unmanned vehicles is more and more increasing; there are a lot of possibilities of their utilization both in the civil and military area. Unmanned vehicles can operate in different types of environment; we can distinguish these vehicles as aerial, ground, surface, underwater and space (satellite).

Together with the development of quality of technical equipment and sensorial systems, the software progress and improvement is being carried out. The tendency to automate as many as possible manual actions, which an operator has had to deal with so far, is enormous. According to the degree of automation, we can define unmanned vehicles as semiautonomous or autonomous.

The main aim is to automate significant processes such as decision support processes, automatic searching, surveying, monitoring and identifying objects and targets, autonomous motion of vehicles in various environments etc. Ground vehicles create real challenges particularly for the last mentioned area; this is a very complex process with many connected steps to be dealt with.

University of Defence in Brno, the Czech Republic, within its long-term research deals with (among others) development of unmanned ground and aerial vehicles determined for tactical utilization. One of many goals of this research consists in solving autonomous motion of ground vehicles indoors and outdoors.

The department of military management and tactics has recently developed an experimental vehicle in order to verify and demonstrate the basic principles of the autonomous motion process. The next parts briefly introduce the design of the motion model applicable also to other types and variants of ground vehicles.

## 2 Experimental unmanned vehicle

The construction of our experimental vehicle is relatively simple. Autonomous motion of this vehicle is based on laser scanning of its surrounding environment. Firstly the configuration of objects and obstructions in the area is created; then it is used for finding an optimal path to a target position. More information about the above mentioned principle can be found in [1], [2].

During the motion of the vehicle along the found path, information about environmental configuration is steadily elaborated and supplied. The whole principle seems to be relatively simple. The main problem, however, consists in keeping the information about the precise position of the vehicle in the area. While moving the vehicle outdoors it can be solved via a GPS receiver integrated with an inertial navigation system elaborating GPS data regularly.

A GPS receiver cannot be used in buildings and closed areas since there is no GPS signal available. Different principles must be used. In our case, it is a set of three activities cooperating together with very precise results:

1. Estimation of an approximated vehicle position via our mathematical model of its motion.
2. Elaboration of the position via a digital magnetometer and accelerometer.
3. Further elaboration of the position using comparison with configurations of the surrounding environment obtained from the initial and new position of the vehicle.

The first step of the whole process, i.e. the design of the mathematical motion model of the vehicle, is elaborated in this article in more detail. The article presents the two basic issues:

1. Estimation of a vehicle position depending on values of parameters of its motion system (so-called manual control).
2. Computation of optimal values of parameters of the motion system, which has to be set in order to move the vehicle into the demanded position (so-called autonomous or position control).

The first model is used for manual control of the vehicle by an operator. The second model is intended for autonomous control when the vehicle moves along the found optimal path step by step.

### 3 Model of manual control

Before concerning the design of the model itself, we need to define the basic architecture of the vehicle. As already mentioned, the architecture is quite simple. The vehicle is propelled by a pair of servomotors made by TG Drives; each servomotor propels one driving wheel. The control is provided via a control unit connected to other peripheral devices such as communication unit, digital magnetometer and accelerometer, laser scanning device and several other devices (electronic elements, storage batteries, voltage converter etc.).

The architecture of our experimental vehicle is presented in figure 1; the size of the chassis is 610 x 500 mm. However, appearance of the vehicle is not the top priority now; the main goal and purpose of the vehicle is to verify and demonstrate the fundamental principles of its autonomous motion.

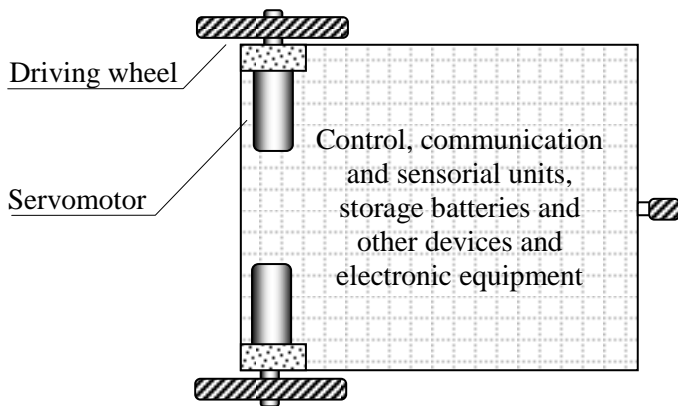


Fig. 1 Architecture of the experimental unmanned vehicle

The model for manual control estimates the position of the vehicle during its motion when controlled by an operator. The operator controls speed and direction of the vehicle by setting rotating speed of both servomotors; i.e. only two variables are set at arbitrary instants of time during manual control of the vehicle.

