Bihar Engineering University, Patna

B.Tech. 1st Semester Examination, 2023

Course: B.Tech. *Time: 03 Hours*
Code: 103102 *Subject: Mathematics-I(Calculus & Differential Equations)* *Full Marks: 70*

Instructions:-
(i) The marks are indicated in the right-hand margin.
*(ii) There are **NINE** questions in this paper.*
*(iii) Attempt **FIVE** questions in all.*
(iv) Question No. 1 is compulsory.
(v) Symbols used (if any) have their usual meanings

Q.1 ***Choose the correct answer of the following (Any seven question only):*** **[2 x 7 = 14]**

(a) The solution of the given differential equation is :
$(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0$
(i) x3 + y3 - 6xy(x + y) = c (ii) $2x^3 + 4y^3 - 4xy = c$
(iii) $x^3 + y^3 - 5xy(x + y) = c$ (iv) $x^2 + y^2 - 6xy = c$

(b) The complementary function for the given differential equation is:
$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = x^2$$
(i) $c_1e^{-x} + c_2e^{-4x}$ (ii) $c_1e^{-2x} + c_2e^{-x}$
(iii) $c_1e^{-3x} + c_2e^{-x}$ (iv) None of the above.

(c) The solution of the differential equation $\frac{dy}{dx} - y\cot(x) = 2x\sin(x)$ is :
(i) $y = x^2 + c$ (ii) $y\sin(x) = 2x + c$
(iii) $y\operatorname{cosec}(x) = x^2 + c$ (iv) $y\cot(x) = 3x + c$

(d) The wronskian of the functions {sin(x), cos(x) } is :
(i) 1 (ii) -1
(iii) 0 (iv) 2

(e) Suppose $\vec{A} = x^2z^2\,\hat{\imath} - 2y^2z^2\,\hat{\jmath} + xy^2z\,\hat{k}$ then the divergence of $\vec{A}$ at point P (1, -1, 1) is :
(i) 4 (ii) 5
(iii) 6 (iv) 7

(f) Let $A = \int\int_R \sin(x + y)\,dx\,dy$, where the region R is given by $\{0 \le x \le \frac{\pi}{2}, 0 \le y \le \frac{\pi}{2}\}$ then the value of A is :
(i) -1 (ii) 1
(iii) 0 (iv) 2

(g) The series $1 + \frac{3}{2!} + \frac{5}{3!} + \frac{7}{4!} + \ldots\ldots\ldots\ldots$ is:
(i) convergent (ii) divergent
(iii) Neither convergent nor divergent (iv) None

(h) If f:R → R be a function defined by $f(x) = \begin{cases} x^2 + \sin\frac{1}{x^2}, & x \neq 0 \\ 0, & x = 0 \end{cases}$
(i) f is differentiable on R (ii) f is differentiable on R – {0}
(iii) f is not differentiable at 0 (iv) None of these

(i) If $f(x, y) = e^{x+y+1}$ then the value of $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}$ is
(i) $(x + y)\,e^{x+y+1}$ (ii) $xe^y + ye^x$
(iii) $2e^{x+y+1}$ (iv) None of the above

(j) The value of $\Gamma\left(-\frac{3}{2}\right)$ is
(i) 1 (ii) $\frac{\pi}{2}$
(iii) $\sqrt{\pi}$ (iv) $\frac{4}{3}\sqrt{\pi}$

Q.2 (a) Show that: [7]

$$\int_0^1 |ln x|^n dx = (-1)^n n!,\ n\epsilon N.$$

(b) If α and β are the roots of the equation $ax^2+bx+c=0$ the evaluate the following limit. [7]

$$lim_{x\to\alpha}\frac{1-\cos(ax^2+bx+c)}{(x-a)^2}$$

Q.3 (a) Test the convergence of [8]

$$\frac{1}{2\sqrt{1}}+\frac{1}{3\sqrt{2}}x^2+\frac{1}{4\sqrt{3}}x^4+\frac{1}{5\sqrt{4}}x^6+\ldots\ldots\ldots\ldots$$

(b) Prove that [6]

$$Curl(f\boldsymbol{v}) = (grad\ f)\times\boldsymbol{v} + f\ curl\ \boldsymbol{v}$$

Q.4 (a) Use Taylor's theorem to prove that [8]

$$1+\frac{x}{2}-\frac{x^3}{8}<\sqrt{1+x}<1+\frac{x}{2}, if\ \ x>0.$$

(b) If p(x) is a polynomial of degree > 1 and k ∈ R, prove that between any two real roots of p(x) = 0 there exists a real root of $p'(x)+kp(x) = 0$. [6]

Q.5

(a) Show that the function f(x,y) = $\begin{cases}\frac{x^3+y^3}{x-y}, x\neq y\\ 0, \quad\quad x=y\end{cases}$, is not continuous at origin. But f_x *and* f_y exists at (0, 0). [6]

(b) Show that $\int_0^{\pi/2} sin^m\theta cos^n\theta d\theta = \frac{\Gamma(\frac{m+1}{2})\Gamma(\frac{n+1}{2})}{2\Gamma(\frac{m+n+2}{2})}$. [8]

Q.6 (a) Find the maxima and minima of the function $u = x^3+3xy^2-15x^2-15y^2+72x$. [7]

(b) Find $u(x,y) = cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right), 0<x,y<1,$ then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+\frac{1}{2}$ cot u = 0. [7]

Q.7 (a) Find the Fourier series to represent the function defined as [6]

$$f(x)=\begin{cases}x+\frac{\pi}{2}, -\pi<x<0\\ \frac{\pi}{2}-x, 0<x<\pi\end{cases}$$

(b) Solve the following differential equations, [4x2=8]

(i) $(x+2)^2\frac{d^2y}{dx^2}-(x+2)\frac{dy}{dx}-3y=0$

(ii) $(y-1)dx+x(x+1)dy=0$

Q.8 (a) Use variation of parameters method to solve [7]

$$(D^3+1)y = e^{x/2}\sin\frac{x\sqrt{3}}{2}$$

(b) Using operator method solve the following differential equation. [7]

$(D^2-2D+5)\ y = e^x \sin\ 2x$

Q.9 (a) Find a complete integral of the following equation [7]

$$p^2x+q^2y=z$$

(b) Find the general solution of the partial differential equation [7]

$$x^2\frac{\partial z}{\partial x}+y^2\frac{\partial z}{\partial y}=(x+y)z$$