The position of the vehicle during its motion is estimated by reading positions of both servomotors at

regular time periods (about 100 ms). Servomotors provide precise values of their actual turning in the real time in the range of 0 to 65,536. Moreover, each servomotor is equipped with a gearbox with the ratio of transmission 1:73; so the rotation of a driving wheel  $n$  in the range of  $0^\circ$  to  $360^\circ$  is defined by values from 0 to  $73 \cdot 65,536$ .

A distance covered by a driving wheel during the servomotor rotation within one short time period  $\Delta t$  is expressed in formula (1).

$$\Delta d = \Delta n \cdot \frac{2\pi \cdot r_w}{73 \cdot 65,536} \quad (1)$$

where  $\Delta d$  is a distance covered by the driving wheel within a time period  $\Delta t$  [m],

$\Delta n$  is a change of the servomotor position within a time period  $\Delta t$  [-],

$r_w$  is a radius of the driving wheels [m].

The position of the vehicle in the area is defined by three parameters  $x, y, \varphi$  according to figure 2. The aim of this model is to determine the new position of the vehicle depending on the distances  $\Delta d_l$  and  $\Delta d_r$  covered by the left and right driving wheel within a time period  $\Delta t$  (see figure 2a). Figure 2b presents the turning radius of the vehicle  $r_r$  during its motion around its right wheel.

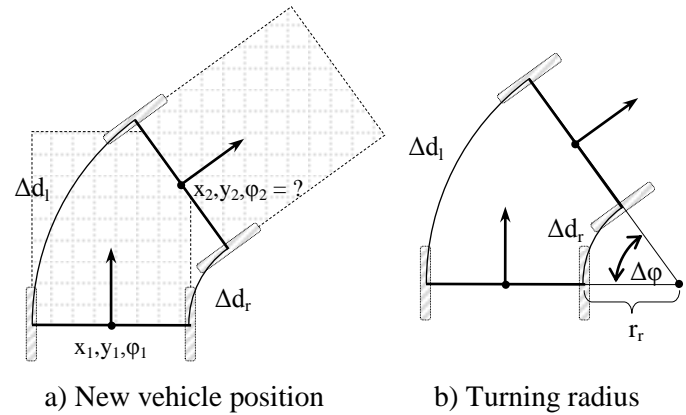


Fig. 2 Position of the vehicle in the area

The first step when computing the new position is to determine the turning radius of the vehicle along one of its driving wheels (see figure 2b). The ratio between distances  $\Delta d_l$  and  $\Delta d_r$  can be expressed according to formula (2). Formula (2) can be rewritten as formula (3), which represents the turning radius of the vehicle  $r_r$  along its right wheel.

$$\frac{\Delta d_l}{\Delta d_r} = \frac{2\pi \cdot (r_r + w)}{2\pi \cdot r_r} \quad (2)$$

$$r_r = \frac{\Delta d_r \cdot w}{\Delta d_l - \Delta d_r} \quad \text{when } \Delta d_l \neq \Delta d_r \quad (3)$$

where  $\Delta d_l$  and  $\Delta d_r$  are distances covered by the left and right wheels at a time interval  $\Delta t$  [m],  
 $r_r$  is the turning radius of the vehicle along its right driving wheel [m],  
 $w$  is the distance between both driving wheels (width) [m].

Formula (4) represents a change of the direction angle  $\Delta\varphi$  of the vehicle in its new position.

$$\Delta\varphi = \begin{cases} 0 & \text{when } \Delta d_l = \Delta d_r \\ \frac{\Delta d_l}{w} & \text{when } \Delta d_l \neq \Delta d_r \text{ and } r_r = 0 \\ \frac{\Delta d_r}{r_r} & \text{when } \Delta d_l \neq \Delta d_r \text{ and } r_r \neq 0 \end{cases} \quad (4)$$

New values of the parameters  $x_i$ ,  $y_i$ ,  $\varphi_i$  at a time interval  $i$  are represented by formula (5).

$$\begin{aligned} \varphi_i &= \varphi_{i-1} + \Delta\varphi_i \\ x_i &= \begin{cases} \left(\frac{\Delta d_l + \Delta d_r}{2}\right) \cdot \sin \varphi_i & \text{when } \Delta d_l = \Delta d_r \\ \left(r_r + \frac{w}{2}\right) \cdot \left[\sin\left(\varphi_{i-1} + \frac{\pi}{2}\right) + \sin\left(\varphi_i - \frac{\pi}{2}\right)\right] & \text{when } \Delta d_l \neq \Delta d_r \end{cases} \quad (5) \\ y_i &= \begin{cases} \left(\frac{\Delta d_l + \Delta d_r}{2}\right) \cdot \cos \varphi_i & \text{when } \Delta d_l = \Delta d_r \\ \left(r_r + \frac{w}{2}\right) \cdot \left[\cos\left(\varphi_{i-1} + \frac{\pi}{2}\right) + \cos\left(\varphi_i - \frac{\pi}{2}\right)\right] & \text{when } \Delta d_l \neq \Delta d_r \end{cases} \end{aligned}$$

Figure 3 presents the simulation of manual control of the vehicle in the area; the figure shows axes of the driving wheels at particular positions of the vehicle and its direction. Table 1 presents values  $\Delta d_l$  and  $\Delta d_r$  at particular steps of the simulation (distance values are in centimeters).

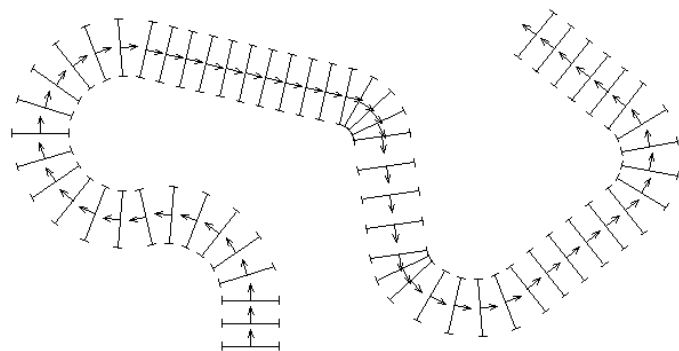


Fig. 3 Simulation of manual control of the vehicle

Step $i$	1-2	3-8	9-20	21-30	31-34	35-38
$\Delta d_l$	20	15	30	18	20	26
$\Delta d_r$	20	30	15	18	5	26
Step $i$	39-40	41-45	46-50	51-55	56-60	
$\Delta d_l$	5	15	22	12	20	
$\Delta d_r$	20	30	22	30	20	

Table 1 Values  $\Delta d_l$  and  $\Delta d_r$  for the simulation of vehicle motion according to figure 3

## 4 Model of autonomous control

This model has been developed for autonomous motion of the vehicle along an arbitrary path which can be determined e.g. via the optimal path-finding algorithm [3], [4]. The path is composed of points, which the vehicle passes through step by step from the initial point to the target point.

Generally, this model ensures motion of the vehicle from the current point to the point next in sequence. When the vehicle reaches that point, the whole process is repeated until reaching the last (target) point in sequence.

Input parameters of the model are composed of the vehicle position  $x_i$  and  $y_i$  at an interval  $i$  and its orientation  $\varphi_i$  as well as the demanded vehicle position at the next step  $i + 1$  of the solution  $x_{i+1}$  and  $y_{i+1}$ . As an output there are distances  $\Delta d_l$  and  $\Delta d_r$  which must be covered by the left and right driving wheels during a time interval  $\Delta t$  provided that rotating speed of both wheels is unchanging; speed can be changed only at the beginning of each time period.

Figure 4 depicts the situation graphically. Notice that the demanded direction angle of the vehicle  $\varphi_{i+1}$  is not an input parameter of the model because this cannot be done within one step. The model would be necessary to extend in order to solve this problem; the simplest solution is to divide each step into two partial sub-steps. The first sub-step prepares the vehicle so that at the end of the second the vehicle is in the demanded position and direction. However, this issue will be the topic of another article in the future. Furthermore, the direction of the vehicle is approaching the path tangent gradually, thus there is no need to extend the model in practice.

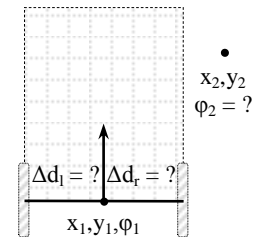


Fig. 4 Input parameters of the model of autonomous control

The problem solution is simple in case that the target point is situated directly on an axis of a direction of the vehicle; formula (6) then expresses distances  $\Delta d_l$  and  $\Delta d_r$ . The sign in the mentioned formula assigns the direction of the wheels rotation and can be determined according to the half-plane given by the axis of driving wheels in which the target point is located. The direction angle is not changed in this case.

$$\Delta d_l = \Delta d_r = \pm \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (6)$$

$$\Delta \varphi = 0$$

The situation is more difficult when the target point is not placed in the direction of the vehicle. Figures 5a and 5b present the solution.

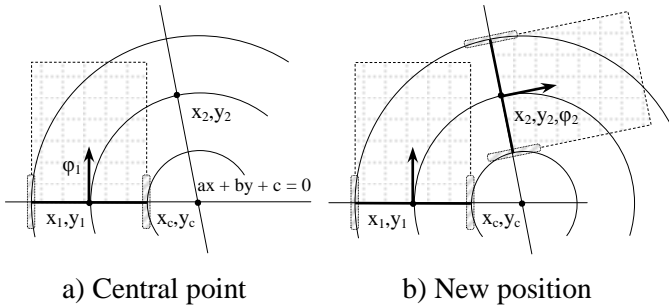


Fig. 5 Features of the model of autonomous control

Firstly, the central point  $x_c, y_c$  should be found; this point means the center of the vehicle rotation and can be found by solution of the equation system (7) with two unknown variables  $x_c$  and  $y_c$ . The equations were created according to the fact that the distance between the central point and the current and target points is equal and the central point lies on the axis of driving wheels (which is known).

$$(x_1 - x_c)^2 + (y_1 - y_c)^2 = (x_2 - x_c)^2 + (y_2 - y_c)^2 \quad (7)$$

$$a \cdot x_c + b \cdot y_c + c = 0$$

Formula (8) for computing the central point  $x_c, y_c$  is found by solving the equation system (7). Analogically, formula (9) can be derived.

$$x_c = \frac{x_2^2 + \frac{2 \cdot c \cdot y_2}{b} + y_2^2 - x_1^2 - \frac{2 \cdot c \cdot y_1}{b} - y_1^2}{\frac{2 \cdot a \cdot y_1}{b} - 2 \cdot x_1 - \frac{2 \cdot a \cdot y_2}{b} + 2 \cdot x_2} \quad \text{when } b \neq 0 \quad (8)$$

$$y_c = -\frac{a \cdot x_c + c}{b} \quad \text{when } b \neq 0$$

$$y_c = \frac{x_2^2 + \frac{2 \cdot c \cdot x_2}{a} + y_2^2 - x_1^2 - \frac{2 \cdot c \cdot x_1}{a} - y_1^2}{\frac{2 \cdot b \cdot x_1}{a} - 2 \cdot y_1 - \frac{2 \cdot b \cdot x_2}{a} + 2 \cdot y_2} \quad \text{when } a \neq 0 \quad (9)$$

$$x_c = -\frac{b \cdot y_c + c}{a} \quad \text{when } a \neq 0$$

When the central point is known we can determine the angle  $\Delta \varphi$  according to formula (10) which is based on the equation for computing the mutual angle of two vectors.

$$\Delta \varphi = \arccos \frac{(x_c - x_1) \cdot (x_c - x_2) + (y_c - y_1) \cdot (y_c - y_2)}{\sqrt{(x_c - x_1)^2 + (y_c - y_1)^2} \cdot \sqrt{(x_c - x_2)^2 + (y_c - y_2)^2}} \quad (10)$$

Distances  $\Delta d_l$  and  $\Delta d_r$  are defined by formula (11). The sign in the mentioned equations can be determined according to the half-plane given by the axis of driving wheels in which the new positions of the left and right wheels are located.

$$\Delta d_l = \pm \Delta \varphi \cdot \sqrt{(x_c - x_l)^2 + (y_c - y_l)^2} \quad (11)$$

$$\Delta d_r = \pm \Delta \varphi \cdot \sqrt{(x_c - x_r)^2 + (y_c - y_r)^2}$$

where  $x_l, y_l$  is the initial position of the left driving wheel,  
 $x_r, y_r$  is the initial position of the right driving wheel.

Figure 6 presents the simulation of a vehicle motion along the predetermined path (the red hatched polygonal line with the yellow points marking demanded positions of the vehicle in a particular step of the simulation). The path contains several sharp breaks; nevertheless the algorithm ensures gradual approaching of the direction of the vehicle towards the curve tangent. Current position of the vehicle is located in its geometric center. Table 2 presents values  $\Delta d_l$  and  $\Delta d_r$  at particular steps during the simulation (distance values are in centimeters).

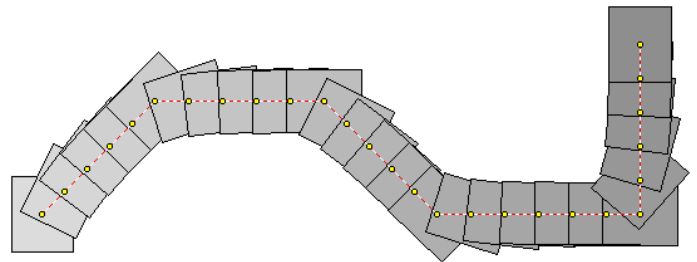


Fig. 6 Simulation of autonomous motion of the vehicle

<b>Step i</b>	1	2	3	4	5	6	7	8	9
<b>Path x</b>	10	20	30	40	50	65	80	95	110
<b>Path y</b>	10	20	30	40	50	50	50	50	50
$\Delta d_l$	33.1	29.7	27.4	26.3	25.8	36.1	31.3	28.8	27.7
$\Delta d_r$	10.4	19.9	23.3	24.6	25.1	11.2	21.5	25.0	26.3
<b>Step i</b>	10	11	12	13	14	15	16	17	18
<b>Path x</b>	125	135	145	155	165	175	190	205	220
<b>Path y</b>	50	40	30	20	10	0	0	0	0
$\Delta d_l$	27.3	33.1	29.8	27.4	26.3	25.8	11.6	21.7	25.1
$\Delta d_r$	26.7	10.3	19.8	23.3	24.6	25.1	35.1	31.2	28.7
<b>Step i</b>	19	20	21	22	23	24	25	26	
<b>Path x</b>	235	250	265	265	265	265	265	265	
<b>Path y</b>	0	0	0	15	30	45	60	75	
$\Delta d_l$	26.3	27.7	26.9	-9.8	12.8	22.1	25.2	26.3	
$\Delta d_r$	27.7	27.3	27.1	32.5	34.8	30.9	28.6	27.6	

Table 2 Values  $\Delta d_l$  and  $\Delta d_r$  for the simulation according to figure 6

## 5 Conclusion

The article presents the basic design for autonomous motion of the unmanned ground vehicle developed at the University of Defence in Brno for experimental purposes. The vehicle demonstrates the algorithm of autonomous motion; this topic is long-term dealt with at the Department of military management and tactics.

In the future, we are going to enhance and verify the algorithm thoroughly. The first step will consist in extending the scanning process of the surrounding environment into the third dimension. The general objective is to apply our algorithm to the autonomous unmanned ground vehicle [2], which is being developed at the University of Defence; this vehicle is used especially for reconnaissance and combat purposes. However, the mathematical model will have to be adjusted to the different physical parameters and features of the vehicle.

The following paper will deal with enhancing the above mentioned model; newly there will be possibility to move the vehicle to the new position following the condition of the demanded direction angle of the vehicle. In this case, we can achieve better and more economic motion along the predetermined path. The path will be replaced with a continuous curve (B-spline, Bezier, Hermite, Akima, polynomial, etc.); the vehicle will copy the tangent of the curve at each point.

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